Unravelling the Carbon Emissions Compliance in Sustainable Supply Chains: The Impacts of Carbon Audit Cooperation

Hongyong Fu^a, Yifeng Lei^a, Shuguang Zhang^{*b}, Kexin Zhao^c, Yanlu Zhao^d

^aResearch Center of Supply Chain Compliance Management, Southwest University of Political Science and Law, Chongqing, China b ^bCentre for Modern Logistics, National University of Singapore (Chongqing) Research Institute, Chongging, China c ^cUniversity of Bristol, Bristol, United Kingdom d Durham University Business School, Durham, United Kingdom

Abstract

As one of the 17 Sustainable Development Goals, reducing carbon emissions is crucial to combat climate change. It has also prompted companies to comply with emission regulations and evaluate the environmental impacts of their supply chains. Yet, news and reports occasionally highlight industrial instances of noncompliance. In particular, supplier's noncompliance is often mistakenly attributed to its downstream manufacturers. Due to this misconception, manufacturers might conduct audits to protect their reputation and sales. Moreover, because a supplier may provide components to multiple competing manufacturers, they may collaborate to share audit findings regarding the common supplier's compliance with carbon emissions regulations. However, studies do not reveal how this audit cooperation affects stakeholder interests. Here, we introduce a stylised model to examine the effects of carbon audit cooperation on the environment, competing manufacturers, and their supplier. We identify two main effects: the free-riding and amplifying effects. The former benefits the supplier but harms the environment and competing manufacturers, while the latter presents the opposite effect. The net impact depends on the balance between these two effects, which challenges conventional beliefs about carbon emissions compliance and highlights the importance of sustainability in the industry. Finally, we explore various extensions to validate the robustness of our findings.

Keywords: Low-carbon supply chain, competing manufacturers, carbon emissions compliance, carbon audit cooperation

1. Introduction

Carbon emissions are the primary contributor to global warming [\(Yang and Chen, 2018;](#page-30-0) [Taleizadeh et al., 2020a\)](#page-29-0). To reduce these emissions, numerous countries and regions have initiated various carbon-related regulations, including carbon emissions caps, cap-and-trade systems, and carbon taxes, to encourage emissions reduction [\(Taleizadeh et al., 2018;](#page-29-1) [Feng](#page-27-0) [et al., 2021\)](#page-27-0). In particular, the carbon cap regulation mandates that a regulatory authority stipulates an emissions cap for firms, who are then obligated to ensure their emissions do not exceed this threshold [\(Chen et al., 2020c\)](#page-27-1). Consequently, this regulation is perceived as a potent and enforceable measure to curtail carbon emissions [\(Ghosh et al., 2017;](#page-28-0) [An et al., 2021\)](#page-26-0).

For example, the US Congressional Budget Office recommends this regulation as a powerful mechanism to reduce carbon emissions [\(Qi et al., 2017\)](#page-29-2).

In recent years, academic interest in carbon cap regulation has surged. In particular, the operations management domain frequently integrates carbon caps into its framework to confine firms' carbon emissions. These models typically delve into determining optimal decisions encompassing optimal production quantities [\(Chen et al., 2020c;](#page-27-1) [An et al., 2021\)](#page-26-0), pricing strategies [\(Qi et al., 2017;](#page-29-2) [Xu et al., 2018\)](#page-30-1), inventory management [\(Benjaafar et al., 2012\)](#page-26-1), and recovery quantities [\(Chen et al., 2020c\)](#page-27-1). The aforementioned studies shed light on firms' decision-making processes in the context of carbon cap regulations. However, a prevailing assumption in these studies is that businesses earnestly curtail their emissions to align with the specified cap, which we term carbon emissions compliance.

While the carbon emissions compliance assumption is prevalent in extant research on the impact of carbon caps [\(Benjaafar et al., 2012;](#page-26-1) [Chen et al., 2020c\)](#page-27-1), its real-world validity is questionable. Instances of non-compliance are frequently reported in the news. For example, Beijing Hyundai Mobis Automotive Parts Co., Ltd., a prominent supplier of Hyundai Motor Company and Kia Motors Corporation, reportedly exceeded its emission limits^{[1](#page-0-0)}. Similarly, the North American cement producer and distributor, Lehigh Southwest Cement Company, has also exceeded its emission limits^{[2](#page-0-0)}. Why do production suppliers frequently breach emission regulations? A plausible explanation is the significant $CO₂$ emissions during production [\(Yu](#page-30-2) [et al., 2020\)](#page-30-2), coupled with suppliers' relatively unsophisticated carbon management capabilities [\(Lee and Choi, 2021\)](#page-28-1). Moreover, suppliers often find emission reductions to be financially burdensome, thereby resulting in cost-inefficiency to comply with caps [\(Shi et al., 2023\)](#page-29-3).

Even though downstream manufacturers (e.g., Hyundai and Kia) are not directly or legally responsible for their suppliers' carbon emissions noncompliance, such as Beijing Hyundai Mobis, manufacturers should take appropriate measures to rectify such deviations. Once a supplier's noncompliance is revealed, the public may believe that downstream manufacturers have failed to effectively manage the supplier [\(Hartmann and Moeller, 2014;](#page-28-2) [Hoffmann et al., 2020\)](#page-28-3). Such negative perceptions can adversely affect manufacturers' sales and reputation. For instance, Toyota was criticised by third-party organisations for its supplier's carbon emissions noncom-pliance^{[3](#page-0-0)}.

Among the new environmental initiatives, carbon audit can validate carbon emissions disclosures and ensure compliance with emission standards [\(Zhang et al., 2020;](#page-30-3) [Zhang, 2020\)](#page-30-4). Directly monitoring carbon emissions is difficult [\(Baksi and Bose, 2007\)](#page-26-2). Furthermore, acquiring carbon emissions information from products is less straightforward [\(Murali et al., 2019\)](#page-29-4). Hence, carbon audit has emerged as an effective solution to address emissions noncompliance. For instance, companies such as L'Oréal and Walmart have adopted carbon audit to assess their suppliers' carbon management performance^{[4](#page-0-0)}. In a supply chain with one supplier (e.g., Beijing

¹https://topic.alibabacloud.com/article/5-units-in-microsoft-china-are-fined-for-exceeding-carbonemissions 2 7 676014.html

²https://bracewell.com/insights/companies-fined-ghg-emissions-reporting-errors

³https://wwwoa.ipe.org.cn/Upload/201610180523002267.pdf

 4 http://www.tanpaifang.com/tanguwen/2018/0523/62019_4.html

Hyundai Mobis) and two competing manufacturers (e.g., Hyundai and Kia), carbon audit can assist manufacturers in ensuring that their suppliers comply with emissions standards.

In terms of carbon audit actions, these manufacturers usually have the following strategic options: they can either work independently on carbon audit or cooperate. Cooperation means that manufacturers share the carbon audit results. This ensures that if the supplier fails the carbon audit inspection even once, all parties are aware about noncompliance and the supplier faces consequences from all manufacturers. Although cooperation can deter suppliers more effectively, we do not know if two rival manufacturers may be open to such cooperation. To elucidate carbon audit strategies and their effects on a supplier's compliance with carbon emissions, we seek to answer the following pivotal question: Is cooperation on carbon audit among competing manufacturers simultaneously advantageous for the environment, manufacturers, and supplier? We approach this decision-making problem as a stylised model and evaluate the equilibrium outcomes under the given circumstances. We offer both theoretical and practical insights by answering this key research question.

1.1. Theoretical Contributions

We develop a stylised model to investigate the implications of carbon audit cooperation between rival manufacturers. Our primary objective is to understand the impacts of such cooperation on various stakeholders, namely, the environment, manufacturers, and their suppliers.

First, we demonstrate that carbon audit cooperation among competing manufacturers might potentially have either detrimental or beneficial effects on the environment. The net effect depends on two critical factors: the free-riding and amplifying effects. The free-riding effect, extensively discussed in the literature (e.g., [Li et al., 2021;](#page-28-4) [Du et al., 2023\)](#page-27-2), elucidates a scenario wherein manufacturers collaborate on carbon audit, allowing one manufacturer to leverage its rival's findings without incurring costs. This dynamic disincentivises each manufacturer from investing in higher carbon audit levels, thereby encouraging the supplier to reduce its carbon emissions compliance level. Consequently, this effect can lead to detrimental outcomes for the environment through carbon audit cooperation. Meanwhile, the amplifying effect, as discussed in [Caro et al.](#page-27-3) [\(2018\)](#page-27-3), highlights that if the supplier fails the carbon audit inspection even once, all parties know about it; this leads to penalties from both manufacturers. This effect can positively impact the environment. Moreover, this deterrence compels the supplier to invest in higher carbon emissions compliance level even if manufacturers opt for a lower carbon audit level under carbon audit cooperation.

Second, carbon audit cooperation among competing manufacturers may negatively impact both parties. Although manufacturers enjoy the amplifying effect, they also suffer from the free-riding effect. Hence, the impact of carbon audit cooperation on a particular manufacturer can be beneficial or detrimental, depending on the interaction of two effects.

Third, carbon audit cooperation may provide both advantages and disadvantages to suppliers also owing to the interplay between the free-riding and amplifying effects. Interestingly, the impact of carbon audit cooperation on the environment is typically opposite that on the suppliers. Thus, if the environment benefits from cooperation, suppliers may face losses, and vice versa. To overcome this, we identify the specific conditions under which a straightforward

transfer payment contract (TPC) can facilitate a triple win scenario. Consequently, all stakeholders can concurrently reap the benefits of carbon audit cooperation, leading to a Pareto improvement.

Finally, we provide several noteworthy insights. Regardless of the manufacturers' decision on cooperation, the optimal carbon emissions compliance level first increases and then decreases as the market size expands. Intriguingly, manufacturers' profits may increase in line with the financial loss caused by suppliers' carbon emissions noncompliance. This paradox arises because both the carbon audit levels set by manufacturers and carbon emissions compliance levels determined by suppliers generally increase with the loss, indirectly profiting the manufacturers. Thus, under specific conditions, manufacturers may experience greater profits despite financial loss.

1.2. Managerial Insights

We provide valuable managerial insights for manufacturers, suppliers, and governments. First, competing manufacturers should exercise caution when they are considering cooperation on carbon audit, especially if they share a common supplier. This cooperation may trigger the free riding effect, potentially leading the supplier to reduce its carbon emissions compliance, thereby harming the manufacturers. Second, managers of suppliers who simultaneously supply parts to competing manufacturers should be vigilant about the possibility of carbon audit cooperation among these manufacturers when deciding the carbon emissions compliance level. Finally, governments should monitor carbon audit cooperation among manufacturers because such cooperation may negatively impact the environment. Regulators should implement effective measures to mitigate this negative effect and protect the environment.

The remainder of this paper is organised as follows. Section 2 reviews the related literature and identifies the research gaps. Section 3 describes the proposed model and presents our basic assumptions. Section 4 deduces the equilibrium decisions and profits in both the noncooperative and cooperative contexts. Section 5 presents a comparative analysis of these two scenarios. Section 6 discusses three directions for extending the base model and presents one contract that achieves Pareto improvement. Finally, Section 7 presents the conclusions and recommendations for future research. All proofs are provided in the Appendix.

2. Literature review

This study is closely related to the literature on carbon policies, supply chain emissions reduction, supplier compliance management, and supply chain audit. We first review the related literature and highlight potential research gaps at the end of this section.

2.1. Carbon Policies

Regulators have enacted various policies related to low-carbon targets to control emissions [\(Chen et al., 2020c\)](#page-27-1). According to [Waltho et al.](#page-30-5) [\(2019\)](#page-30-5) and [Ghosh et al.](#page-28-5) [\(2020\)](#page-28-5), existing carbon policies can be grouped into three primary categories: carbon tax, carbon cap-and-trade, and carbon cap (also called mandatory carbon emissions capacity). Under a carbon tax, firms are required to pay for their carbon emissions. This payment is usually linearly proportional to

the number of carbon units emitted [\(Ghosh et al., 2020\)](#page-28-5). Notably, firms' carbon emissions are not strictly controlled under a carbon tax [\(Benjaafar et al., 2012\)](#page-26-1). Thus, firms do not face the noncompliance problem. Next, under carbon cap-and-trade, regulators assign a carbon quota to firms (also called a carbon emission cap); if their actual emissions are more (less) than this quota, additional quotas can be purchased (sold) from the carbon trade market [\(Xu et al., 2022\)](#page-30-6). Although this policy restricts firms' emissions by a quota, firms can purchase additional quotas if their actual emissions exceed the allocated quota, which mitigates the risk of carbon emissions noncompliance. However, under a carbon cap, the regulatory authority sets an emissions cap and firms must maintain their actual emissions within this cap [\(Chen et al., 2020c;](#page-27-1) [Ghosh](#page-28-5) [et al., 2020\)](#page-28-5). Crucially, firms' actual emissions may exceed the threshold, resulting in carbon emissions noncompliance, which motivates us to investigate carbon emissions noncompliance behavior in supply chain members.

2.2. Supply Chain Emissions Reduction

In contemporary supply chains, an increasing number of companies have recognised the significance of reducing the adverse environmental impacts of their operations. Consequently, they have undertaken substantial efforts to control the carbon emissions from operations [\(Das](#page-27-4) [and Jharkharia, 2018;](#page-27-4) [Heydari et al., 2023\)](#page-28-6). The low-carbon regulations enacted by governments have emerged as primary catalysts, prompting corporations to diminish their carbon footprints [\(Feng et al., 2021\)](#page-27-0). Typical regulations include carbon caps, taxes, and cap-and-trade systems [\(Waltho et al., 2019\)](#page-30-5). A carbon cap implies that the government allocates a specific emissions quota to each enterprise, mandating that their emissions remain below this threshold [\(An](#page-26-0) [et al., 2021\)](#page-26-0). Given its efficacy in real-world applications, this mechanism has not only gained substantial attention within the industry but has also garnered significant academic attention towards optimising emission-related decisions [\(Chen et al., 2020c\)](#page-27-1).

Numerous researchers have examined optimal decision-making strategies aimed at curbing carbon emissions within a supply chain, such as determining the optimal order quantity and reorder points [\(Ghosh et al., 2017\)](#page-28-0), inventory management [\(Benjaafar et al., 2012;](#page-26-1) [Dye and](#page-27-5) [Yang, 2015\)](#page-27-5), production volume [\(An et al., 2021\)](#page-26-0), and pricing models [\(Qi et al., 2017\)](#page-29-2). All these elements operate within the boundaries of a predetermined carbon cap. Furthermore, once carbon emissions are restricted to a stipulated cap, scholars have explored incentive mechanisms that foster collaboration among supply chain entities to further reduce carbon emissions. [Ren](#page-29-5) [et al.](#page-29-5) [\(2015\)](#page-29-5) examine the allocation process to reduce carbon emissions in a supply chain to meet at the strict carbon cap. Meanwhile, [He et al.](#page-28-7) [\(2019\)](#page-28-7) highlight that an effective balance between profitability and emissions control can only be achieved if the carbon cap is set at an intermediate level rather than being overly stringent. To bolster collaborative endeavours, [Xu et al.](#page-30-1) [\(2018\)](#page-30-1) propose a coordination mechanism tailored for dual-channel supply chain stakeholders. [Ding et al.](#page-27-6) [\(2016\)](#page-27-6) examine at government policy incentive mechanism that motivates supply chain members to work together towards reducing carbon emissions.

A prevailing assumption in the extant research on emissions reduction in supply chains under a carbon cap is that primary emission contributors should operate within the defined carbon emission thresholds, commonly referred to carbon emissions compliance. However, instances of noncompliance have been highlighted, such as those related to Hyundai and Kia suppliers. This prompts us to question the ubiquitously accepted notion of carbon emissions compliance and explore the consequences of noncompliance behaviours on supplier emissions.

2.3. Supplier Compliance Management

Supplier compliance management is intrinsically linked to the notion of chain liability, as [Hartmann and Moeller](#page-28-2) [\(2014\)](#page-28-2) indicated. Supply chain liability underscores the consumer perception that companies should shoulder responsibility for their suppliers' noncompliance, which may lead consumers to boycott these companies [\(Hartmann and Moeller, 2014\)](#page-28-2). To mitigate these consequences, firms should adopt proactive strategies to bolster supplier compliance [\(Caro et al., 2018\)](#page-27-3). [Chen and Lee](#page-27-7) [\(2017\)](#page-27-7) state that supplier noncompliance can be divided into two categories: material and process noncompliance.

Material noncompliance pertains to issues such as the use of subpar materials or lead paint in children's products [\(Chen and Lee, 2017\)](#page-27-7). Such violations can be identified via product inspections or tests and treated akin to conventional quality control issues [\(Babich and](#page-26-3) [Tang, 2012\)](#page-26-3). An extensive literature has explored these issues, including studies on inspection modalities to curb supplier product adulteration [\(Rui and Lai, 2015\)](#page-29-6), the relationship between investment in quality and risk of economically motivated adulteration [\(Levi et al., 2020\)](#page-28-8), and overall supplier management [\(Babich and Tang, 2012;](#page-26-3) [Mu et al., 2016;](#page-29-7) [Lee and Li, 2018\)](#page-28-9). Our study does not fall under the scope of material noncompliance as it pertains to carbon emissions, which does not impact the product's physical quality. Meanwhile, process noncompliance encompasses practices such as child labor, substandard working conditions, and mishandling of emissions [\(Chen and Lee, 2017\)](#page-27-7). Such violations are elusive and not easily discernible through product inspections or tests [\(Chen et al., 2020b\)](#page-27-8). In this regard, we perceive this study as a novel example respect to the process noncompliance.

2.4. Supply Chain Audit

Auditing is a systematic process in which evidence is collected and compared using specific criteria to determine compliance. Its primary goal is often to address noncompliance activities [\(Zarei et al., 2020\)](#page-30-7). Some studies indicate that downstream firms can deter suppliers from employing child labour either by providing higher wholesale prices or merging process audits with moderate wholesale prices [\(Cho et al., 2019\)](#page-27-9). Several studies have proposed audit mechanisms for ameliorating working conditions in supplier factories (e.g., [Caro et al., 2018;](#page-27-3) [Zarei](#page-30-7) [et al., 2020\)](#page-30-7). However, unintended consequences may still happen. For instance, if suppliers conceal vital information, downstream companies' audit can be counterproductive [\(Plambeck](#page-29-8) [and Taylor, 2016\)](#page-29-8). Recognising this challenge, researchers have proposed ways to bolster audit efficacy. [Chen et al.](#page-27-8) [\(2020b\)](#page-27-8) construct a game-theoretical model to delve into the effects of supplier-auditor collusion on downstream firms' audit and contracting strategies. The role of audit in increasing supplier compliance, particularly within intricate supply chains, has been another prominent research area [\(Caro et al., 2018;](#page-27-3) [Chen et al., 2020a;](#page-27-10) [Fang and Cho, 2020\)](#page-27-11). Nevertheless, [Caro et al.](#page-27-3) [\(2018\)](#page-27-3) assume that firms neither incentivise suppliers to improve safety nor consider sourcing quantity as a variable. However, in practice, corporations such as Nike and Starbucks have both instituted incentive programs aimed at fostering supplier compliance [\(Porteous et al., 2015\)](#page-29-9) and actively decide on sourcing quantities.

Departing from this literature, we explore the potential benefits of carbon audit cooperation between two competing manufacturers in mitigating supplier carbon emissions noncompliance. We consider both the implementation of an incentive scheme and endogeneity of sourcing quantity. To the best of our knowledge, this aspect of managing supplier carbon emissions noncompliance remains unexplored.

2.5. Research gaps

To distinctly position our study, the differences between our work and comparable existing studies are presented in Table 1.

Table 1: Comparison of this study with related merature.				
Research paper	Supply chain structure	Competition	Carbon Emissions noncompliance	Cooperation on audit
Qi et al. (2017)	$1+2$	Yes		
Rui and Lai (2015)	$1+1$			
Plambeck and Taylor (2016)	$1+1$			
Caro et al. (2018)	$1+2$			Yes
Cho et al. (2019)	$1 + 1$			
Chen et al. $(2020b)$	$1+1$			
Chen et al. $(2020a)$	$3+2$	Yes		
Fang and Cho (2020)	$1+n$			
Zhang et al. (2020)				
This study	$1+2$	${\rm Yes}$	Yes	Yes

Table 1: Comparison of this study with related literature.

Our study differs from research on supply chain carbon emissions reduction in two aspects. First, we focus on a scenario where the primary source of emissions, that is, the supplier, may not comply with carbon-emission targets. In contrast, studies have concentrated on compliance with the carbon emissions target [\(Qi et al., 2017\)](#page-29-2). Our objective is to enhance the supplier's carbon emissions compliance level. Second, supplier compliance management research has typically focused on a single supplier and one downstream firm (e.g., [Rui and Lai, 2015;](#page-29-6) [Lee](#page-28-1) [and Choi, 2021;](#page-28-1) [Cho et al., 2019;](#page-27-9) [Chen et al., 2020b\)](#page-27-8). In this regard, we examine a supply chain with one supplier and two downstream firms, where the two firms are competitors and make separate order quantity decisions.

3. Model setup

Consider a supply chain with one component supplier (denoted by s) and two competing manufacturers (denoted by m_1 and m_2 , respectively). The manufacturers procure components from the supplier to assemble end products. Although imperfectly substitutable, these products are sold in the same market. Specifically, while manufacturers use systematic schemes and advanced technology to minimise carbon emissions, the supplier is less adept at controlling its emissions [\(Lee and Choi, 2021\)](#page-28-1). Consequently, the supplier is more inclined towards carbon emissions noncompliance than the manufacturers. However, from the public's perspective, such noncompliance from the supplier is often perceived as the manufacturers' failure to manage their suppliers. This perception can adversely affect manufacturers' reputation and sales. To reduce the risk of supplier-induced noncompliance, manufacturers may incorporate audit into their collaborations. Following previous research [\(Plambeck and Taylor, 2016;](#page-29-8) [Caro et al., 2018;](#page-27-3) [Lee](#page-28-9) [and Li, 2018;](#page-28-9) [Liu et al., 2022\)](#page-28-10), we adopt a probability model to quantify the decisions of supply chain members and construct their profit functions.

3.1. Supplier's Carbon Emissions Compliance

As mentioned previously, complying with the carbon emissions cap is costly for the supplier. Hence, the supplier must decide whether to invest capital to adhere to the carbon emissions limit. Let $E = c$ indicate that the supplier is in compliance with carbon emissions and $E = c$ n indicate that the supplier's carbon emissions exceed the cap. As carbon emissions exist throughout the production process, it would be difficult and also costly for the supplier to comply with carbon emissions requirement completely. However, the supplier can undertake more effort (e.g., upgrading equipment or adopting sophisticated management tools) to improve the possibility of carbon emissions compliance. We characterise this possibility of complying with the carbon emissions cap as $e = \mathbb{P}(E = c)$, such that $e \in [0, 1]$. By contrast, the possibility of carbon emissions noncompliance is $1 - e$ (i.e., denoted as $\mathbb{P}(E = n)$). Following studies on supplier compliance management (e.g., [Plambeck and Taylor, 2016;](#page-29-8) [Bondareva and Pinker,](#page-26-4) [2019;](#page-26-4) [Fang and Cho, 2020\)](#page-27-11), e can be interpreted as the carbon emissions compliance level. Intuitively, environmental benefits can be positively related to carbon emissions compliance levels. That is, improving the carbon emissions compliance level is beneficial to the environment, and vice versa.

Besides, investing in carbon emissions compliance will have diseconomies of scale or dimin-ishing returns [\(Lee and Park, 2020\)](#page-28-11). Thus, the carbon emissions compliance cost, $C(e)$, has the properties that $\frac{\partial C(e)}{\partial e} > 0$ and $\frac{\partial^2 C(e)}{\partial e^2} > 0$. Furthermore, we adopt a quadratic cost function, $C(e) = \frac{ke^2}{2}$, which satisfies the properties of carbon emissions compliance cost and has been widely used in the literature (e.g., [Chen et al., 2020b;](#page-27-8) [Lee and Park, 2020;](#page-28-11) [He et al., 2021;](#page-28-12) [Taleizadeh et al., 2021\)](#page-30-8), where $k \in \mathbb{R}^+$ is the supplier's cost coefficient of the carbon emissions compliance effort.

3.2. Manufacturers' Carbon Emission Audit

As the supplier may face carbon emissions noncompliance, to mitigate the risk, manufacturers will audit whether the supplier is in compliance. For $i \in \{1,2\}$, let $A_i = p$ indicate that the supplier passes the audit of manufacturer *i*; otherwise, $A_i = f$. In practice, a manufacturer may find it challenging to completely identify a supplier's carbon emissions noncompliance by relying only on audit. Hence, the manufacturer may not be able to detect the supplier's noncompliance if it exists. Still, the manufacturer can invest more effort in improving its audit level which may increase the probability of the supplier failing the manufacturer's audit if supplier truly exhibits the carbon emissions noncompliance. Thus, the higher the manufacturer i's audit level, the lower the probability of the supplier (if in carbon emissions noncompliance) can pass the manufacturer i's carbon audit. Hence, the audit level can be described as $\mathbb{P}(A_i = f | E = n)$, which indicates the probability that the supplier fails to pass the carbon audit if it is in carbon emissions noncompliance. Furthermore, to facilitate the analysis, let a_i represent the audit level (i.e., $a_i = \mathbb{P}(A_i = f | E = n)$ and $a_i \in [0, 1]$). Meanwhile, $1 - a_i = \mathbb{P}(A_i = p | E = n)$ indicates

that the supplier's carbon emissions noncompliance is not identified such that the manufacturer i's carbon audit fails. Further, it is also reasonable to assume that the supplier will always pass all audits if its carbon emissions comply with the cap, namely, $\mathbb{P}(A_i = p | E = c) = 1$.

Similar to the cost structure of carbon emissions compliance effort, carbon audits also suffer from diseconomies of scale or diminishing returns. Therefore, we adopt a quadratic cost function, $C(a_i) = \frac{h_i a_i^2}{2}$, to characterise manufacturer *i*'s audit efforts/costs, where $h_i \in \mathbb{R}^+$ is the manufacturer i 's cost coefficient by implementing the carbon audit. Because we focus on the interaction between carbon emissions compliance and competing manufacturers' audit cooperation throughout the analysis, thus, it is reasonable to assume that the two competing manufacturers have symmetric cost coefficients of carbon audit, that is, $h = h_i, i \in \{1, 2\}$. This assumption has been adopted widely in previous studies (e.g., [Lee and Li, 2018;](#page-28-9) [Fang and Cho,](#page-27-11) [2020\)](#page-27-11).

For ease of exposition, Figure [1](#page-8-0) illustrates the theoretical gaming decisions made throughout the carbon audit procedure. Specifically, if the supplier is compliant with carbon emissions, it will successfully pass the manufacturers' carbon audit. However, if the supplier exhibits carbon emissions noncompliance, it may pass the manufacturer i 's carbon audit with probability a_i and fail to pass the audit with probability $1 - a_i$.

Figure 1: The game model of the carbon audit and $i \in \{1,2\}$

3.3. Profit Function

If the supplier passes manufacturer is carbon audit, the manufacturer will source q_i units of components from the supplier at a unit price w , where w is an exogenously given industry standard price [\(Sim and Kim, 2021\)](#page-29-10). Note that an exogenous sourcing price is a common assumption in supplier compliance management studies (e.g., [Plambeck and Taylor, 2016;](#page-29-8) [Caro](#page-27-3) [et al., 2018;](#page-27-3) [Chen et al., 2020b;](#page-27-8) [Liu et al., 2022\)](#page-28-10). Without loss of generality, we assume that manufacturer i requires one unit of component to assemble one unit of end product and that the manufacturer i's assembly cost is negligible. Similar to [Chen et al.](#page-27-1) [\(2020c\)](#page-27-1), [Lee and Park](#page-28-11) [\(2020\)](#page-28-11), and [He et al.](#page-28-12) [\(2021\)](#page-28-12), we model the market-clearing price, p_i , for manufacturer $i \in \{1, 2\}$ to be linear in production quantities: $p_i = \alpha - q_i - \beta q_{3-i}$, where $\alpha \in \mathbb{R}^+$ is the basic market size and $\beta \in (0,1)$ measures the *competitive intensity* between two manufacturers. In §6.4, we investigate an alternative case in which the demand is uncertain and show that our analytical results are robust.

Note that if the supplier has a noncompliance problem with carbon emissions and passes manufacturer *i*'s carbon audit, the manufacturer will incur a financial loss, L_i . Referring to the literature (e.g., [Lee and Li, 2018;](#page-28-9) [Sim et al., 2019;](#page-29-11) [Fang and Cho, 2020\)](#page-27-11) and focusing on

the impacts of competing manufacturers' carbon audit cooperation, we define $L = L_i$ for all $i \in \{1,2\}$. If the supplier fails to pass manufacturer is carbon audit, manufacturer i does not source any components from the supplier.

Each manufacturer offers an incentive scheme to the supplier to improve its compliance level [\(Porteous et al., 2015\)](#page-29-9). Hence, we consider an incentive mechanism based on the carbon audit outcome. Specifically, when the supplier passes manufacturer i 's carbon audit, the manufacturer offers a reward R_i to the supplier. Otherwise, manufacturer i imposes a penalty F_i on the supplier. Under different reward levels (penalty), the supplier has an incentive to request a higher reward (or lower penalty) from a manufacturer than its rival offers, to motivate the manufacturer to increase the reward (or lower the penalty). Given these bargaining dynamics, according to [Caro et al.](#page-27-3) [\(2018\)](#page-27-3), we have $R_i = R$ and $F_i = F$. For ease of analysis, we explicitly define *incentive intensity* as a summation of the penalty and the reward, that is, $I = R + F$. Next, we separately introduce the profit functions under cooperative and noncooperative carbon audit.

3.3.1. Non-cooperative Carbon Audit

The non-cooperative (denoted by NC in the superscript) case indicates that the two manufacturers do not share their carbon audit results with each other. Therefore, even if the supplier fails to pass one manufacturer's carbon audit, it may pass the other manufacturer's carbon audit. In this case, the profit functions of the supplier and manufacturers are described as follows.

First, the supplier's profit is given by:

$$
\pi_s^{NC} = \underbrace{\sum_{i=1}^2 \left(e\left((w-c) \, q_i + R \right) + (1-e) \, a_i \left(-cq_i - F \right) + (1-e) \left(1 - a_i \right) \left((w-c) \, q_i + R \right) \right)}_{\text{revenue}} - \underbrace{\frac{1}{2} k e^2}_{\text{compliance cost}} \tag{1}
$$

The supplier's profit function consists of two terms: revenue and carbon emissions compliance cost. The revenue term is further decomposed into revenue when carbon emissions are compliant, revenue when carbon emissions noncompliance is not detected by manufacturer i , and loss when carbon emissions noncompliance is identified by manufacturer i.

Moreover, manufacturer i 's profit is given by:

$$
\pi_{m_i}^{NC} = \underbrace{e((p_i - w) q_i - R) + (1 - e) a_i F + (1 - e) (1 - a_i) ((p_i - w) q_i - R - L)}_{\text{revenue}} - \underbrace{\frac{1}{2} h a_i^2}_{\text{audit cost}} \tag{2}
$$

which also includes the *revenue* and *carbon audit cost* terms, respectively. The revenue part consists of the revenue under carbon emissions compliance, revenue when carbon emissions noncompliance is detected by manufacturer i , and loss when carbon emissions noncompliance is not identified by manufacturer i.

3.3.2. Cooperative Carbon Audit

The cooperative (denoted by C in the superscript) case indicates that manufacturers share their carbon audit results with the rival. Therefore, if the supplier fails to pass manufacturer i's carbon audit, the supplier also fails to pass manufacturer $3-i$'s carbon audit for any $i \in \{1,2\}$.

In this case, the profit functions of the supplier and manufacturers can be characterised as follows.

The supplier's profit function is given by:

$$
\pi_s^C = \underbrace{\sum_{i=1}^2 e((w-c)q_i + R) + (1-e)(1-a_i)(1-a_{3-i})((w-c)q_i + R)}_{\text{revenue of compliance and non-detected noncompliance}} - \underbrace{\sum_{i=1}^2 ke^2}_{\text{compliance cost}} + \underbrace{\sum_{i=1}^2 e^{2i} + e^{2i}}_{\text{revenue of detected noncompliance}} + \underbrace{\sum_{i=1}^2 e^{2i}}_{\text{revenue of detected noncompliance}} +
$$

The supplier's profit function comprises the following three parts. In the first part, the first term is the revenue when carbon emissions are compliant; meanwhile, the second term is the revenue when carbon emissions noncompliance is not detected by either manufacturer. The second part is the carbon emissions compliance cost. In the third part, the first term is the revenue when carbon emissions noncompliance is detected by the two manufacturers simultaneously; the second term is the revenue when carbon emissions noncompliance is only detected by manufacturer i ; and the third term is the revenue when carbon emissions noncompliance is only detected by manufacturer 3−i.

The profit function of manufacturer i is given by:

$$
\pi_{m_i}^C = \underbrace{e((p_i - w) q_i - R) + (1 - e) (1 - a_i) (1 - a_{3-i}) ((p_i - w) q_i - R - L)}_{\text{revenue of compliance and non-detected noncompliance}}
$$
\n
$$
\underbrace{(1 - e) (a_i a_{3-i} + a_i (1 - a_{3-i}) + (1 - a_i) a_{3-i})F}_{\text{revenue of detected noncompliance}} - \underbrace{\frac{1}{2} h a_i^2}_{\text{audit cost}}
$$
\n
$$
(4)
$$

The profit function of manufacturer i comprises three parts. In the first part, the first term is the revenue under carbon emissions compliance; and the second term is the revenue when carbon emissions noncompliance is not detected by either manufacturer. In the second part, the first term is the revenue when carbon emissions noncompliance is simultaneously detected by the two manufacturers; the second term is the revenue when carbon emissions noncompliance is detected by manufacturer i ; and the third term is the revenue when carbon emissions noncompliance is detected by the rival manufacturer. The third part is the carbon audit cost.

Throughout this study, we also make the following assumption regarding the model parameters:

Assumption 1. $L > (p_i - w) q_i$

Assumption 1 indicates that the financial loss L should exceed the selling profit of manufacturer i ; otherwise, manufacturer i does not have sufficient incentive to implement the carbon audit. Therefore, it is a reasonable assumption in practical contexts.

3.4. Sequence of the Game

The decision procedure is described into the following two stages: In Stage 1, as the carbon emissions compliance and carbon audit levels are difficult for other firms to observe, we assume that the supplier and each manufacturer make their own decisions independently and simultaneously. Specifically, the supplier determines the carbon emissions compliance level

 e , while each manufacturer decides its own carbon audit level a_i . An alternative sequence is that the supplier decides the carbon emissions compliance level first, while the manufacturers simultaneously set their carbon audit levels after observing the supplier's decision. We study such a sequential game in §6.3 and show that our baseline results are robustly preserved in this extension model. In Stage 2, each manufacturer determines the quantity of components purchased from the supplier and then assembles the components into end products which are sold to consumers if the supplier passes the carbon audit.

For ease of understanding, the key notations are summarised in Table 2.

4. Equilibrium Analysis

In this section, we analyse how carbon audit cooperation among competing manufacturers affects each firm's profit and the environment.

4.1. Non-cooperative Carbon Audit

Under no carbon audit cooperation between two competing manufacturers, each manufacturer audits the supplier's carbon emissions separately and does not share the audit information with the other. Given each firm's profit function presented in $\S 3.3.1$, the equilibrium outcomes are summarised in the following proposition.

Proposition 1. Suppose the two competing manufacturers do not cooperate on carbon audit, (a) the optimal decisions of manufacturer i and the supplier are given by $q_i^{NC*} = \frac{\alpha - w_i}{2 + \beta}$ $\frac{\alpha-w}{2+\beta}$, $a_i^{NC*} =$ $\frac{k(L-V+I)}{2(G+I)(L-V+I)+hk}$, and $e^{NC*} = \frac{2(G+I)(L-V+I)}{2(G+I)(L-V+I)+hk}$; (b) the optimal profit of each firm is given by $\pi_{m_i}^{NC*} = V - R - (1 - e^{NC*}) a_i^{NC*} \times (V - I)$ $(1-e^{NC*}) (1-a_i^{NC*}) L - \frac{1}{2}$ $\frac{1}{2}h\big(a_i^{NC*}\big)^2$, and $\pi_s^{NC*} = 2\left(G - cq_i^{NC*} + R\right) - ke^{NC*} + \frac{1}{2}$ $\frac{1}{2}k\left(e^{NC*\right)^2}$

where $V = \frac{(\alpha - w)^2}{(2 + \beta)^2}$ $\frac{(\alpha-w)^2}{(2+\beta)^2}$ and $G = \frac{(\alpha-w)w}{2+\beta}$ $\frac{\ell-w}{2+\beta}$.

From Proposition [1,](#page-11-0) the optimal sales quantity (sourcing quantity) of manufacturer i decreases with competitive intensity but is independent of the compliance cost coefficient k and carbon audit cost coefficient h. That is, the carbon emissions compliance and carbon audit levels do not affect the manufacturers' sales quantity.

Interestingly, the optimal carbon audit level decreases with the basic market size, which aligns with the trade-off related to the costs that a manufacturer pays for (successful or unsuccessful) audit when the supplier exhibits carbon emissions noncompliance. Specifically, on the one hand, if the manufacturer cannot identify the supplier's carbon emissions noncompliance, it will incur a constant financial loss L and impose a constant penalty F on the supplier. On the other hand, the manufacturer will lose the overall profit which is increasing in the basic market size if it successfully detects supplier's carbon emissions noncompliance. That is, the manufacturer's loss from successfully auditing the supplier's carbon emissions noncompliance increases with the basic market size, but only has a constant loss from unsuccessfully auditing the supplier's carbon emissions noncompliance. Thus, it is strongly motivated to reduce the audit level when the basic market expands. This conclusion is summarised in Corollary $1(a)$.

Corollary 1. Consider no carbon audit cooperation between the two competing manufacturers. Then, the optimal audit level of manufacturer i (a) decreases with the basic market size, α , and (b) increases with the competitive intensity, β .

Corollary [1](#page-12-0) also states that manufacturers' audit levels increase with competitive intensity, which is also consistent with the costs that the manufacturer pays for successful or unsuccessful audit when the supplier exhibits carbon emissions noncompliance. As competitive intensity increases, the manufacturers suffer lower profit from the noncompliant supplier passing the carbon audit. Thus, each manufacturer has an incentive to improve its audit level to identify the supplier's carbon emissions noncompliance, which reduces the possibility that the noncompliant supplier passes the audit inspection and helps the manufacturers reduce their losses. In other words, Corollary [1](#page-12-0) reveals that manufacturers may cover its supplier's carbon emissions noncompliance and that manufacturers should set a high audit level under a competitive environment.

Next, we examine the impact of the basic market size and the competitive intensity on the supplier's optimal carbon emissions compliance level. Corollary [1](#page-12-0) shows that the optimal audit levels decrease with the basic market size, which motivates the supplier to reduce its carbon emissions compliance level. Meanwhile, the supplier's loss increases with the basic market size once its carbon emissions noncompliance is discovered, thereby pushing the supplier to improve its carbon emissions compliance level. In addition, when the basic market size is small, the benefit from improving the carbon emissions compliance level dominates the benefit from reducing it; thus, the optimal carbon emissions compliance level increases with the basic market size. However, when the basic market size is large, the benefit from improving the carbon emissions compliance level is dominated by the benefit from lowering it; thus, the optimal carbon emissions compliance level decreases with the basic market size (See Corollary $2(a)$ $2(a)$).

Corollary 2. Suppose that there is no carbon audit cooperation between the two competing manufacturers. Then, there exist α_1 and β_1 , such that:

(a) if $\alpha \leq \alpha_1$, the optimal carbon emissions compliance level increases with the basic market size; otherwise, the optimal carbon emissions compliance level decreases with the basic market size if $\alpha > \alpha_1$;

(b) if $\beta \leq \beta_1$, the optimal carbon emissions compliance level increases with the competitive intensity; otherwise, the optimal carbon emissions compliance level decreases with the competitive intensity if $\beta > \beta_1$.

Corollary [2\(](#page-12-1)b) also indicates that the optimal carbon emissions compliance level increases first and then decreases with the competitive intensity. First, the optimal audit level increases with the competitive intensity, which motivates the supplier to improve its carbon emissions compliance level. However, the sourcing quantity of each manufacturer decreases with the competitive intensity. That is, the supplier's profit decreases with the competitive intensity, driving it to lower its carbon emissions compliance level. When competitive intensity is low (high), the motivation for the supplier to improve (lower) the carbon emissions compliance level dominates that for the supplier to lower (improve) carbon emissions compliance level.

Corollary 3. Suppose that there is no carbon audit cooperation between the two competing manufacturers. When the financial loss of each manufacturer increases, (a) the optimal audit and carbon emissions compliance levels increase; (b) the manufacturers' profits may increase; and (c) the supplier's profit decreases.

As the financial loss of manufacturer(s), L , increases, manufacturer *i*'s overall loss from unsuccessful audits of the supplier's carbon emissions noncompliance also increases, which increases its audit level. Conjecturing the manufacturer's behaviour, the supplier will also choose a higher carbon emissions compliance level as the financial loss increases (Corollary [3\(](#page-13-0)a)). Interestingly, from Corollary [3\(](#page-13-0)b), the manufacturers' profits may increase with the financial loss because the optimal carbon emissions compliance level is increasing in it. Increasing the carbon emissions compliance level will reduce the risk that one manufacturer is harmed by carbon emissions noncompliance and then (indirectly) benefit it, while the financial loss (directly) harms the manufacturer. Under certain conditions, the indirect benefit dominates the direct harm. Thus, the manufacturer's profit might increase with L. For the supplier, a higher L will lead to a higher carbon audit level and a higher carbon emissions compliance level, both of which are harmful to the supplier. Thus, the supplier's profit decreases in L , as indicated in the Corollary [3\(](#page-13-0)c).

4.2. Cooperative Carbon Audit

Under audit cooperation among competing manufacturers, each manufacturer is willing to share its audit results with the other. That is, once supplier carbon emissions noncompliance is detected by any manufacturer, both will have noncompliant information such that they will penalise the supplier by F simultaneously. Thus, the equilibrium outcomes under carbon audit cooperation are summarized as follows.

Proposition 2. Suppose that the manufacturers cooperate with each other on carbon audit, (a) the optimal decisions of manufacturer i and the supplier are $q_i^{C*} = \frac{\alpha - w_i}{2 + \beta}$ $\frac{\alpha-w}{2+\beta}$, and $e^{C*} =$ $2(G+I)$ $\frac{G^{i+1}}{k} a_i^{C*} (2 - a_i^{C*})$, where $a_i^{C*} \in (0,1)$ is the unique root of equation $f(a) = 0$ and $f(a) = 0$ $\left(1 - \frac{2(G+I)}{k}\right)$ $\frac{K+I}{k}a\left(2-a\right) \Bigl(1-a\right) \left(L-V+I\right) -ha;$

(b) the optimal profit of each firm is given by
$$
\pi_{m_i}^{C*} = V - R - (1 - e^{C*}) a_i^{C*} (2 - a_i^{C*}) (V - I) - (1 - e^{C*}) (1 - a_i^{C*})^2 L - \frac{1}{2} h (a_i^{C*})^2
$$
, and $\pi_s^{C*} = 2 (G - c q_i^{C*} + R) - k e^{C*} + \frac{1}{2} k (e^{C*})^2$.

Proposition [2](#page-13-1) examines each firm's optimal decision(s) and profit under audit cooperation. With the optimality condition, we continue to analyse the impact of the supply chain parameters on optimal decisions.

Corollary 4. Suppose that there is carbon audit cooperation between the two competing manufacturers. The optimal audit level of manufacturer $i(a)$ decreases with the basic market size α , and (b) increases with the competitive intensity β .

As indicated in Corollary [4,](#page-14-0) the optimal audit level of a manufacturer decreases in the basic market size α but increases with the competitive intensity β . The rationale is similar to that of Corollary [1.](#page-12-0) That is, the trade-off between the loss and benefit from audit causes the optimal audit level to decrease with the basic market size and increase with the competitive intensity.

Corollary 5. Suppose that there is carbon audit cooperation between the two competing manufacturers. Then, there exist α_2 and β_2 , such that:

(a) if $\alpha < \alpha_2$, the optimal carbon emissions compliance level increases with the basic market size; otherwise, the optimal carbon emissions compliance level decreases with the basic market size if $\alpha > \alpha_2$;

(b) if $\beta < \beta_2$, the optimal carbon emissions compliance level increases with the competitive intensity; otherwise, the optimal carbon emissions compliance level decreases with the competitive intensity if $\beta > \beta_2$.

Given Corollary [5,](#page-14-1) we know that the optimal carbon emissions compliance level increases first and then decreases with the basic market size (or competitive intensity). This is because the audit level decreases with the basic market size, which motivates the supplier to lower the carbon emissions compliance level. The increasing basic market size also leads to a higher loss from carbon emissions noncompliance. Thus, the motivation for the supplier to lower the carbon emissions compliance dominates the loss from carbon emissions noncompliance if the basic market size is low, and vice versa. In addition, carbon audit levels increase with competitive intensity, which incentivises the supplier to improve its carbon emissions compliance level. This motivation dominates the decreasing profit if competitive intensity is low, and vice versa.

Finally, the impact of financial loss under cooperative carbon audit is quite similar to that under non-cooperative case. That is, the optimal audit and carbon emissions compliance levels increase with financial loss, while the supplier's profit decreases with it. Determined by the indirect benefit from the carbon emissions compliance level increasing with financial loss and the direct loss from higher financial loss, the manufacturers' profits may actually increase with financial loss (Corollary [6\)](#page-14-2).

Corollary 6. Suppose that there is carbon audit cooperation between the two competing manufacturers. When the financial loss of each manufacturer increases, (a) the optimal audit and carbon emissions compliance levels increase; (b) the profit of each manufacturer may increase; and (c) the profit of the supplier decreases.

In summary, carbon audit cooperation between competing manufacturers does not significantly change the rationale for how supply chain design parameters affect each firm's optimal decision(s) and profit.

5. Comparative analysis

In this section, we compare the optimal decisions and profits of supply chain members, given that the manufacturers cooperate or not on carbon audit. We first define manufacturer i's carbon audit efficiency, $E_i = \frac{e}{a_i}$ $\frac{e}{a_i}$, as the return on the environmental benefits generated by manufacturer i 's investment at the carbon audit level.

Theorem 1. When comparing the optimal carbon audit level and efficiency between non-cooperative and cooperative scenarios, the following hold: (a) $a_i^{NC*} > a_i^{C*}$; and (b) $E_i^{NC*} < E_i^{C*}$.

Theorem [1\(](#page-15-0)a) indicates that the carbon audit cooperation is always beneficial for reducing the carbon audit level of an individual manufacturer, thus reducing the carbon audit cost. Specifically, if manufacturers cooperate by sharing audit information, both can take free-riding benefits from the other's auditing activity. That is, one manufacturer is still able to identify the supplier's carbon emissions noncompliance even if it originally fails to detect noncompliance. Considering the free-riding phenomenon and the fact that cooperation acts a deterrent to the supplier, manufacturer may be motivated to reduce investments in carbon audit. The implications of Theorem $1(a)$ is illustrated in Figure [2\(](#page-16-0)a). Notably, from Figure 2(a), we observe that both carbon audit levels under non-cooperative carbon audit and cooperative carbon audit decrease with incentive intensity. This is because the heightened incentive intensity motivates the supplier to increase its carbon emissions compliance level. Consequently, manufacturers decrease their carbon audit levels, regardless of whether they cooperate on carbon audit.

Theorem $1(b)$ shows that carbon audit cooperation has an *amplifying effect* on efficiency, which is caused by the *free-riding*. That is, with carbon audit cooperation, the 'unit' investment of carbon audit level by manufacturer i can always produce higher carbon emissions compliance level. Figure [2\(](#page-16-0)b) presents a graphical interpretation of Theorem [1\(](#page-15-0)b). In addition, Figure [2\(](#page-16-0)b) further illustrates that as the incentive intensity increases, carbon audit efficiency increases regardless of carbon audit cooperation; however, the amplifying effect under cooperation notably increases. In other words, the rate of increase in carbon audit efficiency is faster under carbon audit cooperation than that under non-cooperative carbon audit.

As shown in Theorem [1,](#page-15-0) carbon audit cooperation leads to free-riding and amplifying effects, both of which drive competing manufacturers to decrease their audit levels but improve the audit efficiency. Next, we show the impact of audit cooperation on environmental benefits (i.e., carbon emissions compliance level).

Theorem 2. Comparing the optimal carbon emissions compliance levels between non-cooperative and cooperative scenarios, the following hold: (a) Case $I < \overline{I}$: if $L < \overline{L}$, $e^{NC*} < e^{C*}$, otherwise $e^{NC*} \ge e^{C*}$; and (b) Case $I \geq \overline{I}: e^{NC*} < e^{C*}$ always hold; where $\bar{I} = \frac{k}{\sqrt{5}}$ $\frac{k}{5-1} - G$, and $\bar{L} = \frac{(\sqrt{5}-1)hk}{2[k-(\sqrt{5}-1)(G)]}$ $\frac{(V-1)h}{2[k-(\sqrt{5}-1)(G+I)]}+V-I.$

Theorem [2](#page-15-1) shows that if incentive intensity is high, audit cooperation will lead to a higher carbon emissions compliance level and benefit the environment. In contrast, if incentive intensity is low, the impact of audit cooperation on carbon emissions compliance is determined by the financial loss, L.

Figure 2: Comparison of carbon audit performance between NC and C scenarios Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $h = 15$, $k = 30$, $w = 2$, and $L = 125$. Moreover, considering our assumptions and real-world scenarios, the values of these parameters can be chosen arbitrarily without altering the observed trends in the figure.

A graphical interpretation of Theorem [2](#page-15-1) is presented in Figure [3.](#page-16-1) Specifically, in the blue area of Figure [3,](#page-16-1) where the incentive intensity is high (i.e., $I \geq \overline{I}$), carbon audit cooperation can improve the supplier's carbon emissions compliance level and thus reduce its carbon emissions. The reason is that once carbon emissions noncompliance is detected, both manufacturers can simultaneously impose a higher penalty on the supplier under carbon audit cooperation. Thus, the supplier must raise its carbon emissions compliance level, especially when the incentive intensity is higher.

Figure 3: Comparison of the carbon emissions compliance levels between NC and C Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $k = 30$, and $w = 2$.

In the grey and yellow areas of Figure [3,](#page-16-1) where the incentive intensity is low (i.e., $I < I$). carbon audit cooperation may encourage or discourage the supplier's carbon emissions compliance level. Accordingly, such cooperation produces a positive or negative impact on the environment depending on the financial loss value. Specifically, if the financial loss is high (i.e., $L \geq \overline{L}$, audit cooperation among competing manufacturers harms the environment, and vice versa. The reason lies in the impact of financial loss on free riding through the cooperation. On the one hand, the increase in financial loss aggregates free riding among two competing manufacturers. This expands the audit level gap between cooperative and non-cooperative audit activities, and subsequently, encourages the supplier to reduce the carbon emissions compliance level. On the other hand, the amplifying effect of audit cooperation motivates the supplier to improve carbon emissions compliance level. When financial loss is high, the motivation for the supplier to improve its carbon emissions compliance level from the amplifying effect is dominated by the motivation for the supplier to reduce carbon emissions compliance level from the free-riding effect. Thus, audit cooperation drives the supplier to be reluctant to undertake carbon emissions compliance efforts and negatively affects the environment. Otherwise, if the financial loss is low, the negative motivation from the free-riding effect is dominated by the positive motivation from the amplifying effect. This encourages higher carbon emissions compliance efforts by the supplier, and thus, provides more benefits to the environment. In addition, Figures $3(a)-(c)$ show that the grey area decreases with the carbon audit cost coefficient, h. This indicates that as the unit carbon audit cost increases, carbon audit cooperation becomes more effective in improving carbon emissions compliance level.

In summary, given Theorems [1](#page-15-0) and [2,](#page-15-1) we know that carbon audit cooperation between competing manufacturers enables them to not only invest in a lower carbon audit level (see Theorem $1(a)$, but improves carbon audit efficiency (see Theorem $1(b)$). In addition, it is also worth noting that the high carbon audit efficiency generated by carbon audit cooperation does not always benefit the environment, as indicated in Theorem [2\(](#page-15-1)a).

Theorem 3. Carbon audit cooperation may be detrimental for competing manufacturers.

Figure [4](#page-18-0) graphically illustrates Theorem 3, where the grey area indicates that the cooperation on carbon audit is detrimental to competing manufacturers, and the blue area signifies the opposite. It is apparent that carbon audit cooperation can decrease the audit costs for each manufacturer owing to the amplifying effect, which is a direct benefit for them. However, as mentioned in Theorem $1(a)$, carbon audit cooperation can also result in free-riding effect, potentially prompting the supplier to lower the carbon emissions compliance level, thereby indirectly harming manufacturers' interests. Under specific conditions, the indirect harm caused by the free-riding effect outweighs the direct benefit from the amplifying effect; consequently, cooperation on carbon audit harms both manufacturers. Conversely, the direct benefit dominates the indirect downside effects; thus, cooperation on carbon audit is beneficial to each manufacturer. Additionally, Figures $4(a)-(c)$ show that the grey area shrinks as the carbon audit cost coefficient, h, increases. Thus, the amplifying (free-riding) effect increases (decreases) with a higher unit carbon audit cost. The behind rationale is that a higher unit carbon audit cost leads to greater savings for each manufacturer through cooperative carbon audit, thereby indirectly strengthening the amplifying effect and weakening the free-riding effect.

Theorem 4. Comparing the supplier's profits between non-cooperative and cooperative scenarios, the following hold:

(a) Case
$$
I < \overline{I}
$$
: if $L < \overline{L}$, $\pi_s^{NC*} \geq \pi_s^{C*}$, otherwise $\pi_s^{NC*} \leq \pi_s^{C*}$; and (b) Case $I \geq \overline{I}$: $\pi_s^{NC*} \geq \pi_s^{C*}$ always holds.

Theorem [4](#page-17-0) shows that carbon audit cooperation among manufacturers may have a negative or positive effect on the supplier's profit. Particularly, the impact of carbon audit cooperation on the supplier's profit is the opposite of that on the environment. That is, if cooperation benefits (harms) the environment, it will harm (benefit) the supplier. The reasons for this

Figure 4: The manufacturer i's profit gap between NC and C scenarios Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $k = 30$, $w = 2$, and R=0.

phenomenon can be similarly explained by the interpretation of Theorem [2.](#page-15-1) Specifically, the aggregated free-riding effect harms the environment but benefits the supplier, whereas the amplifying effect benefits the environment but harms the supplier.

Based on Theorems [2,](#page-15-1) [3,](#page-17-1) and [4,](#page-17-0) Figure [5](#page-18-1) graphically illustrates how manufacturers' carbon audit cooperation impacts them, the supplier, and the environment. Specifically, loss-win-loss, win-win-loss, and win-loss-win represent the win or loss outcomes for the manufacturers, supplier, and environment, respectively, in the cooperative case compared to the non-cooperative case. For example, win-loss-win in the blue area indicates that carbon audit cooperation positively affects manufacturers and the environment, but negatively affects the supplier. In addition, Figure [5](#page-18-1) also shows that carbon audit cooperation between competing manufacturers cannot lead to a triple-win or triple-loss. These are the interactive results of the free-riding and the amplifying effects of the cooperative carbon audit. Furthermore, given other conditions, cooperation may benefit both the manufacturers and supplier but harm the environment; or benefit the environment and manufacturers but harm the supplier.

Figure 5: The impact of carbon audit cooperation Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $c=0$, $k = 30$, $w = 2$, and $R=0$.

6. Extensions

In this section, we conduct additional analyses to investigate the impact of carbon audit cooperation on supplier's emissions compliance. As shown in Figure [5,](#page-18-1) the manufacturers and supplier may not simultaneously be better off by cooperating on carbon audit, which are the

'win-loss-win' and 'loss-win-loss' cases. To amend this, we design a new contract that achieves the *Pareto improvement*, enabling the manufacturers and supplier get better off simultaneously in both cases. Furthermore, we extend our base model in three directions for more robust insights. First, we assume that the manufacturers are engaged in price competition (denoted as the superscript ′). Second, we consider a sequential decision framework (denoted as the superscript $'$) in which the supplier first decides the carbon emissions compliance level and the manufacturers then simultaneously react their carbon audit levels. Finally, we explore a scenario with uncertain demand (denoted as the superscript $''$).

6.1. Pareto improvement

Carbon audit cooperation between competing manufacturers leads to three outcomes: winloss-win, win-win-loss and loss-win-loss, where 'win-loss-win' and 'loss-win-loss' deserve further investigation. Specifically, the 'win-loss-win' outcome indicates that the manufacturers and environment benefit from carbon audit cooperation, while the supplier suffers from it. The supplier being worse-off may be perceived as a socially unfair outcome and manufacturers may face negative publicity. To address this concern, when competing manufacturers cooperate on carbon audit, they should help the supplier to benefit from the audit cooperation. Similarly, the 'loss-win-loss' outcome shows that the supplier benefits from carbon audit cooperation but the manufacturers' and environment's interests suffer. This indicates that the supplier may have an incentive for the competing manufacturers to cooperate on carbon audit. Next, we focus on the 'win-loss-win' ('loss-win-loss') scenario and design a contract that can help the supplier (the manufacturers) be better off.

Notably, Hyundai Motor Company assists its suppliers in reducing emissions and helps them comply with local governments' relevant environmental regulations ^{[5](#page-0-0)}. Motivated by this practical example, we design a transfer payment contract (TPC). Specifically, when competing manufacturers cooperate on carbon audit, each manufacturer provides a transfer payment T_m to the supplier to alleviate pressure on the supplier to reduce carbon emissions. Therefore, T_m can be interpreted as part of the carbon emissions compliance cost shared by each manufacturer for the supplier. Similarly, the supplier can offer a transfer payment T_s to each manufacturer to encourage them to cooperate on carbon audit.

Note that the TPC's implementation depends primarily on the supply chain (indexed by a subscript sc) profits. A comparison of the supply chain profit under NC and C cases is summarised in Figure [6](#page-20-0) where areas II, III and IV represent $\pi_{sc}^{NC*} < \pi_{sc}^{C*}$, while area I represents $\pi_{sc}^{NC*} \geq \pi_{sc}^{C*}.$

Echoing the graphical insights, we summarise them into the following theorem.

Theorem 5. (a) If a 'win-loss-win' outcome occurs, each manufacturer always can offer a transfer payment T_m to the supplier, such that both manufacturers and the supplier are better off from the TPC; and

(b) If a 'loss-win-loss' outcome occurs, the supplier can offer a transfer payment T_s to each

⁵https://www.hyundai.com/content/hyundai/ww/data/csr/data/0000000049/attach/english/hmc-2021 sustainability-report-en-v6.pdf

Figure 6: Comparison of supply chain profit between NC and C scenarios Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $c=0$, $k = 30$, $w = 2$, and $R=0$.

manufacturer to encourage cooperation on carbon audit; where $T_m \in$ $\left[\left(\pi_s^{NC*} - \pi_s^{C*} \right) \right]$ $\frac{(-\pi_{s}^{2})}{2}, \pi_{m_{i}}^{C*} - \pi_{m_{i}}^{NC*}$ 1 and $T_s \in \left[\left(\pi_{m_i}^{C*} - \pi_{m_i}^{NC*} \right), \frac{\pi_s^{C*} - \pi_s^{NC*}}{2} \right].$

Combining Figure [5](#page-18-1) with Theorem [5\(](#page-19-0)a), we have the following corollary.

Corollary 7. If a 'win-loss-win' outcome occurs, the TPC can help achieve a triple-win outcome, i.e. a Pareto improvement.

As in the previous analysis, it is impossible to achieve an outcome in which carbon audit cooperation benefits manufacturers, the supplier and the environment simultaneously without additional contracts. However, the proposed TPC can achieve this target (i.e., triple win) when the carbon audit cooperation leads to 'win-loss-win' originally. In addition, the scenarios 'loss-win-loss' and 'win-win-loss' cannot achieve a triple win through the TPC.

6.2. Price Competition

In the base model , we assume that the two manufacturers engage only in quantity competition. Price competition also occurs in the real world. To explore this direction, we investigate the implications of price competition on carbon audit cooperation between competing manufacturers. Following [Qi et al.](#page-29-2) [\(2017\)](#page-29-2) and [Taleizadeh et al.](#page-29-12) [\(2020b\)](#page-29-12), the demand function of manufacturer i under price competition is described as $q_i = \alpha - p_i + \beta p_{3-i}$ $(i \in \{1,2\})$. Then, we obtain the profit functions of the supplier and manufacturer i in the NC and C cases, respectively.

In the NC case, the profit functions of the supplier and manufacturer i are given by:

$$
\pi_s^{NC'} = \sum_{i=1}^2 \left(e\left((w-c)q_i + R \right) + (1-e) a_i \left(-cq_i - F \right) + (1-e) \left(1 - a_i \right) \left((w-c)q_i + R \right) \right) - \frac{1}{2} k e^2 \tag{5}
$$
\n
$$
\pi_{\infty}^{NC'} = e\left((w-c)q_i - R \right) + (1-e) a_i F + (1-e) \left(1 - a_i \right) \left((w-c)q_i - R - I \right) - \frac{1}{2} k a^2 \tag{6}
$$

$$
\pi_{m_i}^{NC'} = e((p_i - w)q_i - R) + (1 - e)a_i F + (1 - e)(1 - a_i)((p_i - w)q_i - R - L) - \frac{1}{2}ha_i^2
$$
(6)

In the C case, the profit functions of the supplier and manufacturer i are characterised as follows:

$$
\pi_s^{C'} = \sum_{i=1}^2 e((w-c)q_i + R) + (1 - e)(1 - a_i)(1 - a_{3-i})((w-c)q_i + R) +
$$

(1 - e)(a_ia_{3-i} + a_i(1 - a_{3-i}) + (1 - a_i)a_{3-i}) (-c(q_i + q_{3-i}) - F) - $\frac{1}{2}$ ke²

$$
\pi_{m_i}^{C'} = e((p_i - w) q_i - R) + (1 - e)(1 - a_i) (1 - a_{3-i}) ((p_i - w) q_i - R - L) + (1 - e)(a_i a_{3-i} + a_i (1 - a_{3-i}) + (1 - a_i) a_{3-i})F - \frac{1}{2} h a_i^2
$$
\n
$$
(8)
$$

Given these profit functions, we first investigate the optimal decisions for each firm under cooperative and non-cooperative cases which are presented in Appendix B. We then discuss how cooperation on carbon audit affects the manufacturers, supplier, and environment under price competition.

Theorem 6. Suppose that the manufacturers are involved in price competition. Then, by comparing the optimal compliance levels of carbon emissions between non-cooperative and cooperative scenarios, the following hold:

(a) Case $I < \overline{I}'$: if $L < \overline{L}'$, $e^{NC'^*} < e^{C'^*}$, otherwise $e^{NC'^*} \ge e^{C'^*}$; and (b) Case $I \geq \overline{I}'$: $e^{NC'^*} < e^{C'^*}$ always holds; where $G' = \frac{[\alpha-(1-\beta)w]w}{2-\beta}$ $\frac{(1-\beta)w}{2-\beta}$, $V' = \frac{[\alpha-(1-\beta)w]^2}{(2-\beta)^2}$ $\frac{(- (1-\beta)w)^2}{(2-\beta)^2},\ \bar{I}'=\frac{k}{\sqrt{5}}.$ $\frac{k}{5-1} - G'$, and $\bar{L}' = \frac{(\sqrt{5}-1)hk}{2[k-(\sqrt{5}-1)(G)]}$ $\frac{(\sqrt{3}-1)h\kappa}{2[k-(\sqrt{5}-1)(G'+I)]}+V'-I.$

Theorem [6\(](#page-21-0)a) indicates that when the incentive intensity is low, carbon audit cooperation between competing manufacturers is detrimental to improving the supplier's carbon emissions compliance level if the financial loss is high. However, when the incentive intensity is high, carbon audit cooperation can improve the supplier's carbon emissions compliance level and benefit the environment, as illustrated in Theorem [6\(](#page-21-0)b).

Theorem 7. Suppose that the manufacturers are involved in price competition. Carbon audit cooperation may be detrimental to the competing manufacturers.

As a graphical presentation of Theorem [7,](#page-21-1) Figure [7](#page-21-2) shows that, even under price competition, carbon audit cooperation should be treated with caution since it may negatively affect manufacturers. The reason lies in that carbon audit cooperation leads to the free-riding and amplifying effects. Among these, the free-riding (amplifying) effect is harmful (beneficial) to manufacturers. Under the interaction of these two effects, carbon audit cooperation may positively or negatively affect manufacturers. Figures [7\(](#page-21-2)a)-(c) show that as the carbon audit cost coefficient h increases, the grey area decreases. This indicates that the free-riding (amplifying) effect weakens (strengthens) as h increases.

Figure 7: Manufacturer i's profit gap between NC and C scenarios under price competition Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $k = 40$, $w = 2$, and R=0.

Theorem 8. Suppose that the manufacturers are involved in price competition. Comparing the supplier's profits between non-cooperative and cooperative scenarios, the following hold: (a) Case $I \lt I^T$: if $L \lt L'$, $\pi_S^{NC'*} \geq \pi_S^{C'*}$, otherwise $\pi_S^{NC'*} < \pi_S^{C'*}$; and (b) Case $I \geq \overline{I}'$: $\pi_s^{NC'^*} \geq \pi_s^{C'^*}$ always holds.

Theorem [8\(](#page-21-3)a) shows that with price competition, when the incentive intensity is low, carbon audit cooperation is detrimental to the supplier's profit if the financial loss is low; otherwise, the supplier benefits from cooperation. When the incentive intensity is high, the supplier does not expect manufacturers to cooperate on carbon audit.

6.3. Sequential Decision

In the base model, we investigate a game in which manufacturers and the supplier decide the carbon audit and carbon emissions compliance levels simultaneously. Here, we continue to explore an alternative decision sequence in which the supplier decides its carbon emissions compliance level at first, and subsequently, the manufacturers decide their audit levels simultaneously. Based on backward induction, we can derive the optimal decisions under the NC and C cases, which are shown in Appendix B. Given this setting, we investigate the impacts of carbon audit cooperation on the environment, manufacturers, and supplier by conducting a numerical analysis, as shown in Figures [8-](#page-22-0)[10,](#page-23-0) respectively.

First, Figure [8](#page-22-0) graphically illustrates the impact of carbon audit cooperation on the environment in a scenario in which the supplier and manufacturers make sequential decisions. Specifically, when the incentive intensity is lower (i.e., $I < 9.8$), carbon audit cooperation positively affects the environment if the financial loss is lower, as shown in the yellow area; otherwise, carbon audit cooperation negatively affects the environment, as shown in the grey area. When incentive intensity is high, carbon audit cooperation positively affects the environment, as shown in the blue area.

Figure 8: Comparison of carbon emissions compliance levels between NC and C scenarios under sequential decision

Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $k = 40$, and $w = 2$.

Second, Figure [9](#page-23-1) illustrates the effect of carbon audit cooperation on manufacturers under the sequential decision-making process. Carbon audit cooperation may be either harmful (shown in the grey area) or beneficial (shown in the blue area) to manufacturers. Addition-ally, Figures [9\(](#page-23-1)a)-(c) demonstrate that, with an increase in the carbon audit cost coefficient h ,

Figure 9: The manufacturer i's profit gap between NC and C scenarios under sequential decision Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $k = 30$, $w = 2$, and $R=0$.

the grey area becomes smaller, indicating that carbon audit cooperation is more attractive to manufacturers.

Finally, Figure [10](#page-23-0) illustrates the impact of carbon audit cooperation on the supplier under the sequential decision-making process in which the supplier may benefit from carbon audit cooperation (as shown in the blue area).

Figure 10: The supplier's profit gap between NC and C under sequential decision Note: This figure is obtained with $\alpha = 15$, $\beta = 0.6$, $k = 30$, $w = 2$, and $R=0$.

In summary, based on the analysis of Figures [8-](#page-22-0)[10,](#page-23-0) we conclude that under sequential decision-making process, the impacts of carbon audit cooperation on the environment, manufacturers, and supplier are robust.

6.4. Uncertain Demand

In the base model, we assume that market demand is deterministic. Here, we explore the scenario with uncertain demand. Recall that if demand is deterministic, the inverse demand function is given by $p_i = \alpha - q_i - \beta q_{3-i}$. Then, we model the uncertain demand by introducing an additive shock to the basic market size α . Specifically, we assume that the parameter α presents two states: $\alpha = \alpha_L$ with probability φ ; and $\alpha = \alpha_H$ with probability $1 - \varphi$, where $\varphi \in [0,1]$. This modeling approach to uncertain demand has been widely used in operations management research (e.g., [Wang and Hu, 2014;](#page-30-9) [Yu and Cao, 2020;](#page-30-10) [Caliskan Demirag et al.,](#page-26-5) [2021;](#page-26-5) [Fu et al., 2022\)](#page-27-12). Moreover, since the carbon emissions compliance and the carbon audit levels are long-term decisions, they should be determined before the realisation of uncertain demand. Meanwhile, decisions regarding the quantity of components purchased are short-term and should be made after the realisation of uncertain demand.

In the NC case, the profit functions of the supplier and manufacturer i are given by:

$$
E\left[\pi_s^{NC'''}\right] = \sum_{i=1}^2 \left\{ e\left(w-c\right) \left(E\left[q_i\right] + R\right) + \left(1 - e\right) a_i \left(-cE\left[q_i\right] - F\right) + \left(1 - e\right) \left(1 - a_i\right) \left(E\left[q_i\right] + R\right) \right\} - \frac{1}{2} k e^2 \tag{9}
$$

$$
E\left[\pi_{m_i}^{NC'''}\right] = e\left[\left(E\left[p_i\right] - w\right)q_i - R\right] + (1 - e)a_i F + (1 - e)\left(1 - a_i\right)\left[\left(E\left[p_i\right] - w\right)q_i - R - L\right] - \frac{1}{2}ha_i^2\tag{10}
$$

In the C case, the profit functions of the supplier and manufacturer i are given by:

$$
E\left[\pi_s^{C'''}\right] = \sum_{i=1}^2 e\left((w-c)E\left[q_i\right] + R\right) + (1-e)\left(1-a_i\right)\left(1-a_{3-i}\right)\left((w-c)E\left[q_i\right] + R\right) + (1-e)\left(a_i a_{3-i} + a_i \left(1-a_{3-i}\right) + (1-a_i) a_{3-i}\right)\left(-c(E\left[q_i\right] + E\left[q_{3-i}\right]) - F\right) - \frac{1}{2}ke^2\tag{11}
$$

$$
E\left[\pi_{m_i}^{C'''}\right] = e\left((E\left[p_i\right] - w)q_i - R\right) + (1 - e)\left(1 - a_i\right)\left(1 - a_{3-i}\right)\left(\left(E\left[p_i\right] - w\right)q_i - R - L\right) + (1 - e)\left(a_i a_{3-i} + a_i \left(1 - a_{3-i}\right) + (1 - a_i) a_{3-i}\right)F - \frac{1}{2}h a_i^2\right)
$$
\n
$$
(12)
$$

Based on the profit functions outlined above, we analyse the optimal decision(s) for each firm, as detailed in Appendix B. Subsequently, we discuss how cooperation on carbon audit impacts manufacturers, the supplier, and the environment in a scenario with demand uncertainty.

Theorem 9. Under the scenario with demand uncertainty, comparing the optimal carbon emissions compliance levels between non-cooperative and cooperative scenarios, the following hold: (a) Case $I \lt I^m$: if $L \lt \overline{L}^m$, $e^{NC''''*} \lt e^{C'''*}$, otherwise $e^{NC''''*} \ge e^{C'''*}$; and (b) Case $I \geq \bar{I}'''$: $e^{NC''''*} < e^{C'''*}$; where $\bar{I}''' = \frac{k}{\sqrt{5}}$ $\frac{k}{5-1} - G''''$ and $\bar{L}''' = \frac{(\sqrt{5}-1)hk}{2[k-(\sqrt{5}-1)(G')]}$ $\frac{(6.05-1)h}{2[k-(\sqrt{5}-1)(G'''+I)]} + V''' - I.$

Theorem [9](#page-24-0) shows that, in most cases, carbon audit cooperation between manufacturers positively affects the supplier's carbon emissions compliance. However, under $I \leq \overline{I}^{\prime\prime\prime}$ and $L > \bar{L}'''$, carbon audit cooperation exerts a negative effect. This indicates that manufacturers may (may not) benefit from carbon audit cooperation, which is further verified by Theorem [10.](#page-24-1)

Theorem 10. Under the scenario with demand uncertainty, carbon audit cooperation may be detrimental for the two manufacturers.

Figure [11](#page-25-0) graphically illustrates the results of Theorem [10.](#page-24-1) The grey (blue) area indicates that carbon audit cooperation is harmful (beneficial) to manufacturers. Moreover, in Figures [11](#page-25-0) (a)-(c), as the carbon audit cost coefficient increases, the grey area becomes smaller, indicating that the more expensive the carbon audit, the greater the benefits of cooperation for the manufacturers.

Theorem 11. Under the scenario with demand uncertainty, comparing the supplier's profits between non-cooperative and cooperative scenarios, the following hold: (a) Case $I \lt I^{\text{III}}$: if $L \lt I^{\text{III}}$, $\pi_s^{\text{NCIII}*} \geq \pi_s^{\text{CIII}*}$, otherwise $\pi_s^{\text{NCIII}*} \leq \pi_s^{\text{CIII}*}$; and (b) Case $I \geq \overline{I}'''$: $\pi_s^{NC'''} \geq \pi_s^{C'''}$ always holds.

Figure 11: The manufacturer i's profit gap between NC and C scenarios with demand uncertainty Note: This figure is obtained with $\alpha_L = 10$, $\alpha_H=20$, $\beta = 0.6$, $\varphi=0.5$, $k = 30$, $w = 2$, and $R=0$.

Theorem [11](#page-24-2) shows that when the incentive intensity is low, the supplier can benefit from carbon audit cooperation between manufacturers if the financial loss is high. Thus, the supplier expects manufacturers to cooperate on carbon audit. Conversely, the supplier cannot benefit from carbon audit cooperation.

Based on Theorems [9,](#page-24-0) [10,](#page-24-1) and [11,](#page-24-2) we conclude that under a uncertain demand scenario, the impacts of carbon audit cooperation on the environment, supplier, and manufacturers are fundamentally consistent as scenarios with deterministic demand.

7. Conclusion

Research on supply chain emissions reduction under carbon cap regulation (e.g., [Benjaafar](#page-26-1) [et al., 2012;](#page-26-1) [Ren et al., 2015;](#page-29-5) [Qi et al., 2017;](#page-29-2) [He et al., 2019;](#page-28-7) [An et al., 2021\)](#page-26-0) has predominantly focused on the carbon emissions compliance scenario. Here, we depart from this premise, inspired by a real-world example of carbon emissions noncompliance by a common supplier of Hyundai Motor Company and Kia Motors Corporation. Instead, we focus on a supply chain consisting of one supplier and two competing manufacturers, and investigate the impact of manufacturers' carbon audit cooperation on the environment, manufacturers, and supplier.

Through our stylised model, we derive several key findings about the effects of manufacturers' cooperation on carbon audit. First, we identify two effects — the free-riding effect and the amplifying effect — that can result from such cooperation. The free-riding effect benefits the supplier but harms the environment and manufacturers; meanwhile, the amplifying effect benefits the environment and manufacturers, but harms the supplier. Depending on how these two effects interact, manufacturers' cooperation on carbon audit can be either harmful or beneficial to the environment, manufacturers, and supplier. It is important to note that a triple-win outcome cannot be achieved because free-riding and amplifying effects have opposite impacts on the environment and supplier. However, a transfer payment contract (TPC) between the supplier and manufacturers can lead to a triple-win outcome in scenarios in which cooperation only harms the supplier.

Second, we show that the optimal carbon emissions compliance level increases and then decreases as the basic market size increases. Further, manufacturers' profits may increase with financial loss caused by the undetected carbon emissions noncompliance. This is because a higher financial loss results in an increase in the carbon audit level, which motivates the supplier

to improve its carbon emissions compliance level. This, in turn, reduces the manufacturers' expected losses from carbon emissions noncompliance. In certain circumstances, the (indirect) benefit from the manufacturers' higher financial loss dominates the (direct) loss from higher financial loss.

Finally, we demonstrate the robustness of our results through three extensions. First, we examined the price competition between the two manufacturers. Second, we investigated the decision sequence in which the supplier sets its carbon emissions compliance level before manufacturers decide on their carbon audit levels. Finally, uncertain demand is investigated. Our findings highlight the importance of carefully considering the factors that affect manufacturers' cooperation on carbon audit, and the potential benefits and drawbacks of such cooperation.

Extant research on low-carbon supply chains primarily examines optimal decision-making under low-carbon policies and incentive mechanisms for emissions reduction. However, few examine the issue of carbon emissions noncompliance within the supply chain. This study proposes that by engaging in carbon audit, manufacturers can collaborate to improve their suppliers' carbon emissions compliance levels, thereby expanding the scope of low-carbon supply chain research. Nonetheless, this study has some limitations as well. First, the efficacy of carbon audit may be compromised if suppliers take measures to evade it. Future research can adopt methods to prevent such evasion. Second, the auditor and supplier may collude, which can impede the effectiveness of carbon auditing in addressing noncompliance. Therefore, designing an effective mechanism to prevent collusion is an important research area. Third, manufacturers may exhibit loss aversion under higher losses, which can be incorporated into the future research analysis.

References

- An, Simin, Bo Li, Dongping Song, Xue Chen. 2021. Green credit financing versus trade credit financing in a supply chain with carbon emission limits. European Journal of Operational *Research* **292**(1) 125–142.
- Babich, Volodymyr, Christopher S Tang. 2012. Managing opportunistic supplier product adulteration: Deferred payments, inspection, and combined mechanisms. Manufacturing $\mathcal C$ Service Operations Management 14(2) 301–314.
- Baksi, Soham, Pinaki Bose. 2007. Credence goods, efficient labelling policies, and regulatory enforcement. *Environmental and Resource Economics* 37(2) 411–430.
- Benjaafar, Saif, Yanzhi Li, Mark Daskin. 2012. Carbon footprint and the management of supply chains: Insights from simple models. IEEE transactions on automation science and *engineering* $10(1)$ 99–116.
- Bondareva, Mariya, Edieal Pinker. 2019. Dynamic relational contracts for quality enforcement in supply chains. Management Science 65(3) 1305–1321.
- Caliskan Demirag, Ozgun, Weili Xue, Jie Wang. 2021. Retailers' order timing strategies under competition and demand uncertainty. Omega 101 102256. doi:https://doi.org/10.1016/j. omega.2020.102256.
- Caro, Felipe, Prashant Chintapalli, Kumar Rajaram, Chris S Tang. 2018. Improving supplier compliance through joint and shared audits with collective penalty. Manufacturing $\mathcal C$ Service Operations Management 20(2) 363–380.
- Chen, Jiayu, Anyan Qi, Milind Dawande. 2020a. Supplier centrality and auditing priority in socially responsible supply chains. Manufacturing $\mathcal B$ Service Operations Management 22(6) 1199–1214.
- Chen, Li, Hau L Lee. 2017. Sourcing under supplier responsibility risk: The effects of certification, audit, and contingency payment. Management Science 63(9) 2795–2812.
- Chen, Li, Shiqing Yao, Kaijie Zhu. 2020b. Responsible sourcing under supplier-auditor collusion. Manufacturing & Service Operations Management 22(6) 1234–1250.
- Chen, Yuyu, Bangyi Li, Guoqing Zhang, Qingguo Bai. 2020c. Quantity and collection decisions of the remanufacturing enterprise under both the take-back and carbon emission capacity regulations. Transportation Research Part E: Logistics and Transportation Review 141 102032.
- Cho, Soo-Haeng, Xin Fang, Sridhar Tayur, Ying Xu. 2019. Combating child labor: Incentives and information disclosure in global supply chains. Manufacturing $\mathcal C$ Service Operations Management **21**(3) 692-711.
- Das, Chiranjit, Sanjay Jharkharia. 2018. Low carbon supply chain: A state-of-the-art literature review. Journal of Manufacturing Technology Management 29(2) 398–428.
- Ding, Huiping, Qilan Zhao, Zhirong An, Ou Tang. 2016. Collaborative mechanism of a sustainable supply chain with environmental constraints and carbon caps. International Journal of Production Economics 181 191–207.
- Du, Hang, Lai Wei, Qinghua Zhu. 2023. Competition of consumables' original brand manufacturers and remanufacturers considering the entry of compatible manufacturers. European Journal of Operational Research 306(2) 679–692.
- Dye, Chung-Yuan, Chih-Te Yang. 2015. Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints. European Journal of Operational Research 244(1) 187–200.
- Fang, Xin, Soo-Haeng Cho. 2020. Cooperative approaches to managing social responsibility in a market with externalities. Manufacturing \mathscr Service Operations Management 22(6) 1215–1233.
- Feng, Hairong, Yinlian Zeng, Xiaoqiang Cai, Qian Qian, Yongwu Zhou. 2021. Altruistic profit allocation rules for joint replenishment with carbon cap-and-trade policy. European Journal of Operational Research 290(3) 956–967.
- Fu, Ke, Ce Wang, Jiayan Xu. 2022. The impact of trade credit on information sharing in a supply chain. Omega 110 102633. doi:https://doi.org/10.1016/j.omega.2022.102633.
- Ghosh, Arindam, JK Jha, SP Sarmah. 2017. Optimal lot-sizing under strict carbon cap policy considering stochastic demand. Applied Mathematical Modelling 44 688–704.
- Ghosh, Arindam, J.K. Jha, S.P. Sarmah. 2020. Production-inventory models considering different carbon policies: a review. International Journal of Productivity and Quality Management $30(1)$ 1–27.
- Hartmann, Julia, Sabine Moeller. 2014. Chain liability in multitier supply chains? responsibility attributions for unsustainable supplier behavior. Journal of operations management $32(5)$ 281–294.
- He, Longfei, Jun Mao, Chenglin Hu, Zhang Xiao. 2019. Carbon emission regulation and operations in the supply chain supernetwork under stringent carbon policy. Journal of cleaner production 238 117652.
- He, Ping, Zheng Wang, Victor Shi, Yi Liao. 2021. The direct and cross effects in a supply chain with consumers sensitive to both carbon emissions and delivery time. European Journal of Operational Research 292(1) 172–183.
- Heydari, Hanieh, Ata Allah Taleizadeh, Fariborz Jolai. 2023. Financing a two-stage sustainable supply chain using green bonds: Preventing environmental pollution and waste generation. Engineering Applications of Artificial Intelligence 117 105583.
- Hoffmann, Nils Christian, Juelin Yin, Stefan Hoffmann. 2020. Chain of blame: A multi-country study of consumer reactions towards supplier hypocrisy in global supply chains. Management International Review 60(2) 247–286.
- Lee, Hsiao-Hui, Cuihong Li. 2018. Supplier quality management: Investment, inspection, and incentives. Production and Operations Management 27(2) 304–322.
- Lee, Jun-Yeon, Sungyong Choi. 2021. Supply chain investment and contracting for carbon emissions reduction: A social planner's perspective. International Journal of Production Economics 231 107873.
- Lee, Seungrae, Seung Jae Park. 2020. Who should lead carbon emissions reductions? upstream vs. downstream firms. International Journal of Production Economics 230 107790.
- Levi, Retsef, Somya Singhvi, Yanchong Zheng. 2020. Economically motivated adulteration in farming supply chains. Management Science 66(1) 209–226.
- Li, Yanran, Bo Li, Wei Zheng, Xue Chen. 2021. Reveal or hide? impact of demonstration on pricing decisions considering showrooming behavior. Omega 102 102329.
- Liu, Weihua, Wanying Wei, Tsan-Ming Choi, Xiaoyu Yan. 2022. Impacts of leadership on corporate social responsibility management in multi-tier supply chains. European Journal of Operational Research 299(2) 483–496.
- Mu, Liying, Milind Dawande, Xianjun Geng, Vijay Mookerjee. 2016. Milking the quality test: Improving the milk supply chain under competing collection intermediaries. Management Science **62**(5) 1259–1277.
- Murali, Karthik, Michael K Lim, Nicholas C Petruzzi. 2019. The effects of ecolabels and environmental regulation on green product development. Manufacturing $\mathcal C$ Service Operations Management **21**(3) 519–535.
- Plambeck, Erica L, Terry A Taylor. 2016. Supplier evasion of a buyer's audit: Implications for motivating supplier social and environmental responsibility. Manufacturing $\mathscr C$ Service Operations Management 18(2) 184–197.
- Porteous, Angharad H, Sonali V Rammohan, Hau L Lee. 2015. Carrots or sticks? improving social and environmental compliance at suppliers through incentives and penalties. Production and Operations management $24(9)$ 1402–1413.
- Qi, Qi, Jing Wang, Qingguo Bai. 2017. Pricing decision of a two-echelon supply chain with one supplier and two retailers under a carbon cap regulation. *Journal of Cleaner Production* 151 286–302.
- Ren, Jie, Yiwen Bian, Xiaoyan Xu, Ping He. 2015. Allocation of product-related carbon emission abatement target in a make-to-order supply chain. Computers \mathcal{C} Industrial Engineering 80 181–194.
- Rui, Huaxia, Guoming Lai. 2015. Sourcing with deferred payment and inspection under supplier product adulteration risk. Production and Operations Management 24(6) 934–946.
- Shi, Jinzhao, Dian Liu, Qiang Du, TCE Cheng. 2023. The role of the procurement commitment contract in a low-carbon supply chain with a capital-constrained supplier. International Journal of Production Economics 255 108681.
- Sim, Jeongeun, Fouad El Ouardighi, Bowon Kim. 2019. Economic and environmental impacts of vertical and horizontal competition and integration. Naval Research Logistics (NRL) 66(2) 133–153.
- Sim, Jeongeun, Bowon Kim. 2021. Regulatory versus consumer pressure and retailer responsibility for upstream pollution in a supply chain. Omega **101** 102250.
- Taleizadeh, Ata Allah, Leila Aliabadi, Park Thaichon. 2020a. A sustainable inventory system with price-sensitive demand and carbon emissions under partial trade credit and partial backordering. Operational Research 22 4471–4516.
- Taleizadeh, Ata Allah, Nima Alizadeh-Basban, Bhaba R. Sarker. 2018. Coordinated contracts in a two-echelon green supply chain considering pricing strategy. Computers \mathcal{C} Industrial Engineering 124 249–275.
- Taleizadeh, Ata Allah, Bita Hazarkhani, Ilkyeong Moon. 2020b. Joint pricing and inventory decisions with carbon emission considerations, partial backordering and planned discounts. Annals of Operations Research 290 95–113.
- Taleizadeh, Ata Allah, Seyed Taghi Akhavan Niaki, Nima Alizadeh-Basban. 2021. Cost-sharing contract in a closed-loop supply chain considering carbon abatement, quality improvement effort, and pricing strategy. RAIRO-Oper. Res. 55 S2181–S2219.
- Waltho, Cynthia, Samir Elhedhli, Fatma Gzara. 2019. Green supply chain network design: A review focused on policy adoption and emission quantification. International Journal of Production Economics 208 305–318.
- Wang, Zizhuo, Ming Hu. 2014. Committed versus contingent pricing under competition. Production and Operations Management $23(11)$ 1919–1936. doi:10.1111/poms.12202.
- Xu, Jianteng, Qi Qi, Qingguo Bai. 2018. Coordinating a dual-channel supply chain with price discount contracts under carbon emission capacity regulation. Applied Mathematical Modelling 56 449–468.
- Xu, Xiaoping, Mengying Zhang, Lai Chen, Yugang Yu. 2022. The region-cap allocation and delivery time decision in the marketplace mode under the cap-and-trade regulation. International Journal of Production Economics 247 108407. doi:https://doi.org/10.1016/j.ijpe. 2021.108407.
- Yang, Huixiao, Wenbo Chen. 2018. Retailer-driven carbon emission abatement with consumer environmental awareness and carbon tax: Revenue-sharing versus cost-sharing. Omega 78 179–191.
- Yu, Man, Erbao Cao. 2020. Information sharing format and carbon emission abatement in a supply chain with competition. *International Journal of Production Research* 58(22) 6775– 6790. doi:10.1080/00207543.2019.1685704.
- Yu, Yugang, Sijie Zhou, Ye Shi. 2020. Information sharing or not across the supply chain: The role of carbon emission reduction. Transportation Research Part E: Logistics and Transportation Review 137 101915.
- Zarei, Hamid, Morteza Rasti-Barzoki, Ilkyeong Moon. 2020. A mechanism design approach to a buyer's optimal auditing policy to induce responsible sourcing in a supply chain. Journal of environmental management 254 109721.
- Zhang, Huanjing. 2020. Construction and application of low-carbon audit evaluation index system based on low-carbon economy. Modern Economy 11(12) 2027–2040.
- Zhang, Yalian, Liyun Gu, Xin Guo. 2020. Carbon audit evaluation system and its application in the iron and steel enterprises in china. Journal of Cleaner Production 248 119204.

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