

Predicting Oil price Movements: A Dynamic Artificial Neural Network Approach

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Abstract

Price of oil is important for the economies of oil exporting and oil importing countries alike. Therefore, insight into likely future behaviour and patterns of oil prices can improve economic planning and reduce the impacts of oil market fluctuations. This paper aims to improve the application of Artificial Neural Network (ANN) techniques to prediction of oil price. We develop a dynamic Nonlinear Auto Regressive model with exogenous input (NARX) as a form of ANN to account for the time factor. We estimate the model using macroeconomic data from OECD countries. In order to compare the results, we also develop a time series and ANN static. We also use the output of time series model to develop NARX model. The NARX model is trained with historical data from 1974 to 2004 and results are then verified with data from 2005 to 2009. The results show that NARX model is more accurate than time series and static ANN models in predicting oil prices in general as well as in predicting the occurrence of oil price shocks.

Keywords: Oil price forecasting; Time series model; NARX model.

JEL Classification: C45, C53, C61, C63, C67.

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1. Introduction

Since 1970s, the oil markets have been subject to strong periodic fluctuations and shocks. Oil price, as a globally traded commodity, is sensitive to changes in economic conditions and political events(Adrangi et al., 2001; Panas and Ninni, 2000). Oil prices also affect the economic prosperity of both oil exporting and oil importing countries. In addition, price of oil, directly and indirectly, impacts various markets including those of other energy carriers. Hence, a better understanding of the likely future behaviour of oil prices can reduce vulnerability of the economy from fluctuations and changing conditions in the oil market.

However, the inherent difficulty to predict the oil price shocks¹ is a major challenge and is reflected in the diversity of the previous studies on the subject. Literature has used several approaches to predicting oil price (Section 2). These have led to different price predictions and levels of accuracy. More precisely, due to the complex interactions between economic and other factors which affect oil price, the traditional approaches for prediction of oil prices have exhibited some shortcomings (Mirirani and Li, 2004; Tang and Hammoudeh, 2002).

The present study aims to improve the modelling and accuracy of predictions of oil prices and shocks. We address this issue mainly through using a time factor, which enables the models to be dynamic and better predict the prices and price shocks. We develop a dynamic Artificial Neural Network (ANN) approach known as Nonlinear Auto Regressive model with eXogenous input (NARX). To our knowledge, the present

¹ Sudden fluctuations in oil price as a result of factors such as political crisis, disturbance in the oil supply, and unilateral decisions by oil exporters (see Appendix 2).

study is one of the few to use the Mackinnon-White-Davison (MWD) test to analyse and compare different models of oil price prediction. The model is optimised by identifying dummy variables which help the inclusion of qualitative factors² and time delays. Additionally, we use a three-step approach (time series, ANN static and NARX) that allows validating the results and assessing the improvement in the accuracy of the model after each stage. We show that the application of the NARX model enhances the dynamic performance of the model and improves the ability of the ANN methodology to predict oil price and in particular the occurrence of price shocks.

The next section provides a brief overview of the previous methods and studies for predicting the price of oil. Section 3 describes the general aspects of the methodology and the data used in this paper. Section 4 describes time Series, ANN and NARX models developed in this study and presents and compares the results obtained from them. Section 5 is the conclusions.

2. Previous Studies

Previous studies of oil price prediction have used a range of different approaches and techniques. Broadly, these approaches can be classified into: (i) Auto-Regressive Conditional Heteroskedasticity (ARCH), (ii) simulation, (iii) value at risk, and (iv) mathematical modelling. Table 1 summarizes a selection of these studies. As shown in the table, these have used different techniques and time spans and have achieved differing results and degrees of accuracy, thus leaving scope for further improvements.

In order to mitigate such deficiency, one can use dynamic models to account for time dependency oil price (Movagharnejad et al., 2011). ANN is a suitable technique for

² In this study, we consider the supply-side factors which affect oil price as qualitative factors. For more details see Section 4.1.2.

such a purpose (Kermanshahi, 1998) and has been applied to modelling and forecasting of the behaviour of nonlinear economic variables. For example, (Nakamura, 2005) has employed a Multi-Layer Perceptron (MLP) method for forecasting inflation and (Zhang and Qi, 2005) explore applicability of neural networks to forecast seasonal time series with a trend component.

To our knowledge, the literature on the application of the ANN method for forecasting the oil price is rather limited. Ghaffari and Zare (2009) forecast the West Texas Intermediate (WTI) crude oil spot prices using a combination of ANNs and Fuzzy Logic. Movagharnejad et al. (2011) used ANN and a time variable as a constant variable; thus the dynamic nature of the process was not accounted for. In order to account for the time dependency of the variables Jammazi and Aloui (2012) applied mathematical models while Yu et al. (2008) used short periods of time for modelling.

Table 1: Previous studies and methods of oil price predictions

Forecasting Method	Approach and Findings	Index of Accuracy	Time Span (years)	Study
Auto Regressive Conditional Heteroskedasticity(ARCH) ^a	Analysis of uncertainties in the oil price	$R^2 < 0.7$	4-6	(Day and Lewis, 1993; Duffie and Gray., 1996; Kang et al., 2009; Xu and Taylor, 1995).
Simulation ^b	Monetary factors such as GDP and import/export rates are the determining factors which affects the oil price.	$0.82 < R^2 < 0.91$	2-30	(Barsky and Kilian, 2001; Bernanke et al., 1997; Finn, 2000; He et al., 2012; Kim and Loungani, 1992; Obstfeld and Rogoff, 1995; Rotemberg and Woodford, 1996; Shin et al., 2012).
Value at Risk ^c	Mont Carlo simulation is used in combination with historical trends of factors such as currency value, oil supply, OECD oil demand etc. This approach seeks to identify factors with highest impact on price of oil.	$R^2=0.95$	43	(Amano, 1987; Busch and Raschky, 2004; Jorion, 1999; Wahrenburg, 1995).
Mathematical Modelling ^d	Different Mathematical Modelling Approaches	$0.87 < R^2 < 0.9$ MAE ^e =12.04% RMSE ^f =8.513	1-22	(Mirirani and Li, 2004; Tang and Hammoudeh, 2002).

^aARCH uses Ordinary Least Squares (OLS) technique and assumes that the errors' variances are constant; this technique is widely applied for predicting oil price.

^bSimulation is based on specific time models and therefore shows static behaviour. It cannot be applied for different time spans.

^cValue at Risk (VAT) operates based on value, risk and reliability of the predictions and results of other models.

^dThese models predict the results based on the price change patterns using pure mathematical theories.

^eMean Absolute Error.

^fRoot Mean Square Error.

3. Methodology and Data

The methodology used in this paper consists of three distinct but complementary stages namely: time series, ANN static, and ANN dynamic (NARX). While each stage (method) could be used to obtain some results (i.e. oil price prediction), applying the chain analysis (to improve the results of previous stage) makes it possible to increase the overall accuracy of the analysis. Details of the procedure applied in this paper are as follows:

- Stage 1: Time series: A time series model is used to identify the meaningful factors affecting oil price and to calculate the number of lags of independent and dependent variables (inputs for ANN static and NARX). The time series model itself will be further developed to obtain the final results (time series oil price prediction).
- Stage 2: ANN static: In order to validate the applicability of the result of the time series (inputs for the NARX model) we develop an ANN static model to verify the data and to prevent possible errors in the NARX model. The static ANN model is developed following the methodology described in (Movagharnejad et al., 2011) and based on the results of time series analysis in Stage 1 (i.e. the factors with the biggest impacts on the oil price). The results of this stage are comparable to those previously reported in (Movagharnejad et al., 2011).

- Stage 3: Using the time series results (i.e. main factors affecting oil price and the number of lags), the NARX model is used to include the factor of time in the analysis.

In each of the stages above, the R-squared was compared to the previous stage to ensure improvement in the accuracy of the results. A description of alternative methodologies is presented in the following subsections. A detailed application of these methods for predicting the oil prices is discussed in Section 4.

3.1. Time Series(TS)

A time series is a stretch of values (observations on the values) that a variable takes at successive points in time. Times series data is usually spaced at uniform time intervals (Brillinger, 2001; Greene, 2003; Gujarati and Madsen, 1998). Time series forecasts the future based on past data. In other words, time series analysis models use previously observed values in a trend to predict the future values (Greene, 2003). A critical step in the time series modelling is to verify the credibility of variables that are considered in the analysis and discover the relations between them. In order to do this, we used Auto Regressive Moving Average (ARMA) and Auto Regressive Integrated Moving Average (ARIMA) approaches. This will improve the accuracy of prediction thorough (1) identifying the most relevant variables and the most accurate models (2) optimising and estimating the selected model and (3) improving the model performance (e.g. through identifying the interconnections between variables, including dummy variables etc.) (Gujarati and Madsen, 1998).

3.2. Artificial Neural Network (ANN)

ANN imitates the learning process in human brain. The fundamental processing element of a neural network is a neuron. A biological neuron receives inputs from external sources, combines them with a nonlinear operation and then produces the final results. The network usually consists of an input layer, some hidden layers, and an output layer (Kalogirou, 2000). These types of networks are generally known as Multi-Layer Perceptron (MLP) neural networks.

An important step in the neural network is to train the model to learn the relationship between input and output parameters (i.e. the interconnecting weights between neurons). In MLP, weights are determined by Error Back-Propagation (EBP) algorithms which minimize a quadratic cost function by a gradient descent method. The interconnecting weights between the neurons are adjusted based on the inputs and desired output during the training phase (Borouhaki et al., 2003). Figure 1 illustrates the main features of an MLP network.

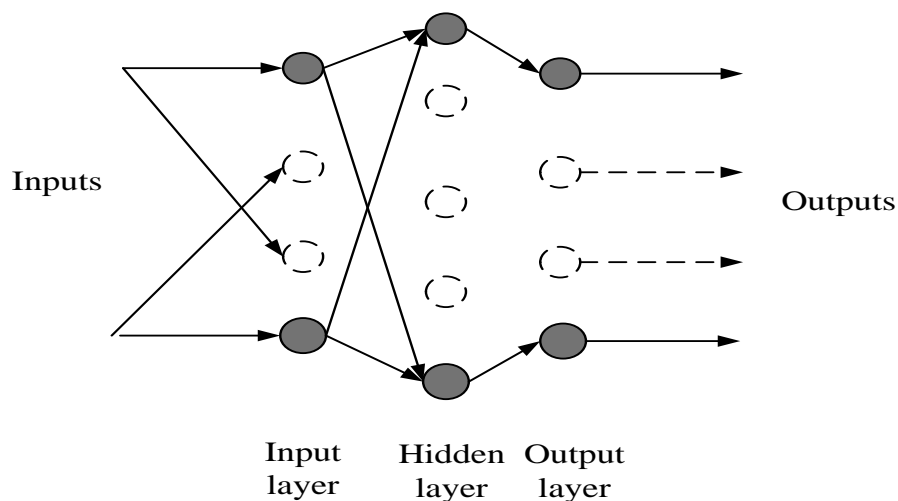


Figure 1: Multi-Layer Perceptron (MLP) Neural Network

Source: (Borouhaki et al., 2003)

At the initial step, the inputs are inserted in the MLP network and propagated forward in order to determine the resulting signal at the output neurons. Desired output targets are actual outputs; and ANN tries to eliminate the difference between them and the computed outputs (Boroushaki et al., 2003). The difference between the computed output vectors and the desired output represents an error that is back propagated through the network in order to adjust the weights. This process is then repeated and the learning continues until the desired degree of accuracy is achieved (Haykin, 1999).

3.3. Nonlinear Auto Regressive Model with eXogenous Input (NARX)

Nonlinear Auto Regressive model with eXogenous input (NARX) is a specific form of ANN which is dynamic and considers the factor of time. The dynamic part (i.e. the signal vector applied to the input layer of the MLP) contains the past and present inputs. These represent the exogenous as well as the model generated outputs on which the model is regressed. The dynamic behaviour of the NARX model is described by Equation 1.

$$\mathbf{y}(n + 1) = F(\mathbf{y}(n), \dots, \mathbf{y}(n - q + 1), \mathbf{u}(n), \dots, \mathbf{u}(n - p + 1)) \quad \text{Equation 1}$$

Where F represents a nonlinear function of its constituent arguments and “ n ” is the time factor which denotes the present value of the model input (i.e. $u(n)$) and the future value of the model output (i.e. $y(n+1)$).” (Boroushaki et al., 2003).

In the present study, the training of the NARX model is carried out by batch learning method in which the entire plant data sets $(1, \dots, T)$, during a transient are then used for

learning, until the total transient output error reaches a certain value³ ($\sum_T E(n, t)$) where t denotes the number of entire data sets in a transient.⁴

3.4.Data

Both supply- and demand-side factors will affect the market price of oil. Considering the interactions between economic growth, energy demand, and oil price, we use macroeconomic indices of OECD countries (as the largest importers of oil) as inputs for the models.

Table 2 summarizes the demand-side factors with potential impacts on oil price. The applied macroeconomic variables such as Gross Domestic Product (GDP) and Final Consumption Expenditure (FCE) directly or indirectly cover indexes such as population, number of cars, development of energy sector etc. In subsequent stages, depending on the effects of these variables on the oil price, some variables will be excluded from the models.

In order to include the impact of supply side factors (e.g., political crisis, disturbance in the oil supply, and unilateral decisions on the amount of oil export, etc.) on oil price dummy variables are included in the analysis (see Section 4.1).

³This value determines the maximum accepted error value of the NARX and was selected to be 10^{-3} .

⁴Each time period or transient is concerned to implementing a set of training data to the neural network between years 1974 to 2004.

Table 2: Variables used in the estimated models

Variable	Abbr.	Description	Min. (during the time period of study)	Max. (during the time period of study)	Unit	Reference
Gross Domestic Product	GDP	Sum of gross value added by all resident producers in the economy plus any product taxes minus any subsidies not included in the value of the products.	4.04E12	4.38E13	US\$	(WDI, 2007)
GDP Growth	GG	Annual percentage growth rate of GDP at market prices based on constant local currency. Aggregates are based on constant 2000 \$US.	-4.04	6.32	% ^a	(WDI, 2007)
Net Energy Import	NEI	NEI is considered in both absolute (kilo tons of oil equivalent) and relative (% of energy use) forms.	20.96	34.5	% ^a	(WDI, 2007)
Final Consumption Expenditure	FCE	FCE is the annual change in the sum of household final consumption expenditure and general government final consumption expenditure. FCE includes any statistical discrepancy in the use of resources relative to the supply of resources and is proportional to the oil price.	-0.723	6.576	% ^a	(WDI, 2007)
Gold Price	GP	Gold price is used to avoid inconsistencies caused by minor economic crises in the model.	124.74	972.35	US\$	(Kitco, 1995; NMA, 2011)
Energy Production	EP	EP accounts for different forms of primary energy (i.e. petroleum, natural gas, solid fuels and combustible renewable and waste) as well as primary electricity.	2.44	3.87	mill. kt. of oil equivalent	(IEA, 2011)
Energy Use	EU	EU refers to primary energy prior to transformation to other end-use products. EU equals to indigenous production plus imports and stock changes, minus exports and fuels supplied to ships and aircraft engaged in international transport.	3.63	5.55	mill. kt. of oil equivalent	(IEA, 2011)
Oil Rent	OIR	OR is the difference between the value of crude oil in international markets and the total costs of production. OR is estimated based on sources and methods described in (Day and Lewis, 1993).	42112.88	415634.96	mill. US\$	(Day and Lewis, 1993)

^a Annual growth (%).

4. Model Development and Results

4.1. Time Series Model

The parameters introduced in Table 2 are used as initial inputs for modelling. As the first step, we develop four models: Linear-Linear (Lin-Lin), Linear-Logarithm (Lin-Log), Logarithm-Linear (Log-Lin) and Logarithm-Logarithm (Log-Log). A time series model is used to (i) identify the variables with highest impact on oil price and (ii) optimise the model. The models' output and the results are presented in Appendix (1-a). It should be noted that at this stage, the results (Appendix 1-a) are not yet optimized and the optimization will be undertaken when the most accurate model is selected (Section 4.1.1).

4.1.1 Model Selection - Using Primary Input Variables

In order to compare the models with linear and logarithmic outputs and in order to choose the most accurate model, a two steps comparison methodology is applied:

- i) R-squared: Is used to compare models with similar outputs (i.e. linear output or logarithmic output). When comparing two models, the larger the R-squared is the more accurate is the model. As shown in Appendix (1-a), R-squared for Lin-Lin, Lin-Log, Log-Lin and Log-Log is 0.9478, 0.7447, 0.9031, and 0.9204 respectively. Therefore, Lin-Lin is chosen when comparing Lin-Log and Lin-Lin. Similarly, Log-Log is identified as the most accurate model between Log-Log and Log-Lin.
- ii) MWD test: In order to compare Lin-Lin and Log-Log models, MDW test is applied. For this we used H_0 and H_1 theories which indicates that Lin-Lin and

Log-Log as the better models respectively. The MWD test consists of the following steps:

- Estimation of Lin-Lin model and the values of Crude Oil price (COP)
- Estimation of Log-Log model and the values of Log (COP)
- Calculating MLN⁵: $MLN = \log(\widehat{COP}) - \log(\widehat{COP})$
- Calculating MLG⁶: $MLG = \text{Anti log}(\widehat{COP}) - \widehat{COP}$
- Estimating COP using MLN; if the t-Statistic of MLN coefficient is less than 0.05 (i.e. probability < 0.05) then H₀ theory is not valid.
- Estimating Log (COP) using MLG; if the t-Statistic of MLG coefficient is less than 0.05 (i.e. probability < 0.05) then H₁ is not valid.

As shown in Appendix (1-b) for Lin-Lin model, the probability of the MLN is smaller than 5%. Therefore, the Lin-Lin estimation is not meaningful. Conversely, in the Log-Log model, since the probability of MLG is more than 5%, the Log-Log estimation is found to be meaningful. Therefore, Log-Log is chosen as the optimum model for time series modelling. In the next stage, we optimise the results of the Log-Log model.

4.1.2 Optimization of the Log-Log Model

In order to improve the accuracy of the chosen model (Log-Log), the input variables with negligible impacts will be identified and excluded from the analysis. In addition, dummy variables are used in order to account for the supply-side factors and actions of

⁵ MLN is the verification factor for the model with linear outputs.

⁶ MLG is the verification factor for the model with logarithmic outputs.

oil suppliers on oil prices. Initially, for each and all of the years in the time period of the study, a dummy variable is included in the Log-Log model. In other words, at the first step, it was considered that the effects of dummy variables exist in every year. In the next step, based on the t-statistic the most non-relevant (meaningless) dummy variables were omitted from the analysis and only the variables with the probability of less than 0.05 remained in the model. The results indicate that only dummy variables for years 1978, 1982, and 1985 are meaningful and therefore remain in the model. The step-by-step analysis is provided in Appendix 4.

These results are compatible with the historical data which show fluctuations in oil price in the same years (see Appendix 2). Therefore, we modify the model to incorporate these dummies as shown in Equation 2.

$Log(COP)$

$$= C + c_2Log(GP1) + c_3Log(EP1) + c_4Log(EU1) + c_5Log(OIR1) + c_6Log(ORP1) + c_7Log(G1) \\ + c_8Log(GG1) + c_9Log(EI1) + c_{10}Log(EIP1) + c_{11}Log(FC1) + c_{12}V78 + c_{13}V82 + c_{14}V85$$

Equation 2

In Equation (2), V78, V82 and V85 are dummy variables for years 1978, 1982, and 1985 respectively. In addition, as suggested by the preliminary results and due to high probability, “GDP” and “Net Energy Import” (NEI) are excluded from the input factors as shown in Equation (3). Credibility of this assumption is justified by reviewing the initial input variables. More precisely, given that the NEI index is equal to the difference between Energy Use and Energy Production, the effects of NEI are implicitly reflected in the analysis. Similarly for GDP, simultaneous consideration of factors such as Energy Use, Final Consumption Expenditure and GDP Growth will cover those

aspects of GDP that potentially impact oil prices. The estimation results of the model are shown Table 3.

$\text{Log}(COP)$

$$= C + c_1 \text{Log}(GP1) + c_2 \text{Log}(EP1) + c_3 \text{Log}(EU1) + c_4 \text{Log}(OILR1) + c_5 \text{Log}(ORP1) \\ + c_6 \text{Log}(CG1) + c_7 \text{Log}(EIP1) + c_8 \text{Log}(FC1) + c_9 V78 + c_{10} V78 + c_{11} V85$$

Equation 3

Table 3: Results - Optimized Log-Log Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-9.596844	9.852510	-0.974051	0.3384
LGP1	0.805901	0.144913	5.561296	0.0000
LEP1	-9.652807	1.523072	-6.337721	0.0000
LEU1	9.613340	1.092572	8.798816	0.0000
LOILR1	0.176623	0.087535	2.017740	0.0533
LGG1	-0.165291	0.076320	-2.165749	0.0390
LFC1	0.554790	0.155645	3.564447	0.0013
V78	0.520849	0.189832	2.743736	0.0105
V82	0.872693	0.197329	4.422532	0.0001
V85	-0.483639	0.166255	-2.909022	0.0070
R-squared	0.960691	Mean dependent var	3.081422	
Adjusted R-squared	0.948056	S.D. dependent var	0.718824	
S.E. of regression	0.163828	Akaike info criterion	-0.559063	
Sum squared resid	0.751511	Schwarz criterion	-0.128119	
Log likelihood	20.62219	F-statistic	76.03452	
Durbin-Watson stat	2.244913	Prob(F-statistic)	0.000000	

4.1.3 Autoregressive Integrated Moving Average (ARIMA) Model

Stochastic processes are powerful tools for analysing the interactions between different variables. These can be represented by time series models such as Auto-Regressive (AR) models, Integrated (I) models, and Moving Average (MA) models. Combinations of these processes produce Auto-Regressive Moving Average (ARMA) and Auto-Regressive Integrated Moving Average (ARIMA) models. The ARIMA model is used to analyse self-dependency and interdependency of variables. We use the data for the

period 1974-2008 in the selected model (i.e. Log-Log model) and then test the model against the ARIMA. The results show zero interrelations for the ARI and 2 interrelations of the MA model (Equation 4).

$Log(COP)$

$$= C + c_1 Log(GP1) + c_2 Log(EP1) + c_3 Log(EU1) + c_4 Log(OLR1) + c_5 Log(ORP1) + c_6 Log(GG1) + c_7 Log(EIP1) + c_8 Log(FC1) + c_9 V78 + c_{10} V82 + c_{11} V85 + [MA(2) = c_{12}, BACKCAST = 1974]$$

Equation 4

As mentioned, the starting year of the analysis is 1974 whereas the first data used in the modelling is 1972. This means that we have two years of delay and the backcast parameter is inserted in Equation 4 to account for this. Table 4 presents the estimated coefficients and results for the ARIMA model in Equation 4. This test uses two time lags for the oil price as a result of price shocks. Note that the shocks represent the value of the variables in each year and the previous year (i.e. COP (t, t-1)).

Table 4: Result of ARIMA test in Log-Log model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-9.599679	9.320089	-1.029999	0.3121
LGP1	0.808139	0.150905	5.355270	0.0000
LEP1	-9.652997	1.557649	-6.197158	0.0000
LEU1	9.613194	1.140825	8.426530	0.0000
LOILR1	0.176683	0.090320	1.956186	0.0609
LGG1	-0.166765	0.079207	-2.105422	0.0447
LFC1	0.554926	0.154254	3.597472	0.0013
V78	0.505121	0.195011	2.590224	0.0153
V82	0.892030	0.205952	4.331249	0.0002
V85	-0.492409	0.164913	-2.985878	0.0059
MA(2)	-0.191796	0.011817	-16.23033	0.0000
R-squared	0.962227	Mean dependent var	3.081422	
Adjusted R-squared	0.948237	S.D. dependent var	0.718824	
S.E. of regression	0.163542	Akaike info criterion	-0.546290	
Sum squared resid	0.722146	Schwarz criterion	-0.072252	
Log likelihood	21.37951	F-statistic	68.78019	
Durbin-Watson stat	2.309223	Prob(F-statistic)	0.000000	
Inverted MA Roots	.44	-.44		

4.1.4 Independent Variables Lags

Having established the relations between the dependent and independent variables, we use Interdependent Variable Lags (IVL) to identify the interdependency of independent variable lags on the dependent variable. A lag of 3 units is used for each independent variable used in the previous section. Next, we exclude variables with probabilities deviating largely from 0.5%. Then, we examine new data in the model (using the same procedure) and the next lag is excluded. In other words, in order to identify the interdependencies between the variables, we exclude one lag at a time. The estimated model is shown in Equation (5) and Table 5 shows the estimation for the Log-Log model.

$$\begin{aligned} \text{Log}(COP) = & C + c_1 \text{Log}(GP(1)) + c_2 \text{Log}(EP(1)) + c_3 \text{Log}(EP(-2)) + c_4 \text{Log}(EU(1)) + \\ & c_5 \text{Log}(OILR(1)) + c_6 \text{Log}(OILR(-1)) + c_7 \text{Log}(GG(-1)) + c_8 \text{Log}(FC1) + c_9 V78 + c_{10} V82 + \\ & c_{11} V85 + [MA(2) = c_{13}, BACKCAST = 1974] \end{aligned} \quad \text{Equation 5}$$

Table 5: Result - Number of Lags in Log-Log Model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-3.190548	5.346005	-0.596810	0.5565
LGP1	0.405229	0.115146	3.519264	0.0018
LEP1	-14.26699	1.557940	-9.157599	0.0000
LEP1(-2)	5.709769	1.470018	3.884149	0.0007
LEU1	7.854597	0.829963	9.463790	0.0000
LOILR1	0.217904	0.051963	4.193450	0.0003
LOILR1(-1)	0.219698	0.076914	2.856420	0.0089
LGG1(-1)	0.144494	0.032759	4.410850	0.0002
LFC1	0.256250	0.049747	5.151060	0.0000
V78	0.727590	0.143993	5.052948	0.0000
V82	0.615448	0.175614	3.504557	0.0019
V85	-0.412285	0.109819	-3.754223	0.0010
MA(2)	-0.948895	0.019937	-47.59421	0.0000
R-squared	0.977194	Mean dependent var		3.194302
Adjusted R-squared	0.965295	S.D. dependent var		0.544130
S.E. of regression	0.101367	Akaike info criterion		-1.465943
Sum squared resid	0.236331	Schwarz criterion		-0.894116
Log likelihood	39.38697	F-statistic		82.12609
Durbin-Watson stat	2.176109	Prob(F-statistic)		0.000000
Inverted MA Roots	.97	-.97		

In order to evaluate the importance of the constant term “C” of the final time series model, we applied adjusted R-squared to compare Equation (5) with and without C. Given that the adjusted R-squared is higher for the case without C, we exclude the constant parameter and the modified model is shown in Equation (6).

$$\begin{aligned} \text{Log}(COP) = & c_1 \text{Log}(GP(1)) + c_2 \text{Log}(EP(1)) + c_3 \text{Log}(EP(-2)) + c_4 \text{Log}(EU(1)) + \\ & c_5 \text{Log}(OILR(1)) + c_6 \text{Log}(OILR(-1)) + c_7 \text{Log}(GG(-1)) + c_8 \text{Log}(FC1) + c_9 V78 + c_{10} V82 + \\ & c_{11} V85 + [MA(2) = c_{13}, BACKCAST = 1974] \end{aligned} \quad \text{Equation 6}$$

Using independent input data for the time period between 1974 and 2008 in Equation 6, time series model predicts the oil price. Figure 2 shows the estimated prices against the historical prices. As shown in the figure, the estimated prices match the actual prices with high accuracy both when they move slowly as well as when they exhibit shocks and sharp changes.

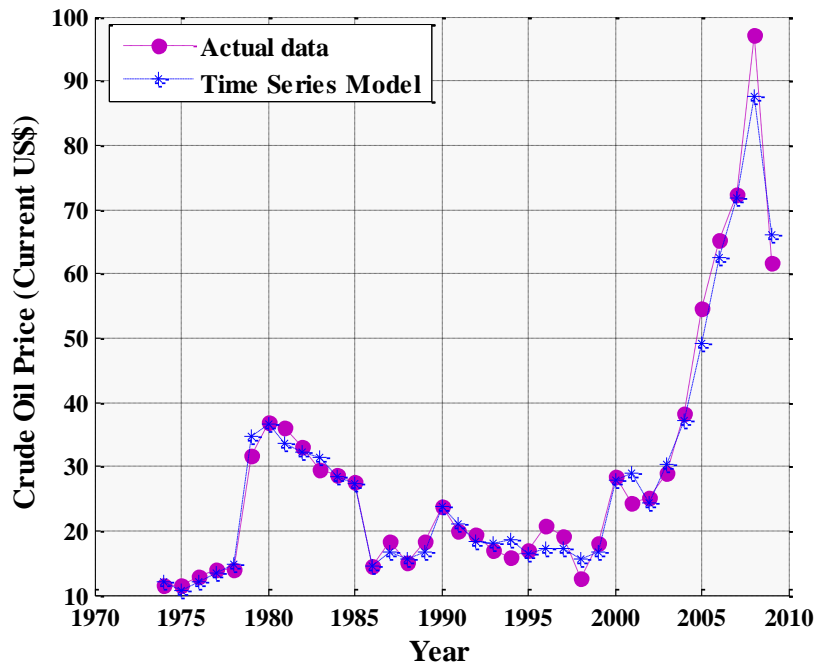


Figure 2: Oil price - Actual data vs. time series results

4.2. The Nonlinear Auto Regressive Model with Exogenous Input (NARX)

In order to develop the NARX model we use feedbacks from time series. Before using these feedbacks, they are verified in ANN static model (see Sections 3 and 4.3). The algorithm is shown in Figure 3 where COP, GP, EP, GG, EU, OIR, FC, V, p_i , and q denote Crude Oil price, Gold Price, Energy Production, GDP Growth, Energy Use, Oil Rent, Final Consumption expenditure, dummy Variable (effects of supply side factors), the number of lags of input i and the number of lags of output (crude oil price) respectively. Figure 3 schematically shows how input factors are inserted in the NARX model and in the delaying factors (shown by Z^{-1} in the Figure). The dynamic behaviour of the NARX network in Figure 3 is presented in Equation 7.

$$\begin{aligned}
 &COP(t + 1) \\
 &= f[COP(t), \dots, COP(t - q), GP(t), \dots, GP(t - p_1), EP(t), \dots, EP(t - p_2), GG(t), \dots, GG(t \\
 &\quad - p_3), EU(t), \dots, EU(t - p_4), OIR(t), \dots, OIR(t - p_5), FC(t), \dots, FC(t - p_6), V, t]
 \end{aligned}$$

Equation 7

For the analysis, we classify the historical data in two categories. More precisely, we use the data for the 1974-2004 period to train the network. In the next instance, the data for the 2005-2009 are used for testing the model (see Appendix 3). In order to obtain more accurate results while reducing the required computing time, we use Equations 8 and 9 to normalize all the input and output data in [-10, 10] and [-1, 1] intervals respectively (NMA, 2011).

$$P_{scal} = \frac{P_{old} - P_0}{P_0 - P_{min}} * \alpha$$

Equation 8

$$P_0 = \frac{P_{max} + P_{min}}{2}$$

Equation 9

Where P_{old} and P_{scal} denote oil price before and after normalization respectively. P_{min} and P_{max} represent the minimum and maximum of the parameters respectively and “a” is a binary parameter which takes a value between 1 and 10.

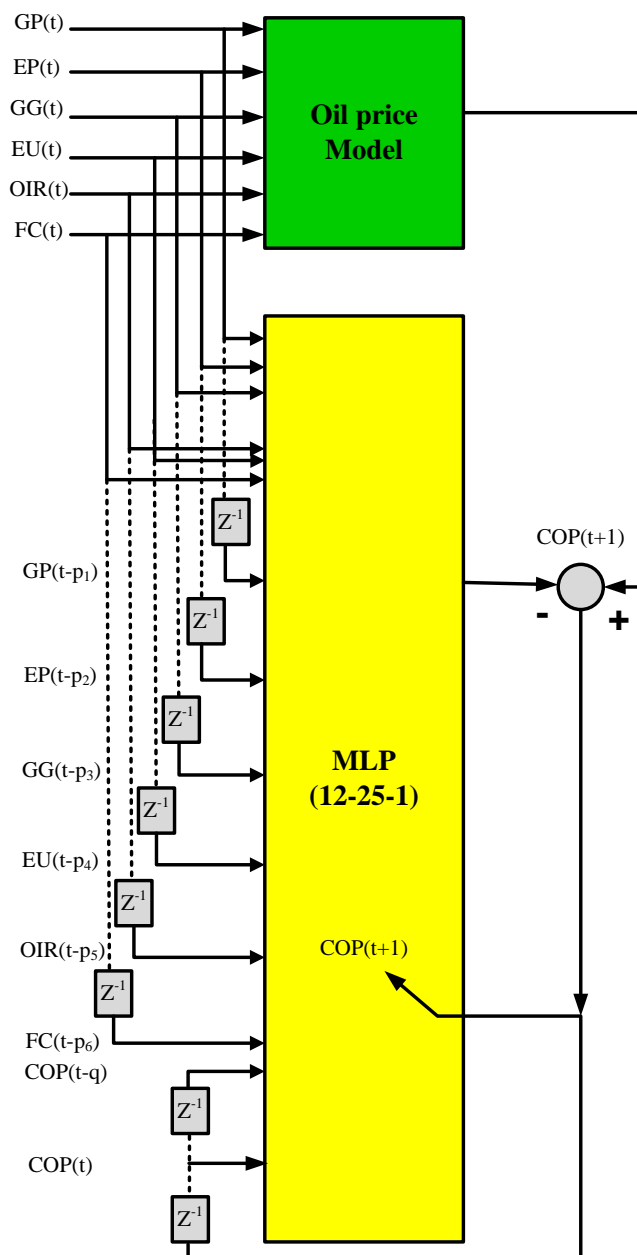


Figure 3: Schematic structure of the NARX model

In this exercise GP, EP, GG, EU, OIR and FC variables were used in the NARX model after 0, 2, 1, 0, 1, and 0 lags respectively; and the oil price in the output is inserted as the feedback to input after 1 lag. The lags for the NARX model are chosen based on the final equation of time series model (i.e. Equation 6), which is one of the unique characteristics of this study as discussed in Section 3. Equation (10) shows the dynamic behaviour of the model.

$$\begin{aligned} COP(t+1) \\ = f[COP(t), COP(t-1), GP(t), EP(t), EP(t-2), GG(t-1), EU(t), OIR(t-1), FC(t), V, t] \end{aligned}$$

Equation 10

Using a small number of hidden neurons results in inaccuracy of the correlation between inputs and outputs, whereas an increase in the number of neurons in hidden layer will saturate the neural network which could result in local optimums (rather than the global optimum). In this case increase in the number of epochs will not necessarily decrease $\sum_k (d_k - O_k)^2$. In other words, the run time of the programme increases and the final result will not necessarily change. Therefore, the number of neurons selected should reflect this trade-off. Optimum number of hidden neurons are found by trial and error.

Figure 4 shows the total number of required epochs versus the number of hidden neurons to determine the data for training the NARX model, where in each epoch all inputs are applied to the ANN model. Variations of the required epoch versus number of hidden neurons are used as index for finding the optimum number of hidden neurons.

More precisely, the optimum value of hidden neurons is reached when the value of the index falls below 0.02%. As shown in the figure, the optimum number of hidden neurons in the first NARX model is reached at 25.

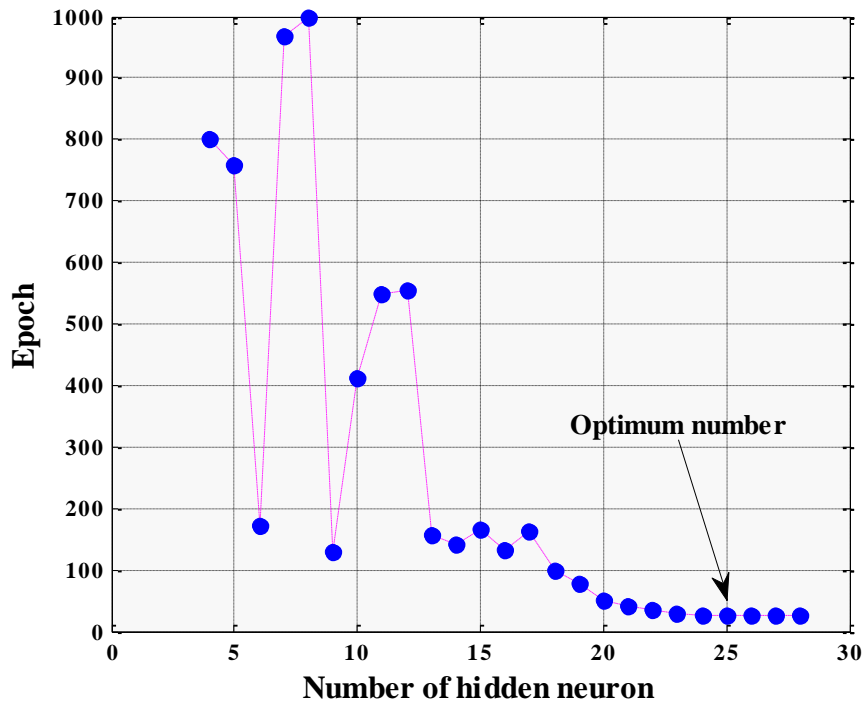


Figure 4: Value of epochs versus number of hidden neurons to train the data for first NARX model

Figure 5 shows the results of training, testing and forecasting phases in the NARX model. As shown in the figure, the estimated prices by the NARX model for the training period 1974-2004 closely match the observed prices. The model also estimates accurate prices for the testing period 2005-2009, which includes both a rather sharp rise as well as decline in oil prices. It predicts the marked oil price rise in 2008 and the subsequent sharp decline in 2009. In addition, the NARX model predicts an oil price of \$80/barrel

for 2010 (which is not part of the model testing period) while the actual market price in that year was \$80.5/barrel.

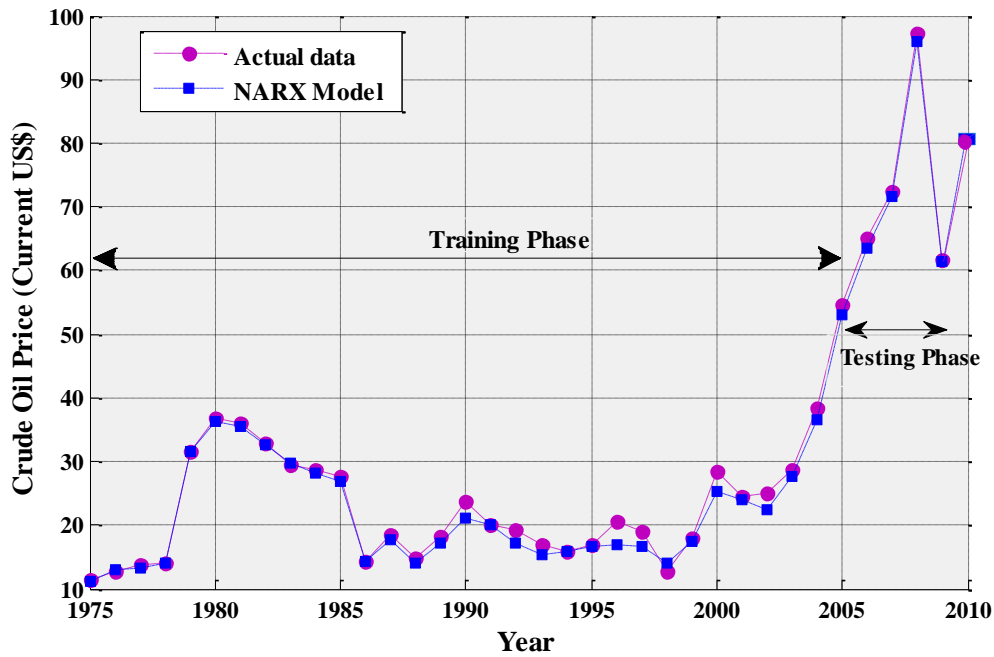


Figure 5: Comparison of NARX predicted oil price vs. Actual price

It is noteworthy that the 2005-2009 period includes both pre and post 2007 worldwide financial and economic crisis which led to a marked decline in economic output and thus the global demand for oil. Although the NARX model appears to produce rather accurate price predictions, we also compare and test the accuracy and performance of the model against those of other approaches.

4.3. Comparison with Static ANN and Time series Results

As mentioned in Section 2, ANN Static has been used in several studies and has resulted in relatively accurate results in predicting the oil price (Ghaffari and Zare, 2009; Jammazi and Aloui, 2012; Movagharnejad et al., 2011; Yu et al., 2008). We developed the

static ANN model following the methodology described in (Movagharnjad et al., 2011) and as presented in Equation (11).

We use a similar approach to that of the NARX model in order to determine the number of the neurons in the hidden layer; and this is calculated to be 15 for the static ANN model.

$$COP(t) = f[GP(t), EP(t), GG(t), EU(t), OIR(t), FC(t), t] \quad \text{Equation 11}$$

As Equation 11 suggests, there is no time lag between the input and the output and the time parameter is not considered in ANN Static. The results of the ANN static model are used as the base for comparison with the results of the NARX model. Moreover, the ANN static is used to verify the validity of inputs of the NARX model that were initially suggested in time series.

We use the Mean Absolute Error (MAE) and R-squared for comparing the results from time series, NARX, and ANN models (Table 6). Equation (12) shows the formula for calculating MAE. In Equation (12) N is the number of outputs obtained from each of the three models. A lower MAE value indicates more accurate results and is preferred to a high value.

$$MAE = Mean\left(\frac{|Pr_i(estimated) - Pr_i(actual)|}{|Pr_i(actual)|}\right) \Bigg|_{i=1 \text{ to } N} \quad \text{Equation 12}$$

Table 6: Comparison of accuracy of the different models

Model	Phase	MAE (%)	R ² (%)
NARX	Training	3.28	98
	Testing	4.96	97
Time series	-	6.47	96
ANN static	Training	6.5	90
	Testing	8	87

Table 6 compares the MAE and R-squared of the results obtained from the three models. As shown in the table, the results from the NARX model shows both the lowest MAE and the highest R-squared and is, therefore, by these measures more accurate than both ANN static and Time series. More precisely, the accuracy of the results from the NARX model is clearly higher than the ANN static model. This is because the NARX model takes into account the “time factor” in the estimations. In addition, the NARX model modifies the output from time series model and, therefore, improves its prediction accuracy.

5. Conclusions

The price of oil is important for the economies of oil-importing- as well as oil-exporting countries. . Therefore, insight into likely future behaviour and patterns of oil prices can improve economic planning and help reduce the impacts of oil price movements and sudden market fluctuations.

While the ANN-Static is a well-established methodology for predicting oil price (e.g., see (Ghaffari and Zare, 2009; Movagharnejad et al., 2011)the main purpose of the current study is to further improve the accuracy of ANN-Static by including the factor of time in the analysis. Therefore, we developed a NARX model in which the parameter of time is included by using the feedbacks from time series model.

We use a set of high-level key economic variables of OECD countries to develop a model for predicting oil prices. In order to assess and compare the accuracy of the NARX results, we also develop a time series model and an ANN static model. We use data for the 1974-2004 period to train the model. The training step was used to calculate

the optimized structure and the MAE of the model. NARX model shows the lowest MAE (3.28 and 4.96% in the training and testing phases respectively) and was, therefore, more accurate than those of time series and ANN static models (MAE values equal to 6.47 and 8% respectively). In other words, as indicated by the results, including the time lags in the analysis by simultaneous application of time series and NARX, has improved the accuracy of the predictions. For example, the NARX model predicts the oil price in 2010 to be \$80/barrel. The actual market price in 2010 was \$80.5/barrel, which represents an increase of \$18 in relation to the previous year.

As an advanced type of recurrent neural network, NARX is used for the first time in this study for oil price prediction. The present study has several advantages compared to the previous works. It is the first study to use the MWD test to develop a basic model for predicting the oil price. The model is optimized by identifying the dummy variables which helps to include qualitative factors such as political events and time delays. Moreover, in another innovative approach, we use the results of time series model in order to determine the time lags and optimise them. Real world data are used for the modelling purpose, and the prediction error of less than 5% (MAE) is obtained in the testing step. In addition, the model produces accurate predictions of the shocks in the oil market.

Results of the NARX model from this study are encouraging. Further studies are needed to determine whether such dynamic models consistently produce more accurate predictions than the alternative methods. Moreover, this approach can be used to predict the effect of oil price changes on the price of other energy carriers such those of coal and natural gas.

Nomenclature

C	Total consumption (US\$)
COP	Crude oil price (US\$)
d_k	Actual value of unit k
E	Error
EP	Energy production (kt of oil equivalent)
EU	Energy use (kt of oil equivalent)
FC	Final consumption expenditure (% Annual)
GDP	Gross domestic product (US\$)
GG	GDP growth (%)
GP	Gold price (US\$)
OIR	Oil rent (US\$)
O_i	Activation of unit i
p_i	Number of delay input i
q	Number of delay output unit
R^2	Adjusted R-squared
r	GDP growth (%)
RSME	Root square mean error
T	Number of entire data
T	Time
w_{ij}	Weight from unit j to unit i
y_i	Activation function of unit i

Subscripts

H	Hidden unit
I	Input unit
j	Hidden unit
k	Output unit

Appendix 1-a: Output of Initial Time series Models

	Lin	Log																																																																																																														
Lin	Lin-Lin Model: COP=C+c ₂ GP1+c ₃ EP1+c ₄ EU1+c ₅ OILR1+c ₆ ORP1+c ₇ G1+c ₈ GG1+c ₉ EI1+c ₁₀ EIP1+c ₁₁ FC1 Equation a	Lin-Log Model: COP=C+c ₂ Log(GP1)+c ₃ Log(EP1)+c ₄ Log(EU1)+c ₅ Log(OILR1)+c ₆ Log(ORP1)+c ₇ Log(G1)+c ₈ Log(GG1)+c ₉ Log(EI1)+c ₁₀ Log(EIP1)+c ₁₁ Log(FC1) Equation b																																																																																																														
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Note: The number “1” in EP1, GP1 etc., show the time lag between the input and output (oil price) variables which is considered to account for time dependency of variables.

Appendix 1-b: Results of MWD Test

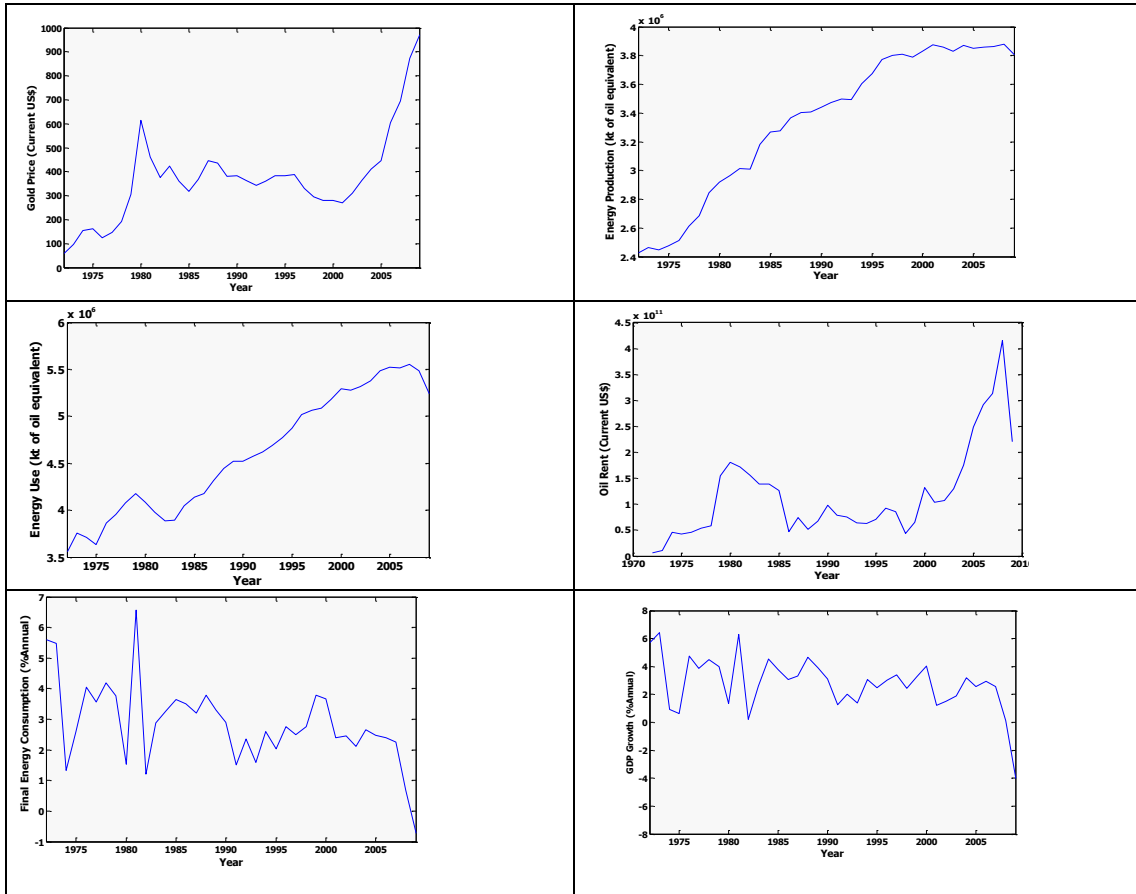
Log-Log model					Lin-Lin model				
Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-13.99515	23.17579	-0.603869	0.5508	C	968.9389	148.3565	6.531153	0.0000
LGP1	0.682410	0.228153	2.991019	0.0057	GP1	0.038018	0.013240	2.871358	0.0077
LEP1	-15.52261	9.769643	-1.588861	0.1233	EP1	-0.000853	0.000120	-7.116823	0.0000
LEU1	16.14495	9.765344	1.653290	0.1094	EU1	0.000612	8.49E-05	7.207601	0.0000
LG1	0.093539	0.554049	0.168829	0.8671	OILR1	-6.24E-11	3.56E-11	-1.754452	0.0903
LOILR1	0.350372	0.111029	3.155688	0.0038	G1	8.63E-13	4.11E-13	2.098831	0.0450
LGG1	-0.170611	0.111597	-1.528819	0.1375	GG1	1.004463	1.418551	0.708091	0.4847
LEI1	-4.042917	3.600306	-1.122937	0.2710	EI1	-33.85960	4.926072	-6.873549	0.0000
LFC1	0.517300	0.220788	2.342966	0.0265	FC1	-0.387600	1.926443	-0.201200	0.8420
MLG	-1.85E-06	9.67E-06	-0.191727	0.8493	MLN	-13.44937	4.941616	-2.721655	0.0110
R-squared	0.920523	Mean dependent var 3.081422			R-squared	0.951831	Mean dependent var 27.32158		
Adjusted R-squared	0.894977	S.D. dependent var 0.718824			Adjusted R-squared	0.936348	S.D. dependent var 19.63238		
		Akaike info criterion 0.144958					Akaike info criterion 6.258847		
S.E. of regression	0.232951	Schwarz criterion 0.575901			S.E. of regression	4.953121	Schwarz criterion 6.689791		
Sum squared resid	1.519454	F-statistic 36.03375			Sum squared resid	686.9354	F-statistic 61.47623		
Log likelihood	7.245804	Durbin-Watson stat 2.088678			Log likelihood	-108.9181	Durbin-Watson stat 2.038922		
Durbin-Watson stat	2.088678	Prob(F-statistic) 0.000000			Durbin-Watson stat	2.038922	Prob(F-statistic) 0.000000		

Appendix 2: Main geopolitical events affecting oil prices in 1978, 1982, and 1985

Source: (HIBPOP, 2011; Williams, 2011; World-Bank, 2011)

Year	Events
1978 V78	From 1974 to 1978, the world crude oil price was relatively flat ranging from \$12.52 to \$14.57 per barrel. When adjusted for inflation world oil prices were in a period of moderate decline. During that period OPEC capacity and production was relatively flat near 30 million barrels per day. In contrast, non-OPEC production increased from 25 million barrels per day to 31 million barrels per day. The resulting excess supply had reduced the prices.
1982 V82	The Iran-Iraq war had led to another round of crude oil price increases in 1979 and 1980. The Iranian revolution resulted in the loss of 2 to 2.5 million barrels of oil per day between November 1978 and June of 1979. In 1980 Iraq's and Iran's crude oil production fell 2.7 million and 600,000 barrels of oil per day respectively. The combination of these two events resulted in the increase in the crude oil prices from \$14 in 1978 to \$35 per barrel in 1981.
1985 V85	From 1982 to 1985, OPEC attempted to set production quotas low enough to stabilize the prices. Repeated failures occurred because various members of OPEC would produce beyond their quotas. Saudi Arabia acted as the swing producer cutting its production to stem the free falling prices. In August of 1985 they tired this role and linked their oil prices to the spot market and in early 1986 increased production from 2 to 5 million barrels per day.

Appendix 3: Variations of NARX inputs- 1974 to 2009



Appendix 4: Initial dummy variables considered in the analysis

Dummy Variables for time period between 1972-1979					Dummy Variables for time period between 1980-1990					Dummy Variables for time period between 1990-2008				
Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.	Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	26.36817	31.20053	0.845119	0.4076	C	-56.75656	39.65521	-1.431251	0.1760	C	-4.806597	51.92701	-0.092564	0.9274
LGP1	0.292648	0.301197	0.971615	0.3423	LGP1	0.855216	0.381120	2.243952	0.0429	LGP1	0.772536	0.254494	3.035575	0.0079
LEP1	-29.61218	16.40107	-1.805502	0.0854	LEP1	2.726179	17.85680	0.152669	0.8810	LEP1	-15.59575	11.18665	-1.394139	0.1823
LEU1	27.05636	15.98533	1.692574	0.1053	LEU1	-1.207902	17.53962	-0.068867	0.9461	LEU1	15.89602	10.03337	1.584315	0.1327
LG1	0.702153	0.625294	1.122917	0.2741	LG1	0.633913	0.949301	0.667768	0.5160	LG1	0.014133	0.860223	0.016430	0.9871
LOILR1	0.364323	0.136690	2.665326	0.0145	LOILR1	-0.034898	0.186783	-0.186837	0.8547	LOILR1	0.294422	0.136462	2.157545	0.0465
LGG1	-0.177576	0.129558	-1.370632	0.1850	LGG1	-0.081860	0.132019	-0.620066	0.5459	LGG1	-0.138084	0.166845	-0.827615	0.4201
LEI1	-8.064361	5.582177	-1.444662	0.1633	LEI1	4.321173	7.177459	0.602048	0.5575	LEI1	-4.320831	3.898157	-1.108429	0.2841
LFC1	0.475752	0.268364	1.772783	0.0908	LFC1	0.427209	0.311036	1.373503	0.1928	LFC1	0.361110	0.274166	1.317119	0.2064
V72	-0.190310	0.351455	-0.541491	0.5939	V80	0.063200	0.421829	0.149825	0.8832	V96	-0.139567	0.319736	-0.436509	0.6683
V73	0.452871	0.369844	1.224492	0.2343	V81	0.045237	0.447636	0.101057	0.9210	V97	-0.216817	0.333971	-0.649210	0.5254
V74	-0.352023	0.390099	-0.902395	0.3771	V82	1.004473	0.528408	1.900941	0.0797	V98	0.551738	0.345990	1.594663	0.1303
V75	0.029283	0.420728	0.069601	0.9452	V83	-0.358781	0.350425	-1.023845	0.3246	V99	0.226519	0.342929	0.660542	0.5183
V76	-0.261524	0.416865	-0.627358	0.5372	V84	0.214065	0.346211	0.618309	0.5471	V00	-0.375816	0.349118	-1.076472	0.2977
V77	0.097066	0.332968	0.291519	0.7735	V85	-0.613459	0.303477	-2.021437	0.0643	V01	0.169405	0.350621	0.483157	0.6355
V78	0.583035	0.326091	1.787955	0.0882	V86	-0.138188	0.401214	-0.344425	0.7360	V02	0.001084	0.333290	0.003252	0.9974
V79	0.359725	0.485999	0.740177	0.4674	V87	-0.524762	0.293179	-1.789905	0.0968	V03	0.089070	0.351082	0.253703	0.8030
R-squared	0.944907	Mean dependent var	3.081422		V88	-0.196956	0.340063	-0.579175	0.5724	V04	0.115511	0.333547	0.346311	0.7336
Adjusted R-squared	0.902931	S.D. dependent var	0.718824		V89	0.200166	0.299143	0.669132	0.5151	V05	-0.059729	0.331567	-0.180143	0.8593
S.E. of regression	0.223956	Akaike info criterion	0.146943		V90	-0.099150	0.313687	-0.316079	0.7570	V06	-0.124872	0.337879	-0.369576	0.7165
Sum squared resid	1.053287	Schwarz criterion	0.879548		V91	0.136927	0.335616	0.407987	0.6899	V07	0.123147	0.333853	0.368867	0.7171
Log likelihood	14.20808	F-statistic	22.51067		V92	-0.336640	0.298168	-1.129028	0.2793	V08	-0.595821	0.390611	-1.525356	0.1467
Durbin-Watson stat	2.198348	Prob(F-statistic)	0.000000		V93	-0.169481	0.319837	-0.529897	0.6051	R-squared	0.954782	Mean dependent var	3.081422	
					V94	-0.054809	0.298330	-0.183720	0.8571	Adjusted R-squared	0.895433	S.D. dependent var	0.718824	
					V95	-0.029263	0.263448	-0.111075	0.9133	S.E. of regression	0.232444	Akaike info criterion	0.212566	
					R-squared	0.972420	Mean dependent var	3.081422		Sum squared resid	0.864486	Schwarz criterion	1.160642	
					Adjusted R-squared	0.921504	S.D. dependent var	0.718824		Log likelihood	17.96125	F-statistic	16.08768	
					S.E. of regression	0.201394	Akaike info criterion	-0.123954		Durbin-Watson stat	2.359641	Prob(F-statistic)	0.000000	
					Sum squared resid	0.527274	Schwarz criterion	0.953405						
					Log likelihood	27.35513	F-statistic	19.09838						
					Durbin-Watson stat	2.139479	Prob(F-statistic)	0.000001						

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