

An OLG model of global imbalances

Sara Eugeni

*University of Warwick, Department of Economics, CV4 7AL, Coventry, UK.
E-mail address: S.Eugeni@warwick.ac.uk. Phone number: +44 (0)24 76584930.*

Abstract

In this paper, we investigate the relationship between East Asian countries' high propensity to save and global imbalances in a two-country OLG model with production. The saving behavior of emerging economies and capital outflows to the United States can be attributed to their poor pay-as-you-go systems. The model predicts that emerging countries run a trade surplus only as long as the long-run growth rate of the economy is higher than the real interest rate (capital overaccumulation case). The low real interest rates in the US is therefore evidence in favour of the hypothesis that there is a "global saving glut" in the world economy. The model can explain why the US current account deteriorated gradually and only in the late 1990s, although the net foreign asset position had already turned negative in the early 1980s. Finally, the analysis also implies that an improvement of the pay-as-you-go system in China would have the effect of reducing the imbalances. In accordance with the theory, we find that the higher is the percentage of the working population covered by the pay-as-you-go system the lower are savings and the current account balance in a cross-section of countries.

Keywords: global imbalances, capital flows, trade dynamics, OLG model, pay-as-you-go-system, saving rates

1. Introduction

The trade imbalances between the United States and East Asian economies, or global imbalances, have attracted an unprecedented amount of attention from both economists and policy makers. The reasons are several. Firstly, the size of these imbalances is unprecedented as they involve one of the richest countries in the world and a large area of emerging, growing economies. Second, global imbalances are not a temporary phenomenon. Figure D.1 shows the steady deterioration of the United States' current account deficit since the late 1990s. The recent adjustment has involved trade with Europe and oil-producing countries, but not other emerging economies. In particular, the US deficit towards China, which mirrors very closely the Chinese surplus, did not shrink with the recession.

One of the most common views on global imbalances is the "global saving glut hypothesis", due to Bernanke [4]. The core of the argument is that the high saving rates in East Asia have created an excess of savings in the world economy, which has resulted in capital flows towards the US and low real interest rates. Bernanke [4] also stressed that the understanding of global imbalances requires a "global perspective" and that they do not "primarily reflect economic policies and other economic developments within the United States itself". In other words, current account imbalances must be thought of as an equilibrium phenomenon.

In this paper, we provide a general equilibrium framework to discuss the global saving glut hypothesis and therefore investigate the relationship between emerging countries' high propensity to save and global imbalances. Our setup is a two-country OLG model with production along the lines of Diamond [10]. The Diamond model is a natural framework to deal with questions related to the presence of excess savings in an economy. The model admits the possibility that, in a perfectly competitive economy, there is capital overaccumulation. Therefore, the concept of "excess savings" has a precise meaning in the OLG model, as it corresponds to the notion of dynamic inefficiency. The advantage of this approach is that we can ask with clarity the question of whether it is true that a global saving glut have led to global imbalances.

Before tackling the issue of excess savings, the first question that we need to answer is why East Asian countries' saving rates are higher than the United States'. Figure D.2 depicts the saving rates of the US and a few East Asian countries over the last 30 years. An interesting - and crucial, to us - aspect of the data is that while global imbalances emerged in the late 1990s, that East Asian countries save more than the United States is certainly not a new fact. This divergence in the countries' saving rates is structural as it can be observed in the data long before global imbalances emerged.

Our explanation behind this fact is that pay-as-you-go social security systems are very poor in East Asian countries as compared to the United States. Officially, many East Asian countries have schemes which are classified as pay-as-you-go. However, the percentage of the working population actually covered by such schemes is much lower than in the US. Table 1 reports the wide difference between the US coverage rate and that of East Asian countries that are reported to have a pay-as-you-go scheme¹.

Table 1: Pay-as-you-go systems' coverage

	Coverage % labour force	Coverage % working population
United States	92.2	71.4
East Asian countries	44.1	31.3
China	33.5	27.7

Notes: See Appendix E for data definitions and sources.

To capture the heterogeneity in the coverage of the pay-as-you-go systems, we study a two-country OLG model with production in which one country has a pay-as-you-go system while the other does not². Cross-sectional evidence supports the idea that countries with pay-as-you-go systems tend to have lower saving rates, especially the more extensive is the coverage (Samwick [26]). Yet, the implications for global imbalances of the fact that East Asian countries save more to finance old age consumption because of their poor pay-as-you-go systems are still unexplored. One of the contributions of this paper is to fill this gap in the literature.

In section 2 and 3, we present the model and characterize the direction of capital flows and trade at and outside steady states. First, we show that the emerging country always lends to the developed country, as the young of the former country save relatively more in the absence of the

¹The fact that the Chinese coverage rate is extremely low is also documented in Asher et al. [2]. Peter Diamond was one of the leading economists who participated at this study on social security reforms in China. See also Boersch et al. [5], OECD [21] and Song et al. [28].

²In reality, neither the US coverage rate is 100% nor the Chinese coverage rate is 0%. Such simplification is only adopted to avoid heavy notation.

pay-as-you-go system³. Yet, the pattern of trade in the consumption good does depend on the long-run efficiency of the world economy. We prove that the direction of trade depends on how the population growth rate compares with the interest rate, and this is also the case outside steady states. The emerging country runs a trade surplus only as long as the world economy is beyond the golden rule level of capital (capital overaccumulation). Otherwise, the emerging country runs a trade deficit despite the fact that it is the lender country. Only in the coincidental case of the golden rule, trade happens to be balanced.

The main implication of these results is that we would not observe the current pattern of trade if there was not an excess of savings in the world economy. In this sense, this paper provides a formal argument in favour of the “global saving glut hypothesis”. Caballero et al. [7] argue that the saving glut story can be interpreted within their framework, by allowing for different saving rates across countries. However, as saving rates are exogenous, a higher saving rate in East Asia does not imply that these countries save too much. In this paper, a global excess of savings can arise endogenously, as a long-term consequence of the financial integration between the United States and countries whose saving rate is higher due to their poor pay-as-you-go-system. Differently from Caballero et al. [7], the OLG structure allows us to relate the amount of savings in the economy to the golden rule benchmark.

Another interesting aspect of the trade balance result is that the developed country runs a trade deficit in the capital overaccumulation case because aggregate consumption is higher than in the other country. The reason is that there is sufficient growth in the economy to compensate interest payments to the emerging country. It is often claimed that global imbalances are due to the fact that emerging countries are consuming too little. This model shows that this is nothing but equilibrium behavior.

Our findings are related to two seminal papers of David Gale [13], [14]. Gale made the important point that countries can run permanent trade imbalances in general equilibrium models. His intuition was that this is especially possible in OLG economies. Gale had discovered that the sign of the balance of trade depends on efficiency properties in a Solow model with heterogenous agents and in a pure exchange OLG economy with inside money. The paper is also related to Polemarchakis et al. [23], which found that trade is balanced at the golden rule in a pure exchange OLG economy with outside money. The seminal paper on international capital mobility in a Diamond setting is Buiter [6], in which countries are heterogenous in the discount factors.

In section 4, we study the dynamics of capital flows and global imbalances for plausible initial conditions of the autarkic economies. It turns out that the model is able to account for the dynamics and the timing of global imbalances, as well as the dynamics of real interest rates and net foreign asset positions. First, the model can rationalize the fact that the US current account and real interest rates deteriorated *gradually* (Figure D.1 and D.7). Second, the model can explain why the accumulation of net foreign liabilities started in the early 1980s (Figure D.6), well *before* the emergence of global imbalances⁴.

The model provides intuitive explanations for these facts. Because of their higher saving rates, emerging countries started to lend abroad soon after they opened to trade with the US. The decline of real interest rates can be read as a consequence of capital accumulation in the world economy (Figure D.7). Global imbalances arose as soon as interest rates fell below the long-run

³Geide-Stevenson [16] found a similar result in a two-country Diamond model in which the pay-as-you-go tax is proportional rather than lump-sum. Her analysis is limited to steady states, in this paper we also study the dynamics of capital flows and trade imbalances.

⁴See section 4 for a comparison with the literature on these stylized facts.

growth rate, implying that the world economy is saving too much.

We also ask whether it is plausible that the world economy is experiencing a saving glut. According to the model, this requires that the long-run growth rate of the world economy is higher than the real interest rate. We find evidence of this in the data. This is hardly surprising, since US real interest rates have hit a historic low in the past decade.

Finally, we provide empirical evidence to support the predictions of the model. In a cross-section of countries, we find a statistically significant negative association between the coverage rate of pay-as-you-go systems and the saving rate, consistently with earlier work of Samwick [26]. In this paper, we also document the fact that a higher coverage rate is associated with a lower current account balance.

This paper is mainly related to the body of literature which puts emphasis on differences in institutions as the main determinant of global imbalances, e.g. Caballero et al. [7], Mendoza et al. [24] and Angeletos et al. [1]. The focus of these papers is on financial markets' different stages of development, and yet the sense of our analysis is similar as the type of pension system enforced in a country affects saving and investment possibilities. Caballero et al. [7] explain global imbalances as the result of a negative shock to emerging countries' level of financial development, while our view is that global imbalances arose as the outcome of the financial integration between the US and emerging economies. In this respect, this paper is closer to Mendoza et al. [24] and Angeletos et al. [1].

Differently from the above papers, this paper contributes to the debate on whether and how the imbalances should be addressed from a policy point of view. While there is widespread agreement that global imbalances must be reduced, this is advocated on the basis of a variety of arguments⁵. In Mendoza et al. [24] and Angeletos et al. [1], global imbalances are a temporary phenomenon, meant to disappear in the long-run. In Caballero [7], global imbalances are a benign aspect of the world economy. In an OLG economy, the presence of excess savings means that there is room for policy interventions. It is often claimed that East Asian countries should introduce policies to boost domestic demand, in view of correcting the imbalances. If we accept that the world economy is overaccumulating capital, long-term policies in this direction are clearly desirable. For instance, reforms aimed at improving the pay-as-you-go system in China would not only be Pareto-improving but also have the effect of reducing the imbalances.

2. The World Economy

In this section, we describe the two-country model, which maintains the basic structure of Diamond (1965). We will refer to country 1 (2) as the developed (emerging) country.

Agents live for two periods and a new generation is born in each country for all t . The size of the population follows $L_{i,t} = L_{i,0}(1 + n)^t$, where $L_{i,0}$ are the young born in country i at date 0 and n is the (common) population growth rate⁶. The only source of growth in the model comes from population⁷.

The two countries only differ in the pension systems. Country 1 has a pay-as-you-go social security system, while the system in country 2 is fully-funded. Country 1's government levies

⁵See a recent collection of papers written by central bankers on the topic [17].

⁶We computed the average growth rate of the working age population (15-64 years old) in China and the US using the UN World Population Prospect. The average is calculated for the past 10 years, the period in which we observe global imbalances. Both countries have a population growth rate very close to 1%.

⁷See the Appendix for an extension of the model with labour augmenting technological progress.

a time-invariant lump-sum tax τ_1 on the young, which is used to finance the old's pension b_1 at each t ⁸. The policy is balanced so that taxes are equal to transfers at each t : $\tau_1 L_{1,t} = b_1 L_{1,t-1}$. It follows that the transfer which the current old receive is equal to $b_1 = (1+n)\tau_1$, i.e. each generation receives a transfer which is bigger than the tax if population is increasing.

Finally, we need to specify which markets are open for international trade. We assume that the consumption good can be costlessly traded between the countries. As our focus is to analyze the pattern of trade in the good, we impose that labour is immobile.

2.1. Firms

Competitive firms use capital and labour to produce the consumption good by means of an identical, constant returns technology: $Y_{i,t} = F(K_{i,t}, L_{i,t})$.

As anticipated above, firms located in country i can only hire workers in the domestic labour market. We consider the production function in its intensive form as the number of workers is given at each t : $y_{i,t} = f(k_{i,t})$. The function f is strictly increasing and concave in $k_{i,t}$. Capital depreciates at the constant rate $0 \leq \delta \leq 1$ in both countries and we assume that the following boundary conditions hold:

$$\lim_{k_{i,t} \rightarrow +\infty} f'(k_{i,t}) = 0 \quad \lim_{k_{i,t} \rightarrow 0} f'(k_{i,t}) = +\infty$$

At time 0, the two autarkic economies open to trade after production has taken place. Their “initial” level of capital will respectively be $k_{1,0}$ and $k_{2,0}$. Starting from period 1, firms' demand for capital is met in the world market and therefore they will face the same path of interest rates $\{r_t\}_{t \geq 1}$. Firms solve the following maximization problem:

$$\max_{k_{i,t}} \pi_{i,t} = f(k_{i,t}) - (r_t + \delta)k_{i,t} - w_{i,t} \quad \forall i, t \geq 1 \quad (1)$$

The necessary and sufficient conditions for a maximum are:

$$r_t = f'(k_{i,t}) - \delta \quad (2)$$

$$w_{i,t} = f(k_{i,t}) - f'(k_{i,t})k_{i,t} \quad (3)$$

Because the countries have access to the same technology, it is immediate that capital stocks per capita are equalized: because $k_{1,t} = k_{2,t} = k_t$, it is also true that $w_{1,t} = w_{2,t} = w_t$ for all t . While the two countries might start with different initial conditions, potential income differences vanish once the two countries open to trade.

This assumption is somewhat strong, but our theory does not aim at explaining per capita income differences between the US and East Asian countries. Moreover, it is convenient to abstract from other potential bases for trade to study how differences in pension systems have an impact on capital accumulation and trade.

2.2. Consumers

Agents get utility from consuming in the two periods of life. Preferences are stationary and identical both within generation and across countries. The utility function is C^2 , strictly increasing, strictly concave and additively separable:

$$U(c_{i,t}^t, c_{i,t+1}^t) = u(c_{i,t}^t) + \beta v(c_{i,t+1}^t) \quad (4)$$

⁸The results of the paper are robust to different types of taxation. The derivation of the model under proportional tax is available under request.

where $c_{i,t}^t$ denotes consumption when young and $c_{i,t+1}^t$ is consumption when old of the generation (born at time) t in country i . Also:

$$\lim_{c_{i,t}^t \rightarrow 0} u'(c_{i,t}^t) = +\infty \quad \lim_{c_{i,t+1}^t \rightarrow 0} u'(c_{i,t+1}^t) = +\infty$$

The budget constraints are:

$$c_{i,t}^t = w_t - \tau_i - s_{i,t} \quad (5)$$

$$c_{i,t+1}^t = s_{i,t}(1 + r_{t+1}) + \tau_i(1 + n) \quad (6)$$

where $s_{i,t}$ denotes savings. As there is no pay-as-you-go system in country 2, $\tau_2 = 0$. In our two-country world, the young are allowed to lend both to domestic and foreign firms. Which country is going to be the borrower (lender) will be established in equilibrium.

The maximization problems of the two consumers are the following:

$$\max_{s_{1,t}} u(w_t - \tau_1 - s_{1,t}) + \beta v(s_{1,t}(1 + r_{t+1}) + \tau_1(1 + n)) \quad (7)$$

$$\max_{s_{2,t}} u(w_t - s_{2,t}) + \beta v(s_{2,t}(1 + r_{t+1})) \quad (8)$$

The necessary and sufficient conditions for a maximum are:

$$u'(w_t - \tau_1 - s_{1,t}) = \beta(1 + r_{t+1})v'(s_{1,t}(1 + r_{t+1}) + \tau_1(1 + n)) \quad (9)$$

$$u'(w_t - s_{2,t}) = \beta(1 + r_{t+1})v'(s_{2,t}(1 + r_{t+1})) \quad (10)$$

The agents' optimal savings are then a function of the wage and the interest rate. In country 1, they also depend upon the taxes and transfers related to the pension system.

In the OLG model, it is well known that savings are lower in presence of a pay-as-you-go system (see e.g. [3], or [25]). Given w_t and r_{t+1} , we have that:

$$\frac{ds_{1,t}}{d\tau_1} = -\frac{u''(c_{1,t}^t) + \beta(1 + n)(1 + r_{t+1})v''(c_{1,t+1}^t)}{u''(c_{1,t}^t) + \beta(1 + r_{t+1})^2v''(c_{1,t+1}^t)} < 0 \quad (11)$$

At each t , the young in country 1 save less than in country 2 as their net wage is lower due to the tax. However, the extent of the fall in saving will depend on how n and r_{t+1} compares. In particular, if $n > r_{t+1}$ ($< r_{t+1}$) then the drop in saving is larger since $\frac{ds_{1,t}}{d\tau_1} < -1$ (> -1). In fact, the income of country 1's consumers is higher (lower) when the rate of return on the pension system is higher (lower) than the interest rate. This can be seen from the consolidated budget constraint:

$$c_{1,t}^t + \frac{c_{1,t+1}^t}{1 + r_{t+1}} = w_t - \tau_1 \frac{r_{t+1} - n}{1 + r_{t+1}} \quad (12)$$

When $n > r_{t+1}$, $c_{1,t}^t$ increases since consumption is a normal good⁹. Therefore, savings will be even lower. Only when $n = r_{t+1}$, savings decrease one for one with the tax as (11) shows.

We also characterize the saving functions by the following assumption.

⁹That savings are increasing in the wage can be derived from the first-order conditions. It can be checked that $\frac{ds_{1,t}}{dw_t} = \frac{1}{1 + \beta(1 + r_{t+1})^2} \frac{v''(c_{1,t+1}^t)}{u''(c_{1,t}^t)}$, therefore $0 < s_w < 1$.

Assumption 1. *Consumption when young and when old are gross substitutes:*

$$s_r > 0$$

where s_r is the partial derivative of the saving function with respect to the interest rate.

2.3. Equilibrium

Given $(\tau_1, k_{1,0}, k_{2,0})$, a competitive equilibrium is a sequence of capital stocks $\{k_t^*\}_{t \geq 1}$, consumption plans $\{c_{i,t}^*, c_{i,t+1}^*\}_{t \geq 0}$ and factor prices $\{r_t^*, w_t^*\}_{t \geq 1}$ such that:

- (i) $\{c_{i,t}^*, c_{i,t+1}^*\}_{t \geq 0}$ maximize the agents' utility function (4) subject to the budget constraints (5),(6) for all i ;
- (ii) $\{k_t^*\}_{t \geq 1}$ maximize the firms' profit function (1);
- (iii) the (world) capital market clears for $t \geq 0$:

$$\sum_i L_{i,t} s_{i,t}^* = \sum_i K_{i,t+1}^*$$

If the capital market clears at each t , the (world) market for the good will clear by Walras' Law. The good market is in equilibrium when the total resources available (after production) are equal to the consumption of the current young and old, and next period's capital stocks of the two countries.

$$\begin{aligned} \sum_i F(K_{i,t}^*, L_{i,t}) + (1 - \delta) \sum_i K_{i,t}^* &= \\ &= \sum_i L_{i,t} c_{i,t}^* + \sum_i L_{i,t-1} c_{i,t}^{t-1*} + \sum_i K_{i,t+1}^* \end{aligned} \quad (13)$$

Equation (13) will be extensively used in the next section to study the pattern of trade between the two countries.

3. The pattern of trade

3.1. Dynamics in the capital market and capital flows

In this section, we analyze the direction of capital flows and trade in the model described above. The first step is to study how capital accumulates in this economy. The capital market is equilibrium in as long as the world demand for capital is equal to the world supply (savings):

$$\begin{aligned} K_{t+1}^* &\equiv \sum_i K_{i,t+1}^* = L_{1,t} s_1(f(k_{1,t}^*) - f'(k_{1,t}^*)k_{1,t}^*, f'(k_{t+1}^*), \tau_1) + \\ &+ L_{2,t} s_2(f(k_{2,t}^*) - f'(k_{2,t}^*)k_{2,t}^*, f'(k_{t+1}^*)) \end{aligned} \quad (14)$$

where K_{t+1}^* denotes the world capital stock at time $t + 1$. We have already established that $k_{1,t} = k_{2,t} = k_t$ for $t \geq 1$, while at $t = 0$ countries might start with different levels of capital. Before proceeding, it is convenient to introduce the following definition:

Definition 1. *Country i 's size is: $\rho_i \equiv \frac{L_{0i}}{L_0}$.*

Because the countries grow at a common rate, ρ_i is constant over time and depends on the countries' initial labour forces. We can now divide (14) by the world labour supply L_t and get:

$$(1+n)k_{t+1}^* = \rho_1 s_1(f(k_{1,t}^*) - f'(k_{1,t}^*)k_{1,t}^*, f'(k_{t+1}^*), \tau_1) + \rho_2 s_2(f(k_{2,t}^*) - f'(k_{2,t}^*)k_{2,t}^*, f'(k_{t+1}^*)) \quad (15)$$

At each t , the world capital stock per capita (which is equivalent to the domestic capital stocks) is determined by the savings of country 1 and 2. Equation (15) shows that each country will contribute to the supply side of the market according to its size.

Hereafter, we study the above difference equation in the capital stock. The world economy is in steady state when $k_t^* = k_{t+1}^* = k^*$:

$$(1+n)k^* = \rho_1 s_1[f(k^*) - f'(k^*)k^*, f'(k^*), \tau_1] + \rho_2 s_2[f(k^*) - f'(k^*)k^*, f'(k^*)] \quad (16)$$

Lemma 1. (i) Given $k_{1,0} > 0$ and $k_{2,0} > 0$, there exists a unique intertemporal equilibrium as long as $\tau_1 < \bar{\tau}_1(k_{1,0})$. (ii) If $\lim_{k_t \rightarrow 0} \frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} > 1$, there exists at least a stable steady state.

Proof. The proof is in the Appendix. ■

Part (i) of Lemma 1 establishes that there exists an equilibrium path only if each country's savings are positive at $t = 0$. It is intuitive that we need a condition on the tax level to avoid circumstances under which income is either zero or negative in the initial period. In other words, a perfect foresight equilibrium will exist only if the level of the tax is compatible with having positive savings in the economy¹⁰. Part (ii) shows that there exist paths converging to a stable steady state. This is important as the focus of the next section will be on the behavior of the economy near a stable steady state.

We can now analyze the pattern of trade between the countries¹¹. We start with the direction of asset trade. Given the capital market equilibrium equation, it is immediate to show which of the two countries has to borrow to sustain the equilibrium capital stock.

Definition 2. The net foreign assets position (per capita) of country i at the end of period t is:

$$a_{i,t+1} \equiv \frac{s_{i,t}}{1+n} - k_{t+1} \quad (17)$$

Proposition 1 (Net foreign assets). Country 1 (2) is the borrower (lender) country for all $t \geq 1$.

Proof. First, substitute equation (15) into Definition 2. In equilibrium, the net foreign assets positions are:

$$a_{1,t+1}^* = -\frac{\rho_2}{1+n}(s_{2,t}^* - s_{1,t}^*) \quad a_{2,t+1}^* = \frac{\rho_1}{1+n}(s_{2,t}^* - s_{1,t}^*) \quad (18)$$

where $\rho_1 a_{1,t+1}^* + \rho_2 a_{2,t+1}^* = 0$ by asset market clearing. From equation (11), we know that country 1 saves less than country 2 since they face the same factor prices. Since it must be true that $s_{2,t}^* > s_{1,t}^*$ for all k_t^*, k_{t+1}^* , the sign of a follows:

$$a_{1,t+1}^* < 0 \quad a_{2,t+1}^* > 0 \quad \forall t \geq 1 \quad (19)$$

¹⁰See De La Croix et al. [9] for a detailed analysis of the (closed economy) Diamond model with lump-sum transfers.

¹¹We postpone the discussion of the pattern of trade at the openness to section 4, where we study the dynamics of net foreign assets and global imbalances for realistic initial conditions of the autarkic economies.

■

Proposition 1 shows that country 2 (the emerging country) will always lend to country 1, it does not matter whether the economy is in a steady state or not. The intuition behind this result is simple. We know that the equilibrium capital stock is combination of savings in the two countries and the developed country saves less than the emerging economy. Therefore, while country 1 has to borrow to sustain k_{t+1}^* , country 2's savings (partly) find an outlet in country 1.

It might be noted that the extent of trade will depend on how large is the difference between the two countries' savings. For instance, countries trade more the bigger is the size of the pay-as-you-go system in country 1. It is worth stressing that the direction of asset trade does not depend on whether we are in the capital overaccumulation case or not. However, this becomes relevant once we consider the countries' balance of trade.

3.2. The balance of trade and efficiency

We can now study the pattern of trade in the consumption good. First, we define the balance of trade of country i as the country's excess supply for the consumption good.

Definition 3. *The (per capita) trade balance of country i is:*

$$tb_{i,t}^* \equiv f(k_{i,t}^*) + (1 - \delta)k_{i,t}^* - c_{i,t}^{t*} - \frac{c_{i,t}^{t-1*}}{1+n} - k_{i,t+1}^*(1+n) \quad (20)$$

If $tb_{i,t}^* > 0$ in equilibrium, then country i is net exporter as output is higher than “domestic absorption”.

Definition 3 stems from the per capita version of (13), the consumption good's market clearing equation. Equation (13) states that the sum of the countries' balances must be zero at each t . By dividing both sides by the world population L_t , we obtain that $\rho_1 tb_{1,t} + \rho_2 tb_{2,t} = 0$. While this equation must hold, that $tb_{i,t}^* \neq 0$ for every i is possible in equilibrium.

An equivalent way to look at the balance of trade is in terms of net capital flows. First, exploit the fact that $f(k_{i,t}^*) = w_t^* + (r_t^* + \delta)k_{i,t}^*$ to get:

$$tb_{i,t}^* = w_t^* + (r_t^* + \delta)k_{i,t}^* + (1 - \delta)k_{i,t}^* - c_{i,t}^{t*} - \frac{c_{i,t}^{t-1*}}{1+n} - k_{i,t+1}^*(1+n)$$

Using the budget constraints of the young and the old living at time t , we obtain:

$$tb_{i,t}^* \equiv [s_{i,t}^* - k_{i,t+1}^*(1+n)] - \left(\frac{1+r_t^*}{1+n} \right) [s_{i,t-1}^* - k_{i,t}^*(1+n)] \quad (21)$$

Next, using Definition 2 we can rewrite (21) as follows:

$$tb_{i,t}^* = (1+n)a_{i,t+1}^* - (1+r_t^*)a_{i,t}^* \quad (22)$$

The above characterization shows that the balance of trade reflects trade in the capital market in period t and $t-1$.

Proposition 2 (Balance of trade and steady states). *At the golden rule allocation ($r^* = n$), trade is balanced.*

If the steady state is inefficient ($r^ < n$), country 2 (the emerging country) is in surplus while country 1 (the developed country) is in deficit.*

If the steady state is efficient ($r^ > n$), the opposite is true.*

Proof. Consider equation (22). Imposing $a_{i,t}^* = a_{i,t+1}^* = a_i^*$ and $r_t^* = r^*$, the trade balance of country i in the steady state is:

$$tb_i^* = (n - r^*)a_i^* \quad (23)$$

It immediately follows that $tb_i = 0 \forall i$ at the golden rule allocation. The other statements are a direct implication of our hypotheses and the sign of a_i^* (Proposition 1). ■

If the world economy converges to a steady state such that $r^* = n$, not only steady state consumption will be maximized but trade will be balanced in the long-run. Yet, that trade is balanced does not imply that the two countries do not trade at all. In fact, borrowing and lending still takes place at the golden rule (by Proposition 1) but exports are completely offset by imports.

However, this can only happen by coincidence. In all other cases, there will be trade imbalances between the two countries. To comment on the result, let us consider the trade balance of country 2:

$$tb_2^* = \underbrace{(1+n)a_2^*}_{\text{exports}} - \underbrace{(1+r^*)a_2^*}_{\text{imports}}$$

We have seen that the young in country 2 lend to firms located in country 1 as they save relatively more (exports). At the same time, the old of country 1 pay the loan back, along with interest payments, to the old of country 2 (imports).

The proposition states that the sign of the balance of trade will depend on how n and r^* compares. Indeed, notice that while a_i^* is constant at the steady state, $A_{i,t}^*$ will grow at the population growth rate. Proposition 2 then says that the lender country will have a surplus as long as the net income from abroad is not enough to compensate the increase in lending induced by population growth. Instead, if the interest rate was higher than the population growth rate, country 2 should be in deficit.

The model implies that the reason why we observe global imbalances is that there is a saving glut in the world economy. Therefore, it provides a formalization of Bernanke's hypothesis. We postpone to section 4 the discussion of whether it is plausible that the world economy is on an inefficient path, with the support of some empirical evidence.

The fact that the sign of the balance of trade of a country depends on whether the world economy happens to be below or beyond the golden rule allocation is not just true at the steady state of the model. Next, we show that this holds outside stationary states too.

To this purpose, it is more convenient to work with equation (20). As technologies are identical, it is intuitive that all the action comes from aggregate consumption. Because pension systems are different, the countries' consumption possibilities are not the same and this will explain the direction of trade in the consumption good.

Lemma 2 (Consumption). *For any generation $t \geq 1$, the agent born in country 1 consume relatively more (less) when $n > r_{t+1}$ ($< r_{t+1}$).*

Proof. The proof is in the Appendix. ■

In Lemma 2, we show that agents born in country 1 consume more in the capital overaccumulation case. It is interesting to note that this result supports the idea that East Asian countries are consuming too little relatively to the United States, and this has something to do with global imbalances. The reason is that country 1's generations have a higher income, despite that they have to pay interest rates to country 2. When $n > r_{t+1}^*$, there is enough growth in the economy

for the pension to compensate interest payments to the foreign country. An examination of the two agents' budget constraints should convince the reader of this fact.

Given Lemma 2, we can analyze the pattern of trade in the consumption good outside steady states:

Proposition 3 (Balance of trade outside steady states). *Country 1 (the developed country) is in deficit at a given t when $n > r_t^*$ and $n > r_{t+1}^*$, while in surplus when $r_t^* > n$ and $r_{t+1}^* > n$. If $r_t^* > n$ and $r_{t+1}^* < n$, the sign is ambiguous.*

Proof. We consider the developed country, the opposite is obviously true for the emerging economy. If country 1 imports, then $tb_{1,t} < tb_{2,t}$. Given Definition 3 and because $k_{1,t} = k_{2,t} \forall t \geq 1$, the following must hold for country 1 to be in deficit:

$$c_{1,t}^{t*} + \frac{c_{1,t}^{t-1*}}{1+n} > c_{2,t}^{t*} + \frac{c_{2,t}^{t-1*}}{1+n}$$

Indeed, Lemma 2 showed that consumption is higher for generations in country 1 as long as next period's interest rate is lower than the population growth rate. Therefore, for $tb_{1,t} < 0$ it is sufficient that $n > r_t^*$ and $n > r_{t+1}^*$. Instead, when $r_t^* > n$ and $r_{t+1}^* > n$ generations of country 2 consume more and $tb_{2,t} < 0$.

Suppose that at a given t , we have that $r_t^* > n$ but next period's interest rate falls below the population growth rate. While $c_{1,t-1}^{t*} < c_{2,t-1}^{t*}$ by $r_t^* > n$, $c_{1,t}^{t*} > c_{2,t}^{t*}$ by $r_{t+1}^* < n$. The net effect will depend on other parameters of the economy (see Appendix C for an illustration in the Cobb-Douglas case). ■

The proposition establishes that the deficit (surplus) country is the country which consumes relatively more (less) at a given t .

At the golden rule, it is worth noting that the consumption allocation of the two representative generations is identical despite the different pension systems (see the proof of Lemma 2 in the Appendix). This gives a different angle to the balanced trade result. Because savings decrease one for one with τ_1 and consumers' wealth is not affected by the pension system when $r^* = n$, consumption choices in the two countries are the same at the golden rule. Indeed, the planner would choose such allocation if giving the same weights to the agents (in fact, we did not allow for heterogeneity in preferences).

4. The dynamics of net foreign assets and global imbalances

The results of section 3 imply that the dynamics of the countries' balance of trade are strongly related to the efficiency of the world economy's capital accumulation path. In particular, we have found that the lender country (the country with no pay-as-you-go pension system) runs a trade surplus only as long as the population growth rate is higher than the interest rate. Therefore, the theory suggests that global imbalances are a signal that the world economy is overaccumulating capital.

In this section, we demonstrate that the model is able to qualitatively replicate the evolution of the US current account and net foreign assets' position since the early 1980s (the time of China's integration into the world economy). Second, we provide some evidence to support the claim that there is a "global saving glut" in the world economy. If we can say that the long-run growth rate of the world economy is higher than the real interest rate, it is then plausible that the

world economy is on an equilibrium path characterized by an excess of savings.

To start with, we need to address the following questions. What are the conditions under which the world economy converges to an inefficient steady state? And are these reasonable enough? To make progress on these issues, we introduce some assumptions on the characteristics of the two countries in autarky. Moreover, we make a conjecture on the two countries' initial conditions at time 0, which would correspond to the financial openness of emerging countries¹².

Assumption 2 (Autarkic steady states). *Suppose country 1 has a locally stable steady state such that $r_1^{autss} = n$. For country 2, the locally stable steady state satisfies $n > r_2^{aut}$.*

Assumption 3 (Initial conditions). *At the time of financial integration $t = 0$, country 1 is at the autarkic steady state $k_{1,0} \equiv k_1^{autss}$.*

Country 2's initial capital stock satisfies $k_{2,0} < k_2^{aut}$. Moreover, $k_{1,0}$ and $k_{2,0}$ are such that $s_1(w(k_{1,0}), r(k_1^), \tau_1) > s_2(w(k_{2,0}), r(k_1^*))$, where k_1^* is the equilibrium capital stock at $t = 1$.*

Our main hypothesis is that the pay-as-you-go system, which has been introduced during the Great Depression, "fixed" the long-run inefficiency of the US economy. This assumption is also consistent with the fact that US trade was balanced before 1980. In general, it is known that a country with no pay-as-you-go system has a higher capital stock and therefore a lower interest rate than a country with a pay-as-you-go system. It follows that the autarkic steady state of the emerging economy is inefficient. This is coherent with our previous analysis, as we treated the two countries as identical except for the pension systems.

That country 2 opened to trade with a relatively low capital stock and along its transition path, while country 1 was already at the autarkic steady state, should not be controversial. We will explain Assumption 3 in more detail in the context of Proposition 5.

We are now ready to characterize the long-run equilibrium of the world economy.

Proposition 4 (World steady state). *Under Assumption 2, the world economy has a locally stable steady state such that $n > r^*$.*

Proof. It suffices to show that the (world) interest rate is between the autarkic interest rates: $r_1^{autss} > r^* > r_2^{aut}$, because we assumed that $r_1^{autss} = n$ (see the Appendix for a proof). ■

Our first step is to study trade at the time of China's financial integration. For instance, $t = 0$ could correspond to 1980. That the world capital market is open means that the young can lend both to domestic and foreign firms. As it might be expected, the pattern of trade at the openness will depend on the two countries' initial conditions.

Proposition 5 (Financial integration). *Under Assumption 3, (i) the developed country is the lender country and runs a trade surplus at $t = 0$; (ii) the developed country runs a trade deficit at $t = 1$.*

Proof. The proof is in Appendix A. ■

The proof shows that k_1^* is pinned down by total savings at $t = 0$, which depend on the two countries' initial conditions. At the outset of financial integration, a realistic scenario is one

¹²Lemma 1 established that there exists at least a stable steady state for the world economy. In this section, we restrict attention to those paths converging to a stable steady state.

in which capital flows to the capital scarce, emerging country. To impose that $k_{2,0} < k_{1,0}$ is not enough because while country 1 has a higher wage, there is the negative partial equilibrium effect of the pay-as-you-go on country 1's savings to take into account. Therefore, we need more stringent conditions for country 1 to save more and therefore lend to country 2 (Assumption 3). Only in the initial period, the balance of trade of country 1 is equivalent to the net foreign asset position. At $t = 1$, the developed country's current account position turns into deficit: the old in country 2 pay off their debt and country 1 now starts to borrow (by Proposition 1).

In the previous section, we established that capital flows to the developed country for $t \geq 1$ and that the sign of the balance of trade depends on whether the interest rate is higher or lower than the population growth rate. Here, we study the dynamics of net foreign assets and the balance of trade in more detail. We restrict our analysis to the case in which both the utility and the production functions are Cobb-Douglas, since it is analytically tractable. Figure D.3 illustrates the path of capital accumulation in the world economy under such assumptions on technology and preferences:

Assumption 4. *The utility and the production functions are Cobb-Douglas:*

$$\begin{aligned} U(c_{i,t}^t, c_{i,t+1}^t) &= \beta \log c_{i,t}^t + (1 - \beta) \log c_{i,t+1}^t \\ F(K_{i,t}, L_{i,t}) &= K_{i,t}^\alpha L_{i,t}^{1-\alpha} \end{aligned}$$

We fully derive the model under Assumption 4 in Appendix C.

Proposition 6 (The dynamics of net foreign assets). *Under Assumptions 2, 3 and 4, country 1 (the developed country) accumulates net foreign liabilities as the world economy converges to the (inefficient) world steady state.*

Proof. In Appendix C, we show that equation (18) for country 1 becomes under Cobb-Douglas preferences:

$$a_{1,t+1}^* = -\frac{\rho_2 \tau_1}{1+n} \left(1 - \beta \frac{\alpha k_{t+1}^{*\alpha-1} - \delta - n}{1 + \alpha k_{t+1}^{*\alpha-1} - \delta} \right)$$

Our goal is to study how $a_{1,t+1}$ changes with k_{t+1} :

$$\frac{\partial a_{1,t+1}}{\partial k_{t+1}} = \frac{\rho_2 \tau_1}{1+n} \frac{\beta \alpha (\alpha - 1) (1+n) k_{t+1}^{\alpha-2}}{(1 + \alpha k_{t+1}^{\alpha-1} - \delta)^2}$$

Assumption 3 implies that the initial conditions of the autarkic economies are such that the initial capital stock of the world economy (k_1^*) is to the left of the world steady state. Given that $\frac{\partial a_{1,t+1}}{\partial k_{t+1}} < 0$, country 1's net foreign liabilities increase as the capital stock accumulates. ■

Proposition 3 established that the sign of country 1's balance of trade at a given t depends on whether the current and next period's interest rates are lower or bigger than n . It should now be evident that trade dynamics depends both on the initial conditions and the long-run properties of the autarkic economies. By Proposition 2 and 4, we already know that country 1 will run a deficit in the long-run. In the next proposition, we study the dynamics of trade imbalances.

Proposition 7 (The dynamics of global imbalances). *Under Assumptions 2, 3 and 4, the balance of trade of country 1 deteriorates as the world economy converges to the (inefficient) world steady state.*

Proof. The balance of trade of country 1 under Cobb-Douglas preferences is (see Appendix C):

$$tb_{1,t}^* = \rho_2 \tau_1 \left[(1 - \beta) \frac{\alpha k_t^{*\alpha-1} - \delta - n}{1 + n} + \beta \frac{\alpha k_{t+1}^{*\alpha-1} - \delta - n}{1 + \alpha k_{t+1}^{*\alpha-1} - \delta} \right]$$

The balance of trade of country 1 decreases with the current and the future capital stock:

$$\frac{\partial tb_{1,t}}{\partial k_t} = \frac{\rho_2 \tau_1 (1 - \beta) \alpha (\alpha - 1) k_t^{\alpha-2}}{1 + n} < 0 \quad \frac{\partial tb_{1,t}}{\partial k_{t+1}} = \frac{\rho_2 \tau_1 \beta \alpha (\alpha - 1) k_{t+1}^{\alpha-2} (1 + n)}{(1 + \alpha k_{t+1}^{\alpha-1} - \delta)^2} < 0$$

As $k_1^* < k^*$, the balance of trade of country 1 decreases as the capital stock converges to the world steady state. ■

We can now compare the time-series of the US current account and net international position with the predictions of the model. Figure D.4 provides an illustration of the dynamics of the balance of trade given the results of this section. After an adjustment period of two years, the US current account deteriorates while converging to the world steady state in which the US runs a trade deficit. Our framework seems to be more successful in capturing the dynamics of global imbalances with respect to other two-country models, e.g. Angeletos et al. [1], Caballero et al. [7], Mendoza et al. [24]. In these papers, the United States run a trade deficit immediately after China's financial integration (or a shock), and then the deficit gradually improves. Our framework is more consistent with the data as it predicts the *gradual* deterioration of the US deficit (Figure D.1).

The second aspect of interest is the dynamics of US foreign assets. Proposition 6 establishes that US net foreign liabilities accumulate over time (see Figure D.5). Figure D.6 shows the time-series of the US net foreign assets: the pattern predicted by the model is indeed very similar. In this respect, the contribution of this paper is to explain why the US net foreign assets position turned negative *before* the emergence of global imbalances. In the above papers, the net foreign assets position and the balance of trade turn negative simultaneously.

Finally, we show that the data provide some support for the hypothesis that there is an excess of savings in the world economy. For the developed country to run a trade deficit, it is required that the world interest rate is below the population growth rate (see equation (23)). Therefore, the "global saving glut hypothesis" can be assessed by looking at the data on these two variables.

Let us start with the real interest rate. Figure D.6 indicates that the deterioration of the US international position is due to a steady increase in net external debt (private and public), which has reached 40% of GDP in 2007. It is known that the US have a positive position as far as FDI and equity holdings are concerned, which mitigates the impact of foreign debt on the overall position. As a matter of principle, the interest rate that the US earn on FDI and equity assets might be different from the interest rate that the US pay on its external debt. However, this does not change the fact that the accumulation of foreign debt drives the net foreign assets position of the US. Therefore, the rates of interest on the US government bonds are reasonable proxies for the world interest rate. Figure D.7 plots the time-series of the real interest rate on US Treasury Bills and 10-years government bonds. While interest rates were quite high in the early 1980s, they have embarked on a negative trend since then. This is consistent with our description of the world economy as in the process of transition towards the world steady state.

As far as growth is concerned, it is obvious that population growth is only one of the sources of output growth. In fact, the model can be easily extended to include labour-augmenting tech-

nological progress where technology grows at a common rate g in the two countries. In this case, we show in Appendix B that equation (23) becomes:

$$\hat{t}b_i^* \approx (n + g - r^*)\hat{a}_i^* \quad (24)$$

where the hat denotes variables per effective worker. In this extended version of the model, the presence of a global saving glut now requires that $n + g > r^*$. In our model, $n + g$ is the long-run output growth rate of the world economy¹³. To check whether the inequality is satisfied, we then calculate the average output growth rate of the world economy between 1980 and 2010. The world economy includes those countries which are the focus of this paper, i.e. China, the United States and the Asian Tigers¹⁴. We find that the average output growth rate of the world economy for this period has been 7.3%. This figure is likely to overestimate $n + g$, as it does also include transitional growth. In fact, our explanation for the decline of real interest rates and the gradual build-up of global imbalances is the consequence of the capital accumulation process that has taken place in the world economy following the financial integration of East Asian countries. Alternatively, we can consider the US growth rate as a lower bound for the long-run growth rate of the world economy. According to Caballero et al. [7], average real output growth in the US is equal to 3.33%. Figure D.7 shows that real interest rates have started to fall below the US growth rate in the late 1990s/early 2000s, which is when global imbalances emerged. The gap between the real interest rate and the growth rate of the world economy has widened during the last decade, which is consistent with the slow deterioration of the US current account deficit against East Asian countries.

We can conclude that there is evidence that the United States have accumulated a trade deficit because high saving rates in East Asian countries (due to their poor pay-as-you-go systems) have been pushing the real interest rate below the long-run growth rate of the world economy.

A final word is due about dynamic inefficiency. In our setup, we assume that the US economy was at the golden rule before integrating with inefficient (emerging) countries. We have shown that the consequence is that the integrated economy is overaccumulating capital. Part of the literature is of the view that actual economies are not dynamically inefficient (see De La Croix [9] for a discussion, p. 84). These statements are often based on early tests on the dynamic efficiency of stochastic OLG economies. More recently, these empirical findings have been questioned by Chattopadhyay [8], who has shown that a widely used criterion to test dynamic efficiency, the net dividend criterion, does not actually give sufficient conditions for optimality. The results of this paper emphasize that the capital overaccumulation case is not only interesting from a theoretical point of view, as it has something to tell us on relevant stylized facts such as global imbalances.

4.1. Country size

In this section, we show that country size has an impact on capital flows and trade dynamics. In the following Proposition, we establish that the steady state capital stock of the world economy is increasing in country 2's size.

Proposition 8. *Let k_{ρ_2, ρ_1}^* and $k_{\tilde{\rho}_2, \rho_1}^*$ be the steady state capital stocks of two economies, for which $\tilde{\rho}_2 > \rho_2$. Then, $k_{\tilde{\rho}_2, \rho_1}^* > k_{\rho_2, \rho_1}^*$.*

¹³In reality, countries do not grow at the same rate. See the section below for a discussion on this point.

¹⁴The Asian tigers include Taiwan, South Korea, Hong Kong and Singapore. We take the countries' PPP-converted GDP at current prices from Heston A., Summers R., Aten B., Penn World Table Version 7.1, Center for International Comparisons of Production, Income and Prices at the University of Pennsylvania, July 2012.

Proof. The logic of the proof is the same as for Proposition 4. Consider equation (A.4) in the Appendix. In Lemma 1(ii), we have proved that there exists a stable steady state k_{ρ_2, ρ_1}^* such that $g(k_{\rho_2, \rho_1}^*) = 0$. Now consider another economy for which $\tilde{\rho}_2 > \rho_2$. It is straightforward that if $k_{\tilde{\rho}_2, \rho_1} = k_{\rho_2, \rho_1}$, then $g(k_{\rho_2, \rho_1}) < 0$. Proposition 4 already showed that the function g is increasing in k if the steady state is stable. Hence, it must be true that $k_{\tilde{\rho}_2, \rho_1}^* > k_{\rho_2, \rho_1}$ for $\tilde{g}(k_{\tilde{\rho}_2, \rho_1}^*) = 0$. ■

Proposition 8 shows that the higher is country 2's size, the more severe is capital overaccumulation in the world economy. Moreover, country size is important as far as the size of capital flows are concerned. First, the net foreign liabilities per capita of country 1 are increasing with the size of country 2 (equation (18)). Together with the fact that $n - f(k^*)$ is increasing with capital, global imbalances are also larger if ρ_2 is higher (equations (23)). If we allowed for technological progress, we would still have that the capital stock of the world economy is increasing in $\hat{\rho}_2$ (see Appendix B).

As a matter of fact, $\hat{\rho}_i$ is the most appropriate measure for country size as country i 's share of world savings would depend on country i 's share of total productivity Z , as well as on the size of the population. Then, a higher productivity in the United States would compensate for China's bigger population to some extent¹⁵. Using that $\frac{Y_i^*}{L_i Z_i} \equiv \hat{y}_i^* = \hat{y}_{i,t}^* \equiv \frac{Y_{i,t}^*}{L_{i,t} Z_{i,t}}$, we can rewrite $\hat{\rho}_i$ as follows:

$$\hat{\rho}_i = \frac{L_{i,t} Z_{i,t}}{L_i Z_i} = \frac{Y_{i,t}^*}{Y_i^*}$$

Therefore, $\hat{\rho}_2$ can be computed by calculating East Asian countries' share of total GDP, where total GDP is the sum of the GDP of the US, China and the Asian tigers. The data show that East Asia's share has exponentially increased over time, from 21% in 1980 to 50% in 2010¹⁶.

Taking into account both productivity and population differentials, the data indicate that country 2's size has increased over time. This is not surprising given the high growth rate of these countries over the recent period. Proposition 8 can then explain why the trade imbalances between the US and East Asian countries are so large: East Asian countries' share of world GDP is increasingly relevant. However, this is only a comparative statics effect. Since we have assumed that $\hat{\rho}_2$ is constant, the model cannot capture the fact country 2's size has grown over time. A way to take into account this empirical observation would be to assume that $g_2 > g_1$ ¹⁷. Under this restriction, $\hat{\rho}_2$ can be written as follows:

$$\begin{aligned} \hat{\rho}_{2,t} &= \frac{L_{2,t} Z_{2,t}}{L_t Z_t} \\ \hat{\rho}_{2,t} &= \frac{L_{2,0}}{L_0} \frac{(1 + g_2)^t Z_{2,0}}{(1 + g_1)^t Z_{1,0} + (1 + g_2)^t Z_{2,0}} \\ \hat{\rho}_{2,t} &= \frac{L_{2,0}}{L_0} \frac{1}{\left(\frac{1+g_1}{1+g_2}\right)^t \frac{Z_{1,0}}{Z_{2,0}} + 1} \end{aligned} \quad (25)$$

¹⁵As a matter of fact, a simple way to account for the fact that China is poorer than the US is to assume that $Z_{2,0} < Z_{1,0}$. We thank Antonia Díaz and Timothy Kehoe for having raised this point.

¹⁶In the Penn World Tables, there are two sets of data for China due to measurement problems. The above shares are derived from China's version 2 of the Penn World Tables. For China version 1, the shares are 15% in 1980 and 48% in 2010.

¹⁷In fact, the average population growth rates have been about 1% in both countries. See footnote 8.

In the limit, the size of country 2 tends to its population size:

$$\lim_{t \rightarrow \infty} \hat{\rho}_{2,t} = \lim_{t \rightarrow \infty} \frac{L_{2,0}}{L_0} \frac{1}{\left(\frac{1+g_1}{1+g_2}\right)^t \frac{Z_{1,0}}{Z_{2,0}} + 1} = \frac{L_{2,0}}{L_0}$$

However, the size of country 1 tends to zero:

$$\lim_{t \rightarrow \infty} \hat{\rho}_{1,t} = \lim_{t \rightarrow \infty} \frac{L_{1,0}}{L_0} \frac{1}{1 + \left(\frac{1+g_2}{1+g_1}\right)^t \frac{Z_{2,0}}{Z_{1,0}}} = 0$$

In the long-run, the United States becomes a small open economy as they do not contribute to the determination of prices in the world economy. This is a standard implication of neoclassical growth models, as Gourinchas and Jeanne [18] observe: “that countries have the same growth rate in the long run is a standard assumption, often justified by the fact that no country should have a share of world GDP converging to 0 or 100 percent.”. In a two-country model with exogenous growth, the question of the importance of China’s spectacular economic growth as far as capital flows and global imbalances are concerned cannot be addressed in a satisfactory way. Understanding the mechanism behind China’s high growth, high saving rate and current account surplus remains one of the most important directions for future research¹⁸.

5. Cross-country evidence

Our theory predicts that the country with no pay-as-you go system lends to the pay-as-you-go country and runs a trade deficit even in the long-run. In Table 1, we presented evidence showing that although most East Asian countries have schemes that are classified as pay-as-you-go systems, the percentage of the labour force or working population actually covered is considerably lower than in the US. Therefore, we argued that the poor coverage of the pay-as-you-go systems is a plausible reason behind the high saving rates in East Asia and the trade imbalances against the US.

In this section, we test the hypothesis that differences in the coverage of pay-as-you-go systems is an important factor towards explaining the cross-country variation in the saving rates and net capital flows. For this purpose, we run two sets of regressions. Firstly, we examine whether an increase in the coverage induces a fall in private savings¹⁹. A similar exercise was done by Samwick [26] for an early-1990s sample. In this paper, we take advantage of the fact that we now have a much richer dataset in terms of sample size and pension systems’ characteristics. Differently from Samwick [26], we also estimate the effect of pay-as-you-go coverage on the current account balance as a percentage of GDP.

Cross-sectional data on pension systems coverage are from the World Bank database²⁰. Country-level observations have been collected or updated over the past ten years for most countries in the world. As the most recent information on coverage was obtained in 2011 for some

¹⁸See Song et al. [27] for an endogenous growth model which deal with these facts in a small open economy setting.

¹⁹Figure 2 shows the time-series of gross national savings as a percentage of GDP instead of private savings due to lack of sufficiently long time-series for the latter. However, as pay-as-you-go systems’ coverage affect private savings and not (at least directly) public savings, we use private savings as a percentage of GDP to assess the quantitative importance of our mechanism in the cross-sectional analysis.

²⁰Appendix E contains details on variable definitions, data sources and summary statistics for the variables used in the regressions.

countries, we run our cross-country regressions for year 2011. The World Bank also provides a three pillars classification of pension systems. Pillar 0 refers to the so-called “social pensions” in “recognition of their social-policy goal of offering a safety-net, minimum income” to the old (Pallares-Miralles et al. [22]). Pillar 1 includes all mandatory publicly managed schemes, while mandatory privately managed schemes are grouped under Pillar 2. It turns out that many countries have pension schemes under more than one pillar. However, no data is available on the funding characteristics of such schemes. Therefore, we need to come up with some criterion to classify countries in two groups: those with a pay-as-you-go system and those without. For robustness purposes, we construct two different samples based on alternative definitions of pay-as-you-go systems.

According to the first definition, we say that a country has a pay-as-you-go system whenever it has any of the following programs: a basic or universal pension (Pillar 0); a defined-benefit (DB) or a notional defined-contribution scheme (NDC) under Pillar 1. Therefore, we work under the assumption that DB and NDC schemes are pay-as-you-go. Pallares-Miralles et al. [22] stress that this is the case for the majority of such publicly managed schemes. The first sample does then include all the countries which have a pay-as-you-go pension system under this definition.

The second definition is somewhat stricter: a country has a pay-as-you-go system if it does not have any provident funds (PF) or privately managed scheme. While countries with a mixed pension system would be included in the sample under the previous criterion, such countries are excluded under the second one as their pension system is not purely pay-as-you-go. Therefore, the second sample only includes countries with a “pure” pay-as-you-go system²¹.

Our baseline regression for private savings is the following:

$$\text{private savings \% GDP}_i = \alpha + \beta \text{ coverage rate \% labour force}_i + \gamma \cdot X_i + \varepsilon_i \quad (26)$$

where X is a vector of control variables. We will also run the same regression using a broader definition of the coverage rate, which include the unemployed in addition to the labour force:

$$\text{private savings \% GDP}_i = \alpha + \beta \text{ coverage rate \% working population}_i + \gamma \cdot X_i + \varepsilon_i \quad (27)$$

Estimates for equations (26) and (27) are reported respectively in Table 2 and 3. The first two columns are the results obtained using the first sample, while the last two column are the estimates for the sample of countries with a “pure” pay-as-you-go system.

The estimates support our theory that the higher is the percentage of the population covered by the pay-as-you-go system, the lower is the saving rate of a country. Coefficients are strongly significant and are not sensitive to alternative classifications of the pay-as-you-go systems or different coverage rates. It is worth stressing that a simple OLS regression of private savings on the coverage rate is not statistically significant and has a positive coefficient. In fact, the coverage rate is strongly correlated with GDP per capita²²: as one might expect, richer countries have a more inclusive pension system. Therefore, omitting GDP per capita would considerably bias our results. We also control for other important determinants of aggregate savings, such demographic factors and financial markets’ development. Our demographic control variable is the

²¹We exclude the countries which do not have a pay-as-you-go system from both samples as the presence of zeros could bias our results. In our original sample of 126 countries, 16% (29%) of the countries do not have a (“pure”) pay-as-you-go system.

²²In the first sample, the correlation coefficient between the coverage rate as a percentage of the labour force (working population) and GDP per capita is 0.79 (0.84).

Table 2: OLS regression of private savings % GDP, 2011

	PAYGO (1)	PAYGO (2)	pure PAYGO (1)	pure PAYGO (2)
<i>Coverage rate % labour force</i>	0.0292 (0.0292)	-0.1156*** (0.0423)	0.0334 (0.0336)	-0.1257** (0.0540)
GDP per capita		0.0001 (0.0001)		0.0001 (0.0001)
GDP per capita growth		0.3254* (0.1782)		0.2993 (0.2082)
Age dependency ratio		-0.2810*** (0.0667)		-0.3090*** (0.0752)
Domestic credit % GDP		0.0141 (0.0220)		0.0146 (0.0250)
Constant	21.6453*** (1.8227)	41.2410*** (5.2597)	21.2901*** (2.0071)	43.1934 *** (5.9532)
R^2	0.0114	0.2744	0.0134	0.2967
No. of countries	89	83	75	69

Notes: The standard errors are reported in parenthesis. *** indicates significance at the 1% level, ** at the 5% level and * at the 10% level.

Table 3: OLS regression of private savings % GDP, 2011

	PAYGO (1)	PAYGO (2)	pure PAYGO (1)	pure PAYGO (2)
<i>Coverage rate % working population</i>	0.0326 (0.0349)	-0.1508*** (0.0568)	0.0363 (0.0401)	-0.1917** (0.0782)
GDP per capita		0.0002* (0.0001)		0.0002* (0.0001)
GDP per capita growth		0.3885** (0.1795)		0.3698* (0.2061)
Age dependency ratio		-0.2138*** (0.0597)		-0.2409*** (0.0671)
Domestic credit % GDP		0.0090 (0.0219)		0.0138 (0.0252)
Constant	21.9366*** (1.5723)	36.3423*** (4.7298)	21.6641*** (1.7198)	43.1934 *** (5.9532)
R^2	0.0091	0.2379	0.0100	0.2637
No. of countries	97	90	83	76

Notes: The standard errors are reported in parenthesis. *** indicates significance at the 1% level, ** at the 5% level and * at the 10% level. Data are for 2011.

Table 4: OLS regression of current account % GDP, 2011

	PAYGO (1)	PAYGO (2)	pure PAYGO (1)	pure PAYGO (2)
<i>Coverage rate % labour force</i>	0.0769*** (0.0290)	-0.0852*** (0.0298)	0.0712** (0.0312)	-0.1114*** (0.0337)
GDP per capita		0.0002** (0.0001)		0.0002** (0.0001)
GDP per capita growth		-0.0527 (0.1774)		-0.0790 (0.2104)
Age dependency ratio		-0.2491*** (0.0673)		-0.2724*** (0.0771)
Domestic credit % GDP		-0.0036 (0.0219)		-0.0013 (0.0245)
Investment % GDP		-0.5563*** (0.1515)		-0.5587*** (0.1551)
Government budget balance % GDP		0.7830*** (0.1901)		0.7777*** (0.2223)
Constant	-6.6864*** (1.8251)	28.3371*** (6.4144)	-6.5336*** (2.3257)	30.3995*** (7.2093)
R^2	0.0740	0.5595	0.0578	0.5569
No. of countries	90	83	75	68

Notes: Robust standard errors are reported in parenthesis. *** indicates significance at the 1% level, ** at the 5% level and * at the 10% level.

Table 5: OLS regression of current account % GDP, 2011

	PAYGO (1)	PAYGO (2)	pure PAYGO (1)	pure PAYGO (2)
<i>Coverage rate % working population</i>	0.1038*** (0.0317)	-0.1023*** (0.0389)	0.0985*** (0.0343)	-0.1626*** (0.0501)
GDP per capita		0.0002*** (0.0001)		0.0003*** (0.0001)
GDP per capita growth		-0.0303 (0.1678)		-0.0417 (0.1931)
Age dependency ratio		-0.1794*** (0.0618)		-0.2008*** (0.0708)
Domestic credit % GDP		-0.0076 (0.0220)		-0.0005 (0.0244)
Investment % GDP		-0.5239*** (0.1425)		-0.5305*** (0.1421)
Government budget balance % GDP		0.8672*** (0.1908)		0.8732*** (0.2175)
Constant	-6.3926*** (1.8549)	23.0271*** (5.7123)	-6.2701*** (1.9354)	25.0292*** (6.4100)
R^2	0.0823	0.5509	0.0657	0.5530
No. of countries	98	90	83	75

Notes: Robust standard errors are reported in parenthesis. *** indicates significance at the 1% level, ** at the 5% level and * at the 10% level.

age dependency ratio, which is defined as the ratio of young and old population to the working population. Consistently with what we would expect, the regression coefficient is negative and statistically significant in all our regressions. The dependency ratio in Sub-Saharan countries is extremely higher than in East Asian and developed countries and this explains why the average saving rate of Sub-Saharan countries is not significantly different from developed countries despite the coverage of their pay-as-you-go system is much worse than in East Asian countries. On the other hand, our proxy for financial market development (domestic credit as a percentage of GDP) is never significant. This is in contrast with most of the theoretical literature on global imbalances, which stresses the importance of the heterogeneity of financial markets for saving rate differentials and net capital flows (Caballero et al. [7], Mendoza et al. [24], Angeletos et al. [1]). As a robustness check, we used as an alternative proxy the percentage of the population older than 15 with a credit card. The higher is this percentage, the higher should be agents' ability to borrow and therefore the lower the saving rate. However, the proxy is not significant and our coefficients are not significantly affected (if not slightly improved). We also controlled for the size of public pension spending, life expectancy and oil rents as a percentage of GDP. However, these variables are neither significant nor their introduction does change our estimates. Therefore, we exclude them from our regressions.

Therefore, these results validate the idea that pension system characteristics are important when it comes to explaining cross-country variations in the saving rate. This is also consistent with the findings of Samwick [26]. We can now discuss the magnitude of the coefficients. For instance, let us consider the coverage rate differential between the US and China. According to Table 1, this differential is as large as 58.7% (43.7%) in relation to the labour force (working population). A decrease in the coverage rate of these magnitudes induces an increase in the saving rate between 6.59% and 8.37%. Since the saving rate differential between the US and China is 24.52%, we can say that our mechanism is also quantitatively important.

We now repeat the same exercise using current account as a percentage of GDP as dependent variable. In this case, we also control for the government budget and investment as a percentage of GDP. We also use robust standard errors as the Breusch-Pagan test suggests that we have to reject the hypothesis of homoskedastic errors. Table 4 and 5 report our estimates under the two coverage rates.

In the first and third column, we can observe that the higher is the pay-as-you-go coverage the higher is the current account balance of a country. As before, this result is biased as it reflects the fact that we omit GDP per capita from this regression. Once we control for GDP per capita and include the rest of the controls, we find a statistically significant negative relationship between the current account and the pay-as-you-go coverage rate. As for private savings, this finding is robust to alternative definitions of pay-as-you-go systems and coverage rates.

To have an idea of the magnitudes, let us consider again the coverage rates' differential between the United States and China. Keeping everything else constant, a lower coverage induces an improvement of the current account balance as a percentage of GDP of the order of 5.9822% to 7.1057%. This is a quantitatively significant channel, considering that the current account balance differential between the two countries is 4.806%.

6. Conclusions and policy implications

This paper takes seriously Bernanke's hypothesis that global imbalances are due to a global saving glut. We have proposed a model in which a global excess of savings arises because of the financial integration between the United States and dynamically inefficient economies, which

have a higher propensity to save than the US because they have a poor pay-as-you-go pension system. The increase in world savings had as long-run effects the drop of real interest rates and the emergence of global imbalances. These and other stylized facts can be read through the lens of this model.

The model indicates that both the current direction of trade and the low real interest rates as compared to growth in the world economy are signals that the world economy is on an inefficient path. If that was not the case, the theory says that United States' trade balance should be either zero or in surplus and we should also observe much higher interest rates. Pension reforms in China in the direction of improving the coverage of their pay-as-you-go system would increase domestic demand and therefore reduce world savings. The US deficit towards China would shrink, which is the outcome that many politicians and economists seem to hope for. In accordance with the theory, our cross-country analysis shows that a higher coverage of the pay-as-you-go system is associated with a lower saving rate and current account balance as percentages of GDP.

This paper clearly abstracts from two important, possibly related, facts: (1) China has a higher investment ratio; (2) while the United States have a negative net international position overall, they have a positive position in foreign direct investments. The key step to understand these facts would be to introduce more assets, which requires the introduction of uncertainty in the model. We leave this for future research.

Acknowledgments

This work was mostly done during my PhD in York and is based on the first chapter of my thesis. I thank my supervisor Subir Chattopadhyay for many useful discussions and comments. I am grateful to the editor Timothy Kehoe and two anonymous referees for suggestions which have considerably improved the paper. I also thank Yves Balasko, Mauro Bambi, Monserrat Pallares-Miralles, Neil Rankin, Gianpaolo Rossini, Gabriel Talmain, participants at the Research Students' Workshop and the General Equilibrium Days 2011 in York, the XVII Workshop in Dynamic Macroeconomics in Vigo, the RES Autumn School 2012, the Warwick PhD Economics Conference 2013, the RES Annual Conference 2013 and the EEA Annual Congress 2013 for the many inputs received and the Department of Economics and Related Studies in York for financial support.

References

- [1] Angeletos G.-M., Panousi V. (2011), Financial integration, entrepreneurial risk and global dynamics, *Journal of Economic Theory*, 146, 863-896.
- [2] Asher et al. (2005), Social Security Reform in China: Issues and Options, Policy Study of the China Economic Research and Advisory Programme, <http://economics.mit.edu/files/691>.
- [3] Azariadis C. (1993), *Intertemporal Macroeconomics*, Blackwell Publishers, Oxford.
- [4] Bernanke B.S. (2005), The Global Saving Glut and the U.S. Current Account Deficit, speech delivered for the Sandridge Lecture at the Virginia Association of Economists, Richmond, March 10, <http://www.federalreserve.gov/boarddocs/speeches/2005/200503102/>
- [5] Boersch A., Finke R. (2008), Pension systems and market trends in Asia-Pacific, *Conference Proceedings of the OECD/IOPS Global Private Pensions Forum*, Beijing, November 2007, 24-34.
- [6] Buiter W.H. (1981), Time Preference and International Lending and Borrowing in an Overlapping-Generations Model, *Journal of Political Economy*, 89 (4), 769-797.
- [7] Caballero R.J., Fahri E., Gourinchas P.O. (2008), An Equilibrium Model of "Global Imbalances" and Low Interest Rates, *American Economic Review*, 98 (1), 358-393.

- [8] Chattopadhyay S. (2008), The Cass criterion, the net dividend criterion and optimality, *Journal of Economic Theory*, 139, 335-352.
- [9] De La Croix D., Michel P. (2002), A Theory of Economic Growth, Cambridge University Press, Cambridge.
- [10] Diamond P.A. (1965), National Debt in a Neoclassical Growth Model, *American Economic Review*, 55(5), 1126-1150.
- [11] Feldstein M., Liebman J.B. (2002), Social Security, in *Handbook of Public Economics*, Chapter 32, Volume 4, edited by A.J. Auerbach and M. Feldstein, 2246-2324.
- [12] Feng J., Lixin H., Sato H. (2011), Public pension and household saving: Evidence from urban China, *Journal of Comparative Economics*, 39, 470-485.
- [13] Gale D. (1971), General Equilibrium with Imbalance of Trade, *Journal of International Economics*, 1 (2), 141-158.
- [14] Gale D. (1974), The Trade Imbalance Story, *Journal of International Economics*, 4 (2), 119-137.
- [15] Galor O., Ryder H.E. (1989), Existence, Uniqueness, and Stability of Equilibrium in an Overlapping-Generations Model with Productive Capital, *Journal of Economic Theory*, 49, 360-375.
- [16] Geide-Stevenson D. (1998), Social Security Policy and International Labour and Capital Mobility, *Review of International Economics*, 6(3), 407-416.
- [17] Global Imbalances and Financial Stability (2011), *Financial Stability Review*, 15, Banque de France.
- [18] Gourinchas P., Jeanne O. (2013), Capital Flows to Developing Countries: The Allocation Puzzle, forthcoming, *Review of Economic Studies*.
- [19] Lane P.R., Milesi-Ferretti G.M. (2007), The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970 - 2004, *Journal of International Economics*, 73 (2), 223-250.
- [20] Lucas R.E. (1990), Why Doesn't Capital Flow from Rich to Poor Countries?, *American Economic Review*, 80, 92-94.
- [21] OECD (2011), Pensions at a Glance 2011: Retirement-income Systems in OECD and G20 Countries, OECD Publishing.
- [22] Pallares-Miralles M., Romero C., Whitehouse E. (2012), International Patterns of Pension Provision II, A Worldwide Overview of Facts and Figures, Social Protection and Labor Discussion Paper No. 1211, World Bank.
- [23] Polemarchakis H.E., Salto M. (2002), Balance-of-Payments Equilibrium and the Determinacy of Interest Rates, *Review of International Economics*, 10 (3), 459-468.
- [24] Mendoza E.G., Quadrini V., Ríos-Rull J.-V. (2009), Financial Integration, Financial Development, and Global imbalances, *Journal of Political Economy*, 117(3), 371-416.
- [25] Samuelson P.A. (1975), Optimum Social Security in a Life-Cycle Growth Model, *International Economic Review*, 16 (3), 539-544.
- [26] Samwick A. (2000), Is Pension Reform Conducive to Higher Savings?, *The Review of Economics and Statistics*, 82 (2), 264-272.
- [27] Song Z., Storesletten K., Zilibotti F. (2011), Growing Like China, *American Economic Review*, 101, 202-241.
- [28] Song Z.M., Tao Yang D. (2010), Life Cycle Earnings and Saving in a Fast-Growing Economy, unpublished.

Appendix A. Proofs

Proof of Lemma 1

(i) Take equation (15) for any $t \geq 1$ and define the function g as follows:

$$g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) \equiv (1+n)k_{t+1} - [\rho_1 s_1(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), \tau_1) + \rho_2 s_2(f(k_t) - f'(k_t)k_t, f'(k_{t+1}))]$$

We want to establish the existence of $k_{t+1} > 0$ given $k_t > 0$, such that $g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) = 0$. To do that, we study the sign of g as k_{t+1} tends to infinity and zero. The first limit tells us that g is positive for k_{t+1} approaching infinity:

$$\lim_{k_{t+1} \rightarrow +\infty} g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) = +\infty \quad (\text{A.1})$$

(savings are always bounded above by w_t). Therefore, for at least a $k_{t+1} > 0$ to exist we need:

$$\lim_{k_{t+1} \rightarrow 0} g(k_{t+1}; k_t, \tau_1, \rho_1, \rho_2) < 0 \quad (\text{A.2})$$

When $\rho_1 = 1$ (closed economy), De La Croix et al. [9] show that it is enough that the young's income after tax is strictly positive for savings to be positive, as savings are increasing in income. In particular, the following condition must hold : $w_t > \tau_1$. It turns out that the same condition is valid in a two-country economy. It is not sufficient that aggregate savings are positive, since we only allow for strictly positive consumption. For an equilibrium to exist, we need both countries' savings to be positive.

Now, define $\bar{\tau}_1(k_t)$ as the level of tax for which savings are zero in country 1 (it is obvious that $\bar{\tau}_1$ is increasing in k_t). Therefore, as long as $\tau_1 < \bar{\tau}_1(k_t)$, equation (A.2) is satisfied and therefore k_{t+1} exists.

We now prove that k_{t+1} is unique given k_t . By Assumption 1, g is increasing in k_{t+1} :

$$g'(k_{t+1}) = 1 + n - s_r f''(k_{t+1}) > 0 \quad \forall k_{t+1}$$

This is enough to ensure uniqueness. We can then write

$$k_{t+1} = \phi(k_t; \tau_1, \rho_1, \rho_2)$$

which is a single-valued, strictly increasing function in k_t ²³.

The above discussion is also valid at $t = 0$. It follows that if $\tau_1 < \bar{\tau}_1(k_{1,0})$ at time 0, $k_1 > 0$ exists given $(k_{1,0}, k_{2,0})$ and is unique. A unique intertemporal equilibrium will exist by induction.

(ii) We know already that the saving locus of the economy is increasing. Suppose that

$$\lim_{k_t \rightarrow 0} \frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} > 1$$

For the saving locus to cross the 45 degree line from above at least once, we need to show that the following is true:

$$\lim_{k_t \rightarrow +\infty} \frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} < 1 \quad (\text{A.3})$$

The argument is the same as for closed economies and relies on the fact that savings can never exceed the wage (see Azariadis [3], p. 84). Since

$$(1 + n)k_{t+1} = \rho_1 s_{1,t} + \rho_2 s_{2,t} \leq w_t$$

that condition (A.3) is satisfied can be shown by dividing both sides of the inequality by k_t and then taking the limit:

$$\lim_{k_t \rightarrow +\infty} \left[\frac{\phi(k_t; \tau_1, \rho_1, \rho_2)}{k_t} \right] \leq \frac{1}{1+n} \lim_{k_t \rightarrow +\infty} \left[\frac{f(k_t)}{k_t} - f'(k_t) \right] = 0$$

This proves the existence of at least one locally stable steady state.

Proof of Lemma 2

²³See Galor et al. [15] for a throughout study of the function ϕ .

Consider the budget constraints of the agents born at t at equilibrium:

$$\begin{aligned} c_{1,t}^{t*} + \frac{c_{1,t+1}^{t*}}{1+r_{t+1}^*} &= w_t^* - \tau_1 \frac{r_{t+1}^* - n}{1+r_{t+1}^*} \equiv I_{1,t}^* \\ c_{2,t}^{t*} + \frac{c_{2,t+1}^{t*}}{1+r_{t+1}^*} &= w_t^* \equiv I_{2,t}^* \end{aligned}$$

It is easy to see that the two agents will always have different budget sets, except in the case $r^* = n$ where $I_{1,t}^* = I_{2,t}^*$. Iff $n > r_{t+1}^*$, $I_{1,t}^* > I_{2,t}^*$. Because of that, note that the budget line of agent 1 is to the right of agent 2's budget line. It is parallel as they face the same interest rate r_{t+1}^* . Marginal rates of substitutions of the two agents are obviously equalized:

$$1 + r_{t+1}^* = \frac{u'(c_{1,t}^{t*})}{\beta v'(c_{1,t+1}^{t*})} = \frac{u'(c_{2,t}^{t*})}{\beta v'(c_{2,t+1}^{t*})}$$

Because utility functions are identical across agents and consumption goods are normal, we can conclude that $c_{1,t}^{t*} > c_{2,t}^{t*}$ and $c_{1,t+1}^{t*} > c_{2,t+1}^{t*}$. If $r_{t+1}^* > n$, the opposite is true.

Proof of Proposition 4

Let k_2^{aut} be the level of capital such that country 2 is at the autarkic steady state, and define the function g_2 as follows:

$$g_2(k_2^{aut}) \equiv (1+n)k_2^{aut} - s_2(f(k_2^{aut}) - f'(k_2^{aut})k_2^{aut}, f'(k_2^{aut})) = 0$$

where

$$g_2'(k_2^{aut}) = 1 + n + s_w f''(k_2^{aut})k_2^{aut} - s_r f''(k_2^{aut})$$

When the steady state is stable, $g_2'(k_2^{aut}) > 0$ as

$$\frac{dk_{2,t+1}}{dk_{2,t}}(k_2^{aut}) = \frac{-f''(k_2^{aut})k_2^{aut} s_w}{1+n - s_r f''(k_2^{aut})} < 1$$

Similarly, let k^* be the steady state world capital stock and define the function g for the world economy:

$$\begin{aligned} g(k^*; \tau_1, \rho_1, \rho_2) &\equiv (1+n)k^* - [\rho_1 s_1(f(k^*) - f'(k^*)k^*, f'(k^*), \tau_1) + \\ &+ \rho_2 s_2(f(k^*) - f'(k^*)k^*, f'(k^*))] = 0 \end{aligned} \quad (A.4)$$

Now suppose that $k^* = k_2^{aut}$. From equation (11), we know that country 1 saves less than country 2 for any k , then $g(k_2^{aut}; \tau_1, \rho_1, \rho_2) > 0$. Note that $g'(k_2^{aut}; \tau_1, \rho_1, \rho_2) = g_2'(k_2^{aut})$, and therefore for g to be zero k must fall. It follows that $k^* < k_2^{aut}$.

Similarly, it can be shown that $k_1^{autss} < k^*$. Diminishing returns to capital implies that $r_1^{autss} > r^* > r_2^{aut}$.

Proof of Proposition 5

(i) At $t = 0$, the world capital market clears if the following equation holds:

$$(1+n)k_1^* = \rho_1 s_1 (f(k_{1,0}) - f'(k_{1,0})k_{1,0}, f'(k_1^*), \tau_1) + \rho_2 s_2 (f(k_{2,0}) - f'(k_{2,0})k_{2,0}, f'(k_1^*))$$

Under Hypothesis 2, $s_{1,0}^* > s_{2,0}^*$. By Proposition 1, it follows that:

$$a_{1,1}^* > 0 \quad a_{2,1}^* < 0$$

Because of no trade in the previous period, the countries' trade balances will only reflect the current trade in the capital market: $tb_{i,0}^* = (1+n)a_{i,1}^*$. Hence:

$$tb_{1,0}^* > 0 \quad tb_{2,0}^* < 0$$

(ii) Let us write the balance of trade of country 1 at $t = 1$:

$$tb_{1,1}^* = (1+n)a_{1,2}^* - (1+r_1^*)a_{1,1}^*$$

Because $a_{1,2}^* < 0$ (Proposition 1) and we have shown that $a_{1,1}^* > 0$, then $tb_{1,1}^* < 0$.

Appendix B. Technological progress

The aim of this section is to show how to get the condition for country 1 to run a trade deficit in the long-run under labour-augmenting technological progress (equation (24)). Under this assumption, the production function is still homogeneous of degree one in the two arguments:

$$Y_{i,t} = F(K_{i,t}, Z_{i,t}L_{i,t}) \quad Z_{i,t} = (1+g)Z_{i,t-1}$$

where, in principle, $Z_{1,0} \neq Z_{2,0}$.

We define $\hat{k}_{i,t} \equiv \frac{K_{i,t}}{Z_{i,t}L_{i,t}}$ as capital per effective worker. The first-order conditions of the firms now become:

$$\begin{aligned} r_t &= f'(\hat{k}_{i,t}) - \delta \\ \hat{w}_t &= f(\hat{k}_{i,t}) - f'(\hat{k}_{i,t})\hat{k}_{i,t} \end{aligned}$$

where $\hat{w}_t \equiv \frac{w_{i,t}}{Z_{i,t}}$.

Taxes must grow at the same rate of technological progress, for the tax to have an impact on savings in the long-run: $\tau_{1,t} = (1+g)\tau_{1,t-1}$. At each t , because $L_{1,t}\tau_{1,t} = L_{1,t-1}b_{1,t}$ must hold, $b_{1,t} = \tau_{1,t}(1+n)$. Therefore, the budget constraints are:

$$\begin{aligned} c_{i,t}^f &= w_{i,t} - \tau_{i,t} - s_{i,t} \\ c_{i,t+1}^f &= s_{i,t}(1+r_{t+1}) + \tau_{i,t}(1+n)(1+g) \end{aligned}$$

where $\tau_2 = 0$. The market clearing condition for capital expressed in capital per effective worker becomes:

$$\hat{k}_{t+1}^*(1+n)(1+g) = \hat{\rho}_1 \hat{s}_{1,t}^* + \hat{\rho}_2 \hat{s}_{2,t}^*$$

where $\hat{\rho}_i \equiv \frac{L_{i,t} Z_{i,t}}{L_i Z_i}$. Following the same steps as in section 3, we derive the balance of trade per effective worker for country 1:

$$\hat{t}b_{1,t}^* \equiv [\hat{s}_{1,t}^* - (1+n)(1+g)\hat{k}_{t+1}^*] - \frac{1+r_t^*}{(1+n)(1+g)}[\hat{s}_{1,t-1}^* - \hat{k}_t^*(1+n)(1+g)]$$

which at the steady state simplifies as follows:

$$\hat{t}b_1^* = [(1+n)(1+g) - (1+r^*)]\hat{a}_1^* \approx \hat{a}_1^*(n+g-r^*)$$

where $\hat{a}_1^* \equiv \frac{A_{1,t}^*}{Z_{1,t} L_{1,t}}$.

Appendix C. A Cobb-Douglas Example

In this section, we derive the model for Cobb-Douglas utility and production functions:

$$U(c_{i,t}^t, c_{i,t+1}^t) = \beta \log c_{i,t}^t + (1-\beta) \log c_{i,t+1}^t \quad (\text{C.1})$$

$$f(k_t) = k_t^\alpha \quad (\text{C.2})$$

We study this example in some detail as our variables of interest have a simpler dynamics with Cobb-Douglas functions.

From profit maximization, the factor prices are:

$$r_t = \alpha k_t^{\alpha-1} - \delta \quad (\text{C.3})$$

$$w_t = (1-\alpha)k_t^\alpha \quad (\text{C.4})$$

The saving functions in the two countries are:

$$s_{1,t} = (1-\beta)(w_t - \tau_1) - \beta\tau_1 \frac{1+n}{1+r_{t+1}} \quad (\text{C.5})$$

$$s_{2,t} = (1-\beta)w_t \quad (\text{C.6})$$

It is known that, with log-utility, savings are a constant fraction of the wage and do not depend on the rate of interest. In country 1, the young also consume a fraction of the discounted future transfer.

Overall, the impact of the pay-as-you-go system on country 1's savings is:

$$\frac{\partial s_{1,t}}{\partial \tau_1} = -(1-\beta) - \beta \frac{1+n}{1+r_{t+1}} = -1 + \beta \frac{r_{t+1} - n}{1+r_{t+1}} \quad (\text{C.7})$$

The market clearing equation for capital is:

$$\begin{aligned} K_{t+1}^* &= L_{1,t} \left[(1-\beta)((1-\alpha)k_t^{*\alpha} - \tau_1) - \beta\tau_1 \frac{1+n}{1+\alpha k_{t+1}^{*\alpha-1} - \delta} \right] + \\ &+ L_{2,t}(1-\beta)(1-\alpha)k_t^{*\alpha} \end{aligned} \quad (\text{C.8})$$

The capital stock evolves over time as follows:

$$(1+n)k_{t+1}^* = (1-\beta)(1-\alpha)k_t^{*\alpha} - \rho_1 \tau_1 \left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k_{t+1}^{*\alpha-1} - \delta} \right] \quad (\text{C.9})$$

while the steady state capital stock satisfies:

$$(1+n)k^* = (1-\beta)(1-\alpha)k^{*\alpha} - \rho_1\tau_1 \left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k^{*\alpha-1} - \delta} \right] \quad (C.10)$$

For any given $k_t > 0$, it can be verified that $k_{t+1} > 0$ exists as long as $(1-\alpha)k_t^\alpha - \tau_1 > 0$ (see Lemma 1) and that the higher is τ_1 , the lower k_{t+1} will be given k_t . It can also be checked that the saving locus is increasing (here, $s_r = 0$):

$$\frac{dk_{t+1}}{dk_t} = \frac{(1-\beta)\alpha(1-\alpha)k_t^{\alpha-1}}{(1+n) - \frac{\rho_1\tau_1\beta(1+n)\alpha(\alpha-1)k_{t+1}^{\alpha-2}}{(1+\alpha k_{t+1}^{\alpha-1} - \delta)^2}} > 0 \quad (C.11)$$

The specific feature of this example is that the saving locus is concave as $\frac{d^2 k_{t+1}}{(dk_t)^2} < 0$. Figure D.3 illustrates the dynamics of capital stock per capita in the world economy.

With $\tau_1 = 0$, it is known that there exists a globally unique steady state with Cobb Douglas utility and production function. With $\tau_1 > 0$, the number of steady states depends on how big is the tax. If the tax is small enough, then there are two steady states (one unstable and one stable). At a certain threshold for the tax, the steady state is not hyperbolic and above that we have non-existence of steady states. See De La Croix et al. [9] for a detailed discussion²⁴.

We can now compute the net foreign assets position of country 1:

$$a_{1,t+1}^* = -\frac{\rho_2\tau_1}{1+n} \left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k_{t+1}^{*\alpha-1} - \delta} \right] \quad (C.12)$$

At the golden rule k^{GR} and other stationary allocations, a_1 is respectively:

$$a_1^{GR} = -\frac{\rho_2\tau_1}{1+n} \quad (C.13)$$

$$a_1^* = -\frac{\rho_2\tau_1}{1+n} \left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k^{*\alpha-1} - \delta} \right] \quad (C.14)$$

Using the capital flows definition (22), we can plug equation (C.12) in and compute the balance of trade of country 1:

$$tb_{1,t}^* = \rho_2\tau_1(1-\beta) \left[\frac{(\alpha k_t^{*\alpha-1} - \delta) - n}{1+n} \right] + \rho_2\tau_1\beta \left[\frac{(\alpha k_{t+1}^{*\alpha-1} - \delta) - n}{1+\alpha k_{t+1}^{*\alpha-1} - \delta} \right] \quad (C.15)$$

When both interest rates are bigger than the population growth rate, it is evident that $tb_{1,t}^* > 0$. Suppose now at a given \bar{t} , k_t^* and k_{t+1}^* are such that $r_t^* > n$ and $r_{t+1}^* < n$. The first part of the equation is positive and reflects the fact that the old in country 2 are consuming more (exports). But part two is negative as the young in country 2 are now consuming less (imports). Which of the two is bigger will also depend on β .

In the long-run, the balance of trade satisfies:

$$tb_1^* = \frac{(\alpha k^{*\alpha-1} - \delta) - n}{1+n} \rho_2\tau_1 \left[(1-\beta) + \frac{\beta(1+n)}{1+\alpha k^{*\alpha-1} - \delta} \right] \quad (C.16)$$

²⁴They discuss a closed economy, but the substance of the argument does not change.

The two representative agents' consumption obeys:

$$c_{1,t}^{f*} = \beta \left[(1 - \alpha)k_t^{\alpha*} - \tau_1 \frac{(\alpha k_{t+1}^{\alpha-1*} - \delta) - n}{1 + \alpha k_{t+1}^{\alpha-1*} - \delta} \right] \quad (C.17)$$

$$c_{1,t+1}^{f*} = (1 + \alpha k_{t+1}^{\alpha-1*} - \delta)(1 - \beta) \left[(1 - \alpha)k_t^{\alpha*} - \tau_1 \frac{(\alpha k_{t+1}^{\alpha-1*} - \delta) - n}{1 + \alpha k_{t+1}^{\alpha-1*} - \delta} \right] \quad (C.18)$$

$$c_{2,t}^{f*} = \beta(1 - \alpha)k_t^{\alpha*} \quad (C.19)$$

$$c_{2,t+1}^{f*} = (1 + \alpha k_{t+1}^{\alpha-1*} - \delta)(1 - \beta)(1 - \alpha)k_t^{\alpha*} \quad (C.20)$$

As we established in Lemma 2, agents born in country 1 consumes more (less) when the world economy happens to be beyond (below) the golden rule allocation.

Appendix D. Figures

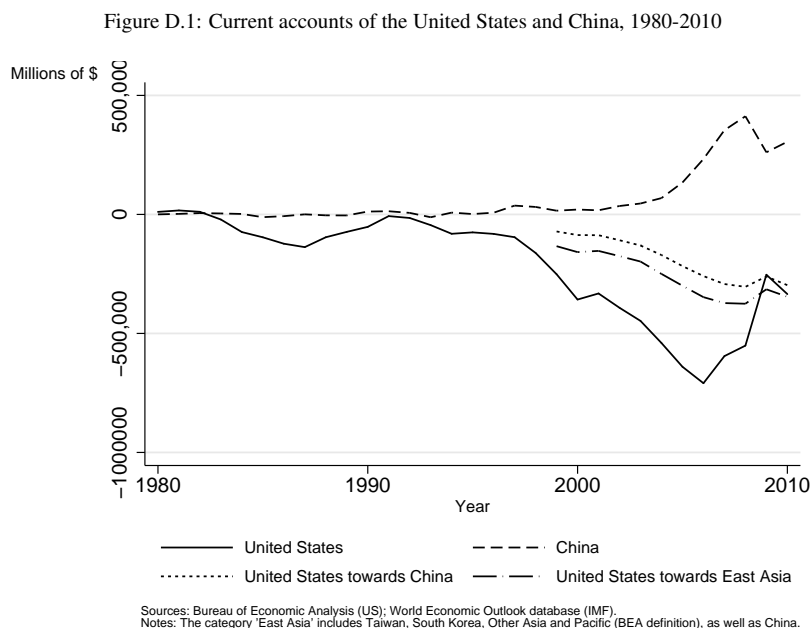


Figure D.2: Gross national savings as percentage of GDP, 1980-2010

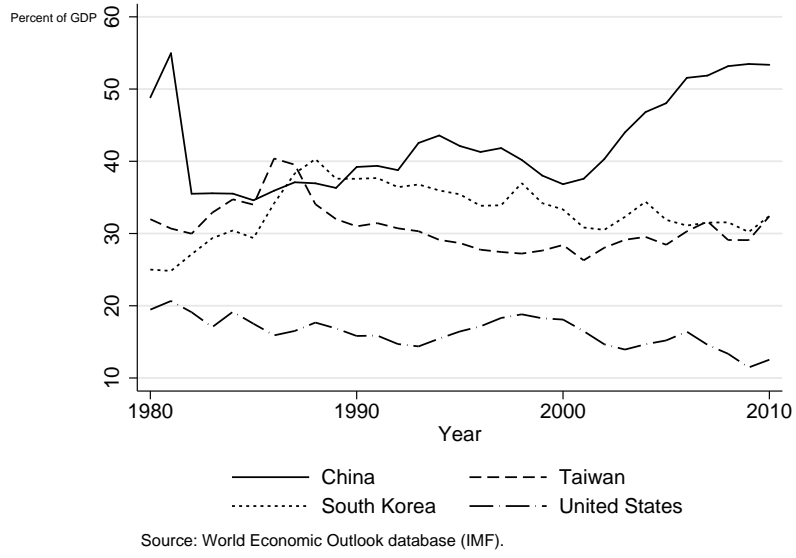


Figure D.3: Capital accumulation in the world economy under Cobb-Douglas utility and production functions

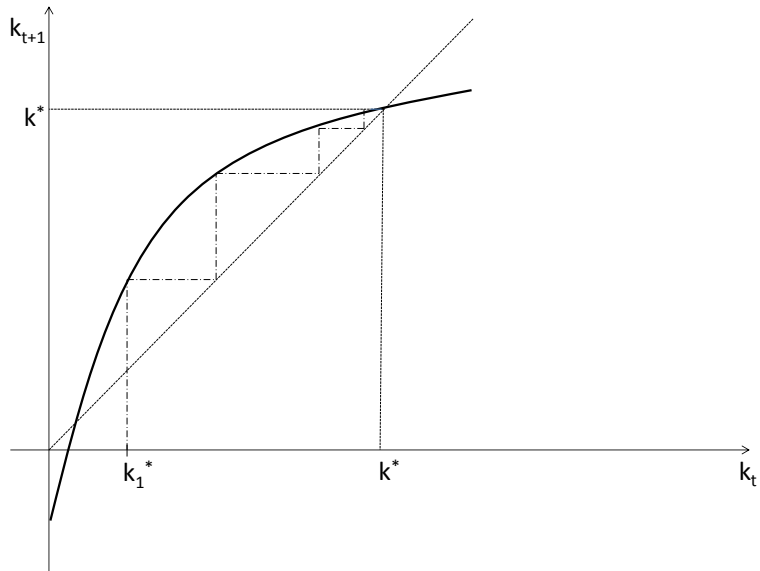


Figure D.4: The dynamics of country 1's balance of trade under Cobb-Douglas utility and production functions

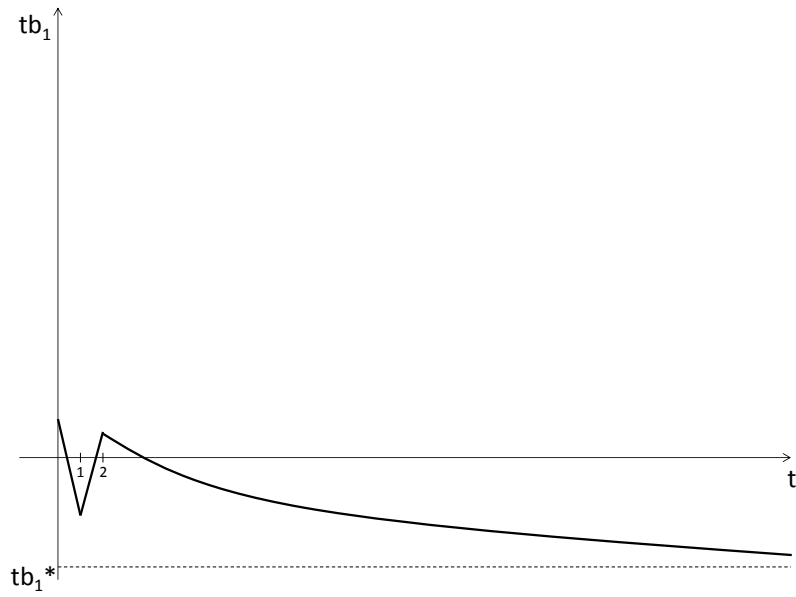


Figure D.5: The dynamics of country 1's net foreign assets under Cobb-Douglas utility and production functions

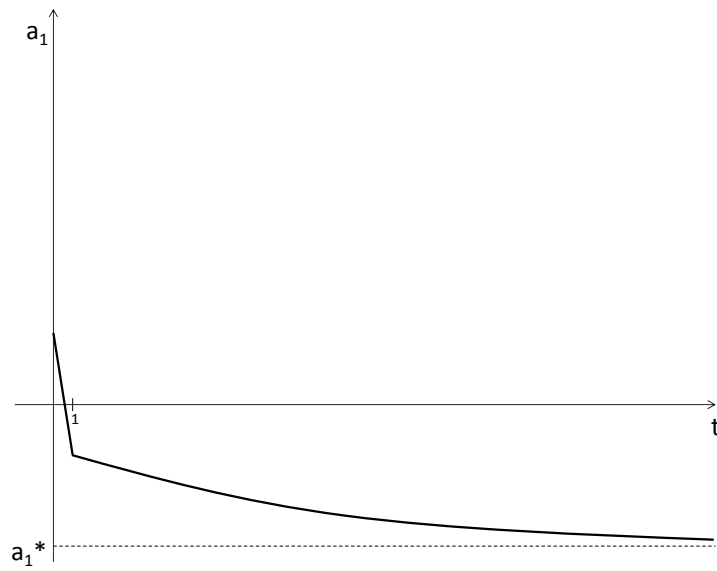
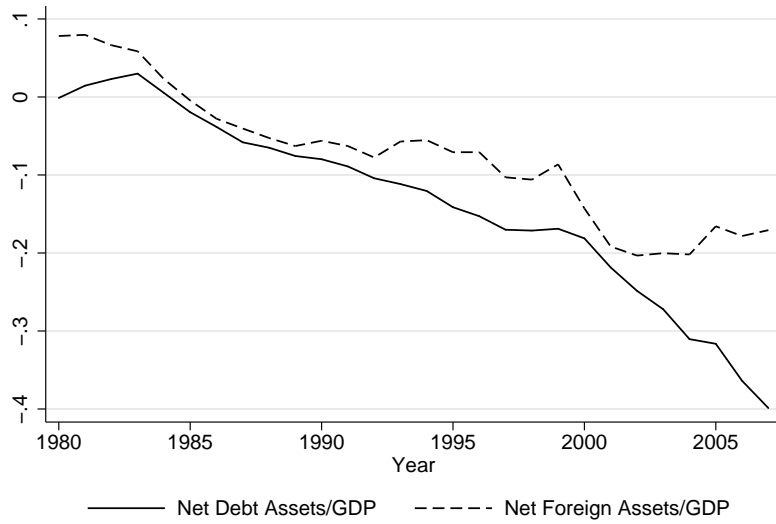
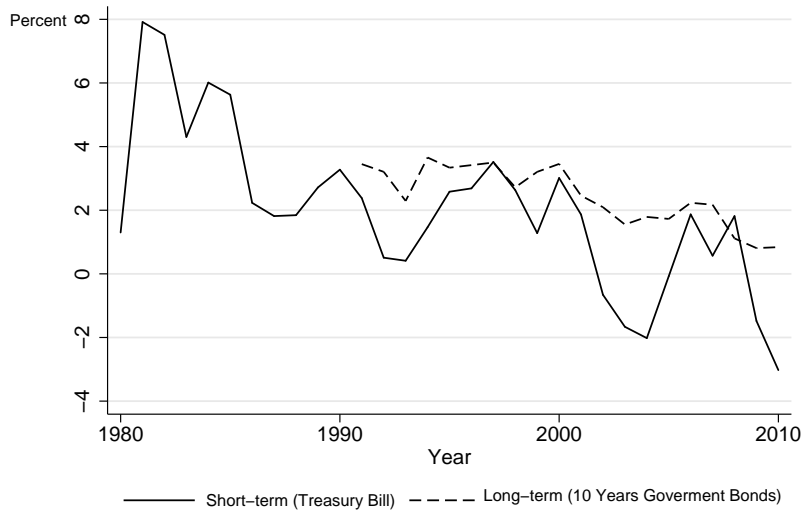


Figure D.6: The United States' net international position, 1980-2007



Sources: Lane and Milesi-Ferretti's database (updated to 2007).

Figure D.7: Real interest rates in the United States, 1980-2010



Notes: IFS data, yearly rates. The Treasury Bill rate is adjusted for the actual CPI. The long-term yield is adjusted for expected inflation from the Survey of Professional Forecasters as in Caballero et al. (2008).

Appendix E. Data

Cross-country data on the pillars' classification and coverage are from the World Bank Pensions Database. Data on aggregate savings, investment, government budget balance, GDP per capita and GDP per capita growth are taken from the IMF World Economic Outlook Database. Data on age dependency ratios, domestic credit and the variables used for robustness checks (mentioned in the text but not included in the tables) are all from the World Bank databases.

Hereafter, we include additional information on the variables used as well as summary statistics:

East Asian countries (Table 1): China, Hong Kong SAR, Republic of Korea, Mongolia, Philippines, Thailand, Vietnam.

Coverage rate % labour force: Total number of active contributors % labour force.

Coverage rate % working population: Total number of active contributors % working age population.

Private savings % GDP: Gross national savings % GDP minus general government net lending/borrowing % GDP.

GDP per capita: Gross domestic product based on purchasing-power-parity (PPP) per capita GDP.

GDP per capita growth: is yearly growth calculated on GDP per capita.

Age dependency ratio: is the ratio of dependents - people younger than 15 or older than 64 - to the working-age population - those aged 15-64. Data are shown as the proportion of dependents per 100 working-age population.

Domestic credit % GDP: Domestic credit to private sector refers to financial resources provided to the private sector, such as through loans, purchases of nonequity securities, and trade credits and other accounts receivable, that establish a claim for repayment. For some countries these claims include credit to public enterprises.

Investment % GDP: Total investment as % GDP

Government budget balance % GDP: General government net lending/borrowing % GDP.

Table E.6: Summary statistics

Variable	Observations	Mean	Standard deviation	Min	Max
Private savings % GDP	104	23.26222	9.124567	-16.919	52.111
Current account % GDP	105	-2.044086	10.73842	-44.853	41.809
Coverage % labour force	91	52.93187	33.65384	1.9	100
Coverage % working population	99	36.57172	26.84493	1.3	93.6
GDP per capita	105	17311.36	15241.74	599.509	78079.91
GDP per capita growth	105	4.612147	4.574742	-13.59288	23.83932
Age dependency ratio	106	57.43044	16.77064	27.9922	110.5901
Domestic credit % GDP	96	69.45359	56.83403	4.913907	208.4063
Investment % GDP	104	23.38087	8.083654	5.476	62.509
Government budget balance % GDP	82	-0.5491707	5.307461	-10.37	25.399