

Intersecting Brane Worlds

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Abstract

It is known that chiral fermions naturally appear at certain intersections of branes at angles. Motivated by this fact, we propose a string scenario in which different standard model gauge interactions propagate on different (intersecting) brane worlds, partially wrapped in the extra dimensions. Quarks and leptons live at brane intersections, and are thus located at different positions in the extra dimensions. Replication of families follows naturally from the fact that the branes generically intersect at several points. Gauge and Yukawa couplings can be computed in terms of the compactification radii. Hierarchical Yukawa couplings appear naturally, since amplitudes involving three different intersections are proportional to $e^{-A_{ijk}}$, where A_{ijk} is the area of a string world-sheet extending among the intersections. The models are non-supersymmetric but the string scale may be lowered down to 1-10 TeV. The proton is however stable due to a set of discrete symmetries arising from world-sheet selection rules, exact to all orders in perturbation theory. The scenario has some distinctive features like the presence of KK, winding and other new excited states ('gonions'), with masses below the string scale and accessible to accelerators. The models contain scalar tachyons with the quantum numbers of standard $SU(2) \times U(1)$ Higgs doublets, and we propose that they induce electroweak symmetry breaking in a somewhat novel way. Specific string models with D4-branes wrapping on $\mathbf{T}^2 \times (\mathbf{T}^2)^2/\mathbf{Z}_N$, leading to three-family semirealistic spectra, are presented in which the above properties are exemplified.

1 Introduction

Two of the most important aspects of the observed fermion spectrum of the standard model (SM) are its chirality and the family replication. Any fundamental theory explaining the structure of the SM should thus give an understanding of these two very prominent features. With the developments of string theory of the last five years we have learnt that a natural setting to understand gauge interactions in this context is that of Type II Dp -branes, which contain gauge fields localized in their world-volume. However, Dp -branes isolated on a smooth space have extended supersymmetry, and hence do not lead to chiral fermions. Thus, for example, Type IIB D3-branes at a smooth point in transverse space have $\mathcal{N} = 4$ supersymmetry on their four-dimensional world-volume.

A simple possibility to obtain chirality is to locate the D3-branes on some singularity in transverse space, the simplest possibility being a $\mathbf{C}^3/\mathbf{Z}_N$ orbifold singularity [1, 2]¹. There is however an interesting alternative to obtain chiral fermions, which has not been very much exploited in the past from the phenomenological viewpoint. As first pointed out in [8], when Dp -branes intersect at non-vanishing angles, open string stretched between them may give rise to chiral fermions living at the intersection. Our purpose in the present article is to study the phenomenological potential of this kind of configurations, in which the observed quarks and leptons are associated to intersections among Dp -branes. In our setting the different SM gauge interactions propagate on different branes, and chiral fermions propagate at their intersections. That is, we have gauge bosons propagating on *intersecting brane worlds*, with quarks and leptons populating the intersections.

Explicit string theory compactifications with branes intersecting at angles have appeared in [9], and more extensively in [10]. We will concentrate in this paper on the simplest non-trivial case, corresponding to D4-branes with one of their world-volume dimensions wrapped on a circle inside a two-torus [10]. Thus the model contains different stacks of D4-branes for the different SM gauge groups, wrapping on the two-torus, and intersecting on four-dimensional subspaces, on which chiral fermions propagate. For example, left-handed quarks appear at the intersection of the $SU(3)$ D4-branes with the $SU(2)_L$ D4-branes. A pictorial depiction of this type of configuration is shown in Fig 1. Interestingly enough, two non-parallel D4-branes on a torus typically intersect at

¹Specific semirealistic string models based on this possibility with the gauge group of the SM or a left-right extension were recently constructed in [3]. See e.g. [4] for other attempts to build realistic string models of the brane world scenario [5, 6, 7]

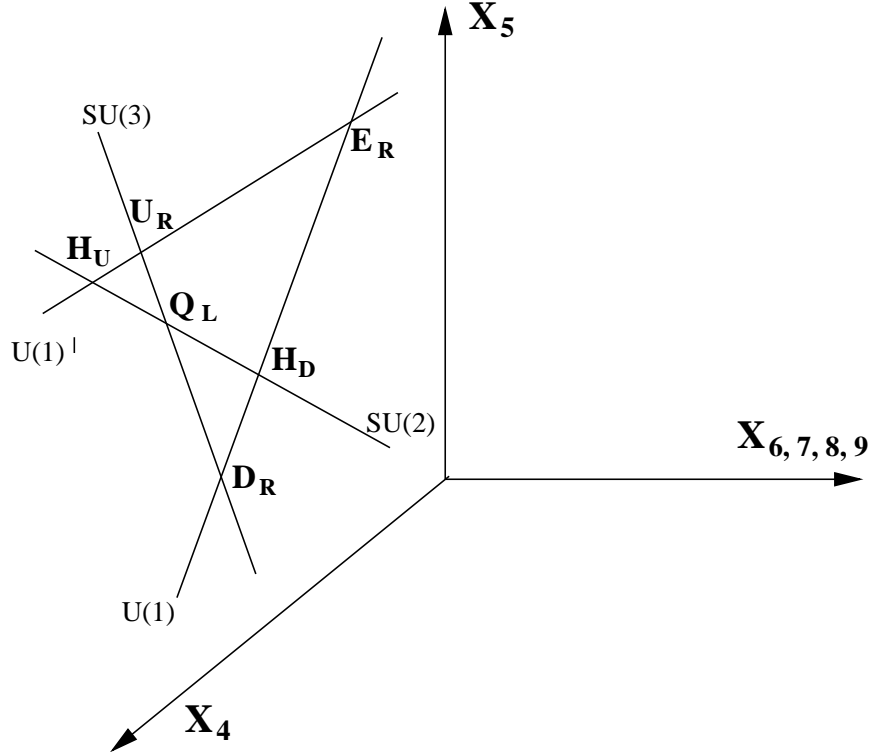


Figure 1: A simplified picture of the intersecting brane world scenario. Each gauge interaction propagates along a D-brane with four flat dimensions (not shown in the figure), and partially wrapped on a cycle in the internal space parameterized by X_4 , X_5 (a two-torus in our models). All branes are transverse to the space parameterized by X_6 , X_7 , X_8 , X_9 . Chiral fermions, such as quarks and leptons, are localized at the intersections of the wrapped branes (for simplicity, we have shown only one such intersection, even though generically multiple intersection points exist).

more than one point, leading to several copies of the same matter content. Thus *replication of quark-lepton generations is a generic property* in this kind of configurations. In particular, it is easy to construct models with a triplication of generations.

Another interesting feature of these constructions is the structure of Yukawa couplings. Some intersection give rise also to scalar fields, which may transform with the quantum numbers of Weinberg-Salam Higgs doublets. Their Yukawa couplings with left(right)-handed fermions $F_L(F_R)$ will be proportional to $\exp(-A_{ijk})$, where A_{ijk} is the area of the worldsheet extending among the intersections where the Higgs, F_L and F_R live. Due to this fact, it is easy to obtain a hierarchical structure of quark and lepton masses, as we show in some specific models.

The models we are describing are generically non-supersymmetric. In order to avoid

the gauge hierarchy problem, one may lower the string scale down to 1-10 TeV in the usual way [5], by having some or all of the four extra dimensions transverse to the two-torus large enough ². An important property of these models is that they do not predict gauge coupling unification at the string scale. Rather, the gauge coupling of each gauge group is inversely proportional to the length of the wrapped cycle. The values of the coupling constants can therefore be computed in terms of the compact radii, leading to results which may be made compatible with the experimental values. We also show that a generic difficulty in models with a low string scale, proton stability, is naturally solved in these configurations, where quarks and leptons live on intersecting branes. The reason is that a proton decay process requires an overall interaction with three incoming $SU(3)$ triplets (and no outgoing ones). Such processes require worldsheets with an odd number of quark insertions, which do not exist (to any order in perturbation theory).

The scenario we propose has additional specific features. We show that there exist Kaluza-Klein (KK) and/or winding excitations of the SM gauge bosons, which may have masses well below the string scale. Moreover there is a new class of extra excited modes of fields at intersections (with spin=1/2,0,1). They correspond to excited open strings stretching in the vicinity of the intersections of the branes at angles. Their masses are proportional to the brane angles, hence we refer to them as ‘gonions’. They may have masses just above the weak scale, and thus could provide the first signatures of a low-scale string theory.

To show that the properties advertised above are indeed possible within the context of string theory, we construct a class of specific string compactifications yielding the above general structure. In particular one can easily construct a large set of three-generation models based on D4-branes with one dimension wrapped on circles in $\mathbf{T}^2 \times (\mathbf{T}^4/\mathbf{Z}_3)$ [10]. They are non-supersymmetric, and typically involve extra heavy leptons beyond those in the SM. In these specific examples, in addition to the quarks and leptons, some intersections also contain scalar tachyons. They are a reflection of the absence of supersymmetry in the configuration, and signal an instability against the rearrangement of the D4-branes, which tend to align parallel. Interestingly enough, in some cases these tachyons have the quantum numbers of Higgs fields, and we propose that their presence just signals electroweak symmetry breaking.

²See [11] for early proposal of large volume (heterotic) compactifications, and [12] for an early proposal of a low string scale.

2 Intersecting Standard Model brane-worlds

In order to explore the building of models with quarks and leptons at brane intersections, we are going to consider the simplest case of sets of D4-branes wrapping different circles on a two-torus. More specifically, we consider the compactification of Type IIA string theory on a compact variety of the form $\mathbf{T}^2 \times \mathbf{B}_4$, where \mathbf{B}_4 is a four-dimensional variety whose specific form is not necessary for the moment ³. We will skip the more technical details here and postpone issues like tadpole cancellation and the form of the variety \mathbf{B}_4 to section 7. We do this to simplify the presentation, but also because the main physical issues we are discussing are present in other more complicated string constructions with intersecting branes [9, 10]. Thus, we consider several sets of D4-branes with one world-volume dimension wrapped on different circles within a two-torus. Consider first a square two-torus, obtained by quotienting two-dimensional flat space \mathbf{R}^2 by the lattice of translations generated by the two vectors $e_1 = (1, 0)$, $e_2 = (0, 1)$. Thus one makes the identification $X = X + l2\pi e_1 + p2\pi e_2$, $l, p \in \mathbf{Z}$. The corresponding two circles are taken with arbitrary radii R_1 and R_2 , respectively. We denote by (n, m) a non-trivial cycle winding n times around the cycle defined by e_1 and m times around the cycle defined by e_2 . Different stacks of D4-branes wrap around different (n, m) cycles.

Consider now a stack of N_i overlapping D4-branes with wrapping numbers (n_i, m_i) and a second stack of N_j D4-branes with wrapping numbers (n_j, m_j) . As is well known, each set of branes gives rise to a unitary gauge factor, giving a gauge group $U(N_i) \times U(N_j)$. Notice that these gauge interactions live in Minkowski space plus one extra bulk dimension, which is different for each gauge factor. Matter multiplets arise at the intersections between the two sets of D4-branes. The number of intersections in the two-torus is given by

$$I_{ij} = n_i m_j - n_j m_i \tag{2.1}$$

At those intersections there arise chiral fermions ⁴ which transform in the bi-fundamental representation (N_i, \overline{N}_j) of $U(N_i) \times U(N_j)$. These bi-fundamentals correspond to open strings stretching between both stacks of branes, and hence localized

³More generally, one can consider Type IIA compactified on a six-dimensional variety (e.g. a CY manifold), which is a torus bundle over a base \mathbf{B}_4 . That is, for any small patch U in \mathbf{B}_4 the local geometry factorizes as $\mathbf{T}^2 \times U$, but the global topology is not $\mathbf{T}^2 \times \mathbf{B}_4$.

⁴Actually, in order for the fermions at the intersection to be chiral, the transverse variety \mathbf{B}_4 mentioned above has to fulfill certain conditions, namely it must be singular, as we describe in Section 7. We assume in this section that this is the case.

near the intersections. Thus *chiral fermions are localized in the six compact dimensions*. Due to the multiple number of intersections, we obtain I_{ij} copies of such fermion content ⁵, hence replication of the spectrum is a generic feature in this type of construction. In fact, it is quite easy to obtain configurations with three generations. To see that, let us discuss the following example

Example 1

We choose a configuration of D4-branes at angles leading to a left-right symmetric model. With that purpose, we consider four sets of branes with $N_1 = 3$, $N_2 = 2$, $N_3 = 2$ and $N_4 = 1$, and wrapping numbers

$$N_1 : (1, 0) ; N_2 : (1, 3) ; N_3 : (1, -3) ; N_4 : (-1, 0). \quad (2.2)$$

The resulting gauge group is $U(3) \times U(2)_L \times U(2)_R \times U(1)$. The intersection numbers (2.1) computed using the wrappings (2.2), give rise to a set of chiral fermions transforming under the non-abelian factors as

$$3(3, 2, 1) + 3(\bar{3}, 1, 2) + 3(1, 2, 1) + 3(1, 1, 2) + 6(1, 2, 2) \quad (2.3)$$

Notice that the fermion content is that of three generations of quark and leptons. In addition there are ‘‘Higgsino-like’’ fermions transforming in $(1, 2, 2)$.

The model contains four $U(1)$ gauge symmetries, from the $U(N_i)$ factors in the different sets of branes, with generators Q_i , $i = 1, \dots, 4$. In fact, all fields in the model are neutral under the diagonal combination $Q_{diag} = \sum_i Q_i$, which therefore decouples. Moreover some of the remaining $U(1)$ symmetries are anomalous (with anomaly cancelled by a generalized Green-Schwarz mechanism). Their detailed discussion [10] requires an explicit construction within string theory, to be performed in Section 7. For our purposes here, the main conclusion from the analysis is that the anomalous $U(1)$ ’s gain a mass of the order of string scale, and that one of the surviving anomaly-free linear combinations can be identified with the standard (B-L) symmetry of left-right symmetric models (see section 7).

This D4-brane configuration is depicted in Fig. 2. In that figure opposite sides of the square are identified to recover the topology of a two-torus. Gauge fields are localized along the straight lines within the square, which represent the wrapped D4-branes. For example, the $SU(2)_L$ branes are wrapping three times around e_2 and once

⁵Actually (2.1) gives the intersection number counted with orientation, which agrees with the naive intersection number up to a sign. A negative I_{ij} indicates that the intersections give rise to $-I_{ij}$ fermions of opposite chirality.

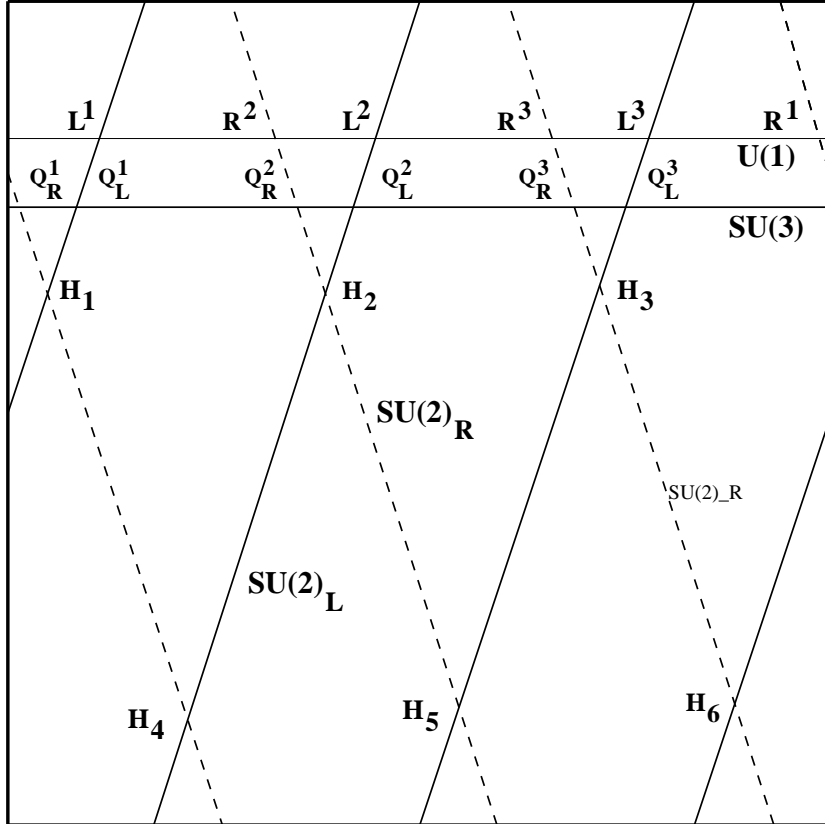


Figure 2: D4-branes wrapping on a two-torus yielding a three-generation $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ model, example 1. Gauge bosons propagate along one world-volume internal dimension, depicted as lines. Quarks and leptons, appearing in three copies, are located at the intersection points of different pairs of branes.

around e_1 . Chiral fermions are localized at the intersection points of the different lines, and transform as bi-fundamental representations under the gauge symmetries on the corresponding branes. Notice the important point that, since intersections take place at different points in the two-torus, the different quarks and leptons sit at distant locations in the extra dimensions. This turns out to be important when studying the structure of Yukawa couplings in this kind of models (see Section 4).

Example 2

There is in fact a wealth of possibilities ⁶. For instance, we can construct a Standard Model configuration, based on four sets of branes with $N_1 = 3$, $N_2 = 2$, $N_3 = N_4 = 1$, and wrapping numbers

$$N_1 : (1, 0) ; N_2 : (1, 3) ; N_3 : (0, -3) ; N_4 : (1, -3) . \quad (2.4)$$

The intersection numbers (2.1) corresponding to these wrapping numbers are ± 3 , or ± 6 . The resulting chiral fermions transform as

$$3(3, 2) + 3(\bar{3}, 1) + 3(\bar{3}, 1) + 3(1, 2) + 3(1, 1) + 6(1, 2) \quad (2.5)$$

under $SU(3) \times SU(2)_L$. This correspond to three quark-lepton generations plus an extra set of three vector-like leptons (“Higgsinos”). This D4-brane configuration is depicted in Fig. 3. Concerning $U(1)$ ’s, again the diagonal combination decouples but there is however an anomaly-free combination, roughly of the form

$$Q_Y = -\frac{1}{3}Q_1 - \frac{1}{2}Q_2 - Q_4 , \quad (2.6)$$

which can be identified with standard hypercharge ⁷. Here Q_i is the $U(1)$ generator of the i^{th} stack of D4-branes.

Up to this point, we have not mentioned whether there are scalar fields at the intersections. In general there are such fields, as we describe in Section 7. Their existence depends on the geometry of the transverse compact space \mathbf{B}_4 . Phenomenological models require the existence of Higgs scalars, which in our models should arise at the intersections of the $SU(2)_L$ branes with some $U(1)$ (or $SU(2)_R$) branes. This is certainly the case in many explicit string theory models, as we discuss in Section 7. Leaving their detailed study for later sections, we proceed, assuming for the time being that the models under study indeed contain appropriate scalars to play the role of standard model Higgs fields.

⁶One can in fact classify different families of models (wrapping numbers) leading to three generations. See section 7.

⁷There are in this particular model additional anomalous and anomaly-free $U(1)$ ’s whose discussion is postponed to section 7.

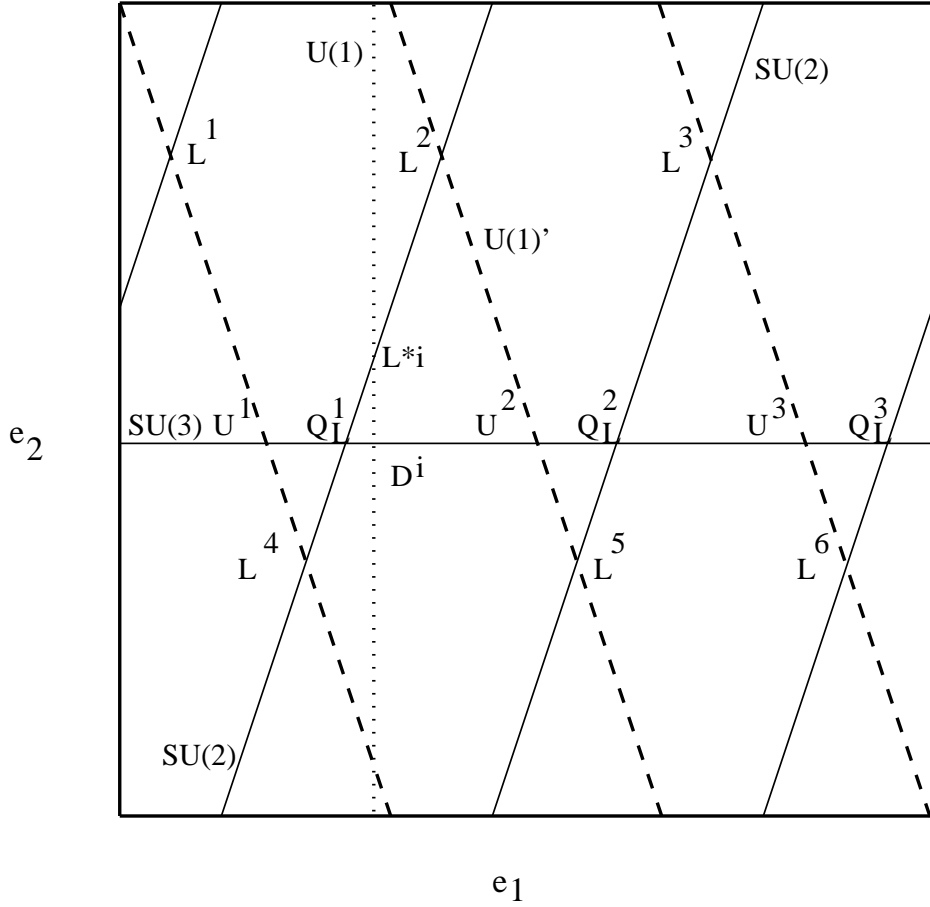


Figure 3: D4-branes wrapping on a torus yielding a three-generation standard model, example 2. Gauge bosons propagate along the lines, which indicate the wrapped D4-brane world-volumes. Quarks and leptons are however localized at the intersection points among the different branes. The vertical $U(1)$ brane is wrapped three times along the depicted cycles, hence leads to three (coincident) intersections with each of the remaining branes.

3 The gauge coupling constants

Unlike what happens in other string scenarios, the couplings for the different gauge factors in the model do not have the same value at the string scale, so there is no unification of gauge couplings⁸. The gauge fields on different sets of wrapping D4-branes have different gauge couplings g_i , with fine structure constant inversely proportional to the length of the wrapped cycle

$$\frac{4\pi^2}{g_i^2} = \frac{M_s}{\lambda_{II}} |(n_i, m_i)| \quad (3.1)$$

where M_s is the string scale, λ_{II} is the Type II string coupling, and $|(n, m)|$ is the length of the cycle (n, m) . Here we will consider the case of a general metric for the torus. This length depends on the compactification radii R_1, R_2 , and the angle θ between the two vectors defining the torus lattice. Distances on a flat torus can be seen as a scalar product of vectors with the metric

$$g = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = (2\pi)^2 \begin{pmatrix} R_1^2 & R_1 R_2 \cos \theta \\ R_1 R_2 \cos \theta & R_2^2 \end{pmatrix} \quad (3.2)$$

The length of a cycle $v = (n, m)$ is

$$|(n, m)| = (g_{ab} v^a v^b)^{1/2} = 2\pi \sqrt{n^2 R_1^2 + m^2 R_2^2 + 2nm R_1 R_2 \cos \theta} \quad (3.3)$$

Thus the relative size of the different coupling constants is governed by the wrapping numbers (n_i, m_i) , the compactification radii R_1, R_2 and $\cos \theta$. In the case of an anomaly free $U(1)$ defined by a linear combination

$$Q = \sum_i c_i Q_i \quad (3.4)$$

the corresponding coupling is given by

$$\frac{1}{g_{U(1)}^2} = \sum_i c_i \frac{1}{g_i^2}. \quad (3.5)$$

In the case of models analogous to that of example 2, one finds

$$\alpha_{QCD}^{-1} = \frac{1}{\pi \lambda_{II}} |(n_1, m_1)| \quad (3.6)$$

$$\alpha_2^{-1} = \frac{1}{\pi \lambda_{II}} |(n_2, m_2)| \quad (3.7)$$

$$\alpha_Y^{-1} = (3\alpha_{QCD})^{-1} + (2\alpha_2)^{-1} + \frac{1}{\pi \lambda_{II}} |(n_4, m_4)| \quad (3.8)$$

⁸The question of gauge couplings in multiple brane scenarios has also been considered in [13, 14, 15, 3].

where lengths are measured in string units. This leads to a weak angle

$$\sin^2 \theta_W = \frac{g_y^2}{g_y^2 + g_2^2} = \frac{6}{(9 + 2\xi_1 + 6\xi_4)} \quad (3.9)$$

where $\xi_1 = g_2^2/g_1^2$ and $\xi_4 = g_2^2/g_4^2$.

These are the values of the couplings at the string scale, which, since the models are non-supersymmetric, should be of the order of 1-10 TeV to avoid a hierarchy problem. In order to compare the values (3.8) with low-energy data, running from the string scale to the weak scale should be taken into account. The details of this running depend on the precise low-energy content of the model⁹. There seems to be enough freedom in this class of models to accommodate the experimental values by appropriately varying the choice of (n_i, m_i) , the radii $R_{1,2}$, and the angle θ . A detailed analysis of possibilities is beyond the scope of this paper. For illustration, an estimation of the coupling constants values is performed in Section 7 for an explicit string SM example.

4 The structure of Yukawa couplings

As we have seen in previous sections, quarks, leptons and Higgs fields live in general at different intersections. Yukawa couplings among the Higgs H^i and two fermion states F_R^j, F_L^k arise from a string worldsheet stretching among the three D4-branes which cross at those intersections. The worldsheet has a triangular shape, with vertices on the relevant intersections, and sides within the D4-brane world-volumes. The area of such world-sheet depends on the relative locations of the relevant fields, and some couplings may even require world-sheets wrapped around some direction in the two-torus.

The size of the Yukawa coupling is, for a square torus, of order¹⁰

$$Y_{ijk} = \exp\left(-\frac{R_1 R_2}{\alpha'} A_{ijk}\right) \quad (4.1)$$

where A_{ijk} is the adimensional area (the torus area has been scaled out) of the worldsheet connecting the three vertices. Since the areas involved are typically order one in string units, corrections due to fluctuations of the worldsheet may be important, but we expect the qualitative behaviour to be controlled by (4.1). This structure makes

⁹As studied in Section 6, there may be KK/winding and other type of excitations in the region between M_Z and M_s . They may lead to important modifications of the coupling running to some extent analogous to those in [16].

¹⁰For a general metric one just has to replace $R_1 R_2 \rightarrow R_1 R_2 |\sin\theta|$.

very natural the appearance of hierarchies in Yukawa couplings of different fermions, with a pattern controlled by the radii and the size of the triangles.

The cycle wrapped by the i^{th} D4-brane around a rectangular torus is given by a straight line equation

$$X_2^i = a_i(2\pi R_2) + \frac{m_i R_2}{n_i R_1} X_1^i, \quad (4.2)$$

and the i^{th} and j^{th} D4-branes intersect at the point:

$$(X_1, X_2)_{ij} = \frac{2\pi}{I_{ij}} (n_i n_j (a_i - a_j) R_1, (a_i n_i m_j - a_j n_j m_i) R_2) \quad (4.3)$$

where I_{ij} is the intersection number for the two D-branes. Hence, the area of each triangle depends not only on the wrapping numbers (n_i, m_i) but also on the a_i 's.

It is clear from the above structure that one can easily generate hierarchies of Yukawa couplings and possibly interesting textures for suitable choices of the free parameters in the models, i.e. the wrapping numbers, the compact radii (and the angle between axes for non-square tori), and the parameters a_i of each stack of branes. A systematic search for phenomenologically interesting textures is beyond the scope of this paper. However, let us illustrate the idea by considering as an example the left-right symmetric model considered in section 2 (example 1).

The configuration is shown for the case of a square lattice in Fig. 4, where in order to get a better visualization, we include several fundamental domains of the torus. The left(right)-handed quarks are denoted by $Q_L^i(Q_R^i)$ and the left(right)-handed leptons by $L^i(R^i)$. Scalars transforming as $(1, 2, 2)$ appear at the intersection of the $SU(2)_L$ and $SU(2)_R$ branes and are denoted by H_i . Let us first consider the structure of quark Yukawa couplings to one of the Higgs fields, say H_3 . For the choice of brane positions shown in Fig. 4, the couplings of H_3 to the three generations of quarks

$$h_3 H_3 Q_L^3 Q_R^3 \quad ; \quad h_2 H_3 Q_L^2 Q_R^2 \quad ; \quad h_1 H_3 Q_L^1 Q_R^1 \quad (4.4)$$

with $h_3 \gg h_2 \gg h_1$ for sufficiently large radii. In particular, notice that the example in the figure would give rise to a hierarchy (before QCD loop corrections) $m_b > m_\tau > m_\mu > m_s > m_d > m_e$ and $m_t > m_c > m_u$, corresponding to the relative sizes of the triangles.

In this example there are additional Yukawa couplings, which are perhaps more evident in the representation in Fig. 2, involving other Higgs fields like H_4 , H_5 and H_6 . In particular one has additional couplings of the form

$$h' H_4 Q_L^1 Q_R^2 \quad + \quad h'' H_4 Q_L^2 Q_R^1 \quad +$$

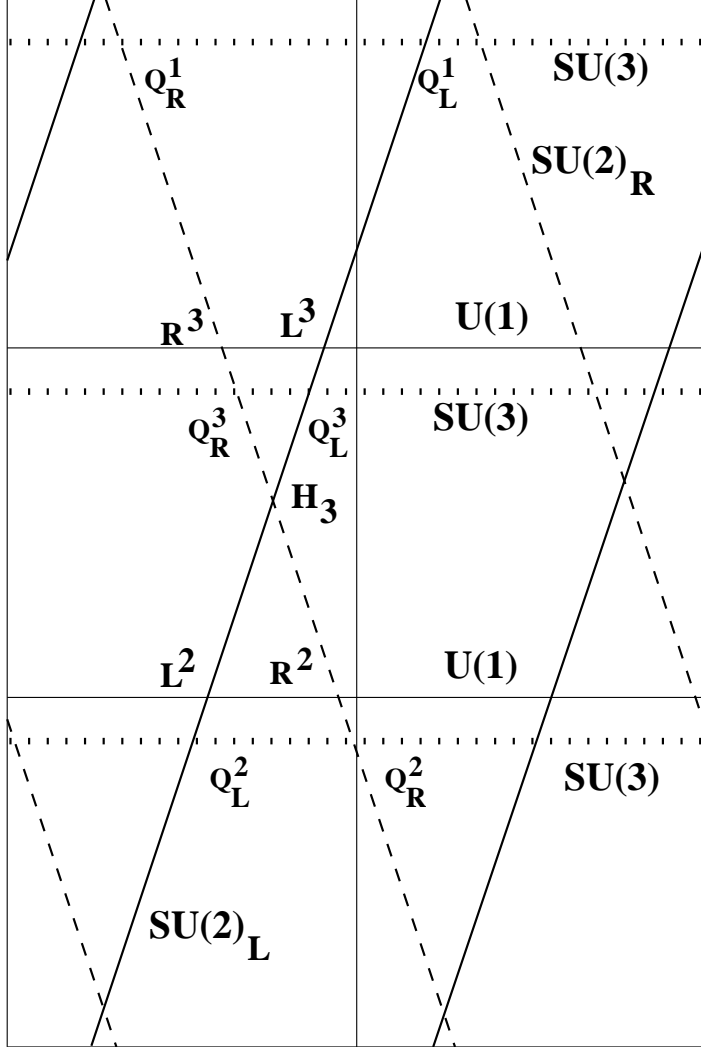


Figure 4: The $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ model of Fig. 2. Several torus fundamental domains are shown to highlight the relative size of the different Yukawa couplings. To avoid clutter, we do not show all the copies of the branes. Also, we only highlight the Yukawa couplings involving the Higgs field H_3 , hence do not show other fields living at the relevant intersections. World-sheets giving rise to quark (lepton) Yukawa couplings correspond to triangles with one vertex (H_3) containing the Higgs and other two vertices Q_L^i, Q_R^i (L^i, R^i) containing the quarks (leptons).

$$\begin{aligned}
& h' H_5 Q_L^2 Q_R^3 + h'' H_5 Q_L^3 Q_R^2 + \\
& h' H_6 Q_L^3 Q_R^1 + h'' H_6 Q_L^1 Q_R^3
\end{aligned} \tag{4.5}$$

Hence, assuming H_3 has the dominant vev as above, vevs for H_4 , H_5 or H_6 would contribute to non-diagonal entries in the quark mass matrix, giving rise to generation mixing. Clearly, a similar pattern holds for leptons.

In fact, the existence of mixing is generic in this class of brane models. This is explicit also in the SM example of Figure 3. If we assume that the Higgs fields which couple to the u-type quarks arise at the intersections labeled L^i , it is clear from the figure that the scalars in the locations L^4, L^5, L^6 couple diagonally to the quarks whereas those in L^1, L^2, L^3 generate off-diagonal couplings.

In the left-right symmetric models the Yukawa couplings of u-type and d-type quarks are equal, although the masses are different if the vevs of the Higgs fields coupling to u- and d-quarks are different. In the case of SM configurations the Yukawa couplings of u- and d-quarks are in general different. For example, one may consider a SM-like configuration obtained from the left-right model depicted in Fig. 4 by replacing the two $SU(2)_R$ D4-branes by two parallel branes next to each other, as shown in Fig. 5. In this case, the areas of the different triangles corresponding to u- and d-quark Yukawa couplings are different, leading to different hierarchical patterns. This example illustrates how the location of the different branes allows for different patterns (textures) for fermion masses. It would be very interesting to study the different general classes of quark and lepton textures which can be accommodated in schemes of this type. Notice that the origin of hierarchies in this class of models is somewhat similar to that suggested for heterotic orbifolds in ref.[17] (see also [18]). For a recent proposal in the context of brane worlds see [19].

5 Mass scales and nucleon stability

The models we are considering are in general non-supersymmetric and hence, we must set the string scale close to the weak scale to avoid a hierarchy problem. The four-dimensional Planck scale M_p is related to the string scale M_s and the compact volumes by (see e.g. [14])

$$M_p = \frac{2\sqrt{V_2 V_4}}{\lambda_{II} \alpha'^2} \tag{5.1}$$

where V_4 is the volume of the compact variety \mathbf{B}_4 transverse to the torus where the D4-branes wrap and $V_2 = R_1 R_2 |\sin \theta|$ is the area of the torus. In order to have not too small

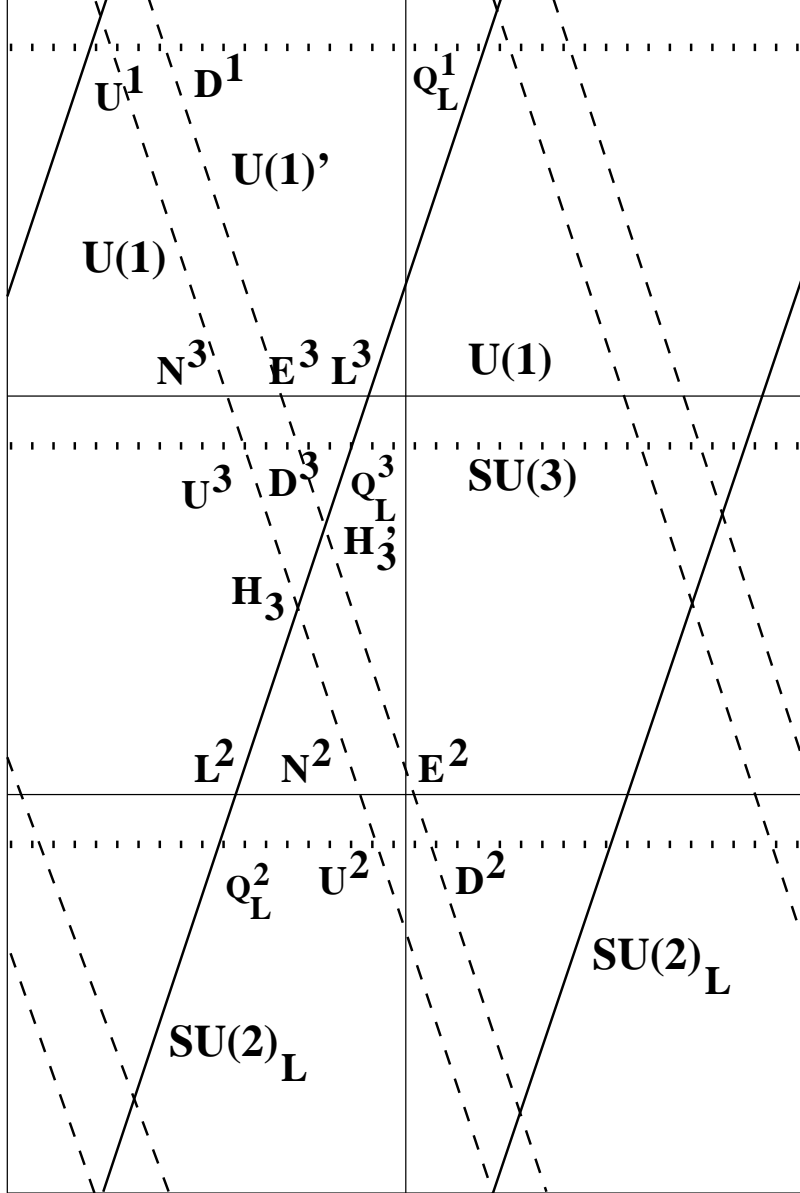


Figure 5: A standard model-like configuration obtained from that in Fig. 4 by splitting the $SU(2)_R$ D4-branes into two parallel $U(1)$ -branes. Now the size of the triangles corresponding to u- and d-quark Yukawa couplings are different.

gauge and Yukawa couplings $\sqrt{V_2}/(\lambda_{II}\alpha')$ cannot be very large. Still, one can obtain the required value for M_p by appropriately choosing a large value for V_4 . In particular, setting the string scale $M_s = 1-10$ TeV, one should choose $V_4 \approx 10^{16}-10^{10}$ (GeV) $^{-4}$. For isotropic compactifications, this requires $M_c \approx 3 \times 10^{-4} - 10^{-2}$ GeV, but this is not the only choice. In fact, two of the dimensions inside \mathbf{B}_4 could be kept of order the string length, while the remaining two are taken in the millimeter range, leading to a phenomenology similar to some brane-world scenarios considered in the recent literature.

One of the main problems for the construction of brane-worlds with a low scale of order 1-10 TeV is proton stability. If the fundamental scale of the theory is that low, one expects (unless some symmetry forbids it) the existence of four-fermion dimension six operators mediating proton decay, which would be suppressed only by powers of $1/M_s^2$. Interestingly enough, nucleon decay is automatically forbidden (to all orders in perturbation theory) in intersecting brane world models. In order for proton decay to proceed, there must be an effective operator involving three incoming quarks and no (net) outgoing ones. In our case, this would require a string amplitude, with e.g. the topology of a disk, with boundary on the intersecting D-branes, and involving just three vertex operator insertions associated to the quarks. These arise at intersections of the $SU(3)$ branes with some other $SU(2)$ or $U(1)$ stack of branes. On the world-sheet boundary, each such insertion changes the worldsheet boundary conditions from those associated to $SU(3)$ branes to those associated to $SU(2)$ or $U(1)$ branes (or viceversa). Hence, any amplitude must involve an even number of such insertions, so there is no disk configuration which can contribute to proton decay. The argument in fact is valid for other string worldsheet topologies, with any number of holes and boundaries, hence the result is exact to all orders in perturbation theory.

In other words, the above argument applied to any stack of branes shows that there is an exact discrete symmetry $(\mathbf{Z}_2)^K$, where K is the number of brane stacks. Under this symmetry, any state arising from an open string stretched between the i^{th} and j^{th} stacks of branes is odd under the i^{th} and j^{th} \mathbf{Z}_2 's, and even under the rest. The \mathbf{Z}_2 associated to the $SU(3)$ stack of branes prevents proton decay. Notice that Higgs scalars are neutral under this \mathbf{Z}_2 , hence their vevs do not break this symmetry. These discrete symmetries are expected to be broken by non-perturbative effects, but their violations are presumably negligible.

Thus the nucleon is stable in this kind of brane intersection models. This is a remarkable fact, which is important for scenarios in which the string scale is close to

the weak scale, say at $M_s \propto 1 - 10$ TeV. Let us also emphasize that this automatic proton stability is not generic in other brane world scenarios, such as D3-branes at singularities [3], but depend on the particular model considered. This feature makes the intersecting brane world scenario a very interesting proposal.

6 Low energy spectrum and signatures at accelerators

The models we are considering have standard quarks and leptons, arising at the intersections, but are non-supersymmetric and in general squarks and sleptons are not present. However, the models typically contain extra particles beyond the content of the minimal SM, which can be rather light. In this Section we review the main type of extra particles present in generic models of this type.

1) Excited KK gauge bosons

The gauge interactions of the standard model are sensitive to the presence of the toroidal extra dimensions around which the D-branes wrap. Hence in these models *there are Kaluza-Klein replicas of gluons and electroweak gauge bosons*. In our models of D4-branes, these Kaluza-Klein gauge-boson excitations have masses (for a general torus metric) given by :

$$M_{KK}^i = \frac{|k|}{\sqrt{n_i^2 R_1^2 + 2n_i m_i \cos \theta R_1 R_2 + m_i^2 R_2^2}} \quad \text{with } k \in \mathbf{Z} \quad (6.1)$$

where i labels the different stacks of branes. This formula is interesting because it can be used to relate the masses of the Kaluza-Klein replicas to the gauge coupling constants in (3.1) *at the string scale*. Indeed, masses of KK states are integer multiples of

$$M_{KK}^i = \frac{2\alpha_i(M_s)}{\lambda_{II}} M_s \quad (6.2)$$

Thus these replicas are expected to be lighter than the string scale for $(\lambda_{II}/2) \geq \alpha_i$. The expression (6.2) also shows that the masses of the KK replicas are on the ratios of the fine structure constants (at the string scale) for the corresponding gauge bosons. Thus the electroweak excited W's, γ and Z's will be in general the lightest KK modes, and could be the first experimental signature of extra dimensions (see e.g. [7]).

Notice that if the excited gauge bosons are relatively light, one has to include their effect in the running of the gauge coupling constants from M_s down to the electroweak scale. The effect of these excited gauge bosons would be to make the $SU(3)$ and $SU(2)$

inverse couplings to decrease faster as we increase the energies. The overall effect of this particular contribution would be analogous to the accelerated running suggested in [16].

2) Excited gauge bosons from windings

Depending on the values of the radii R_1, R_2 and the wrapping numbers (n_i, m_i) , some string winding states may be below the string scale. Indeed, for the case of branes *multiply wrapped* around $R_{1,2}$, there may be open strings stretching between different pieces of the brane in the fundamental region. For example, there exist such states associated to open strings stretched between the $SU(2)_R$ D4-brane lines in Fig. 2, or between the $SU(2)_L$ or $U(1)'$ D4-brane lines in Fig. 3. These states are massive excited gauge bosons in the corresponding brane, with masses proportional to the separation of the different pieces of the D4-brane under consideration. The masses of these winding modes are (for coprime n, m)

$$M_{\text{stretch}}^i = 2\pi p M_s^2 \frac{R_1 R_2 |\sin \theta|}{\sqrt{n_i^2 R_1^2 + 2n_i m_i \cos \theta R_1 R_2 + m_i^2 R_2^2}} \quad (6.3)$$

with p a positive integer. Thus, for large wrapping numbers n_i, m_i or small radii $R_{1,2}$ or $\sin \theta$ some modes may be below the string scale. Notice that, unlike the KK modes, these states are stringy in nature, and hence their mass *depends explicitly on the string scale*. For relatively small radii (and for the case of multi-wrapped D4-branes) these excited gauge bosons may be lighter than the corresponding KK mode (see also [21]), so that either one or the other may be lighter than the string scale. In particular, for the case of a square torus ($R_1 = R_2, \cos \theta = 0$) one can derive the bound for the KK and winding replicas of each gauge boson,

$$M_{\text{stretch}}^i M_{KK}^i = 2\pi(n_i^2 + m_i^2)^{-1} M_s^2 \quad (6.4)$$

so that one or the other could be found at accelerators before reaching the string threshold.

Unlike the gauge sector, quarks and leptons are localized in the six extra dimensions and do not have this type of KK excitations. Consequently, their interactions do not conserve KK quantum numbers, i.e. there exist in principle couplings of the type $q\bar{q} \rightarrow G^*, W^*, B^*$, of quarks to KK excitations of gauge bosons (see [7] and references therein). Thus KK excitations need not be produced in pairs. Similar statements can be made about the winding states.

3) Gonions: Light string excitations of chiral fields

We have described how the groundstates of open strings stretched between intersecting branes give rise to chiral fermions. There are also additional (vector-like) states corresponding to excited open strings (with oscillator excitations) stretched between the intersecting branes [8]. Such modes are also localized at the vicinity of the intersection. They give rise to towers of excited states, with spacing controlled by the intersection angle times the string scale (rather than any compactification scale), and are somewhat new in their behaviour. To distinguish them from the KK and winding excitations of the gauge bosons, we call these fields *gonions*, being associated to branes at angles. There may exist gonions with spin=1/2,0 and 1. At *all* the intersections there are in general fermionic (vector-like) gonions with masses given by

$$m_{ij}^2(\text{fermion}) = q \frac{|\alpha_{ij}|}{\pi} M_s^2 \quad (6.5)$$

where $q > 0$ is an integer and α_{ij} is the angle formed between the corresponding pair of branes. On the other hand at *some* of the intersections (concretely, at those at which Higgs-like fields reside, see sections 7,8) there are in addition scalar and vector gonions with masses

$$m_{ij}^2(\text{scalar}) = (q - 1/2) \frac{|\alpha_{ij}|}{\pi} M_s^2 ; \quad m_{ij}^2(\text{vector}) = (q + 1/2) \frac{|\alpha_{ij}|}{\pi} M_s^2 \quad (6.6)$$

where q is a non-negative integer ¹¹. Thus, the size of these masses depends on the intersection angles. We will argue in section 8 that these angles may be relatively small, in order to suppress the weak scale relative to the string scale. Notice that, the intersection angle α_{ij} depends on the shape of the torus,

$$\begin{aligned} \cos \alpha_{ij} &= \frac{g_{ab} v_i^a v_j^b}{|v_i| |v_j|} = \\ &= \frac{a^2 n_i n_j + a \cos \theta (n_i m_j + n_j m_i) + m_i m_j}{\sqrt{(a n_i)^2 + 2 a n_i m_i \cos \theta + m_i^2} \sqrt{(a n_j)^2 + 2 a n_j m_j \cos \theta + m_j^2}} \end{aligned} \quad (6.7)$$

where $v_i = (n_i, m_i)$ and $a = R_1/R_2$. Thus, e.g. for θ close to π , the angle α_{ij} becomes close to zero. So, if $\alpha_{ij} M_s^2$ is of order the weak scale, one should see the first excited (vector-like) replicas of the observed quarks and leptons not much above the weak scale. These masses will be generation independent, but differ from one type of standard model fermion to the other since their masses are proportional to the corresponding intersection angles.

¹¹For $q = 0$ there are tachyons which will be discussed in Section 8. They are associated to Higgs-like fields.

These gonion excitations of the chiral fields in the intersections are the most likely signature of the present scheme at accelerators. They have the same quantum numbers under the gauge group as the corresponding quark or lepton living at the corresponding intersection. Thus, for example, coloured gonions should be produced by gluon fusion at a hadronic collider, and would look very much like new vector-like quark generations with generation independent masses. In addition all type of gonions have couplings to the ordinary quarks and leptons which will be of order of the usual Yukawa couplings. For example, a scalar or vector gonion in the same intersection as a Higgs field, will have couplings to quarks and leptons proportional to the corresponding Yukawa couplings. This is because the coupling would be proportional to $\exp(-A_{ijk})$, with A_{ijk} the area of the worldsheet stretched among the gonion and the two fermion intersections, very much like in standard Yukawa couplings. Thus, bosonic gonions will typically decay into third generation quarks and leptons. Again, note that if these gonions have masses not much above the weak scale (as suggested in section 8), they will contribute to the running of the gauge couplings in between the weak and the string scales.

4) **Extra massless states in the brane bulk**

The massless sector of each of the D4-branes of course includes the gauge bosons of the corresponding gauge group, but may contain extra particles. In particular, although the complete theory is non-supersymmetric due to the presence of the intersections, the gauge sector living on the bulk of the D4-branes (i.e. within the brane, but away from the intersections) may be supersymmetric, even with $\mathcal{N} = 2$ or $\mathcal{N} = 4$ supersymmetry. In this case, besides the gauge bosons, there exist fermionic and/or bosonic partners transforming in the adjoint of each gauge group. The presence or not of these enhanced SUSY sectors depends on the geometry of the transverse compact variety \mathbf{B}_4 ¹².

The simplest possibility from the phenomenological perspective is having no SUSY in the bulk. Even in this case, there may be additional scalars and vector-like fermions transforming in the adjoint of each gauge group, and massless at tree level. Indeed, the presence of these scalars would signal the possibility of separating the branes within a stack (i.e. like the two $SU(2)_R$ D4-branes of left-right symmetric models) into a set of parallel branes. They would lead to e.g. $SU(3)$ octet scalars and $SU(2)_L$ triplet scalars. Although massless at the tree-level, both scalars and fermions would acquire one-loop masses, see eq. (8.5), of order $\approx \alpha_i M_s$. If present, they could also provide

¹²In Section 7 we construct specific string models in which $B_4 = \mathbf{T}^4/\mathbf{Z}_N$, with an enhanced $\mathcal{N} = 2$ supersymmetry in the bulk of the D4-branes. Analogous models with $\mathcal{N} = 0$ may be obtained by performing a \mathbf{Z}_N twist breaking all SUSY's. See [10] for details.

interesting signatures at colliders.

In addition to the above signatures, one may have the standard signature of extra dimensions of graviton emission to the bulk (corresponding to the large transverse space \mathbf{B}_4), which has been extensively analyzed in the literature [20]. Obviously, if the string scale is reached, explicit string modes would be accessible. However, as pointed out above, in the present scenario the KK/winding excitations of gauge bosons, and gion excitations of chiral fields are expected to be lighter, and much more accessible. A detailed phenomenological analysis of their production at colliders would be interesting.

7 Explicit string models

In this section ¹³ we would like to present specific Type IIA string models, with D4-branes wrapping on a torus, yielding structures very similar to the ones sketched in the previous sections.

The kind of configurations we consider here have been recently studied in [10], to which we refer the reader interested in the more technical details. Here we will merely present several of these string constructions, providing explicit realizations of the scenario discussed in section 2. As explained in [10], D4-branes in flat space lead to non-chiral matter content in their intersection. One is therefore led to consider D4-branes (with one direction wrapped on one-cycles in a two-torus) sitting at singular points in a transverse space, which we take to be $\mathbf{B}_4 = (\mathbf{T}^2)^2/\mathbf{Z}_N$.

For concreteness we center on \mathbf{Z}_3 orbifolds (extension to the general case being straightforward [10]), generated by a geometric action θ with twist vector $v = \frac{1}{3}(1, -1, 0, 0)$. We consider K different stacks of D4-branes, each one containing N_i branes, with wrapping numbers around the 2-torus given by (n_i, m_i) . We set the four transverse coordinates of the D4-branes at the fixed point at the origin in $(\mathbf{T}^2)^2/\mathbf{Z}_3$. The \mathbf{Z}_3 action may be embedded in the $U(N_i)$ gauge degrees of freedom of the i^{th} stack of D4-branes, through a unitary matrix of the form

$$\gamma_{\theta,i} = \text{diag}(\mathbf{1}_{N_i^0}, e^{2\pi i \frac{1}{3}} \mathbf{1}_{N_i^1}, e^{2\pi i \frac{2}{3}} \mathbf{1}_{N_i^2}) \quad (7.1)$$

with $\sum_a N_i^a = N_i$. Due to this twist the initial gauge group $\prod_{i=1}^K U(N_i)$ is broken to $\prod_{i=1}^K \prod_{a=1}^3 U(N_i^a)$.

¹³Readers not familiar with technicalities of string theory may skip to the following section.

Cancellation of twisted tadpoles in the theory imposes the constraints ¹⁴

$$\sum_{i=1}^K n_i \text{Tr} \gamma_{\theta^k, 4_i} = 0 \quad ; \quad \sum_{i=1}^K m_i \text{Tr} \gamma_{\theta^k, 4_i} = 0 \quad (7.2)$$

These conditions guarantee, as usual, the cancellation of gauge anomalies. At the intersections of the different D4-branes, there appear massless fermions transforming under $\prod_{i=1}^K \prod_{a=1}^3 U(N_i^a)$ as [10]

$$\sum_{i < j} \sum_{a=1}^3 I_{ij} \times [(N_i^a, \overline{N}_j^{a+1}) + (N_i^a, \overline{N}_j^{a-1}) - 2(N_i^a, \overline{N}_j^a)] \quad (7.3)$$

with the usual convention for negative multiplicities (see footnote 5). One easily checks that tadpole cancellation conditions indeed imply that this fermion spectrum is free of non-Abelian gauge anomalies. Concerning mixed $U(1)$ anomalies, some of the $U(1)$ gauge symmetries have triangle anomalies, as is often the case in string theory constructions. The theories are nevertheless consistent, due to the cancellation of the anomaly by a generalized Green-Schwarz mechanism, involving twisted closed string states. The corresponding gauge bosons become massive, with mass of the order of the string scale, by combining with certain twisted closed string scalars, whereas the orthogonal linear combinations are anomaly-free and remain massless (see [10] for details). Armed with the above information, we can now construct explicit string compactifications similar to the examples given in section 2.

Before showing specific models, notice that, once the wrapping numbers have been specified, an infinite number of models can be constructed by acting on all the wrapping vectors (n_i, m_i) with (the same) $SL(2, \mathbf{Z})$ transformation ¹⁵. This kind of transformations preserves the intersection numbers between different sets of branes, i.e. the chiral spectrum. Distances are also preserved if the metric transforms accordingly ¹⁶. Two models in the same $SL(2, \mathbf{Z})$ family represent the same physics: the spectrum is related to the intersection matrix and the masses are related to the metric of the torus.

¹⁴We do not impose cancellation of untwisted tadpoles, assuming they are properly cancelled by an additional set of D4-branes away from the origin in $(\mathbf{T}^2)^2/\mathbf{Z}_3$. Such extra branes do not change the field theory spectrum in the sector at the origin, and hence are irrelevant for our discussion.

¹⁵Matrices of the form,

$$C = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad (7.4)$$

where a, b, c, d are integers and $\det(C) = 1$

¹⁶If g_A is metric of the original torus, the transformed metric should be of the form $g_B = (C^{-1})^T g_A C^{-1}$.

It is therefore interesting to classify all non-equivalent models leading, to the same intersection matrix. This number turns out to be just the sum of the divisors of the number of generations, e.g. for three generations there are four non-equivalent families of three generation models $(1 + 3)$. To obtain all non-equivalent families with a given intersection matrix, one would proceed as follows.

- Consider a pair of D-brane stacks, i and j , with intersection $I_{i,j}$, and find all non-equivalent pairs of wrapping numbers with such intersection.
- For each fixed choice of wrapping numbers, the remaining wrapping numbers are determined by imposing the intersection numbers with i and j , which are now linear equations.
- Finally, one should check the intersections among branes different from i and j . Also, solutions with non-integer wrappings should be rejected.

With an intersection number $I_{ij} > 0$, we can use $SL(2, \mathbf{Z})$ to bring the wrappings of the stacks i and j to the form $(n_i, 0)$ and (n_j, m_j) , with $n_i > 0$, $m_j > n_j \geq 0$, and $n_i m_j = I_{ij}$. The number of solutions is just the sum of all the divisors of I_{ij} . Each solution then determines the remaining wrapping numbers in terms of intersection numbers. Leaving a full study of the characteristics of the different families, we turn to studying a couple of examples of three generation models.

Example 1

Consider five sets of D4-branes with multiplicities $N_1 = 3$, $N_2 = 2$, $N_3 = 2$ and $N_4 = N_5 = 1$, and wrapping numbers

$$N_1 : (1, 0) ; N_2 : (1, 3) ; N_3 : (1, -3) ; N_4 : (-1, 0) ; N_5 : (3, 0) . \quad (7.5)$$

Notice that this choice is identical to the one in example 1 of Section 2, except for one additional D4-brane. The latter will be required in the present example in order to cancel the twisted tadpole conditions, and render the string configuration consistent. The twists acting on CP indices are taken to be

$$\begin{aligned} \gamma_{\theta,1} &= \mathbf{1}_3 \\ \gamma_{\theta,2} = \gamma_{\theta,3} &= \alpha \mathbf{1}_2 \\ \gamma_{\theta,4} &= \alpha \\ \gamma_{\theta,5} &= \alpha^2 \end{aligned} \quad (7.6)$$

where $\alpha = \exp(2\pi i/3)$. One can easily check that these choices of wrapping numbers and CP twist matrices verify the tadpole cancellation conditions (7.2). The gauge group is $U(3) \times U(2)_L \times U(2)_R \times U(1)_4 \times U(1)_5$. Using (2.1) and (7.3), one easily obtains the massless chiral fermion spectrum displayed in Table 1.

Intersection	Matter fields	Q_1	Q_2	Q_3	Q_4	Q_5	$B-L$	X
(12)	$3(3, 2, 1)$	1	-1	0	0	0	1/3	0
(13)	$3(\bar{3}, 1, 2)$	-1	0	1	0	0	-1/3	0
(23)	$12(1, 2, 2)$	0	1	-1	0	0	0	0
(24)	$6(1, 2, 1)$	0	-1	0	1	0	1	1
(34)	$6(1, 1, 2)$	0	0	1	-1	0	-1	-1
(25)	$9(1, 2, 1)$	0	-1	0	0	1	-1	-2/3
(35)	$9(1, 1, 2)$	0	0	1	0	-1	1	2/3

Table 1: Spectrum of the $SU(3) \times SU(2)_L \times SU(2)_R$ model. We present the quantum numbers of the chiral fermions under the $U(1)^5$ group, as well as the charge under the $B-L$ linear combination and the anomaly-free generator Q_X .

Non-abelian cubic anomalies automatically cancel, while there are two anomalous $U(1)$'s which become massive. The diagonal sum of the five $U(1)$ generators is anomaly-free, but actually it decouples since all particles have zero charge under it. In addition there are two anomaly-free linear combinations

$$\begin{aligned}
Q_{B-L} &= -\frac{2}{3}Q_1 - Q_2 - Q_3 - 2Q_5^D \\
Q_X &= Q_4 - \frac{2}{3}Q_5^D
\end{aligned}
\tag{7.7}$$

We have displayed the charge under these two generators in Table 1. The first linear combination plays the role of $B-L$ symmetry. The model contains three quark-lepton chiral generations, plus some additional vector-like leptons¹⁷. There is an additional subtlety here concerning the fifth brane with wrapping $(3, 0)$. It turns out that whenever n and m are not coprime as in this case, a brane gives rise not to a single $U(1)$ field but to several copies. In the present case a brane wrapping with $(n, m) = (3, 0)$ gives rise to three $U(1)$ fields with generators Q_5^a , $a = 1, 2, 3$ corresponding to open

¹⁷Notice that the number of generations arises from the intersection number between the cycles (7.5), and is completely unrelated to the order of the orbifold group \mathbf{Z}_3 .

strings stretching between the first wrapping of the brane and the first, second and third wrappings. Thus in addition to the above two anomaly-free $U(1)$'s (with $Q_5^D = \sum_a Q_5^a$) there are other two ($Q_5^1 - Q_5^2$ and $Q_5^2 - Q_5^3$) which only couple to the fields in the intersections (25) and (35) which we have not displayed in the table.

Comparing with example 1 in Section 2, besides these extra leptons there are the additional anomaly-free $U(1)$'s mentioned above. They arise from the additional D4-brane we have introduced for technical reasons, namely in order to achieve cancellation of twisted tadpoles. Their presence should not be considered as a necessary consequence of the present scenario but rather from its particular realization.

Example 2

Consider five different stacks of D4_{*i*}-branes, with multiplicities $N_1=3$, $N_2 = 2$ and $N_3 = N_4 = N_5 = 1$, and wrapping numbers ¹⁸

$$N_1 : (1, 0) ; N_2 : (1, 3) ; N_3 : (0, -3) ; N_4 : (1, -3) ; N_5 = (3, 0) . \quad (7.9)$$

This choice is similar to example 2 in Section 2, differing only in the introduction of one additional D4-brane, required to achieve cancellation of twisted tadpoles in the model. The twists acting on CP indices are taken to be

$$\begin{aligned} \gamma_{\theta,1} &= \mathbf{1}_3 \\ \gamma_{\theta,2} &= \alpha \mathbf{1}_2 \\ \gamma_{\theta,3} = \gamma_{\theta,4} &= \alpha \\ \gamma_{\theta,5} &= \alpha^2 \end{aligned} \quad (7.10)$$

Again, one can easily check that these choices of wrapping numbers and CP twist matrices verify the tadpole cancellation conditions (7.2). The gauge group is $U(3) \times U(2)_L \times U(1)_3 \times U(1)_4 \times U(1)_5$. From (2.1) and (7.3), the spectrum of chiral fermions is easily computed, and the result is shown in Table 2.

¹⁸Indeed, there are other three $SL(2, \mathbf{Z})$ families with the same intersection numbers,

$$\begin{aligned} N_1 : (1, 0) ; N_2 : (0, 3) ; N_3 : (1, -3) ; N_4 : (2, -3) ; N_5 = (3, 0) \\ N_1 : (1, 0) ; N_2 : (2, 3) ; N_3 : (-1, -3) ; N_4 : (0, -3) ; N_5 = (3, 0) \\ N_1 : (3, 0) ; N_2 : (0, 1) ; N_3 : (3, -1) ; N_4 : (6, -1) ; N_5 = (9, 0) . \end{aligned} \quad (7.8)$$

Notice however that only the second family leads to a gauge group $SU(3) \times SU(2) \times U(1)^n$. In the remaining families, color or weak branes are multiply wrapped and lead to a replication of the corresponding gauge factor.

There are two anomaly-free $U(1)$ linear combination (apart from the diagonal one, which decouples) given by

$$\begin{aligned} Q_Y &= -\frac{1}{3}Q_1 - \frac{1}{2}Q_2 - Q_4 - Q_5 \\ Q_X &= Q_3 - Q_4 - \frac{2}{3}Q_5 \end{aligned} \tag{7.11}$$

Table 2 also provides the charges under these linear combinations. Interestingly, we see that the first of these generators can be identified with standard weak hypercharge. Again, the model contains three quark-lepton generations plus some vector-like leptons and additional $U(1)$ gauge factors. As happened in the left-right symmetric model, in the present case the third brane with wrapping $(0, -3)$ and the fifth with $(3, 0)$ give rise to 2+2 additional anomaly-free $U(1)$'s whose charge we have not displayed in the table. Comparing with example 2 of Section 2, we find the model is very similar, the differences being due to the presence of an additional D4-brane, which we have been forced to introduce in order to satisfy twisted tadpole cancellation conditions.

Intersection	Matter fields	Q_1	Q_2	Q_3	Q_4	Q_5	Y	X
(12)	$3(3, 2)$	1	-1	0	0	0	1/6	0
(13)	$3(\bar{3}, 1)$	-1	0	1	0	0	1/3	1
(14)	$3(\bar{3}, 1)$	-1	0	0	1	0	-2/3	-1
(23)	$6(1, 2)$	0	1	-1	0	0	-1/2	-1
(24)	$12(1, 2)$	0	1	0	-1	0	1/2	1
(25)	$9(1, 2)$	0	-1	0	0	1	-1/2	-2/3
(34)	$6(1, 1)$	0	0	-1	1	0	-1	-2
(35)	$9(1, 1)$	0	0	1	0	-1	1	5/3
(45)	$9(1, 1)$	0	0	0	1	-1	0	-1/3

Table 2: Spectrum of a standard model. We present the quantum numbers of the chiral fermions under the $U(1)^5$ group, as well as the hypercharge linear combination and the additional Q_X generator.

The above two examples illustrate how the general properties described in the previous sections may in fact be obtained in the context of string theory. Although in these particular examples, due to technical reasons, we were forced to add an extra brane, which led to extra $U(1)$'s and additional leptons, our discussion of gauge and Yukawa couplings, structure of mass scales, proton stability, and the possible presence of light

KK/winding gauge boson excitations and gonions remains valid for these explicit string examples.

As an illustration we can estimate the possible values of coupling constants as discussed in section 3. Recall that, since the hypercharge generator (7.11) involves the additional D4-brane, not present in (2.6) in the toy model in Section 2, we must replace $|(n_4, m_4)| \rightarrow |(n_4, m_4)| + |(n_5, m_5)|$ in (3.8). For instance, by choosing $\cos \theta \simeq -1$ and $R_2/R_1 = 1.57$, we obtain the α_i 's are in the ratios $1 : 0.27 : 0.09$ which coincide, within less than 6 % with experimental ratios $1 : 0.268 : 0.0861$. A more precise determination of low-energy would require taking into account the effect of different thresholds as discussed above. In any event, as claimed in Section 3, there seems to be enough freedom to reproduce experimental values of coupling constants in the present setup.

The specific examples discussed in this section have however a potential problem, regarding the scalar sector, as pointed out in [10]. In the class of models with D4-branes wrapping on $\mathbf{T}^2 \times (\mathbf{T}^2)^2/\mathbf{Z}_N$ that we are discussing, there are tachyonic scalars appearing at *some* of the D4-brane intersections. In particular, for a general set of D4-branes at a \mathbf{Z}_3 orbifold, there appear complex scalars at intersections involving D4-branes with the same eigenvalue in the CP twist matrix $\gamma_{\theta^k, 4_i}$. They transform under $\prod_{i=1}^K \prod_{a=1}^3 U(N_i^a)$ as

$$\sum_{i < j} \sum_{a=1}^3 I_{ij} \times (N_i^a, \bar{N}_j^a) \quad (7.12)$$

Their masses are given by

$$M_{ij}^2 = -\frac{M_s^2}{2} \left| \frac{\alpha_{ij}}{\pi} \right| \quad (7.13)$$

where $|\alpha_{ij}|$ is the angle at which the corresponding pair of D4-branes intersect on the torus. Thus the model contains tachyons at those intersections. Their properties are discussed in more detail in next Section.

8 Tachyons and electroweak symmetry breaking

In the specific string compactifications described in previous section, besides the chiral fermions present at every intersection, there exist complex scalars at *some* of them. For example, as one can read from (7.12), in the standard model example 2 of previous Section there are complex scalars in the intersections (23), (24) and (34), transforming as $(1, 2)_{-1/2}$, $(1, 2)_{1/2}$ and $(1, 1)_{-1}$ under $SU(3) \times SU(2) \times U(1)_Y$ respectively. In

the case of the left-right model, example 1, there are complex scalars at the same intersections, transforming as $(1, 2, 2)$, $(1, 2, 1)$ and $(1, 1, 2)$ under $SU(3)_c \times SU(2)_L \times SU(2)_R$.

As we mentioned in the previous chapter their masses are given by (7.13), and hence they are tachyonic. This signals an instability of the brane configuration which tends to favour the alignment of the D4-branes along parallel directions. On the other hand, the fact that in these examples some of the *tachyons have precisely the quantum numbers of Higgs fields* suggests that perhaps what these tachyons indicate is some stringy version of a Higgs mechanism [10] (see also [22] for an early proposal of the SM Higgs as tachyon, in a different (but related) context). Since many of the theoretical aspects of the tachyon potential and dynamics are still under study (see [23, 24, 25] for some recent references on tachyon condensation in brane-antibrane systems), our discussion in this Section is tantalizing, but to some extent qualitative.

A possible obstacle for this interpretation is that naively tachyonic masses are of the order of the string scale. In the case of the Standard Model, that would require a string scale of the order of the weak scale, a possibility not consistent with experimental observations. The situation would be better for the case of tachyonic $SU(2)_R$ doublets in left-right symmetric models, since $SU(2)_R$ breaking at the TeV scale would require a string scale in the region 1-10 TeV, which can be achieved without contradiction with experiment.

However, the situation is better, even for SM configurations. In fact, as follows from (7.13), the mass of tachyons may be substantially smaller than the string scale if the intersection angles α_{ij} are sufficiently small (but non-vanishing, so that the branes intersect to yield a chiral model). In particular, by varying the shape (complex structure) of the torus one can make all these angles arbitrarily small.

In particular consider the case of a squashed torus with θ close to π , so that $\cos \theta = -1 + \epsilon^2/2$. In that case one can check using (6.7) that the angles between the different D4-branes are proportional to ϵ , and hence be made arbitrarily small. In particular it is easy to find in that limit:

$$M_{ij}^2 = -\frac{M_s^2}{2} \left| \frac{\alpha_{ij}}{\pi} \right| = -\frac{M_s^2}{2} \frac{a\epsilon |I_{ij}|}{|an_i - m_i||an_j - m_j|} \quad (8.1)$$

where, if $m_i \neq 0$, $a = R_1/R_2 > m_i/n_i, m_j/n_j$. Here I_{ij} is the intersection matrix described in chapter 2. Thus we see that the size of the negative tachyonic mass may be made arbitrarily low by fixing ϵ (or in some cases a) to a sufficiently small value.

In terms of the effective field theory, this negative mass square signals the breaking of the gauge symmetry. Consider first the SM example 2. The $SU(2)_L$ doublets at the

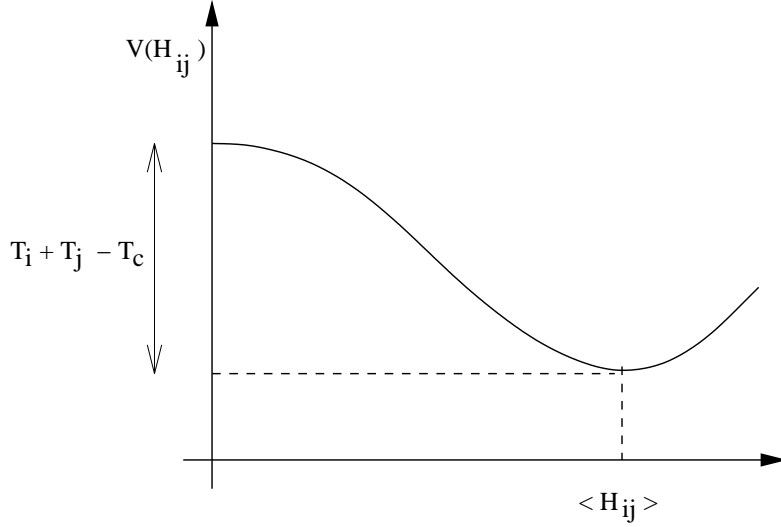


Figure 6: Qualitative form of the tachyon (Higgs) potential originated by intersecting brane instability.

intersections (24) and (23) and the $SU(2)_L$ singlet at the intersection (34) have masses

$$M_{24}^2 = -M_s^2 \frac{3a\epsilon}{(a^2 - 9)} ; M_{23}^2 = -M_s^2 \frac{a\epsilon}{2(a - 3)} ; M_{34}^2 = -M_s^2 \frac{a\epsilon}{2(a + 3)} \quad (8.2)$$

with $a > 3$. Consider for example a value $a = R_1/R_2 = 10/3$. Then these negative masses would be in the ratios $54/19 : 3 : 3/19$ respectively. Thus the negative mass square of the Higgs doublets in the intersections are much larger than that of the charged singlet and hence standard electroweak breaking would be preferred ¹⁹.

This would certainly be an intriguing origin for electroweak symmetry breaking. Whereas in the standard model a negative $(mass)^2$ is put by hand for the Higgs doublet, in the present scheme it appears naturally due to the presence of tachyons at brane intersections. Hence chirality and gauge symmetry breaking are linked in these models: chirality requires intersecting branes, which yield tachyonic modes which in turn trigger electroweak symmetry breaking.

From the point of view of string theory the interpretation goes as follows. The presence of tachyons in two intersecting D4-branes signal an instability of the system under recombination of both into a single D4-brane. For example, consider again the SM construction, example 2 above. There are two parallel D4-branes with wrapping numbers $(n, m) = (1, 3)$ which give rise to $SU(2)_L$ gauge interactions. They intersect

¹⁹As discussed below, loop effects tend to give positive contributions to the scalar masses, which can easily overcome the tiny tachyonic mass of the singlet scalar.

with another brane with wrapping number $(0, -3)$, and at the intersections we get tachyonic scalars with masses as in (8.2). Their presence indicates an instability of the system against the recombination of e.g. one of the $(1, 3)$ branes with the $(0, -3)$ brane, giving rise to a single D4-brane with wrapping numbers $(1, 3) + (0, -3) = (1, 0)$. The string theory construction shows that the recombination process corresponds to the tachyon field rolling to a minimum, which is reached in the final configuration. In the process, the tachyon condensate breaks the gauge symmetry. Namely, the non-Abelian $SU(2)_L$ generators disappear from the massless spectrum since there only remains one $(1, 3)$ brane instead of two. Thus, with the *tachyon at the minimum of its potential* two intersecting D4-branes have merged into a single one.

The detailed form of this scalar potential is not known, although the properties of similar tachyons in brane-antibrane configurations have been studied e.g. in [23, 24, 25]. For instance, adapting the results in [23], one concludes that, if a D4-brane i combines with a D4-brane j to form a combined D4-brane c , the depth of the potential is given by the difference of the D-brane tensions (after compactification on their corresponding cycles)²⁰. That is, $\Delta V = T_c - (T_i + T_j)$, where [27]

$$T_i = \frac{M_S^4}{(2\pi)^4 \lambda_{II}} |(n_i, m_i)| = M_s^4 / (16\pi^3 \alpha_i(M_s)) \quad (8.3)$$

and analogously for the branes j and c . Here $|(n, m)|$ is the length (3.3), and α_i the fine structure constant for the corresponding group. This is schematically shown in Fig. 6.

In the regime of small interbrane angles discussed above, the potential depth is small. Specifically, for the recombination discussed above $(1, 3) + (0, -3) \rightarrow (1, 0)$, one obtains

$$\Delta V = \frac{M_S^4}{(2\pi)^4 \lambda_{II}} \frac{3(R_1 M_S)}{2(a-3)} \epsilon^2 \quad (8.4)$$

so for R_1 of order one in string units, ΔV is of the order of $\epsilon^2 M_S^4$. Even though the detailed form of the potential is not known, one can make a rough estimate of the tachyon vev at its minimum (by computing at which vev the mass term cancels the tension difference) to be of order $\sqrt{\epsilon} M_s$.

If we communicate an amount of energy larger than $M_s \sqrt{\epsilon}$ to the system, the vev of the tachyon becomes irrelevant. This means that we are able to resolve the combined brane into the original pair of branes, and produce W -bosons. This is certainly a

²⁰See [26] for a similar statement in a different (but related) context.

quite intriguing interpretation of the process of electroweak symmetry breaking in the standard model ²¹.

The tachyonic scalar masses given in (8.2) are tree-level results. In addition all scalars receive corrections to their $(mass)^2$ from loop effects. One can estimate those corrections from the effective field theory. In particular, one gauge boson exchange gives corrections of order

$$\Delta M^2(\mu) = \sum_a \frac{4C_F^a \alpha_a(M_s)}{4\pi} M_s^2 f_a \log(M_s/\mu) + \Delta M_{KK/W}^2 \quad (8.5)$$

where the sum on a runs over the different gauge interactions and C_F^a is the eigenvalue of the quadratic Casimir in the fundamental representation. Here $\Delta M_{KK/W}^2$ denotes further contributions which may appear from the KK/W and gonion excitations if they are substantially lighter than the string scale M_s . The function f_a is given by

$$f_a = \frac{2 + b_a \frac{\alpha_a(M_s)}{4\pi} t}{1 + b_a \frac{\alpha_a(M_s)}{4\pi} t} \quad (8.6)$$

where $t = 2 \log(M_s/\mu)$ and b_a are the coefficients of the one-loop β -functions. These corrections are positive and may overcome in some cases the tachyonic masses if the latter are small. Extra KK/winding excitations may contribute to this effect if they lie between the weak and the string scales. In particular, notice that since the intersection angles are small, as suggested above, there will be relatively light *gonion* excited fields, of the type discussed in Section 6, just above the weak scale, and contributing to one-loop corrections.

In addition, a doublet scalar should have a large Yukawa coupling to the top quark, giving rise to a negative one-loop contribution to the $(mass)^2$ of the doublet. This would contribute further to inducing electroweak symmetry breaking, very much as in the radiative symmetry breaking mechanism [28]. A full description of electroweak symmetry breaking in this class of models would thus require an understanding of these loop corrections which may compete with the tree-level ones.

9 Final comments and outlook

In this paper we have presented a string scenario in which there is one brane-world per SM gauge interaction. At the intersections of the branes live the quarks and leptons,

²¹As pointed out in [10] the tachyon condensation process is analogous to a standard Higgs mechanism as long as no other gauge symmetry enhancements are available in the probed energy range. In our case, this would require that other sets of branes with total wrapping $(1, 0)$ are heavier than the considered pair $(1, 3) + (0, -3)$. Suitable choices of geometric moduli lead to this behaviour.

which are the zero modes of open strings close to each intersection. Our original motivation for this proposal was the fact that brane intersections is one of the few known ways to obtain chirality in the brane world context in string theory. In addition it offers an explanation for quark-lepton family replication, since generically branes can intersect at multiple points.

While studying the proposal we have found a number of interesting aspects of this scheme. For instance, hierarchical Yukawa couplings naturally appear due to the fact that the quarks, leptons and Higgs fields are located at different points in the compact dimensions. The Yukawas are proportional to $e^{-A_{ijk}}$, where A_{ijk} is the area of the worldsheet extending among the intersections where the fermions and the Higgs live. Due to this fact, it is easy to obtain hierarchical results for the different Yukawa couplings. Next, the models are non-supersymmetric, but the hierarchy problem may be solved by lowering the string scale down to $1 - 10$ TeV, and taking the dimensions transverse to the branes large enough. Interestingly enough, even though the string scale is so low, the proton is naturally stable to all orders in perturbation theory, due to discrete symmetries following from worldsheet selection rules. The proton is stable because its decay would require an overall interaction with three incoming quarks and no outgoing ones. Such process would require worldsheets with an odd number of quark insertions, which do not exist. Finally, concerning gauge coupling constants, we have found that they do not unify in this setup, since each brane comes along with its own coupling constant. However, they may be computed in terms of the compactification radii, and may be made compatible with the observed values.

One of the interesting aspects of the intersecting brane-worlds scenario is that it predicts the existence of certain particle excitations in the energy region between the weak and the string scales. There are KK (and/or winding) replications of the gauge bosons, which could be directly produced at colliders by quark-antiquark annihilation. In addition there is a new class of states, which we have baptized as *gonions*, which have masses proportional to the string scale times the intersection angles, (hence the name gonions). They correspond to excited strings stretched close to the intersection of two branes. They include massive vector-like copies of quarks and leptons. In addition there are bosons with spin=0,1 close to some of the intersections. All of them come in towers starting about the weak scale. It should be interesting to study in more detail the experimental signatures of these new fields as well as setting limits on their masses from present data.

Like in many non-supersymmetric models, the spectrum contains scalar tachyons.

Interestingly enough, in the specific string models that we construct, those tachyons have precisely the quantum numbers of Higgs fields. Thus it is tempting to propose that these tachyonic states are just signaling the presence of spontaneous gauge symmetry breaking. It should be interesting to explore in more detail the theoretical viability of this exciting possibility.

In this article we have concentrated on the simplest possibility of D4-branes wrapping at angles on a torus. We would like to emphasize, however, that most of the general structures we find apply more generally, to any configuration involving collections of branes intersecting at angles in more general varieties²². Another point worth mentioning is that the case of D4-branes admits an interesting M-theory lift. Indeed, D4-branes correspond to M-theory 5-branes wrapping on the eleventh dimension, compactified on a circle \mathbf{S}^1 . Thus the models discussed in the paper may be regarded as M-theory compactifications on $\mathbf{S}^1 \times \mathbf{T}^2 \times \mathbf{B}_4$ with M5-branes wrapping on $\mathbf{S}^1 \times \mathbf{T}^2$.

There are a number of issues to be further studied. On the theoretical side, the brane configurations we have considered are non-supersymmetric, and hence the question of their stability deserves further study. In this regard, it is worth mentioning that (meta)stable configurations on analogous models using wrapping D6-branes have been recently discussed in [10]. Also, consideration of more general string configurations with branes at angles could lead to improvements in model building in this setup. On the more phenomenological side, it should be interesting to carry out a general study of possible three-generation models leading to interesting gauge coupling predictions, and fermion mass textures, using the built-in mechanism for the generation of hierarchies in this class of models. There are other aspects that we have not discussed, such as the question of neutrino masses, or the strong CP problem. It should be interesting to examine whether this scenario provides some new understanding for these questions. Finally, the study of signatures of the different KK, windings and gion particles at accelerators should also be interesting. Unlike other string scenarios, this seems to be amenable to direct experimental test.

In summary, we believe that the intersecting brane worlds setup provides new ways to look at the specific physics of brane world scenarios with a low string scale. It also suggests natural solutions to some of its potential problems, like proton stability and predicts the presence of new KK/winding and gion particles in between the weak and the string scales which should be accessible to future colliders. It would be interesting

²²For more general possibilities involving higher dimensional branes see [9], and the more extensive analysis in [10]. See also [29] for systems of D6-branes on 3-cycles in general Calabi-Yau spaces.

to work out in more detail the predictions of this scenario which could perhaps provide an exciting alternative to the much more studied case of low-energy supersymmetry.

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