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Gaussian distributions on a logarithmic scale. As in England, width commonly exceeds length. Vertical dimensions correlate with length more than with width. Cirque form varies with geology, but also with relief as both vary between mountain groups. The main contrast is between larger, better-developed cirques and higher relief on volcanic rocks in the north-west, and smaller, less-developed cirques and lower relief on sedimentary rocks in the south.

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**Allometric development of glacial cirque form:  
geological, relief and regional effects on the cirques of Wales**

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**Abstract**

Headward and downward erosion near glacier sources, at rates exceeding fluvial erosion, is important in recent discussions of orogen development and the limits to relief. This relates to a long history of debate on how the form of glacial cirques develops, which can be advanced by relating shape to size in large data sets. For 260 cirques in Wales, this confirms different rates of enlargement in the three dimensions: faster in length than in width, and slower in vertical dimension whether expressed as overall height range, axial height range or wall height. Maximum gradient, plan closure and number of cols increase with overall size. This allometric development applies over different cirque types, regions and rock types. Headwall retreat, often by collapse following glacial erosion at the base, is faster than downward erosion. Welsh cirques form a scale-specific population and, as in other regions, size variables follow Gaussian distributions on a logarithmic scale. As in England, width commonly exceeds length. Vertical dimensions correlate with length more than with width. Cirque form varies with geology, but also with relief as both vary between mountain groups. The main contrast is between larger, better-developed cirques and higher relief on volcanic rocks in the north-west, and smaller, less-developed cirques and lower relief on sedimentary rocks in the south.

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## 1. Introduction

### 1.1. Aims and objectives

The erosion of glacial cirques takes tens to hundreds of thousands of years and is thus not susceptible to direct observation. Clearly cirques start from glacial occupation of already indented topography, often fluvial but sometimes volcanic or following landsliding. Several schemes have been proposed for cirque development (e.g. Gordon, 1977; Bennett, 1990): all are based on the concept that cirques enlarge over time, as the bedrock removed cannot be replaced in the same form. This substitution of spatial variation for change over time is known as the ergodic hypothesis (Cox, 1977; Thorn, 1988).

Cirque distribution is excellent evidence of former glaciation and is fairly well established, but data on the precise form of glacial cirques are limited and based on varied definitions. Here I report on an effort to provide comprehensive and comparable data, relevant to hypotheses of cirque development.

The aim of this paper is to use extensive data on spatial variation in cirque form to show how cirques may develop. This involves relating both morphometric and qualitative data to a number of possible controls: geology, relief, position and timing or type of last glacial occupation. The objectives are to present a high-resolution data set for all cirques in Wales, to use it to test hypotheses of changes in cirque shape with size and of environmental controls on both, and to set the statistical results in the context of a long historical debate on downward and headward cirque erosion. The data set is sufficiently large to be subdivided on various criteria, with numerous cirques in each class of geology or grade of development. This permits more reliable models, narrower confidence intervals, and statistical tests of greater power. The data also provide a sampling frame for future detailed studies of glacial morphology and Quaternary chronology, comparable to that provided for the English Lake District by Evans and Cox (1995).

### 1.2. Review of cirque development models: wearing down and wearing back

There is no obvious reason why downward erosion by glacial plucking, abrasion and meltwater should be equal to headwall recession by these and other processes (frost action and slope collapse). Contradictory views have been published, and it is worth considering the development of ideas on this controversial subject before presenting morphometric data permitting a test of their applicability. Fuller historical depth is provided in Evans (in press).

In the early twentieth century, cirques were considered to wear back by ‘sapping’ of the base of the headwall, usually by frost weathering and rockfall. Local glaciers flatten and deepen cirque floors, but by removing rockfall they constantly attack headwalls at their bases (de

Martonne, 1901). Richter (1900) emphasized cirque enlargement by headwall retreat through frost weathering, with glaciers abrading cirque floors but protecting them from fluvial incision; this produced a high terrace which could eventually truncate a mountain range. Richter's hypothesis that in Norway this retreat had gone so far as to create the high mountain plateaus (the Palaeic surface) was regarded as inconceivable by Ahlmann (1919, p. 220).

Johnson's (1904) bergschrund hypothesis explained headwall retreat by vigorous frost weathering in a narrow zone. This provided debris that aided glacial abrasion downstream. Hobbs (1910, 1911) proposed four stages of sculpture by mountain glaciers, from youthful 'channelled and grooved' uplands through adolescent 'early fretted' and mature 'fretted' to senile 'monumented' upland, as cirques enlarged laterally at the expense of 'pre-glacial' upland. This 'Cycle of Mountain Glaciation', emphasizing headward recession, was discussed at some length in Embleton and King (1968). It involves, however, a dangerous substitution of time by space, the more so as Hobbs' main examples of the four stages are widely separated over space, respectively in the Bighorn Mountains of Wyoming, in northwest Wales, in the (Swiss) Alps, and in Glacier National Park, Montana. Hobbs suggested that cirques become more complicated with age: others attribute the complexity of broader cirques to coalescence. Davis (1911, pp. 56-61) described the enlargement and coalescence of cirques in Colorado and supported Richter's idea of mountain truncation at the snowline by prolonged glaciation.

Blache (1952, pp. 112-5) rejected Hobbs' four stages: instead of duration of glaciation, he related such differences to increased altitude, giving greater slope and greater ice discharge. The stages seem to relate also to dissection of the initial topography. Blache (1960, p. 30) denied that cirques expand laterally as in Hobbs' scheme. Derbyshire and Evans (1976) related Hobbs' 'stages' to tectonic environment and degree of dissection, especially the spacing of valleys.

Lewis (1938, 1940) suggested that meltwater forced its way down far below the bergschrund, explaining the great height of some cirque headwalls (over 300 m, even in Britain). Following later fieldwork, in the Jötunheimen, Norway (Clark, 1951; Clark and Lewis, 1951), Lewis emphasized the importance of rotational flow in cirque development (Lewis, 1949, 1960). Rotational flow maximises basal sliding but requires a certain range of ice surface gradients (see section 3.2). Lewis (1949) calculated that gradients of 15 - 20° and ice thicknesses of 80 - 100 m gave the greatest probabilities of rotational slip, and thus erosion of rock basins, both in cirques and below valley steps. Rotational flow has now been clearly demonstrated in Norway on Veslgjuv-breen (Grove, 1960) and Vesl-Skautbreen (McCall, 1960), and in Colorado on the Arapaho Glacier (Waldrop, 1964). Overall centre-line gradients were some 17°, 27° and 19° respectively. Lewis (1960) calculated that the 63 m deep rock basin of Blea Water, in a cirque in the English Lake

District, was excavated by a rotationally flowing glacier sloping at  $13^{\circ}$  to  $16^{\circ}$ , and that at least  $7^{\circ}$  was required to overcome basal friction.

A few authors have given more prominence to the downward erosion of cirque floors. Strøm (1945) contrasted the rapid cirque backwall recession in the quartzite of Rondane (southeast Norway) with the apparent vertical erosion (producing rock basins up to 170 m deep) in the plutonic rocks of Moskenesøy, Lofoten Islands. Galibert (1962, p. 16) proposed that in the Alps, crest retreat could be negligible compared with vertical incision of glacial cirques where jointing was most pronounced. Incision was due to greater abrasion under thick ice, accompanied by collapse of the headwall base by pressure release on deglaciation (see also Birot, 1968): high summits are deeply frozen and ‘paralysed’.

Referring mainly to the Uinta Mountains of Utah, and to some Norwegian cirques, White (1970, p. 123) maintained that “In many mountainous areas... arêtes between opposing cirques are so narrow and steep that very slight additional headward erosion by either of the opposing glaciers would have breached the arêtes to form a col or a pass. Yet these arêtes persist, in rarely broken continuity threading their precarious way for great distances between the headwalls of successions of opposing cirques”. Avalanche ravines on some headwalls would not have survived undermining by active sapping. White concluded that cirque floors are lowered by glacial erosion more rapidly than walls are cut back, a view that provides the opposite extreme to Richter.

Evidence of strong headward erosion was found in south-central Alaska by Tuck (1935), who proposed divide migration of up to 1 km due to stronger glacial erosion in northward-facing valley heads; the effects of differential insolation on glacier balance were reinforced by south winds. A similar interpretation of southward divide migration had been applied in west-central British Columbia where north-facing glaciers “have cut short northerly-trending valleys on the north side of the higher mountains” (Hanson 1924, p.31). Evans (1972) confirmed Hanson’s hypothesis for several ranges in the Coast Mountains of southwest British Columbia. The main divides of the Bendor and Tatlow Ranges appear to have been shifted 1 to 2 km southward by cirque headwall retreat.

Evans (1997) emphasized the role of slope collapse in cirque headwall retreat, and listed examples of historical rock avalanches from cirque headwalls. The 1873, 1991 and other collapses around Mount Cook can be added to this list. These were controlled by stress-release joints slightly steeper than the cliffs, in cohesionless, closely jointed rock. “Many occur high on mountain slopes where gravitational collapse is the only operative erosion process to keep pace with glacial and fluvial valley incision” (McSaveney, 2002, p. 69). In the Ben Ohau Range, also in New Zealand, Brook et al. (2006) suggest that cirques lengthen and deepen faster than they widen, increasing in

maximum gradient and reaching well-developed shapes after 600 ka. Brocklehurst and Whipple (2002) suggested that cirque headwall retreat is important on the east slope of the Californian Sierra Nevada, causing divide recession of up to 2.5 km beyond that expected from fluvial erosion.

Two recent publications based on analyses of DEMs (Digital Elevation Models) emphasize the importance of glacial erosion in cirques, in mid-latitude mountains. In part of the Washington Cascades, Mitchell and Montgomery (2006) note a parallelism between three trends: non-volcano summit altitudes, cirque floors and modern glacier median altitudes. Each rises eastward at 9 to 15 m km<sup>-1</sup>. Only about 10% of each subdivision rises above the highest cirque floors, and few peaks rise more than 600 m above. Rock exhumation rates suggest vertical erosion of 2 to 5 km in the last 15 Ma, and are greatest 30 to 40 km west of the highest peaks. This is not the pattern expected from fluvial or slope erosion, so Mitchell and Montgomery propose a 'glacial buzzsaw' of greatest glacial erosion at the average Quaternary glacial equilibrium line represented by the cirque floors, where ice discharge and velocity were greatest. This increased the slope gradients above cirque floors to over 30°, causing slope failure. Thus both vertical and headward erosion in cirques is considered to dominate landscape development at high altitudes.

Uplift rates are greater in the Kyrgyz Range of the Tien Shan, where Oskin and Burbank (2005) use the sub-Cenozoic unconformity to suggest an east - west spatial gradient of uplift and thus of landform development. As mountains are taken above the snowline, glacial erosion both deepens and widens fluvial valleys, increasing local relief. This starts from the north slope where the snowline is some 200 m lower, pushing the divide 0.9 to 4.4 km southward. Erosion is localized at the bases of cirque headwalls, and cirque headwall retreat is two to three times the rate of vertical erosion. Glacial erosion is thus not simply a function of ice flux, but requires further help, perhaps from subglacial water pressure fluctuations in cirques. "Cirque retreat can effectively bevel across an elevated alpine plateau ..." (Oskin and Burbank, 2005, p. 936).

Discussions of cirque development often assume that the cirque is isolated. In many areas, however, cirque headwalls intersect: cirques are side by side along one side of a ridge or more rarely (where glaciation has been more symmetrical) back to back across a ridge. This may limit further enlargement of cirques despite ongoing erosion, as arêtes are lowered along with cirque floors. Also cirque thresholds may recede by erosion of a glacial trough downstream. Most authors regard both modes of development as important, and Evans (1997) emphasized the need for erosion to be greatest at the base of the headwall, to account for development of the characteristic break of slope and a low-gradient floor. This may be facilitated by water pressure fluctuations, or by a transition from cold to warm, sliding ice as the glacier thickens: both mechanisms encourage plucking of blocks of rock. Erosion of a rock basin is even stronger

evidence of glacial erosion and probably requires rotational flow, but only a minority of cirques have a rock basin or even a lake (Derbyshire and Evans, 1976). If cirque development proceeds differently in different regions, the factors controlling this need to be established. Cirques are eroded both downward and headward. They are initiated by glaciers filling hollows of diverse types, and positive feedbacks (Graf, 1976) prevent them from escaping this inheritance: they remain diverse (Evans and Cox, 1995).

[Fig. 1 about here]

## **2. Data**

### *2.1. Study Area: Wales*

“I cannot imagine a more instructive and interesting lesson for any one who wishes (as I did) to learn the effects produced by the passage of glaciers, than to ascend a mountain like one of those south of the upper lake of Llanberis...convex domes or bosses of naked rock, generally smoothed, but with their steep faces often deeply scored in nearly horizontal lines...” (Darwin, 1842, p. 188). Darwin’s early work on the glaciation of Wales (in the classic area of northern Snowdonia) is of historical importance in the acceptance of the glacial theory, and is set in its full context by Herbert (2005, pp. 277-284).

Choice of Wales as a study area provides a large number of cirques on a variety of rock types from Cambrian to Carboniferous (Silesian) in age, and in relief varying from the narrow mountainous ridges of Snowdon, through the more massive Carneddau Mountains, to the plateaus of central Wales and the sandstone escarpments of the south. Wales is on the west coast of Great Britain, between England and Ireland, and between 51.4 ° and 53.4 ° N. The morphology of Wales defies brief summarization, but the highest ground is in the northwest (northern Snowdonia), followed by the Aran – Cadair Idris Range in the western part of north-central Wales, and the Brecon Beacons in the south (Fig. 1). Wales and its English borderland form the Cambrian Massif, a Caledonian massif with a strong northeast-southwest grain and a Hercynian accretion in the south with a roughly east-west grain. Thomas (1970) has shown many structural influences in the topography. Quaternary tectonic movements are believed to have been subdued.

Position is used as a surrogate for climate (Evans, 1999) as there are few direct climatic observations in cirques and none for the glacial periods when they were developing. In Wales temperature varies mainly with altitude (Sumner, 1997), but at present precipitation varies with exposure, related mainly to distance from the west and south coasts but with a lag so that the wettest mountains (Snowdon, Arenigs, Arans, Pumlumon, the Rhondda area and the western Brecon



Beacons) are 20 to 30 km from the coast. In areas with cirques, modern annual precipitation (1916-50) varied from 4500 mm on Snowdon and 2500 mm on Pumlumon (also spelled Plumlumon and Plynlimon) and the western Brecon Beacons to 1300 mm in the southeast (in the Black Mountains, the Radnor Forest, and near Abergavenny), and in Yr Eifl in northwest Wales (MHLG, 1967). The driest mountains are the Clwydian Range in the northeast, which show no clear cirques. Almost all areas with modern annual precipitation over 2000 mm support glacial cirques. Precipitation in the winter half-year is mainly 54-58% of the total on the mountains of North Wales, and 57-61% on South Wales mountains.

[Fig. 2 about here]

British and Irish cirques are believed to have developed over a series of glaciations, each intermediate in extent between the present non-glacial conditions and glacial maxima when most of Wales, Ireland, Scotland and northern England was covered by a coalescent ice sheet. Wales has undergone both ice-cap and local glaciation (Campbell and Bowen, 1989; Lewis and Richards, 2005). 'Irish Sea' ice from Scotland and Cumbria covered the north coast and western peninsulas, but the local Welsh Ice Cap was strong enough to prevent exotic ice from penetrating the areas with cirques: it built up to at least 850 m altitude from Snowdon to Carnedd Llewelyn (McCarroll and Ballantyne, 2000), and 750 m around Cadair Idris (Ballantyne, 2001) and the Rhinog mountains. Jansson and Glasser (2004) recognize varying ice flow patterns, and extents of cold-based ice, during build-up, maximum, and decline of the Last Glaciation. Cold ice covered the highest summits at the maximum. Ice streams formed during deglaciation, but did not affect areas with cirques (Jansson and Glasser 2004, Fig. 3). As these areas were mainly ice sources, and often covered by ice frozen to the bed, cirques suffered little modification by ice-cap glaciation except in the Moelwyn, Rhinog and Migneint areas, between Snowdon and Cadair Idris.

Wales covers 20,760 km<sup>2</sup>, and the 260 cirques occur in an area 180 x 50 km (Fig. 2). Thus the distribution is much less dense than in the Maritime Alps, where Federici and Spagnolo (2004) measured 432 glacial cirques in 67 x 26 km (1742 km<sup>2</sup>). There is, however, a large gap between the cirques of Mid Wales and those of the Brecon Beacons, and there are few cirques in northeast Wales. The greatest concentrations are in northern Snowdonia (103 cirques in 30 x 18 km), Aran – Cadair Idris (48 cirques in 30 x 11 km) and the Brecon Beacons (30 cirques in 29 x 7 km) (Fig. 2). With one cirque per 5.2, 6.9 and 6.8 km<sup>2</sup> respectively, each of these sub-regions has a density of cirques rather less than the Maritime Alps (one per 4.0 km<sup>2</sup>). The English Lake District (excluding Black Combe) has 155 cirques in 33 x 31 km (Evans and Cox, 1995, Fig. 1a); one per 6.6 km<sup>2</sup>.

In Wales approximately 35% of cirques are named ‘Cwm...’, although this term is also applied to steep-sided fluvial valleys especially below confluences. The classic cirques of the Snowdon area have been well known since Davis (1909), and mapped and illustrated by Addison (1987). Lewis (1938) considered Llyn Cau on Cadair Idris, and Embleton and Hamann (1988) illustrated Glaslyn on Snowdon, but the cirques in many other parts of Wales have received little attention. Distributions of aspect (azimuth) are summarised by vector mean and vector strength (Evans, 2006), also known as mean direction and mean resultant length. Fig. 3 confirms that most cirques face north or east (Evans, 1999): for all 260 cirques the vector strengths are 53% for axis aspect and 58% for headwall aspect; vector means are  $049^\circ$  for both.

Many, but far from all, cirques were occupied by glaciers in the Devensian Late Glacial, the Loch Lomond Stadial (Gray, 1982; Ballantyne, 2001). Evans (1999) showed that the distribution of cirques occupied by these glaciers or by snowpatches was comparable to that of the whole set, but floors averaged 68 m higher and crests, 90 m higher; larger and better-developed cirques are more likely to have been occupied. For the present study, data have been updated from the work of Lynas (1996), Lowe and Larsen (in Ballantyne, 2001), Carr (2001) and Hughes (2002), mainly extending the Late Glacial occupation of cirques. However, the occupation of 83 cirques remains uncertain, pending considerable further fieldwork. Further references on Welsh moraines and glaciation are given in Evans (1999), Walker and McCarroll (2001) and Lewis and Richards (2005).

[Fig. 3 about here]

## 2.2. *Data variables and definitions*

The main data set used here covers all identified cirques in Wales and supersedes the provisional data set used in a previous study of cirque distribution (Evans, 1999). Cirque form has been re-measured and particular attention has been paid to marginal (debatable) features. One objective is to provide a high-quality data set for comparison with that for the Lake District (Evans and Cox, 1995). The Lake District data have been used by a number of investigators (e.g. Cox, 2004, 2005a) and no measurements have been challenged. The only challenge to the definition of cirques there has come from Wilson (2002) who proposed one extra cirque at Blindtarn Moss west of Grasmere: the headwall had been rejected as relating to a transverse ice flow.

If the development of cirques is to be studied, it is essential to consider all cirques from the most debatable to the most classic and from the smallest to the largest. Hence a complete inventory is needed. Cirques were graded 1 to 5, from classic to marginal or debatable (Fig. 2). Field checks have been extensive, but given the large area over which the 260 cirques are spread in Wales,

checks could not be as thorough as those in the Lake District, and it is likely that detailed investigations will add some further cirques and reject some grade 5 (marginal, debatable) cirques. It is supposed that all definite (grade 3 and better) cirques are included here, and that all measurements are accurate. From the present data, the robustness of results can be tested by repeating analyses with exclusion of marginal cirques, or also of poor ones; or more stringent thresholds for floor or headwall gradient can be set.

Numbers in each grade, 1 to 5, are 23, 51, 68, 59 and 59 respectively. Fig. 4 illustrates the characteristics of differently graded cirques, from less well-known areas. The classic and well-defined cirques (Llyn Lluncaws and Craig Trum y Ddysgl) have sharp contrasts between steep headwalls and flat floors; post-glacial talus accumulation has produced or extended intermediate slopes. Cwm Cwareli has a gentler headwall of alternating sandstone and shale and is less deeply enclosed, but still a definite (average) cirque. Craig Rhiw-erch would also be definite but for the steepness of its floor, which makes it marginal and caused hesitation over its inclusion.

Increasingly, such measurements will be made on-line from scanned maps in GIS, and more automatically by processing DEMs. High-quality DEMs are becoming available and their use is particularly appealing as more broadly-based variables can be defined (Evans, 1987; e.g. gradient-weighted vector mean aspect, and vector strength instead of plan closure). Also profiles and surfaces can be fitted within a cirque. At present, the initial stage of defining and delimiting cirques is subjective and involves air photo interpretation and fieldwork; it takes longer than measurement of the variables used here. The availability of accurate manually measured data sets such as this for Wales should provide useful calibration for future GIS-derived measurements.

Cirques are defined according to the agreed definition reported by Evans and Cox (1974). This is compatible with that of de Martonne (1901). Compared with previous definitions of Snowdonian cirques, the present approach appears to be more stringent in requiring at least part of the headwall to be steep, and more tolerant in accepting sloping floors, up to just over 20°. Thus cirques mapped by both Unwin (1973) and Bennett (1990) at Cwm Bychan (Conwy), Cwm Tŷ-du (Llanberis), Cwm Merch (east of Snowdon), Cwm Planwydd (northeast of Nantlle) and Cwm Ciprwrth (above Cwm Pennant, south of Nantlle) are excluded because they lack cliffs, as is Bennett's Cwm yr Afon Goch (Aber). Their cirque on the northwest slope of Moel Cynghorion, west of Snowdon, is retained in spite of a large landslide from its headwall: before this postglacial event, the cirque floor was lower. Addison (1987, and in Addison et al., 1990, p. 13) was also more demanding than Unwin or Bennett, but omitted most of the Moelwyn-Siabod cirques (which have been modified by the over-riding Welsh ice cap). In the South Wales coalfield numerous landslides are mapped, but their distribution is mainly outside cirques.

[Fig. 4 –photos- about here]

Cirques were classified also by type, most being valley-side (157, including those on escarpments) or valley-head (75, of which two thirds have thresholds). All these are simple, as opposed to nested cirques. Seventeen inner (upper) cirques are contained within 11 outer (lower) cirques: the southernmost are on the north slope of Pumlumon, in the middle of Wales. Ten of the inner cirques are contained in five outer cirques on the Snowdon (Yr Wyddfa) range. This concentration arises because here the summits rise highest above the former glacier equilibrium lines. Furthermore, only on Snowdon itself are three nested levels recognized, for Cwm Llydaw and Cwm Llan; Snowdon has cirques on all sides (Davis, 1909; Addison, 1987; Addison et al., 1990). The criterion for recognizing outer cirques around lower floors is that each should have significant additional headwall that served as an ice source, as opposed to a trough side. When measuring the characteristics of outer cirques, their inner cirques are included except for floor characteristics.

### *2.3. Measurements and their accuracy*

Measurement accuracy depends of course on the care taken by those measuring, and on the quality and resolution (e.g. contour interval) of the maps used. The main differences come from use of different definitions in delimiting cirques, as discussed above. Full definitions of the variables measured and calculated are given in Evans and Cox (1995, Tables 1 and 2); many are in common with those used earlier by Gordon (1977) and by Andrews and Dugdale (1971), facilitating comparisons. Using these tried and tested methods, with a given cirque outline and the accurate photogrammetric contours at 10 m interval on Ordnance Survey 1:10,000 scale maps enlarged for use here, measurements are quite accurate and reproducible. Field experience has increased confidence in the accuracy of these contours.

Once the cirque outline is established it is best next to delimit the floor from the headwall. A spacing of 20 m for 10 m contours gives a slope of 1 in 2, or 26.6°, which usefully defines the boundary between cirque floor (ideally <20°: Evans and Cox, 1974) and headwall (ideally >33°, steeper than talus). (A similar spacing is used in delimiting the headwall crest, i.e. the cirque outline, wherever there are gentler slopes above.) Generalizing slightly to give a fairly simple boundary, this permits estimation of maximum floor altitude. It is also needed for locating maximum (head)wall height, along a single slope line, which gives an ancillary measure of vertical dimension.

Outlining the headwall is also useful when it comes to estimating the cirque focal point, the mid-point of the threshold or sill. This is straightforward for cirques which are internally symmetrical, but many cirques deviate from this and many thresholds are sloping, so the mid-point deviates from the lowest point. In these cases there is an inevitable subjectivity, as it is necessary to compromise between the mid-points given by several contours on the headwall, and that half-way between the intersections of the headwall-floor boundary with the cirque outline. Differences in the focal point shift the median axis, changing its aspect and length.

The median axis (Unwin, 1973) has been visually estimated as leaving half the cirque map area to the left, and half to the right. Use of tracings to superimpose the two sides suggests that initial visual estimates give axial aspects within  $5^\circ$  of the final value, which in turn is within  $2^\circ$  of the true value. Thus such manual estimates are fully comparable with the exact measurements now possible in GIS (Federici and Spagnolo, 2004). Estimates of headwall aspect are less precise, but they are unaffected by uncertainty over the focal point and less affected by variations in cirque outline definition, such as those due to possible headwall extensions with marginal gradients. Here the future use of DEMs will permit calculation of a headwall resultant vector based on point values of slope aspect, weighted by gradient in excess of the  $26.6^\circ$  threshold. The present manual estimates are considered accurate within  $10^\circ$ . It is reassuring also that the difference between headwall and axis aspect is symmetrically distributed, with a standard deviation of only  $20^\circ$ ; its range is  $\pm 65^\circ$ .

Six altitude variables are defined. Estimates are accurate within 5 m, but considerable variance may come from differences in cirque outline, floor boundary and median axis. Thus the most reproducible variables are probably modal floor altitude and maximum crest altitude. The maximum altitude above, draining into a cirque, is often given exactly by a spot height, but estimates could be out by tens of metres where the location is uncertain on a mountain shoulder with divergent flow.

Again given the focal point and outline, length and width measurements are accurate within 10 m. Larger differences come from shifting the focal point or changing the cirque boundary. Visual estimates of the perpendicular nature of length and width were within  $2^\circ$ . Plan closure (Evans, 1969; Gordon, 1977) is a variable that requires some experience, to avoid problems with quadrants and complementary angles, but after careful measurement reproducibility within a few degrees is achieved.

With these methods of measurement and estimation from maps, and given a map with the cirque outline, all directly measured variables can be established within about an hour. This varies with cirque size, between about 30 and 90 minutes. Once a data set is complete and entered into a

statistical program, hours should be spent on consistency and outlier checks, extending to days if numerous errors are discovered and corrected. Before the final measurement by the author, most of these cirques were measured at different times, from different maps, and/or by different people, under the author's supervision. This improved the establishment of the best cirque outline, and aided avoidance of substantial errors.

[Fig. 5 about here]

### **3. Results**

#### *3.1. Size*

Welsh cirques are comparable in size to those described from Scotland and the English Lake District. They do not fall into discrete size classes, but give fairly smooth continua on all measures of size. Positive skewness ranges from 1.22 for wall height to 2.45 for length. After logarithmic transformations these are reduced to  $-0.06$  and  $0.32$ . For variables measuring components of vertical dimension, generated as differences between altitude variables, skewness of  $0.76$  to  $2.49$  is reduced to  $-0.22$  to  $-0.76$ . Fig. 5a shows how well-behaved the main size variables are on a logarithmic scale. These quantile plots (Cleveland, 1994, pp. 143-9) permit the observed distribution to be compared with a model probability distribution, in this case the log-Gaussian (also known as log-normal). Compared with histograms, they emphasize the overall shape and show every observation, unaffected by binning (classing), and they 'stack' onto one plot to facilitate comparisons (Cox, 2005b). Their linearity confirms that each is well approximated by the log-Gaussian model, and thus all further analyses deal with logarithms of these size variables. Note that each variable is ranked separately, so vertical comparisons on Fig. 5a generally concern different cases.

As is usual, extreme values are more erratic than those in the middles of the distributions. One notable effect is that ranked length is less than ranked width except for the four longest cirques. These are in fact the outer cirques of Cwms Llydaw, Llan and Dwythwch, all on Snowdon, plus Cwmffynnon on the Glyderau. (Except for Cwm Dwythwch these are also the three widest cirques, although the ranking process for the plots is independent for each variable.) Omission of all 11 'outer' cirques reduces skewness, but when relations between variables are considered (below) these cirques are 'on trend' and strengthen rather than weaken correlations, so it is best to retain them.

As in other regions, for example the Rocky Mountains of the USA (Graf, 1976) or Canada (Trenhaile, 1976), Welsh cirques have width close to length so that plan form is compact. Median

(and mean) values for Wales are 610 (667) m for length, 700 (772) m for width and 215 (236) m for amplitude. Spreads represented by the 5 and 95 percentiles (used here in preference to the range of extreme values) are 310-1235 m for length, 375-1502 m for width and 112-431 m for amplitude. These dimensions are comparable to those in England and in Scotland (Gordon, 1977), but rather smaller than those in northern Scandinavia (Hassinen, 1998) and considerably smaller than those in the Canadian Rockies and Columbia Mountains (Trenhaile, 1976) and the Antarctic Dry Valleys (Aniya and Welch, 1981). Cirques in the Maritime Alps (Federici and Spagnolo, 2004), the central Spanish Pyrenees (García-Ruiz et al., 2000) and the Cayoosh Range of British Columbia (Evans, 1994, Table 26.3) are comparable to Welsh cirques in area, but have rather greater vertical dimensions.

Confidence intervals show that cirque width is significantly greater than length, more so in Wales than in the Lake District. Width exceeds length also in the central Pyrenees (García-Ruiz et al., 2000), in central Sweden (Vilborg, 1984) and in northern Scandinavia (Hassinen, 1998) but in most other studies (northwest Scotland, the Maritime Alps, the whole Italian Piemonte, the Cayoosh Range, Baffin Island, the Canadian Rockies and Columbia Mountains and the Antarctic Dry Valleys) length is commonly greater. Width/length ratio averages 1.22 in Wales, and ranges from 0.49 to 3.47, so only one cirque is twice as long as broad. Cirques with width over twice length tend to be poor or marginal. Six of the 16 are in the Rhondda – Hirwaun group, and a further six are also in South Wales; all but one of these are on sandstone.

[Fig. 6 about here]

Fig. 6 shows, in five equal classes, the spatial distribution of cirque (overall) size, a combination of the three orthogonal dimensions. Size is defined as the cube root of (length x width x amplitude), so that it too has units of metres: it is required for analyses later in the paper. Larger cirques are concentrated in northern Snowdonia, and smaller ones are more common in South Wales. Cadair Idris and most northern regions have means above 460 m while the Brecon Beacons average 448 m and Corris and the rest of South Wales have means below 420 m, except that Pumlumon (mean 486 m) can compete with the northern regions and Migneint and Upper Dyfi bring down their regional group means.

These regional variations in the size and shape of Welsh cirques are further demonstrated by the variables in Figs. 7 and 8. The point-and-box plots show both the individual data and the quartile summaries (25, 50 and 75 percentiles). They are preferred over conventional box plots,

which can mislead by masking the number of cases on which they are based, and which have difficulties handling log scales because inter-quartile range is dependent on the metric.

As expected, the greatest vertical dimensions are in the Snowdon and Glyder groups, followed by Carneddau, Nantlle – Hebog, Cadair Idris and the Aran Range (Fig. 1), with mean amplitudes greater than 250 m. All these are within the main Ordovician volcanic belt, although some cirques are on intercalated siltstones. The least deep vertically are in South Wales (means less than 195 m) and in the Migneint area (between the Arenig and Moelwyn mountains), followed by Corris and Pumlumon (Fig. 7a). These are on sedimentary or lightly metamorphosed rocks. The low outlier for Arenig – Migneint is Llynnau Barlwyd, where the median axis intersects a very low central col, an unusual situation.

Length too is greater in North Wales than in the South (Fig. 7b) but Cadair Idris, Nantlle – Hebog and the Carneddau have surprisingly low medians and the cirques of Mid Wales (Pumlumon) are surprisingly long. There is, however, much less North – South contrast in width (not illustrated), which is greatest in the Western Brecon Beacons and Moelwyn – Siabod and least in the Black Mountains, Abergavenny and Corris.

These results are affected by inclusion of marginal cirques, which form 30% of the total from Corris southward, but only 18% farther north. Overall, in spite of some impressive cirques on sandstone in South Wales, cirques on sedimentary, slaty and greywacke rocks are smaller than those on volcanic and igneous rocks. For analyses of variance over the 17 regional groups,  $R^2$  values are 0.294, 0.093 and 0.014 for the logarithms of amplitude, length and width respectively. Length is significant at  $P=0.0007$ , whereas width is quite insignificant.

Fig. 5b shows that altitude variables are well distributed without transformation, across broad ranges, except that ‘high’ altitudes cannot exceed the limiting value of 1065 m, the highest point in Wales.

[Fig. 7 about here]

### 3.2. Shape and gradient

Gradient and closure variables are symmetrically distributed (Fig. 5c and d) except for minimum gradient where proximity to the lower bounding value of zero inevitably produces positive skew. Reversed slopes in lakes and bogs are excluded because of inadequate data. As in the Lake District (Evans and Cox, 1995, Fig. 5), quantile plots for plan closure (Fig. 5d) and axial gradient are very linear, confirming well-behaved Gaussian distributions with no need for



transformation. Profile closure and maximum and minimum gradient have tails shorter than the Gaussian, giving mildly S-shaped quantile plots.

Measuring how deeply cirques cut into mountains, closure in plan is greatest (mean 149°) in the Pumlumon group, where most cirques are at the heads of valleys incised into the plateau. Otherwise it is greater in groups containing igneous rocks, including the Berwyns (Fig. 7c). The high outlier for Moelwyn is Cwmorthin (295°), which has a complicated headwall. Easily the lowest plan closures are in the Black Mountains (61°), where the four cirques are shallow recesses in long, smooth valley sides. Other sandstone groups, including the Rhinogs, have relatively low closures. Over the 17 regional groups,  $R^2$  is 0.073,  $P = 0.0043$ .

Maximum gradients (not illustrated) show a clear contrast between the five mountain groups of northern Snowdonia, plus Cadair Idris and the Arans, and all the rest. Low values are found south of Cadair Idris, especially in the Western Brecon Beacons. Minimum gradients are lowest throughout Snowdonia. High minima for Corris (mean 9.4°) and the six Upper Dyfi cirques reflect the inclusion of marginal cirques on the weak Silurian and Upper Ordovician siltstones.  $R^2$  is 0.191 for maximum and 0.158 for minimum gradient; both are highly significant.

Axial gradient (Fig. 7d;  $R^2$  is 0.180) is a more representative measure of overall cirque steepness. All groups have values both above and below 20°, but Corris and the four north-western Snowdonia groups have means above 21°. Moelwyn – Siabod joins the three groups immediately to its south in having low gradients, together with all of South Wales where the Brecon Beacons have means just below 18°. This mixed picture arises because cirques are steep both where relief is highest (around Snowdon) and where cirque development is more marginal (as around Corris).

As a glacier filling a cirque is likely to start near the top of the median axis, axial gradients are usefully compared with the gradients of cirque glaciers. Glaciers not reaching that high up the headwall, and those overflowing the cirque threshold, would have gentler gradients. Only 10% of Welsh axial gradients are less than 13°, which is sufficient to support rotational flow (section 1.2). Only three (1% of cirques) are gentler than 8°; 10% are steeper than 30°, and the maximum axial gradient is 38°. Thus the great majority of cirques can be related to cirque glaciers or glacier sources capable of rotational flow.

As in the Lake District, correlations within this group of shape and gradient variables are weak, except that axial gradient correlates +0.63 with minimum gradient. Maximum and minimum gradients correlate only –0.26. Plan closure correlates –0.35 with axial, but only  $\pm 0.19$  with the other two gradients (still significant, at the 0.005 level), and –0.20 with width/length. Principal component analysis of six shape and gradient variables confirms the weakness of their interrelations; the components have 40, 26, 19, 12, 3 and 0 percent of the total variance. This

contrasts with the six altitude variables, for which the first component has 86%, and the seven logarithmic size variables, 71%.

### 3.3. Geological effects

All erosional landforms are affected by the material into which they cut, but it is often difficult to pin down precise morphometric contrasts between lithologies (Evans, 1994). In Snowdonia, Unwin (1973, p. 87) noted structural control of detail but “considerable disregard for geological structure” with cirques cross-cutting different lithologies in the Nantlle – Hebog group. In the Snowdon group and especially in Y Glyderau, some cirques are elongated along the strike. Floors are often on weaker strata, giving exaggerated forms deep in profile in several cirques of Y Carneddau. Bennett (1990) developed the concept of ‘strike cirques’ further.

The importance of joints, faults and other planes of parting in cirque development was emphasized for example by Haynes (1968) and Addison (1981). In Snowdonia, the fracture network disregards lithological boundaries, but lithology controls fracture spacing and rock mass strength at smaller scales (Addison, 1981). However, data are not available on a broad basis and for the present survey the units mapped by the British Geological Survey and others are used. Thirteen units are distinguished in Fig. 8; 22% of Welsh cirques are on Devonian and Carboniferous sandstones (with intercalated shales), 14% are on weak Ordovician and Silurian siltstones and greywackes, 19% are on tuffaceous Ordovician sediments, 29% are on Ordovician volcanic rocks, 8% are on Cambrian rocks and 6% are on intrusive granitic rocks and dolerites.

For each variable, a one-way analysis of variance gives the variability accounted for by the 13 classes of geology. Table 1 ranks the 14 variables, and shows that the most closely related to geology are vertical dimensions and altitudes, followed by gradients (especially maximum) and plan closure. The relations are weaker than those over regional groups, but their ranking is comparable. The variation of length with geology is small (Fig. 8a), and that of width is insignificant. Coal Measures cirques (which have mainly Pennant sandstone headwalls) are shorter than others, and have lower amplitudes, but are just as wide. Lengths are greatest on tuffs.

[Table 1 about here]

The strongest relation to geology comes from relief (within 2 km radius), which is another controlling factor rather than a cirque characteristic. This suggests considering the joint effects of the two on the other variables. Relief is in fact the greater control for maximum gradient, axial gradient, size and length, with no additional effect from geology. It is also the greater for maximum

crest altitude, wall height, height range and amplitude, for which geology has small but significant further effects. On the other hand, relief is quite unrelated to plan closure, width and minimum gradient, and has no significant addition for lowest altitude. Thus it seems that for vertical dimensions, related gradients, length and thus size, relief is the direct control, but is itself affected by geology. Geology affects plan closure and minimum gradient (floor development) directly. For a selection of well-developed Austrian and British cirques, Embleton and Hamann (1988) also found that relief was more important than geology in controlling cirque form.

Rock basin lakes are a characteristic more closely related to geology (Haynes, 1968; Evans, 1994). In Wales, 11 of the 21 major rock basin lakes are on 'ignimbrite, lava and rhyolite', forming 29% of the cirques on that geological class. There are none on Lower Ordovician siltstones or on Silurian or younger rocks, although there are three major moraine-dammed lakes on Coal Measures sandstones and three more on Old Red Sandstone (Devonian).

[Fig. 8 about here]

Fig. 8b shows greater cirque amplitude (height range along the median axis) for volcanic and igneous rocks and tuffaceous sediments than for other rock types; amplitude is lowest on Devonian and Carboniferous rocks. As amplitude is more variable between rock types than are length and width (Table 1), variation in the compound variable 'size' is similar to that for amplitude. Relief (Fig. 8c) shows a similar pattern (and relief at 1 km radius correlates +0.89 with relief at 2 km), with clearly lower values on the younger, sedimentary rocks of the South, including the Silurian, and highest values on the three volcanic rock types and on microgranite; tuffaceous siltstones rank lower than for amplitude.

Maximum gradient (Fig. 8d) shows a clear dichotomy around  $64^\circ$ , between steep headwalls on volcanic and igneous rocks and gentle ones on sedimentary rocks – including Cambrian grits, and tuffaceous siltstones but not tuffaceous sandstones. Plan closure shows a different pattern (Fig. 8e), being greater on Silurian and Lower Ordovician siltstones, mudstones and greywackes (with numerous valley-head cirques), and lowest on the intrusive rocks (microgranite and dolerite) which are difficult to incise. It is fairly low on sandstones: around  $100^\circ$ , whether Cambrian, Devonian or Carboniferous.

### 3.4. *Effects of other controls*

As expected, the altitude and size groups of variables each have strong intercorrelations. The three 'high' altitude variables all correlate +0.91 or more: the three 'low' ones, +0.93 or more,

and all six, +0.68 or more. Width and length, as logarithms, correlate +0.72. Length correlates +0.62 to +0.71 with the three measures of vertical dimension, whereas width correlates only +0.36 to +0.45.

Thus it is necessary to be selective in relating characteristics to combinations of others. Length, width and amplitude all increase with (any of the three) maximum altitudes, and they increase as lowest altitude decreases. Regressions including dummy variables for categories show that length is a function first of maximum altitude above, followed by lowest altitude, cirque type and occupation, with  $R^2 = 0.54$ , to which relief adds a little but geology adds nothing. Width gives  $R^2 = 0.36$  for the same four, with lowest altitude being more important than maximum: relief increases this to 0.41. Amplitude relates more to maximum altitude than to lowest, and as it is calculated as the difference between a high and a low variable, including both gives a spuriously strong prediction.  $R^2 = 0.41$  for amplitude as a function of maximum altitude above, cirque type and geology; relief increases this to 0.47 but occupation gives no further improvement.

[Table 2 about here]

#### 4. Allometry

##### 4.1. Variations of shape with size

Table 2 shows that maximum headwall gradient increases with all five size variables, and minimum floor gradient decreases. But maximum increases mainly with vertical dimensions, to which minimum is barely related. Overall axial gradient, as expected, rises with vertical dimensions and falls with increasing length and width. The width/length ratio has modest correlations with all size variables; these are negative for vertical dimensions, which as noted above relate more to length than to width. Plan closure also relates first to length, then to width. Larger cirques are likely to have more cols (over 30 m deep) in their crests; average sizes are 440 m for no cols, 566 m for one, 743 m for two, 754 m for three, and 1501 m for the one cirque with four cols.

[Fig. 9 about here]

A more graphic way of relating shape to size is to divide size into five equal classes and draw representative profiles for each (Fig. 9). The choice of five classes gives a reasonable number, 52, in each class for reliable estimation of mean values. For each size class, modal and maximum floor altitude, median and maximum crest altitude, and maximum altitude above are plotted relative to lowest altitude, and against horizontal coordinates based on length, minimum gradient (lowest

segment), maximum gradient (either side of the dot), and an assumed  $10^\circ$  above the crest. This is comparable to the developmental diagrams in Gordon (1977) and Bennett (1990), but based more directly on the data. Assuming a space-time transformation, headwall recession is greater than vertical enlargement, but the latter is considerable.

Development in length and width can be portrayed by a similar series (Fig. 10), using mean plan closures to dictate the curvatures. The full width is used, but as plan closure is measured along the mid-height contour which does not extend to either end of the median axis, only half the length is represented here: these are not cirque outlines. Shape changes are more subtle than size changes, but there is a progressive broadening out.

[Fig. 10 about here]

#### *4.2. Allometric development*

The concept of allometry, widely applied in developmental biology, simply implies that shape changes as organisms grow. Thus it is opposed to isometry, the maintenance of constant shape. These alternatives are commonly assessed on logarithmic scales, so that a given interval on an axis represents multiplication by a given factor. Applications to surveys of landforms at a single time are based on the assumption that the landforms have grown over time. This is not unreasonable for bedrock landforms, especially those such as cirques where positive feedback is involved. Application of allometry to glacial cirques was initiated by Olyphant (1981, p. 681), who took 'volume' (length x width x depth) as a reference measure of size to which other size measures could be related. He defined length as from cirque lip to headwall midpoint, width as the average of four measurements equally spaced along the length axis, starting at the cirque lip, and depth as the vertical difference between the cirque lip and the average cirque rim altitude. The width and depth measures are more averaged than those used here, but this is likely to affect the base constants rather than the gradients of logarithmic relations.

Here, following Evans and Cox (1995) and Evans and McClean (1995), I use median-axial length, maximum width at right angles to that axis, and vertical amplitude as the difference between crest altitude at the median axis and lowest altitude. 'Volume' is the product of the three linear dimensions, and 'size' is its cube root, useful in giving a linear measure comparable in magnitude to the three original measures. As  $\text{volume} = \text{size}^3$ , the power exponents (gradients of logarithmic regressions) are three times as great for size as for volume. That is, while length, width and amplitude exponents sum to 1.0 for predictions from volume, they sum to 3.0 for predictions from size.

An advantage of the present data set is that there are two further definitions of vertical dimension, whose exponents can be compared with those for amplitude. Wall height is measured independently as the greatest drop along any headwall slope line, from crest to floor at right angles to contours. Overall height range is the extreme, relating maximum crest altitude to lowest altitude: it cannot be less than amplitude or wall height. It is also possible to compare exponents for different geologies or grades. Confidence intervals are considered the best way of comparing the feasible variations in estimates of exponents, and the significance of differences between them.

[Table 3 about here]

Table 3 confirms that exponents for the three components of size do indeed sum to 3.0. It shows that length increases (with size) faster than does width, and width probably increases faster than amplitude. Width and wall height show exponents not significantly different from 1.0 throughout; thus they are isometric, increasing in proportion to overall size. Length has significantly greater exponents; it increases faster than size. The confidence interval on the length exponent does not overlap with any other, except wall height. The results for overall height range are very similar to those for amplitude, except for valley-head cirques. The ‘better’ column, where poor and debatable cirques are excluded, gives very similar exponents except for wall height. Although ‘outer’ cirques form a distinctly larger population, their exclusion makes no difference to the exponents obtained. Valley-side cirques show almost the same exponents as the total set, but the smaller set of valley-head cirques does give a steeper variation in amplitude with size, and a reduced variation in length. Fig. 11 shows that the scatter of data points around these regressions is well-behaved, with near constant variance on a logarithmic scale.

[Fig. 11 about here]

These results provide a powerful confirmation of cirque allometry, robust over different cirque types. Length increases faster than size, and vertical dimension increases more slowly whether axial or overall height range is used (Fig. 11); this is consistent with Gordon’s (1977, Fig. 7) model. Large cirques are relatively longer and less deep than if growth were isometric. The isometry of wall height is interesting, but it does show considerably more scatter than the other vertical variables in both dimensionless ( $R^2$ ) and dimensional (RMS error) terms.

[Table 4 about here]

From Table 4 it seems that allometry of cirques in Wales is less pronounced than in the other areas. This may reflect the varied relief and geology of Wales. The greater increase of length than width is shared with the English Lake District and the Cayoosh Range. It is contrary to what would occur if allometry were due to lateral coalescence, which would increase width rather than depth.

Relations within different regions and broad rock types will now be considered. As confidence intervals are too broad for the 13 classes, three broader divisions of geology are used: igneous and volcanic; other, that is Cambrian and Ordovician sediments including tuffaceous; and Silurian and younger rocks. In relation to Fig. 8, these three are defined as the first five geological classes, the next four, and the final four which equate to South Wales and parts of Mid Wales.

In Table 5, the strongly overlapping confidence intervals for each variable show that none has exponents differing significantly between these three rock types. The high  $R^2$  and low RMS errors confirm the strengths of these relations. The poorer result for amplitude on 'other' rocks is due to the outlying but real value for Llynau Barlwyd (on siltstones), noted above: thus overall height range gives a better result for 'other'. Wall height shows greater scatter than the other variables, and unlike height range and amplitude its exponents for igneous and other do not differ significantly from 1.0.

[Table 5 about here]

Throughout Table 5, length exponents consistently exceed those for width, except on the Silurian and younger rocks, and all vertical dimensions produce lower exponents. Thus the results of Table 3 apply across the three main rock divisions. A similar analysis (not shown) for three broad regional divisions, North (Carneddau to Moelwyn-Siabod), Mid (Arenig-Migneint to Pumlumon) and South (Brecon Beacons to Rhondda) further reinforced the results. Here all three vertical dimensions gave exponent values significantly less than 1.0 for North and South, but closer to 1.0 for Mid Wales, which mixes high and low relief. Length and width gave highly overlapping confidence intervals, with the length exponent greater in North but the width exponent slightly greater in South.

## 5. Summary and Conclusions

In Wales, width exceeds length in 66% of the 260 cirques. This ratio applies across different cirque types, but rises to 91% for the 56 cirques on Devonian and Carboniferous sandstones, often on escarpments or straight valley-sides. Width averages 772 m, length 667 m,

(axial) amplitude 236 m, overall height range 269 m and wall height 197 m. Cirque dimensions are thus comparable to those in other mid-latitude studies, and especially to those in the English Lake District. Axial gradients are compatible with rotational flow of former glaciers in the great majority of these cirques.

Although differences between regions in length and width and their variations could arise from different practices of cirque definition, this is not the case for the contrast between Wales and the Lake District, with mean length/width ratios of 0.90 and 0.94, and the Cayoosh Range with 1.14, as I am responsible for all these definitions. The differences probably reflect regional topographic and tectonic settings, especially the degree of dissection of the landscape and the available relief. Although length and width behave somewhat differently, the slower increase of vertical dimension with cirque size is observed in all these regions.

If size (a combination of length, width and amplitude) is divided into five quintiles, each of its components (and the other vertical dimensions) increases monotonically, but there are changes in shape. Length increases most, vertical dimensions least. Plan and profile closure increase, and maximum gradient increases faster than minimum decreases.

These data provide the strongest test yet of allometry in glacial cirques. As elsewhere, vertical dimensions – amplitude, height range and wall height – increase with overall size, but significantly less than do horizontal dimensions. In Wales, only length has an exponent significantly greater than 1.0; that is, it increases faster than size in general. With minor variations, these results hold over different cirque types, regions and rock types. This type of allometry is logical given the role of glacial erosion in cirque development; the glacier rises higher against the backwall and moves away from it faster than from the sidewalls, thus increasing length faster than width. It is also reasonable that headwall retreat, often by collapse following glacial erosion at the base (Evans, 1997), is faster than downward erosion. Such importance of cirque headwall retreat has been emphasized recently by Mitchell and Montgomery (2006) and especially by Oskin and Burbank (2005), raising the question whether glacial erosion in mid-latitude mountains works largely as a ‘buzzsaw’ at the snowline or mainly by the calibration of glacial troughs to ice discharge. The divide retreat proposed by Evans (1972), Oskin and Burbank (2005) and others, shows that lateral coalescence of cirques cannot explain the greater increase of horizontal dimensions, especially since length increases with size faster than does width.

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## References

Addison, K., 1981. The contribution of discontinuous rock-mass failure to glacial erosion. *Annals of Glaciology* 2, 3-10.

Addison, K., 1987. *Snowdon in the Ice Age*. K. & M. K. Addison, Broseley, Shropshire, 28 pp. ISBN 0 9511175 1 3.

Addison, K., Edge, M.J., Watkin, R., (Eds.), 1990. *The Quaternary of North Wales: Field Guide*. Quaternary Research Association, Coventry, 190 pp.: see pp. 11-19, 77-81, and 99-103.

Ahlmann, H.W., 1919. Geomorphological studies in Norway. *Geografiska Annaler* 1, 1-148.

Andrews, J.T., Dugdale R.E., 1971. Factors affecting corrie glacierization in Okoa Bay. *Quaternary Research* 1, 532-551.

Aniya, M., Welch, R., 1981. Morphometric analyses of Antarctic cirques from photogrammetric measurements. *Geografiska Annaler* 63 A, 41-53.

Ballantyne, C.K., 2001. Cadair Idris: a Late Devensian palaeonunatak. In: Walker, M.J.C., McCarroll, D. (Eds.), *The Quaternary of West Wales: Field guide*. Quaternary Research Association, London, pp. 126-131.

Bennett, M., 1990. The cwms of Snowdonia: a morphometric analysis. Dept. of Geography, Queen Mary & Westfield College, London, Research Paper, 2, 48pp.

Biot, P., 1968. Les développements récents des théories de l'érosion glaciaire. *Annales de Géographie* 77, 1-13.

Blache, J., 1952. La sculpture glaciaire. *Revue de Géographie Alpine* 40, 31-123.

Blache, J., 1960. Les résultats de l'érosion glaciaire. *Méditerranée* 1, 5-31.

Brocklehurst, S.H., Whipple, K.X., 2002. Glacial erosion and relief production in the Eastern Sierra Nevada, California. *Geomorphology* 42, 1-24.

Brook, M.S., Kirkbride, M.P., Brock, B.W., 2006. Cirque development in a steadily uplifting range: rates of erosion and long-term morphometric change in alpine cirques in the Ben Ohau Range, New Zealand. *Earth Surface Processes and Landforms*, in press.

- Campbell, S., Bowen, D.Q., 1989. Quaternary of Wales. Geological Conservation Review. Peterborough, Nature Conservancy Council.
- Carr, S., 2001. A glaciological approach for the discrimination of Loch Lomond Stadial glacial landforms in the Brecon Beacons, South Wales. *Proceedings, Geologists' Association* 112, 253-262.
- Clark, J.M., 1951. The investigation of a possible method of cirque erosion. U.G.G.I. Association Internationale d'Hydrologie Scientifique, Publication 32 (Bruxelles Symposium, v.1), 215-221.
- Clark, J.M., Lewis, W.V., 1951. Rotational movement in cirque and valley glaciers. *Journal of Geology* 59, 546-566.
- Cleveland, W.S., 1994. The elements of graphing data. Hobart Press, Summit N.J., 297 pp.
- Cox, N.J., 1977. Allometric change of landforms: discussion. *Geological Society of America, Bulletin* 88, 1199-2000.
- Cox, N.J., 2004. Graphing distributions. *Stata Journal* 4, 66-88.
- Cox, N.J., 2005a. Density probability plots. *Stata Journal* 5, 259-273.
- Cox, N.J., 2005b. The protean quantile plot. *Stata Journal* 5, 442-460.
- Darwin, C.R., 1842. Notes on the effects produced by the ancient glaciers of Caernarvonshire, and on the boulders transported by floating ice. *London, Edinburgh and Dublin Philosophical Magazine and Journal of Science* 21, 180-188.
- Davis, W.M., 1909. Glacial erosion in North Wales. *Quarterly Journal, Geological Society of London* 65, 281-350.
- Davis, W.M., 1911. The Colorado Front Range. *Annals, Association of American Geographers* 1, 21-84.
- Derbyshire, E., Evans, I.S., 1976. The climatic factor in cirque variation. In: Derbyshire, E. (Ed.), *Geomorphology and Climate*. J. Wiley, New York and London, pp. 447-494.
- Embleton, C., Hamann, C., 1988. A comparison of cirque forms between the Austrian Alps and the Highlands of Britain. *Zeitschrift für Geomorphologie, Supplementband* 70, 75-93.
- Embleton, C., King, C.A.M., 1968. *Glacial and Periglacial Geomorphology*. Edward Arnold, London, 608 pp [2<sup>nd</sup> Edition: 1975].
- Evans, I.S., 1969. The geomorphology and morphometry of glaciated mountains. In: Chorley, R. J., (Ed.), *Water, Earth and Man*. Methuen, London, pp. 369-380.

- Evans, I.S., 1972. Inferring process from form: the asymmetry of glaciated mountains. In: Adams, W.P., Helleiner, F.M. (Eds.), *International Geography 1972 v 1*, University of Toronto Press, Toronto, pp. 17-19.
- Evans, I.S., 1987. The morphometry of specific landforms. In: Gardiner, V. (Ed.), *International Geomorphology 1986 Part II*. J. Wiley, Chichester, pp. 105-124.
- Evans, I. S., 1994. Lithological and structural effects on forms of glacial erosion: cirques and lake basins. In: Robinson, D. A., Williams, R.B.G. (Eds.), *Rock Weathering and Landform Evolution*. J. Wiley, Chichester, pp. 455-472.
- Evans, I.S., 1997. Process and form in the erosion of glaciated mountains. In: Stoddart, D.R. (Ed.), *Process and Form in Geomorphology*. Routledge, London, pp. 145-174.
- Evans, I. S., 1999. Was the cirque glaciation of Wales time-transgressive or not? *Annals of Glaciology* 28, 33-39.
- Evans, I.S., 2006. Local aspect asymmetry of mountain glaciation: A global survey of consistency of favoured directions for glacier numbers and altitudes. *Geomorphology* 73 (1-2), 166-184.
- Evans, I. S., in press. Glacial erosional processes and forms: mountain glaciation and glacier geography. In: Burt, T.P., Brunnsden, D., Goudie, A.S., Cox, N.J. (Eds.), *The History of the Study of Landforms or the Development of Geomorphology*, v. 4: Quaternary and Recent Processes and Forms (1890-1960s) and the Mid-Century Revolutions. Routledge, London.
- Evans, I.S., Cox, N.J., 1974. Geomorphometry and the operational definition of cirques. *Area* (IBG, London), 6, 150-153.
- Evans, I.S., Cox, N.J., 1995. The form of glacial cirques in the English Lake District, Cumbria. *Zeitschrift für Geomorphologie*, N.F. 39, 175-202.
- Evans, I.S., McClean, C.J., 1995. The land surface is not unifractal: variograms, cirque scale and allometry. *Zeitschrift für Geomorphologie*, N.F. Supplementband 101, 127-147.
- Federici, P.R., Spagnolo, M., 2004. Morphometric analysis on the size, shape and areal distribution of glacial cirques in the Maritime Alps (Western French-Italian Alps). *Geografiska Annaler* 86 A, 235-248.
- Galibert, G., 1962. Recherches sur les processus d'érosion glaciaires de la Haute Montagne Alpine, *Bulletin, Association des Géographes Françaises* 303-304, 8-46.
- García-Ruiz, J.M., Gómez-Villar, A., Ortigosa, L., Martí-Bono, C., 2000. Morphometry of glacial cirques in the Central Spanish Pyrenees. *Geografiska Annaler* 82A, 433-442.
- Gordon, J., 1977. Morphometry of cirques in the Kintail-Affric-Cannich area of NW Scotland. *Geografiska Annaler* 59A, 177-194.
- Graf, W.L., 1976. Cirques as glacier locations. *Arctic & Alpine Research* 8, 79-90.

- Gray, J.M., 1982. The last glaciers (Loch Lomond Advance) in Snowdonia, North Wales. *Geological Journal*, 17, 111-133.
- Grove, J.M., 1960. A study of Veslgjuv-breen. In: Lewis, W.V. (Ed.), *Norwegian Cirque Glaciers*. R.G.S. Research Series, 4, Royal Geographical Society, London, pp. 69-82.
- Hanson, G., 1924. Reconnaissance between Skeena River and Stewart, British Columbia. *Geol. Survey of Canada, Summary Report for 1923, A*, 29-45.
- Hassinen, S., 1998. A morpho-statistical study of cirques and cirque glaciers in the Senja – Kilpisjärvi area, northern Scandinavia. *Norsk geografisk Tidsskrift* 52, 27-36.
- Haynes, V.M., 1968. The influence of glacial erosion and rock structure on corries in Scotland. *Geografiska Annaler* 50A, 221-234.
- Herbert, S., 2005. *Charles Darwin, geologist*. Cornell University Press, Ithaca, N.Y., 485 pp.
- Hobbs, W.H., 1910. The cycle of mountain glaciation. *Geographical Journal* 35, 146-163 and 268-284.
- Hobbs, W.H., 1911. *Characteristics of Existing Glaciers*. Macmillan, New York, 301pp.
- Hughes, P.D., 2002. Loch Lomond Stadial glaciers in the Aran and Arenig Mountains, North Wales, Great Britain. *Geological Journal* 37, 9-15.
- Jansson, K.N., Glasser, N.F., 2004. Palaeoglaciology of the Welsh sector of the British-Irish Ice Sheet. *Journal of the Geological Society, London* 161, 1-13.
- Johnson, W.D., 1904. The profile of maturity in alpine glacial erosion. *Journal of Geology* 12, 569-578. (Reprinted in Embleton, C. (Ed.), 1972, *Glaciers and Glacial Erosion*. Macmillan, London, pp. 70-78.)
- Lewis, C. A., Richards, A.E. (Eds.), 2005. *The Glaciations of Wales and Adjacent Areas*. Herefordshire, Logaston Press, 228 pp.
- Lewis, W.V., 1938. A melt-water hypothesis of cirque formation. *Geological Magazine* 75, 249-265.
- Lewis, W.V., 1940. The function of meltwater in cirque formation. *Geographical Review* 30, 64-83. [Discussion 1949, v. 39, pp. 110-128]
- Lewis, W.V., 1949. Glacial movement by rotational slipping. *Geografiska Annaler* 31, 146-158.
- Lewis, W.V., 1960. The problem of cirque erosion. In: Lewis, W.V. (Ed.), *Norwegian Cirque Glaciers*. R.G.S. Research Series, 4, Royal Geographical Society, London, pp. 97-100.
- Lynas, B., 1996. *Snowdonia Rocky Rambles: Geology beneath Your Feet*. Sigma Press, Wilmslow, 273 pp.

- de Martonne, E., 1901. Sur la formation des cirques. *Annales de Géographie* 10, 10-16.
- McCall, J.G., 1960. The flow characteristics of a cirque glacier and their effect on glacial structure and cirque formation. In Lewis, W.V. (Ed.), *Norwegian Cirque Glaciers*. R.G.S. Research Series, 4, Royal Geographical Society, London, pp. 39-62. (Reprinted in Embleton, C. (Ed.), *Glaciers and Glacial Erosion*. Macmillan, London, pp. 205-228.)
- McCarroll, D., Ballantyne, C.K., 2000. The last ice sheet in Snowdonia. *Journal of Quaternary Science* 15, 765-778.
- McSaveney, M.J., 2002. Recent rockfalls and rock avalanches in Mount Cook National Park, New Zealand. In: Evans, S.G., DeGraff, J.V. (Eds.), *Catastrophic Landslides: Effects, Occurrence, and Mechanisms*. Geological Society of America, *Reviews in Engineering Geology* XV, pp. 35-70.
- MHLG (Ministry of Housing and Local Government), 1967. *Rainfall: Annual Average 1916-1950, "Ten Mile" Map of Great Britain*. Ordnance Survey: Southampton.
- Mitchell, S.G., Montgomery, D.R., 2006. Influence of a glacial buzzsaw on the height and morphology of the Cascade Range in central Washington State, USA. *Quaternary Research* 65, 96-107.
- Olyphant, G.A., 1981. Allometry and cirque evolution. *Geological Society of America Bulletin Part I*, 92, 679-685.
- Oskin, M., Burbank, D.W., 2005. Alpine landscape evolution dominated by cirque retreat. *Geology* 33, 933-936.
- Richter, E., 1900. *Geomorphologische Untersuchungen in den Hochalpen*. Petermann's (Geographische) Mitteilungen, Ergänzungsband 29, 1-103.
- Sumner, G., 1997. Wales. In: Wheeler, D., Mayes, J. (Eds.), *Regional Climates of the British Isles*. London, Routledge, pp. 131-157.
- Strøm, K.M., 1945. Geomorphology of the Rondane area. *Norsk Geologisk Tidsskrift* 25, 360-378.
- Thomas, T.M., 1970. The imprint of structural grain on the micro-relief of the Welsh uplands. *Geological Journal* 7, 69-100.
- Thorn, C.E., 1988. *An Introduction to Theoretical Geomorphology*. Unwin Hyman, Boston, 247pp.
- Trenhaile, A.S., 1976. Cirque morphometry in the Canadian Cordillera. *Annals, Association of American Geographers* 66, 451-462.
- Tuck, R., 1935. Asymmetrical topography in high latitudes resulting from alpine glacial erosion. *Journal of Geology* 43, 530-8.
- Unwin, D.J., 1973. The distribution and orientation of corries in northern Snowdonia, Wales. *Transactions, Institute of British Geographers* 58, 85-97.
- Vilborg, L., 1984. The cirque forms of central Sweden. *Geografiska Annaler* 66A, 41-77.

Waldrop, H.A., 1964. Arapaho Glacier: A Sixty-year Record. University of Colorado Studies, Geology, 3, 37pp.

Walker, M.J.C., McCarroll, D., (Eds.), 2001. The Quaternary of West Wales: Field Guide. Quaternary Research Association, London, 184 pp.

White, W.A., 1970. Erosion of cirques. Journal of Geology 78, 123-6.

Wilson, P., 2002. Morphology and significance of some Loch Lomond Stadial moraines in the south-central Lake District, England. Proceedings, Geologists' Association 113, 9-21.

## Figures

Fig. 1. Wales: altitudes of the highest summits in each of the 17 cirque groups used here (e.g. Fig. 7), plus important secondary summits. Names of the groups are used (or parts of compound names such as Moelwyn – Siabod and Nantlle – Hebog), rather than summit names. Here, and on Figs. 2 and 3, the grid lines are kilometres on the Ordnance Survey National Grid, a variant of the Universal Transverse Mercator.

Fig. 2. The distribution of cirques in Wales, by grade.

Grades are defined as:

1. Classic, with all textbook attributes, – a steep headwall curving around a deeply excavated floor;
2. Well-defined, with the headwall curving around the floor and both clearly developed;
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Fig. 8. Point-and-box plots stacked by geology, showing all data values (crosses), medians, inter-quartile ranges (shaded), and overall median (vertical line) for (a): length and (b): amplitude, on logarithmic scales; also for (c): relief, (d): maximum gradient and (e): plan closure. Ord. = Ordovician. (Graphic design by N.J. Cox.)

Fig. 9. Cirque profile development over five classes of size. This is a data-based generalization, using floor and crest altitudes and maximum and minimum gradients. The values used are means for the 52 cirques in each of five equal size classes. Each dot represents altitude of the top end of the median axis.

Fig. 10. Cirque plan development over five classes of size, represented by a hypothetical average mid-height contour. This is a data-based generalization, using plan closure, width, and half the length. The values used are means for the 52 cirques in each of five equal size classes.

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**Table 1**

Analysis of variance results, in rank order, for relation of cirque characteristics to the 13 classes of geology in Fig.8

<i>Variable</i>	<i>adjusted <math>R^2</math></i>	<i>variance ratio, <math>F</math></i>	<i>P-value</i>
log(Relief in 2 km)	.506	23.1	.0000
log(Relief in 1 km)	.438	17.8	.0000
Max. crest altitude	.340	12.1	.0000
log(Wall height)	.250	8.2	.0000
log(Height range)	.234	7.6	.0000
Lowest altitude	.208	6.7	.0000
log(Amplitude)	.199	6.4	.0000
Max. gradient	.165	5.3	.0000
Plan closure	.083	3.0	.0007
Min. gradient	.054	2.2	.0113
log(Size)	.046	2.0	.0220
Axial gradient	.045	2.0	.0231
log(Length)	.038	1.9	.0398
log(Width)	-.020	0.6	.8688

Note that adjustment for the 12 fitted constants has reduced  $R^2$  values considerably.

**Table 2**

100x correlations of five size variables with five shape or gradient variables

	<i>max. gradient</i>	<i>min. gradient</i>	<i>axial gradient</i>	<i>plan closure</i>	<i>width/length</i>
<i>log length</i>	38	-51	-49	51	-38
<i>log width</i>	30	-55	-48	39	34
<i>log amplitude</i>	53	-00	36	22	-35
<i>log heightrange</i>	58	-12	20	33	-35
<i>log wall height</i>	57	-20	23	29	-33



**Table 3**

Exponents for logarithmic (power) regressions of size variables on overall cirque size in Wales

<i>Variable</i>	<i>expon.</i>	<i>95% conf.</i>	$R^2$ , %	<i>st.dev.</i>	<i>better</i>	<i>no outer</i>	<i>valley-side</i>	<i>valley-head</i>
Length	1.12	1.07-1.18	86	0.16	1.10	1.12	1.13	1.01
Width	0.98	0.89-1.06	68	0.17	0.98	0.99	0.97	0.94
Amplitude	0.90	0.81-0.99	61	0.16	0.91	0.89	0.90	1.05
Height range	0.91	0.83-0.99	67	0.15	0.89	0.90	0.90	0.93
Wall height	0.97	0.86-1.09	52	0.19	0.85	0.97	1.02	0.99

95% confidence intervals and  $R^2$  measures of fit for all 260 cirques in Wales are given on the left. These are followed by the standard deviation of each variable, and exponents for 142 better cirques (graded definite, well-defined or classic), for the 249 cirques excluding 'outer' cirques, for 157 valley-side and for 75 valley-head cirques

**Table 4**

Comparative exponents for logarithmic (power) regressions of size variables on overall cirque size

<i>Variable</i>	<i>Wales</i>	<i>Lake D.</i>	<i>Cayoosh</i>	<i>Mar. Alps</i>	<i>Blanca</i>
<i>number of cirques</i>	260	158	198	432	15
Length	1.12	1.17	1.10	1.08	1.14
Width	0.98	1.10	1.05	1.08	1.20
Amplitude	0.90	0.74	0.85		
Height range	0.91	0.75	0.83	0.84	0.66

Results for the whole of Wales are compared with those for the English Lake District (Evans and Cox, 1995, p. 195), the Cayoosh Range of British Columbia (Evans and McClean, 1995, p. 136), the Maritime Alps of Italy and France (Federici and Spagnolo, 2004, p. 240), and the Blanca Massif in the southern Colorado Rockies. The two former are in 'old massifs': the others, in active orogenic belts.

**Table 5**

Exponents, with 95% confidence intervals, for logarithmic (power) regressions of size variables on overall size, for three mapped geologies

<i>Variable</i>	<i>geology</i>	<i>expon.</i>	<i>95% conf.</i>	<i>R<sup>2</sup>, %</i>	<i>RMS error</i>
Length	igneous	1.11	1.04-1.18	92	.054
	other	1.18	1.06-1.30	81	.077
	younger	1.10	0.98-1.21	83	.067
Width	igneous	1.04	0.93-1.16	78	.093
	other	0.96	0.82-1.10	66	.093
	younger	1.13	0.97-1.29	73	.093
Amplitude	igneous	0.85	0.71-0.96	65	.104
	other	0.87	0.68-1.05	48	.122
	younger	0.77	0.66-0.87	70	.067
Height range	igneous	0.87	0.74-0.99	67	.101
	other	0.82	0.69-0.95	64	.083
	younger	0.83	0.71-0.95	71	.071
Wall height	igneous	0.87	0.69-1.05	51	.141
	other	0.91	0.72-1.10	50	.124
	younger	0.82	0.64-1.00	51	.107

'Igneous' are intrusive and volcanic (91 cirques), 'other' are Cambrian and Ordovician sediments including tuffaceous (93 cirques), and 'younger' are Silurian, Devonian and Carboniferous rocks (76 cirques).

**FIGURES for “Allometric development of glacial cirque form: geological, relief  
and regional effects on the cirques of Wales”**

**Ian S. Evans**

(submitted 28 Aug 2005, revised 19 January & corrected 22 February 2005)

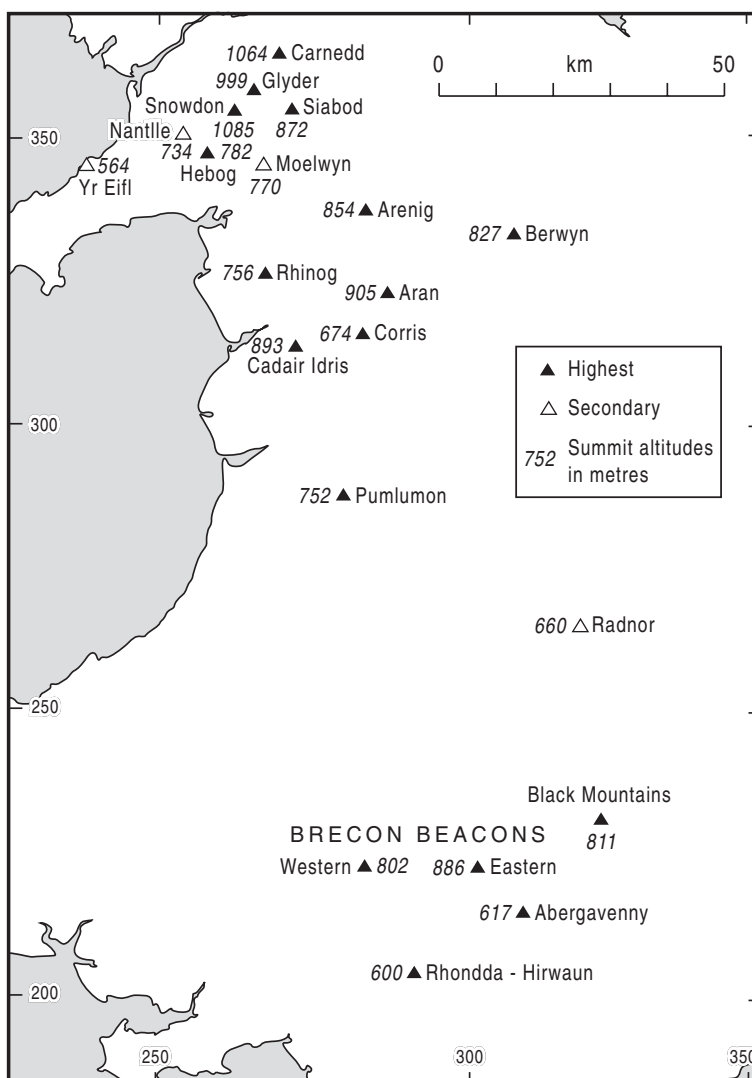


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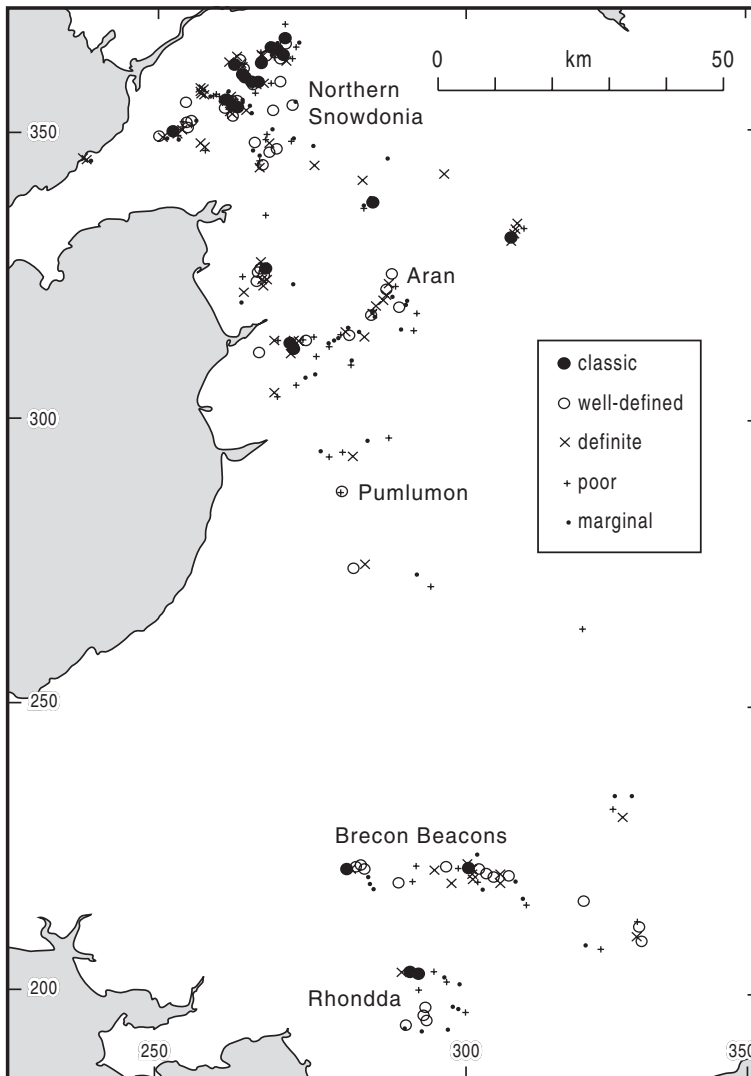


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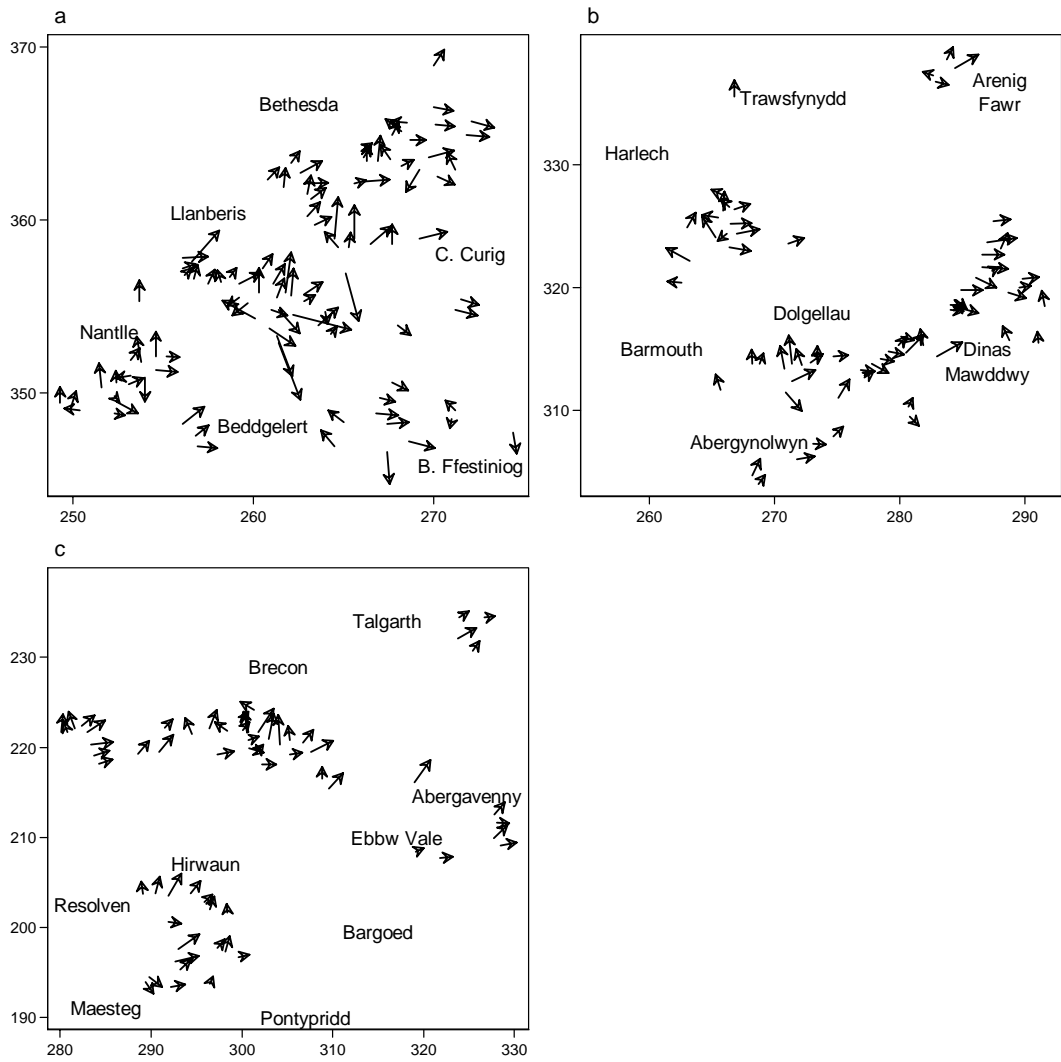


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a:

b:



c:

d:

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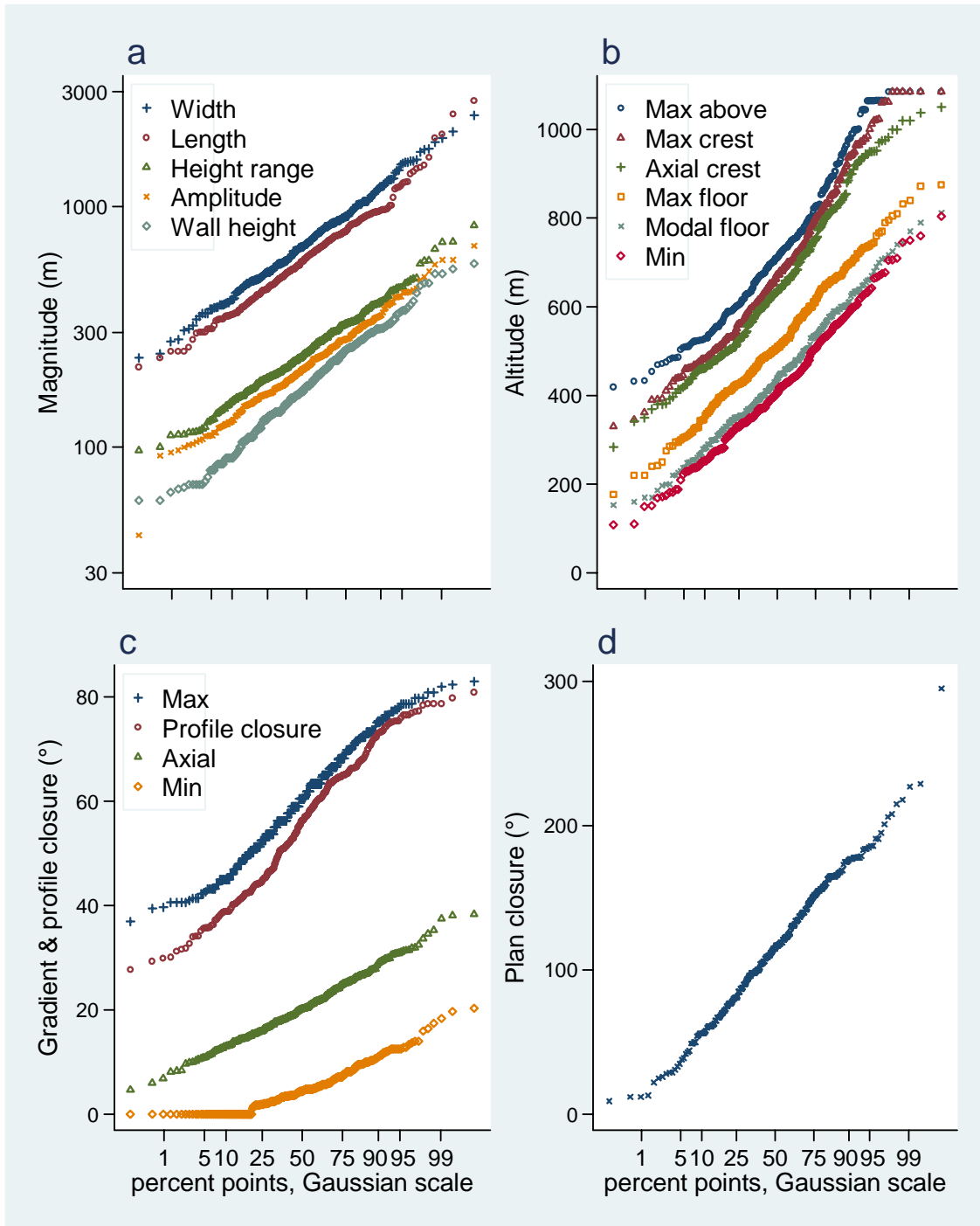


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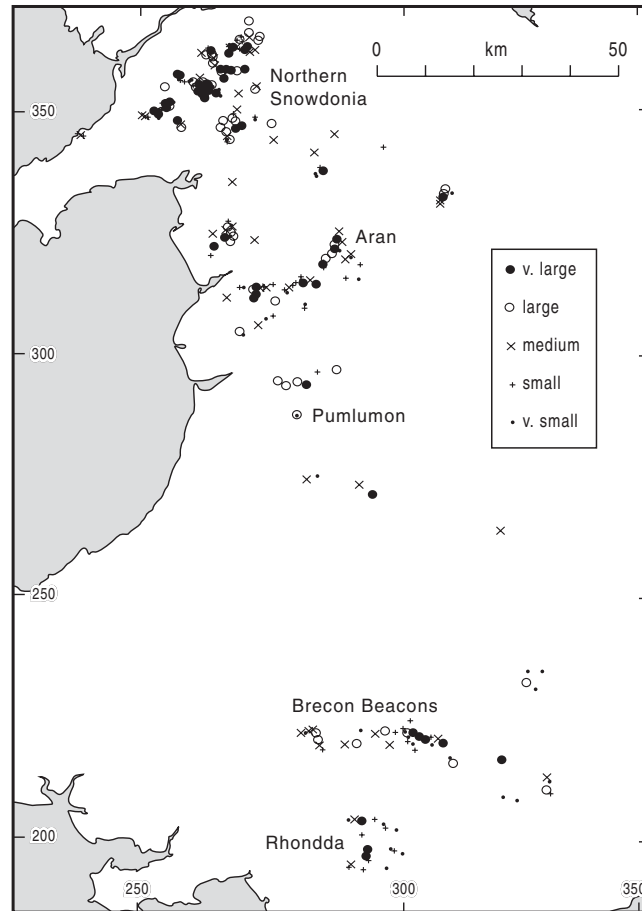


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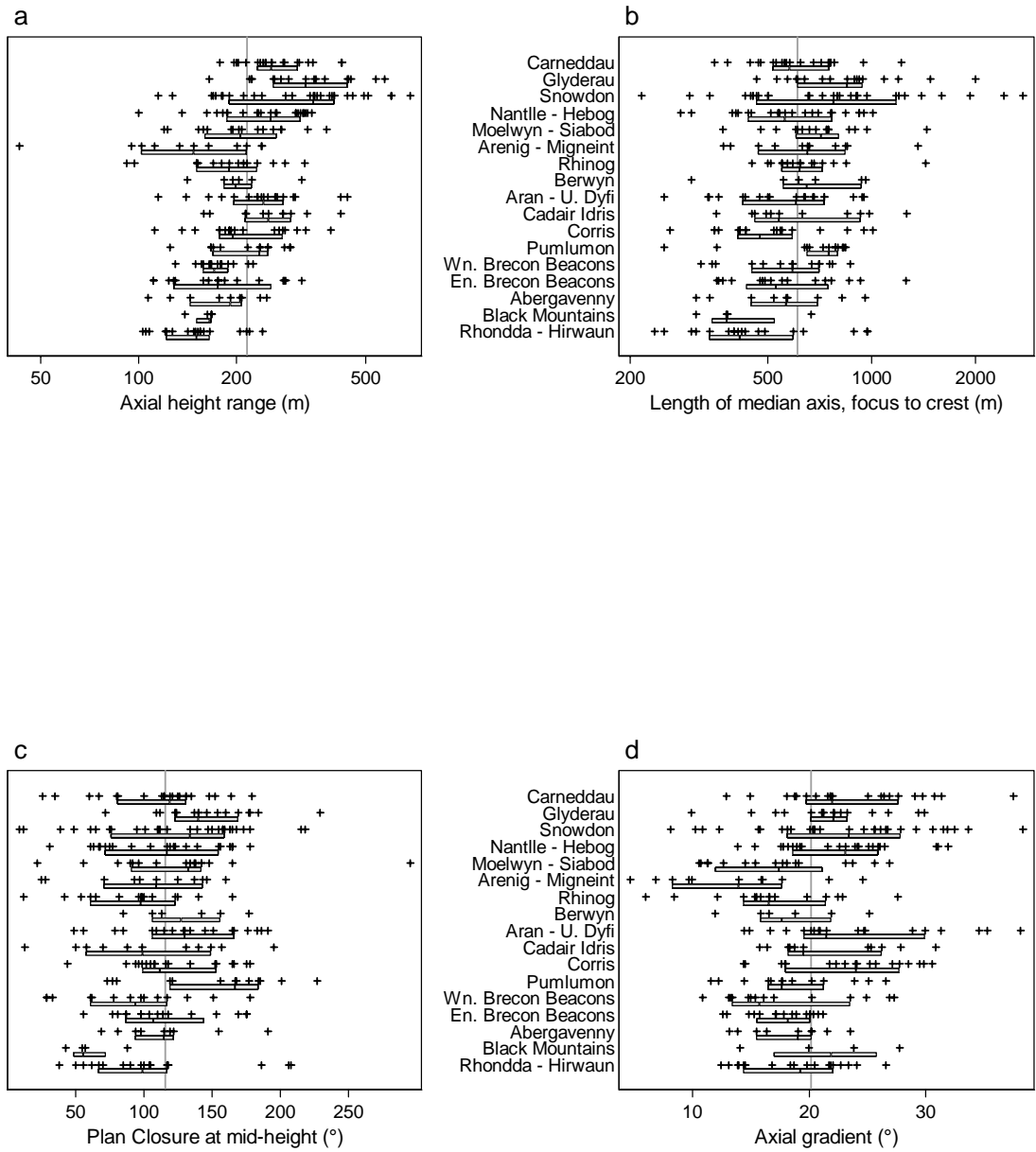
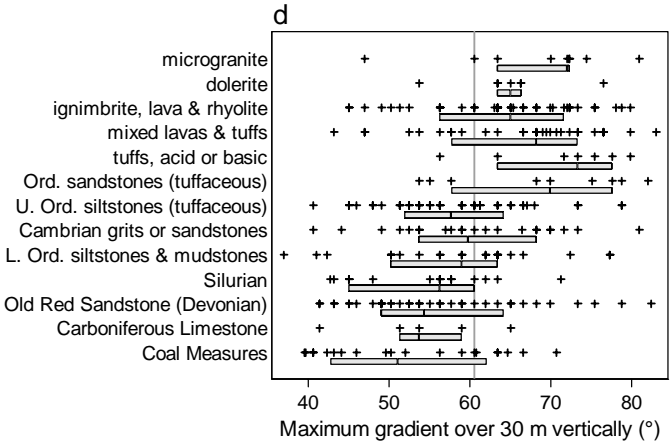
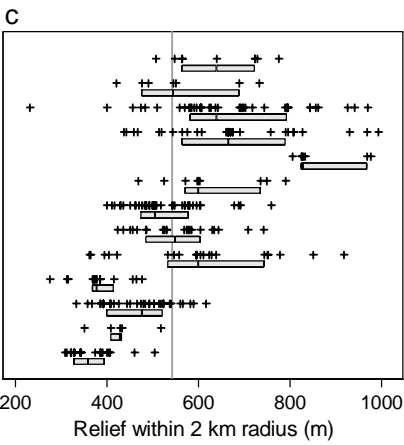
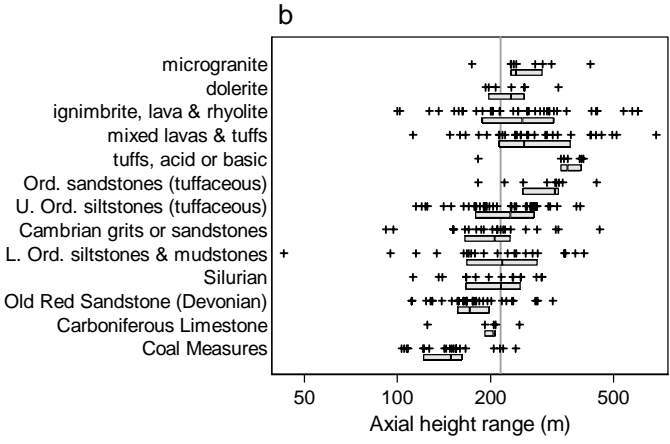
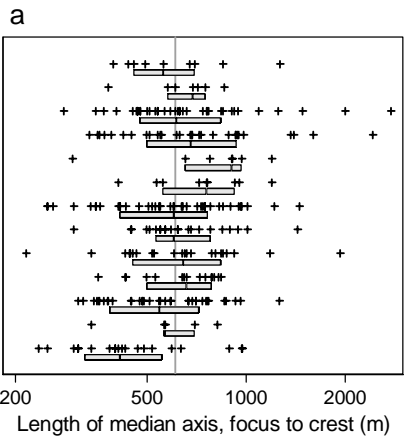


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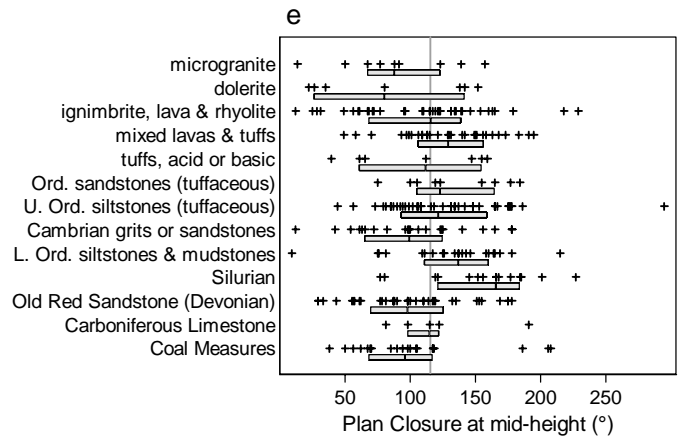


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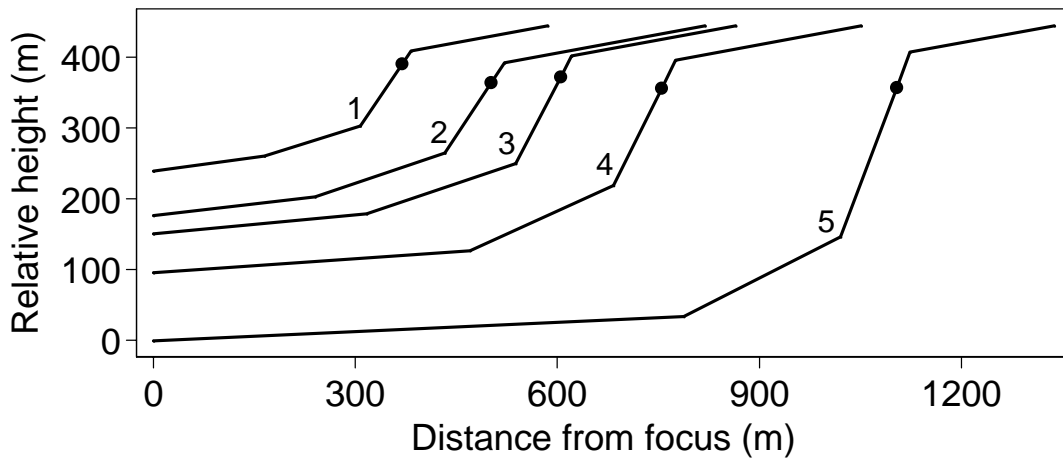


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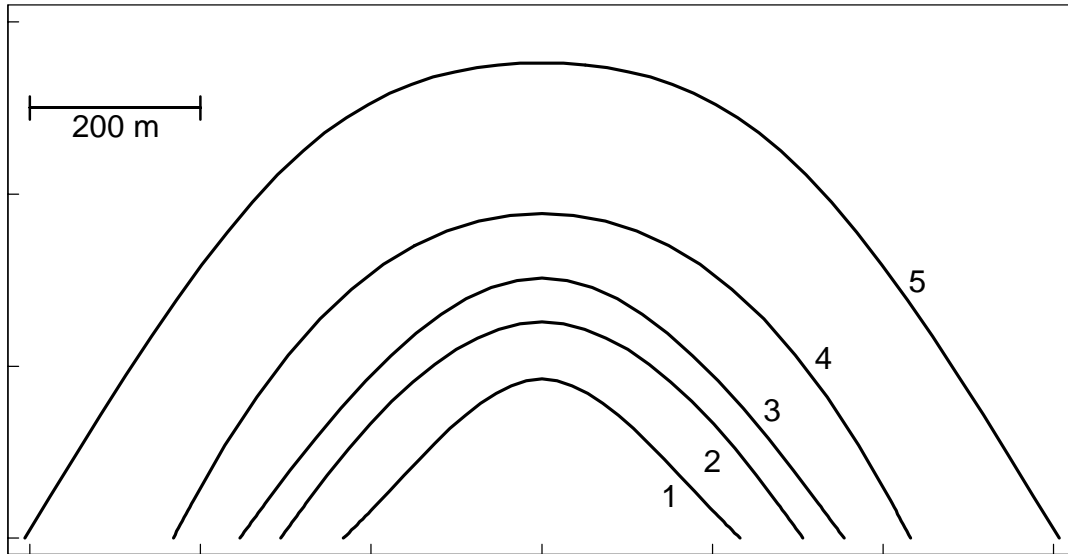


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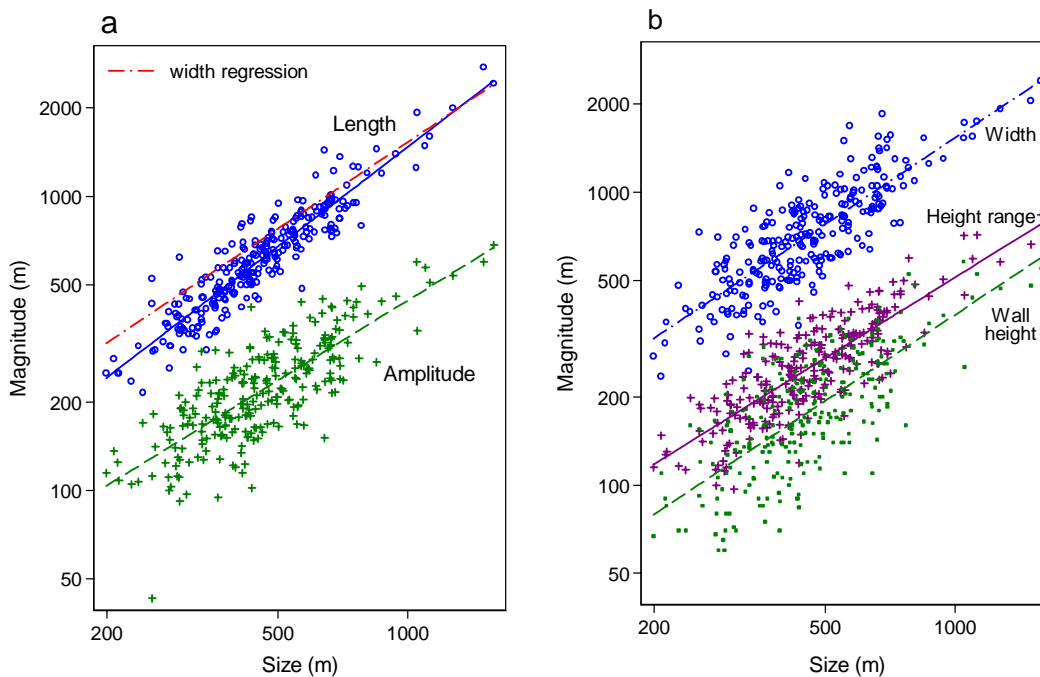


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Dear Takashi,

I have made the necessary figure corrections and just some tiny further changes to the text - removing one more sentence (after Davis 1911 sentence) in 1.2. If you wish I can send the corresponding text with changes.

Many thanks for your help,

Ian.





241	2919	2035	330	365	440	559	600	545	975	1408	180	64.6	0	100	319	385	major mori	0	classic	LLS	glacie	valley-side	Rhondta-F	Coal	Meas	37	31	L. Fawr, C	215	110	12.43544	64.6	1.444103	2.95E+08	665.8073	229	2.989005	3.149603	2.295272	2.332438	2.823349	2.041393	2.359835	0	0
242	2944	2038	370	399	438	512	515	492	470	1050	88	60.8	2.7	118	255	320	major bog	0	poor	LLS	glacie	valley-side	Rhondta-F	Coal	Meas	45	38	C-y-Bwlch	122	68	14.55138	58.1	2.234043	6.02E+07	391.9365	142	2.872088	3.021189	1.944483	2.08038	2.59216	1.832509	2.152288	0	0
243	2959	2028	358	400	440	475	510	461	310	700	60	39.7	3.4	38	256	329	drift	0	marginal	uncertain	valley-side	Rhondta-F	Coal	Meas	82	82	C-y-Ysgo	103	82	18.37948	36.3	2.626965	2.24E+07	281.682	117	2.491362	2.645098	1.778151	2.012837	2.449786	1.913814	2.068186	0	0	
244	2965	2020	237	248	312	393	510	392	430	680	130	42.3	3.7	98	257	342	drift	0	poor	uncertain	valley-head	Rhondta-F	Coal	Meas	24	18	Tarren-y-B	155	75	19.82246	38.6	1.581395	4.53E+07	356.5357	156	2.633488	2.832509	2.113943	2.190332	2.552103	1.875081	2.193125	0	0	
245	2984	2016	275	290	346	391	433	380	235	480	70	44.1	10.1	86	276	308	drift	0	marginal	uncertain	valley-side	Rhondta-F	Coal	Meas	0	385	Cefnhebe-r	105	71	24.07355	34	2.042553	1.16E+07	227.9464	116	2.371068	2.681241	1.845098	2.021189	2.387833	1.851558	2.054458	0	0	
246	2919	2006	330	395	398	490	522	480	300	880	105	56.3	5.7	62	309	343	drift	0	poor	uncertain	valley-side	Rhondta-F	Coal	Meas	100	66	C. Blaerth	150	66	26.65565	50.6	2.933333	3.96E+07	340.8514	160	2.477121	2.944483	2.021189	2.176991	2.532525	1.815644	2.20412	0	0	
247	2930	1978	253	330	417	514	523	494	970	1224	143	60.5	4.4	206	333	386	drift	0	well-defne	LLS	glacie	valley-side	Rhondta-F	Coal	Meas	44	55	Saertern	241	164	13.95282	56.1	1.261656	2.86E+08	658.9565	261	2.986772	3.087781	2.155338	2.382017	2.818857	2.214844	2.416641	0	0
248	2927	1962	321	350	412	544	559	541	890	1184	165	66.6	2	208	308	373	major bog	0	well-defne	LLS	glacie	valley-side	Rhondta-F	Coal	Meas	80	78	Franc. Cwr	220	91	13.88487	64.6	1.330337	2.32E+08	614.3108	223	2.94939	3.073352	2.217484	2.342423	2.798381	1.959041	2.348305	0	0
249	2932	1953	337	350	410	508	524	503	410	765	146	63.4	3.1	104	337	368	major bog	0	well-defne	LLS	glacie	valley-side	Rhondta-F	Coal	Meas	43	48	Graig Fied	166	73	22.04194	60.3	1.914634	5.34E+07	376.6349	171	2.816784	2.88487	2.164353	2.220108	2.579521	1.983324	2.233996	0	0
250	2898	1945	308	320	427	537	565	460	594	854	130	49.6	4.4	186	329	399	drift	1	well-defne	LLS	glacie	valley-head	Rhondta-F	Coal	Meas	146	128	Blaengawr	152	119	14.35354	45.2	1.437771	7.71E+07	425.6271	229	2.773787	2.931458	2.113943	2.181844	2.582029	2.075847	2.359835	1	1
251	2965	1937	330	350	378	460	472	438	250	360	85	40.6	11.3	105	259	390	drift	0	marginal	none	valley-side	Rhondta-F	Coal	Meas	33	21	Bwlfa, Daw	108	48	23.36434	29.3	1.44	9720000	213.4136	130	2.93794	2.556303	1.929419	2.033244	2.339222	1.681241	2.113943	0	0	
252	2973	1977	350	395	413	471	481	471	400	500	65	46	4.8	70	259	328	drift	0	marginal	uncertain	valley-side	Rhondta-F	Coal	Meas	29	40	Mawdy, D	121	63	16.83057	41.2	1.25	2.42E+07	289.249	121	2.60296	2.69897	1.812913	2.082785	2.401272	1.795341	2.082785	0	0	
253	2982	1973	282	302	364	462	475	424	424	1165	100	59	2.4	67	252	321	drift	0	marginal	uncertain	valley-side	Rhondta-F	Coal	Meas	25	15	Tarren Ma	142	82	18.68015	56.6	2.773809	6.95E+07	411.1067	180	2.623249	3.086326	2	2	1.52288	2.613955	1.913814	2.255272	0	0
254	2986	1967	253	254	315	390	419	380	340	593	110	63.4	0	50	231	329	major mori	0	poor	uncertain	valley-side	Rhondta-F	Coal	Meas	73	77	C. Rhodri	127	62	20.48214	63.4	1.744118	2.56E+07	294.7445	137	2.531479	2.73055	2.041393	2.103804	2.469444	1.792392	2.138721	0	0	
255	2984	1939	320	359	396	530	555	525	490	471	135	39.5	8.3	117	367	407	drift	0	marginal	uncertain	valley-side	Rhondta-F	Coal	Meas	145	145	Dwyn, Cw	205	78	22.70284	31.2	0.961225	4.73E+07	361.6793	210	2.691198	2.870021	2.133334	2.311754	2.59324	1.889814	2.322219	0	0	
256	2922	1934	370	390	434	532	537	525	386	850	130	50.2	3.7	85	339	404	drift	0	marginal	uncertain	valley-side	Rhondta-F	Coal	Meas	73	80	Forch, Tai	155	64	21.87815	46.5	2.202073	5.09E+07	370.4924	162	2.586587	2.929419	2.113943	2.190332	2.588779	1.80618	2.209515	0	0	
270	2368	3456	110	153	177	330	564	284	440	580	170	80.9	5.7	139	541	564	drift	0	poor	uncertain	valley-side	Rhondta-F	Coal	Meas	34	25	Llwyd-y-gri	174	67	21.57655	75.2	1.318182	4.44E+07	354.1142	220	2.843453	2.783428	2.230449	2.249549	2.549143	1.826075	2.342423	0	0	
271	2374	3453	150	170	220	410	564	382	675	684	207	63.4	3.8	91	512	564	drift	0	definite	uncertain	valley-side	Nantlle-He	microgranil	16	23	Celloig, Ll	232	70	18.96803	59.8	0.983704	1.04E+08	470.2404	260	2.825304	2.821168	2.319597	2.365488	2.87232	1.845098	2.414973	0	0		
272	2379	3451	108	160	240	345	485	342	562	538	140	47	8.1	77	431	547	drift	0	marginal	uncertain	valley-side	Nantlle-He	microgranil	69	60	Merbwl, Ll	234	132	22.60538	38.9	0.953737	7.05E+07	413.0846	237	2.749738	2.92765	2.146128	2.388216	2.616039	1.20574	2.374748	0	0		
273	2718	3649	372	372	405	695	790	630	754	1015	295	66.3	0	80	384	477	major mori	0	poor	LLS	glacie	valley-side	Carneddau	dolerite	105	95	S.C. Eglau	258	33	18.88987	66.3	1.346154	1.97E+08	582.3075	323	2.877371	3.008468	2.469822	2.41162	2.765152	1.518514	2.509202	0	0	
274	2721	3657	372	372	395	651	729	629	715	1120	250	66.3	0	26	390	420	major mori	0	marginal	LLS	glacie	valley-side	Carneddau	dolerite	106	106	N. C. Eglau	257	23	15.77049	66.3	1.568434	2.06E+08	590.4922	259	2.854308	3.049216	2.4133	2.409933	2.771152	1.361728	2.4133	0	0	
281	2643	3534	431	460	495	614	619	613	297	310	120	56.3	15.9	61	628	628	outslipping	0	marginal	uncertain	valley-side	Snowdon	luffs, acid r	28	28	S. Wenall, G	182	64	31.49974	40.4	1.043771	1.68E+07	255.8958	183	2.472756	2.491362	2.079181	2.260071	2.408063	1.80618	2.282451	0	0		
283	2628	3557	396	420	507	675	921	635	660	625	370	70.2	11.8	49	731	941	outslipping	0	marginal	uncertain	valley-side	Snowdon	ignimbrite	67	66	Beudy Ma	439	111	33.62992	58.4	0.946997	1.81E+08	565.7564	479	2.818544	2.79588	2.568202	2.642465	2.75265	2.045323	2.680336	0	0		
284	2628	3553	588	610	640	916	921	885	502	245	276	52.5	6.1	12	694	928	outslipping	0	marginal	uncertain	valley-side	Snowdon	ignimbrite	67	60	Cil Coch I	297	52	30.60996	44.4	0.488048	3.65E+07	331.1993	328	2.700704	2.389166	2.440903	2.472756	2.520875	1.716033	2.515874	0	0		
290	3267	2344	452	480	521	615	664	615	310	438	106	51.3	12.5	43	330	407	drift	0	marginal	uncertain	valley-side	Black Mou	Old Red St.	79	83	Choch, B	163	69	27.73568	38.8	1.412903	2.21E+07	280.7638	163	2.491362	2.641474	2.025308	2.212188	2.448341	1.838849	2.212188	0	0		
291	3239	2344	503	515	563	642	671	642	382	504	80	43.2	7	55	281	333	drift	0	marginal	none	valley-side	Black Mou	Old Red St.	79	60	Efengyl S.,	139	60	19.99515	36.2	1.319372	2.68E+07	299.1136	139	2.582063	2.70243	1.90309	2.143015	2.478586	1.778151	2.143015	0	0		
292	3254	2307	392	400	445	555	640	560	380	458	132	60.5	3.3	57	364	408	major bog	0	definite	LLS	glacie	valley-side	Black Mou	Old Red St.	39	35	Maw-y-ffw	168	53	23.85447	57.2	1.205263	2.92E+07	308.0724	173	2.578784	2.669866	2.120574	2.225309	2.488653	1.724276	2.230486	0	0	
293	3238	2321	430	440	488	602	680	598	669	1494	140	65	1.5	88	328	392	major bog	0	poor	LLS	glacie	valley-side	Black Mou	Old Red St.	367	60	Esgob, Tai	168	58	14.09668	63.5	2.233184	1.68E+08	351.6903	172	2.825428	3.174351	2.146128	2.225009	2.741695	1.763428	2.235528	0	0	
294	3013	2242	339	390	451	525	590	515	570	575	110	41.4	11.3	110	403	521	drift	0	marginal	uncertain	valley-side	En. C. & B. Old Red St.	307	300	Dyfnan, Ll	178	112	17.15922	30.1	1.008772	5.77E+07	388.3834	186	2.758875	2.759688	2.041393	2.245513	2.587018	2.048218	2.269613	0	0			
295	2943	2182	273	300	343	453	635	448	345	760	130	52.5	3.7	33	379	513	drift	0	marginal	uncertain	valley-side	Wn. Breco	Old Red St.	71	73	S. Tawe F.	175	70	26.89624	48.8	2.202699	4.59E+07	358.006	180	2.537819	2.880814	2.113943	2.243038	2.55389	1.845098	2.273754	0	0		
296	2837	2191	420																																										