Dynamic Capacity Allotment Planning for Airlines via Travel Agencies

Nurşen Aydın * Nalan Gülpınar † Yijun Zheng ‡

Abstract

In this paper, we study the dynamic capacity allocation problem of an airline using allotment contracts among travel agencies. The capacity allocation problem is formulated as a dynamic programming model where the initial fixed allotment is determined on the basis of a contract between agencies and the airline while the variable allotment is distributed over the planning horizon. We develop approximation methods based on linear programming and Lagrangian relaxation to solve the underlying dynamic programming model. The computational experiments are designed to illustrate the performance of the proposed approaches using real data. The numerical results show that the capacity allotment policy in terms of fixed and variable allotments is significantly affected by price and demand patterns. Even though fixed allotments can provide a cushion against the loss due to potential empty seats, they may not be in the best interest of the airline. Moreover, the booking rules and the airline's capacity allocation strategy have an impact on the airline's revenue and the agency's profit.

Keywords: OR in airlines, capacity allotment, dynamic programming, approximation methods

1 Introduction

Airlines sell flight tickets via different channels. They can either sell directly through their proprietary reservation system, sales offices, and call centres, or indirectly through traditional brick-and-mortar stores, tour operators and travel agencies. Since the 1970s, airlines have been selling flight tickets using travel agencies as their distribution channels (Buhalis, 2004). With technological advances and the spread of the internet, online ticket sales for flights have been heavily used by airlines, online travel agencies and tour operators. Fiig et al. (2015) and Vinod (2009) discuss the roles and industrial evolution of distribution channels in airline revenue management. According to Fiig et al. (2015), airlines and agencies are considered as partners in the air travel industry and traditional travel agencies will continue to play an important intermediary role between airlines and customers in the future. Individual and corporate customers prefer agencies since they can offer products from different providers. According to the U.S. airline market report published by PhocusWright (2022), travel agencies are expected to process more than 40% of gross airline bookings in 2023. Blackler (2023) discusses that demand for travel has gradually returned to pre-pandemic level and passengers prioritize flexibility provided by travel agencies over other factors such as cost due to the uncertainty faced during the pandemic. A recent study on the UK travel

*The University of Warwick, Warwick Business School, CV4 7AL, UK. Email: naydin@wbs.ac.uk
†Durham University Business School, Mill Hill Lane, Durham, DH1 3LB, UK. Email: nalan.gulpinar@durham.ac.uk
‡Corresponding Author; Capital Normal University, Beijing, China. Email: yijun.zheng@cnu.edu.cn

market shows that 54% of UK passengers prefer a travel agency or a travel management company to book their flights rather than booking independently (Flight Centre, 2022). Despite their market share in airline sales, revenue management with travel agencies has received very limited attention within the extensive research in airline revenue management.

In distribution channels, airlines and travel agencies share a similar booking control system. This system allows travel agencies to access fares, booking rules, and information related to seat availability in real-time (Fiig et al., 2015; Isler and D'Souza, 2009). Travel agencies can either dynamically sell tickets as customers arrive or reserve a block of seats (so-called allotments) from flights (Amadeus, 2023; Gunther et al., 2012; Hitit, 2023). Allotments are negotiated between the airline and travel agencies for a specific traveling season. The airline sets contracts with travel agencies before the selling season starts. The contract specifies a list of flights with requested seat capacities and associated pricing schemes. There are two main types of capacity allotments, *fixed* and *variable* (Bravo, 2023; IBS Software, 2023). In fixed allotment, agencies reserve seats prior to the start of the selling season, during which the airline is unable to modify the initial allocation. Moreover, the unsold fixed allotment seats cannot be returned to the airline without incurring charges. Thus, a fixed allotment contract enables agencies to secure seat capacity while it allows the airline to mitigate risk of unsold seats under demand uncertainty. In variable allotment (also known as prorata or soft allotment), on the other hand, agencies reserve seats for a certain period of time and have flexibility to return the unsold seats to the airline. Throughout the selling season, the airline periodically monitors the bookings recorded in their own system and those made by agencies. Based on the sale performance and expected future bookings, the remaining variable allotment capacities allocated to each agency can be updated. Insufficient capacity allocation to different agencies may result in rejecting high-fare class requests arriving towards the end of the reservation period, which in turn may lead to a significant revenue loss. Thus, the airlines need to develop an effective capacity planning policy by taking into account customer demand to maximize the total revenue received by different sales channels.

In this paper, we formulate the capacity planning problem of an airline selling tickets of a single-leg flight through multiple travel agencies, where the seat capacity can be further classified into fixed and variable allotments. The underlying capacity allotment planning problem is motivated by a real-world case study introduced by a travel technology company providing commercial and operational IT systems for the airline and travel industry Hitit (2023). We consider a stream of customers requesting tickets during the booking horizon of the flight. Customers can book flight tickets via either travel agencies or the airline reservation system. Each customer can request a ticket from exactly one sales channel. We assume that the airline has full information about how flight tickets will be priced during the booking horizon by travel agencies and in its own reservation system. At the beginning of the booking horizon, the airline allocates fixed and variable allotments to all contracted travel agencies and its reservation system. During the booking horizon, the airline periodically reviews and re-allocates the remaining variable allotment capacity among all agencies and the reservation system as well as to accept/reject customers' booking requests for a flight ticket through its reservation system. The allocated capacity of fixed allotment cannot be changed and the agencies cannot return any unsold tickets to the airline. On the other hand, the variable allotment seats allocated to each sales channel can be updated during the planning horizon. Given the allocated fixed and variable allotment capacities, each agency independently makes the seat booking decisions for customers arriving between two consecutive review periods without knowing how the other sales channels behave. At the end of the booking horizon, the total revenue of the airline is calculated as the total of the immediate revenue gained by assigning fixed allotment seats to sales channels and the revenue obtained from selling variable allotment seats through all sales channels including the airline's reservation system.

To the best of our knowledge, the fixed and variable capacity allotment problem with travel agencies has not been previously studied in the passenger airline literature. The contribution of this paper lies in development of the modelling and solution approaches.

- We introduce a dynamic programming formulation of the capacity allotment problem from the airline's perspective where the airline anticipates agencies' capacity management strategies based on their remaining seats to maximize their revenue. Unlike the traditional single-leg capacity management problem, we consider two types of seat capacities in terms of fixed and variable allotments. We assume that the fixed allotment capacities are decided at the beginning of booking horizon and cannot be changed during the booking horizon whereas the variable allotment allocations can be updated at specific review periods. As the airline and travel agencies manage their own reservation (booking) systems independently between two consecutive review periods, they apply different decision rules for customer requests arriving to each individual sale channels. Each travel agency's contribution to the airline's revenue is evaluated from the perspective of the airline by considering the agency's capacity management strategy.
- The airline's dynamic capacity planning model requires keeping track of the bookings performed by each agency as well as the type of capacity; thus it has a high-dimensional state variable. Consequently, the dynamic program model becomes intractable. Due to curse of dimensionality of the stochastic dynamic programming model, we propose two different approximation methods to obtain the dynamic capacity allotment policy for the airline. In both approximation methods, the problem is decomposed into smaller sub-problems with respect to the sales channels by excluding the review periods. In the first approximation method, the expected revenue obtained from travel agencies is computed by using a deterministic programming approach while the airline's reservation system is formulated as a dynamic programming model. The resulting optimisation problem corresponds to a large linear programming model that can be efficiently solved. As the second approach, we introduce a Lagrangian relaxation-based decomposition method. We formulate the capacity allotment problem as a binary programming model where binary variables denote each capacity allotment decision. Similar to the first approximation method, each sales channel is treated as a single subproblem by relaxing the linking constraints for sales channels with a Lagrange multiplier. Due to the special structure of the problem, one can easily obtain the capacity allotment policy using the sub-gradient method. The proposed decomposition method utilizes the dynamic programming formulation. Hence, it captures the temporal dynamics of the customer arrival process.
- We design a series of computational experiments to illustrate performance of the proposed approaches using a real agency data obtained from a European airline information technology provider through a consulting project. We analyse capacity management strategies of agencies considering different decision rules for the seat booking strategy including the dynamic policy given the fixed and variable allotment capacities. Our numerical results demonstrate that under certain conditions,

fixed and variable capacity allocations are not always in the best interest of the airline.

The remainder of the paper is organised as follows. In Section 2, we review the most relevant literature on airline capacity management. In Section 3, we present the dynamic programming formulation of the capacity allotment problem for an airline selling tickets for one single-leg flight through multiple travel agencies. In Section 4, we focus on the solution methodologies for the dynamic programming model and introduce two channel based approximation methods. In Section 5, we first describe our real data and design of computational experiments and then present the numerical results. Section 6 summarises the main findings and concluding remarks.

2 Review of Related Literature

In this section, we will review the literature on capacity management through agency channels and outline the most related studies on the capacity allocation problems in airline revenue management. There is a large body of literature on capacity allocation problems in revenue management. For a comprehensive review of this area, we refer the reader to Phillips (2005), Talluri and van Ryzin (2004), and Klein et al. (2020). We classify the most relevant research papers developing policies for the capacity allotment problem into two groups in terms of application areas in Table 1.

References	Capacity Type	Agency Contract	Solution Approach
Cargo Transportation			
Amaruchkul et al. (2007)	Weight and volume	—	One-dimensional approximation
Gupta (2008)	Weight and volume	Flexible allotment	Single period solution
Levin et al. (2012)	Weight and volume	Fixed allotment	Benders decomposition
Moussawi (2014)	Weight and volume	Fixed allotment	Two-stage approach
Moussawi et al. (2021)	Number of containers	Fixed allotment	Two-stage approach
Airline Passenger Booking			
Gunther et al. (2012)	Number of seats	—	Mixed-integer program
Wang et al. (2018)	Number of seats	—	Bid prices - DP model
Our paper	Allotment size	Fixed and variable allotment	Channel-based decomposition

Table 1
Comparison of the most relevant literature

Only a few studies focus on the capacity control problem with agency distribution channels in airline revenue management. These studies mainly investigate effects on sales of offering tickets through agency channels. Isler and D'Souza (2009) review the development of global distribution systems for travel agencies over years. They discuss various difficulties of integrating a pricing mechanism between airline booking control and travel agencies. Castillo-Manzano and Lopez-Valpuesta (2010) explore the recent customer trends and analyse the profile of airline passengers according to the chosen purchase channel. Gunther et al. (2012) present a comprehensive overview of airline distribution channels by discussing several applications in this area such as customer behaviour prediction in choice models and airline channel management. In the airline channel management problem, the authors consider the effects of offering a promotion in an airline's own web system and on the other distribution channels. In particular, they develop a model using deterministic demand and analyse a customer shift between competing airlines and agencies. One of the challenges in this problem is to estimate model parameters affecting the demand shift between competing airlines. Gunther et al. (2012) also point out several research directions such as the seat allotment problem in airline distribution systems. Roma et al. (2014) focus on price dispersion in the airline industry. They perform an empirical analysis to investigate which factors in airlines' direct sales channel and the online travel agencies affect daily price dispersion. Wang et al. (2018) study the capacity control problem of an airline selling seats through travel agencies and its own online reservation system. They assume that the airline has full control of the reservation system and decides to accept or reject each arriving agency request. They present a dynamic programming (DP) model to find an optimal booking policy and analyse the demand shift between agencies. The DP model can be expressed as the standard capacity allocation model introduced by Lautenbacher and Stidham (1999) for single-leg airline revenue management as there is no pre-defined capacity limit for an agency. Song et al. (2021) work on the dynamic pricing problem in a multi-channel setting by considering the demand substitution effect among channels. They formulate the problem as a stochastic dynamic programming model and investigate the optimal pricing policies.

Unlike the passenger airline systems, capacity management with agency contracts is an active area of research in cargo transportation. Most carriers sell a significant portion of their cargo capacity through long-term contracts to agencies (so-called forwarders). This contracted capacity, known as *allotments*, was first studied by Kasilingam (1996). Considering spot market demand and allotment contracts, he investigates the differences between air cargo and passenger revenue management over different problems such as forecasting, overbooking and seat allotment. Similarly, Billings et al. (2003) compare the air cargo system with the passenger airline and discuss the benefits and challenges of adopting revenue management systems in air cargo. Amaruchkul et al. (2007) consider an airline selling cargo capacity solely through the spot market and formulate a dynamic programming model for managing the spot booking requests for a single-leg problem. Due to the high-dimensional state space of the dynamic programming model, they propose a one-dimensional approximation to the value function. Delgado et al. (2019) focus on spot bookings in network cargo transportation under demand and capacity uncertainty. They propose a multistage stochastic programming model in which uncertainty is modelled as a scenario tree. Several studies in air cargo revenue management focus on the allotment contracts. Allotment contracts typically cover a period of few months up to a year and contract conditions remain the same during this duration. Gupta (2008) studies flexible allotment contracts for a single period that allow adjustments in contract parameters depending on realised demand. He divides the interaction between an airline and forwarders into three stages. In the first stage, the airline decides on the allocated cargo capacity and the associated fee for the forwarders. In the second stage, forwarders return the unused capacity to the airline while in the third stage, the airline executes a new contract considering the given information. Gupta (2008) discusses whether flexible allotment contracts increase profitability compared to fixed contracts.

There are studies exploring the coordination between allotment contracts and spot market booking decisions. For instance, Levin et al. (2012) study the allotment problem for parallel flights. The airline can either sell its capacity through the spot market or by long-term contracts. The problem is addressed in two stages. In the first stage, the airline sets the long-term fixed allotment contracts and in the second stage, the remaining capacity is sold through the spot market. The spot market capacity control problem is formulated as a dynamic programming model. Since the resulting model is computationally intractable, they propose a value function approximation to tackle the curse of dimensionality within the dynamic programming model and the overall problem is solved by Benders decomposition. Moussawi (2014) follows a two-stage approach to solve the allotment problem for a single-leg flight. Similar to the model proposed by Levin et al. (2012), the airline first decides how much capacity to allocate to each contract. Then, given the initial decision, a dynamic capacity allocation policy is devised for requests from the spot market. In order to tackle the curse of dimensionality in the dynamic programming model, they approximate the two-dimensional state space as a one-dimensional space to obtain a spot market booking policy. Moussawi et al. (2021) study the allotment problem in sea cargo transportation. Following Moussawi (2014), they also formulate the problem as in two stages. Considering the specific features of sea cargo transportation, they propose a heuristic method for the spot market booking control. Wang et al. (2022) study the risk-averse capacity management strategies in sea cargo transportation considering both spot market booking and long-term allotment contracts. They develop a two-stage stochastic programming model to decide which contracts to select and how to allocate capacity for the selected contracts and spot market customers.

It is important to note that the allotment problem shows different characteristics in cargo and passenger airlines from the booking and capacity management perspectives (Meng et al., 2019). To begin with, there are two types of allotment contracts in passenger airlines which are fixed and variable. Fixed allotment is similar to the long-term contracts in cargo airlines. The travel agency purchases a fixed amount of capacity at a given price through a fixed allotment agreement. Unlike cargo reservations, the purchased capacity cannot be returned to the passenger airline. On the other hand, in a variable allotment agreement, the travel agency reserves some capacity from the passenger airline, and the reserved capacity may change during the reservation period. In addition, the price of variable capacity is not fixed and can change throughout the reservation period. Compared to fixed allotment, variable allotment is more flexible; hence, the travel agencies generally prefer this type of agreement. The variable allotment system is not common in cargo airlines. Also, the studies in cargo airlines generally assume that the contracted cargo bookings arrive or are realised just before the flight departure. Therefore, unlike the passenger airlines, there is no interaction between long-term cargo allotments and spot market bookings during the reservation period. Due to these features, it is crucial to develop novel modelling and solution approaches specific to the allotment problem in passenger airlines. To the best of our knowledge, there are no studies examining the fixed and variable capacity allotment problem in the passenger airlines. We formulate the dynamic capacity allotment problem from the airline's perspective and investigate the interaction between fixed and variable allotments.

The modelling and solution approaches considered in this study build upon the literature on the capacity control problems in revenue management. Approximation and decomposition methods are widely used to construct various capacity control strategies such as the bid-prices and booking limits for complex capacity control problems in revenue management. Topaloglu (2009) proposes a decomposition approach to divide the network revenue management problem into single-leg sub-problems. Although resulting single-leg problems can be efficiently solved, the method itself is computationally demanding for large-scale problems. Erdelyi and Topaloglu (2009) study the network capacity control problem with overbooking and develop separable approximations to decompose the problem by individual flight legs. They consider an iterative and simulation-based method to generate approximations. Thus, the quality of the solution depends on the number of iterations and samples. Kunnumkal and Topaloglu

(2010) introduce an approximation method to allocate flight itinerary prices to the itinerary's flight legs and obtain a single-leg problem for each flight leg in an airline network. Birbil et al. (2014) present a general decomposition framework based on origin-destination path decomposition for the network revenue management problem. The proposed method first allocates network capacities to each origin-destination path. A capacity allocation policy is then determined within each path. Using the structural properties of the decomposed problem, they reformulate the model as a linear programming model. The optimal solution of the linear programming model can be used for bid-price and capacity control policies. The reader is referred to Gonsch et al. (2013), Hosseinalifam et al. (2016), Kunnumkal and Talluri (2016) and Dai et al. (2019) for alternative strategies for decomposing and approximating the complex capacity control problems in revenue management. In this paper, we introduce a stochastic dynamic programming model for the capacity allotment problem considering different capacity restrictions due to fixed and variable allotments. We utilize decomposition methods to tackle the high-dimensional state vector of the proposed dynamic programming model. While developing the decomposition methods, we consider the interaction between fixed and variable allotments and different sales channels.

3 Dynamic Programming Formulation

In this section, we present a dynamic programming (DP) formulation of an airline's capacity allotment problem using a discrete state Markov Decision Process. The airline distributes the capacity allotments of a single-leg flight among different sale channels as travel agencies and their own reservation system. The airline and agencies use their own selling channels operating at different markets. Let $\mathcal{I}$ represent a set of sales channels including n contracted agencies and the airline's online reservation system (denoted by o). Let C denote the total capacity of the single flight in terms of number of seats.

We consider a planning horizon consisting of T stages denoted by discrete times $t \in \mathcal{T} = \{1, \ldots, T\}$. The flight departs at the beginning of $T + 1$. Let $\mathcal{S} = \{s_1, \ldots, s_K\}$ denote a set of K review periods at which variable allotments are reallocated by the airline, where $s_1 = 1$ and $\mathcal{S} \subset \mathcal{T}$. The remaining time periods $\mathcal{T} \backslash \mathcal{S}$ are assumed to be sufficiently small such that the probability of more than one customer requesting a ticket through any sales channel during each time period is negligible. As shown in Figure 1, the capacity allotment process involves decision stages where (re)allocation of capacity allotments and customer bookings are implemented.

At the beginning of the planning horizon s_1, the airline allocates fixed and variable allotments to all travel agencies and its online reservation system. During the booking horizon, the airline reviews and re-allocates the remaining variable allotment capacity among all agencies and the online reservation system at each review period s_l for $l = 2, \cdots, K$. The airline and travel agencies accept or reject arriving customers' seat requests by considering these allocated capacities during the booking horizon. A description of notation is summarised in Table 2.

We assume that each agency's request regarding the number of fixed allotments is received before the booking/seat allocation procedure begins. Thus, the airline does not allocate more than the requested seat capacity. Let d_i denote the fixed allotment capacity requested by agency $i \in \mathcal{I} \backslash \{o\}$. We assume that each channel sets seat selling prices to their own customers independently. The airline receives pre-defined payment from travel agencies through assigning a seat capacity. Agency i pays f_i and r_{it} to the airline

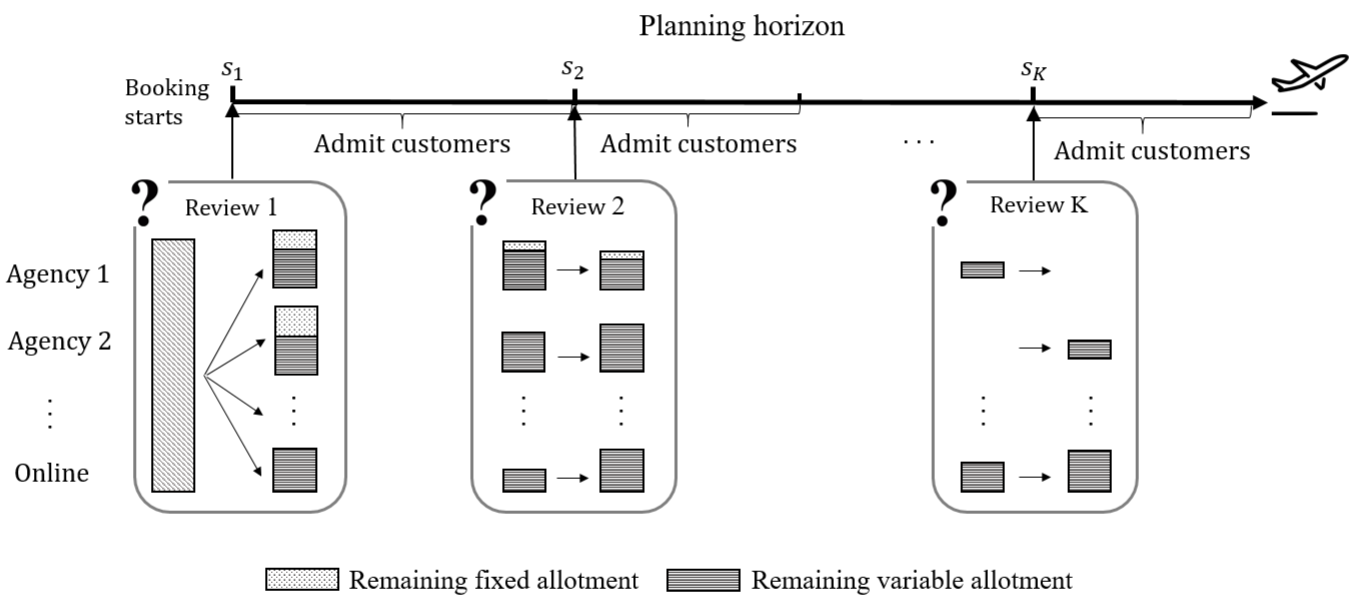


Figure 1
Decision stages of the dynamic capacity allotment problem

for selling fixed and variable allotments, respectively. Travel agencies are not allowed to return unsold fixed allotments to the airline (Bravo, 2023). In practice, the payment of the fixed allotment is collected according to the initial agreement between the airline and travel agencies. In this study, we assume that the agencies pay the cost of unsold fixed allotment capacity at the end of the planning horizon. A customer's request for a flight ticket may arrive to any of sales channels at $t \in \mathcal{T}\backslash\mathcal{S}$, that is defined as an intermediate time between any two review periods. We assume that each sales channel targets a different market. Let p_{it} represent the probability of a customer purchasing a ticket through sales channel i at time t such that $\sum_{i\in\mathcal{I}} p_{it} \leq 1$.

A state of the dynamic system is defined as the remaining seat capacities of sales channels at time t. The remaining fixed and variable allotments of sales channel i at time t are denoted by x_{it} and y_{it}, respectively, and written in a vector form as $(\mathbf{x}_t, \mathbf{y}_t) \in \mathbb{N}^{|\mathcal{I}|\times|\mathcal{I}|}$. In particular, the initial state of the dynamic model arising at the first review period $t = 1$ is denoted by $(\mathbf{0}_1, \mathbf{0}_1)$. The airline reservation system does not use fixed allotment capacity at any time period; thus, we set $x_{ot} = 0$ for $t \in \mathcal{T}$.

Airline's Capacity Allotment Model: The airline decision process involves both capacity allocation decisions at review periods and also seat booking decisions for their own online reservation system between two consecutive review periods. At time $t = 1$, the total seat capacity C of a specific flight needs to be allocated among agencies (through fixed and variable allotments) and its own online reservation system. Let $\boldsymbol{\omega} \in \mathbb{N}^{|\mathcal{I}|}$ and $\boldsymbol{\sigma} \in \mathbb{N}^{|\mathcal{I}|}$ denote vectors of decision variables ω_i and σ_i, respectively, representing the number of fixed and variable allotments allocated to channel i. The search space at review period $t = 1$

$$\mathcal{H}(\mathbf{0}_1, \mathbf{0}_1) = \big\{(\boldsymbol{\omega},\ \boldsymbol{\sigma}) \in \mathbb{N}^{|\mathcal{I}|} \mid \omega_i \leq d_i,\ \forall i \in \mathcal{I}\backslash\{o\};\ \omega_o = 0;\ \sum_{i\in\mathcal{I}} \omega_i + \sigma_i = C\big\},$$

Notation	Description
Sets	
$\mathcal{I} = \{1, \ldots, n, o\}$	sales channels where o indicates airline's online reservation system
$\mathcal{T} = \{1, \ldots, T\}$	discrete time periods over the booking horizon
$\mathcal{S} = \{s_1, \ldots, s_K\}$	K review periods specified over the booking horizon
$\mathcal{H}(\cdot)$ and $\mathcal{U}(\cdot)$	search spaces for allocating capacity and accepting customers, respectively
$\mathcal{Y}(\cdot)$	search spaces for accepting customers in approximation method
W_k	search spaces for variable allotment at review period k in approximation method
$P_k(\cdot)$	search spaces for allocating capacity at review period k in approximation method
Parameters	
d_i	amount of fixed allotment capacity requested by agency i
f_i and r_{it}	revenue to receive from sale of each fixed and variable allotments by channel i
g_{it}	price of flight ticket sold by agency i at time t
p_{it}	probability of customer purchasing a ticket through sales channel i at time t
x_{it} and y_{it}	remaining fixed and variable allotments of sales channel i at time t
C	total capacity of a single flight in terms of number of seats
C'_t	available capacity at time t
θ_{ik}	upper bound on fixed allotment capacity for agency i at review period k
γ_l and ρ_l	slope and intercept values for the piece-wise linear function
e_t	$\|\mathcal{I}\|$-dimensional unit vector with $e_{it} = 1$ if channel i accepts customer demand at t
Discrete Decision Variables	
ω_i and σ_i	amount of fixed and variable allotments allocated to agency i at $t = 1$
δ_{it}	amount of variable allotments reallocated to channel i at $t \in \mathcal{S}\backslash\{1\}$
$\kappa_t \in \{1, 0\}$	denotes whether a customer receives variable allotment from the airline at t, or not
$\mu_{it} \in \{1, 0\}$	denotes whether a customer receives fixed allotment from agency i at t, or not
$\nu_{it} \in \{1, 0\}$	denotes whether a customer receives variable allotment from agency i at t, or not
$\alpha_i \in \{1, 0\}$	represents whether customer demand is satisfied from fixed allotment, or not
$u_{i\omega\sigma} \in \{1, 0\}$	represents whether ω fixed and σ variable allotments are allocated to channel i or not

Table 2
A description of notation

determines the optimal allocation of fixed and variable allotments for all channels. Given the allocation decisions $\boldsymbol{\omega}$ and $\boldsymbol{\sigma}$ at $t = 1$, a state at $t = 2$ is $(0 + \boldsymbol{\omega},\, 0 + \boldsymbol{\sigma})$.

In any other review periods $t \in \mathcal{S}\backslash\{1\}$, the airline first reviews the remaining fixed and variable allotment capacities, and then reallocates the currently available variable capacity C'_t among all channels, including the airline's own reservation system. Let $\boldsymbol{\delta}_t \in \mathbb{Z}^{|\mathcal{I}|}$ denote a vector of decision variables δ_{it}, which represents the number of reallocated variable allotments to channel i. Note that δ_{it} is unrestricted in sign; this implies that the airline may increase or decrease the number of variable allotments to be assigned to any sales channel at each review period. Given state $(\mathbf{x}_t, \mathbf{y}_t)$ at review period $t \in \mathcal{S}\backslash\{1\}$, the search space for $\boldsymbol{\delta}_t$ is represented as

$$\mathcal{H}(\mathbf{x}_t, \mathbf{y}_t) = \big\{\boldsymbol{\delta}_t \in \mathbb{Z}^{|\mathcal{I}|} \mid -y_{it} \le \delta_{it} \le C'_t,\ \forall i \in \mathcal{I};\ \sum_{i \in \mathcal{I}} \delta_{it} = 0\big\}.$$

This basically ensures the best allocation of total available variable capacity C'_t among different sales channels. Once the reallocation decision $\boldsymbol{\delta}_t$ is taken at t, the state at $t + 1$ then becomes $(\mathbf{x}_t, \mathbf{y}_t + \boldsymbol{\delta}_t)$.

During the seat booking process, the airline dynamically accepts customers to maximize the total revenue from its own reservation system. At any intermediate time period $t \in \mathcal{T}\backslash\mathcal{S}$, the action set involves the booking decision for the airline's reservation system. Let κ_t denote a binary decision variable representing whether a customer request arriving at the airline's reservation system at time t is satisfied or not. We also define an $|\mathcal{I}|$-dimensional unit vector $\mathbf{e}_t$ whose i-th element corresponding to sales channel $i \in \mathcal{I}$ is set to one if channel i accepts a customer's request at t. The feasible set $\mathcal{U}_t(\mathbf{x}_t, \mathbf{y}_t)$ at state $(\mathbf{x}_t, \mathbf{y}_t)$ can be described as follows;

$$\mathcal{U}_t(\mathbf{x}_t, \mathbf{y}_t) = \Big\{\kappa_t \in \{0,1\} \mid \kappa_t \le y_{ot}\Big\}.$$

Depending on the airline's action κ_t taken at state $(\mathbf{x}_t, \mathbf{y}_t)$ at time t, the next state of the system $(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})$ at time $t+1$ is $(\mathbf{x}_t, \mathbf{y}_t - \mathbf{e}_t\kappa_t)$. The airline receives revenue r_{ot} from the sale of a seat in its own reservation system at time t.

For any time between any two consecutive review periods, the airline also collects payments from agencies as they sell their own fixed and variable allotments. Note that the airline does not have any influence on the agencies' decisions to accept or reject the customer's request; thus, agency i can fulfill a customer demand from available fixed or variable allotment capacities. Let μ_{it} and ν_{it} denote the booking (binary) decisions for fixed and variable allotment capacities of agency i at time t, respectively. When agency i fulfills a customer demand from fixed or variable allotment capacity at time t (i.e., $\mu_{it} = 1$ or $\nu_{it} = 1$), the price of fixed or variable allotment capacities (f_i or r_{it}) is paid to the airline. The next state of the system $(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})$ at $t+1$ is given by $(\mathbf{x}_t - \mathbf{e}_t\mu_{it}, \mathbf{y}_t - \mathbf{e}_t\nu_{it})$. Note that, as the fixed allotment payment is pre-defined and unsold fixed allotments are not allowed to return to the airline, the same amount of total expected revenue is collected if the payment is received either at the time of customer reservation or at the end of the planning horizon.

The airline anticipates each agency's capacity management strategy based on their remaining fixed and variable allotment capacity to maximize their revenue. In order to formulate the expected revenue obtained from sales channel $i \in \mathcal{I}\backslash\{o\}$, we introduce indicator functions $\mathbb{1}_{(\mu_{it}=1)}$ and $\mathbb{1}_{(\nu_{it}=1)}$ for fixed and variable allotment capacity consumption, respectively. Note that the indicator functions are computed on the basis of the booking decisions received from each agency. Then we can compute the expected revenue $A_t(\mathbf{x}_t, \mathbf{y}_t)$ to be collected from all agencies at time t as follows;

$$\begin{aligned} A_t(\mathbf{x}_t, \mathbf{y}_t) = \sum_{i \in \mathcal{I}\backslash\{o\}} p_{it}\Big(\Big[r_{it} + L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\Big]\mathbb{1}_{(\nu_{it}=1)} + \Big[f_i + L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\Big]\mathbb{1}_{(\mu_{it}=1)}\Big) \\ + p_{it}L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\mathbb{1}_{(\mu_{it}=0)}\mathbb{1}_{(\nu_{it}=0)} + (1 - p_{it})L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}). \end{aligned}$$

Given a state $(\mathbf{x}_t, \mathbf{y}_t)$ of remaining allotment capacities at time $t \in \mathcal{T}$, the value function $L_t(\mathbf{x}_t, \mathbf{y}_t)$ computes the airline's expected total revenue received by all sales channels from t to the end of planning horizon T and can be formulated as the following dynamic optimisation model;

$$L_t(\mathbf{x}_t, \mathbf{y}_t) = \begin{cases} \max\limits_{\boldsymbol{\omega}, \boldsymbol{\sigma} \in \mathcal{H}(\mathbf{0}, \mathbf{0})} \Big\{L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\Big\}, & \text{for } t = 1 \\ \max\limits_{\boldsymbol{\delta}_t \in \mathcal{H}(\mathbf{x}_t, \mathbf{y}_t)} \Big\{L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\Big\}, & \text{for } \ t \in \mathcal{S}\backslash\{1\} \\ \max\limits_{\kappa_{ot} \in \mathcal{U}_t(\mathbf{x}_t, \mathbf{y}_t)} \Big\{p_{ot}\Big[r_{ot}\kappa_t + L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\Big] + (1 - p_{ot})L_{t+1}(\mathbf{x}_{t+1}, \mathbf{y}_{t+1})\Big\} + A_t(\mathbf{x}_t, \mathbf{y}_t), & \text{for } t \in \mathcal{T}\backslash\mathcal{S}. \end{cases} \tag{1}$$

The corresponding transition function updates the state space as follows:

$$(\mathbf{x}_{t+1}, \mathbf{y}_{t+1}) = \begin{cases} (\boldsymbol{\omega}, \boldsymbol{\sigma}), & \text{if } t = 1 \\ (\mathbf{x}_t, \mathbf{y}_t + \boldsymbol{\delta}_t), & \text{if } t \in \mathcal{S}\backslash\{1\} \\ (\mathbf{x}_t, \mathbf{y}_t - \mathbf{e}_t\kappa_t), & \text{if airline receives customer request at } t \in \mathcal{T}\backslash\mathcal{S} \\ (\mathbf{x}_t - \mathbf{e}_t\mu_{it}, \mathbf{y}_t - \mathbf{e}_t\nu_{it}), & \text{if agency } i \text{ receives customer request at } t \in \mathcal{T}\backslash\mathcal{S} \\ (\mathbf{x}_t, \mathbf{y}_t), & \text{otherwise.} \end{cases} \tag{2}$$

The airline collects the unsold fixed allotment fee at the end of the booking horizon. Then the boundary condition at state $(\mathbf{x}_{T+1}, \mathbf{y}_{T+1})$ for time $T+1$ is represented as

$$L_{T+1}(\mathbf{x}_{T+1}, \mathbf{y}_{T+1}) = \sum_{i \in \mathcal{I}\backslash\{o\}} f_i x_{i,T+1}. \tag{3}$$

The dynamic capacity allotment model in (1)-(3) becomes computationally intractable because of exponential growth in the state space as the number of contracted agencies increases. Although this dynamic model cannot be directly solved by an analytical approach, it still provides motivation in designing a capacity planning policy for airlines with multiple sales channels. If a relationship between the capacity allocated to a specific sales channel and the expected revenue was somehow established (by independently considering the capacity planning problem of each individual sales channel), then the overall capacity planning problem of the airline (that allocates the seat capacity over all sales channels by maximizing the total expected revenue) could be solved. In Section 4, we will present solution approaches based on decomposition of the dynamic programme with respect to sales channels.

4 Channel-Based Decomposition Methods

In order to obtain the capacity allotment policy for the airline, we propose two alternative approximation methods by relaxing review periods and decomposing the dynamic programming model (1)-(3) with respect to sales channels. The approximation methods are based on linear programming (LP) and Lagrangian relaxation. Both approaches treat each sales channel as a single sub-problem for given fixed and variable allotment capacities. The first approximation method estimates the expected revenue obtained from agencies deterministically and solves an optimisation problem over all possible allocations of these capacities.The resulting optimisation problem corresponds to a large linear programming model that can be efficiently solved. However, the optimal solution of this linear programming model may not be integer. The second approximation method, on the other hand, is based on an integer (binary) programming model, where binary variables denote each capacity allotment decision. Similar to the first method, the capacity allotment problem is decomposed into many single capacity sub-problems by relaxing the linking constraint with a Lagrange multiplier. The proposed decomposition method utilizes the dynamic programming formulation. The performances of these approaches are numerically tested by using real flight booking data in Section 5. Next, we will briefly describe the linear programming and Lagrangian relaxation based approximation methods.

4.1 LP Based Approximation

The fundamental idea behind this approach is to decompose the dynamic programming model of the airline's capacity allotment problem (1)-(3) into sub-problems corresponding to sales channels and to re-optimize the allotment decision at each review period. For a generic description of the decomposition method, let us consider a review period $k \in \mathcal{S}$. For ease of notation, we define variables x_{ik} and y_{ik} to represent fixed and variable allotment capacities, respectively, to be allocated to channel i for $i \in \mathcal{I}$ at the beginning of review period k. We assume that at review period k, the remaining capacity C'_k needs to be allocated among all sales channels and fixed allotment capacity d_{ik} is requested by channel i. Notice that the DP model (1)-(3) is developed under the assumption that fixed allotment capacities are determined at the beginning of the reservation period and cannot be reallocated at future review periods. As we wish to present a decomposition model for the generic formulation of the capacity allotment problem, we use d_{ik} as the initial fixed allotment capacity at review periods $k \in \mathcal{S}\backslash\{1\}$.

Let y_{ok} denote the variable allotment capacity to be allocated to channel o at review period k. We introduce $\vartheta_{ok}(y_{ok})$ to define the expected optimal revenue obtained through the airline reservation system from review period k to time T given the variable allotment capacity y_{ok}. We develop an optimization model to approximate the expected total revenue obtained through all sales channels without considering any specific booking strategy of travel agencies. Let μ_{it} and ν_{it} denote the satisfied demand from the fixed and variable allotment capacities through sales channel $i \in \mathcal{I}\backslash\{o\}$ at time $t \in \mathcal{T}\backslash\mathcal{S}$. Due to the linear approximation, we can relax the binary conditions on decision variables μ_{it} and ν_{it}. Then the optimization model at review period k can be formulated as follows;

$$
\begin{aligned}
\max \quad & \sum_{i\in\mathcal{I}\backslash\{o\}} f_{ik}x_{ik} + \sum_{i\in\mathcal{I}\backslash\{o\}}\sum_{t=k}^{T} r_{it}\nu_{it} + \vartheta_{ok}(y_{ok}) && && (4)\\
\text{subject to} \quad & \sum_{i\in\mathcal{I}} x_{ik} + y_{ik} \le C'_k, && && (5)\\
& x_{ik} \le d_{ik}, && i \in \mathcal{I} && (6)\\
& \sum_{t=k}^{T} \mu_{it} = x_{ik}, && i \in \mathcal{I}\backslash\{o\} && (7)\\
& \sum_{t=k}^{T} \nu_{it} \le y_{ik}, && i \in \mathcal{I}\backslash\{o\} && (8)\\
& \mu_{it} + \nu_{it} \le p_{it}, && i \in \mathcal{I}\backslash\{o\},\ t = k,\dots,T && (9)\\
& \mu_{it}, \nu_{it} \le 1, && i \in \mathcal{I}\backslash\{o\},\ t = k,\dots,T && (10)\\
& \mu_{it}, \nu_{it} \ge 0, && i \in \mathcal{I}\backslash\{o\},\ t = k,\dots,T && (11)\\
& x_{ik}, y_{ik} \in \mathbb{Z}_+, && i \in \mathcal{I} && (12)
\end{aligned}
$$

where the total expected revenue obtained through sales channels from review period k to the end of the planning horizon T is maximized. Constraint (5) ensures that the total allocated fixed and variable seats do not exceed the available flight capacity at review period k. Constraints (6) guarantee that the fixed allotment requests from agencies are not violated. Constraints (7) and (8) satisfy that the accommodated demand from the fixed and variable seats does not exceed the allocated fixed and variable capacities, respectively. Constraints (9) guarantee that the accepted reservation requests do not exceed the expected number of arrivals at each time period.

Notice that for $k = 1$, the solution of model (4)-(12) provides an optimal size of both fixed and variable allotments allocated to each sales channel to achieve the maximum expected revenue given the initial flight capacity C. On the other hand, at any review period $k \in \mathcal{S}\backslash\{1\}$, we re-optimise the problem (i.e., resolve the optimisation model (4)-(12) for given fixed allotment capacity) to determine only the size of variable allotment allocation over all sales channels given the remaining C'_k capacity. Since the airline receives the fixed allotment revenue at $k = 1$, we set $f_{ik} = 0$ for other review periods $k \in \mathcal{S}\backslash\{1\}$. The model (4)-(12) does not consider the booking strategy of agencies and hence, it overestimates the expected revenue obtained by the airline. In our numerical experiments, we discuss the impact of different booking strategies used by agencies on the airline's expected revenue.

After presenting the approximation model (4)-(12), let us explain further the expected revenue of airline given the variable allotment capacity y_{ok}, $\vartheta_{ok}(y_{ok})$. Similar to the DP model (1)-(3), we introduce κ_t to denote booking decision for the airline reservation channel at time t. If $\kappa_t = 1$, then the reservation request is accepted by the airline using the variable allotment capacity. The set of feasible capacity booking decisions at time $t \in \mathcal{T}\backslash\mathcal{S}$ can be written as follows;

$$\mathcal{Y}_{ot}(y_{ot}) = \Big\{\kappa_t \in \{0, 1\} \mid \kappa_t \leq y_{ot}\Big\},$$

where y_{ot} is the available variable allotment capacity at time t. Then, the dynamic programming model, that calculates the expected revenue obtained through airline reservation system from time t to T, becomes

$$\vartheta_{ot}(y_{ot}) = \max_{\kappa_t \in \mathcal{Y}_{ot}(y_{ot})} \Big\{p_{ot}\Big[r_{ot}\kappa_t + \vartheta_{o,t+1}(y_{ot} - \kappa_t)\Big] + (1 - p_{ot})\vartheta_{o,t+1}(y_{ot})\Big\}, \tag{13}$$

with boundary conditions of $\vartheta_{o,T+1}(y_{o,T+1}) = 0$ for all states $y_{o,T+1}$ at time $T + 1$, and $\vartheta_{ot}(0) = 0$ for all $t \in \mathcal{T}\backslash\mathcal{S}$. The optimisation problem (4)-(12) is difficult to solve efficiently due to the nonlinear objective function of the airline and the integrality conditions. The following proposition displays a structural property for airline's objective function.

Proposition 4.1. *The expected revenue function of airline, $\vartheta_{ot}(y_{ot})$, is discrete concave in variable y_{ot}.*

Proof. The proof is provided in Appendix A.3. □

Using this proposition, the nonlinear expected revenue function of airline $\vartheta_{ok}(y_{ok})$ in (4)-(12) can be reformulated as a piece-wise linear function. The remaining available capacity C'_k provides a bound on the number of pieces for the concave function. Given the remaining capacity C'_k at review period $k \in \mathcal{S}$, we define set $\mathcal{W}_k = \{1, \ldots, C'_k\}$ to store the feasible variable allotments. We also introduce parameters γ_l and ρ_l for $l \in \mathcal{W}_k$ to denote the slope and intercept values for the piecewise linear function. Due to the concavity property, we have $\gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_{C'_k}$ and $\rho_1 \leq \rho_2 \leq \cdots \leq \rho_{C'_k}$. Then, the expected revenue function of airline $\vartheta_{ok}(y_{ok})$ can be rewritten as

$$\vartheta_{ok}(y_{ok}) = \min_{l \in \mathcal{W}_k} \{\gamma_l \, y_{ok} + \rho_l\}.$$

To linearize the airline's objective function, we introduce an auxiliary decision variable z_k and move the piece-wise linear function into constraints. Then the linear optimization model at review period k can be reformulated as

$$\max \quad \sum_{i\in\mathcal{I}\backslash\{o\}} f_{ik}x_{ik} + \sum_{i\in\mathcal{I}\backslash\{o\}}\sum_{t=k}^{T} r_{it}\nu_{it} + z_k \tag{14}$$

$$\text{subject to} \quad \text{Constraints } (5)-(11), \tag{15}$$

$$z_k \le \gamma_l\, y_{ok} + \rho_l, \qquad l \in \mathcal{W}_k \tag{16}$$

$$x_{ik}, y_{ik} \ge 0, \qquad i \in \mathcal{I}. \tag{17}$$

Note that the number of variables in the resulting model can be quite large even for a moderate-size single-leg problem after piece-wise linear approximation since we consider all feasible variable allotments. To efficiently solve this capacity allotment problem, the integrality constraints can be relaxed. In this case, the optimal fixed and variable allotment capacities obtained by solving the optimization model (14)–(17) may not satisfy the integrality conditions. By simply rounding down the non-integer allotment allocation decisions, we can obtain a suboptimal solution. Next, we present a Lagrangian relaxation-based approximation method that provides integer fixed and variable allotment capacities.

4.2 Lagrangian Relaxation Based Approximation

Similar to the LP based decomposition approach described in the previous section, the Lagrangian relaxation method first decomposes the capacity allotment problem into subproblems corresponding to sales channels, each of that provides the expected revenue through the sale of allocated fixed and variable allotment capacities. This method uses a binary programming model where the binary variables denote the capacity allocation decisions for sales channels. In general, binary optimisation models are considered impractical for large-scale problems due to their computational complexity. Therefore, most of the existing studies develop a linear programming model of the network revenue management problem. Instead, we introduce an alternative solution method using the Lagrangian relaxation approach in this paper. Our approximation method builds upon the approach presented in Section 4.1 and estimates the expected revenue from agencies dynamically. Recall that the model (4)-(12) estimates the expected revenue obtained from travel agencies using a deterministic approach.

We use x_{ik} and y_{ik} to denote the size (in terms of units) of fixed and variable allotment capacities allocated to sales channel i at review period k, respectively. Since there are at most $\theta_{ik} = \min\{d_{ik}, C'_k\}$ units of fixed allotment to be assigned to sales channel i, we have $x_{ik} = 1, \cdots, \theta_{ik}$. In addition, y_{ik} varies between 0 and the remaining capacity C'_k at review period k such that $x_{ik} + y_{ik} \le C'_k$.

In addition, we define a binary decision variable for each sales channel $i \in \mathcal{I}$ to represent the capacity allotment decisions. For ease of notation, we use x and y instead of x_{ik} and y_{ik} to denote the fixed and variable allotment capacities in definition of the binary decision variable. Let u_{ixy} be a binary variable representing whether x fixed and y variable allotment capacities are allocated to sales channel i or not. In other words, $u_{ixy} = 1$ only when fixed and variable capacities to be allocated to channel $i \in \mathcal{I}$ are given as x and y, respectively. Otherwise, we have $u_{ixy} = 0$. Notice that $u_{ixy} = 0$ for $x + y > C'_k$ due to the violation of the flight capacity restriction. The feasibility set P_k of binary decisions at review period

$k \in \mathcal{S}$ is defined as

$$P_k = \Big\{ u_{ixy} \ \Big| \ \sum_{x=0}^{\theta_{ik}} \sum_{y=0}^{C'_k} u_{ixy} = 1, \ \ i \in \mathcal{I}; \ \ u_{ixy} \in \{0, 1\}, \ \ 0 \le x \le \theta_{ik}, \ \ 0 \le y \le C'_k, \ \ i \in \mathcal{I} \Big\}.$$

The first set of the constraints ensures that exactly one combination of fixed and variable allotments (that is represented as (x, y)) is to assign to each sales channel $i \in \mathcal{I}$. The second set of constraints corresponds to binary restrictions. Let $\bar{\vartheta}_{ik}(x, y)$ denote the expected revenue obtained by allocating x fixed and y variable allotment capacities to channel $i \in \mathcal{I}$ at review period $k \in \mathcal{S}$. Then the capacity allotment model at review period k can be formulated as follows;

$$\begin{aligned} \max_{u_{ixy} \in P_k} \quad & \sum_{i \in \mathcal{I} \backslash \{o\}} \sum_{x=0}^{\theta_{ik}} \sum_{y=0}^{C'_k} \bar{\vartheta}_{ik}(x, y) u_{ixy} + \sum_{x=0}^{\theta_{ok}} \sum_{y=0}^{C'_k} \bar{\vartheta}_{ok}(x, y) u_{oxy} \\ \text{subject to} \quad & \sum_{i \in \mathcal{I}} \sum_{x=0}^{\theta_{ik}} \sum_{y=0}^{C'_k} (x + y) u_{ixy} \le C'_k. \end{aligned} \tag{18}$$

This optimisation model determines the best fixed and variable capacity allocation policy for all channels to maximise the total expected revenue. The main constraint ensures that the allocated allotment capacities cannot exceed the available capacity at review period $k \in \mathcal{S}$. Note that the airline's reservation system does not use a fixed allotment capacity. Thus, we set $\theta_{ok} = 0$ for channel o.

Although the predefined review periods are not explicitly considered in this formulation, the optimal capacity allotment policy can be updated at each review period through re-optimisation of (18). Recall that the fixed allotment capacities allocated at the initial review period cannot be revised throughout the planning horizon. Therefore, the optimisation model (18) can be resolved at review period $k \in \mathcal{S} \backslash \{1\}$ to revise variable allotment capacities for all channels. Next, we will focus on how to estimate the expected revenue contributions $\bar{\vartheta}_{ok}(x, y)$ and $\bar{\vartheta}_{ik}(x, y)$ obtained by the airline's reservation system and travel agencies for $i \in \mathcal{I} \backslash \{o\}$, respectively.

Airline's Expected Revenue Contribution: Suppose that 0 fixed and y variable allotment capacities are assigned to the airline's booking system. The expected revenue $\bar{\vartheta}_{ot}(0, y)$ obtained through the airline's reservation system from time t to T can be computed by the following dynamic programming model.

$$\bar{\vartheta}_{ot}(0, y) = \max_{\kappa_t \in \mathcal{Y}_{ot}(y)} \Big\{ p_{ot} \Big[r_{ot} \kappa_t + \vartheta_{o,t+1}(0, y - \kappa_t) \Big] + (1 - p_{ot}) \vartheta_{o,t+1}(0, y) \Big\} \tag{19}$$

Notice that (19) is formulated by adapting the model (13) where fixed and variable allotment capacities are set as $x = 0$ and y, respectively. In addition, the boundary conditions are $\bar{\vartheta}_{o,T+1}(0, y) = 0$ for all y at time $T + 1$, and $\bar{\vartheta}_{ot}(0, 0) = 0$ for all $t \in \mathcal{T} \backslash \mathcal{S}$.

Agency's Expected Revenue Contribution: The expected revenue $\bar{\vartheta}_{ik}(x, y)$ obtained by allocating x fixed and y variable allotment capacities to agency channel $i \in \mathcal{I} \backslash \{o\}$ is estimated from the airline's perspective by solving the agency capacity management model presented in Appendix A.1. Next, a brief description of this process follows.

Let μ^*_{it} and ν^*_{it} denote binary variables representing the estimated booking decisions for agency i at time t given the available fixed and variable capacities (x, y), respectively. If $\mu^*_{it} = 1$, then it is expected that agency i will use the fixed allotment capacity to satisfy demand at time t, generating revenue $f_i \times \mu^*_{it}$ for the airline. Conversely, $\nu^*_{it} = 1$ indicates that agency i is expected to use the variable allotment capacity to meet demand, which in turn generates revenue $r_{it} \times \nu^*_{it}$ for the airline. Given the available fixed and variable allotment capacities (x, y) at time t, the expected revenue obtained from agency i from t to T can be formulated as follows;

$$\bar{\vartheta}_{it}(x, y) = p_{it}\Big[f_i\mu^*_{it} + r_{it}\nu^*_{it} + \bar{\vartheta}_{it+1}(x - \mu^*_{it}, y - \nu^*_{it})\Big] + (1 - p_{it})\bar{\vartheta}_{i,t+1}(x, y) \tag{20}$$

where the boundary condition is $\bar{\vartheta}_{i,T+1}(x, y) = f_i x$ for all states (x, y) at time $T+1$. Note that the booking decisions, μ^*_{it} and ν^*_{it}, for agency i depend on the allocated fixed and variable allotment capacities.

As in the DP model (1)–(3), the expected revenue contribution of each agency is also considered from the airline's perspective. We can compute the estimated booking decisions (μ^*_{it} and ν^*_{it}) and expected revenue contribution $\bar{\vartheta}_{ik}(x, y)$ received via channel i by solving the agency capacity management model (25) outlined in Appendix A.1. It is worthwhile to highlight that although we use dynamic programming to estimate the expected revenue contribution through channel i, the framework outlined in (18) is versatile and can be generalised to other cases where the agency contribution is derived by using a deterministic approach. In this case, $\bar{\vartheta}_{ik}(x, y)$ for all agencies $i \in \mathcal{I}$ by considering all possible combinations of x and y are input to the optimization model (18). Since the state space of dynamic programming recursion for channel $i \in \mathcal{I}$ is defined for $x = 0, \cdots, \theta_{ik}$ and $y = 0, \cdots, C'_k$, the optimal solution table for $x = \theta_{ik}$ and $y = C'_k$ already includes the expected revenues for all possible values of x and y. Consequently, the expected revenue $\bar{\vartheta}_{ik}(x, y)$, $i \in \mathcal{I}$ for all $x = \theta_{ik}$ and $y = C'_k$ can be easily computed.

The next proposition establishes a lower bound on the optimal expected revenue over the whole planning horizon.

Proposition 4.2. *The optimal objective value of the binary programming model (18) at the first review period provides a lower bound on the optimal expected revenue of the dynamic programming model (1)-(3).*

Proof. The proof is provided in Appendix A.4. □

We next describe the Lagrangian relaxation method to efficiently solve the binary optimisation model (18). The constraint linking the capacity allocation decisions across all sales channels is transferred into the objective function with a Lagrange multiplier. The Lagrangian function is obtained as

$$\mathcal{L}(\mathbf{u}, \Gamma) = \sum_{i \in \mathcal{I}\setminus\{o\}} \sum_{x=0}^{\theta_{ik}} \sum_{y=0}^{C'_k} \Big(\bar{\vartheta}_{ik}(x, y) - \Gamma(x + y)\Big)u_{ixy} + \sum_{x=0}^{\theta_{ok}} \sum_{y=0}^{C'_k} \Big(\bar{\vartheta}_{ok}(x, y) - \Gamma(x + y)\Big)u_{oxy} + C'_k\Gamma,$$

where Γ represents a Lagrange multiplier associated with the linking constraint. The Lagrangian relaxation problem can be stated as

$$Z(\Gamma) \quad = \quad \max_{u_{i\omega\sigma} \in P_k} \mathcal{L}(\mathbf{u}, \Gamma) \tag{21}$$

whose optimal solution clearly depends on the Lagrange multiplier Γ whereas the dual problem of the Lagrangian relaxation model is

$$\min_{\Gamma \geq 0} Z(\Gamma). \tag{22}$$

Note that the constraint set P_k associated with review period $k \in \mathcal{S}$ has a totally unimodular structure since each column has exactly one +1. Therefore, for any Lagrange multiplier Γ, we can relax the binary requirements on decision variables u_{ixy} without affecting the optimal solution. The resulting relaxation model is convex in Γ and also separable over channels for a given Lagrange multiplier. Therefore, we can efficiently compute the optimal value of the Lagrange multiplier by using a standard subgradient optimisation algorithm. A brief description of the subgradient algorithm is provided below. For further information, the reader is referred to Bertsekas (2015).

Let $\Gamma^{(n)}$ represent the value of the Lagrangian multiplier and $\varphi^{(n)}$ denote the predetermined step size at iteration n. The algorithm starts with an initialisation of $\Gamma^{(1)} = 0$ at the first iteration ($n = 1$). At iteration $n + 1$, we update the value of $\Gamma^{(n+1)}$ based on the value of $\Gamma^{(n)}$ obtained in the previous iteration n as follows;

$$\Gamma^{(n+1)} = \Gamma^{(n)} - \varphi^{(n)}\Big(C'_k - \sum_{i \in \mathcal{I}\setminus\{o\}} \sum_{x=0}^{\theta_{ik}} \sum_{y=0}^{C'_k} (x+y) u_{ixy}^{(n)} - \sum_{x=0}^{\theta_{ok}} \sum_{y=0}^{C'_k} (x+y) u_{oxy}^{(n)}\Big).$$

The algorithm stops when the iteration limit is reached.

We close this section with a remark on our approximation method. The number of binary variables in (18) increases with the number of sales channels and flight capacity. In order to solve this model efficiently, we propose the Lagrangian relaxation method that considers the temporal dynamics of the customer arrival process since the expected revenues in (18) are obtained from the dynamic programming model. Unlike the LP-based decomposition approach, the Lagrangian relaxation method guarantees the integrality condition due to the total unimodularity property.

5 Computational Experiments

We design computational experiments in order to illustrate performance of the proposed airline's dynamic capacity allotment model and the approximation approaches to determine fixed and variable allotments among different sales channels for a single-leg flight. In particular, we aim to answer the following questions:

- What are the benefits of introducing fixed and variable allotments to travel agencies within airline revenue management?
- How do different factors such as pattern of demand, price of flight ticket and acceptance rule impact on performance of dynamic capacity planning decisions in terms of fixed and variable allotments?
- How do different capacity allotment policies of the airline and booking acceptance rules of sales channels impact on airline's revenue and agency's profit?

In this section, we first describe the design and data structure used for numerical experiments and then present the computational results of the dynamic capacity planning model of airlines with contracted travel agencies.

5.1 Data and Design of Experiments

For the computational experiments, we use real booking data for a single-leg flight of an international airline provided by our industrial partner. The flight has a fixed seat capacity of $C = 219$. The airline tickets are sold via two kinds of sale channels: travel agencies and an online reservation system owned by the airline. In this case study, we consider five travel agencies (labelled as $A1, A2, \cdots, A5$) that are selling flight tickets of domestic and international airlines. As stated in the problem description, travel agencies bid on the initial seat allocation (so-called 'fixed allotment') before the selling season starts. A contract signed between the airline and travel agencies specifies the size of fixed allotment and its price. According to the contract, travel agencies are not allowed to return any unsold seats (initially assigned as fixed allotment). Similarly, they cannot demand more fixed allotments than the limit set by the airline. In our numerical experiments, it is set as at most 20 seats. On the other hand, unlike fixed allotments, variable allotments provide full flexibility in terms of reassignment of seat capacity among all sales channels and the airline. The booking horizon for the flight consists of $T = 2000$ discrete time periods in which at most one customer may arrive and request a flight ticket.

We assume that the arrival of customers is stochastic and the customer requests arrive based on a homogeneous Poisson process with rate λ. Let parameter ρ denote the load factor that indicates different arrival intensities. It is computed as the ratio of total expected demand over the flight capacity, $\rho = \frac{T(1-exp(-\lambda))}{C}$. In our computational experiments, we examine two distinct scenarios for the load factor, $\rho = 1.0$ and $\rho = 1.2$ by adjusting the arrival rate λ to assess the potential impact of demand and price patterns on capacity allocation decisions.

In order to illustrate impact of demand pattern on performance of policies, we consider two different demand patterns in our experiments. For the first demand pattern, we assume that the probability of a customer requesting capacity from a specific sales channel increases or decreases over time (abbreviated as 'D_{fixed}'). We generate these arrival probabilities following the approach proposed by (Aydin et al., 2013). Let a customer request from sales channel i arrive at time t with probability $\psi_i(t)$ such that $\sum_{i\in\mathcal{I}} \psi_i(t) = 1$. These multinomial probabilities for demand pattern D_{fixed} are given as follows;

$$\psi_i(t) = \frac{\pi_i(t)}{\sum_{i\in\mathcal{I}} \pi_i(t)}, \quad i \in \mathcal{I}, t \in \mathcal{T}\backslash\mathcal{S},$$

where $\pi_i(t)$ are predetermined piece-wise linear functions. In the second demand pattern (abbreviated as 'D_{random}'), the arrival probabilities are generated by Dirichlet distribution with parameters $\eta_i(t)$, $i \in \mathcal{I}$. Notice that, the parameters of the Dirichlet distribution vary over time. We define $\eta_i(t)$ at time $t \in \mathcal{T}\backslash\mathcal{S}$ for channel $i \in \mathcal{I}$ as $\eta_i(t) = v_i + (\bar{v} - v_i)(1 - exp(-|\mathcal{I}|t/T))$ where $\bar{v}$ and v_i are non-negative constants. Then, the multinomial probabilities for demand pattern D_{random} are given as follows;

$$\phi_i(t) = \frac{\eta_{it}}{\sum_{i\in\mathcal{I}} \eta_{it}}, \quad i \in \mathcal{I}, t \in \mathcal{T}\backslash\mathcal{S}.$$

We refer to Birbil et al. (2009) for the details of this arrival process. Given the customer requests arriving according to a homogeneous Poisson process with rate λ, no arrival probability at time t is $p_{0t} = exp(-\lambda)$. Using the multinomial probabilities defined for demand patterns D_{fixed} and D_{random}, the arrival probability p_{it} for sales channel i is given by $p_{it} = \psi_i(t)(1 - p_{0t})$ and $p_{it} = \phi_i(t)(1 - p_{0t})$, respectively. Figure 2 illustrates the arrival probabilities over time for these two demand processes.

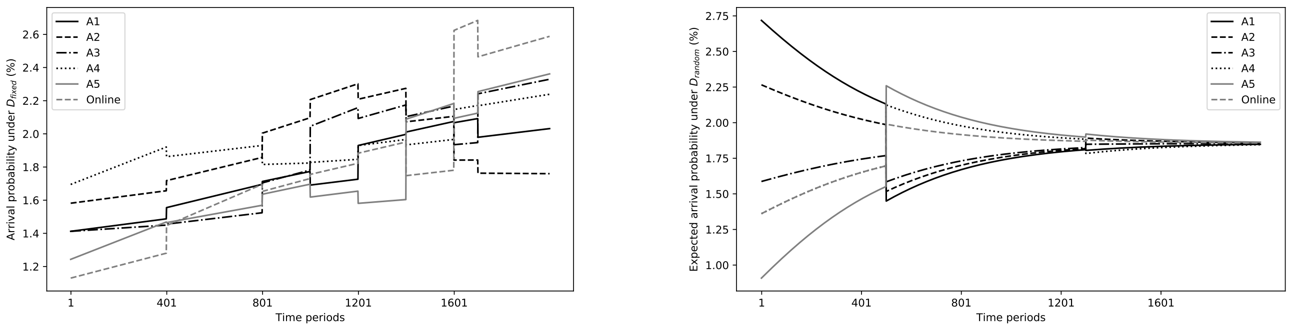


Figure 2
Fixed (left) and random (right) passenger arrival probabilities over the booking horizon

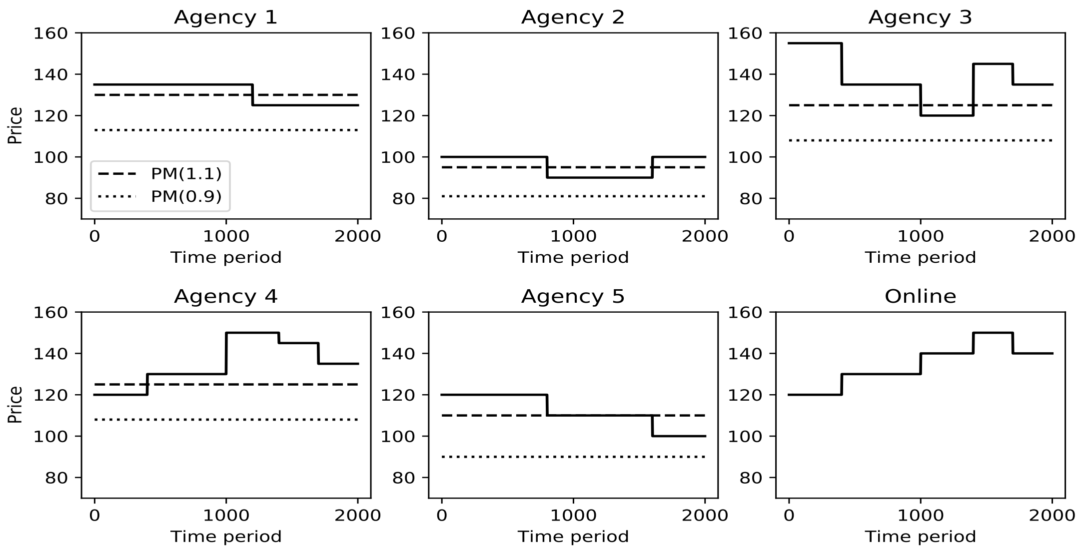
In practice, the prices of fixed allotments (for each travel agency) are set at the beginning of the selling season by the airline. Since pricing information for the fixed allotment was not available in the real booking data provided by our industrial partner, we set fixed allotment prices based on the lowest ticket price $r_i^{\min}$ to charge agency i. Then the fixed allotment price for agency i is determined as $f_i = \theta r_i^{\min}$ where parameter $\theta \in \{0.9, 1.1\}$ represents the price multiplier. The prices (r_{it}) of variable allotments for the airline and agencies are taken from the real dataset. On the other hand, the selling price (g_{it}) used by travel agencies was not available in the real booking data. Thus, the ticket selling price for agency i at time t is determined in terms of the variable allotment price (r_{it}) as $g_{it} = 1.2 \times r_{it}$.

For the computational experiments, we consider two different price patterns set by the airline and they are used by each sales channel over the booking horizon. As presented in Figure 3, Panel A displays the price pattern observed from the real data set while Panel B shows a hypothetical price pattern where flight prices for all channels are assumed to be increasing over time during the booking horizon. In Figure 3, the variable and fixed allotment prices are displayed by solid and dotted lines, respectively.

The proposed dynamic capacity allocation model is solved by the optimization based decomposition methods as described in Section 4. The airline's capacity planning policies are obtained by solving the dynamic capacity allotment problem using the linear programming and Lagrangean relaxation based approximation methods and abbreviated as *'LP-Apr'* and *'Lag-Rel'*, respectively. In addition to these optimization based decomposition methods, we also consider *performance-based capacity allocation approach* that is widely used in practice as a benchmark. In this approach, the airline determines the variable allotment capacities to be distributed among agencies at review time periods. At the beginning of the planning horizon, the airline decides the fixed allotment capacity for each travel agency (i.e., ω_i for $i \in \mathcal{I}\backslash\{o\}$) and the variable allotment capacity for its reservation system (σ_o) while the remaining capacity is evenly distributed among agencies as their initial variable allotment capacities. Then, at each review period, the airline reallocates the variable allotment capacity considering the percentage of sold flight tickets by each agency since the previous review period. The variable allotment capacity for the airline's reservation system is determined by either *LP-Apr* or *Lag-Rel.*

All optimization algorithms are implemented in MATLAB and the computational experiments are run on a desktop computer Intel(R) Core(TM) i5-700 CPU 3.40GHz.

Panel A: Fixed and variable allotment prices observed from real data



Panel B: Fixed and variable allotment prices generated with increasing pattern

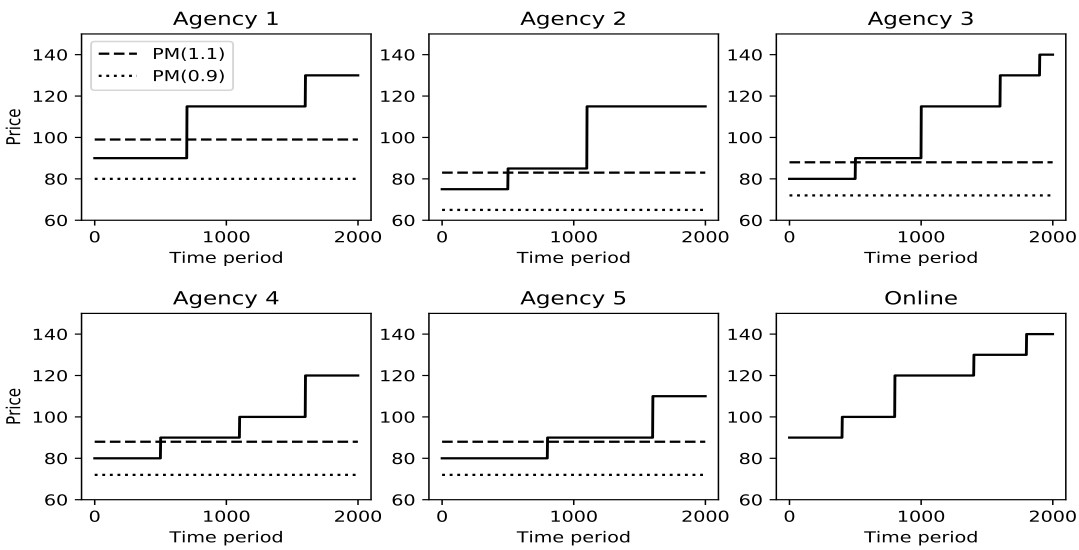


Figure 3
Patterns of real (Panel A) and generated (Panel B) prices over the booking horizon

5.2 Simulation Study

We conduct a simulation study to evaluate performance of capacity allotment policies by reducing the potential impact of uncertain customer arrivals and also customer preferences. We first randomly generate a stream of customers arriving during the discretised booking horizon based on the probability distribution defined by different demand patterns. Then we apply the capacity allotment policy to allocate fixed and variable allotments to all sales channels at the beginning of the booking horizon. During the booking horizon, the remaining variable allotment capacities are reallocated among sales channels at every review period. At each time period within the booking horizon, one customer requests a ticket from a specific sales channel and an appropriate acceptance rule is adopted to reject or accept the customer.

The total expected revenue of the airline from selling both fixed and variable allotments is collected at the end of the booking horizon. Algorithm 1 (presented in Appendix A.2) summarises the simulation procedure used in our computational experiments.

For the airline's capacity allotment model, the expected revenue obtained from each agency is estimated from the airline's perspective where each agency may adopt different decision rules as the booking strategy. Given the available seat capacity, the acceptance rules are adopted by travel agencies as well as the airline's reservation system to determine whether a customer request for a flight ticket is fulfilled or not. We specifically implement two acceptance rules in our simulation study.

- *First-Come-First-Serve (abbreviated as 'FCFS') Rule:* In practice, this acceptance rule has been widely used by travel agencies. Given the allocated fixed and variable capacities, travel agencies intend to accept all customers arriving during the booking horizon as long as they have sufficient capacity. Considering that agencies purchase the fixed allotment capacity in advance, they use fixed allotment capacity first and then variable allotment capacity to accommodate customer requests.

- *Dynamic Opportunity Cost (abbreviated as 'DP') Rule:* Given the allocated fixed and variable allotments, each agency solves the dynamic capacity planning model (23)-(24) presented in Appendix A.1 at each review period by considering the whole booking horizon. When there is sufficient capacity, the value functions are compared between the cases where the request is accepted or rejected. The acceptance decision is made when the case of accepting the request has a higher value than the alternative one. The travel agencies first use fixed allotment capacity to fulfil customer demand.

5.3 Numerical Results

We present results of our numerical experiments under three main categories to illustrate i) performance and robustness of the capacity allotment policies determined by two approximation methods, ii) impact of various model parameters (such as demand and price patterns, and fixed allotment price) on the fixed and variable allotment allocation decisions and and the airline's revenue, and iii) impact of airline's capacity allocation strategies on agencies' expected profit.

Performance and Robustness of Capacity Planning Policies: We are first interested in illustrating *performance* and *robustness* of the optimization-based approaches introduced to solve the dynamic capacity allotment problem of the airline. The performance of an approach is measured in terms of maximum revenue achieved by the airline over the booking horizon of a single flight, whereas the robustness is determined by the regular performance of an approach over different demand scenarios, load factors and price patterns. We use two specific settings for load factor parameter (LF = 1.0 and 1.2) and price multiplier (PM = 0.9 and 1.1).

Table 3 presents the airline's revenue obtained by *LP-Apr* and *Lag-Rel* policies using different sale channels as travel agencies (selling fixed and variable allotments) and the online reservation system. The maximum revenue is highlighted in bold to show the best policy among all different problem settings under different demand and price patterns. We also investigate the case where the airline does not allocate any fixed allotment (abbreviated as 'No-Fixed') under both *LP-Apr* and *Lag-Rel* policies to demonstrate the benefit of using fixed versus only variable allotments.

Overall, these results show that the Lag-Rel policy outperforms to the LP-Apr one under all settings.

Table 3
Airline revenues obtained by different policies

		Stable Demand				Random Demand			
		PM(1.1)		PM(0.9)		PM(1.1)		PM(0.9)	
Policy	*Allotment*	LF(1.0)	LF(1.2)	LF(1.0)	LF(1.2)	LF(1.0)	LF(1.2)	LF(1.0)	LF(1.2)
Panel A: Real Price Pattern									
LP-Apr	*Fixed*	7220	6260	0	0	5545	5295	0	0
	Variable	18384	21022	25990	27703	19943	22102	25854	27822
	Total	25604	27282	25990	27703	25488	27397	25854	27822
Lag-Rel	*Fixed*	875	875	0	0	750	625	0	0
	Variable	25511	26980	26356	27821	25528	27382	26253	27986
	Total	**26386**	**27855**	**26356**	**27821**	**26278**	**28007**	**26253**	**27986**
LP-Apr	*No-Fixed*	25990	27703	25990	27703	25854	27822	25854	27822
Lag-Rel	*No-Fixed*	26356	27821	26356	27821	26253	27986	26253	27986
Panel B: Increasing Price Pattern									
LP-Apr	*Fixed*	4030	2684	0	0	4691	3630	0	0
	Variable	17536	19914	21272	22397	16401	18663	20730	22008
	Total	21566	22598	21272	22397	21092	22293	20730	22008
Lag-Rel	*Fixed*	4455	7925	3287	3008	6055	6560	4319	3766
	Variable	18546	15974	19011	20134	16676	17316	17566	19263
	Total	**23001**	**23899**	**22298**	**23142**	**22731**	**23876**	**21885**	**23029**
LP-Apr	*No-Fixed*	21272	22397	21272	22397	20730	22008	20730	22008
Lag-Rel	*No-Fixed*	21710	22595	21710	22595	21710	22595	21710	22595

This is mainly because the Lagrangean-based optimization model considers future demand uncertainty of agency channels. It allows the airline to dynamically obtain higher total revenue by updating variable allotment allocations. We investigate the impact of various factors such as load factor, demand and price patterns on the airline's expected revenue. Both Lag-Rel and LP-Apr policies perform better as the load factor increases. When the load factor is high, more customers request flight tickets and both policies benefit from the increase in the number of higher paying customers. When we compare the results for different price patterns, we observe that the airline allocates less fixed allotment capacity to agencies under the real price pattern and low fixed price (PM = 0.9). This provides more flexibility for the airline to redistribute the remaining capacity at the review periods. Considering the low price of fixed allotment, allocating less fixed capacity under this scenario could increase the total revenue obtained by the airline. On the other hand, when fixed price is high (PM = 1.1), the airline allocates more fixed allotment capacity since fixed allotment price is higher than the lowest variable allotment price. The effect of fixed allotment price is more striking under the increasing price pattern. We observe that both methods allocate more fixed allotment under both demand patterns. This is mainly due to the increasing variable prices over time. Since variable allotment price is low at the beginning of the booking horizon, the airline allocates more fixed allotment which could potentially increase the airline's revenue.

We also investigate benefit of allocating fixed allotment capacity among agencies by implementing the LP-Apr and Lag-Rel policies with no fixed allotment capacity (i.e., only variable allotment is used). We observe that both policies provide lower revenue when no fixed allotment is considered in comparison to the ones using fixed allotment. Since the agencies cannot return the unsold fixed allotment capacity, fixed allotments help the airline to reduce the loss in revenue especially when the load factor is low. Having fixed allotment reduces impact of uncertainty in the booking process and it could generate more revenue for the airline.

In order to analyse potential impact of capacity allocation policies on overall performance of sales channels, we present the fixed allotment allocations to each travel agency provided by both solution methods under demand realisations using the real and generated price patterns in Table 4. Across all demand scenarios and price patterns, when the price of fixed allotment for any sales channel is lower than its variable allotment price (with price multiplier 0.9), both LP-Apr and Lag-Rel policies become more conservative in using fixed allotments. Thus, we only present results obtained by high fixed price case when PM = 1.1. As we discussed before, the fixed allotment provides a cushion for the airline against the loss in total revenue. In other words, when the airline expects to generate less revenue under low demand or low price situations, they tend to increase the allocation of fixed allotment capacity to travel agencies. The results presented in Table 4 show that under real price pattern, the LP-Apr policy tends to allocate more fixed allotments than the Lag-Rel policy does under both demand scenarios. Similar to our previous discussion, both policies allocate more fixed allotments as the load factor decreases. However, each policy behaves differently in allocating fixed allotments under any load factors and different price patterns as they assess the agencies' ability and performance in generating revenue varies. Specifically, Lag-Rel tends to allocate more fixed allotments compared to LP-Apr under the increasing price pattern, whereas LP-Apr allocates more fixed allotment under the real price pattern. This behaviour can be attributed to the lower variable prices at the beginning of the reservation period in the increasing price pattern. Considering that Lag-Rel outperforms LP-Apr by generating higher revenue, we can conclude that Lag-Rel can anticipate the changes in the demand pattern much better than LP-Apr when making fixed allotment decisions.

Table 4
Fixed allotments allocated by different policies under high fixed capacity price and different demand scenarios

		Real Price Pattern					Increasing Price Pattern				
Policy	Agencies	A1	A2	A3	A4	A5	A1	A2	A3	A4	A5
Panel A: Stable demand pattern											
LP-Apr	LF (1.0)	16	17	8	7	15	10	8	7	9	11
	LF (1.2)	19	-	10	8	14	12	-	5	7	5
Lag-Rel	LF (1.0)	-	-	-	7	-	4	5	5	13	11
	LF (1.2)	-	-	-	7	-	19	4	7	16	20
Panel B: Random demand pattern											
LP-Apr	LF (1.0)	14	14	7	6	7	15	10	8	7	12
	LF (1.2)	17	-	8	7	11	18	-	10	-	11
Lag-Rel	LF (1.0)	-	-	-	6	-	7	6	6	13	20
	LF (1.2)	-	-	-	5	-	8	6	7	15	20

Figure 4 illustrates how variable allotment capacity varies among all sales channels at each review period using the real price pattern. In this figure, the load factor and price multiplier are set as 1.0 and 1.1, respectively. Overall, we observe that both LP and Lagrangian relaxation based approximation methods allocate allotment capacity to all sales channels at each review period. In fact, the Lag-Rel policy allocates more variable allotments (especially to agencies) than LP-Apr under both demand patterns. For example, under stable demand scenario, agencies 2 and 5 receive less variable allotment in the LP-Apr policy. This is mainly because these two agencies have relatively low prices for variable allotment as

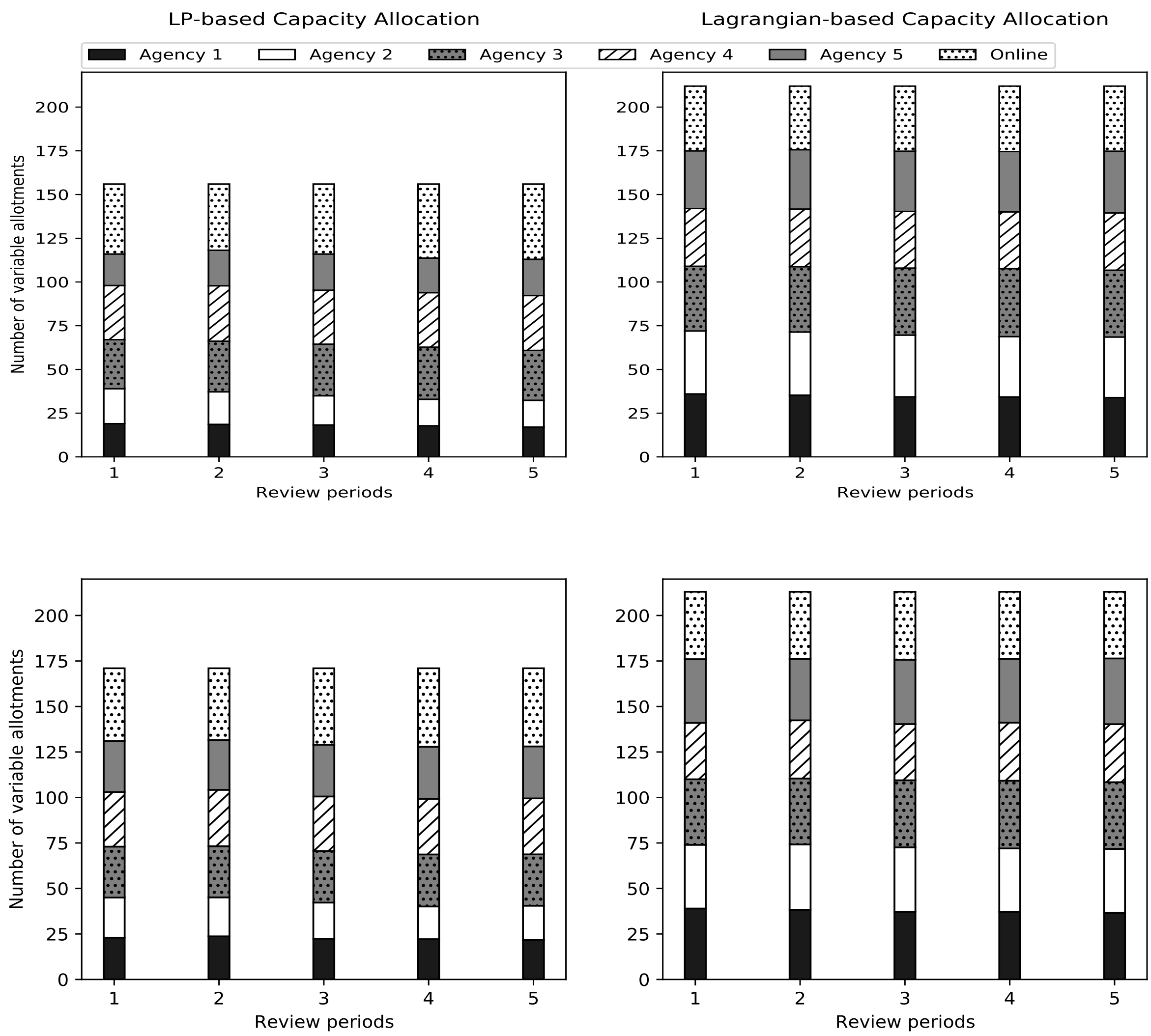


Figure 4
Variable allotment capacity allocated by LP (left) and Lagrangian relaxation (right) based approaches at each review period using real price pattern under stable (top) and random (bottom) demand scenarios

illustrated in Figure 3. Similarly, under both demand patterns, we notice that the LP-Apr policy tends to allocate more variable allotment to airline's online channel, whereas the Lag-Rel policy distributes variable allotment capacity more evenly among all sale channels. Since the LP based approximation method uses the expected agency demand, it tends to allocate less variable allotment to travel agencies.

Impact of Capacity Allotment Policies and Acceptance Rules: The computational experiments conducted so far aim to show performance of the optimization-based capacity planning approaches assuming that all travel agencies apply strategic acceptance decision rule (described as dynamic opportunity cost *'DP'*) for booking the customer requests. Now we wish to analyse impact of different capacity allotment strategies on the airline's revenue if the airline implements a performance-based allotment allocation approach and the agencies use less strategic and less forward looking FCFS acceptance rule. Figure 5 displays the airline's expected revenue obtained by the optimization-based (left) and performance-based (right) capacity allotment strategies over all sale channels under both price patterns with high priced fixed allotment. Note that, as the customer arrival probability changes for each sale channel under random demand pattern, we analyse the results under this setting with high load factor. We present numerical

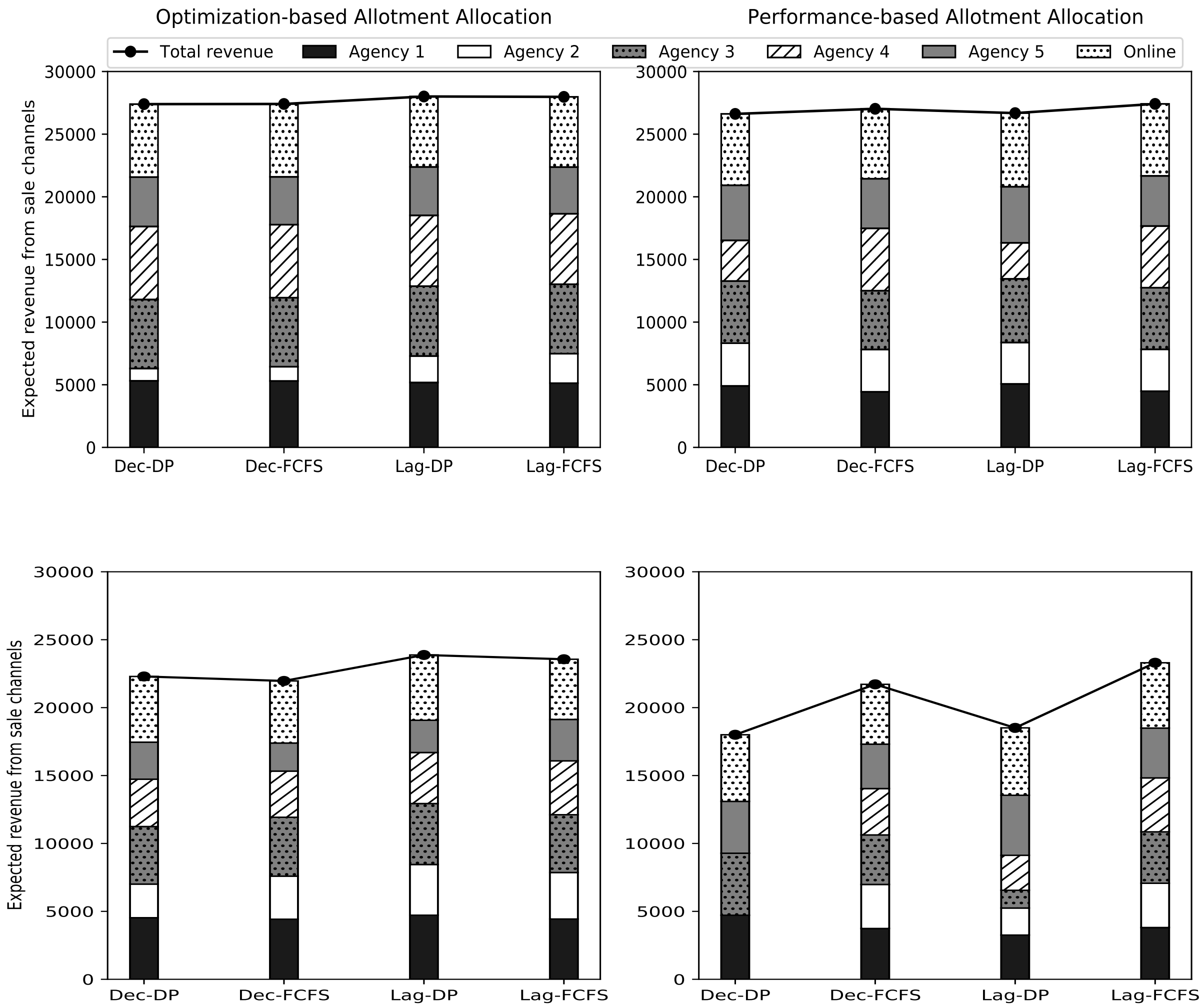


Figure 5
Impact of different allotment allocation strategies and booking rules on airline's expected revenue using random demand with real (top) and increasing (bottom) price patterns

results for optimization-based (left) and performance-based (right) capacity allocation policies obtained by solving the dynamic programming model at review periods by using the decomposition method with linear programming and Lagrangean relaxation approaches. We differently label each combination of specific capacity allocation policies of the airline and acceptance rules of agencies. For example, 'Dec-DP' under optimization-based allocation indicates that the airline uses LP-Apr policy and agencies apply for the DP booking rule while 'Lag-FCFS' under performance-based allocation indicates that the airline uses Lag-Rel with performance based allocation to distribute the capacity among sale channels and agencies apply for a FCFS booking rule.

From the numerical results presented in Figure 5, we observe that the airline obtains higher total revenue by the optimization-based capacity allocation compared to the one obtained by the performance-based capacity allocation. As the performance-based policy only considers the past sale performances of agencies, it neglects the future demand uncertainty when allocating variable allotment capacity. Therefore, it performs worse than the optimization-based capacity allocation policy under all scenarios. Considering the expected revenues received from each sale channel, we observe that the revenue contribution from agencies largely depends on the price pattern and the capacity allocation policy. For example, under the real price pattern, the airline obtains higher revenues from agencies 1, 3 and 4 than agencies 2 and 5 when it allocates capacity allotment by the optimization-based method. This is mainly because the three agencies generally charge higher price than the other two agencies. On the other hand, under the performance-based allocation policy, the airline receives more revenue from agencies 1, 3 and 5 since these agencies sell more at the beginning of the reservation period. Thus, they receive more capacity when the performance-based capacity allocation procedure is followed. Notice that the airline does not receive any revenue from agencies 2 and 4 in 'Dec-DP' case using the performance-based capacity allocation.

Under the increasing price pattern, these two agencies do not sell any tickets until the next review period since they expect higher paying customers may arrive later during the reservation horizon. Since the airline reallocates the variable allotment capacity considering the total sales of each agency, these two agencies do not receive any variable allotment capacity at review periods. On the other hand, if these two agencies use the FCFS approach (Dec-FCFS), they behave less strategically and accept any customers as long as they have sufficient capacity. Considering the agencies' perspective, we can conjecture that the FCFS booking rule is generally more advantageous under the performance-based allocation policy.

Table 5
Expected profit of travel agencies and accepted/rejected customer requests

Policy-Method		*Agency 1*		*Agency 2*		*Agency 3*		*Agency 4*		*Agency 5*	
Panel A: Real Price Pattern											
OptP	Dec-DP	**1165**	(41/-/3)	197	(10/4/30)	**1377**	(42/-/2)	**1141**	(43/-/2)	**920**	(36/-/7)
	Dec-FCFS	1162	(41/-/3)	228	(12/-/32)	**1377**	(42/-/2)	**1141**	(43/-/2)	896	(35/-/8)
	Lag-DP	1036	(39/-/-)	423	(21/9/-)	1115	(40/-/-)	1105	(42/-/-)	773	(34/-/-)
	Lag-FCFS	1025	(39/-/-)	**471**	(24/-/-)	1108	(40/-/-)	1100	(41/-/-)	746	(33/-/-)
PerA	Dec-DP	**1083**	(38/-/6)	**682**	(35/3/5)	**1268**	(37/2/5)	625	(23/13/9)	**1012**	(40/-/3)
	Dec-FCFS	989	(34/-/10)	677	(35/-/9)	1213	(36/-/8)	**970**	(37/-/8)	923	(36/-/7)
	Lag-DP	1013	(38/-/-)	662	(34/-/-)	1016	(36/-/-)	568	(21/-/1)	895	(40/-/-)
	Lag-FCFS	897	(34/-/-)	669	(35/1/-)	983	(36/2/-)	960	(36/8/-)	801	(35/-/-)
Panel B: Increasing Price Pattern											
OptP	Dec-DP	**777**	(41/-/2)	495	(23/19/2)	756	(41/1/2)	**697**	(36/7/2)	**440**	(29/10/3)
	Dec-FCFS	755	(40/-/4)	**634**	(35/-/9)	**776**	(42/-/2)	679	(37/-/8)	308	(24/-/19)
	Lag-DP	560	(41/1/-)	602	(38/4/-)	522	(40/1/-)	144	(33/10/-)	130	(21/20/-)
	Lag-FCFS	504	(39/-/-)	541	(36/-/-)	473	(39/-/-)	59	(36/-/1)	-63	(29/-/-)
PerA	Dec-DP	**813**	(43/-/1)	0	(-/11/33)	**822**	(43/-/-)	0	(-/7/38)	**658**	(41/1/-)
	Dec-FCFS	618	(35/-/9)	**648**	(36/-/8)	638	(36/-/7)	**682**	(37/-/8)	549	(37/-/6)
	Lag-DP	507	(26/18/-)	542	(18/25/-)	256	(10/31/-)	140	(21/24/-)	212	(43/-/-)
	Lag-FCFS	378	(34/-/-)	508	(35/-/-)	379	(35/-/-)	60	(36/-/-)	61	(36/-/-)

Finally, we would like to investigate the impact of capacity allocation policies (based on optimisation and agency performances - abbreviated as "OptP" and "PerA", respectively) using different acceptance rules on the performance of travel agencies in terms of expected profit and number of customer requests admitted/rejected. Note that a customer's request can be only rejected due to a value function acceptance rule and a lack of capacity. Table 5 presents each agency's expected net profit as well as the number of customer requests accepted/rejected (due to value and lack of capacity). Specifically, we report the number of accepted customers, customers rejected by the acceptance decision rule, and customers rejected due to a lack of capacity, respectively, in a parenthesis in terms of three components as (-/-/-). The maximum profit obtained by each agency under different capacity allocation policies and the acceptance rules is highlighted in **bold**. The results in Table 5 show that the DP acceptance rule is usually more profitable for agencies than FCFS. This result can be attributed to the impact of forward looking feature of the DP rule. Recall that DP considers the potential customer requests arriving in future time periods when making acceptance decisions. However, one can notice that this acceptance rule may not always work well, especially when the airline allocates capacities using the performance-based policy. As we discussed before, the performance-based policy considers past sales of agencies when allocating the capacity. In terms of number of rejected customers, we observe that DP tends to reject more customers under performance-based policy than the optimization-based policy. Since the DP rule does not consider past sales, it may perform poor when the airline uses performance-based allocation. Note that FCFS accepts customers based on the available capacity, thus there is no rejected customers due to the acceptance rule in FCFS. Although agencies accept more customers by FCFS, it may not be always profitable. For example, agency 5 has a significant profit loss in 'Lag-FCFS' case under increasing price pattern. Considering the variable prices of agency 5 given in Figure 3, we observe that this agency accepts more low paying customers due to FCFS which results in loss of profit. The results in Table 5 also point out that performance of an acceptance rule used by agencies is significantly affected by the capacity allocation policy used by the airline.

6 Conclusions

In this paper, we studied the dynamic capacity planning problem of an airline using (fixed and variable allotment) contracts with travel agencies selling flight tickets. Finding the optimal fixed and variable allotment policies requires solving a complex stochastic dynamic program due to a high-dimensional state vector. To address the curse of dimensionality in the dynamic programming model, we propose two approximation methods for determining the allocation of capacity allotments across all sales channels. We conducted a series of computational experiments in order to investigate impact of various factors on the fixed and variable allotment decisions and overall performance of the capacity allotment policies. We illustrated the added value of introducing fixed and variable allotments among travel agencies considering different factors such as pattern of demand, ticket prices and booking rules of agencies using the real airline data. Moreover, we investigated impact of different capacity allotment strategies of the airline on their revenue and agency's profit and implemented optimization-based and performance-based strategies to allocate the capacity to sale channels. Our results show that the optimization-based capacity allocation performs better than the performance-based strategy since it considers the future demand uncertainty when making the capacity allocation decisions. We also analysed capacity management strategies of agencies considering different booking rules. We implemented the first-come-first-serve booking rule as a benchmark and compared its performance with the forward looking optimisation based booking rule. We observed that the forward looking strategy is generally more profitable for agencies than the first-come-first-serve rule. Having said that the performance of booking strategy is significantly affected by the price pattern and the airline's capacity allocation policy.

Our findings lead to important managerial insights. First, the airline generally allocates more fixed allotment capacity when customer demand (low load factor) is low since fixed allotment provides a cushion against the loss due to potential empty seats. On the other hand, the variable allotment capacity obtained from different allocation policies is significantly affected by the price pattern. Secondly, airlines should take into account demand uncertainty when making capacity allotment decisions among travel agencies. In particular, the reoptimisation of capacity allotments at specific review periods is crucial for the airline to respond promptly to changes in customer demand. Similarly, agencies must consider the airline's capacity allocation strategy and customer demand uncertainty in addition to their acceptance (or booking) strategy when making capacity allotment requests from the airline. A possible future avenue is to investigate the optimal policy selection for travel agencies. In this paper, we focus on the airline's capacity planning problem where each agency's contribution is evaluated from the airline's perspective. Developing a model that accounts for dynamic interactions between an airline and multiple travel agencies presents a challenging yet interesting area for further exploration.

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A Appendix

A.1 Agency Capacity Management Model

In a dynamic setting, agencies make their own booking decisions about accepting or rejecting arriving customer requests. Given the seat capacity being allocated by the airline at each review period, an agency's capacity management problem becomes a standard revenue management problem. A state of the dynamic system for agency i at time t is determined by the remaining seat capacity in terms of fixed and variable allotments (denoted by x_{it} and y_{it}, respectively). At the first review period $t = 1$, travel agency i receives fixed (ω_i) and variable (σ_i) allotments, where $(\omega_i, \sigma_i) \in \mathcal{H}(\mathbf{0}_1, \mathbf{0}_1)$. Given state (x_{it}, y_{it}) at any review period $t \in \mathcal{S}\backslash\{1\}$, travel agency i receives variable allotments δ_{it} as the airline re-allocates the remaining seat capacity among all sales channels without violating any constraints of $\mathcal{H}(\mathbf{x}_t, \mathbf{y}_t)$. Then the next state for agency i at time $t+1$ is obtained as $(x_{it}, y_{it} + \delta_{it})$.

A customer's request for a flight ticket may arrive to agency i with purchasing probability p_{it} at time $t \in \mathcal{T}\backslash\mathcal{S}$ between any consecutive two review periods. In this case, agency i sells the flight ticket at price of g_{it} and pays f_i or r_{it} to the airline depending on the use of fixed or variable allotment capacity at time t, respectively. Let us define binary variables μ_{it} and ν_{it}, respectively, to represent whether the fixed and variable allotments are used by channel i to make booking decisions at time t. If $\mu_{it} = 1$ (or $\nu_{it} = 1$), then fixed (or variable) allotment capacity is used to source the seat for the customer demand at time t. Given decisions μ_{it} and ν_{it} taken at state (x_{it}, y_{it}) of time t, the next state at time $t+1$ can be obtained as $(x_{it} - \mu_{it}, y_{it} - \nu_{it})$. In this case, the agency's profit is calculated as $g_{it}(\mu_{it} + \nu_{it}) - f_i\mu_{it} - r_{it}\nu_{it}$.

Travel agencies intend to first use fixed allotment capacity in order to fullfil the customer demand so that the potential risk of unsold capacity and consequently potential revenue loss can be eliminated. In order to impose this situation into decision-making, we introduce an auxiliary (binary) decision variable α_i. If $\alpha_i = 1$ (or 0), then the customer demand is satisfied from fixed (or variable) allotment. When no fixed allotment is left, the customer requests are satisfied from variable allotments. Therefore, booking decisions at state (x_{it}, y_{it}) at time t need to satisfy the following set of linear constraints:

$$\mathcal{U}_{it}(x_{it}, y_{it}) = \Big\{\mu_{it}, \nu_{it} \in \{0,1\} \mid \nu_{it} \le y_{it},\ \mu_{it} \le x_{it},\ \nu_{it} \le 1 - \alpha_i,\ x_{it} \le M\alpha_i,\ \alpha_i \in \{0,1\}\Big\},$$

where M is a sufficiently large number that can be specified as C. Let $V_{it}(x_{it}, y_{it})$ denote the value function for agency i at state (x_{it}, y_{it}) at time t. Given allocated capacity (ω_i, σ_i and δ_{it}) by the airline as well as fixed and variable allotment capacities (x_{it} and y_{it}), the value function at $t \in \mathcal{T}$ computes the expected net revenue/profit from time t to the end of planning horizon T as follows;

$$V_{it}(x_{it}, y_{it}) = \begin{cases} V_{i,t+1}(x_{i,t+1}, y_{i,t+1}), & \text{for } t = 1 \\ V_{i,t+1}(x_{i,t+1}, y_{i,t+1}), & \text{for } t \in \mathcal{S}\backslash\{1\} \\ \max\limits_{\mu_{it}, \nu_{it} \in \mathcal{U}_{it}(x_{it}, y_{it})} \Big\{p_{it}\Big[g_{it}(\mu_{it} + \nu_{it}) - f_i\mu_{it} - r_{it}\nu_{it} + V_{i,t+1}(x_{i,t+1}, y_{i,t+1})\Big] & \\ \qquad + (1 - p_{it})V_{i,t+1}(x_{i,t+1}, y_{i,t+1})\Big\}, & \text{for } t \in \mathcal{T}\backslash\mathcal{S}. \end{cases} \tag{23}$$

The corresponding transition function can be described as follows:

$$(x_{i,t+1}, y_{i,t+1}) = \begin{cases} (\omega_i, \sigma_i) & \text{if } t = 1,\ (\omega_i, \sigma_i) \in \mathcal{H}(\mathbf{0}_1, \mathbf{0}_1) \\ (x_{it}, y_{it} + \delta_{it}), & \text{if } t \in \mathcal{S}\backslash\{1\},\ \delta_{it} \in \mathcal{H}(\mathbf{x}_t, \mathbf{y}_t) \\ (x_{it} - \mu_{it}, y_{it} - \nu_{it}), & \text{if customer request arrives at } t \in \mathcal{T}\backslash\mathcal{S} \\ (x_{it}, y_{it}) & \text{otherwise.} \end{cases} \tag{24}$$

At the end of the booking horizon, agency i pays the cost of unsold fixed allotment to the airline. Then the boundary condition for the agency i's capacity management problem at time $T+1$ is described at state $(x_{i,T+1}, y_{i,T+1})$ as $V_{i,T+1}(x_{i,T+1}, y_{i,T+1}) = -f_i x_{i,T+1}$. Note that the DP model (23)-(24) cannot be solved due to involving the airline's future allotment decisions in review periods. We overcome this problem without considering the review periods in our numerical experiments. We also investigate different booking strategies for the agencies.

Next, we discuss the agency capacity management model with no review periods. Similar to the DP model in (13), we introduce $\vartheta_{it}(x_{it}, y_{it})$ to represent the expected revenue obtained by channel i from t to T given the available fixed and variable capacity allotments (x_{it}, y_{it}) at time t. By excluding review periods from model (23), we obtain the following dynamic programming function to calculate the expected revenue of channel i, for $i \in \mathcal{I}\backslash\{o\}$, at time $t \in \mathcal{T}\backslash\mathcal{S}$ as

$$\begin{aligned} \vartheta_{it}(x_{it}, y_{it}) = \max_{\mu_{it},\, \nu_{it} \,\in\, \mathcal{U}_{it}(x_{it},\, y_{it})} \quad & \Big\{ p_{it} \Big[g_{it}(\mu_{it} + \nu_{it}) - f_i \mu_{it} - r_{it} \nu_{it} + \vartheta_{i,t+1}(x_{it} - \mu_{it}, y_{it} - \nu_{it}) \Big] \Big\} \\ & + (1 - p_{it}) \vartheta_{i,t+1}(x_{it}, y_{it}) \end{aligned} \tag{25}$$

with boundary conditions of $\vartheta_{i,T+1}(x_{i,T+1}, y_{i,T+1}) = -f_i x_{i,T+1}$ for all states $(x_{i,T+1}, y_{i,T+1})$ at time $T+1$. In addition, $\vartheta_{it}(0,0) = 0$ for all $t \in \mathcal{T}\backslash\mathcal{S}$ with the boundary condition $\bar{\vartheta}_{i,T+1}(x_{it}, y_{it}) = f_i x_{i,T+1}$ for all states $(x_{i,T+1}, y_{i,T+1})$ at time $T+1$. The optimal value function, $\vartheta_{it}(x_{it}, y_{it})$, given the allocated fixed and variable capacities (x_{it}, y_{it}) can be efficiently calculated using the standard backward dynamic programming approach.

A.2 Simulation Procedure

We conduct a simulation study to evaluate the performance of capacity allotment policies. The pseudocode for the general simulation process is provided in Algorithm 1 and it can be summarised summarised under three main steps:

Step 1: At time $t = 1$, we determine fixed and variable allotment capacities and compute the booking policy for each sales channel $i \in \mathcal{I}$ using one of the following methods.

- For the LP-based approximation method, the linear optimisation model (14)-(17) is solved.
- For the Lagrangian relaxation based approximation method, the dual problem (22) is solved by a standard subgradient algorithm.

Step 2: Between review periods, $t \in \mathcal{T} \backslash \mathcal{S}$, the sale channels receive customer requests. Considering the remaining capacity, each channel applies an acceptance rule for an arriving customer request to decide whether to fulfil the request or not.

- The remaining fixed and variable allotment capacities are updated accordingly.

Step 3: At intermediate review period $t \in \mathcal{S} \backslash \{1\}$, we resolve the optimisation models (14)-(17) and (22) for given values of remaining allotment capacities. This process carries on in the same manner until the end of the planning horizon.

Algorithm 1: Simulation Procedure

Initialization: $t = 1$; $C_t = C$; $TotalRev = 0$; $AP_i = 0$, for $i \in \mathcal{I} \backslash \{o\}$
  for $t = 1, \cdots, T$ **do**
    if $t = 1$ **then**
      – Solve model (14)-(17) or (22) to obtain (ω_i, σ_i) for $i \in \mathcal{I}$.
      – Set $x_{it} := \omega_i$ and $y_{it} := \sigma_i$, for $i \in \mathcal{I}$
      – Update capacity level $C_t := C_t - \sum_{i \in \mathcal{I} \backslash \{o\}} \omega_i$
      – Compute total revenue as $TotalRev := TotalRev + \sum_{i \in \mathcal{I} \backslash \{o\}} f_i \omega_i$
      – Calculate agency's profit as $AP_i := AP_i - f_i \omega_i$, for $i \in \mathcal{I} \backslash \{o\}$
    end if
    if $t \in \mathcal{S} \backslash \{1\}$ **then**
      – Solve model (14)-(17) or (22) to obtain δ_i, for each agency $i \in \mathcal{I}$ given C_t and x_{it}
      – Set $y_{it} := \delta_i$ for $i \in \mathcal{I}$
    else
      – Apply acceptance rule of sale channel $i \in \mathcal{I}$ for each customer request
      if fixed allotment is used by channel i, for $i \in \mathcal{I} \backslash \{o\}$ **then**
        – Update remaining capacity $x_{it} := x_{it} - 1$
        – Compute agency's profit as $AP_i := AP_i + g_{it}$
      else if variable allotment is used by channel $i \in \mathcal{I}$ **then**
        – Update remaining capacity $y_{it} := y_{it} - 1$ and $C_t := C_t - 1$
        – Calculate total revenue as $TotalRev := TotalRev + r_{it}$,
        – Compute agency's profit as $AP_i := AP_i + (g_{it} - r_{it})$, for $i \in \mathcal{I} \backslash \{o\}$
      end if
    end if
  end for

A.3 Proof of Proposition 4.1

The proof follows from the concavity result presented by Lippman and Stidham (1977). For ease of notation, we define $\Delta\vartheta_{ot}(y_o) = \vartheta_{ot}(y_o) - \vartheta_{ot}(y_o - 1)$. The value function $\vartheta_{ot}(y_o)$ is discrete concave in state variable y_o, if the differences $y_o \mapsto \vartheta_{ot}(y_o) - \vartheta_{ot}(y_o - 1)$ are non-increasing (Lippman and Stidham, 1977). In other words, we should show that $\Delta\vartheta_{ot}(y_o) \geq \Delta\vartheta_{ot}(y_o+1)$ using an induction over time periods.

The result holds at time $T+1$ since $\vartheta_{oT+1}(y_o) - \vartheta_{oT+1}(y_o - 1)$ and $\vartheta_{oT+1}(y_o + 1) - \vartheta_{oT+1}(y_o)$ are both zero for any given y_o. We assume that the result holds for time period $t+1$. Our proof is completed once we show that $\Delta\vartheta_{ot}(y_o) \geq \Delta\vartheta_{ot}(y_o + 1)$. The value function $\vartheta_{oT+1}(y_o)$ can be given as follows:

$$\begin{aligned} \vartheta_{ot}(y_o) &= p_{ot} \max\{r_{ot} + \vartheta_{ot+1}(y_o - 1), \vartheta_{ot+1}(y_o)\} + (1 - p_{ot})\vartheta_{ot+1}(y_o) \\ &= p_{ot} \max\{r_{ot} + \vartheta_{ot+1}(y_o - 1) - \vartheta_{ot+1}(y_o), 0\} + \vartheta_{ot+1}(y_o) \\ &= p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} + \vartheta_{ot+1}(y_o). \end{aligned} \tag{26}$$

Following the value function in (26), we can obtain $\Delta\vartheta_{ot}(y_o)$ and $\Delta\vartheta_{ot}(y_o + 1)$ as follows;

$$\begin{aligned} \Delta\vartheta_{ot}(y_o) &= p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} + \vartheta_{ot+1}(y_o) \\ &\quad - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o - 1), 0\} - \vartheta_{ot+1}(y_o - 1), \\ &= p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o - 1), 0\} + \Delta\vartheta_{ot+1}(y_o), \quad \text{and} \\ \Delta\vartheta_{ot}(y_o + 1) &= p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o + 1), 0\} + \vartheta_{ot+1}(y_o + 1) \\ &\quad - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} - \vartheta_{ot+1}(y_o), \\ &= p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o + 1), 0\} - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} + \Delta\vartheta_{ot+1}(y_o + 1). \end{aligned}$$

Then, we investigate $\Delta\vartheta_{ot}(y_o) \geq \Delta\vartheta_{ot}(y_o + 1)$ as follows;

$$\begin{aligned} &\Delta\vartheta_{ot}(y_o) \; - \; \Delta\vartheta_{ot}(y_o + 1) \\ &\quad = p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o - 1), 0\} + \Delta\vartheta_{ot+1}(y_o) \\ &\quad\quad - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o + 1), 0\} + p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} - \Delta\vartheta_{ot+1}(y_o + 1) \\ &\quad \geq p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} - p_{ot} \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o + 1), 0\} + \Delta\vartheta_{ot+1}(y_o) - \Delta\vartheta_{ot+1}(y_o + 1) \geq 0, \end{aligned}$$

where the first inequality follows from the induction assumption. For the second inequality, we consider two cases. We first investigate the case: $r_{ot} - \Delta\vartheta_{ot+1}(y_o) \geq 0$ and $r_{ot} - \Delta\vartheta_{ot+1}(y_o + 1) \geq 0$.

$$\begin{aligned} &p_{ot}(\max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o), 0\} - \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o + 1), 0\}) + \Delta\vartheta_{ot+1}(y_o) - \Delta\vartheta_{ot+1}(y_o + 1) \\ &\quad = p_{ot}(r_{ot} - \Delta\vartheta_{ot+1}(y_o) - r_{ot} + \Delta\vartheta_{ot+1}(y_o + 1)) + \Delta\vartheta_{ot+1}(y_o) - \Delta\vartheta_{ot+1}(y_o + 1) \\ &\quad = (1 - p_{ot})(\Delta\vartheta_{ot+1}(y_o) - \Delta\vartheta_{ot+1}(y_o + 1)) \geq 0, \end{aligned}$$

where the last inequality holds due to induction assumption. Given that $\vartheta_{ot+1}(y_o) \geq \vartheta_{ot+1}(y_o + 1)$, we also investigate the case where $r_{ot} - \Delta\vartheta_{ot+1}(y_o) \leq 0$ and $r_{ot} - \Delta\vartheta_{ot+1}(y_o+1) \geq 0$ as follows;

$$\begin{aligned}
p_{ot}(\max\{r_{ot} - &\Delta\vartheta_{ot+1}(y_o), 0\} - \max\{r_{ot} - \Delta\vartheta_{ot+1}(y_o+1), 0\}) + \Delta\vartheta_{ot+1}(y_o) - \Delta\vartheta_{ot+1}(y_o+1) \\
&= -p_{ot}(r_{ot} - \Delta\vartheta_{ot+1}(y_o+1)) + \Delta\vartheta_{ot+1}(y_o) - \Delta\vartheta_{ot+1}(y_o+1) \\
&= \Delta\vartheta_{ot+1}(y_o) - (1-p_{ot})\Delta\vartheta_{ot+1}(y_o+1) - p_{ot}r_{ot} \\
&= p_{ot}\Delta\vartheta_{ot+1}(y_o) + (1-p_{ot})\Delta\vartheta_{ot+1}(y_o) - (1-p_{ot})\Delta\vartheta_{ot+1}(y_o+1) - p_{ot}r_{ot} \geq 0.
\end{aligned}$$

The last inequality follows from the condition $r_{ot} - \Delta\vartheta_{ot+1}(y_o) \leq 0$. Following the induction assumption, it is clear that $\Delta\vartheta_{ot}(y_o) \geq \Delta\vartheta_{ot}(y_o+1)$. Therefore, the value function $\vartheta_{ot}(y_o)$ is discrete concave in y_o.

A.4 Proof of Proposition 4.2

The proof follows from the approximation bound result presented by Erdelyi and Topaloglu (2009). Assume that x_i^* and y_i^* denote the optimal amount of fixed and variable allotment capacity allocated to channel $i \in \mathcal{I}$ at the first review period $t = 1$ under the optimal solution of the binary programming model (18). Suppose that X_{it} and Y_{it} denote the number of reservations accepted for sales channel $i \in \mathcal{I}$ at time period t using fixed and variable allotment capacity under the optimal policy, respectively. Due to the feasibility restrictions in model (18), we have

$$\begin{array}{ll}
\sum_{t\in\mathcal{T}\setminus\mathcal{S}} \sum_{i\in\mathcal{I}} X_{it} + Y_{it} \leq C & \\
\sum_{t\in\mathcal{T}\setminus\mathcal{S}} X_{it} \leq x_i^* & i \in \mathcal{I} \\
\sum_{t\in\mathcal{T}\setminus\mathcal{S}} Y_{it} \leq y_i^* & i \in \mathcal{I}
\end{array}$$

where the first constraint ensures that the total number of accepted reservations do not exceed the capacity at time period $t = 1$. Similarly, the second and third constraints guarantee that fixed and variable capacity restrictions are satisfied, respectively. The total revenue under the optimal policy is

$$\sum_{t\in\mathcal{T}\setminus\mathcal{S}} \sum_{i\in\mathcal{I}} f_i X_{it} + \sum_{i\in\mathcal{I}} f_i(x_i^* - \sum_{t\in\mathcal{T}\setminus\mathcal{S}} X_{it}) + \sum_{t\in\mathcal{T}\setminus\mathcal{S}} \sum_{i\in\mathcal{I}} r_{it} Y_{it}.$$

Since the number of accepted customers is restricted by the total number of requests arrived for channel i over the time periods $t \in \mathcal{T}\setminus\mathcal{S}$, we obtain the total expected revenue under the optimal policy as follows:

$$\sum_{t\in\mathcal{T}\setminus\mathcal{S}} \sum_{i\in\mathcal{I}} f_i \mathbb{E}(X_{it}) + \sum_{i\in\mathcal{I}} f_i(x_i^* - \sum_{t\in\mathcal{T}\setminus\mathcal{S}} \mathbb{E}(X_{it})) + \sum_{t\in\mathcal{T}\setminus\mathcal{S}} \sum_{i\in\mathcal{I}} r_{it} \mathbb{E}(Y_{it})$$

This solution is feasible for the airline's dynamic programming model (1)-(3). However, the exact dynamic programming model considers capacity updates at the review periods and hence, the objective value of the binary programming model provides a lower bound on the optimal expected revenue.