Pollution Tax, Partial Privatization and Environment

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Abstract

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JEL Classifications: H23, Q50, Q58, L13, L33

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Abstract

Considering a differentiated mixed duopoly we show that when privatization and pollution tax are used together environmental damage will be non-monotone in the level of privatization, and optimal privatization is always partial privatization. Whether privatization will improve the environment or not depends on the public firm's concern for environment. If the public firm is unconcerned about environment, the socially optimal privatization will also damage the environment most. But when the public firm is concerned about environment, privatization will improve the environment. Generally, the relationship between optimal privatization and product substitutability is also non-monotone and inverted U-shaped.

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1 Introduction

We consider a mixed duopoly with differentiated products to study the environmental impact of optimal privatization and pollution tax. Existing studies have considered product differentiation and optimal privatization only in isolation, but not together. It is undeniable that both product variety and privatization are important features of modern economy. Partial or complete privatization of state-owned enterprises has been a feature of government policy in many developing as well as developed countries since 1980's (Megginson and Netter, 2001; Maw, 2002). For example, Boubakri et al. (2008) document that partial privatization of state-owned enterprises has been the most prevalent phenomenon in a sample of 120 developing countries during the period from 1988 to 2005. Evidence of partial privatization is also found by Gupta (2005) and Fana et al. (2007) in the case of India and China, respectively. Bortolotti and Faccio (2009) document that at the end of 2000 governments retained control of 88 out of 141 privatized firms in 'Organization for Economic Co-operation and Development' countries. Recent examples of partial privatization include UK's 'Bio Products Laboratory' (80 percent) in 2013, New Zealand's 'Mighty River Power Limited' (48.22 percent) and 'Meridian Energy Limited' (49 percent) in 2013 and New Zealand's 'Genesis Energy Limited' (49 percent) in 2014 (Wikipedia, 2014). It has also been noted that in transition economies many state-owned industries were reliant on highly polluting technologies. Hence, omitting these considerations from formal models invariably leaves a gap in the literature.

There is a growing literature on environmental policy in strategic settings involving privately owned firms. A wide range of issues has been covered in this literature, from product differentiation (Canton et al., 2008; Fujiwara, 2009) and strategic delegation (Barcena-Ruiz and Garzon, 2002; Pal, 2012) to foreign trade (Barrett, 1994; Bhattacharya and Pal, 2010). Alongside there is a separate literature on mixed oligopoly devoted to studying the strategic impact of full or partial public ownership in one of the competing firms, generally without environmental implications (deFraja and Delbono, 1989; Matsumura, 1998). This literature shows that optimal privatization can be partial or zero depending on many considerations such as production technology, firm entry, foreign trade and product variety.

Recently an overlap of the above-mentioned two literatures has emerged where the impact of privatization on environment is sought to be analyzed.[1] Barcena-Ruiz and Garzon (2006) and Wang and Wang (2009) have examined the effects of privatization on environmental outcomes, by comparing equilibrium outcomes under full privatization with that under full nationalization. While Barcena-Ruiz and Garzon (2006) considered homogeneous goods, Wang and Wang (2009) allowed product differentiation. But, none of them allowed the possibility of partial privatization and, thus, failed to analyze how the *optimal* privatization affects the environment. Saha (2009) studied social optimality of

[1] In a monopoly set-up Beladi and Chao (2006) and Saha (2013) have examined the effects of privatization on pollution, and argued that privatization of a public firm may increase environmental damage in some cases. However, these models ignored abatement measures, and of course output competition. Ohori (2012) extends this framework to vertical relationship. Cato (2008) have demonstrated that desirability of mixed oligopoly over private oligopoly depends on the extent of negative externalities generated through production.

partial privatization in one or both firms in a setting of differentiated mixed duopoly with external cost (such as pollution); but his model did not allow for pollution tax or abatement measures. Using a setting of international duopoly with homogeneous products, Ohori (2006) has shown that partial privatization will be socially optimal, but the environmental damage will also be higher. In contrast, Naito and Ogawa (2009) and Wang et al. (2009) argued that partial privatization will improve the environment if all firms were domestically owned (assuming homogeneous products).[2] On the other hand, considering tax-subsidy scheme that allows for the possibility of the tax on the output and the subsidy on the abatement to be different, in a homogeneous products mixed duopoly, Pal and Saha (2014) have shown that the government can implement the socially optimal output and abatement by keeping the public firm fully public. However, it is optimal for the government to partially privatize the public firm, unless the private firm is fully owned by a domestic party.

From the above literature it appears that the relationship between privatization and environmental damage is complex. Ordinarily, in the presence of environmental concerns optimal privatization will be greater, because privatization tends to reduce the industry output and the associated pollution. But a pollution tax also does the same by encouraging abatement (and possibly in addition restricting output). Yet, such a simple (negative and

[2] Kato (2013) analyzed implications of pollution by firms on socially optimal level of privatization, without allowing for any environmental policy instrument and ignoring the possibility of abatement by firms. Whereas, Kato (2006) examined effects of emission permits, tradable vis-a-vis non-tradable, on social welfare in the case of a mixed oligopoly.

monotonic) relationship holds when these two policies (privatization or tax) are used in isolation. When they are used together, greater privatization will call for a lower tax, and their combined impact on the environmental damage is far from clear. Additional complications arise if other considerations, such as foreign ownership or product variety, are taken into account. For this reason perhaps to the best of our knowledge no paper has studied optimal privatization in a differentiated oligopoly allowing for pollution tax and abatement.

We try to disentangle the privatization-environment relationship in a two-firm-two-product setup; both firms are domestically owned but one is public. A general treatment of this problem proves difficult, and the main stumbling block appears to be the objective function of the public firm, in particular whether it shares the government's concern for environment or not. Empirical evidence suggests that public firms' concern for the environment differs widely across countries. For example, Chang et al. (2013) demonstrate that state-owned firms tend to invest more in pollution reducing activities than privately owned firms in China.[3] A survey of managers' attitudes also reveals that Chinese state-owned firms' managers are more concerned about the environment than their counterparts in privately owned firms (Fryxell and Lo, 2001). On the other hand, Hettige et al. (1996) document that pollution intensities of state-owned 'pulp and paper' plants are much higher than privately owned plants in Bangladesh, India, Indonesia and Thailand. Therefore, we

[3]This result is based on data from Chinese firms in the eight most polluting industries (mining, textiles and clothing, metal and non-metal, biomedicine, petrochemicals, food and beverage, water-electricity-gas and pulp-paper-printing) for the period 2001-2010.

consider two scenarios – one in which the public firm does not take into account the environmental damage and the other where it does.

The first scenario provides some clear-cut and interesting results. The relationship between privatization and environmental damage presents an inverted U pattern. The environmental damage first increases up to a critical point, and thereafter declines steadily. It turns out that at this critical level of privatization social welfare (which is given by economic welfare minus environmental damage) is also maximum. Essentially, the economic welfare of the society and the environmental damage are both inverted U-shaped when plotted against privatization, and both reach their maximum at the same level of privatization. Moreover, when rising economic welfare rises at a faster rate than the environmental damage, and likewise when declining it declines also at a faster rate. Therefore, when privatization is implemented at the socially optimal level, not only will the environmental damage be higher than the pre-privatization level, but it will be at its maximum.

The intuition behind this result is as follows. An increase in the level of privatization leads to (i) a decrease in the output of the public firm and an increase in the output of its rival firm (which depends on the degree of substitutability), and lower emission of pollutants in the aggregate. This is the *direct effect* of privatization. (ii) But higher privatization calls for a lower environmental tax, which triggers an *indirect effect* by inducing both firms to increase their production and cut back on abatement. Thus, the relationship between privatization and environmental damage depends on the combined impacts of the direct and indirect effects on production as well as abatement. We show that though the combined impact on the industry output is negative, the indirect effect (via lower tax and its effect on abatement) will outweigh the combined effects on output until privatization reaches a critical level, and thereafter the output effect will outweigh the abatement effect. This happens mainly because the products are not perfect substitutes. With perfect substitutes the direct effect is larger and thus at all levels of privatization, environmental damage is falling, and hence privatization will always improve the environment. This can be shown as a special case of our model where the optimal privatization is zero privatization. Government in that case will combat pollution only through taxation. We also observe that optimal privatization itself bears an inverted U relationship with the degree of substitutability between the two products.

The second scenario where the public firm does care about the environment proves to be too intractable to provide any clear result in the general case. Since the public firm cares about environment, its optimal abatement will depend not only on the pollution tax but also on the level of privatization, and in addition its output will tend to rise with the pollution tax if the level of privatization is low. These two effects complicate the relationship between the two policy instruments – privatization and tax. Intuitively, if privatization is sufficiently high, the public firm is effectively a private firm having no concern for environment, and we might expect an inverted U relationship between privatization and environmental damage as witnessed in the first scenario. But when the level of privatization is low this relationship might get reversed. Thus, the relationship between privatization and environmental damage may start as U-shaped and then switches to an inverted U shape. While this is still a non-monotone relationship as in the first scenario, the absence of a single pattern renders analytical solution intractable. Therefore, we consider several examples and demonstrate that the privatization-environment relationship indeed is a mixture of the two patterns. However, optimal privatization generally occurs at the U-shape range and at the declining phase. Thus, after privatization environmental damage generally will be lower in sharp contrast to the first scenario. A comparison of the equilibrium outcomes indicates that in the second scenario the optimal level of privatization and tax rate may be smaller, but social welfare and environmental damage may be higher. That social damage can be higher in the second scenario suggests that public firm's concern for environment does not ensure environmental improvement. Further, in both scenarios there is an inverted U relationship between optimal privatization and product substitutability.

For the sake of comparison we have considered a special case where products are homogeneous, firms do not undertake abatement measures and the environmental damage function is linear.[4] When the public firm does not care about the environment, the government needs to impose an environmental tax to combat pollution. However, on the issue of privatization, it prefers full public ownership. The reason is that when firms have constant and identical marginal cost of production and goods are perfect substitutes, economic welfare is maximized by turning the industry into a public monopoly. Subsequently, the public firm is disciplined through an environmental tax.

On the other hand, when the public firm cares about environment, it internalizes the

[4]We are thankful to a referee for suggesting this special case.

social cost. Therefore, here not only does the government retain full public ownership, but also sets a zero tax. The environmentally concerned public firm does not need to be disciplined through tax. However, zero tax and environmental concerns of the public firm imply that the public firm will have a higher effective marginal cost (social marginal cost) than the private firm. Therefore, economic welfare is maximized if both firms coexist. The voluntary output contraction by the environmentally conscious public firm is partially offset by the private firm's output. As can be seen, from this special model we learn about different implications of the tax choice, but its privatization prediction is not interesting. Our general framework with differentiated products, abatement and quadratic damage function yields more realistic predictions.

The rest of the paper is organized as follows. The next section presents the basic framework of the model. Section 3 characterizes the equilibrium outcomes when firms do not care about environment. A comparison of the equilibrium outcomes under two alternative types of government, environment concerned $vis-á-vis$ environment unconcerned, is also presented in Section 3. The case of social welfare maximizing public firm is analyzed in Section 4. Section 5 discusses a special case of homogeneous products. Section 6 concludes.

2 The model

Let us consider a differentiated oligopoly with two firms. Firm 1 is partially public and firm 2 is entirely privately owned. The inverse demand function faced by firm i $(= 1, 2)$ is assumed to be linear:

$$p_i = A - q_i - \gamma\, q_j, \quad i,\, j\, = 1,\, 2,\, i \neq j;$$

where q_i and p_i denote quantity and price, respectively, of the product of firm i, and γ $(0 < \gamma < 1)$ denotes the degree of product differentiation. Lower value of γ indicates higher degree of product differentiation, i.e., lower degree of substitutability between products. The underlying utility function of the representative consumer is $U = Aq_1 + Aq_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) + m$, where m is the quantity of the numeraire good produced in a competitive sector. This specification of the representative consumer's utility function is similar to that of Singh and Vives (1984). We assume, for convenience, that both firms have identical marginal cost of production c, which is constant, and that there is no fixed cost of production.[5]

Production processes in both firms pollute the environment. We assume, for simplicity, that production of each unit of output emits one unit of pollutant. However, pollution can be reduced by undertaking abatement measures. We consider that firm i chooses the abatement level a_i (≥ 0) and, thus, it emits $(q_i - a_i)$ units of pollutant. Following Ulph (1996), we assume that the cost of pollution abatement of firm i is $C_i = \frac{a_i^2}{2}$ and the total environmental damage due to pollution by the industry is as follows.

$$ED = \frac{1}{2}d(q_1 - a_1 + q_2 - a_2)^2, \tag{1}$$

where d is the increment in marginal environmental damage due to pollution, which is

[5]Qualitative results of this paper go through, if we consider increasing marginal costs of production.

assumed to be greater than $\frac{1}{6+2\gamma}$.[6] Implicitly we are assuming that both products emit the same type of pollutants. This is justifiable on the ground that the products are closely related.

The government being concerned about environment imposes an environmental tax t ($0 \leq t < A-c$) on each firm per unit of pollution emitted.[7] The resulting total tax revenue is $T = t(q_1 - a_1 + q_2 - a_2)$. Firm i's profit is $\pi_i = (p_i - c)q_i - t(q_i - a_i) - \frac{a_i^2}{2}$.

The government's objective is to maximize social welfare $SW = U - \sum_i p_i q_i + \sum_i \pi_i + T - ED$, which takes the following expression

$$\begin{aligned} SW =& Aq_1 + Aq_2 - \frac{1}{2}(q_1^2 + q_2^2 + 2\gamma q_1 q_2) - c(q_1 + q_2) \\ & - \frac{a_1^2}{2} - \frac{a_2^2}{2} - \frac{1}{2}d(q_1 - a_1 + q_2 - a_2)^2. \end{aligned} \tag{2}$$

Clearly, if the government did not care about environment it would not charge a pollution tax, and if it indulges in privatization it must be to maximize only the economic welfare $S_0 = U - \sum_i p_i q_i + \sum_i \pi_i$, as is the case with standard mixed oligopoly models.

The public firm, firm 1, may or may not share the same objective as the government even if it is fully owned by the government. Though commonly it is assumed to be the same, several papers have allowed divergence on empirical grounds as the firm level decisions are more decentralized and the privatization decision is taken at a much higher level of government. Hence, some divergence may be permitted (see Saha (2009) and Wang and

[6] This form of environmental damage function is widely considered in the literature. $d > \frac{1}{6+2\gamma}$ ensures that the optimal environmental tax rate is positive, irrespective of the level of privatization.

[7] Since emission subsidy is not politically viable, we restrict our focus on non-negative emission tax.

Wang (2009)). Here, we assume that a fully nationalized firm 1's objective function is

$$R = SW - \rho[T - ED], \quad \rho \in \{0, 1\}. \tag{3}$$

The case of $\rho = 0$ corresponds to the standard case in which the public firm's objective is to maximize the social welfare – same as the government. But if $\rho = 1$, the public firm does not care about the environmental damage and the tax revenue of the government. We will consider both scenarios to examine how the privatization decision and environmental outcomes may vary between the two cases.

Following the practice of the mixed duopoly literature (see Matsumura (1998)) suppose the government privatizes $\theta \in [0, 1]$ proportion of the ownership of firm 1. The private partner of the public firm will be naturally interested in maximizing profit, while the public partner will be interested in maximizing R. Therefore, we write the partially privatized firm 1's objective function as

$$O_1 = \theta\pi_1 + (1 - \theta)R. \tag{4}$$

The stages of the game involved are as follows.

Stage 1: The government decides on the level of privatization (θ).

Stage 2: The government chooses the environmental tax rate (t).

Stage 3: Each firm simultaneously and independently decides on the output (q_i) and abatement (a_i).

We should note that stage 1 and stage 2 can be clubbed together as the same player is making the decisions without any intervening strategic interactions, and hence the optimal polices of the government will not change. However, to ascertain whether environmental damage rises or falls with privatization sequencing of the two decisions will be helpful.

3 Firms do not care about environmental damage

First we assume $\rho = 1$ which implies that the public firm does not care about environment, and of course the private firm is not expected to be concerned about environment. Therefore, $R = U - \sum_i p_i q_i + \sum_i \pi_i$. We start from the third stage by noting that given any $(\theta, t(\theta))$ the outcome of the strategic interactions between the two firm is given by the following equations:

$$\begin{aligned}
\frac{\partial O_1}{\partial q_1} &= \theta MR_1 + (1-\theta)p_1 - (c+t) = 0 \\
\frac{\partial \pi_2}{\partial q_2} &= MR_2 - (c+t) = 0 \\
\frac{\partial O_1}{\partial a_1} &= t - a_1 = 0, \quad \frac{\partial \pi_2}{\partial a_2} = t - a_2 = 0.
\end{aligned}$$

In the above $MR_i = p_i + q_i(\partial p_i/\partial q_i)$ represents the usual marginal revenue of firm i. Both firms produce up to where its (weighted or un-weighted) marginal revenue equals marginal cost inclusive of the tax. Since the public firm does not take the government's tax revenue into account, it perceives the tax as an added marginal cost. Both firms also abate up to the point where its marginal abatement cost equals the tax rate. There is no asymmetry between the firms in their abatement choice, because neither has concern for environment. Due to the linear demand curves, we can explicitly solve for the outputs and abatements as

$$
\begin{aligned}
q_1 &= \frac{(A - c - t)\ (2 - \gamma)}{2\ (1 + \theta) - \gamma^2}, \\
q_2 &= \frac{(A - c - t)\ (1 - \gamma + \theta)}{2\ (1 + \theta) - \gamma^2}, \qquad (5) \\
\text{and} \quad a_1 &= a_2 = t.
\end{aligned}
$$

In the above, though t is implicitly dependent on θ (due to sequential choice of θ and t), it may be useful to note the *ceteris paribus* effects of t and θ. We note $\frac{\partial q_1}{\partial t} < \frac{\partial q_2}{\partial t} < 0$ when θ is assumed to be unchanged[8], and $\frac{\partial q_1}{\partial \theta} < 0 < \frac{\partial q_2}{\partial \theta}$, $\frac{\partial (q_1+q_2)}{\partial \theta} < 0$ when any effect of θ on t is ignored.[9]

An increase in the environmental tax rate leads to output reductions in both firms, but the reduction is larger in the public firm. On the other hand, if the level of privatization rises, its direct effect (ignoring any effect on the tax) is predictably negative on the public firm's output, positive on firm 2's output, but negative on the aggregate output. Therefore, it is apparent that if we consider only the direct effects, both privatization and pollution taxation will improve environment by reducing the aggregate outputs and increasing each firm's abatement.

Of course, to see the full effect of privatization on the environment we need to take into account the indirect effects as well that are occurring through the tax rate. Alternatively, if θ and t are decided at the same time, we need to see the combined effect of optimal θ and t on the environment and compare it with the pre-privatization level.

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8 $\frac{\partial q_1}{\partial t} = -\frac{2-\gamma}{2-\gamma^2+2\,\theta} < -\frac{1+\theta-\gamma}{2-\gamma^2+2\,\theta} = \frac{\partial q_2}{\partial t}$, since $\theta \in (0, 1)$ in case of partial privatization.

9 $\frac{\partial q_1}{\partial \theta} = -\frac{2\,(A-c-t)\,(2-\gamma)}{(\gamma^2-2\,(1+\theta))^2} < 0 < \frac{\partial q_2}{\partial \theta} = \frac{\gamma\,(A-c-t)\,(2-\gamma)}{(\gamma^2-2\,(1+\theta))^2}$, since $0 < \gamma < 1$. $\frac{\partial (q_1+q_2)}{\partial \theta} = -\frac{(2-\gamma)^2\,(A-c-t)}{(\gamma^2-2\,(1+\theta))^2} < 0$.

Total effects of privatization: Now we consider stage 2 of the game. Given θ from the first stage the government maximizes $SW(t;\theta)$ with respect to t with the perfect foresight of the stage 3 outcome.

$$\begin{aligned}\frac{\partial SW}{\partial t} &= \sum_i \frac{\partial SW}{\partial q_i}\frac{\partial q_i}{\partial t} + \sum_i \frac{\partial SW}{\partial a_i}\frac{\partial a_i}{\partial t} = 0 \\ &= \sum_i [p_i - c - ED'(.)]\frac{\partial q_i}{\partial t} + \sum_i [ED'(.) - a_i] = 0 \\ &= \sum_i [p_i - c]\frac{\partial q_i}{\partial t} + ED'(.)\left[2 - \sum_i \frac{\partial q_i}{\partial t}\right] - 2t = 0.\end{aligned}$$

It can be verified that $\partial^2 SW/\partial t \partial \theta < 0$. Hence, with the help of the second order condition $\partial^2 SW/\partial t^2 < 0$ we ascertain $dt/d\theta < 0$ from the following equation

$$\frac{\partial^2 SW}{\partial t^2}\frac{dt}{d\theta} + \frac{\partial^2 SW}{\partial t \partial \theta} = 0.$$

That $dt/d\theta$ indicates that privatization and environmental tax are substitutes in nature; in other words they are similar instruments for improving social welfare when there are environmental concerns.

Now consider the effect of privatization on environmental damage $ED(t,\theta) = \frac{1}{2}d[q_1(t,\theta) + q_2(t,\theta) - a_1(t,\theta) - a_2(t,\theta)]^2$.

$$\begin{aligned}\frac{dED}{d\theta} =& \frac{\partial ED}{\partial \theta} + \frac{\partial ED}{\partial t}\frac{\partial t}{\partial \theta} \\ =& \underbrace{\underset{(+)}{\frac{\partial ED}{\partial (q_1+q_2)}}\underset{(-)}{\frac{\partial (q_1+q_2)}{\partial \theta}}}_{\substack{(-)\\ \text{Direct effect}}} + \underbrace{\underset{(+)}{\frac{\partial ED}{\partial (q_1+q_2)}}\underset{(-)}{\frac{\partial (q_1+q_2)}{\partial t}}\underset{(-)}{\frac{\partial t}{\partial \theta}}}_{\substack{(+)\\ \text{Indirect effect, via tax through output}}} + \underbrace{\underset{(-)}{\frac{\partial ED}{\partial (a_1+a_2)}}\underset{(+)}{\frac{\partial (a_1+a_2)}{\partial t}}\underset{(-)}{\frac{\partial t}{\partial \theta}}}_{\substack{(+)\\ \text{Indirect effect, via tax through abatement}}}\end{aligned}$$

It is evident that privatization has two opposing effects on environmental damage: (a) privatization reduces the environmental damage by restricting the overall industrial production (*direct effect*) and (b) it calls for lower environmental tax, to take account of the economic surplus, which in turn increases environmental damage by encouraging higher production and lower abatement by firms (*indirect effects*).

Clearly, the overall effect can go either way depending on whether the direct effect dominates, or the indirect effects dominate. The disagreement among many authors which we discussed in the introduction stems from this ambiguity. For example, in Beladi and Chao (2006) there is only one firm and there is no abatement. Hence, the third term is absent and the second term is smaller. As a result, we get a negative overall effect (i.e. privatization leads to an improvement of environment). Similarly, in Ohori (2012) there is abatement, and hence the third term is present. But because it is a monopoly set up the direct effect dominates and we get the same effect as in Beladi and Chao (2006). We also know that in some duopoly models with homogeneous product such as Naito and Ogawa (2009) and Wang et al. (2009) the same result holds. Then it must be the case that the direct effect of θ dominates over the combined indirect effects.

But can we make the same conclusion with differentiated products? Clearly, in the general case that is not possible, and we need to study the explicit solution of our model. Though it is cumbersome and tedious, fortunately we can derive an explicit solution of t in terms of θ and ascertain its first and second order derivatives. Then we are also able to derive the expression of $ED(.)$ in terms of θ, which then can be studied to see if environment improves or deteriorates with privatization.

The explicit solution of t is

$$t = \frac{(A-c)\left[d\,(3-2\gamma+\theta)\,\{7-2\gamma\,(1+\gamma)+5\theta\}+\gamma\,(2+6\theta)-\gamma^2\,(1+\theta)-\theta\,(6+\theta)-1\right]}{13+2\gamma^3+2\gamma^4-2\gamma\,(1-\theta)+9\theta\,(2+\theta)-2\gamma^2\,(6+5\theta)+d\{7-2\gamma\,(1+\gamma)+5\theta\}^2} = t(\theta). \quad (6)$$

It can be checked that optimum t is always less than the marginal environmental damage. Given any θ, marginal environmental damage is given by $ED'(.) =$

$$\frac{(A-c)\;d\;(23+4\,\gamma^3-24\,\gamma\;(1+\theta)+\theta\;(34+7\,\theta))}{13+2\,\gamma^3+2\,\gamma^4-2\,\gamma\;(1-\theta)+9\,\theta\;(2+\theta)-2\,\gamma^2\;(6+5\,\theta)+d\,(7-2\,\gamma\;(1+\gamma)+5\,\theta)^2}.$$

Using this we can write $t - ED'(.) =$

$$-\frac{(A-c)\;(1+2\,d)\;\left((1-\gamma)^2+(6-(6-\gamma)\;\gamma)\;\theta+\theta^2\right)}{13+2\,\gamma^3+2\,\gamma^4-2\,\gamma\;(1-\theta)+9\,\theta\;(2+\theta)-2\,\gamma^2\;(6+5\,\theta)+d\,(7-2\,\gamma\;(1+\gamma)+5\,\theta)^2}.$$

Always $t < ED'(.)$, except in the unique special case of $\gamma = 1$ and $\theta = 0$; $t = \frac{\partial ED}{\partial(q_1-a_1+q_2-a_2)}$ only if $\gamma = 1$ and $\theta = 0$. In other words, $t < ED'(.)$ is generally true in oligopolies, with the exception of homogeneous products and full nationalization. But for differentiated products we always have $t < ED'(.)$. Moreover, we can also ascertain that $\frac{\partial t}{\partial \theta} < 0$, $\frac{\partial^2 t}{\partial \theta^2} > 0$. We summarize these results in Lemma 1.

Lemma 1: *For differentiated products, the optimal pollution tax is always strictly less than the marginal environmental damage, and it decreases at a decreasing rate with an increase in the level of privatization.*

Now, consider the explicit expression of $ED(\theta)$ as follows.[10]

$$ED(\theta) = \frac{1}{2}d\{q_1(\theta) + q_2(\theta) - 2t(\theta)\}^2$$
$$= \frac{(A-c)^2 d\{23 + 4\gamma^3 - 24\gamma\,(1+\theta) + \theta\,(34+7\theta)\}^2}{2[13 + 2\gamma^3 + 2\gamma^4 - 2\gamma\,(1-\theta) + 9\theta\,(2+\theta) - 2\gamma^2\,(6+5\theta) + d\{7 - 2\,\gamma\,(1+\gamma) + 5\theta\}^2]^2}$$

We observe that, if $\theta < \frac{1+\gamma-2\,\gamma^2}{9-7\,\gamma} = \theta^*$ ($0 < \theta^* < 1$, since $0 < \gamma < 1$), $\frac{dED(\theta)}{d\theta} > 0$. Alternatively, if $\theta \geq \theta^*$, $\frac{dED(\theta)}{d\theta} \leq 0$. That is, environmental damage due to pollution increases (decreases) with the increase in level of privatization, unless privatization is greater (less) than the critical level θ^*. Also, note that the environmental damage is maximum at $\theta = \theta^*$.[11] The underlying reason is as follows. When the level of privatization is relatively less ($\theta < \theta^*$), an increase in the level of privatization decreases overall production, but the environmental tax rate and, thus, pollution abatement by firms decrease more than proportionately than the decrease in overall production due to increased privatization ($\mid \frac{d\,[2a\,(\theta)]}{d\theta} \mid > \mid \frac{d[q_1(\theta)+q_2(\theta)]}{d\theta} \mid$). As a result, the higher level of privatization leads to higher pollution and higher environmental damage. The converse is true, if the level of privatization is more than a critical level. Alternatively, we can say that the indirect effects of privatization on environmental damage dominates its direct effect, unless the level of privatization is more than a critical level. These are interesting findings.

Proposition 1: *In the case of a differentiated mixed duopoly when the public firm*

[10] $q_1(\theta) = \frac{(A-c)\,(1+2\,d)\,(-2+\gamma)\,(-7+2\,\gamma\,(1+\gamma)-5\,\theta)}{13+2\,\gamma^3+2\,\gamma^4+2\,\gamma\,(-1+\theta)+9\,\theta\,(2+\theta)-2\,\gamma^2\,(6+5\,\theta)+d\,(7-2\,\gamma\,(1+\gamma)+5\,\theta)^2}$ and

$q_2(\theta) = \frac{(A-c)\,(1+2\,d)\,(-7+2\,\gamma\,(1+\gamma)-5\,\theta)\,(-1+\gamma-\theta)}{13+2\,\gamma^3+2\,\gamma^4+2\,\gamma\,(-1+\theta)+9\,\theta\,(2+\theta)-2\,\gamma^2\,(6+5\,\theta)+d\,(7-2\,\gamma\,(1+\gamma)+5\,\theta)^2}$

[11] $\frac{dED(\theta)}{d\theta} = 0 \Rightarrow \theta = \theta^*$, and $\frac{d^2ED(\theta)}{d\theta^2} \mid_{\theta=\theta^*} < 0$. Also, note that, if products are homogeneous, i.e., if $\gamma = 1$, $\theta^* = 0$. Therefore, in case of homogeneous products, the only feasible case is $\frac{dED}{d\theta} < 0$, which is in line with the findings of Naito and Ogawa (2009) and Wang et al. (2009).

is unconcerned about environment, the environmental damage will be non-monotone in the level of privatization, θ. An increase in θ will adversely affect the environment, if $\theta < \frac{1+\gamma-2\gamma^2}{9-7\gamma} = \theta^*$ ($0 < \theta^* < 1$), and improve the environment if $\theta > \theta^*$. The environmental damage is maximum at θ^*.

The above proposition is in contrast to many of the existing papers such as Beladi and Chao (2006), Ohori (2012), Naito and Ogawa (2009) and Wang et al. (2009). In contrast to these papers we show that when the products are differentiated and the public firm is unconcerned about environment, privatization can further damage the environment, if privatization is still below a critical level. It is also clear from Proposition 1 that if the degree of substitutability γ is very high, say close to 1, the critical θ is zero. In that case, whatever level of privatization is chosen it will result in a smaller environmental damage. In this sense, we are able to generate the result of Naito and Ogawa (2009) and Wang et al. (2009) as a special case of our model. Our model can also generate the monopoly model of Beladi and Chao (2006). Further, the results of Barcena-Ruiz and Garzon (2006) and Wang and Wang (2009) can be replicated by considering two values of θ: $\theta = 0$ and $\theta = 1$.

Optimal privatization: Now we turn to privatization decision in stage 1 with the correct anticipation of the subsequent decisions. The government will maximize $\underset{\theta}{Max}\, SW(\theta)$, where $SW(\theta)$ is obtained by substituting $t(\theta)$, $q_i(\theta)$ and $a_i(\theta)$ in the expression of SW. Fortunately we can get an explicit solution for θ, and that is exactly the same critical θ

(θ^*).

$$\theta_{pp} = \frac{1 + \gamma - 2\gamma^2}{9 - 7\gamma} = \theta^*.$$

Clearly, $0 < \theta_{pp} < 1$, since $0 < \gamma < 1$. That is, socially optimal privatization is partial privatization. In the special case of $\gamma = 1$ we have $\theta_{pp} = 0$, i.e. no privatization. Also, note that the socially optimal level of privatization coincides with the environmental damage maximizing level of privatization. Then it must be the case that at the social optimum the economic welfare is also at its maximum, because social welfare is the economic welfare minus environmental damage. For the social welfare to achieve its maximum at θ_{pp} we must have the economic welfare rising at all $\theta < \theta_{pp}$, and it is rising at a rate faster than the environmental damage. On the other hand, at all $\theta > \theta_{pp}$ economic welfare must be declining faster than the environmental damage.

Further, $\theta'(\gamma) \geq (<) 0$ if $\gamma \leq (>) 4/7 (= 0.57)$. That is to say, θ_{pp} also exhibits an inverted U-shaped relationship with the product substitutability parameter γ, which is depicted in Figure 1. θ_{pp} attains its maximum value 9/49 (=0.18) at $\gamma = 0.57$.

[–Figure 1 is about here –]

Figure 1: Product substitutability and optimal privatization in the first scenario

The partial privatization result is in line with the existing literature on privatization in case of differentiated products oligopoly (see, for example, Saha (2009) and Fujiwara (2007)). To illustrate it further, note that a fully nationalized firm sets the price at marginal cost. Privatization induces it to be less aggressive in the product market and to increase its price above the marginal cost. As a result, private firm's output increases at the expense of public firm's output leading to fall in total output. Thus, privatization adversely affects consumer surplus, but it leads to higher industry profit. Moreover, consumer surplus (industry profit) decreases (increases) at an increasing (decreasing) rate with the increase in level of privatization. For low levels of privatization, increases in privatization raise industry profit more than associated decrease in consumer surplus, when products are differentiated. The reverse is true, if the level of privatization is sufficiently high. As a result, social welfare first increases and then decreases with increase in privatization. Therefore, in case of differentiated products mixed duopoly, partial privatization is optimal.[12] Environmental pollution due to production, together with environmental tax and pollution abatement by firms, adds another dimension to it, as discussed before.

Now we report below the optimal tax rate and some key equilibrium variables. The optimal tax rate is positive given our assumption $d > \frac{1-\gamma}{14-12\gamma}$ which makes $H > 0$. This suggests pollution tax is worthwhile only if pollution exceeds a minimum threshold. It is also worth noting that the partially public firm's output will be greater than the private

[12]In case of homogeneous product mixed duopoly/oligopoly also partial privatization is socially optimal, provided that there is increasing marginal cost of production (see Matsumura (1998)).

firm's output, as $9 - 7\gamma > 5 - 5\gamma$.

$$
\begin{aligned}
t_{pp} &= \frac{(A-c)\ (\gamma + 2\,d\ (7 - 6\,\gamma) - 1)}{H},\\
ED_{pp} &= \frac{2\,(A-c)^2\,d\,(8 - 7\,\gamma)^2}{H^2},\\
q_{1,pp} &= \frac{(A-c)\ (1 + 2\,d)\ (9 - 7\,\gamma)}{H},\\
q_{2,pp} &= \frac{(A-c)\ (1 + 2\,d)\ (5 - 5\,\gamma)}{H},\ \text{where } H = 9 - 7\,\gamma^2 + 2\,d\ (17 - 7\,\gamma\ (1+\gamma))\,.
\end{aligned}
$$

Proposition 2: *Socially optimal privatization will be partial privatization, and the combination of privatization and pollution tax will damage the environment most, though they will also lead to maximum economic welfare. The partially privatized firm will produce more than the private firm in the equilibrium.*

To benchmark our results, we check the level of privatization that would have been optimal had the government not been concerned for environment. In that case, it would not impose any pollution tax and consequently firms will not do any abatement. The government still would choose θ to maximize only the economic welfare of the society $S_0 = U - \sum_i p_i q_i - \sum_i \pi_i$. In such a scenario, social welfare will be predictably lower; government's optimal privatization will be $\theta_0 = \frac{(1-\gamma)\gamma}{4-3\gamma}$ leading to an associated environmental damage of $ED_0 = \frac{(A-c)^2 d (6-5\gamma)^2}{2(4-3\gamma^2)^2}$. It can be easily checked $\theta_0 < \theta_{pp}$ and $ED_0 > ED_{pp}$. This is not surprising. When the government is not concerned about environment, it will choose a smaller level of privatization and consequently the environmental damage will be higher. But note that ED_0 corresponds to a single policy regime (only privatization is pursued), while ED_{pp} corresponds to a regime of dual policies (privatization and pollution tax), and as already noted it is the highest within this regime. So, we can say that even if the public firm does not care about environment, we still achieve a Pareto superior outcome if the government cares about environment.

It might be useful to consider some numerical examples to gain further insights into the effects of increased product differentiation, which are not obvious from the analytical expressions above. In Table 1 (see Appendix), we report some numerical simulations assuming $A = 10$, $c = 0$ and $d = 1/2$, but permitting incremental variations in γ starting from $\gamma = 1/10$ and ending with $\gamma = 9/10$. Apart from reporting our main variables of interest θ and ED, we report a host range of variables. We notice that as γ increases, i.e. the products are becoming more homogeneous, overall social welfare and social damage will both fall; the tax rate will also get smaller. With reduced product variety consumers experience a loss in (gross) utility, and firms lose profit due to greater intensity of competition.

4 Social welfare maximizing public firm

Now we consider the second scenario where the public firm cares about environment and thus tries to maximize full social welfare. So in this case $\rho = 0$ and, the objective function of the partially privatized firm is $O_1 = \theta\pi_1 + (1 - \theta)SW$. We would like to see how the results in this case differs from the previous case.

As before we start from the last stage of the game where firms simultaneously choose their outputs and abatements. Given any (θ, t) the firms' actions are determined from the following equations:

$$\begin{aligned}
\frac{\partial O_1}{\partial q_1} &= \theta(MR_1 - t) + (1-\theta)[p_1 - ED'(.)] - c = 0 \\
\frac{\partial \pi_2}{\partial q_2} &= MR_2 - (c+t) = 0 \\
\frac{\partial O_1}{\partial a_1} &= \theta t + (1-\theta)ED'(.) - a_1 = 0, \quad \frac{\partial \pi_2}{\partial a_2} = t - a_2 = 0.
\end{aligned}$$

The public firm now takes into account the environmental damage in its output choice, and also it abates up to a point where its marginal abatement cost is equal to the weighted average of the pollution tax and the marginal environmental damage. If θ is close to 1, the outputs and abatements will behave in the same way as in the previous section. But if θ is close to zero, the tax rate does not directly affect the public firm's output or abatement. But as the private firm's output will fall with the tax, the public firm's output will increase (strategic effects). The increased tax will also force the private firm to increase its abatement and will thus reduce the total as well as the marginal environmental damage. Since the public firm will equate the marginal abatement cost (which is the abatement itself) with the marginal environmental damage, it will choose a lower level of abatement. Thus, we expect to see the partially privatized firm to behave very differently in this case, if the level of privatization is sufficiently low. These intuitions can be verified from the analytical solutions:

$$
\begin{aligned}
q_1 =& \frac{(A-c)\,\{2-\gamma+d\ (1-\gamma)\ (1-\theta)\}+t\{\gamma+d\ (3+\gamma)\ (1-\theta)-2\,\theta\}}{2-\gamma^2+2\,\theta-d\ (1-\theta)\,\{\gamma+\gamma^2-2\ (2+\theta)\}}, \\
q_2 =& \frac{(A-c)\,\{1-\gamma+\theta+d\,(1-\theta)\,(2-\gamma+\theta)\}-t\{1+(1-\gamma)\ \theta+d\ (1-\theta)\ (2+\gamma+\theta)\}}{2-\gamma^2+2\theta+d\,(-1+\theta)\,\{\gamma+\gamma^2-2\,(2+\theta)\}}, \quad (5') \\
a_1 =& \frac{(A-c)\,d\,(1-\theta)\,(3-2\,\gamma+\theta)+t\{\left(2-\gamma^2-d\gamma^2\right)\theta+(2+3\,d)\,\theta^2-d\left(3-\gamma-\gamma^2+\gamma\theta\right)\}}{2-\gamma^2+2\,\theta-d\ (1-\theta)\,\{\gamma+\gamma^2-2\ (2+\theta)\}}, \\
a_2 =& t
\end{aligned}
$$

From equation (5') we can see that if θ is close to zero, $\partial q_1/\partial t > 0$ and $\partial a_1/\partial t < 0$; the opposite is true if θ is close to 1.[13] We also see, when $\theta = 0$ and t is sufficiently small, say $t = 0$, the public firm *may* produce less than the private firm (i.e. $q_1 < q_2$) if the damage parameter d is greater than 1. Being environmentally concerned the public firm tries to restrict pollution by producing less. It also abates more than the private firm (i.e. $a_1 > a_2$) for all plausible values of d. Even if we allow positive tax rate t (but not very high t), these results largely hold .[14]

However, these are only *ceteris paribus* effects. We need to take into account the indirect effects of θ occurring through the tax as well. As in the earlier case, the government can restrict the private firm's output by imposing higher emission tax. But a higher tax may induce the public firm to produce more, and the industry output may increase or decrease. Similar ambiguity may also arise for the aggregate abatement when the public firm reduces

[13] Always $\frac{\partial q_2}{\partial t} < 0$; but $\frac{\partial q_1}{\partial t} > 0$ if $0 \leq \theta < \frac{3\,d+\gamma+d\,\gamma}{2+3\,d+d\,\gamma} < 1$.

[14] If $\theta = 0$, (i) $q_1 < q_2$ when $d > 1$ and $0 \leq t \leq \frac{(A-c)\,(d-1)}{1+5\,d+\gamma+2\,d\,\gamma}$ and (ii) $a_1 > a_2$ when $0 \leq t < \frac{-3\,A\,d+3\,c\,d+2\,A\,d\,\gamma-2\,c\,d\,\gamma}{-2-7\,d+2\,d\,\gamma+\gamma^2+2\,d\,\gamma^2}$.

its abatement in response to a rise in the emission tax. Despite these ambiguities we find that when the expressions for q_1, q_2, a_1 and a_2 from (5') are substituted in the expression for social welfare (SW) in (2), SW is concave in t and it has a unique maximum, say $t = \hat{t}(\theta)$. However, the expression of $\hat{t}(\theta)$ is too long to report here. The sign of $\frac{\partial \hat{t}(\theta)}{\partial \theta}$ also depends on too many parameters, such as A, c, d,γ, and also on the level of privatization. Because of this difficulty no clear prediction can be made about the relationship between privatization and environmental damage. In turn the analysis of the privatization decision also becomes intractable. Therefore, we need to attempt some simulations guided by the following intuition. Intuitively, when θ is sufficiently high we should expect the same inverted U-relationship as in the previous section, and when θ is sufficiently small we should expect a U-relationship. Thus over the entire range of θ the relationship might be a mixture of the two patterns. The indirect effects of privatization may dominate only in the middle range of θ while the direct effects may dominate at both low and high values of θ.

Example: Assume that $A = 10$, $c = 0$, $d = \frac{1}{2}$, $\gamma = \frac{2}{3}$. Then the stage 2 equilibrium tax rate, for any given level of privatization, is $\hat{t}(\theta) = \frac{20\left(229+362\,\theta-299\,\theta^2+616\,\theta^3+192\,\theta^4\right)}{3341+2674\,\theta+5837\,\theta^2+2880\,\theta^3+3968\,\theta^4}$.

Corresponding outputs and abatements of firms are as follows.

$$\begin{aligned}
q_1 =& \frac{40\ (521 - 6\,\theta + 489\,\theta^2 + 496\,\theta^3)}{3341 + 2674\,\theta + 5837\,\theta^2 + 2880\,\theta^3 + 3968\,\theta^4}, \frac{\partial q_1}{\partial \theta} < 0; \\
q_2 =& \frac{40\ (165 + 246\,\theta + 621\,\theta^2 + 20\,\theta^3 + 448\,\theta^4)}{3341 + 2674\,\theta + 5837\,\theta^2 + 2880\,\theta^3 + 3968\,\theta^4}, \frac{\partial q_2}{\partial \theta} > 0; \\
a_1 =& \frac{20\ (381 - 62\,\theta + 1021\,\theta^2 - 560\,\theta^3 + 320\,\theta^4)}{3341 + 2674\,\theta + 5837\,\theta^2 + 2880\,\theta^3 + 3968\,\theta^4}, \frac{\partial a_1}{\partial \theta} < 0; \\
a_2 =& \hat{t}(\theta), \frac{\partial a_2}{\partial \theta} < 0 \text{ if } 0.132896 < \theta < 0.86557; \\
& \frac{\partial (q_1 + q_2)}{\partial \theta} < 0 \text{ and } \frac{\partial (a_1 + a_2)}{\partial \theta} < 0.
\end{aligned}$$

Note that, in contrast to Lemma 1, the tax rate is not always decreasing in level of privatization. Nonetheless, for any given level of privatization, optimum tax rate is less than marginal environmental damage ($= \frac{20\left(381+90\,\theta+749\,\theta^2+488\,\theta^3+192\,\theta^4\right)}{3341+2674\,\theta+5837\,\theta^2+2880\,\theta^3+3968\,\theta^4}$). Now, substituting the above expressions for q_i and a_i ($i = 1, 2$) in (1), we get the environmental damage as follows.

$$ED(\theta) = \frac{400\,(381 + 90\,\theta + 749\,\theta^2 + 488\,\theta^3 + 192\,\theta^4)^2}{(3341 + 2674\,\theta + 5837\,\theta^2 + 2880\,\theta^3 + 3968\,\theta^4)^2}.$$

It is easy to check that $\frac{\partial ED(\theta)}{\partial \theta} > 0$, if $\theta \in (\theta_1, \theta_2)$ where $\theta_1 = 0.451642$ and $\theta_2 = 0.7745632$; otherwise, $\frac{\partial ED(\theta)}{\partial \theta} < 0$. It implies that environmental damage is non-monotone in level privatization.

[– Figure 2 is about here –]

Figure 2: Privatization and environmental damage

Figure 2 depicts the relation between environmental damage and level of privatization. It is easy to check that environmental damage is maximum (minimum) at $\theta = 0$ ($\theta = 1$). However, in $0 < \theta < 1$, environmental damage has a local minimum (maximum) at $\theta = \theta_1$

($\theta = \theta_2$). Clearly, we can say that environmental damage is non-monotone in privatization, even if the public firm cares for social welfare (at least for some plausible parametric values), though it is no longer is inverted U-shaped. Instead, it is a mixture of the U shape and the inverted U-shape, as we have speculated in the introduction. The inverted U shape comes back at higher values of θ.

When this relation is substituted in the social welfare function, its expression becomes

$$SW(\theta) = \frac{400\ (357 + 306\,\theta + 517\,\theta^2 + 400\,\theta^3 + 320\,\theta^4)}{3341 + 2674\,\theta + 5837\,\theta^2 + 2880\,\theta^3 + 3968\,\theta^4}.$$

The government maximizes the above in the first stage by choosing optimal $\theta = 0.10093 = \theta_s^*$. So, in this case also, partial privatization is optimal. But in contrast to the first scenario, optimal privatization does not damage the environment most; on the contrary, it improves the environment, as θ_s^* is clearly less than θ_1 (See Fig. 2). In this particular example, from the environmental point of view the best policy is full privatization; obviously that will substantially reduce the economic welfare.

We also run some simulations by varying the product differentiation parameter γ and the emission intensity parameter d. These results are reported in Table 2 (see Appendix). It is evident that, partial privatization is optimum from social welfare point of view and environmental damage is also non-monotone in privatization. Further, as in the first scenario, here also the optimal privatization exhibits an inverted U relationship with the product substitutability (see Figure 3). But now this relationship is sensitive to several model parameters. Social damage and social welfare are both declining in γ as before.

[– Figure 3 is about here –]

Figure 3: Product substitutability and optimal privatization in the second scenario

Comparing the equilibrium outcomes with those of the first scenario as shown in Table 1, we find that in the second scenario the optimal privatization and tax are both lower, and the public (private) firm produces less (more) and abates more (less); social welfare is also higher. This is not surprising, because the public firm is environmentally concerned in the second scenario. But surprisingly, environmental damage is also higher in the second scenario. On close scrutiny, we see that aggregate abatements in both scenarios are more or less same, but the aggregate output in the second scenario is much higher due to lower tax, which encourages the private firm to raise its output significantly. The net effect results in greater environmental damage, despite the fact that the public firm is now environmentally concerned. This suggests that firms' environmental concern does not ensure a better environmental outcome. We also see that when the emission intensity parameter d increases, predictably social welfare falls.

Proposition 3: *When the public firm shares the same environmental concern as the government, environmental damage may first decrease with privatization, then increase, and then decrease again displaying an inverted U pattern at higher levels of privatization. In equilibrium, privatization once again will be partial. But, in contrast to the first scenario, the environmental damage will be lower than the pre-privatization level. Also, the optimal level of privatization and tax rate may be smaller, but social welfare and environmental damage may be higher, compared to that in the first scenario.*

5 A special case

For the sake of comparison, let us now consider a special case in which (a) products are homogeneous ($\gamma = 1$), (b) no firm undertakes abatement measures and (c) the environmental damage function is linear $ED = d(q_1 + q_2)$, while all other things remain the same as before.

In the first scenario where firms do not care about the environment, the Stage 3 equilibrium outputs of firm 1 and firm 2, environmental damage and social welfare are, respectively, as follows.

$$\begin{aligned}
q_1 =& \frac{A - c - t}{2\theta + 1}, \\
q_2 =& \frac{(A - c - t)\theta}{2\theta + 1}, \\
ED =& \frac{d(\theta + 1)(A - c - t)}{2\theta + 1}, \text{ and} \\
SW =& \frac{(\theta + 1)(A - c - t)\{\theta(3A - 3c - 4d + t) + A - c - 2d + t\}}{2(2\theta + 1)^2}.
\end{aligned}$$

Now, it can be checked that, for any given θ, the social welfare maximizing tax rate is $t = \frac{d - \theta(A - c - 2d)}{\theta + 1} = \hat{t}$, say. It is evident that, if $t = \hat{t}$, $ED = d(A - c - d)$ and $SW = \frac{1}{2}(A - c - d)^2$, which are independent of θ. Clearly, zero privatization ($\theta^* = 0$) and tax rate equal to marginal environmental damage ($t^* = d$) is an optimal choice of the government. In other words, when firms do not care about the environment, the government can achieve the maximum possible level of social welfare through appropriately designed tax policy, without privatizing the public firm. The industry is turned into a public monopoly. The private firm is driven out, because its marginal cost (which is production cost plus tax) is same as the public firm's, and it cannot compete with the fully public firm for their goods being perfect substitutes.

Next, in the second scenario where the public firm does care about the environment, the Stage 3 equilibrium outputs of firm 1 and firm 2, environmental damage and social welfare are, respectively, as follows.

$$\begin{aligned} q_1 =& \frac{A - c - 2d(1-\theta) + (1-2\theta)t}{1+2\theta}, \\ q_2 =& \frac{\theta(A-c) + d(1-\theta) - t}{1+2\theta}, \\ ED =& \frac{(A-c)d(1+\theta) - d^2(1-\theta) - 2d\theta t}{1+2\theta}, \text{ and} \\ SW =& \frac{\{(A-c-d) + \theta(A-c+d-2t)\}\{(A-c-d) + \theta(3A-3c-5d+2t)\}}{2(1+2\theta)^2}. \end{aligned}$$

Clearly, environmental tax will affect on environmental damage and social welfare, provided that there is privatization ($\theta > 0$). It is also easy to check that, if $\theta > 0$, the social welfare maximizing tax rate is $t = -\frac{1}{2}(A - c - 3d)$, which is independent of θ. Thus, $\frac{dED}{d\theta} = \frac{\partial ED}{\partial\theta} = -\frac{d(A-c-3d+2t)}{(2\theta+1)^2} < 0$. That is, environmental damage is monotonically decreasing in privatization. However, for any given tax rate (t), we have $\frac{\partial SW}{\partial\theta} = -\frac{\theta(A-c-3d+2t)^2}{(1+2\theta)^3} < 0$, which implies that no-privatization ($\theta = 0$) is socially optimal. Also, note that under no-privatization, industry output, environmental damage and social welfare do not depend on the rate of environmental tax: when $\theta = 0$, $(q_1 + q_2) = A - c - d$, $ED = (A - c - d)d$ and $SW = \frac{1}{2}(A - c - d)^2$. Therefore, when the public firm cares about the environment, it is optimal for the government (a) to keep the public firm fully public and (b) not to impose any environmental tax.

However, unlike the first scenario, here the private firm is not driven out of the market.

Zero tax and environmental concerns of the public firm imply that the public firm will have a higher effective marginal cost (social marginal cost) than the private firm. Therefore, economic welfare is maximized if both firms coexist. The voluntary output contraction by the environmentally conscious public firm is partially offset by the private firm's output. Interestingly, in this special model, the equilibrium social welfare and environmental damage are identical under the two alternative scenarios. Thus, the lack of environmental concern of a public firm can be overcome by alternative policy instruments to arrive at the same level of social welfare.

Clearly, if we consider this special version of our model, the analysis is much simpler and results are clear-cut, but these results do not correspond to the widely observed phenomena of partial privatization and government interventions to control pollution. Our general model with product differentiation, abatement and non-linear environmental damage function generates more realistic predictions.

6 Conclusion

This paper shows that environmental damage is non-monotone in the level of privatization, irrespective of whether the public firm is concerned about environment or not. If firms are not concerned about environment, the relationship between privatization and environmental damage exhibits an inverted U pattern. Whereas, if the public firm does care about environment, the above relationship may start as U shaped and then changes to inverted U shape. These results are in sharp contrast to the findings of the existing studies. Moreover, this paper demonstrates that the optimal privatization is always partial privatization, and the level of privatization is higher due to environmental concern of the government. However, the nature of impact of privatization on environmental damage crucially depends on the public firm's concern for environment. Socially optimal level of privatization damages the environment most, if the public firm does not care about environment. In contrast, when the public firm is environment concerned, environmental damage at socially optimal level of privatization is lower than pre-privatization level. We also observe that socially optimal privatization seems to bear an inverted U relationship with the degree of product substitutability in both the cases. Results of numerical simulations indicate that environmental concern of the public firm may lead to lower tax rate and less privatization, but higher social welfare as well as higher environmental damage.

It seems that there is a conflict of interests between the 'green lobby' and the 'economic welfare lobby' as far as privatization is concerned. Environmental awareness of citizens and institutional factors are likely to play crucial roles in determining the relative strengths of the parties involved. It would be useful to extend this paper to a more general framework that allows for endogenous determination of the government's objective function through bargaining between the 'green lobby' and the 'economic welfare lobby'. It also remains an open problem to determine the optimal degree of environmental concern of the public firm. It might also be interesting to extend the present analysis by considering consumption related pollution.

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Appendix:

Table 1: Equilibrium outcomes when firms are not environment concerned

Variables	$A=10,\ c=0,\ d=\frac{1}{2}$								
	$\gamma=\frac{9}{10}$	$\gamma=\frac{4}{5}$	$\gamma=\frac{3}{4}$	$\gamma=\frac{7}{10}$	$\gamma=\frac{13}{20}$	$\gamma=\frac{3}{5}$	$\gamma=\frac{1}{2}$	$\gamma=\frac{1}{4}$	$\gamma=\frac{1}{10}$
θ^*_{pp}	**0.1037**	**0.1529**	**0.1667**	**0.1756**	**0.1809**	**0.1833**	**0.1818**	**0.1552**	**0.1301**
ED^*_{pp}	**4.1351**	**4.4012**	**4.5622**	**4.7392**	**4.9319**	**5.1407**	**5.6094**	**7.1492**	**8.4183**
SW^*_{pp}	**40.6699**	**41.9580**	**42.7184**	**43.5393**	**44.4158**	**45.3461**	**47.3684**	**53.4759**	**58.0286**
t^*_{pp}	**1.7943**	**1.7483**	**1.7476**	**1.7556**	**1.7702**	**1.7900**	**1.8421**	**2.0321**	**2.1860**
$q^*_{1_{pp}}$	**6.4593**	**5.9441**	**5.8252**	**5.7584**	**5.7290**	**5.7279**	**5.7895**	**6.2032**	**6.5978**
$q^*_{2_{pp}}$	**1.1962**	**1.7483**	**1.9418**	**2.1067**	**2.2530**	**2.3866**	**2.6316**	**3.2086**	**3.5771**
$a^*_{1_{pp}}=a^*_{2_{pp}}$	**1.7943**	**1.7483**	**1.7476**	**1.7556**	**1.7702**	**1.7900**	**1.8421**	**2.0321**	**2.1860**
$\pi^*_{1_{pp}}$	**5.9365**	**6.9319**	**7.1826**	**7.3642**	**7.5042**	**7.6170**	**7.7909**	**8.0357**	**8.0535**
$\pi^*_{2_{pp}}$	**3.0405**	**4.5846**	**5.2974**	**5.9795**	**6.6427**	**7.2980**	**8.6219**	**12.3595**	**15.1850**
CS^*_{pp}	**28.5307**	**27.5075**	**27.3353**	**27.2910**	**27.3384**	**27.4548**	**27.8393**	**29.3632**	**30.5233**
T^*_{pp}	**7.2972**	**7.3353**	**7.4654**	**7.6438**	**7.8625**	**8.1168**	**8.7258**	**10.8668**	**12.6851**

Table 2: Equilibrium outcomes in the case of SW maximizing public firm

Variables	$A=10,\ c=0,\ d=\frac{1}{2}$									$A=10,\ c=0,\ \gamma=\frac{1}{2}$	
	$\gamma=\frac{9}{10}$	$\gamma=\frac{4}{5}$	$\gamma=\frac{3}{4}$	$\gamma=\frac{7}{10}$	$\gamma=\frac{13}{20}$	$\gamma=\frac{3}{5}$	$\gamma=\frac{1}{2}$	$\gamma=\frac{1}{4}$	$\gamma=\frac{1}{10}$	$d=1$	$d=\frac{3}{2}$
θ^*_s	**0.067**	**0.095**	**0.101**	**0.1033**	**0.1029**	**0.101**	**0.091**	**0.051**	**0.022**	**0.089**	**0.088**
θ_1	**0.375**	**0.430**	**0.452**	**0.470**	**0.485**	**0.498**	**0.517**	**0.525**	**0.495**	**0.490**	**0.494**
θ_2	**0.828**	**0.793**	**0.775**	**0.756**	**0.737**	**0.719**	**0.682**	**0.613**	**0.599**	**0.811**	**0.863**
ED^*_s	**4.188**	**4.515**	**4.700**	**4.898**	**5.108**	**5.331**	**5.820**	**7.369**	**8.614**	**4.452**	**3.500**
SW^*_s	**40.724**	**42.070**	**42.855**	**43.700**	**44.598**	**45.550**	**47.616**	**53.845**	**58.488**	**44.116**	**42.552**
t^*_s	**1.585**	**1.460**	**1.433**	**1.418**	**1.413**	**1.433**	**1.415**	**1.540**	**1.641**	**2.061**	**2.342**
q^*_{1s}	**6.338**	**5.835**	**5.726**	**5.669**	**5.648**	**5.655**	**5.729**	**6.164**	**6.566**	**5.304**	**5.114**
q^*_{2s}	**1.355**	**1.936**	**2.136**	**2.307**	**2.458**	**2.596**	**2.851**	**3.460**	**3.851**	**2.643**	**2.550**
a^*_{1s}	**2.016**	**2.062**	**2.094**	**2.131**	**2.173**	**2.219**	**2.323**	**2.654**	**2.907**	**2.902**	**3.162**
a^*_{2s}	**1.585**	**1.460**	**1.433**	**1.418**	**1.413**	**1.415**	**1.433**	**1.540**	**1.641**	**2.061**	**2.342**
π^*_{1s}	**6.595**	**7.633**	**7.903**	**8.111**	**8.285**	**8.440**	**8.722**	**9.389**	**9.788**	**8.734**	**8.894**

π^*_{2s}	**3.093**	**4.815**	**5.591**	**6.327**	**7.039**	**7.741**	**9.157**	**13.154**	**16.179**	**9.111**	**9.247**
CS^*_s	**28.737**	**27.934**	**27.850**	**27.882**	**27.995**	**28.168**	**28.645**	**30.310**	**31.504**	**24.572**	**22.851**
T^*_s	**6.487**	**6.203**	**6.212**	**6.278**	**6.387**	**6.532**	**6.912**	**8.360**	**9.631**	**6.152**	**5.060**

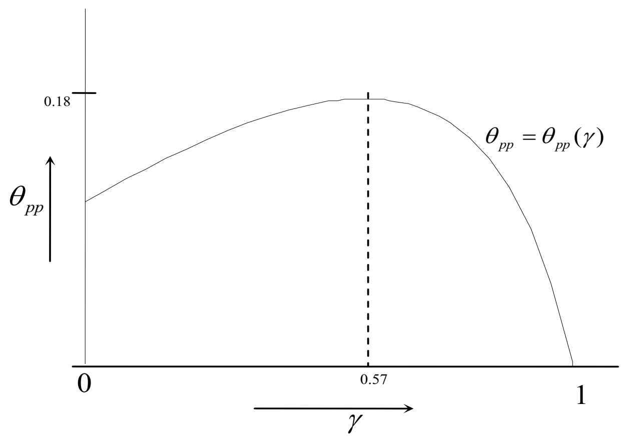

Figure 1: Product substitutability and optimal privatization in the first scenario

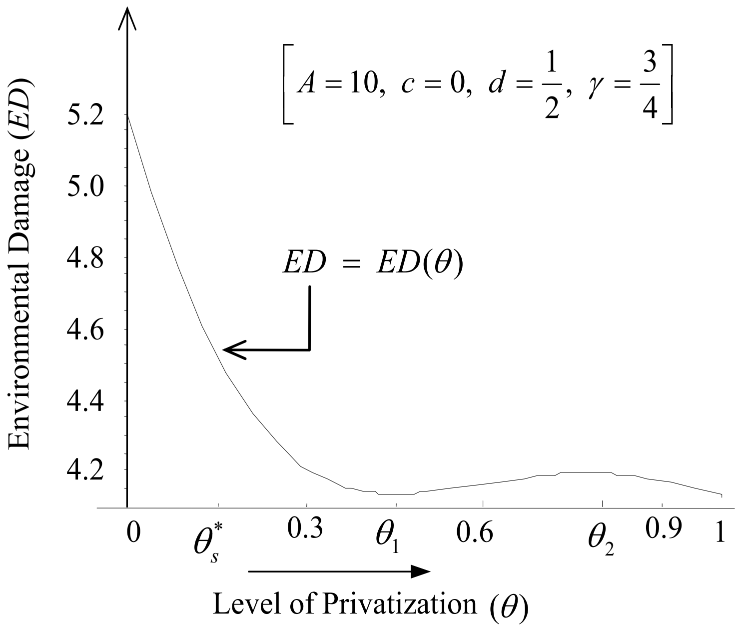

Figure 2: Privatization and environmental damage

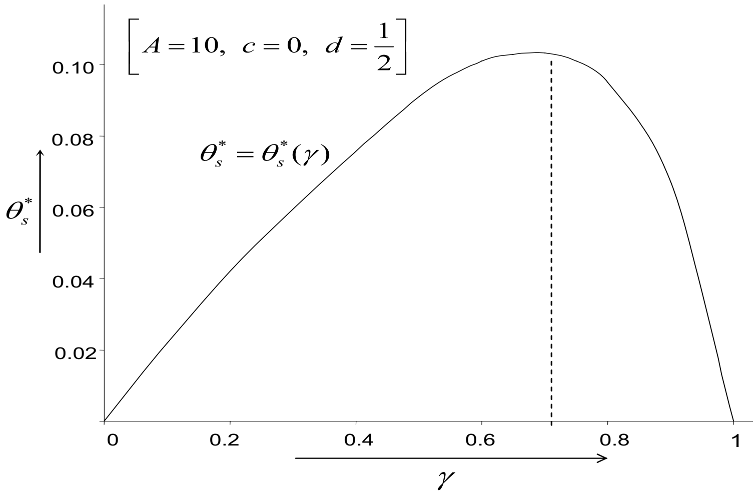

Figure 3: Product substitutability and optimal privatization in the second scenario