The instability theory of drumlin formation and its explanation of their varied composition and internal structure

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Abstract:

Despite their importance in understanding glaciological processes and constraining large-scale flow patterns in palaeo-glaciology, there is little consensus as to how drumlins are formed. Attempts to solve the 'drumlin problem' often fail to address how they are created from an initially flat surface in the absence of obvious cores or obstacles. This is a key strength of the instability theory, which has been described in a suite of physically-based mathematical models and proposes that the coupled flow of ice and till causes spontaneous formation of relief in the till surface. Encouragingly, model predictions of bedform height and length are consistent with observations and, furthermore, the theory has been applied to a range of subglacial bedforms and not just drumlins. However, it has yet to confront the myriad observations relating to the composition and internal structure of drumlins and this could be seen as a major deficiency. This paper is a first attempt to assess whether the instability theory is compatible with the incredible diversity of sediments and structures found within drumlins. We summarise the underlying principles of the theory and then describe and attempt to explain the main types of drumlin composition (e.g. bedrock, till, glacio-fluvial sediments, and combinations thereof). Contrary to a view which suggests that the presence of some sedimentary sequences (e.g. horizontally stratified cores) is inconsistent with the theory, we suggest that one would actually expect a diverse range of constituents depending on the inheritance of sediments that pre-date drumlin formation, the duration and variability of ice flow, and the balance between erosion and deposition (till continuity) at the ice-bed interface. We conclude that the instability theory is compatible with (and potentially strengthened by) what is known about drumlin composition and, as such, offers the most complete and promising solution to the drumlin problem to date.

1. Introduction

Drumlins are one of the most widely studied landforms on Earth, with >1300 contributions (papers, abstracts and theses) in the literature and >400 scientific papers since 1980 (Clark *et al.*, 2009). Their importance stems from their relevance to both glaciology and palaeoglaciology. In glaciology, they are important because they are formed at the ice-bed interface and may exert a modulating effect on ice flow (Schoof, 2002a). However, our understanding of the subglacial processes that occur at this interface is incomplete, largely because of the inaccessibility of this environment under extant glaciers and ice sheets, which limits our observations to geophysical surveys or borehole sampling that cover relatively small areas (e.g. King *et al*., 2007; 2009; Tulaczyk *et al.* 2000). Thus, investigation of drumlins on former ice sheet beds has the potential to uncover important new insights regarding the mechanisms and feedbacks that act to sustain and/or inhibit ice flow and, importantly, formulate and test models of subglacial processes at the ice-bed interface (e.g. Schoof, 2007a, b; Fowler, 2000; 2009a; Hindmarsh, 1998a; 1998b; 1999). Ultimately, the success of such models to account for drumlin formation will improve our ability to predict the rate at which ice and sediment is transported from continents to the oceans, with important implications for future ice sheet stability (e.g. Schoof, 2002; 2004).

In palaeo-glaciology, drumlins also record key information relating to ice sheet flow history, e.g. ice flow direction and changes through time (cf. Boulton & Clark, 1990; Clark, 1993; Kleman and Borgström., 1996; Kleman *et al*., 1997, 2006), and even ice velocity (cf. Hart, 1999; Stokes and Clark, 2002). Thus, they are a vital ingredient for glacial inversion techniques that use the geological record of former ice sheet beds to reconstruct their time-dependent behaviour (see Kleman and Borgström, 1996; Kleman *et al*., 2006). It could be argued, however, that their use is yet to fulfil its true potential. If, for example, we knew the specific conditions under which drumlins of different shapes and sizes developed (e.g. specific ranges of ice thickness, velocity, effective pressures, etc.), then their importance to palaeoglaciology would be considerably magnified.

With the above considerations in mind, the quest for a physically-based model of drumlin formation takes on huge importance and yet, despite this, their origin remains enigmatic and controversial. Numerous hypotheses of drumlin formation have been espoused and include accretion around obstacles (e.g. Fairchild, 1929), dilatant till behaviour (e.g. Smalley and Unwin, 1968), catastrophic meltwater floods (e.g. Shaw, 1983), deformation of till around more competent cores (e.g. Boulton, 1987), lee-side cavity infillings (e.g. Dardis, 1985) and an instability at the ice-till interface (e.g. Hindmarsh, 1998a). As noted by Clark (2010), however, most ideas/hypotheses of drumlin formation fail to address how bumps (drumlins) are created from a flat surface in the absence of obvious obstacles or cores. This appears to be a critical aspect of the 'drumlin problem' because although some drumlins possess an obvious core (e.g. of bedrock or 'stiffer' material), there are numerous reports in the literature of those that do not (see reviews in Patterson and Hooke, 1995; Stokes *et al.*, 2011). Furthermore, most of ideas regarding drumlin genesis are restricted to qualitative descriptions/explanations and very few have progressed to physically-based mathematical models that are capable of making predictions that can be tested against observations.

One theory of drumlin formation that does address relief amplification from an initially featureless surface is the instability theory and, significantly, the last decade or so has seen it described in numerical models of ice flowing over a layer of deforming sediment (e.g. Hindmarsh, 1998a, b; 1999; Schoof, 2007a, b; Fowler, 2000; 2009a; 2010a). It proposes that the coupled flow of ice and till causes the spontaneous formation of relief in the till surface, whereby local highs at the bed will accumulate till by deposition, and lows will be preferentially eroded. This leads to the creation of pattern and structure in the bed that is manifest in a wide range of features termed subglacial bedforms. Indeed, a further appeal of this theory is that it has the potential to provide a unifying explanation for the production of a continuum of subglacial bedforms (cf. Aario, 1977; Rose, 1987) and not only drumlins, having been applied to ribbed moraine (Dunlop *et al.*, 2008; Chapwanya *et al.*, 2011) and recently adapted to address the formation of mega-scale glacial lineations (Fowler, 2010b).

Although models of the instability theory are yet to generate fully three-dimensional drumlins (see section 2.2), predictions of bedform height and length in two-dimensional treatments are consistent with observations (e.g. Fowler, 2000; 2009), which is encouraging (see discussion in Clark, 2010). Perhaps more serious, however, is that the theory has yet to confront the multitude of observations relating to the composition and internal structure of drumlins. This was recently highlighted by Hiemstra *et al*. (2011) who noted that "theoretical studies of flow instability have yet to provide a solution for the sedimentological and structural-architectural variability in drumlins as recorded in the field". For some, this might be viewed as a deficiency: Hart (2005: p. 194), for example, notes that "any model of drumlin formation needs to be related to the sedimentology and structural geology of the drumlins themselves" and Schoof (2007a) questions whether the instability theory can be reconciled with observations of drumlins with stratified cores of glaciofluvial material (e.g. Easterbook, 1986; Sharpe, 1987). Indeed, the oft-cited complexity of drumlin composition (e.g. Menzies, 1979; Patterson and Hooke, 1995) has frequently been seen as a major obstacle for a unifying theory of their formation, although this pessimism may be misplaced (see Stokes *et al.*, 2011).

Whilst the instability theory is principally concerned with the evolution of the ice-till interface, it is important that it can explain observed sedimentary sequences within drumlins (at least qualitatively, but with further progress one anticipates quantitatively). If it is unable to accommodate common sedimentary architectures that are produced or, more commonly, inherited and preserved in a drumlin, then its credibility is damaged. With this in mind, this paper is the first attempt to assess the compatibility of the instability theory of drumlin formation with observations of their composition and internal structure. The underlying principles of the theory are introduced and we then outline a new framework for considering the composition of drumlins, before summarising the main types of drumlin composition reported in the literature (cf. Menzies, 1979; Patterson and Hooke, 1995; Stokes *et al.,* 2011).

Given the conceptual basis of the instability theory, we then consider what kinds of sediments and structures might be expected to occur based on firm physical principles, i.e. we consider the formation of drumlins in its physical context and use what we know or can reasonably infer about this context to provide an explanation of what has been observed in the field.

2. The Instability Theory for Drumlin Formation

2.1. Underlying Principles

A system can be described as 'unstable' when positive feedbacks act to amplify small disturbances, such that small 'natural' variations (perturbations) become larger. A simple illustration of this process can be seen on a flat sand surface (e.g. beach) where a small perturbation (e.g. subtle change in sand thickness) encourages local sediment accretion and the growth of a sand ripple. Such instabilities generally grow at different wavelengths and exponentially, at least initially, and tend to grow fastest at a preferred wavelength. This wavelength of maximum growth rate is determined by the physical operation of the system and, significantly, because one wavelength tends to emerge as dominant, the result is often a pattern of similarly sized and spaced ripples (bedforms) in a field. Such relief amplification from an unstable interface is considered a fundamental mechanism for creating bedforms/waveforms into recognisable patterns, e.g. dunes and ripples in aeolian and fluvial landscapes (e.g. Prigozhin, 1999; Fowler, 2011), which resemble subglacial bedforms, see Figure 1. As noted, the regularity of relief amplification (i.e. the spacing of bedforms at a dominant wavelength) arises because an instability in the system determines that one wavelength will usually grow more quickly than others and that patterning will further develop from bedform interactions, e.g. migration, merging, lateral linking, and cannibalisation might push the system towards fewer larger more widely spaced bedforms (Kocurek *et al*., 2010).

Clark (2010) provides a detailed review of the development of ideas relating to the instability theory for drumlin formation, which can be traced back to the seminal work of Smalley and Unwin (1968), who argued that drumlins might be the product of the flow of sediments beneath ice sheets. However, the notion that subglacial bedforms or, more specifically, wave-forms (instabilities), might arise spontaneously from fluid dynamics at the ice-bed interface was first explored by Hindmarsh (1996; 1998a, b), following the 'paradigm shift' in glaciology in the 1980s (Boulton, 1986), which recognised the importance of the coupled flow of ice and till and its potential to create bedforms (e.g. Boulton, 1987; Boulton and Hindmarsh, 1987). Further analytical developments were made by Fowler (2000, 2009, 2010a) and Schoof (2002a, b; 2007a, b) who confirmed the likelihood of instabilities and, crucially, found that they were largely independent of whether a plastic or viscous till rheology is used, including the highly nonlinear shear-thinning ones typically thought most appropriate for the description of 'nearly plastic' sediment (e.g. Schoof, 2007a).

The basic ingredients and underlying principles of the instability mechanism are shown in Figure 2. The base of the ice is assumed to be at the melting point, and producing sufficient water through basal melting that the till is unfrozen and water saturated. It is then assumed that the till will deform if subjected to a sufficiently high shear stress. The model then considers the flow of ice to be Newtonian viscous, that there is a sliding law relating basal shear stress (τ) to basal velocity (u) and basal effective pressure (N: overburden pressure minus till pore water pressure); and similarly that sediment flux (q) is a function of shear stress and till effective pressure. It is worth noting that the work of Dunlop *et al.* (2008) used a non-linearly viscous model of ice flow, with little qualitative effect on the results.

It is also assumed that, as a granular material, till will only deform if $\tau > \mu N$, where μ is a coefficient of friction; and it is assumed that τ increases with u and N, while q increases with τ but decreases as N increases. In particular, because the effective pressure in the till increases with depth below the ice-till interface, deformation of the till will be limited to a thin mobile layer whose thickness is expected to lie in the range of tens of centimetres to metres. It is then found that the flow of ice over a level substrate is unstable and, for most reasonable choices of sliding law, bedforms grow and equilibrate at finite amplitude (e.g. Hindmarsh, 1999; Fowler, 2009), the height and length of which are consistent with observations, i.e. 10s of metres (cf. Fowler, 2009). The instability occurs because when ice moves over a shallow bump in the interface, it generates a higher compressive stress on the bump's upstream side than in its lee. If, in addition, the effective sediment viscosity is low compared with that of ice, interfacial velocity remains approximately constant, and this then implies that more sediment flows into the bump than out of it, causing it to grow (Schoof, 2007a). The preceding discussion represents the physical context to the theory and previous work provides further details and justification for these underlying assumptions (e.g. Hindmarsh 1998a; Fowler, 2000).

2.2. Recent developments and current limitations

The theory put forward initially by Hindmarsh (1998a) and Fowler (2000) provides an explicit theoretical mechanism for an instability in the flow of ice over deformable sediments, which can generate a pattern of bedforms from an initially planar surface. It is important, however, to outline the current limitations of the theory, which arise partly from the difficulty of the problem (a physically-based model of drumlin formation is clearly not trivial), but perhaps also from the view that makes genetic distinctions between different 'types' of bedforms (e.g. ribbed moraine, drumlins, mega-scale glacial lineations). An alternative view, implicit within the instability theory, is that they sit along a 'bedform continuum' (cf. Aario, 1977; Rose, 1987), such that in two dimensions there may be little physical distinction between ribbed moraines with a short wavelength in the along flow direction and drumlins with a longer wave-length.

In relation to this, the original theory (Hindmarsh, 1998) was a two-dimensional one, and was then used by Dunlop *et al.* (2008) to explain ribbed moraine, but a distinction has often been made between drumlins and ribbed moraine, with the implication being that they are different bedforms, and hence may have different origins. Furthermore, it has been noted that the instability theory has, thus far, failed to generate fully three dimensional drumlins (see discussions in Schoof, 2007a; Pelletier, 2008; Clark, 2010).

Clearly, the fact that three-dimensional bedforms have not yet been predicted by the theory represents a significant challenge to it, but recent developments look promising in this regard. Finite amplitude calculations have been undertaken (Fowler 2009) and three-dimensional modelling of evolving ribbed moraine (Chapwanya *et al.* 2011) have generated 'drumlin-like' culminations (see 'terrain' shaded red in Fig. 1f), although the model failed to produce the expected evolution from drumlinised ribbed moraine to just drumlins (Clark, 2010). The differences between these two implementations lies in the assumption by Fowler (2009) that the water pressure in the deforming till layer is at hydrostatic equilibrium with the subglacial stream system, while Chapwanya *et al.* (2011) assumed a slowly relaxing hydrostatic disequilibrium. In particular, the slow relaxation was due to an assumed till permeability of 10^{-15} m^2, comparable with silt. For a sandy till, or if water flows off drumlins by interfacial rivulets, the present low value may be unwarranted. A working hypothesis relevant in the context of this paper would be that drumlins may be stationary if the till is well-drained, but that they move if the till is poorly drained (we shall discuss both possibilities below). Thus, it would seem that the treatment of subglacial water is vital, which is reminiscent of the views of Shaw and co-workers (e.g. Shaw, 1983; Shaw and Kvill, 1984), although for very different reasons. Indeed, the theory has been modified to allow an active subglacial hydraulic system (Fowler 2010), and this development led to the discovery of a rilling instability, which generated features whose lateral spacing is consistent with observations of mega-scale glacial lineations.

In summary, it is important to stress that none of the recent implementations of the theory are the last word on the subject, since none of them properly solve the coupled ice-till-water flow problem posed by Fowler (2010b). Thus, we would argue that whilst the theory has yet to produce fully three-dimensional drumlins (they emerge as bumps of finite amplitude in 2D models), it is not yet developed to a state where it could be rejected on this basis.

2.3. A new framework for explaining drumlin composition and internal structure

In relation to drumlin composition and internal structure, it is important to stress that the instability theory, thus far, has purposely ignored realistic complications which are nonetheless immaterial to the development of a wavy interface. For example, till is modelled as a homogeneous material and not at the grain-to-grain scale. As such, the model is incapable (in its present form) of making predictions of the kinds of detailed micro-scale sedimentary features that might be generated and observed. This is not necessarily a problem because, as noted by Menzies (1979, p. 350), “it is critical that if any unifying drumlin theory is to be developed it must not be created around unique or special conditions either within the ice mass or drumlin material”. It is important, however, that the theory is not intrinsically contradicted by observations of the commonly observed contents found inside drumlins (e.g.

till, stratified glaciofluvial material, etc.) and nor by the structure or architecture of these contents (e.g. conformable/unconformable with the drumlin surface). However, before confronting the theory with observations, we must first consider a new framework which allows for the vertical movement of the unstable (wavy) interface.

To date, the instability theory provides a framework in which the bed elevation (s) is described by a partial differential equation, known as the Exner equation, which takes the form (in two dimensions),

$$\mathrm{s}_{\mathrm{t}} + q_x = 0 \tag{1}$$

In which (t) represents time, (x) represents downstream distance, (q) represents the till flux, and the subscripts denote partial derivatives. Additional assumptions are that ice flow is continuous and constant, the sediment is constantly saturated, the subglacial hydraulic regime is uniform and constant in time, and the sediment supply upstream is constant, and equal to that downstream. These assumptions are made not because we believe them to be true, but because a theory, any theory, has to make some assumptions, and these are the simplest that we can make in the context of drumlin formation. However, in our present intention of addressing how the theory might explain observations of drumlin composition, we have to allow for relaxation of these assumptions, and we now discuss these in turn.

In consideration of any specific segment of an ice flow line, it is unlikely that sediment influx and efflux will be in balance. The generalisation of equation (1) to describe such situations is the modified equation:

$$s_t + q_x = -E \qquad (2)$$

where E represents the net rate of erosion of sediments. We conceive of the actively deforming till layer as having a (fixed) thickness controlled by the depth at which the yield stress is reached as effective pressure increases. Net removal of the sediment above will thus cause entrainment of the sediment below, and thus an effective erosion of the till bed. We thus distinguish two cases: $E > 0$, an erosional environment, and $E < 0$, a depositional environment. Both will play a part in our interpretations of observed drumlin stratigraphy.

The theory considers the smooth, continual evolution of the bed under constant external conditions. In reality, since nothing happens during dormant periods, the model can also describe the more likely scenarios where evolution occurs in discrete periods, due to distinct drumlin-forming events. One reason for supposing this is that, as a granular material, till will not deform at all unless the effective pressure (overburden minus hydraulic pressure) N is less than τ /μ, where τ is shear stress and μ is a coefficient of friction. In practice, this means $N <$ 1 bar. Such low effective pressures are known to occur under ice streams (cf. Kamb, 1991), but may not be common where channelized drainage occurs (as recorded by eskers formed in Röthlisberger channels), which typically have much higher effective pressures. Moreover, till deformation implies water saturated sediments, which requires not only that the basal temperature be at the melting point, but also that there is net production of water. So it seems natural to suppose that as an ice sheet evolves, basal conditions change so that drumlins are not built continuously, but episodically, and probably quite rapidly (i.e. few decades: cf. Smith *et al*, 2007), which is a further prediction of the instability theory (see Fowler, 2009).

As stated above, the instability theory is still in a state of development and this section indicates our best present understanding. We introduce the minimum ingredients and find what they explain: principally the size and wavelength of subglacial bedforms. To explain further features (their composition and internal structure), we introduce further plausible ingredients. In this preliminary discussion of drumlin composition, we are simply exploring the most likely possibilities that we construe will emerge as the theory is developed.

3. Observations of Drumlin Composition and Internal Structure

Within the vast drumlin literature, numerous papers (>200) report on their composition and internal structure and it is true to say that they are composed of a range of different sediments, exhibit a variety of different structures (e.g. horizontally stratified versus conformable with landform surface), and show evidence of a variety of styles and extent of deformation (see reviews in Menzies, 1979; Patterson and Hooke, 1995; Hart, 1997; Stokes *et al.*, 2011). Perhaps unfortunately, this diversity has led to a large range of explanations/hypotheses of drumlin formation and it has famously been noted that “there are almost as many theories of drumlin formation as there are drumlins” (Sugden and John, 1976, p. 239). Indeed, although drumlin morphology is also variable (though recently shown to have unimodal distributions of length, width, height and shape: Clark *et al.,* 2009; Spagnolo *et al*., 2010; 2011; 2012), it is likely that had drumlins only ever been observed to contain the same contents, the “drumlin problem” (Menzies and Rose, 1987; p. 7) would not be so much of a problem.

Based on a systematic review of the literature and in an attempt to reduce the oft-cited complexity of drumlin composition, Stokes *et al.* (2011) have recently suggested that there are, essentially, just five basic types, albeit with subtle variants, shown in Figure 3. These are:

i. Mainly bedrock
ii. Part bedrock/part till

iii. Mainly till
iv. Part till/part sorted sediments
v. Mainly sorted sediments

Whilst acknowledging the inherent limitations of such a classification, Stokes *et al*. (2011) argue that explaining these basic types provides a more realistic goal for theories or numerical models of drumlin formation to address. They go on to suggest (as others have done, e.g. Dionne, 1987) that the first type (type 1, Fig. 3), purely bedrock forms, could be viewed as genetically different from drumlins formed of unconsolidated sediment (whaleback may be a more appropriate term, cf. Evans, 1996).

Stokes *et al*. (2011) also postulate that because of the unimodal distribution of drumlin dimensions (which suggests a single population of landforms, rather than different types: Clark *et al*., 2009) and because the other four types of drumlin content can often occur within the same drumlin field (e.g. Hill, 1971), and sometimes in a continuum (e.g. Boyce and Eyles, 1991), they are probably genetically related, i.e. their differing contents should not be seen as an obstacle to a unifying theory of drumlin formation. The challenge for the instability theory therefore, is whether it can explain all of the remaining four types of drumlin (listed above). The next section addresses this issue and takes each type of drumlin in turn (excluding purely bedrock forms) and assesses whether the physical principles and mechanisms that underlie the instability theory can explain/predict such observations.

4. Qualitative and Quantitative Explanations of Drumlin Composition and Architecture Predicted by the Instability Theory

4.1. Drumlins composed of part bedrock/part till

4.1.1. Observations

Drumlins composed of part bedrock/part till (often called 'rock-cored drumlins: type 2, Fig. 3) are commonly reported in the literature (e.g. Crosby, 1934; Dionne, 1987; Möller, 1987; Boyce and Eyles, 1991). In their inventory, Stokes *et al*. (2011) list 28 papers that describe this type of drumlin but point out that they are probably far more common than is reported, compared to other types, because of the bias towards sampling drumlins away from regions underlain by crystalline bedrock (see their Table 1 and Figure 17) and partly because they are sometimes called crag-and-tails. The bedrock 'core' can be located at the stoss (e.g. Glückert 1973), middle (Tavast, 2001), or lee side of the drumlin (Tavast, 2001), although it is most common to be positioned at the stoss side (Stokes *et al*., 2011). To distinguish these features from 'crag and tails', Dionne (1987) suggested that till should account for at least 25% of the entire drumlin volume and cover at least portion of the stoss end.

4.1.2. Model Explanation/Prediction

From the point of view of the instability theory, part bedrock/part till drumlins are relatively easy to explain because an instability will form a drumlin due to any small perturbation in the till thickness or bed topography. The instability theory predicts the formation of drumlins as waves which grow from a pre-existing (level) interface. Instabilities grow in nature because there are always perturbations present. Thus, a bedrock protuberance is just an obvious perturbation, and since the theory in one version of its current form (Fowler, 2009) predicts growth of a finite amplitude stationary state, it is a consequence that such perturbations will give rise to drumlins. In short, the theory predicts that bedrock bumps are sufficient but not necessary to seed drumlins. Dynamical analogies abound: the formation of standing waves in rivers at bedrock steps, atmospheric lee waves behind mountains, sand dunes formed behind or in front of obstacles.

More specifically, the theory assumes that till flux (q) increases with basal shear stress (τ) but decreases as effective pressure (N) increases (section 2). Thus, depending on the shear stress (which is, in turn, related to ice velocity and effective pressure), it is obvious that till flux varies both spatially and temporally under different conditions. In some circumstances, therefore, till fluxes will be relatively high through the system and in others, they might be relatively low. Given a system of high till fluxes over an underlying surface of bedrock undulations and where till flux from up-ice is insufficient to maintain continuity (or local erosion is too low), it is natural to expect that the till thickness will decrease and bedrock will become exposed at the surface of the till. Depending on the nature of the bedrock surface and the pre-existing sediment thickness, drumlin forms will thus emerge with varying degrees of bedrock 'control' through time, see Figure 4. The situation of bedrock bumps perhaps seeding some drumlins still holds (Fig. 4b), but as till is preferentially removed from the system (erosion dominates over deposition), bedrock bumps are likely to emerge (Fig. 4c and d) and the ultimate progression sees the system evolve to an entirely bedrock surface (Fig. 4e). Similar erosional processes within a deforming bed were envisaged by Boyce and Eyles (1991) in the Peterborough drumlin field, Ontario. They noted drumlins with bedrock cores in areas where the length of time available for erosion was greatest and sediments were thinnest.

The observation that bedrock cores might be found in various positions within drumlins (cf. Stokes *et al.*, 2011) is also fairly readily understood. Rapid ice flow over a bedrock bump will cause a large cavity to form in its lee, and in the presence of an adequate till supply, the cavity will be infilled by sediments (Fig. 4d). On the other hand, if the ice flow is relatively slow, then we would expect little cavitation, but till dragged towards the obstacle will pile up, causing a stoss-side cavity infill (which some workers have reported and termed 'pre-crags', e.g. Haavisto-Hyvärinen, 1997). In the absence of plentiful till cover, we may expect bedrock bumps to emerge above the till veneer (and the extreme case of this is the whaleback).

Additionally, because effective pressure increases with bed elevation, and thus till mobility decreases, till may simply not be able to reach the summits.

4.2. Drumlins composed of mainly till

4.2.1. Observations

It is no surprise that there are numerous reports of drumlins composed mainly of till (type 3, Fig. 3) (e.g. Wright, 1962; Nenonen, 1994; Menzies *et al*., 1997; Rattas and Piotrowski, 2003). Indeed, Stokes *et al*. (2011) noted that this is by far the most common constituent of drumlins *reported* in the literature and previous studies draw the same conclusion (e.g. Menzies, 1979; Patterson and Hooke, 1995). The emphasis is on '*reported'* because we do not have a large enough sample size to judge whether observations to date are a representative sample of drumlin composition (see discussion in Stokes *et al*., 2011). In some cases, the entire drumlin appears to consist of an essentially structureless/homogeneous unit of till (Habbe, 1992), whereas others exhibit several units; sometimes horizontally bedded (e.g. Stea and Brown, 1989) and sometimes conformable with the drumlin relief (e.g. Nenonen, 1994). The degree to which till units (or any sedimentary units for that matter) are conformable with the drumlin surface is often viewed as a key issue in drumlinology and is discussed in section 5.1. Drumlins composed mainly of till also show a variety of features/structures related to both ductile and brittle deformation (e.g. Menzies *et al*., 1997), although others do not and, again, this issue is true for other types of drumlin.

4.2.2. Model Explanation/Prediction

In many ways, this type of drumlin is the least problematic for the instability theory. In effect, the instability theory models the surface of the till as a sinusoidal wave of varying thickness

(Fig. 2c), but geomorphologists have tended to only map the landform above the mean till surface (e.g. Spagnolo *et al*., 2012) at some (possibly arbitrary: see Smith *et al*., 2006), level; and it is these landforms that sedimentologists have tended to focus on in terms of sampling the sub-surface. Because of the dependence of the theory on a mobile till unit that grows in thickness to form the body of the drumlin that scientists map and sample, an obvious prediction of the instability is, therefore, that the high-points of the sinusoidal wave (which we map as drumlins) should be composed of this mobile till unit.

That the most commonly reported drumlins are those composed of till would seem to serve the theory well, especially where ice flows over a metres thick sequence of tills. However, it is less obvious how such drumlins can be formed in the absence of a pre-existing deep (several metres) till layer, though whether this is a significant problem needs exploration through mathematical modelling. Two mechanisms emerge from our previous discussion of the instability model. When hydraulic connectivity is poor, we may expect bedforms to grow as travelling waves, and these waves will sweep the underlying sediments together as they move. Alternatively, or as well, in net depositional environments, till is gradually deposited as a thickening layer on top of any pre-existing sediments.

4.3. Drumlins composed of part till/part sorted sediments

4.3.1. Key observations

The second most commonly reported type of drumlin (cf. Stokes *et al*., 2011), after those composed mainly of till, are those composed of large amounts of both till and sorted (often glaciofluvial) sediments (type 4, Fig. 3). The location of the sorted sediments can vary. In some cases they form a centrally-positioned core or ‘pod’ (e.g. Rattas and Piotrowski, 2003), whilst in others they form a horizontal unit that separates two till units (e.g. Kerr and Eyles,

2007) or interbedded with till or *vice versa* (e.g. Whittecar and Mickleson, 1979). Arguably, the most commonly reported architecture, however, is where the sorted sediments simply underlie the till unit (e.g. Clapperton, 1989; Boyce and Eyles, 1991; Habbe, 1992; Jorgenson and Piotrowski, 2003). In this situation, there are reports of the sorted sediments showing evidence of glaciotectonic deformation and being incorporated into the overlying till unit, e.g. drag folds and rafts/lenses of underlying sediments (Boyce and Eyles, 1991) from either ductile or brittle deformation. In other cases, the erosional contact with the sorted sediments may show minimal evidence of deformation (e.g. Habbe, 1992; Hart, 1995a).

A commonly reported sub-type of part till/part sorted drumlins are those where the sorted sediments are preferentially found on the lee side of the drumlin. These observations are dominated from locations in Ireland (e.g. Dardis, 1985; Dardis and McCabe, 1983; Dardis *et al.*, 1984; Hanvey, 1987, 1989) but not exclusively (see Fisher and Spooner, 1984).

4.3.2. Model explanation/prediction

Whilst drumlins with components of bedrock and till are relatively easy to explain, the presence of stratified sediments is seen by some (e.g. Schoof, 2007a) as introducing additional complexity that is, perhaps, incompatible with the instability theory. As discussed in section 2.2, in the simplest scenario ($E = 0$) the mean interface level remains constant, but this is not an essential ingredient of the model. If there is limited sediment flux from upstream, perhaps because there is exposed bedrock there, or the effective pressure is too high (or shear stress too low) to promote till deformation, then the erosion rate $E > 0$ and the mean level of the interface will lower, even as the wavy interface (i.e. drumlinised surface) evolves.

The hydraulic potential of the water at the bed is lowest at the lowest parts of the ice-till interface (i.e. around the base of drumlins and in inter-drumlin areas), and so we expect meltwater to be concentrated there (e.g. Fowler, 2010b), see Figure 5. Moreover, because the low levels of effective pressure associated with sediment deformation are consistent with a description of stream flow as a distributed system (Walder and Fowler, 1994), the most likely form of the basal water flow is a slow trickle through a swamp-like basal platform. Conversely, the effective pressure at the tops of drumlins should be higher, so that the till there is stiffer. The higher/thicker drumlin material will be eroded around the base where the till is softer, and the excavated material can then be removed through the meltwater system. In this way, the transport (erosion) of till is enhanced by both the ice motion-induced sediment flux and through meltwater erosion, with the overall effect of a wavy interface cutting vertically into underlying units. It follows therefore, that a way in which to build part-stratified drumlins as shown in Figure 5d is to first build till-filled drumlins in a depositional environment ($E < 0$), and then later have these drumlins subjected to a net erosional environment ($E > 0$), assuming they overlie pre-existing sorted sediments.

Thus, in certain circumstances, the instability theory would predict a wavy interface cutting down into any pre-existing sediments (Fig. 5). The opposite case is where upstream sediment flux is larger than can be excavated out of the drumlin field and, in this way, pre-existing units of sorted sediments may be buried by till units and, in some cases, show evidence of being deformed upwards into the till (e.g. Boyce and Eyles, 1991). In this manner, pods or cores of glaciofluvial material may be incorporated into the till layer and, because they are generally coarser-grained (e.g. sands and gravels) compared to till, such sediments are likely to be better drained and more likely to act as competent material within a deforming layer of till. In this way, they act as boudins around which the deforming till will flow. This idea is not new in the drumlin literature (e.g. Smalley and Unwin, 1968) and was encapsulated most notably in Boulton's (1987) 'theory of drumlin formation by subglacial sediment deformation', see Figure 6. The appeal of the instability theory therefore, is that it may be able to explain drumlins with or without such cores, with the major accomplishment being that such cores (which are not always present: Stokes *et al.*, 2011) are not a necessary pre-requisite.

The presence of deformation features at the contact with underlying or en-drumlin units is also to be expected in that any bump created at the ice-bed interface is likely to induce large stress gradients (cf. Morland and Boulton, 1975). More specifically, units with differing rheologies (e.g. till overlying glaciofluvial sediments) are especially conducive to the production of both ductile folds, and faults caused by fracturing of non-yielding material. Thus, whilst deformation fields are likely to be complex and vary from drumlin to drumlin (depending on their constituents), the observed manifestations of deformation are entirely consistent with the instability theory.

The mechanical properties of the till itself also creates horizontal variations in its properties. The effective pressure in the till at the ice-till interface increases with elevation of this interface, having a vertical gradient $\Delta\rho_{\mathrm{wi}}g$, where $\Delta\rho_{\mathrm{wi}}$ is the density difference between water and ice (g is the acceleration due to gravity); while the effective pressure N increases with depth below the ice-till interface at a rate $\Delta\rho_{\mathrm{sw}}(1-\phi)g$, where ϕ is the sediment porosity and $\Delta\rho_{\mathrm{sw}}$ is the density difference between sediment and water. Consequently, N will increase along a horizontal level as we move from the stoss face to a position immediately under the crest, and then will decline thereafter to the lee face. The higher value of N must cause lower values of μ; tensional stresses will be generated towards the lee of drumlins, while compressive stresses are likely to be generated towards the stoss face of the drumlin (cf. Morland and Boulton, 1975), and failure of the till may also lead to thrust faults, which again have also been observed in numerous drumlins (e.g. Hart, 1995a; Hart 1997; McCabe and Dardis, 1994).

In terms of stratified sediments found only on the lee-side of drumlins, their presence is usually ascribed to deposition by meltwater that is preferentially routed towards cavities behind the developing drumlin (e.g. Dardis *et al*., 1984). As noted above, most proponents suggest that such deposition requires a pre-existing drumlin in place and is, therefore, unlikely to explain the drumlin-forming mechanism. In this sense, it is not a drumlin-forming mechanism (although see related arguments in Shaw, 1983; Fisher and Spooner, 1994, etc.). Thus, we simply note that the presence of stratified glaciofluvial sediments on the lee-side of drumlins is to be expected as a result of cavitation, which (although initially regarded as an undesirable feature), the instability model always predicts to occur (see Schoof, 2007a; b; Fowler, 2009), and meltwater routing towards these low pressure cavities.

4.4. Drumlins composed of mainly sorted sediments

4.4.1. Key observations

Although they are the least commonly reported drumlins in the literature (cf. Stokes *et al.*, 2011), it has been known for a long time that some drumlins are simply composed of sorted (typically glaciofluvial) sediments or have only a thin veneer of till, and they often lack any evidence of widespread deformation (Alden, 1905, Gravenor, 1953; Shaw, 1983; Shaw and Kvill, 1984; Sharpe, 1987; Menzies and Brand, 2007). In many cases, the sorted sediments are horizontally bedded but sorted sediments show a range of architectures and their presence has been attributed to a range of factors. Menzies and Brand (2007), for example, observed pre-existing proglacial and deltaic sediments which acted as an obstacle around which a thin veneer of till was emplaced and which showed evidence of thin-skinned deformation within the till but minimal disturbance of the underlying sediments, see Figure 7. In other cases, evidence of undisturbed sorted sediments has been interpreted as being intimately linked to drumlin formation by subglacial mega-floods, e.g. drumlins represent the glaciofluvial infillings of subglacial cavities (e.g. Shaw, 1983). Such an interpretation assumes that the sediments inside a drumlin are unquestionably linked to the drumlin forming mechanism, which is not always obvious, e.g. Fig. 7 (Menzies and Brand, 2007; Knight and McCabe, 1997a; Stokes *et al*., 2011).

4.4.2. Model explanation/prediction

As explained in section 4.3.2, the most obvious way in which the instability theory can explain the presence of sorted sediments (irrespective of whether they form a part of or a whole drumlin) is through the vertical erosion of a deforming till layer into pre-existing sedimentary units. In such cases, the instability theory would have it that the sorted sediments are often unrelated to drumlin formation, other than their potential to act as a stiffer core (see section 4.3.2 and Fig. 6). Whilst it may be easier to conceptualise this down-cutting as producing part till/part sorted drumlins (Fig. 5), it is perhaps more difficult to envisage how the instability theory might explain drumlins composed of mainly sorted sediments and with only a minimal veneer of till and with minimal disturbance of underlying units (e.g. Fig. 7). We conceive of these drumlins forming in the following way.

Given an ice sheet building up over a layer of stratified sediments with a largely flat surface, the ice will thicken and it may reach the melting point and begin to produce basal water and slide. The water saturates the underlying sediments, which then begin to deform in a thin (e.g. cm to metre) layer. The thickness of the dilating active layer is not simply a property of the sediments, but is also a consequence of the effective pressure and applied shear stress

(Boulton and Hindmarsh, 1987, eqn. (25); Hart *et al*., 1990, eqn (1)). Thin layers are associated with high effective pressures, which themselves are suggestive of very well drained sediments. As noted in section 2.1, subglacial instabilities are largely independent of whether a plastic or viscous till rheology is used (Schoof, 2007a). Thin dilatant layers are consistent with the instability theory and exhibit instabilities (Dunlop *et al*., 2008).

The instability theory predicts that bedforms grow, and we can expect that as they do so, they begin to obliterate the structure of the underlying sediments. If the underlying sediments are sufficiently porous (e.g. sands or gravels) and the overlying active till layer is sufficiently thin, or non-cohesive, we may expect not only that the effective pressures are relatively high, but also that the water in the till layer to be hydrostatic, and it is in this situation that the evolving drumlins may be expected to be stationary (Fowler 2009). As discussed earlier, this leads to a situation in which the internal sediment architecture is maintained. We thus envisage a suite of bedforms consisting of hard resistant material residing in a basal platform of soft swampy sediments, where the basal water flow is situated. The soft material should be erodible, and as it erodes, we may imagine the drumlins collapsing as their foundations are removed (Fig. 5b).

Figure 8 shows the result of a numerical simulation in which this evolution is demonstrated, the details of which are given in the appendix. The initially stratified sediments are indicated by the horizontal coloured bands and, as the bedform descends (its initial range is from -5 to 0 m on the vertical axis), the near surface sediments are distorted and move in a thin veneer along the ice/till interface. The figure shows the resulting stratification after a period of ten years, when the wave form has eroded five metres of sediments (horizontal axis also in metres). Such high erosion rates are compatible with recent observations from under an active ice stream in W. Antarctica (Smith *et al*., 2012). A brief movie of the evolution is included in the supplementary online material.

We have included these calculations because, although internal stratification is a simple and inexorable consequence of stationary drumlin formation in an erosive environment, it has caused perceived difficulties with regard to acceptance of the theory (Schoof 2007a; Pelletier 2008).

5. Discussion

5.1. Erosional versus depositional drumlins: a false dichotomy?

It is clear from the preceding sections that both the composition and structure of drumlins are important aspects for the instability theory to explain. In particular, the extent to which drumlin composition conforms to the drumlin surface is an issue which has attracted much attention and one which has often led to them being classified as 'erosional' or 'depositional' (cf. Patterson and Hooke, 1995); or 'destructional' versus 'constructional' (Hart, 1995b; 1997).

Structures (e.g. layered units of till) that are conformable with the surface of the drumlin form have been noted in a number of studies (e.g. Hill, 1972; Nenonen, 1994; Hanvey, 1992) and are usually interpreted as reflecting a mechanism of formation that involves accretion of material around a core that builds up incrementally, layer by layer (e.g. Fairchild, 1929). Such interpretations are usually supported by reports of clast macro-fabrics that show expected patterns of divergence and convergence around the drumlin as till was deposited and emplaced around a growing obstacle (e.g. Savage, 1968; Goldstein, 1989). As such, drumlins with these surface conformable structures are often referred to as 'depositional' drumlins. In contrast, those with structures that are unconformable with the surface (e.g. beds of sorted glaciofluvial or till units, e.g. Fig. 7) are, in most cases, assumed to reflect pre-existing material that has been left behind by some form of erosional process – hence the term ‘erosional’ drumlins. This dichotomy between erosional and depositional drumlins pervades the early drumlin literature and, as Patterson and Hooke (1995) note, “any general theory of drumlin formation must accommodate both possibilities” (p. 33). The alternative requires two theories: one to explain depositional drumlins and one to explain erosional drumlins.

We note that although serious questions have been raised over the plausibility of the meltwater flood theory for drumlin formation (see Shaw, 1983; Shaw and Kvill, 1984; and Benn and Evans, 2006 versus Shaw and Munro-Stasiuk, 2006), a notable strength of this theory is its ability to explain both erosional and depositional drumlins. Shaw and co-workers were some of the first to recognise the inherent patterning in glacial landscapes and develop a unifying mechanism to create a surface of bedforms. In this sense, it is similar to the instability theory, with the major difference being in terms of the ‘fluid’ media through which bedforms are created.

Crucially, and like the meltwater flood theory, a strength of the instability theory is that it predicts both depositional and erosional drumlins, depending on whether deposition or erosion dominates in particular settings (e.g. section 4.3: Figure 4 and 5). Where till build-up (deposition) is greater than till transport out of the system then drumlins will build-up, accrete, migrate and deform; and this is likely to result in both homogenous and surface conformable (accretionary) structures depending on the duration of ice and sediment flow and the effective pressures on both developing drumlins and inter-drumlin areas. Generally speaking, these depositional environments might be expected in slower flowing areas and/or towards the margins of an ice sheet, where large sediment depocentres are inherited from previous glaciations. In contrast, where sediment supply is limited from upstream, then the mean level of the ice-bed interface will lower as the higher/thicker drumlin material is eroded around the base. These environments might be more common down-ice from the core areas of an ice sheet where previous glaciations may have stripped away the sediments and/or beneath fast-flowing ice streams, where subglacial erosion is often focussed (cf. Smith et al., 2012).

Similar processes of net sediment removal or deposition were described by Schoof and Clarke (2008) in a numerical model of flute formation, which also allowed for observations of both erosional and depositional bedforms. As described above (section 4.3, 4.4 and appendix), net sediment removal will produce drumlins that largely reflect the pre-existing sedimentary units that pre-date drumlin formation (cf. Knight and McCabe, 1997a; Menzies and Brand, 2007) and such units are unlikely to mimic the surface form of the down-cutting drumlinised surface (e.g. Fig. 7). Using these sediments and structures to deduce the fundamental mechanism of drumlin formation is, therefore, largely flawed (Stokes *et al.*, 2011).

The issue of a depositional versus erosional origin is also linked to the observation that drumlins show varying degrees of deformation structures within them. It is very clear from the preceding discussion that sediments inside drumlins show a range of features that attest to both brittle and ductile deformation and which might occur extensively and throughout the entire drumlin thickness (e.g. Menzies *et al.*, 1997) or in discrete locations or very thin layers (e.g. Menzies and Brand, 2007). Such features might occur in till (e.g. Hart, 1995a; Hart 1997) or initially sorted sediments (e.g. Ellwanger, 1992), or at the interface between the two (e.g. Boyce and Eyles, 1992); or even between till and bedrock (e.g. McCabe and Dardis, 1994).

A further strength of the instability theory is that it predicts this range of deformation histories. Where till flux into the system is greater than till flux out of the system, it is likely that bumps (drumlins) will grow through accretionary mechanisms and compressive stresses will develop on their stoss faces, whilst tensional forces develop towards their lee side, and these simple concepts (see also Morland and Boulton, 1975, for a fuller treatment) can explain a range of observed deformation structures that might result during drumlin formation (e.g. boudins, drag folds, thrust faults, etc.). As noted above, however, if the till flux out of the system is greater than that moving into the system then the wavy interface is likely to cut-down through pre-existing sediments to create drumlins as erosional remnants that might show minimal evidence of deformation and/or deformation structures that pre-date drumlin formation. As noted earlier, these ideas are not new in the drumlin literature, with Hart (1995b; 1997) invoking similar scenarios to account for both 'constructional' and 'destructional' deformation to describe drumlins formed by the build-up or removal of material within a deforming layer.

In summary, whilst the concept of erosional and depositional drumlins is useful, it does not justify a view that suggests they have a different mechanism of formation. In this sense, it is a false dichotomy (cf. Schoof and Clarke, 2008) and it is, perhaps, more helpful to view them as end members of a continuum. The appeal of the instability theory is that it accounts for both end members and various intermediate forms along this continuum (Fig. 4 and 5).

5.2. Timescales of drumlin formation: inheritance, preservation and prediction

It is clear from numerous studies described above that the material found within drumlins can be related to processes representing several ice flow phases, as well as being inherited from previous sedimentary environments that were not associated with ice flow in any sense. An excellent example of this can be found in Stea and Pe-Piper (1999) who used whole rock geochemistry to locate the source of igneous erratic material inside drumlins in Nova Scotia. This provenance analysis revealed that the material inside the drumlins was delivered by at least two ice flow phases with different source areas. Furthermore, Stea and Brown (1989) noted that till units in some drumlins represented erosional remnants from older drumlins, around which material was emplaced, see Figure 9. These processes are also expressed in the surface morphology of drumlin fields, many of which are known to reflect a palimpsest landscape of 'cross-cutting' ice flow landforms (cf. Clark, 1993). Furthermore, there are several reports of different types of bedforms being superimposed on each other, e.g. ribbed moraine superimposed on top of drumlins (Dunlop and Clark, 2006) or mega-scale glacial lineations (Stokes *et al.*, 2008); or drumlins superimposed on top of ribbed moraine (e.g. Dunlop and Clark, 2006; Knight and McCabe, 1997b; Hättestrand and Kleman, 1999) or mega-scale glacial lineations (Clark, 1993). Thus, whilst it is clear that some material inside drumlins might be unrelated to the drumlin forming mechanism (the 'erosional' drumlins described above in section 5.1) it is also important to appreciate that material may also reflect a time-integrated record of several ice flow phases and bedforming events, which observations clearly support (e.g. Stea and Brown, 1989).

A key prediction of the instability theory (see Fowler, 2009) that is entirely consistent with these observations is that the time-scale for growth of drumlins is of the order of years (see also Smith *et al.*, 2007), whereas the time-scale of ice sheet occupation and associated changes in ice dynamical behaviour is of the order of hundreds to thousands of years. An obvious consequence is that drumlins are likely to be remoulded by episodic changes in ice flow direction and, in some cases, completely erased. The instability theory would thus predict that, under most circumstances, pre-existing drumlin sediments will form the cores of drumlins from a younger ice flow, if the vertical erosion of the wavy interface or the till flux into the system is not great enough to remove them altogether. In other cases, all evidence of pre-existing drumlins might be removed and yet in other cases, the time window for drumlin formation might be so small as to conduct minimal landform/bedform creation, leaving a pre-existing drumlin field barely modified at all.

These simple concepts suggest that the most important factors controlling drumlin composition and internal structure are: (i) the pre-existing sediments; (ii) the balance between till deposition and erosion; and (iii) the timing and duration of ice flow, which includes episodic changes in flow direction, basal thermal regime, and subglacial water conditions. These controls are encapsulated in Figure 10. If we take a common situation often found towards the margins of an ice sheet with thick layers of till overlying pre-existing glaciofluvial sediments (bottom left in Fig. 10a), the system could evolve to produce drumlins with quite different contents based on the duration and variability of ice flow (Fig. 10a). A similar situation might occur with till overlying bedrock (Fig. 10b). Indeed, these ideas are applicable to a range of settings and, furthermore, provide a useful framework for interpreting drumlin composition (e.g. Boyce and Eyles, 1991).

With the above in mind, it is possible to make some general predictions about where, under a continental ice sheet, drumlins with different compositions are more likely to occur based on the pre-existing substrate (largely influenced by previous cycles of ice sheet erosion/deposition). Figure 11 shows a simplified terrain from a previous glaciation with a core of pre-existing crystalline bedrock (zone 1) surrounded by a transitional zone of bedrock and thin till (< few metres: zone 2) that progressively thickens towards the ice sheet margins (zones 3 and 4). Such a terrain is not dissimilar to an idealised Laurentide Ice Sheet bed during the Late Pleistocene, which is thought to have changed from an all soft-bedded to a mixed hard-soft bedded ice sheet during the Middle Pleistocene through glacial erosion of a thick regolith and resulting exposure of unweathered crystalline bedrock (Clark and Pollard, 1998). Given the waxing and waning of ice sheets during both glacial-interglacial and stadial-interstadial time-scales, it would also be expected that the periphery of the previous ice sheet extent would be characterised by thicker sequences of proglacial/deglacial sediments for subsequent overriding (zone 5).

Given the pre-existing terrain in Figure 11, it is not hard to predict (at a general level) which of the main types of drumlins might be expected to occur in each zone as a result of the last glaciation. Zone 1 would be characterised by purely bedrock forms (type 1; Fig. 3); zone 2 by part bedrock/part till (type 2); zone 3 by mainly till (type 3); zone 4 by part till/part sorted sediments (type 4); and zone 5 by mainly sorted sediments (type 5). This is, of course, a generalisation and localised variations are bound to exist but it emphasises the importance of the pre-existing substrate conditions in influencing drumlin composition.

Unfortunately, it is not yet possible to ascertain whether the spatial patterns in drumlin types shown on Fig. 11 actually exist (e.g. on the Laurentide Ice Sheet bed). This is because observations of drumlin composition are typically restricted to just a small sample of drumlins within a particular drumlin field, and because most observations are tightly clustered towards the southern marginal areas of the last mid-latitude ice sheets (see Fig. 17 in Stokes *et al.*, 2011). However, observations from these regions suggest that 'mainly till' (type 3) and 'part till/part sorted' (type 2) are by far the most commonly reported (Stokes et al., 2011), which may lend support to the ideas encapsulated in Figure 11. Furthermore, whilst there are very few observations of drumlin composition from interior regions of former ice sheets, reports from Fennoscandia suggest that drumlins dominated by bedrock components (zone 2, type 2 on our Fig. 11) are commonly found towards interior zones (e.g. Glückert, 1973; Minell, 1979; Möller 1987; Haavisto-Hyvärinen, 1997).

6. Summary and Conclusions

The instability theory proposes that a range of subglacial bedforms (including drumlins) arise from an instability that occurs at the ice-bed interface as the result of the coupled flow of ice and till and is one of the few explanations to be described in physically-based numerical/analytical models (cf. Fowler, 2000; 2010a; Hindmarsh, 1998a; b; Schoof, 2002b;

Chapwanya *et al.*, 2011). Predictions from these models have been shown to match observations of bedform dimensions (e.g. Dunlop *et al.*, 2008; Fowler, 2000; 2009; 2010b; Chapwanya *et al.*, 2011). A key strength of the instability theory, therefore, is that it offers a promising unifying explanation for a range (continuum?) of subglacial bedforms (cf. Aario, 1977; Rose, 1987) of which drumlins are the most ubiquitous. Observations of drumlin composition and internal structure, however, are incredibly diverse and this is often seen as a major obstacle to a unifying theory. In this paper, we have compared the key observations of drumlin composition and internal structures in the literature and considered, theoretically, how they might arise based on firm physical principles that form the basic ingredients of the instability theory. Contrary to a view which suggests that certain observations (e.g. the presence of undeformed stratified sediments) are inconsistent with the instability theory, we suggest that one would actually *expect* a range of drumlin constituents, including at least some occurrences of drumlins with stratified cores of glaciofluvial material.

In terms of the five main types of drumlin composition (Figure 3) identified in the literature (Stokes *et al.*, 2011) and excluding mainly bedrock forms (type 1) the instability theory suggests:

- Drumlins composed of part-bedrock/part till (type 2) occur because bedrock bumps act as perturbations that give rise to drumlins
- Drumlins composed of mainly till (type 3) occur because of the dependence on a mobile till unit that grows in thickness in a depositional environment to form the body of the drumlin
- Drumlins composed of part till/part sorted sediments (type 4) occur through the advection of till across and erosion into pre-existing sorted sediments and around cores of sorted sediments

- Drumlins composed of mainly sorted sediments (type 4) through the vertical erosion of both till and meltwater into pre-existing sorted sediments at the wavy ice-bed interface

Related to the above, and with specific reference to the structure of the sedimentary units, the instability theory predicts:

- Drumlins which are built by successive episodes of till influx and deposition will naturally build a structure in which the separate till units are conformable with the drumlin surface, i.e. accretionary or depositional drumlins where more sediment flows into bumps than out of them
- Drumlins with internal structures that are unconformable with the drumlin surface in conditions where till flux out of the system is greater than till supply into the system, i.e. erosional cores may be preserved as the wavy interface cuts vertically downwards
- Drumlin formation and shaping can occur rapidly (few decades), such that changes in ice flow direction will lead to inherited cores from previous flow directions.

Within the framework of the instability theory, the varied content of drumlins can be explained by three key factors: (i) the pre-existing sediments; (ii) the balance between till erosion and deposition; and (iii) the variability and duration of ice flow. These simple concepts offer an interpretative and predictive framework for where specific types of drumlin composition might be found on an ice sheet bed and how they might be interpreted in terms of ice dynamics and sediment flux. We conclude that the instability theory represents the most promising solution to the ‘drumlin problem’ thus far and offers a unifying explanation for the creation of a range of subglacial bedforms.

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Figure Captions:

Figure 1: Instabilities create familiar patterned surfaces of bedforms in a range of environments not only restricted to Earth: (A) HiRISE image of the Herschel Crater, Mars, showing a field of barchan dunes (from Kocurek *et al*., 2010); (B) hillshade image from LiDAR-derived DEM of laterally linked barchans dunes to form crescentric dunes in the White Sands Dune Field, New Mexico (from Kocurek *et al*., 2010); (C) multi-beam bathymetric images of Mississippi River bed at Audubon Park, Louisiana, during high discharge of 34,300 $m^3 s^{-1}$ (left) and low discharge of 8,900 $m^3 s^{-1}$ (right) (from Kocurek *et al*., 2010); (d) DEM of down-ice transition from barchan-like ribbed moraine (left) to drumlins (right) in north central Ireland (from Clark, 2010); (E) aerial photograph of classical-type ribbed moraine located at Lake Rogen, Härjedalen, central Sweden; (F) modelled subglacial bedforms using the Hindmarsh-Fowler instability theory as formulated in Chapanwanya *et al*. (2011), taken from Clark (2010). Note that these modelled features are almost identical to the 'real' ribbed moraine in (E), in terms of their dimensions and wavelengths, with 'drumlin-like' culminations appearing in red shading (~ 1 m high) after 50 years).

Figure 2: Schematic diagram showing the basic ingredients and underlying principles of the instability theory. When ice and sediment are allowed to deform and sliding can occur at the ice-till interface (a), the system is prone to the development of an along-flow instability which creates waveforms (bedforms) at the ice-till surface (b) that emerge as drumlins of dominant wavelength (c). See section 2 for detail.

Figure 3: Schematic illustration of the five main types of drumlin (and sub-types) identified in a systematic review of the literature reported in Stokes *et al.* (2011), who further suggest

that purely bedrock forms (type 1) should be referred to as symmetrical or asymmetrical whalebacks, rather than drumlins (cf. Dionne, 1987; Evans, 1996).

Figure 4: Schematic illustration of how drumlins can emerge with varying degrees of bedrock ‘control’ through time in a system where till continuity is inhibited, e.g. where till transport > till supply. The instability theory predicts that the ice-till interface, under certain circumstances, is unstable and becomes wavy (Fig. 2). Depending on till continuity (i.e. the balance between till transport within the deforming layer and till supply from erosion and or advection from up-ice) the wavy interface can cut downwards. Given a setting with a metres-thick layer of pre-existing till overlying bedrock, drumlins will emerge from the instability with minimal bedrock control (A). As till is removed from the system (because till transport > till supply), the till thickness will be reduced and some drumlins will be anchored by pre-existing perturbations that might act as cores in a variety of locations (B). Further till exhaustion might lead to more obvious drumlin cores, as in (C), crag-and-tail features (D), and an entirely bedrock surface (E).

Figure 5: Schematic illustration of how drumlins can emerge with stratified cores of glaciofluvial material (or similar units) through time in a system where till continuity is inhibited, e.g. where till transport > till supply. Given a wavy interface, the effective pressure (*N*) is predicted be highest at the tops of the drumlins and lowest around the base of the drumlin and in inter-drumlin areas, where the hydraulic potential of the water is also likely to be lowest and where meltwater is likely to be concentrated (A). We envisage a slow trickle through a swamp-like basal platform that erodes around the stiffer drumlins (B). As in Figure 4, if the wavy interface cuts vertically into pre-existing sediments, it is likely that pre-existing sediments will be incorporated into the drumlins and one might envisage a situation evolving

through time from drumlins composed of mainly till (C); part till/part sorted sediments (D), to mainly sorted sediments (E). This schematic illustrates pre-existing glaciofluvial sediments but applies to any pre-existing sedimentary units (e.g. till units, deltaic deposits, etc.).

Figure 6: Schematic illustration of how pre-existing sediments may influence drumlin formation (from Boulton, 1987). In this case, areas of 'stiffer' well-drained glaciofluvial material act as cores around which till deforms. Clark (2010) refers to these as drumlin clones (or obstacle drumlins where an obvious bedrock protuberance occurs) to distinguish them from drumlins formed purely from the instability (emergent drumlins). The formation of drumlin clones is consistent with the instability theory but a further appeal of the instability mechanism is that it can also account for drumlins without obvious cores based on purely fluid dynamical principles.

Figure 7: Cross section of the Port Byron drumlin, New York State, USA (redrawn from Menzies and Brand, 2007) that clearly illustrates the presence of mainly stratified sediment overlain with only a thin veneer of till. These observations clearly show minimal disturbance of pre-existing sediments that are unrelated to drumlin sediments but probably acted as a stiffer core (see Fig's 5 and 6).

Figure 8: Evolution of an initially stratified layer of sediments in an erosive environment, as described in the appendix and shown schematically in Figure 5. Horizontal and vertical axes are in metres and parameters used are for a 5 m high drumlin eroding down at E = 0.5 m y^{-1} for 10 years. Deformable till depth (d_T) = 0.5 m and ice velocity (u_0) = 18 m y^{-1}. The drumlin profile is $S = \frac{1}{2}a_0 \cos kx$ where a_0 is the drumlin height, x is the distance along flow in metres, and k is the wave-number (= $2pi/l$, where l is the length (period) of the

drumlin in metres). A brief movie of the evolution is available online as supplementary information.

Figure 9: Changes in ice flow direction over time are likely to result in complex stratigraphies within drumlins, where pre-existing material from older bedforms may be wholly or partially removed. This is detailed in Stea and Brown (1989) who interpreted material from some drumlins in central Nova Scotia as relicts from older bedforms, shown in (A). Shaded areas under stratigraphy and form are thought to represent till units formed at the same time as the drumlin shaping process whereas unshaded areas under stratigraphy represent erosional remnants of earlier units (redrawn from Stea and Brown, 1989). A satellite image of cross-cutting bedforms from Wollaston Peninsula, Victoria Island, Canadian Arctic Archipelago, is shown in (B), depicting three populations of drumlins (selected bedforms highlighted with red (inferred oldest), green and yellow (youngest) lines: from Stokes *et al*. (2006).

Figure 10: The instability theory suggests that the most important determinants of drumlin composition are: (i) the pre-existing sediments (shown bottom left in each panel); (ii) the balance between till erosion and deposition (y-axis on each panel); and (iii), the duration of flow (x-axis on each panel). Variations in the above are predicted to produce a variety of drumlin types from initial substrate conditions, depicted here as till overlying sorted sediments (A) and till overlying bedrock (B), although the concepts apply to any pre-existing terrain. The drumlin types refer to those described in section 4 and Figure 3.

Figure 11: Schematic illustration of the predicted occurrence of drumlins with different composition under an idealised ice sheet, which bears some similarity to the Laurentide Ice

Sheet bed, but is used to make the general point that drumlin composition is likely to largely reflect pre-existing sediments and their position in relation to the ice sheet margin. The drumlin types refer to those described in section 4 and Figure 3.

Appendix

In order to simulate the evolution of subsurface stratified sediments under the evolution of the instability which causes drumlins to grow, it is necessary to specify a subsurface transport field. In the development of the theory (Fowler 2009), no reference was made to any specific rheology, other than that the sediment flux q was assumed to increase with increasing basal stress τ and decrease with increasing effective pressure N. In a two-dimensional region (coordinates x and z) of sediment bounded above by the ice-till interface at $z = s$, we may write

$$q = \int_{-\infty}^{s} u \, dz, \tag{A.1}$$

where the till velocity has horizontal and vertical components u and w.

In a complete theory such as that of Fowler (2009), we derive an evolution equation for s based only on a prescription for q. In order to facilitate our present objective, we will *prescribe* $s(x, t)$, and use its form to infer subglacial sediment transport patterns, based on a realistic assumption about the till velocity. Specifically, we make the assumption that

$$u = u_0 \exp[-b(s - z)], \tag{A.2}$$

where u_0 is the sliding velocity at the ice-till interface, and may be taken to be constant.[1] The exponent b measures the depth ($\sim b^{-1}$) of the deforming till layer, and we expect values $b \sim 1$ m^{-1}, although necessarily, b cannot be constant. In fact, the assumption (A.2) implies that

$$b = \frac{u_0}{q}. \tag{A.3}$$

Notice in particular that q must remain positive (as we expect).

In the present situation, we are interested in the case where net erosion of the sediment causes downcutting of the ice into the sediment, and in this case we pose a modified form of the Exner equation as

$$s_t + q_x = -E, \tag{A.4}$$

where E represents a net erosion rate with units of metres per year. We do not conceive of this erosion as being the plucking and grinding of bedrock (which would not present such a term in the Exner equation), but rather a superfluous removal term by subglacial stream flow.

Given the horizontal velocity u in (A.2), we can solve for w to find (bearing in mind that the kinematic condition at $z = s$ is $w = s_t + us_x + E$)

$$w = us_x + (s_t + E)[1 + b(s - z)] \exp[-b(s - z)], \tag{A.5}$$

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[1] Equation (A.2) is of course inconsistent with a finite thickness of deforming till, but the distinction is only cosmetic, and the present assumption is made purely for algebraic convenience.

and individual particles can be tracked by solving the ordinary differential equations

$$\dot{x} = u, \quad \dot{z} = w. \tag{A.6}$$

In particular, we can track the evolution of different layers of sediment by assigning a variable c which is an indicator for material content. For example, we might take $c = -1$ for clay, $c = 0$ for sand, and $c = 1$ for till. If (ξ, ζ) marks the initial location of a particle, then the sediment type is given by a function

$$c = c(\xi, \zeta), \tag{A.7}$$

and ξ and ζ are the initial values at $t = 0$ for x and z, i. e.,

$$x = \xi, \quad z = \zeta \quad \text{at} \quad t = 0. \tag{A.8}$$

In practice we sequentially plot the surface (x, z, c) parametrically at successive times in terms of the parameters ξ and ζ, using Matlab's `scatter` command.

Choice of parameters

To be specific, we choose the interface position s to be given by the function

$$s = -Et + a \cos k(x - vt), \tag{A.9}$$

where a is the interfacial amplitude, and v is the interfacial wave speed. Generally, a is a function of time, and a representative choice is the function

$$a = \tfrac{1}{2} a_0 (1 - e^{-rt}), \tag{A.10}$$

where a_0 is the final drumlin elevation, and r is a measure of the growth rate.[2] The wavenumber k is defined in terms of the wavelength l by

$$k = \frac{2\pi}{l}, \tag{A.11}$$

and the inlet sediment flux is chosen as

$$q_0 = \tfrac{1}{2} u_0 d_T, \tag{A.12}$$

where d_T is an estimate of deformable till thickness. From (A.4) and (A.9), we have the expression for q,

$$q = q_0 - \frac{\dot{a}}{k}[\sin(k(x - vt) + \sin kvt] + av[\cos k(x - vt) - \cos kvt], \tag{A.13}$$

where we apply the condition $q = q_0$ at $x = 0$.

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[2] More realistic choices might be made to reflect initial exponential growth, but there is little purpose to this.

Symbol	Meaning	Typical value
a_0	elevation	10 m
b^{-1}	till shear exponent	$\sim d_T \sim 1$ m
d_T	till deformation thickness	1 m
E	erosion rate	0.1 m y^{-1}
l	wavelength	300 m
q_0	upstream sediment flux	50 m^2 y^{-1}
r^{-1}	growth time scale	10 y
u_0	sliding velocity	100 m y^{-1}
v	wave speed	$0, 50$ m y^{-1}

Table 1: Typical values of the parameters in the model.

Table 1 gives our estimate of typical values of the parameters. An awkwardness occurs in making the simulations. Because q must remain positive, we see from (A.13) that we must have $q_0 > \dfrac{2\dot{a}}{k}$, and thus the growth time

$$r^{-1} > \frac{a_0}{\pi d_T}\frac{l}{u_0}, \tag{A.14}$$

which is roughly the time $t_D = \dfrac{l}{u_0}$ for a sediment particle at the ice-till interface to move one drumlin length. Equally, it is necessary that $q_0 > a_0 v$, and thus

$$a_0 < \frac{u_0 d_T}{2v}. \tag{A.15}$$

A more thorough analysis of (A.13) shows that the precise condition is that both (A.14) and (A.15) must be satisfied, or simply

$$a_0 < d_T \min\left(\frac{u_0}{2v}, \frac{\pi u_0}{rl}\right). \tag{A.16}$$

The awkwardness lies in the fact that since we typically expect the growth time $r^{-1} \sim \dfrac{l}{u_0}$, and for wave instabilities, $v \sim u_0$, the constraint on amplitude is that $a_o \lesssim d_T$, yielding unnecessarily small amplitude drumlins. This point was one of Schoof's (2007a) objections to the instability theory of drumlin formation, and can be seen to be a purely kinematic consequence of the Exner equation. In reality, cavities form for $a_0 > d_T$, and the Exner equation cannot be applied in the same way (Fowler 2009).

We have sidestepped this issue here by considering only the case where the waves are stationary ($v = 0$) as found by Fowler (2009). It is essentially obvious that a downcutting drumlin with only a thin veneer of mobile till will maintain subsurface stratification; but an illustration nevertheless illuminates the point. It is also obvious that a travelling drumlin will churn up the subsurface sediments, and in fact we consider this to be a mechanism to provide till-formed drumlins, despite only having near-surface mobility. As explained in the text, we may associate travelling drumlins with non-hydrostatic water pressure, i. e., less well-drained material (Chapwanya *et al.* 2011). Figure 8 in the main text shows the results of a computation as described above.

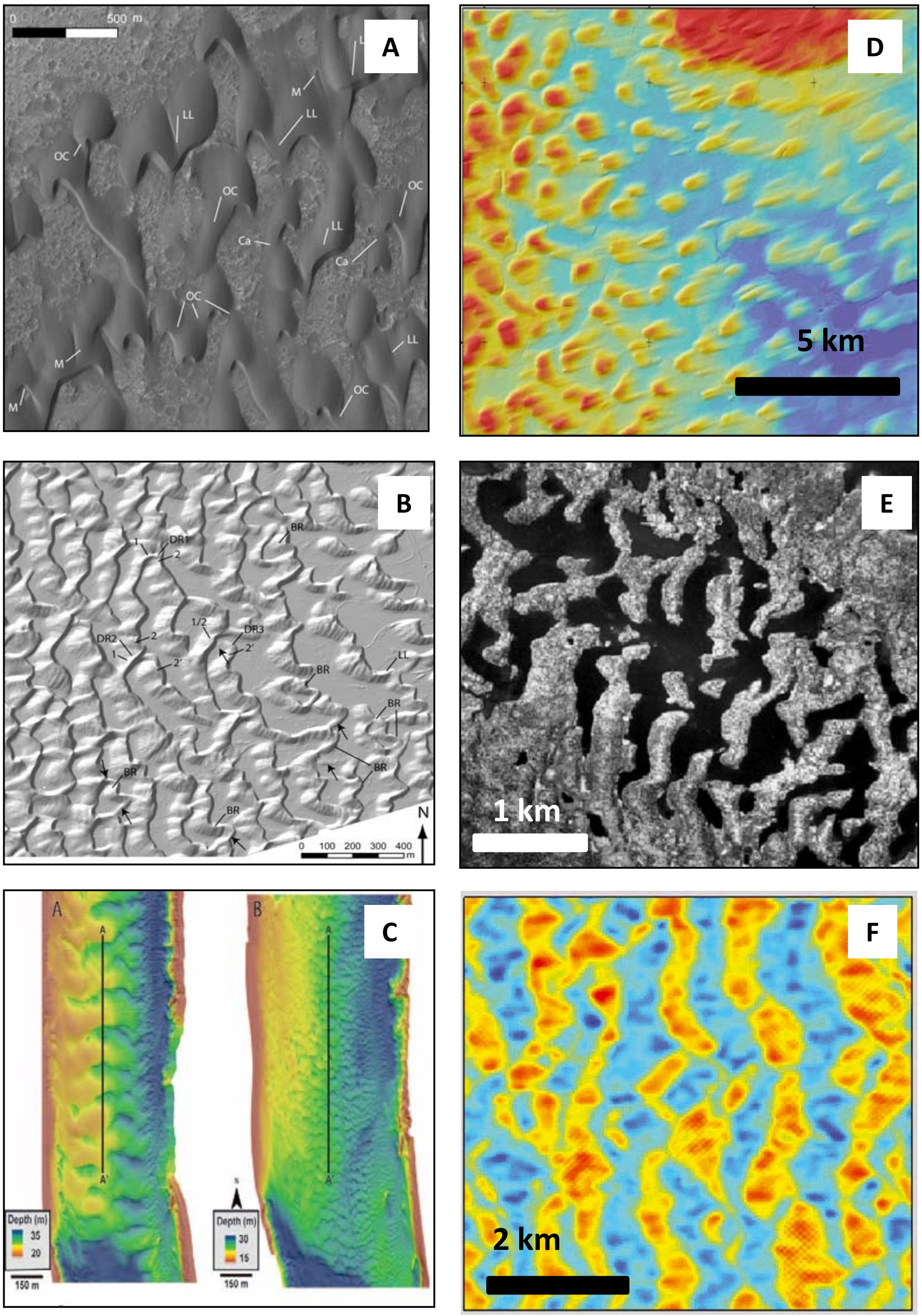
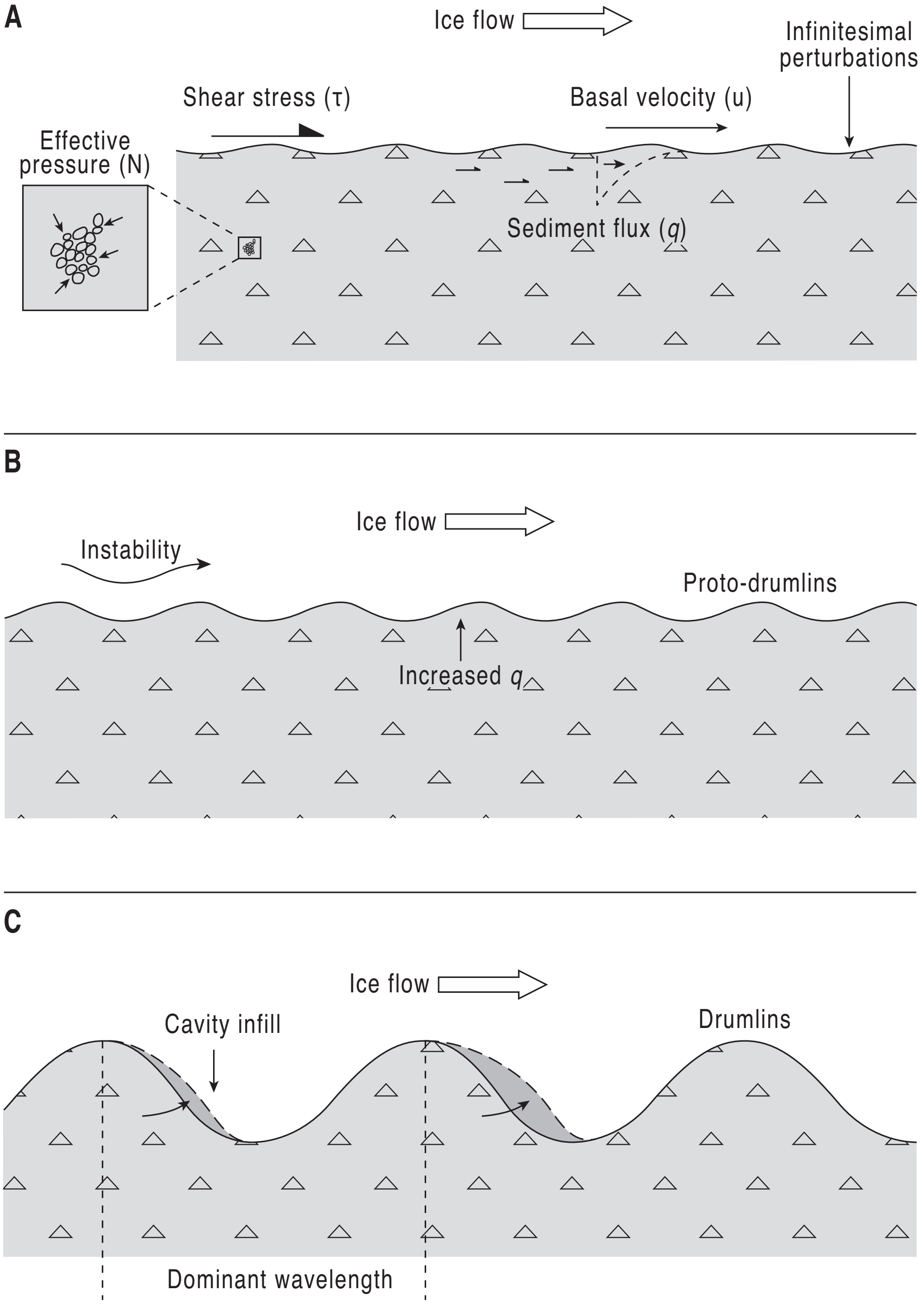
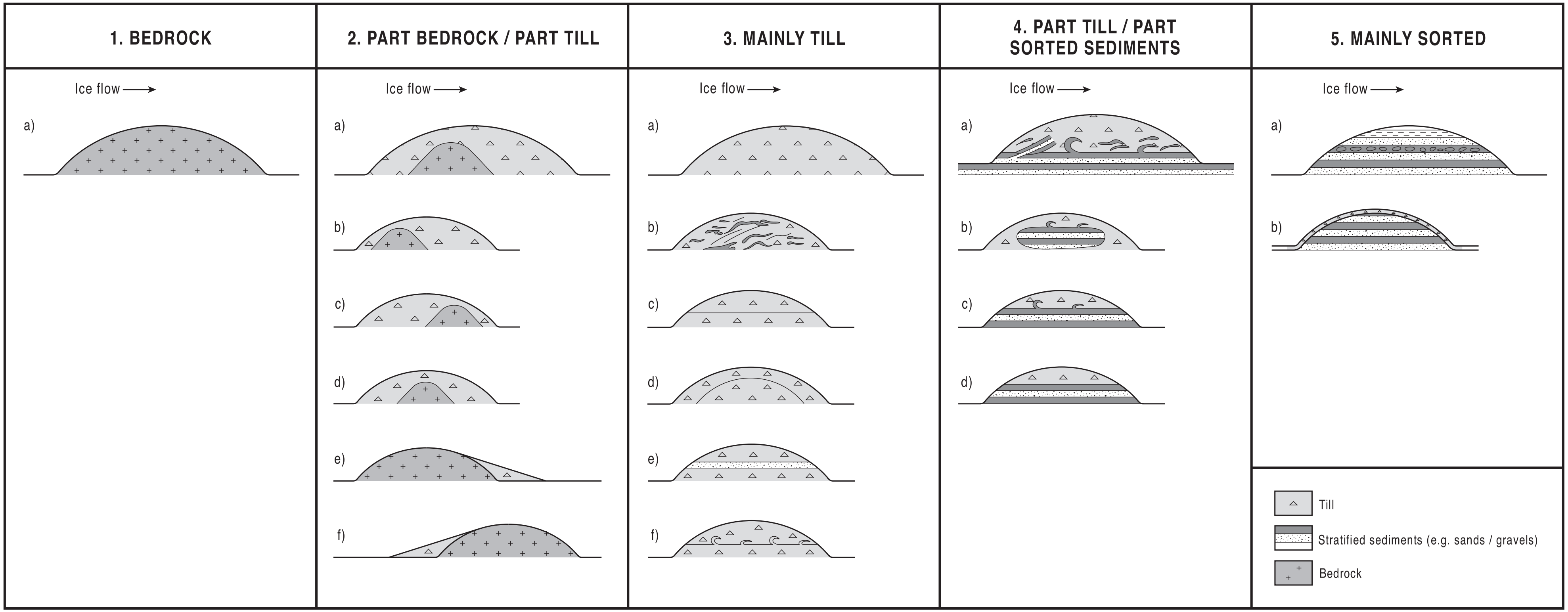
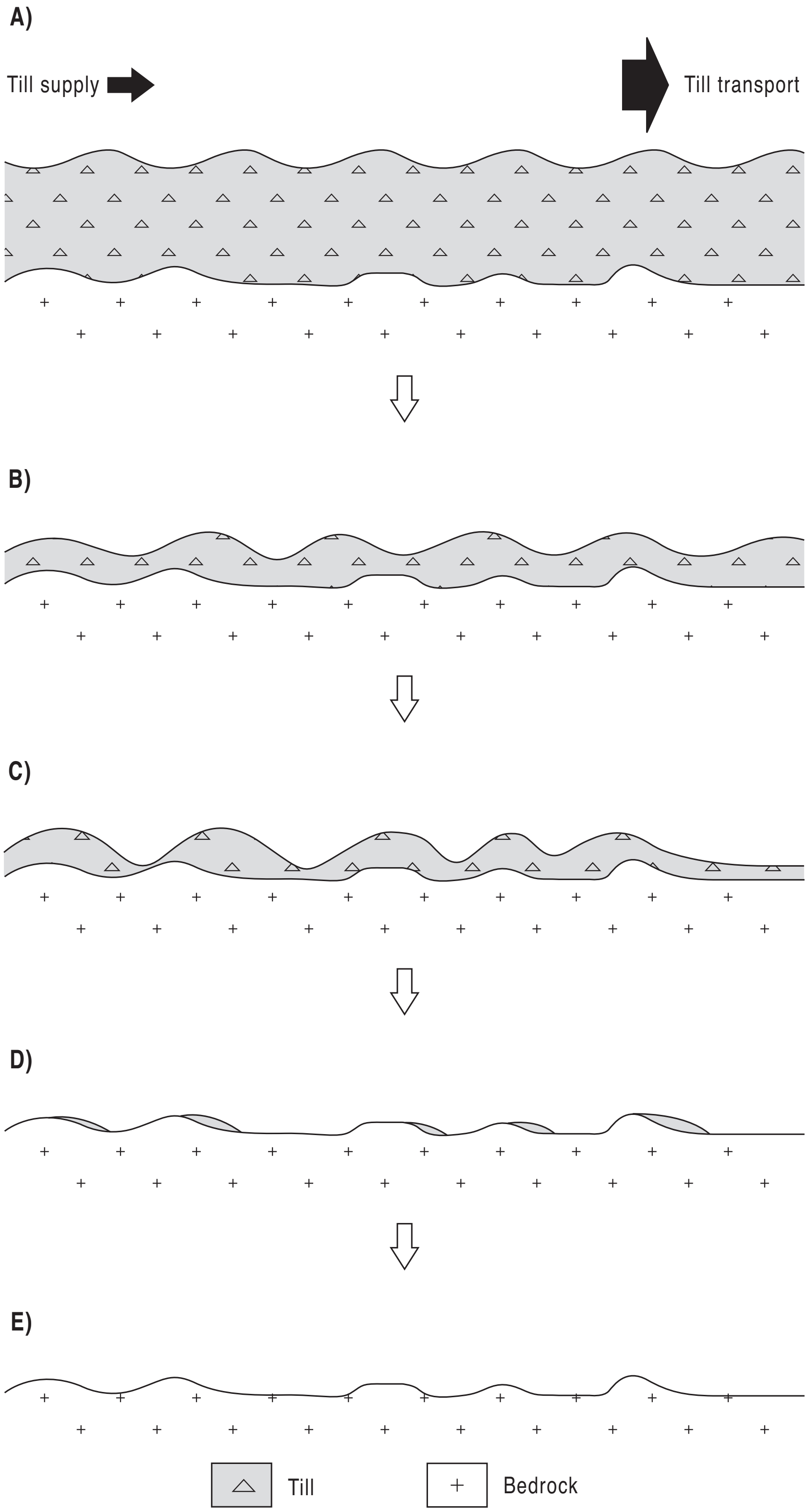
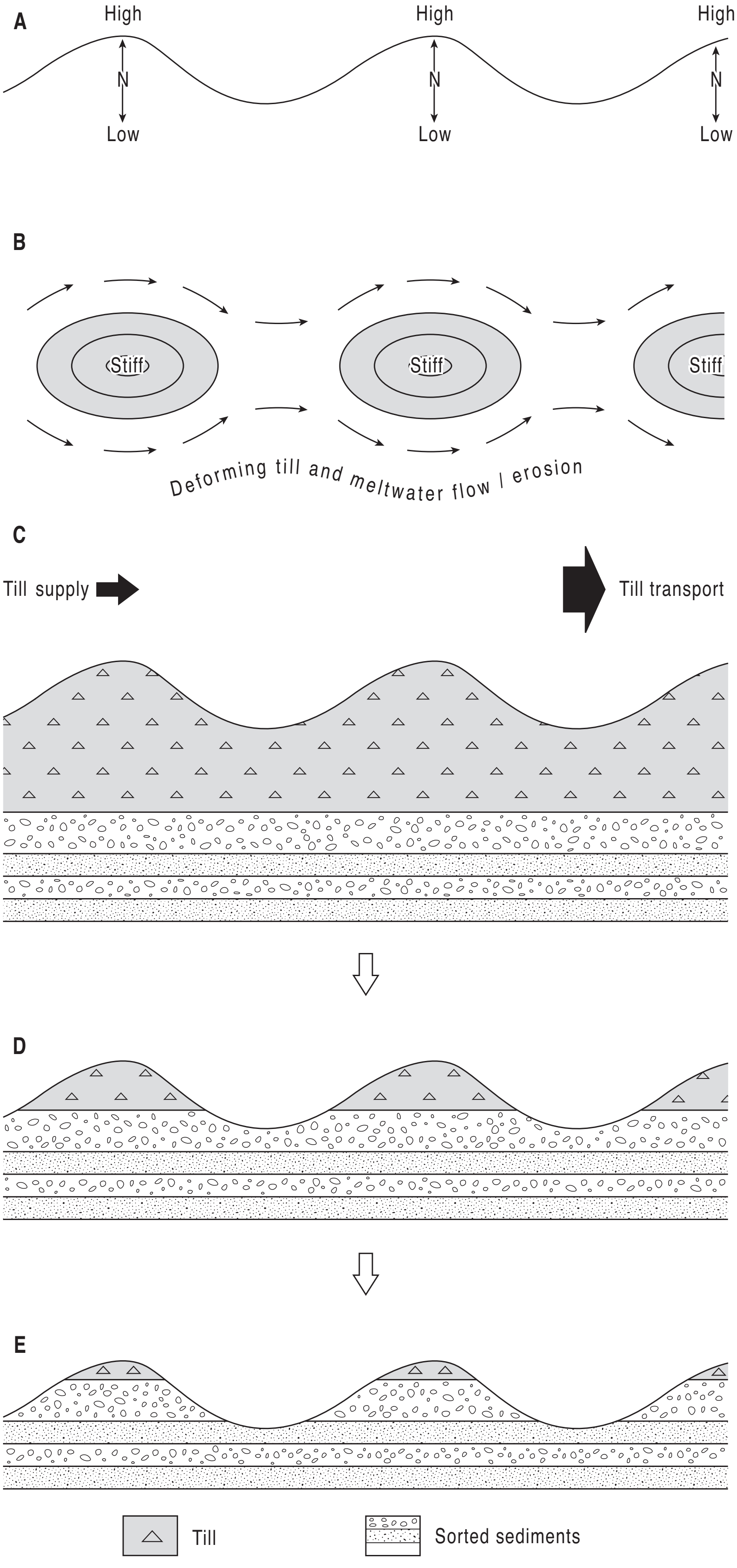

A
0
500 m
L
M
LL
LL
OC
OC
OC
LL
Ca
Ca
OC
LL
M
M
OC
D
5 km
B
BR
1
DR1
2
1/2
DR3
DR2
2
1
2'
2'
LL
BR
BR
BR
BR
BR
N
0
100
200
300
400
m
E
1 km
C
A
B
A
A'
A
A'
Depth (m)
35
20
150 m
Depth (m)
30
15
150 m
F
2 km

Figure 1


A
Ice flow
Infinitesimal perturbations
Shear stress (τ)
Basal velocity (u)
Effective pressure (N)
Sediment flux (q)
B
Ice flow
Instability
Proto-drumlins
Increased q
C
Ice flow
Cavity infill
Drumlins
Dominant wavelength


1. BEDROCK
Ice flow
a)
2. PART BEDROCK / PART TILL
Ice flow
a)
b)
c)
d)
e)
f)
3. MAINLY TILL
Ice flow
a)
b)
c)
d)
e)
f)
4. PART TILL / PART SORTED SEDIMENTS
Ice flow
a)
b)
c)
d)
5. MAINLY SORTED
Ice flow
a)
b)
Till
Stratified sediments (e.g. sands / gravels)
Bedrock


A)
Till supply
Till transport
B)
C)
D)
E)
Till
Bedrock


A
High
High
High
N
N
N
Low
Low
Low
B
Stiff
Stiff
Stiff
Deforming till and meltwater flow / erosion
C
Till supply
Till transport
D
E
Till
Sorted sediments

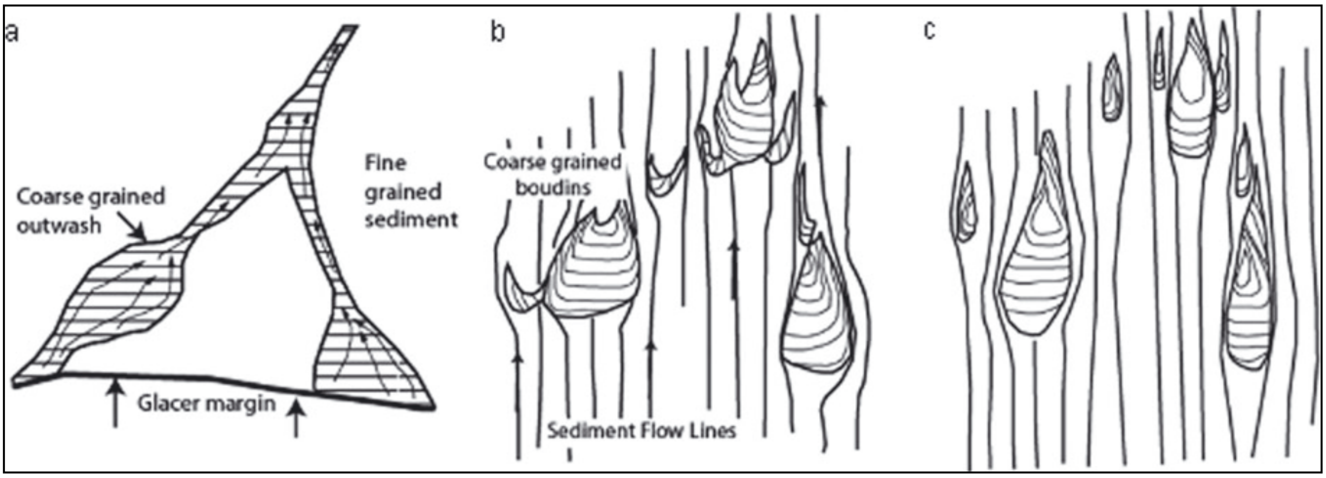
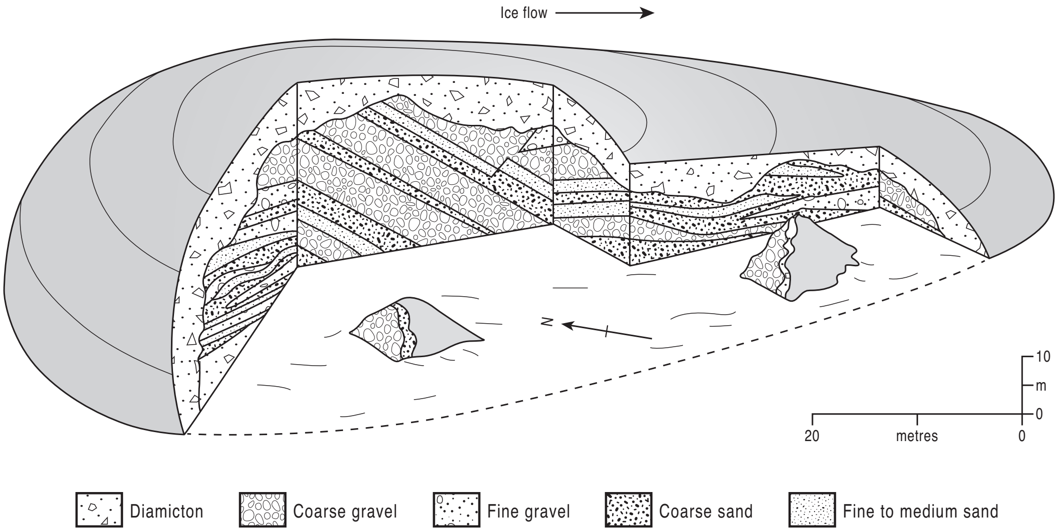
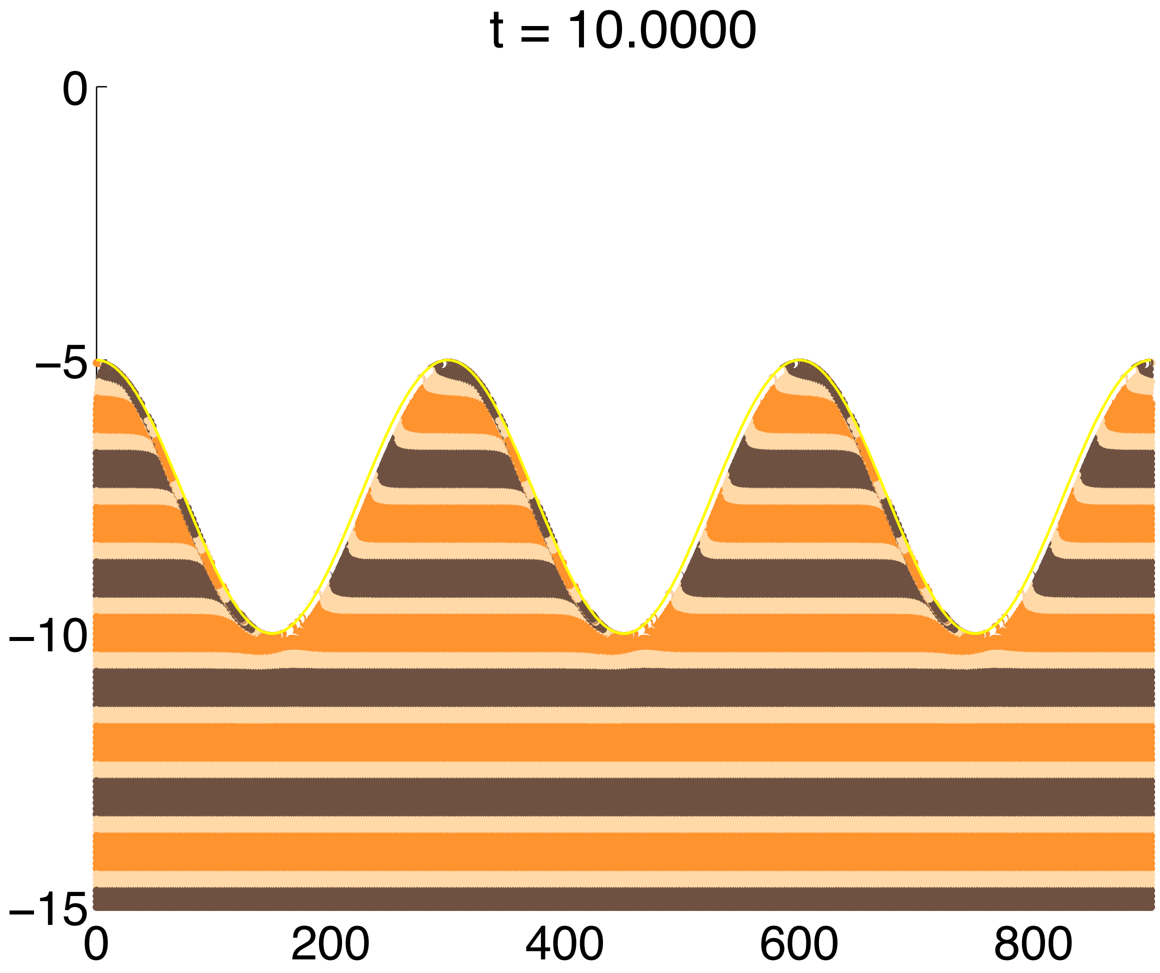

a
Coarse grained outwash
Fine grained sediment
Glacer margin
b
Coarse grained boudins
Sediment Flow Lines
c

Figure 6

PORT BYRON DRUMLIN




t = 10.0000
0
−5
−10
−15
0
200
400
600
800

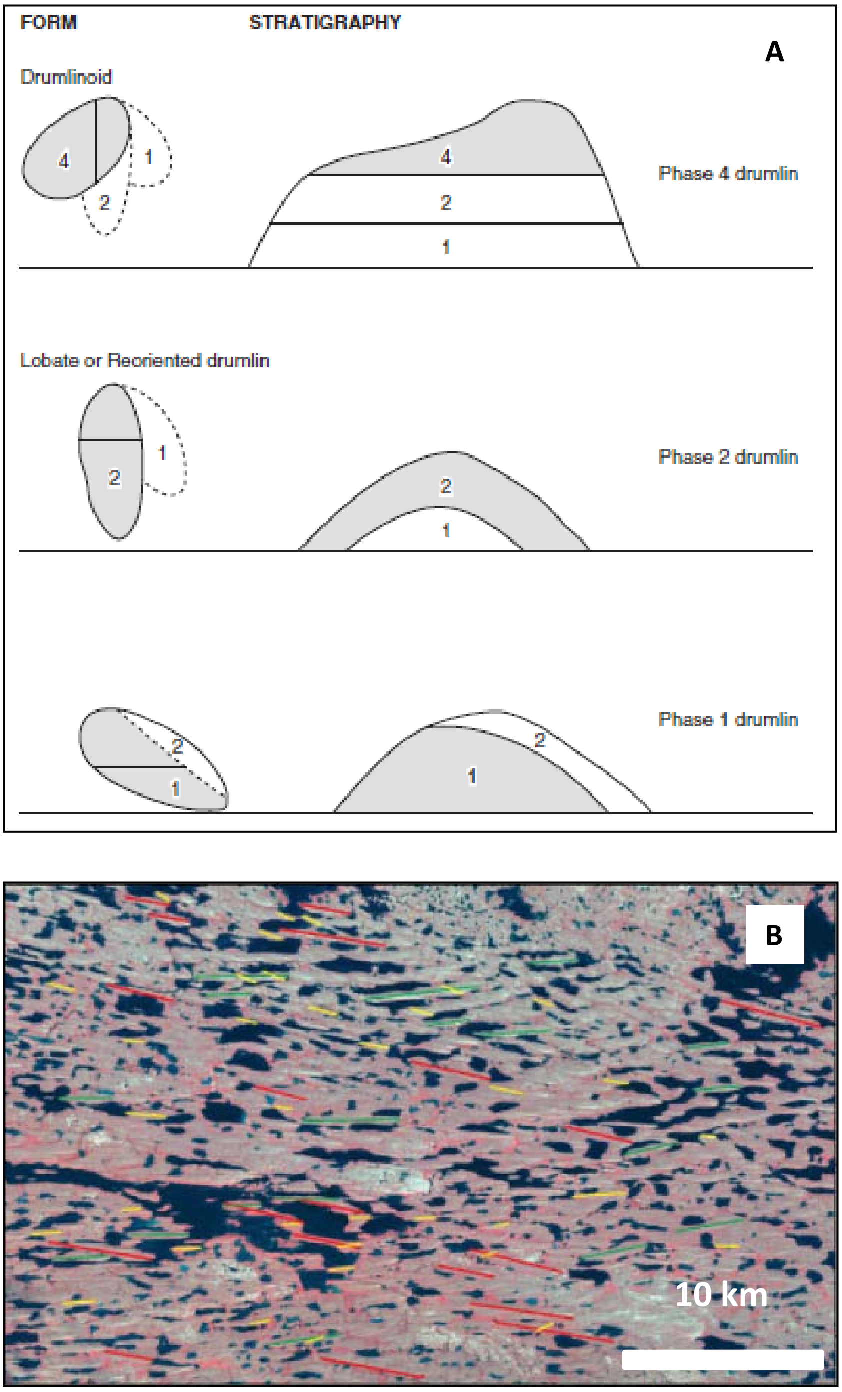
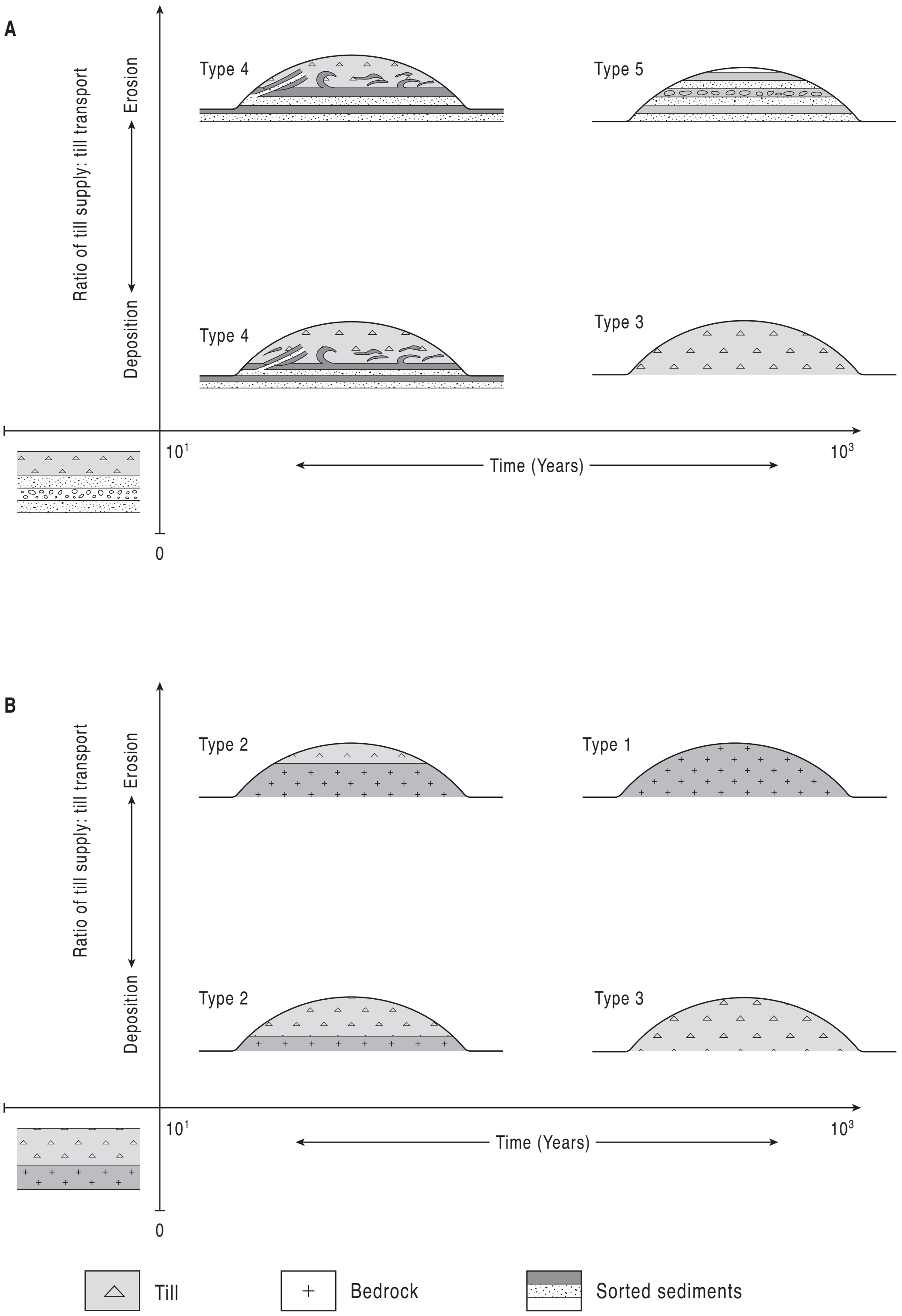
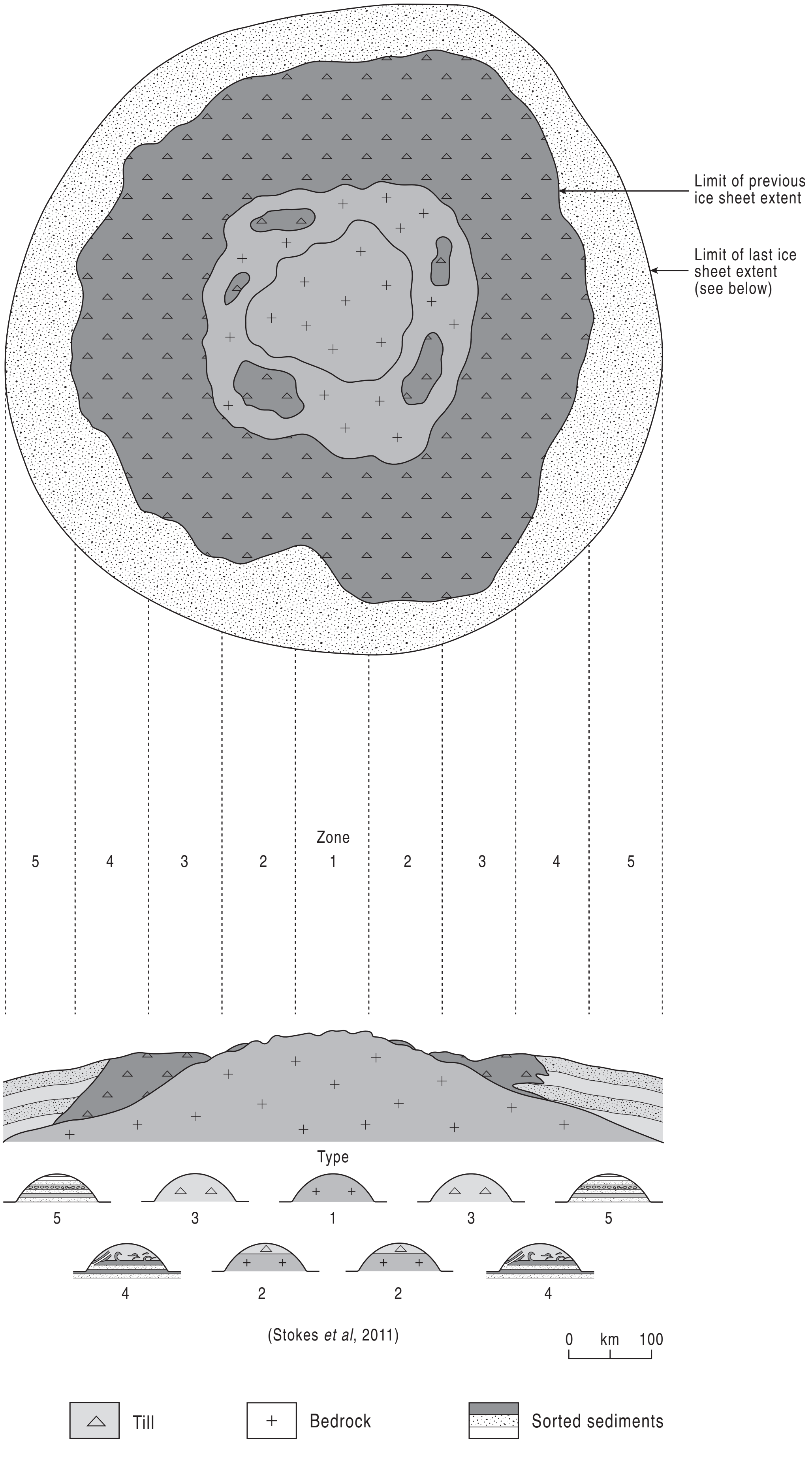

FORM
STRATIGRAPHY
A
Drumlinoid
4
1
2
4
2
1
Phase 4 drumlin
Lobate or Reoriented drumlin
1
2
2
1
Phase 2 drumlin
2
1
2
1
Phase 1 drumlin
B
10 km

Figure 9


A
Ratio of till supply: till transport
Erosion
Deposition
Type 4
Type 5
Type 4
Type 3
10¹
10³
Time (Years)
0
B
Ratio of till supply: till transport
Erosion
Deposition
Type 2
Type 1
Type 2
Type 3
10¹
10³
Time (Years)
0
Till
Bedrock
Sorted sediments



(Stokes *et al*, 2011)