

A NEW APPROACH TO MEASURING THE REBOUND EFFECT ASSOCIATED TO ENERGY EFFICIENCY IMPROVEMENTS: AN APPLICATION TO THE US RESIDENTIAL ENERGY DEMAND

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This version: 17 March 2015

Abstract

This paper brings attention to the fact that the energy demand frontier model introduced by [Filippini and Hunt \(2011, 2012\)](#) is closely connected to the measurement of the so-called rebound effect associated with improvements in energy efficiency. In particular, we show that their model implicitly imposes a zero rebound effect, which contradicts most of the available empirical evidence on this issue. We relax this restrictive assumption through the modelling of a rebound-effect function that mitigates or intensifies the effect of an efficiency improvement on energy consumption. We illustrate our model with an empirical application that aims to estimate a US frontier residential aggregate energy demand function using panel data for 48 states over the period 1995 to 2011. Average values of the rebound effect in the range of 56-80% are found. Therefore, policymakers should be aware that most of the expected energy reduction from efficiency improvements may not be achieved.

Keywords: US residential energy demand; efficiency and frontier analysis; state energy efficiency; rebound effect.

JEL Classification: C5, Q4, Q5.

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Acknowledgments: The authors are indebted to the participants at the EMEE 2014 Workshop in Aachen (Germany) for their helpful suggestions. We would like to thank Thomas Triebs and Beatriz Tovar for their comments on earlier drafts of this paper, and the members of the Oviedo Efficiency Group for their support at various stages during the elaboration of the paper. This research is partially funded by the Government of the Principality of Asturias through its predoctoral grants program. The authors also gratefully acknowledge the financial support of the Spanish Ministry of Science and Innovation through the project ECO2010-17240.

1. Introduction

Reducing energy consumption and emissions is a key policy objective for most governments across the globe and the promotion of energy efficiency policies is seen as a key activity to achieving this goal. In practice, the achievement of savings in energy consumption depends on two issues. First, it is vital that policy makers be able to clearly measure the relative energy efficiency across states and over time. Second, the actual savings in energy consumption might not coincide with the expected savings due to the so-called rebound effect, a phenomenon associated with the consumption of energy and energy services. When the production of an energy service becomes more efficient, then the cost per unit of this service decreases. This cost reduction can produce an increase in the consumption of the energy service that might (at least partially) offset the expected savings in energy consumption derived from the energy efficiency improvements. Measuring the rebound effect is thus crucial in order to properly evaluate the effectiveness of any energy policy instrument that aims to promote energy efficiency improvements.

Regarding the first issue, [Filippini and Hunt \(2011, 2012\)](#) point out that defining and measuring energy efficiency and creating statistical measures as descriptors is a challenging task. They propose the use of a Stochastic Frontier Analysis (SFA) approach to control for characteristics such as the structure of the economy that might bias the usual energy efficiency indicators. These authors illustrate their proposal by estimating an aggregate energy demand frontier model for the total energy consumption of a sample of OECD countries and for the residential energy consumption of the US states. The SFA approach allows them to obtain a “pure” measure of the inefficient use of energy (i.e. ‘waste of energy’) for each country or state.

Concerning the second issue, there is a large number of empirical studies that use econometric methods to estimate the rebound effect. In their review of the literature, [Sorrell and Dimitropoulos \(2008\)](#) have found a lack of consensus with regard to a consistent method to measure the rebound effect. In principle, it could be *directly* obtained from the elasticity of demand for energy services with respect to changes in energy efficiency. However, relatively few studies follow this approach because data on either energy services or energy efficiency are unavailable or are limited in terms of accuracy. As a consequence the rebound effect is often indirectly measured through the estimate of different elasticities that are considered measures of energy efficiency elasticities of the demand for energy, such as the own-price elasticity of the demand for energy.

The main contribution of this paper is to link the energy demand frontier approach with the estimation of the rebound effect. We first bring attention to the fact that the frontier model introduced by [Filippini and Hunt \(2011, 2012\)](#) also provides a direct measure of the rebound effect. However, we point out that a traditional specification of this model implicitly imposes a zero rebound effect, which contradicts most of the available empirical evidence. We next suggest estimating a more comprehensive model to relax the zero rebound effect assumption and examine compliance with some of the restrictions used in previous studies focused on estimating the rebound effect using econometric techniques.

The paper is organized as follows. The next section defines the rebound effect and provides a brief review of the empirical literature on measuring it using econometric models. Both standard and extended energy demand frontier models and the econometric specification of our model are introduced in Section 3. The data and

results of the estimates are presented in Section 4 with a summary and conclusions in the final section.

2. Measuring the rebound effect: a short review of the empirical literature

The rebound effect is a phenomenon associated with energy consumption. This concept has to do with the idea that an increase in the level of efficiency in the use of energy decreases the marginal cost of supplying a certain energy service and hence may lead to an increase in the consumption of that service. This consumer reaction might therefore partially or totally offset the predicted reduction in energy consumption attributed to energy efficiency improvements using engineering models. Measuring the rebound effect is thus crucial in order to properly evaluate the effectiveness of any energy policy instrument that aims to promote energy efficiency improvements. This issue is particularly relevant for the US residential sector since it accounts for 37% of the national electricity consumption, 17% of greenhouse gas emissions and 22% of primary energy consumption ([International Risk Governance Council \(IRGC\), 2013](#)).

The definition of the rebound effect encompasses different mechanisms that may reduce potential energy savings derived from the improvements in energy efficiency. Frequently, three types of rebound effect are distinguished in the specialized literature (see for instance [Greening *et al.*, 2000](#); or [Sorrell and Dimitropoulos, 2008](#)). The first one is the *direct* rebound effect, which measures the increase in the use of the product or service that has experienced the efficiency gain. For instance, a homeowner may employ a portion of the energy savings from using an efficient heater to use the heater for longer periods during the winter to warm the house. The second type is the so-called *indirect* rebound effect and measures the reallocation of energy savings to spending on other goods and services that also require energy. For instance, the savings derived from the use of energy-efficient appliances at home can be spent on travel holidays which may lead to an increase in energy consumption and greenhouse gas emissions. The third type is the *economy-wide* rebound effect and captures the structural changes in the economy due to the variation of prices of goods and services as a consequence of energy efficiency improvements. These changes may produce a new equilibrium in the consumption of goods and services (including energy) in the economy.

The previous taxonomy of rebound effects is focused on end-use energy consumption. However, it should be mentioned that another type of rebound can arise from the productive side of the economy. In fact, [Saunders \(2013\)](#) argues that the energy employed to produce goods and services in the US, represents two-thirds of total energy consumption and hence efficiency gains on this side affect the majority of energy actually consumed. This author finds extremely large rebound effect magnitudes (some of them even greater than 1000%) across 30 US productive sectors, which illustrates the importance of the phenomenon. As this type of effect can be considered a subset of economy-wide rebound and this paper is focused on direct and indirect rebounds of household energy demand, this issue is not going to be analysed in this section.

There is an extensive literature on the concept and measurement of the rebound effect and several approaches have been applied with the aim of quantifying this phenomenon. For instance, in their report for the UK Energy Research Centre, [Sorrell and Dimitropoulos \(2007\)](#) find a wide range of methods that have been applied to measure the direct rebound effect. They identify at least four empirical approaches - single equation models, structural models, discrete/continuous models, and household

production models - and several estimation techniques including ordinary least squares, instrumental variables or maximum likelihood. In addition, several empirical strategies have also been used to indirectly measure this rebound effect. An outline of these approaches can be found in [Table 1](#). This table shows three theoretical relationships between two elasticities. The left-hand side elasticity is the energy efficiency elasticity of the demand for energy, which is used to calculate the clearest and most direct measure of the rebound effect (see [Saunders, 2000](#), and Section 3 below). The lack of accurate data on energy services or energy efficiency typically precludes a direct measurement of the rebound effect based on this elasticity, so that its estimation is usually carried out using the right-hand side of the equations in [Table 1](#).

[Insert Table 1 here]

The first empirical approach relies on estimating the energy efficiency elasticity of the demand for energy services or useful work that is often available in personal transportation studies. For this reason, this engineering-based approach is generally used to measure the direct rebound effect associated with travelling by private cars (see for instance [Greene *et al.*, 1999b](#); or [Small and Van Dender, 2005](#)). More studies follow the second empirical strategy, based on an estimate of the energy cost elasticity of the demand for useful work. This approach has been advocated by [Khazzoom \(1980\)](#), [Greene *et al.* \(1999a\)](#), [Berkhout *et al.* \(2000\)](#) and [Binswanger \(2001\)](#) and, unlike the first approach, it provides a way to estimate the magnitude of the rebound effect even when the available data provides little or no variation in energy efficiency. However, the validity of this approach relies on the assumption that consumers respond in the same way to decreases in energy prices as they do to improvements in energy efficiency (and vice versa). As [Sorrell and Dimitropoulos \(2008\)](#) pointed out, this assumption is likely to be flawed in many cases. These two approaches require accurate measures of the demand for useful work. This restriction has biased research studies towards personal transportation and household heating, where data about energy services can be easily calculated, e.g., vehicle kilometres in the case of transportation.

It is also possible to estimate the direct rebound effect from the own-price elasticity of the demand for energy, i.e., the third approach. While obtaining measures of useful work can be difficult, data on energy demand is more commonly available. The main advantage of the third approach over previous approaches is that data on either useful work or energy efficiency is not required. This explains why the approach based on the own-price elasticity of the demand for energy is the most popular empirical strategy to measure the rebound effect in other energy commodities or sectors (see for instance [Zein-Elabdin, 1997](#); [Berkhout *et al.*, 2000](#); [Roy, 2000](#) and [Bentzen, 2004](#)). However, [Sorrell and Dimitropoulos \(2008\)](#) pointed out that this empirical strategy might also yield biased estimates for the rebound effect if energy efficiency is not explicitly controlled for.¹ In this paper, we propose another approach based on the estimation of an energy demand frontier function. In this framework, the rebound effect is directly estimated from the elasticity of the demand for energy with respect to changes in the level of energy efficiency.

There is a huge variety of estimated rebound effects in the literature not only because different methodological/empirical approaches have been used but also because they have been used to analyse the rebound effect for different energy commodities, sectors, countries or different levels of data aggregation. Since our paper is focused on

¹ In particular, this approach relies on the assumption that energy efficiency is unaffected by changes in energy prices.

residential energy demand, we pay attention mainly to the results of papers on household energy demand. [Sorrell and Dimitropoulos \(2007\)](#) find that for household heating the rebound effect usually ranges from 10 to 58% in the short-term and from 1.4 to 60% in the long-term.² Household energy demand is dominated by the use of fuel and electricity for heating space. Focusing specifically on papers in which the price elasticity of total household electricity demand is estimated, the estimated values suggest an upper bound for the short-term rebound effect in the range of 20 to 35% and between 4% and 225% for the long-term rebound effect. Regarding other household energy services, the reviewed studies suggest a rebound effect up to about 26% for space cooling. Other studies produce rather different results. For instance, [Guertin et al. \(2003\)](#) estimate long-term rebound effects for both water heating and appliances/lighting and obtain values between 32% and 49%.

A recent survey can be found in a report on energy efficiency carried out by the [IRGC \(2013\)](#). This survey is based on the reviews of [Greening et al. \(2000\)](#), [Sorrell \(2007\)](#) and [Jenkins et al. \(2011\)](#) and summarises the large variety of results obtained from papers that measure rebound effects in the residential sector. This report shows that while for residential lighting there is a narrow range of results of the rebound effect from 5 to 12%, in the rest of energy services there is a wider range of values: for space heating the range goes from 2 to 60%, for space cooling from 0 to 50%, for water heating from less than 10 to 40%, and for other consumer energy services from 0 to 49%. As it can be seen, this more updated survey shows very similar values to the report previously mentioned.

However it should be noted that in our paper we estimate a demand function aggregated at state-level for the US residential energy. Therefore our estimated rebound effect captures an overall effect composed of the sum of direct and indirect effects and hence the ideal lower and upper bounds for our estimates are not entirely clear. The literature has identified large positive as well as negative values for the indirect rebound effect, as found in [Thomas and Azevedo \(2013\)](#) for the household case. There are some papers that exhibit large direct rebound effects, such as [Mizobuchi \(2008\)](#) where a rebound effect of about 27% is found for Japanese households although the effect increases to 115% when capital costs are ignored in the analysis. Indirect rebound effects are usually larger than direct rebound effects and it is less ‘uncommon’ to find indirect rebound effects larger than 100%. Some examples can be found in [Lenzen and Dey \(2002\)](#) with an indirect rebound effect of 123% for Australia, [Alfredsson \(2004\)](#) with an indirect rebound effect up to 300% in Sweden or [Brännlund et al. \(2007\)](#) with an indirect rebound effect between 107-115% in CO₂ emissions in a simulation of an efficiency improvement in heating and transport sectors. In some cases this rebound measures can reach extremely large values, as in [Druckman et al. \(2010\)](#) who found indirect rebound effects up to 515% for the case of the UK. Another interesting example can be found in [Fouquet and Pearson \(2012\)](#). These authors estimate price and income elasticities of the demand for lighting in the UK over recent centuries. They found that efficiency improvements in lighting technology resulted in backfire during the nineteenth century, while rebound effects during last century have still shown large values (between 50% and 70%).

² These rebound effects indicate the percentage (expressed in relation to the predicted energy saving) by which the actual energy consumption is larger than the predicted energy consumption after an efficiency improvement. The measuring of the rebound effect is explained in detail in the next section.

3. Measuring rebound effects using energy demand frontier models

In this section, firstly we summarize the aggregate energy demand frontier model proposed by [Filippini and Hunt \(2012\)](#) to measure the level of “underlying energy efficiency” in the US residential sector. Subsequently, we link this model to the literature on the rebound effect and we introduce a more comprehensive model that allows estimating ‘non-zero’ rebound effects using an SFA approach. Once the econometric specification of the model is presented, we finally discuss new econometric issues that appear when the more general SFA approach is used to estimate rebound effects.

3.1. The standard energy demand frontier model

This approach treats energy as a production factor used in combination with other inputs to produce energy services, and attempts to measure inefficiency in the use of input energy as (positive) deviations from an energy demand frontier function that can be estimated for the whole economy or for a given sector. In general terms, the aggregate energy consumption can be written as follows:

$$q = F(Y, P, X, E, \beta)e^v \quad (1)$$

where q is the aggregate energy consumption, Y is the real income, P is the real energy price, β are parameters to be estimated, and X is a set of control variables such as population, average household size, heating degree days, cooling degree days, the share of detached houses, or time dummy variables. While v is the conventional noise term, E is the level of energy efficiency of a particular state. Since the energy efficiency level is not observed by the researcher, [Filippini and Hunt \(2012\)](#) made use of two assumptions in order to estimate equation (1). Firstly, they implicitly assumed that the energy demand function is *separable* in the sense that $F(Y, P, X, E, \beta)$ in (1) is decomposed into a function that does not depend on energy efficiency and an energy-efficiency function, that is:

$$F = f(Y, P, X, \beta)h(E) \quad (2)$$

where $h(E)$ is in turn assumed to be equal to $1/E$ and implies the existence of neutral efficiency gains in the model. The second assumption is that the unobserved energy efficiency term is bounded (i.e. $0 \leq E \leq 1$). These two assumptions allow using the stochastic frontier approach as the model to be estimated can now be written in logs as:

$$\ln q = \ln f(Y, P, X, \beta) + v + u \quad (3)$$

where $u = -\ln E \geq 0$. The error term in (3) thereby comprises two independent parts. The first part, v , is the classical symmetric random noise, often assumed to be normally distributed, i.e. $v \sim N(0, \sigma_v^2)$. The second part, u , is a one-sided error term capturing the level of underlying energy inefficiency that can vary across states and over time. Following [Aigner et al. \(1977\)](#) it is often assumed to follow a half-normal distribution, i.e. $u \sim N^+(0, \sigma_u^2)$. The identification of both random terms in this model (ALS henceforth) relies on the asymmetric and one-sided distribution of u . If the inefficiency term could take both positive and negative values, it cannot be distinguishable from the noise term, v .

Equation (3) is the basic specification of the energy demand frontier that is estimated in [Filippini and Hunt \(2011, 2012\)](#) in order to obtain state-specific energy

efficiency scores.³ In the case of an aggregate residential energy demand function, $f(Y, P, X, \beta)$ reflects the demand of the residential sector of a state that has *and* uses fully efficient equipment and production processes. If a state is not on the frontier, the distance from the frontier measures the level of energy consumption above the minimum demand of reference, i.e. the level of energy inefficiency. Nevertheless, from an empirical perspective, the aggregate level of energy efficiency of US residential appliances is not observed directly, and therefore has to be estimated simultaneously with other parameters of the model. For this reason [Filippini and Hunt \(2011, 2012\)](#) use the expression ‘underlying energy efficiency’.⁴

3.2. The (implicit) rebound effect in the standard energy demand frontier model

Although the basic concept of the rebound effect is not controversial, several mathematical definitions of this effect have been employed in the literature according to the availability of price and efficiency data.⁵ Here we use the definition mentioned by [Saunders \(2000\)](#) which, in our opinion, provides one of the clearest and most direct measurements of the rebound effect. Following this author, the rebound effect is obtained as:

$$R = 1 + \varepsilon_E \quad (4)$$

where ε_E is the elasticity of energy demand with respect to changes in energy efficiency, i.e. $\varepsilon_E = \partial \ln q / \partial \ln E$. [Table 2](#) shows the different rebound effects that we can find in a particular empirical application. The actual saving in energy consumption will only be equal to the predicted saving from engineering calculations when this elasticity is equal to minus one and hence there is no rebound effect ($R=0$). The rebound effect would be positive ($R>0$) if actual savings in energy consumption are less than expected, i.e. $-1 < \varepsilon_E$. The rebound effect could be larger than one ($R>1$) if improvements in energy efficiency increase energy consumption and hence the elasticity of energy demand with respect to changes in energy efficiency is positive, i.e. $\varepsilon_E > 0$. This somewhat counterintuitive outcome is termed ‘backfire’ in the literature ([Saunders, 1992](#)). In practice, negative rebound effects ($R<0$) can also be found for some observations if the improvements in energy efficiency produce larger decreases in energy use than predicted, i.e. $\varepsilon_E < -1$. [Saunders \(2008\)](#) labelled this -also rather counterintuitive- outcome as ‘super-conservation’.⁶

[Insert Table 2 here]

As the one-sided error term in (3) is measuring the level of underlying energy inefficiency, the elasticity of energy demand with respect to changes in energy efficiency is simply $\varepsilon_E = -\partial \ln q / \partial u$. Given the rebound effect definition provided by

³ The estimation of (3) can be performed using either cross-sectional or panel data as in [Filippini and Hunt \(2011, 2012\)](#). They also propose a relatively simple log-log functional form.

⁴ [Filippini and Hunt \(2011, 2012\)](#) advocate using panel data techniques to control for potential endogeneity problems caused by omitted variables or unobserved heterogeneity, an issue that is briefly discussed later on.

⁵ See, for instance, [Sorrell and Dimitropoulos \(2008\)](#).

⁶ For a more extended definition and some examples about this counterintuitive phenomenon see [Saunders \(2008\)](#).

equation (4), we can then conclude that any energy demand frontier model that includes an inefficiency term as an explanatory variable implicitly provides a *direct* measure of the rebound effect. However, since ε_E in (3) is equal to -1 , the standard SFA energy demand frontier model implicitly imposes a zero rebound effect, which contradicts most of the available empirical evidence surveyed in Section 2.

So far we have shown the implications of the standard SFA energy demand frontier model on the measurement of rebound effects. Next we will discuss the implications of the rebound effect story on both identification and measurement of the underlying energy efficiency. A key conclusion that one can get from the extensive literature focused on measuring the relationship between energy efficiency and energy demand is that the rebound effect tends to attenuate, exacerbate, or even reverse the effect of improvements in energy efficiency on energy consumption.⁷ Therefore, the rebound effect issue can be introduced in an *energy* demand application of the SFA approach as a *correction factor* $(1-R)$ that interacts with the energy inefficiency term (u) that is appended to the stochastic energy demand frontier. That is:

$$\ln q = \ln f(Y, P, X, \beta) + v + (1-R)u \quad (5)$$

where again $u = -\ln E \geq 0$. In this model, the effect on energy consumption is not necessarily proportional to the reduction in u ; its effect is attenuated when the rebound effect is partial (i.e. when $0 < R < 1$), exacerbated in case of super-conservation outcomes (i.e. when $R < 0$), or reversed in case of extremely large rebound effects or backfire outcomes (i.e. when $R > 1$).

Another interesting conclusion that can be inferred from the above equation is that any effort to improve energy efficiency of the current set of appliances (or their use) would not produce any change in energy consumption if consumers' reaction completely offset the potential energy savings, and hence the rebound effect is full. This implies that, in an energy demand setting, the underlying level of energy efficiency cannot be identified and estimated if $R=1$, since the energy demand model only have one (and symmetric) error term in this case. The other way around, this discussion suggests that it only makes sense to estimate a stochastic energy demand frontier model when we believe that the rebound effect is not 100%.

3.3. An energy demand frontier model with non-zero rebound effects

Let us move to the estimation of a frontier energy demand model with non-zero rebound effects. To achieve this objective we should deal with several practical issues. The first one has to do with R in (5) that, like the energy inefficiency level, is not observed by the researcher because it is linked to the *demand for energy services*, ES , again a latent variable. To deal with this issue R can be approximated with a set of determinants of the demand for energy services, such as income and energy prices, i.e. $z=(Y,P)$. This seems to be reasonable as most of the literature on the rebound effect

⁷ It is not easy to find a similar phenomenon in efficiency and productivity works where SFA models have traditionally been applied. In that literature any improvement in firms' efficiency is assumed to have a proportional effect on firms' performance (outputs, cost, etc.). Just to conjecture an example, a sort of rebound effect might appear in public firms where employees' salary is not linked to their productivity. In this case, an employee who works efficiently could become "lazy" after a salary improvement since his earnings do not depend on his effort. Another example of rebound effect may also occur when labour productivity increases but this does not lead to one-for-one reductions in employment.

associates the rebound effect with energy prices, and the theory often predicts that the rebound effect declines with income.⁸

If we replace the rebound effect variable R by a rebound-effect function, $R(\gamma'z)$, the model that can be estimated in practice is:

$$\ln q = \ln f(Y, P, X, \beta) + v + [1 - R(\gamma'z)]u \quad (6)$$

where γ are new parameters to be estimated. Several interesting remarks should be made regarding this specification. First, if the rebound-effect function does not depend on any covariate, our model simply collapses to the basic stochastic frontier demand model used in [Filippini and Hunt \(2011, 2012\)](#) that imposes zero rebound effects. In contrast, if $R(\gamma'z)$ varies across observations or states, the above equation allows us to get state-specific rebound effects that can be used for further analyses. Interesting enough, if z includes income and energy prices, the estimated γ can also be used to test whether both income and price elasticities of energy demand depend on energy efficiency.⁹

Second, unlike in production economics where a similar correction factor to our R function is often treated as part of firms' inefficiency, we point out in this paper that $R(\gamma'z)$ is also -or mainly- capturing a rather different in nature phenomenon, i.e. the rebound effects associated to improvements in energy inefficiency.

Third, several specifications of $R(\gamma'z)$ can be used in a particular empirical application. [Saunders \(2008\)](#) recommends using extremely comprehensive (flexible) functional forms such as the Gallant and Fourier forms, which can depict the full range of rebound values. These forms are however intractable in our framework as they would interact with the stochastic part of the model and, hence, the maximum likelihood function would be highly non-linear in parameters. In this sense, as the choice of a particular function in this setting is limited by both methodological and practical issues, we propose exploring two simple rebound-effect functions:

$$R(\gamma'z) = \frac{e^{\gamma'z} - 1}{e^{\gamma'z}} \quad (7)$$

$$R(\gamma'z) = \frac{e^{\gamma'z}}{1 + e^{\gamma'z}} \quad (8)$$

Whereas the rebound-effect function in (7) can depict any value lower than full rebound and even allows to obtain *super-conservation* (SC) outcomes (i.e. $R < 1$), the rebound-effect function in (8) precludes this somewhat counter-intuitive outcome as it only allows for *partial* (PA) rebounds-effects (i.e. $0 < R < 1$). In both cases, a positive

⁸ [Wang et al. \(2012\)](#) point out, for instance, that the marginal utility of energy service consumption will decline as household income increases. Thus, energy efficiency improvements may not induce people to consume as much energy services as before. This means that the direct rebound effect might decline with the increase in household income. This is also confirmed in a limited number of studies, e.g. [Small and Van Dender \(2007\)](#) and [Wang et al. \(2012\)](#) that have found evidence of a negative relationship between the rebound effect and income.

⁹ Indeed, both elasticities can be respectively written as:

$$\varepsilon_Y = \frac{\partial \ln q}{\partial \ln Y} = \frac{\partial \ln f(Y, P, X, \beta)}{\partial \ln Y} + \frac{\partial R(Y, P, \gamma)}{\partial \ln Y} \ln E$$

$$\varepsilon_P = \frac{\partial \ln q}{\partial \ln P} = \frac{\partial \ln f(Y, P, X, \beta)}{\partial \ln P} + \frac{\partial R(Y, P, \gamma)}{\partial \ln P} \ln E$$

(negative) value of γ indicates that the rebound effect increases (decreases) with z . It is worth noting that the SC and PA functions are respectively equal to 0 and 0.5 when $\gamma'z=0$. This might occur when either all γ parameters are zero, or when R does not include a constant term and $z=0$.¹⁰

It should be noted that both specifications (7) and (8) of the rebound effect preclude the existence of backfire and full rebound outcomes. This is not a coincidence as we must impose the restriction $R < 1$ to our rebound-effect functions in order to distinguish inefficiency from noise. Otherwise, the second error term in equation (6) would no longer have a one-sided distribution and then we would not be able to take advantage of the asymmetric distribution of u to decompose the overall error term into two different stochastic components.

Other specifications have been examined in previous versions of this paper, such as the simple cumulative density function of a standard normal variable, Φ , which like the PA function lies between zero and one, or the ratio $\Phi/(1-\Phi)$, which allows for super-conservation outcomes as does the SC model. The results of these models are not shown in the paper as they are very similar to those obtained with the proposed models.

Finally, equations (6) with specification (7) or (8) for the rebound-effect function cannot be estimated if R includes a separated intercept and we assume that $\sigma_u = e^{\delta_0}$, where δ_0 is a parameter to be estimated as in the ALS model. This problem can be easily noticed in the case of the SC function, where our correction factor is also a simple exponential function and hence $(1-R) = e^{-\gamma_0 - \gamma'z}$. In this case, the overall one-sided error term $(1-R) \cdot u$ includes two intercepts (γ_0 and δ_0), but only one can be estimated. In other words, the estimated intercept of the rebound-effect function is *biased* because it also captures the parameter δ_0 that measures the standard deviation of the energy inefficiency term, u , that is, $\hat{\gamma}_0^{SC} = \gamma_0 - \delta_0$.¹¹ As only one constant is allowed, a simple empirical strategy is proposed to deal with this issue. This strategy relies on the assumption that our energy inefficiency term follows the same distribution in both equations (3) and (6), so that the ALS estimate of δ_0 is used to adjust the estimated intercept of $(1-R)$ accordingly.¹²

4. Data and results

Our empirical application is based on a balanced US panel data set for a sample of 48 states over the period 1995 to 2011. That is, we have added four years to the data set used in [Filippini and Hunt \(2012\)](#). For the purposes of this paper attention is restricted to the contiguous states (i.e. Alaska and Hawaii are excluded) except Rhode Island because of incomplete information: The District of Columbia is included and

¹⁰ Most explanatory variables are centred at the sample mean to attenuate convergence problems when estimating the model using maximum likelihood techniques. Hence, $z=0$ for the representative observation.

¹¹ It is worth mentioning that this issue is not important in production economics as both options would yield exactly the same results when modelling *overall* firm inefficiency. However, if we estimate an energy demand frontier function, it matters as we would be either magnifying or diminishing the rebound effect.

¹² For similar grounds, the PA model cannot be estimated with the two intercepts. The adjustment procedure here is not as trivial as when we use the SC model because R in (8) is an index, and the adjustment formula should keep R to take values between zero and one. Further information on these adjustments can be found in Appendix A.

considered as a separate ‘state’. The dataset is based on information taken from three sources. Residential energy consumption quantities and prices are provided by the Energy Information Administration (EIA). Population and real disposable personal income are from the Bureau of Economic Analysis of the US Census Bureau and the heating and cooling degree days are obtained from the National Climatic Data Center at NOAA. The number of housing units comes from the US Census Bureau and the share of detached houses for each state is based on the year 2000 census also obtained from the Census Bureau. Descriptive statistics of the key variables are presented in [Table 3](#).

[Insert Table 3 here]

If we assume a Cobb-Douglas demand function, the econometric specification of the model can be written as:

$$\ln q_{it} = (\beta_0 + \beta_Y \ln Y_{it} + \beta_P \ln P_{it} + \beta_X \ln X_{it}) + [1 - R(\gamma' z_{it})] u_{it} + v_{it} \quad (9)$$

where subscript i stands for state, subscript t is time, $v_{it} \sim N^+[0, \sigma_v]$, and $u_{it} \sim N^+[0, \sigma_u]$. Our dependent variable (q_{it}) is each state’s aggregate residential energy consumption for each year in trillion BTUs. The income variable (Y_{it}) is each state’s real disposable personal income for each year in million 1982 US\$. The price variable (P_{it}) is each state’s real energy price for each year in 1982 US\$ per million BTU. The set of control variables X_{it} includes Population (POP_{it}), the heating and cooling degree days (HDD_{it} and CDD_{it}), the average size of a household (AHS_{it}) obtained by dividing population by the number of housing units, and the share of detached houses for each state (SDH_i).

Regarding the rebound-effect function, it is modelled as a function of potential economic determinants of households’ demand for energy services, such as household size, per capita income, and the price they must pay for energy. That is, $\gamma'z$ is specified as:

$$\gamma_0 + \gamma_{PCI} \ln(Y/POP)_{it} + \gamma_P \ln P_{it} + \gamma_{AHS} \ln AHS_{it} \quad (10)$$

If we impose that the rebound-effect function does not depend on any covariate, we obtain the standard energy demand frontier model estimated in [Filippini and Hunt \(2011, 2012\)](#). Since the PA rebound-effect function prevents unlikely rebound effect outcomes, it is our preferred model. However the specification allowing for super-conservation outcomes, i.e. the SC model, is also estimated for robustness purposes. All models are estimated by maximum likelihood.¹³

We show in [Table 4](#) the estimation results of our preferred frontier energy demand models. The standard ALS model that imposes a zero rebound effects is also shown for comparison grounds. Simple Likelihood Ratio (LR) tests indicate that both the PA and SC models outperform the ALS model. In general, both models perform quite well as most coefficients have the expected sign and almost all are statistically significant at the 5% level. This indicates that the results in terms of the estimated coefficients tend to be robust across the two different specifications of the rebound effect.

[Insert Table 4 here]

Regarding the energy demand frontier, the estimated coefficients can be directly interpreted as elasticities as most of the variables are in logarithmic form. The estimated

¹³ The complete econometric specification of the three energy demand models with all variables included is shown in Appendix B.

magnitudes of both price and income elasticities are quite reasonable from a theoretical point of view. The estimated frontier coefficients suggest that US residential energy demand is price-inelastic, with estimated elasticities of -0.10, -0.12 and -0.11 for the ALS, PA and SC models respectively. The results also suggest that US residential energy demand is income-inelastic, with an estimated elasticity of around 0.36 for the ALS model but only about 0.24 for the models allowing for non-zero rebound effects.

The positive coefficient on population obtained in all models suggests that energy consumption increases with population, given the total amount of disposable income in a particular state. For weather, the estimated cooling degree day elasticities for all three models are rather high, whereas the estimated heating degree day elasticities are much lower. The estimated coefficient of average household size suggests that as family size increases there is a tendency to use less energy, indicating that there are economies of scale with an estimated elasticity larger than unity in absolute terms. For the share of detached houses, the results suggest that there is only a marginal positive but significant influence on US residential energy demand.¹⁴

Table 5 provides descriptive statistics of the estimated energy efficiency for all US states. The ALS values are obtained directly using the Jondrow *et al.* (1982) formula. For the PA and SC models, the efficiency scores are computed dividing the estimated value of the overall one-sided term, i.e. $(1-R)u$, by (one minus) the estimated values of the rebound-effect function. We show three types of results in Table 5 in accordance with different adjustments of the intercept in the rebound-effect function. The first set of efficiency scores is obtained assuming that equation (10) has “no intercept” and hence u contains the whole estimated intercept. The second set of efficiency scores labelled “ALS-adjusted” follows the empirical strategy that uses the ALS estimate to adjust the estimated intercept of equation (10). The third efficiency scores are obtained following the opposite strategy to the first one, so in this case the rebound-effect function is “not adjusted” as it is assumed here that u does not contain an intercept. These three strategies are explained with more detail in Appendix A.

[Insert Table 5 here]

Table 5 shows that the estimated *average* efficiency is between 45.5 and 98.7%. However, this wide range of results is due to the models that consider that the intercept may either be in the rebound-effect function or in the inefficiency term. If we focus on the ALS-adjusted results, the values obtained with the PA and the SC models are much more reasonable (91.1% and 93.8% respectively). Similar results were obtained by Filippini and Hunt (2012) using several specifications of the homoscedastic model. It is worth mentioning that the ALS model produces similar efficiency scores to those obtained when the intercept is properly adjusted. The efficiency scores clearly decrease when the intercepts of the PA and SC rebound-effect functions are not adjusted. By contrast, the largest efficiency scores are obtained when no intercept is considered in the rebound-effect function. These two cases define the lower and upper bound in the efficiency score estimates.

Regarding the rebound-effect function, recall from Table 4 that the coefficients of per capita income, price and average household size are always statistically significant. The theory on rebound effects often predicts that they should decline with

¹⁴ As in Filippini and Hunt (2012), the estimated coefficients of the time dummies (not shown) are significant in all models and although the overall trend in the coefficients is generally negative, they do not fall continually over the estimation period, reflecting the ‘non-linear’ impact of technical progress and other exogenous variables.

income, and the coefficient of this variable is negative in both models.¹⁵ This implies that the states with larger income levels have larger energy efficiency elasticities in absolute values, and therefore their rebound effects are lower. This seems to confirm the aforementioned hypothesis and is in line with the little available evidence on this issue in the empirical literature measuring rebound effects. On the other hand, the positive coefficient obtained for the price variable suggests that energy-inefficient states have more elastic energy demands.¹⁶ This result is expected in theory as energy-inefficient states tend to spend a larger share of their income on energy *ceteris paribus*, and hence the so-called income effect is more intense. With respect to the third variable, average household size, given the negative value of the coefficient, a similar interpretation to per capita income can be done in this case. This result also seems reasonable. In large households, the awareness to achieve potential energy savings that come from energy efficiency improvements is probably larger than in small households because it implies a greater overall benefit, which results in the existence of lower rebound effects.

Our comprehensive frontier model of energy demand allows us to examine the compliance with some of the restrictions often assumed in previous studies devoted to estimating rebounds effects, but with different econometric techniques. For instance, most studies estimate the own-price elasticity of the demand for energy to obtain an indirect measure of the rebound effect. The validity of this papers hinges upon the assumption that consumers respond in the same way to decreases in energy prices as they do to improvements in energy efficiency. In particular, most of the empirical literature on rebound effects assumes that:

$$\varepsilon_E = -\varepsilon_P - 1 \quad (11)$$

We label this restriction as the *assumption of equivalence in responses*. Previous papers assume that equation (11) is fulfilled for all observations. As our model provides elasticities for both energy prices and energy efficiency, it allows us to examine (or even test) this issue in a very simple way. Thus, let us rewrite equation (11) as follows:

$$\varepsilon_P = a + b\varepsilon_E \quad , \quad a = b = -1 \quad (12)$$

Testing that $a = b = -1$ in an auxiliary regression allows us to examine the fulfilment of this assumption. In Appendix C we show that if we use a PA specification of the rebound-effect function, it is possible to directly test this assumption. In this sense, the Wald test carried out using the estimated parameters of our model suggests that energy and price elasticities are statistically different in our case. As a consequence, the absolute value of the price elasticity in the frontier cannot be used for measuring the rebound effect as suggested by equation (11).

On the other hand, [Sorrell and Dimitropoulos \(2008\)](#) pointed out that the estimated price elasticities in previous studies might be biased if energy efficiency is not explicitly controlled for. The nature of this endogeneity problem is clear in our framework if the rebound-effect function depends on the energy price and efficiency is ignored because the overall error term in this case would include R and hence it would be correlated with the energy price in the frontier. In this sense, our extended frontier

¹⁵ Footnote #9 provides the direct effect of per-capita income and price variables on the overall price and income elasticities. Taking into account that $\ln E < 0$, a negative coefficient of per capita income in R reduces the overall income elasticity, ε_Y .

¹⁶ A positive coefficient for the price variable in R implies that $\partial R / \partial \ln P > 0$. As this derivative is multiplied by $\ln E$ in footnote #9 and $\ln E$ is “more” negative in more energy-inefficient states, their overall price elasticity ε_P is larger, in absolute terms, than the overall price elasticity of more efficient states.

model clearly shows that it makes sense to follow [Filippini and Hunt \(2012\)](#) and estimate a standard energy demand model using the empirical strategy proposed by [Mundlak \(1978\)](#) to control for potential endogeneity problems.

We have also estimated our energy demand model including the Mundlak's adjustment but this adjustment does not affect our estimated rebound effects.¹⁷ This is an expected result because our specification of the rebound-effect function already controls for potential endogeneity problems that would appear if we ignore that R is correlated with some of the energy demand drivers. The robustness of our results might also indicate that, given our specification of R , there are not significant traces of endogeneity associated to the inefficiency term, u , and hence there is no need to further extend our model to deal with this extra and cumbersome difficulty.¹⁸

[Table 6](#) provides descriptive statistics for the overall US estimated rebound effects using the PA and SC models. It should be recalled that there are no values larger than unity in the two models because both specifications prevent backfire outcomes. In addition to the “ALS-adjusted” specification, for comparative purposes we show the estimated rebound effects that are obtained if the rebound-effect functions do not contain an intercept or if the estimated intercept is not adjusted (i.e. it completely belongs to the rebound-effect function). This table shows that the average rebound effect is 79% when our preferred PA model is used and the intercept is adjusted using the standard deviation of u of the ALS model. It decreases to 56% when the SC model is used.

[Insert Table 6 here]

Generally speaking, our rebound effects tend to be larger than those obtained in the empirical literature using micro-data on the direct rebound effects of household energy demand (see our discussion in Section 2). Two different issues can partially explain this result. First, note that our estimated rebound effects involve more than one energy service, and hence they are not only capturing direct but also indirect effects. In addition, it should be pointed out that our results are even lower than those obtained in several papers -such as [Lenzen and Dey \(2002\)](#), [Alfredsson \(2004\)](#) or [Mizobuchi \(2008\)](#)- that also get large direct and indirect rebound effects, even reaching effects larger than 100%, i.e. backfire outcomes. A second reason has to do with the curvature of the estimated rebound-effects functions. In [Figure 1](#) it is shown that the proposed rebound-effect functions are concave, at least when the value of $\gamma'z$ in (7) and (8) tends to be positive, as happens in our case due to the positive value of the intercept and the fact that all variables have been centred with respect to the sample mean. Thus, our rebound effect estimates are likely to be upwardly biased because the curvature imposed on our R functions “forces” the rebound effect to increase rapidly when we move away from the zero value. Research devoted to finding more flexible yet still simple rebound-effect functions that relax this curvature would be desirable in the near future.

[Insert Figure 1 here]

Despite the above comments, our large rebound effects may simply indicate that most of the expected savings from improvements in energy efficiency are not reached

¹⁷ Only two of the estimated coefficients lose statistical significance.

¹⁸ This additional source of endogeneity could be addressed if we allow u to depend on a set of covariates (such as income and energy price). Actually, we have tried to estimate some versions of this model without success. This can be taken as evidence of the lack of additional endogeneity problems, but it also might be caused by the fact that the resulting likelihood function is much more complex (i.e. non-linear) than when u is homoscedastic.

for the case of residential energy demand in the US. As enhancements on efficiency in this sector seem to fail for reducing energy consumption, energy taxes could have a prominent role within the energy and climate policies raised to control this issue as happens in the transport sector (Sterner, 2007; Frondel *et al.*, 2012).

On the other hand, it is worth mentioning that the estimated rebound effects in Table 6 is about 97% in both models when it is assumed that the estimated intercept completely belongs to R . Hence, contrary to what happens in the efficiency estimates this procedure gives an upper bound for the rebound effect. These extremely large values probably suggest that R is upward biased, so that the estimated σ_u is also upward biased (i.e. the true σ_u is likely to be less than 1). Assuming, by contrast, that the rebound-effect function does not contain an intercept, the estimates produce an average rebound effect of about 50% for the PA rebound-effect function (and negative for the SC model). This outcome is, however, due to the fact that all variables have been centred with respect to the sample mean, and thus we are imposing that $R=0.5$ for the *average* state. These results thus point out the importance of adjusting the estimated intercepts when computing rebound effects using an SFA approach.

Regarding the issue of allowing or not for super-conservation outcomes, Figure 2 shows the relationship between the ALS-adjusted rebound effects obtained using our proposed models. This figure reveals that the rebound effects in which super-conservation outcomes are not restricted (SC model) are in practice monotonic transformations of the rebound effects obtained using models that only allow for partial rebound effects (PA model). In other words, allowing for super-conservation outcomes only has an effect on the magnitude of the rebound effects, but not on the relative values across observations. Overall, these results indicate that the ranking of rebound effects tend to be robust across different specifications of the R function.

[Insert Figure 2 here]

In Table 7 we show the parameter estimates of both the PA and SC models when they are estimated without time dummies for robustness analysis. These models are presented to check the sensitivity of the approach proposed to measure rebound effects, as these dummies are likely to be capturing - among other common temporal effects - technological improvements in the energy efficiency of households' equipment and appliances over time.

[Insert Table 7 here]

Again, both models perform quite well as most coefficients have the expected sign and almost all of them are statistically significant. Secondly, the income per capita, price and average household size variables of the rebound-effect function again have the expected signs and their coefficients are statistically significant. However, while the remaining coefficients are approximately in the same order of magnitude, the income and price elasticities in the frontier vary notably. This result is particularly striking and highlights the importance of a proper specification of technical progress (using a time trend or temporal dummies) in order to obtain unbiased estimates of the price and income elasticities. This may be a significant problem especially in those analyses aiming at estimating rebound effects through the own price elasticity. Moreover, in the rebound-effect function the coefficient of the price variable is positive and the coefficient of the income variable is negative, indicating that well-off states have lower rebound effects. Likewise, the estimated coefficient for average household size is still significant and positive. In Table 8 we can see that both efficiency scores and rebound effects hardly change, indicating that the specification of technical progress in our

model does not affect our results. As we have seen previously, the rebound-effect function without adjustment and the rebound effect without an intercept show the lower and upper bounds respectively for both the efficiency score estimates and the rebound effect estimates. Encouragingly, these results indicate that, overall, the estimated efficiencies and rebound effects tend to be robust to the different specifications of the technical progress in the frontier.

[Insert Table 8 here]

Finally, our results might help policy makers to design more effective energy saving schemes. For instance, [Figure 3](#) shows the overall relationship between energy efficiency and the rebound effect using our preferred model, the PA specification. If we sort in increasing order the US states according to their average efficiency scores and then check their average rebound effects, we can get an idea about the correlation between these two measures. The average energy efficiency of the states in the fourth quartile is 86.3%. As usual in a frontier analysis framework, energy savings are *potentially* larger in those states with lower efficiency scores. Unlike standard SFA models, our models allow us to know whether the potential reductions in energy inefficiency are passed on entirely to final energy savings. As the states of the fourth quartile have also the lowest rebound effect (56.7%) we have more reasons to encourage energy efficiency improvements in these states. On the other hand, it is worth mentioning that although efficiency and rebound effects tend to increase as we move down the quartiles, the gap between both measures decreases and reaches a minimum difference in the first quartile where the most energy-efficient states (93.3%) are also those with the largest rebound effect (90.9%). This result indicates that as the efficiency of US states increases, households are less sensitive to changes in efficiency and they do not reduce their energy consumption as much as would be expected if we are swayed by what happens to the states with lower levels of efficiency.

[Insert Figure 3 here]

Focusing on the minimum rebound effects on [Table 6](#), we can see that although the rebound effect is large on average, some US states have very small rebound effects compared to others. It can be seen in [Figure 3](#) that there is a clear correlation between energy efficiency and rebound effects, but this does not mean that large energy efficiencies necessarily imply large rebound effects. [Figure 4](#) reveals the heterogeneity that exists in our US sample. Those states with low energy efficiency (below the median) and a low rebound effect (also below the median) are highlighted in dark orange. These states are identified here as priority targets for energy policies, since improvements of energy efficiency in these states may yield large reductions in energy consumption (and probably greenhouse gas emissions).¹⁹ On the other hand, those states marked in the lightest orange have large energy efficiencies as well as large rebound effects and therefore they should be labelled as the lower-priority targets. The intermediate orange highlights those states that have either low energy efficiency or a low rebound effect and hence cannot be identified as priority objectives. In summary, a sound policy would be not only focused on the most inefficient states but also on those with low rebound effects where the policy would have a greater overall effect over energy consumption.

[Insert Figure 4 here]

¹⁹ It should be stressed that if the average value is used instead the median to classify the states, just seven (Connecticut, Illinois, Maryland, Massachusetts, New Jersey, New York and Utah) would be below the average value of both efficiency and rebound effect, and hence only these states would be primary targets.

To finish up this section, we next summarize the main insights derived from our empirical application:

- The proposed approach to measure rebound effects is applied to a panel data sample of residential energy demand of 48 US states over the period 1995-2011.
- There have been found average rebound effects of large magnitude (56-80%).
- The average energy efficiency ranges from 91 to 94%.
- As it was formerly noticed by other researchers, the rebound effect decreases with (per capita) income. The same relationship has been found between the rebound effect and the average household size.
- On the other hand, energy price shows a positive relationship with the rebound effect, which implies that more inefficient states have more elastic demands respect to changes in energy price.
- Energy inefficient states tend to have small rebound effects compared with energy efficient states.
- The suggested approach to obtain rebound effects does not suffer from bias when technical progress is ignored.

5. Conclusions

This paper highlights that the energy demand frontier model, originally proposed by [Filippini and Hunt \(2011, 2012\)](#) to get country-specific energy efficiency scores, is closely linked with the so-called rebound effect, a phenomenon widely examined in the literature on energy economics. In particular, we have shown that the standard specification of the energy demand frontier model basically imposes a rebound effect equal to zero, something that clashes with the empirical evidence obtained in the literature on the rebound effect.

Based on the stochastic frontier approach, a new empirical strategy is proposed to measure the rebound effect associated with energy efficiency improvements. Our comprehensive energy demand frontier model avoids the ‘zero’ rebound-effect assumption through the estimation of a rebound-effect function that regulates the final effect of potential efficiency improvements on energy consumption. Two specifications for the rebound-effect function that preclude backfire outcomes are presented in the paper. While the SC model allows for super-conservations outcomes, the PA model only allows for partial rebound effects. We however advocate using the latter model because it avoids obtaining too large (negative) rebound effects for some observations that are difficult to justify in economic terms.

We illustrate the approach proposed to measuring rebound effects with an empirical application of US residential energy demand data for 48 states over the period 1995-2011. The coefficients of the variables included in the models are highly significant, show the expected signs and have a quite reasonable magnitude regardless of the specification of the rebound-effect function used.

If we pay attention to the coefficients of the variables introduced in the rebound-effect function, several comments can be stressed. First, the rebound effect declines with (per capita) income, a result that corroborates previous results in the literature. Second, the positive coefficient for energy price indicates that the more inefficient states face more elastic energy demands. Finally, the negative coefficient found for average household size suggests that easier potential energy savings can be achieved in states with large households. Regarding the efficiency scores there is not much variation

between estimated (PA and SC) models and they do not change much in response to the different options used to obtain the intercept of the rebound-effect function.

In relation to the rebound effects, values that are too large and too low are obtained if we ignore or do not adjust the estimated intercept of the rebound-effect functions. A simple strategy based on ALS model estimates is proposed to minimize the effects of this issue. The average rebound effects obtained using this strategy are 56% and 80%, which indicates that this phenomenon should not be considered a minor issue in energy and environmental policies for the case of residential energy consumption in the US. Although the estimated rebound effects vary with the functional form, the position of each observation does not change as the SC rebound effects is a monotonic transformation of the rebound effects obtained with the PA model. This is an important result as the relative position of each state in terms of both energy efficiency and rebound effect rankings permits the identification of states where the enforcement of policies with the aim of promoting energy efficiency would be more effective. Compared to those analyses aiming at estimating rebound effects through the own price elasticity, our empirical approach suffers less from biases when technical progress is ignored.

We would like to emphasise that this paper is the first attempt to use the stochastic frontier framework to measure rebound effects associated with energy efficiency improvements. In this sense, we have identified a few number of research areas that can be explored by other researchers in the future in order to better estimate the rebound effects using a similar empirical strategy than the proposed here. This likely would imply the use of more sophisticated techniques than those proposed in this paper.

For instance, a key issue is the identification of the true intercept of the rebound-effect function. We have proposed a simple empirical strategy to split the estimated intercept into its two components but other alternative approaches could be used to deal with this problem. A promising strategy could be treating the correction factor as an additional one-sided random term and, hence, estimating a model with *two* multiplicative one-sided random terms.

Another issue has to do with the concavity problems of the proposed rebound-effect functions, which tend to overestimate the rebound effect. Although this is likely an issue related to our data set, future research can be focused on the use of alternative parametric specifications of the rebound function. In this sense, it should be also explored the potential use of semiparametric regression methods to relax the current concavity constraints. We also encourage specific research focused on the lack-of-backfire assumption used in our energy demand frontier model.

References

- Aigner, D., Lovell, C.A.K. and Schmidt, P. (1977), "Formulation and estimation of stochastic frontier production function models", *Journal of Econometrics*, 6, 21-37.
- Alfredsson, E.C. (2004), "'Green' consumption—no solution for climate change", *Energy*, 29, 513-524.
- Bentzen, J. (2004), "Estimating the rebound effect in US manufacturing energy consumption", *Energy Economics*, 26(1), 123-34.
- Berkhout, P.H.G., Muskens, J.C. and Velthuisen, J.W. (2000), "Defining the rebound effect", *Energy Policy*, 28(6-7), 425-32.
- Binswanger, M. (2001), "Technological progress and sustainable development: What about the rebound effect?", *Ecological Economics*, 36(1), 119-32.
- Brännlund, R., Ghalwash, T. and Norström, J. (2007), "Increased energy efficiency and the rebound effect: effects on consumption and emissions", *Energy Economics*, 29, 1-17.
- Druckman, A., Chitnis, M., Sorrell, S. and Jackson, T. (2010), *An investigation into the rebound and backfire effects from abatement actions by UK households*, RESOLVE Working Paper 05-10, Guildford, University of Surrey.
- Filippini, M. and Hunt, L.C. (2011), "Energy demand and energy efficiency in the OECD countries: A stochastic demand frontier approach", *The Energy Journal*, 32(2), 59-80.
- Filippini, M. and Hunt, L.C. (2012), "US residential energy demand and energy efficiency: A stochastic demand frontier approach", *Energy Economics*, 34(5), 1484-149.
- Fouquet, R. and Pearson, P.J.G. (2012), "The long run demand for lighting: Elasticities and rebound effects in different phases of economic development", *Economics of Energy and Environmental Policy*, 1(1), 83-100.
- Frondel, M., Ritter, N. and Vance, C. (2012), "Heterogeneity in the Rebound Effect: Further Evidence for Germany", *Energy Economics*, 34, 461-467.
- Greene, D.L., Kahn, J.R. and Gibson, R.C. (1999a), *An econometric analysis of the elasticity of vehicle travel with respect to fuel cost per mile using the RTEC survey data*, Oak Ridge National Laboratory, Oak Ridge, Tennessee.
- Greene, D. L., Kahn, J.R. and Gibson, R.C. (1999b), "Fuel economy rebound effect for US household vehicles", *Energy Journal*, 20(3), 1-31.
- Greening, L.A., Greene, D.L. and Difiglio, C. (2000), "Energy efficiency and consumption – the rebound effect – A survey", *Energy Policy*, 28(6-7), 389-401.
- Guertin, C., Kumbhakar, S., and Duraiappah, A. (2003), *Determining demand for energy services: Investigating income-driven behaviours*, International Institute for Sustainable Development.
- International Risk Governance Council (IRGC) (2013), *The Rebound Effect: Implications of consumer behaviour for robust energy policies*, A review of the

literature on the rebound effect in energy efficiency and report from expert workshops, Lausanne: International Risk Governance Council.

- Jenkins, J., Nordhaus, T. and Shellenberger, M. (2011), *Energy Emergence. Rebound & backfire as emergent phenomena*, Oakland, The Breakthrough Institute.
- Jondrow, J., Lovell, K., Materov, I. and Schmidt, P. (1982), "On the estimation of technical inefficiency in the stochastic frontier production function model", *Journal of Econometrics*, 19, 233-238.
- Khazzoom, J. D. (1980), "Economic implications of mandated efficiency in standards for household appliances", *Energy Journal*, 1(4), 21-40.
- Kumbhakar, S.C. and Lovell, C.A.K. (2000), *Stochastic frontier analysis*, Cambridge University Press, Cambridge.
- Lenzen, M. and Dey, C.J. (2002), "Economic, energy and greenhouse emissions impacts of some consumer choice, technology and government outlay options", *Energy Economics*, 24, 377-403.
- Mizobuchi, K. (2008), "An empirical study on the rebound effect considering capital costs", *Energy Economics*, 30, 2486-2516.
- Mundlak, Y. (1978), "On the pooling of time series and cross section data", *Econometrica*, 46, 69-85.
- Roy, J. (2000), "The rebound effect: some empirical evidence from India", *Energy Policy*, 28(6-7), 433-38.
- Saunders, H.D. (1992), "The Khazzoom–Brookes postulate and neoclassical growth", *The Energy Journal*, 13(4), 130-148.
- Saunders, H.D. (2000), "A view from the macro side: Rebound, backfire, and Khazzoom-Brookes", *Energy Policy*, 28, 439-449.
- Saunders, H.D. (2008), "Fuel conserving (and using) production functions", *Energy Economics*, 30, 2184-2235.
- Saunders, H.D. (2013), "Historical evidence for energy consumption rebound in 30 US sectors and a toolkit for rebound analysts", *Technological Forecasting and Social Change*, 80(7), 1317-1330.
- Small, K.A. and Van Dender, K. (2005), *A study to evaluate the effect of reduced greenhouse gas emissions on vehicle miles travelled*, Prepared for the State of California Air Resources Board, the California Environment Protection Agency and the California Energy Commission, Final Report ARB Contract Number 02-336, Department of Economics, University of California, Irvine.
- Small, K.A. and Van Dender, K. (2007), "Fuel efficiency and motor vehicle travel: The declining rebound effect", *The Energy Journal*, 28(1), 25-51.
- Sorrell, S. (2007), *The rebound effect: an assessment of the evidence for economy-wide energy savings from improved energy efficiency*, A report produced by the Sussex Energy Group for the Technology and Policy Assessment function of the UK Energy Research Centre, UK Energy Research Centre.
- Sorrell, S. and Dimitropoulos, J. (2007), *UKERC review of evidence for the rebound effect. Technical Report 2: Econometric studies*, Working Paper, UK Energy Research Centre.

- Sorrell, S. and Dimitropoulos, J. (2008), "The rebound effect: Microeconomic definitions, limitations and extensions", *Ecological Economics*, 65(3), 636-649.
- Stern, T. (2007), "Fuel taxes: An important instrument for climate policy", *Energy Policy*, 35(6), 3194-3202.
- Thomas, B. and Azevedo, I.L. (2013), "Estimating direct and indirect rebound effects for U.S. households with Input-Output analysis. Part 1: Theoretical framework", *Ecological Economics*, 86, 199-210.
- Zein-Elabdin, E.O. (1997), "Improved stoves in Sub-Saharan Africa: The case of the Sudan", *Energy Economics*, 19(4), 465-75.
- Wang, H., Zhou, P. and Zhou, D.Q. (2012), "An empirical study of direct rebound effect for passenger transport in urban China", *Energy Economics*, 34(2), 452-460.

Table 1. Approaches for measuring the direct rebound effect

Approach 1	$\varepsilon_E(q) = \varepsilon_E(S) - 1$
Approach 2	$\varepsilon_E(q) = -\varepsilon_P(S) - 1$
Approach 3	$\varepsilon_E(q) = -\varepsilon_{P_q}(q) - 1$

Notes: Letters in parentheses stand for elasticity numerators and subscripts for elasticity denominators.
E: Energy efficiency; *q*: Energy; *S*: Useful work; *P_S*: Energy cost of useful work; *P_q*: Energy price.

Table 2. Possible values for the rebound effect and the energy efficiency elasticity

$R > 1$	Backfire	$\varepsilon_E > 0$
$R = 1$	Full rebound	$\varepsilon_E = 0$
$0 < R < 1$	Partial rebound	$-1 < \varepsilon_E < 0$
$R = 0$	Zero rebound	$\varepsilon_E = -1$
$R < 0$	Super-conservation	$\varepsilon_E < -1$

Table 3. Summary statistics of variables

<i>Variable</i>	<i>Description</i>	<i>Mean</i>	<i>Std. Dev.</i>	<i>Min.</i>	<i>Max.</i>
Q	Energy consumption (trillion BTUs)	229.60	209.42	19.02	932.92
Y	Real disposable personal income (million 1982US\$)	92,620	105,635	6,072	654,780
P	Real price of energy (per million BTU)	16.86	5.11	8.22	35.18
POP	Population (1000)	5,977	6,407	485	37,692
HDD	Heating degree days (base: 65 °F)	5,134	2,007	555	10,745
CDD	Cooling degree days (base: 65 °F)	1,147	805	128	3,870
AHS	Average household size (No. of people per housing unit)	2.33	0.17	1.83	2.99
SDH	Share of detached houses (%)	62.27	9.74	13.20	74.00

Table 4. Parameter estimates (models with time dummy variables)

<i>Variable</i>	<i>Parameter</i>	ALS		PA		SC					
		<i>Est.</i>	<i>Std. E.</i>	<i>Est.</i>	<i>Std. E.</i>	<i>Est.</i>	<i>Std. E.</i>				
<i>Frontier</i>											
Intercept	β_0	5.012	*** 0.022	5.043	*** 0.018	5.042	*** 0.018				
ln (Y)	β_Y	0.364	*** 0.037	0.238	*** 0.046	0.236	*** 0.046				
ln (P)	β_P	-0.101	*** 0.025	-0.117	*** 0.030	-0.114	*** 0.030				
ln (POP)	β_{POP}	0.670	*** 0.038	0.797	*** 0.047	0.799	*** 0.047				
ln (AHS)	β_{AHS}	-1.117	*** 0.053	-1.480	*** 0.086	-1.469	*** 0.088				
ln (HDD)	β_{HDD}	0.373	*** 0.013	0.347	*** 0.013	0.348	*** 0.013				
ln (CDD)	β_{CDD}	0.084	*** 0.007	0.080	*** 0.008	0.080	*** 0.008				
SDH	β_{SDH}	0.005	*** 0.001	0.005	*** 0.001	0.005	*** 0.001				
<i>Noise term</i>											
ln (σ_v)	-	-2.633	*** 0.120	-2.554	*** 0.036	-2.555	*** 0.037				
<i>Rebound-effect</i>											
Intercept	γ_0			4.281	*** 0.714	4.124	*** 0.670				
ln (Y/POP)	γ_{PCI}			-7.014	*** 2.242	-6.148	*** 2.034				
ln (P)	γ_P			1.577	* 0.862	1.446	* 0.769				
ln (AHS)	γ_{AHS}			-14.283	*** 3.640	-12.187	*** 3.165				
<i>Inefficiency term (homoscedastic)</i>											
ln (σ_u)	δ_0	-2.530	*** 0.258								
Log-likelihood		842.183		875.919		874.951					

Notes: ***, ** and * indicate that the coefficient are significant at 1%, 5% and 10%.

Table 5. Energy efficiency scores using the PA and SC models
(models with time dummy variables)

	<i>Mean</i>	<i>Std. Dev.</i>	<i>Min.</i>	<i>Max.</i>
ALS	0.939	0.025	0.831	0.977
PA				
No intercept ($\tilde{\gamma}_0^{PA} = 0$)	0.964	0.040	0.703	0.989
ALS-adjusted ($\tilde{\gamma}_0^{PA} = 1.833$)	0.911	0.041	0.651	0.958
Not adjusted ($\tilde{\gamma}_0^{PA} = 4.281$)	0.455	0.078	0.202	0.847
SC				
No intercept ($\tilde{\gamma}_0^{SC} = 0$)	0.987	0.003	0.974	0.997
ALS-adjusted ($\tilde{\gamma}_0^{SC} = 1.593$)	0.938	0.013	0.880	0.987
Not adjusted ($\tilde{\gamma}_0^{SC} = 4.124$)	0.455	0.079	0.200	0.848

Table 6. Rebound effects using the PA and SC models
(models with time dummy variables)

	<i>Mean</i>	<i>Std. Dev.</i>	<i>Min.</i>	<i>Max.</i>
PA				
No intercept ($\tilde{\gamma}_0^{PA} = 0$)	0.505	0.269	0.033	0.982
ALS-adjusted ($\tilde{\gamma}_0^{PA} = 1.833$)	0.791	0.208	0.177	0.997
Not adjusted ($\tilde{\gamma}_0^{PA} = 4.281$)	0.966	0.052	0.713	1.000
SC				
No intercept ($\tilde{\gamma}_0^{SC} = 0$)	-1.178	3.131	-17.866	0.969
ALS-adjusted ($\tilde{\gamma}_0^{SC} = 1.593$)	0.557	0.637	-2.835	0.994
Not adjusted ($\tilde{\gamma}_0^{SC} = 4.124$)	0.965	0.051	0.695	1.000

Table 7. Parameter estimates (models without time dummy variables)

<i>Variable</i>	<i>Parameter</i>	ALS		PA		SC	
		<i>Est.</i>	<i>Std. E.</i>	<i>Est.</i>	<i>Std. E.</i>	<i>Est.</i>	<i>Std. E.</i>
<i>Frontier</i>							
Intercept	β_0	4.937 ***	0.009	4.992 ***	0.008	4.990 ***	0.008
ln (Y)	β_Y	0.259 ***	0.033	0.114 ***	0.042	0.113 ***	0.041
ln (P)	β_P	-0.207 ***	0.017	-0.198 ***	0.021	-0.196 ***	0.021
ln (POP)	β_{POP}	0.776 ***	0.035	0.921 ***	0.043	0.923 ***	0.043
ln (AHS)	β_{AHS}	-1.113 ***	0.058	-1.430 ***	0.080	-1.422 ***	0.081
ln (HDD)	β_{HDD}	0.353 ***	0.013	0.326 ***	0.012	0.326 ***	0.012
ln (CDD)	β_{CDD}	0.079 ***	0.007	0.070 ***	0.007	0.069 ***	0.007
SDH	β_{SDH}	0.004 ***	0.001	0.004 ***	0.001	0.004 ***	0.001
<i>Noise term</i>							
ln (σ_v)	-	-2.738 ***	0.108	-2.518 ***	0.036	-2.520 ***	0.037
<i>Rebound-effect</i>							
Intercept	γ_0			4.014 ***	0.585	3.881 ***	0.539
ln (Y/POP)	γ_{PCI}			-6.855 ***	1.930	-5.979 ***	1.720
ln (P)	γ_P			1.326 *	0.743	1.190 *	0.651
ln (AHS)	γ_{AHS}			-12.592 ***	3.101	-10.719 ***	2.696
<i>Inefficiency term (homoscedastic)</i>							
ln (σ_u)	δ_0	-2.239 ***	0.117				
Log-likelihood		804.455		839.947		839.194	

Notes: ***, ** and * indicate that the coefficient are significant at 1%, 5% and 10%.

Table 8. Energy efficiency scores and rebound effects using the preferred PA model (model without time dummy variables)

	<i>Mean</i>	<i>Std. Dev.</i>	<i>Min.</i>	<i>Max.</i>
Energy Efficiency Scores				
No intercept	0.957	0.044	0.671	0.985
ALS-adjusted	0.886	0.046	0.603	0.955
Not adjusted	0.456	0.083	0.150	0.861
Rebound effects				
No intercept	0.506	0.258	0.042	0.970
ALS-adjusted	0.805	0.190	0.224	0.995
Not adjusted	0.961	0.054	0.708	0.999

Figure 1. Curvature of the estimated rebound-effect functions

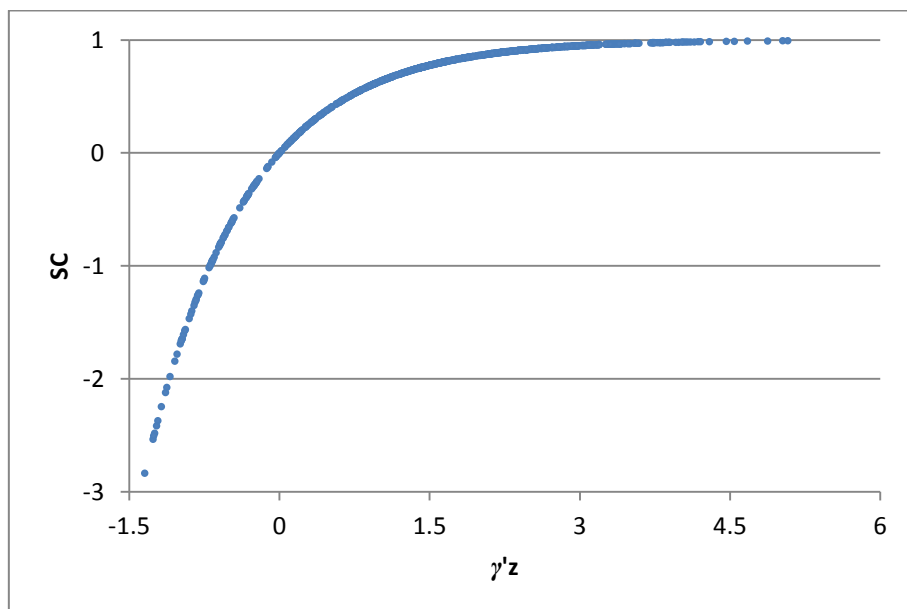
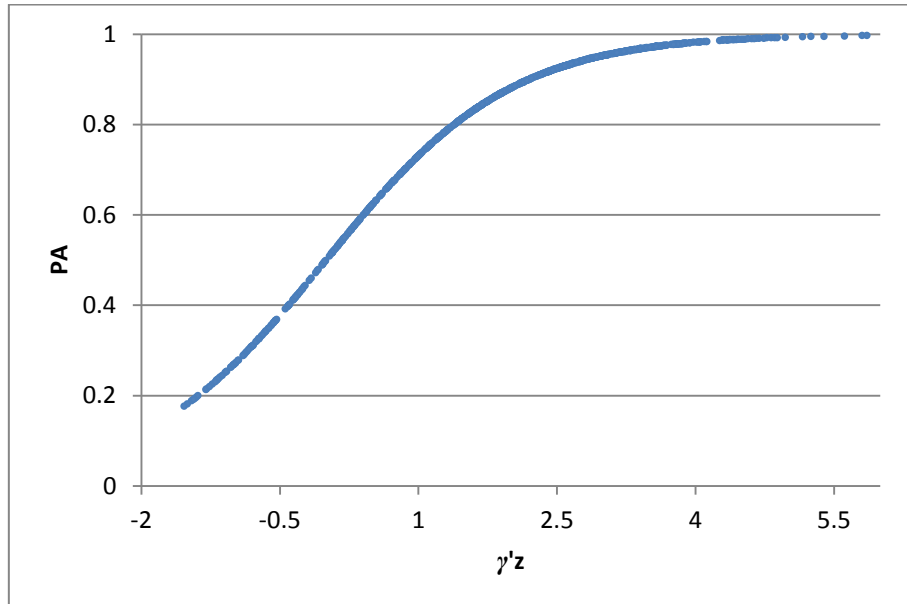


Figure 2.
Rebound effects with and without super-conservation outcomes (ALS-adjusted intercepts)

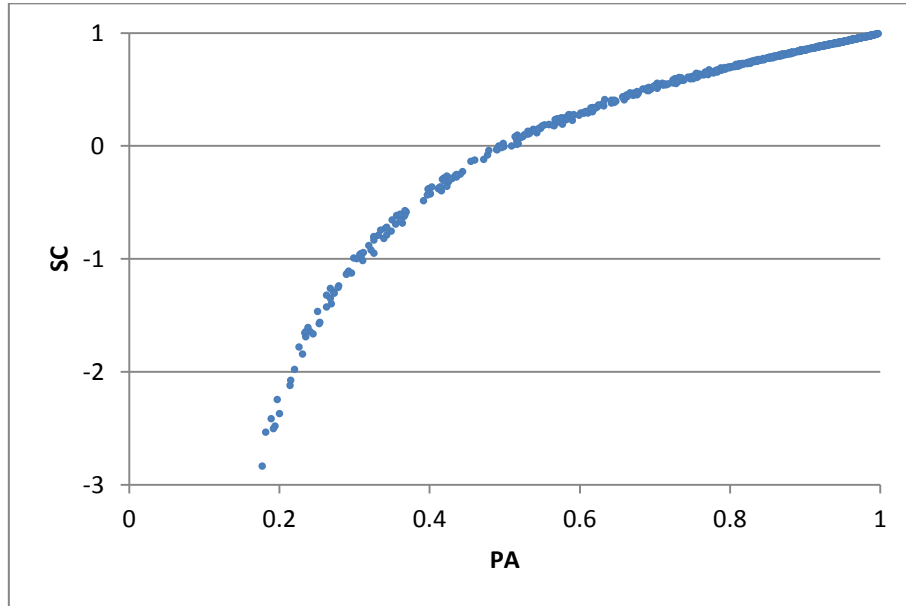


Figure 3. Average energy efficiency scores and rebound effects using the PA model

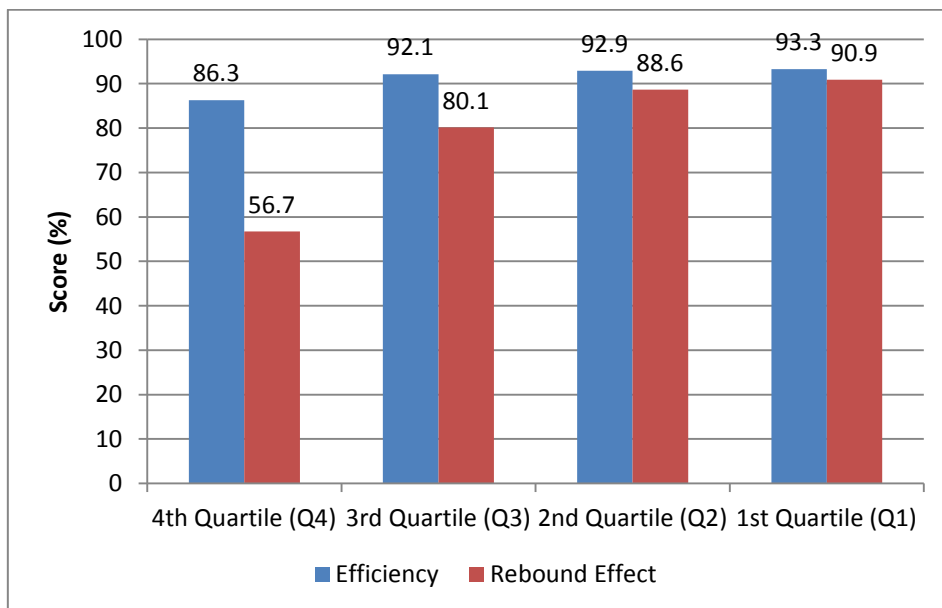
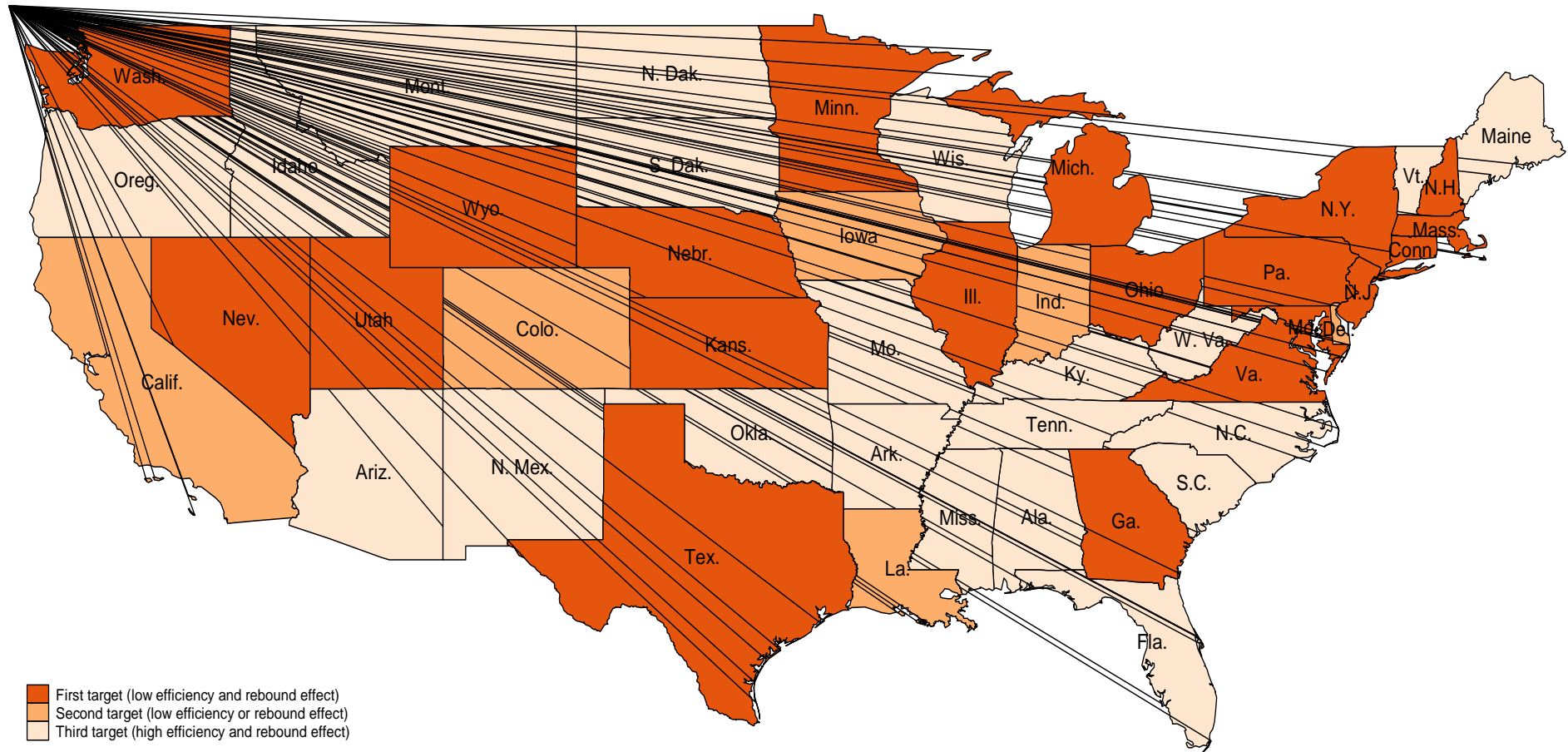


Figure 4. Map of US states in which priority targets to reduce energy consumption are identified



APPENDIX A

Procedure to adjust the intercept of the rebound-effect function and calculation of the energy efficiency scores

In this appendix we explain the details of the proposed adjustments that should be carried out in order to decompose the estimated values of γ_0 from the SC and PA specifications (i.e. $\hat{\gamma}_0^{SC}$ and $\hat{\gamma}_0^{PA}$) into two intercepts: the ‘true’ intercept of the rebound effect function (γ_0), and the logarithm of the standard deviation of the inefficiency, $\delta_0 = \ln \sigma_u$. We next explain based on these decompositions how the state-specific energy efficiency scores should be computed.

The procedure to adjust the intercept of the SC rebound-effect function in (7) is straightforward because the correction factor $(1-R)$ in this case is a simple exponential function. This implies that the estimated intercept of the rebound-effect function is just the simple different between the true parameter γ_0 and δ_0 , that is:

$$\hat{\gamma}_0^{SC} = \gamma_0 - \delta_0 \quad (\text{A1})$$

If we use the ALS estimate of $\ln \sigma_u$, i.e. $\hat{\delta}_0^{ALS}$, as a proxy for δ_0 , the value of γ_0 is then obtained by adjusting the estimated intercept as follows:

$$\tilde{\gamma}_0^{SC} = \hat{\gamma}_0^{SC} + \hat{\delta}_0^{ALS} \quad (\text{A2})$$

The adjusted intercept $\tilde{\gamma}_0^{SC}$ is subsequently used to calculate R , and the computed value of R , hereafter $R(\tilde{\gamma}_0^{SC})$, is used later on to get state-specific energy efficiency scores using the SC model.

A similar procedure is followed in the PA specification of the rebound-effect function as it also cannot be estimated with two intercepts. The adjustment procedure in this case is not as straightforward as when we use the SC model because the adjustment formula should keep R to take values between zero and one. To comply with this restriction, we first try to find a *constant* correction factor $(1-R)$ that would yield the same σ_u . That is, we compute the value of γ_0 in (8) that satisfies the following restriction:

$$1 - R(\gamma_0) = \frac{1}{1 + e^{\gamma_0}} = e^{\delta_0} \quad (\text{A3})$$

This restriction yields the following value for γ_0 :

$$\gamma_0' = \ln \left(\frac{1 - e^{\delta_0}}{e^{\delta_0}} \right) \quad (\text{A4})$$

where γ_0' represents the bias in the estimation of the intercept of the PA rebound-effect function, that is:

$$\hat{\gamma}_0^{PA} = \gamma_0 + \gamma_0' \quad (\text{A5})$$

If we had used the procedure in (A3) to adjust the intercept of the SC rebound-effect function in (7), we would get that $\gamma_0' = -\delta_0$ and hence equation (A1) holds. As in

(A2), if we again use $\hat{\delta}_0^{ALS}$ as a proxy for δ_0 , the estimate of γ_0 in PA model is then obtained by adjusting the estimated intercept as follows:

$$\tilde{\gamma}_0^{PA} = \hat{\gamma}_0^{PA} - \ln\left(\frac{1 - e^{\hat{\delta}_0^{ALS}}}{e^{\hat{\delta}_0^{ALS}}}\right) \quad (\text{A6})$$

The adjusted intercept $\tilde{\gamma}_0^{PA}$ can then be used to compute $R(\tilde{\gamma}_0^{PA})$ in order to get state-specific energy efficiency scores using the PA specification of the model.

Next we explain how we obtain the estimates of energy efficiency for each state. In the ALS model, the composed error term in (3) is simply the sum of v and u . Hence, we can directly use the [Jondrow et al. \(1982\)](#) formula to get the ALS energy efficiency scores. However for the PA and SC models, the efficiency scores are computed in two steps because we do not have estimates of $v+u$, but estimates of $\varepsilon=v+(1-R)\cdot u$. The problem here is to extract the information that ε contains on u , given our estimate of R .

In the first step we take advantage of [Jondrow et al. \(1982\)](#) in order to estimate the *overall* asymmetric random term $\tilde{u} = (1-R)u$ using the conditional distribution of \tilde{u} given the composed error term ε . Both the mean and the mode of the conditional distribution can be used as a point estimate of \tilde{u} . However, the conditional expectation $E(\tilde{u}|\varepsilon)$ is, by far, the most employed in the stochastic frontier analysis literature (see [Kumbhakar and Lovell, 2000](#)).

Given our distributional assumptions, and adding state and time subscripts, the analytical form for $E(\tilde{u}_{it} | \varepsilon_{it})$ can be written as follows:

$$E(\tilde{u}_{it} | \varepsilon_{it}) = \bar{\mu}_{it} + \bar{\sigma}_{it} \left[\frac{\phi(-\bar{\mu}_{it}/\bar{\sigma}_{it})}{1 - \Phi(-\bar{\mu}_{it}/\bar{\sigma}_{it})} \right] \quad (\text{A7})$$

where

$$\sigma_{it}^2 = \sigma_v^2 + (1 - R_{it})^2 \sigma_u^2 \quad (\text{A8})$$

$$\bar{\mu}_{it} = \frac{\varepsilon_{it} (1 - R_{it})^2 \sigma_u^2}{\sigma_{it}^2}$$

$$\bar{\sigma}_{it} = \frac{(1 - R_{it}) \sigma_u \sigma_v}{\sigma_{it}}$$

It should be noted here that the first step does not depend on the above-mentioned adjustments because it relies on the overall asymmetric random term. For this reason, we should get the same point estimator for \tilde{u}_{it} regardless we assume, for instance, that the rebound effect does not contain an intercept but we allow for a non-zero δ_0 parameter, or instead we assume that u_{it} follows a standard half normal distribution with $\sigma_u=1$ or $\delta_0=0$, but now R_{it} includes a separate constant, γ_0 .

The second step uses the state-specific values of $R(\tilde{\gamma}_0^{SC})$ and $R(\tilde{\gamma}_0^{PA})$ to get point estimators for the energy inefficiency term u_{it} (hereafter \hat{u}_{it}), once we have a point estimator for \tilde{u}_{it} . Indeed, if we use the SC model, the point estimator of u_{it} can then be obtained as follows:

$$\hat{u}_{it}^{SC} = E(\tilde{u}_{it} | \varepsilon_{it}) / (1 - R_{it}(\tilde{\gamma}_0^{SC})) \quad (A9)$$

If a PA specification of the model is used, state-specific point estimators of u_{it} are similarly obtained as:

$$\hat{u}_{it}^{PA} = E(\tilde{u}_{it} | \varepsilon_{it}) / (1 - R_{it}(\tilde{\gamma}_0^{PA})) \quad (A10)$$

On the contrary, when the ALS-adjustment is not followed and the “no intercept” strategy is applied, u contains the whole estimated intercept. In this case γ_0 is equal to zero, and R is computed without any intercept. Therefore, the point estimator of u_{it} should be computed as:

$$\hat{u}_{it}^{NI} = E(\tilde{u}_{it} | \varepsilon_{it}) / (1 - R_{it}(\gamma_0 = 0)) \quad (A11)$$

In the “not adjusted” strategy, all the intercept is assumed to belong to the rebound-effect function and u does not contain any intercept. Hence, the point estimator of u_{it} is computed using the original estimate of γ_0 either from the SC or the PA specification, that is:

$$\hat{u}_{it}^{NA} = E(\tilde{u}_{it} | \varepsilon_{it}) / (1 - R_{it}(\gamma_0 = \hat{\gamma}_0)) \quad (A12)$$

Regardless the strategy followed to deal with the intercept issue, the final state-specific energy efficiency indices are computed as follows:

$$EF_{it} = \exp(-\hat{u}_{it}) \quad (A13)$$

APPENDIX B

*Complete econometric specification of the estimated models
(without including time dummy variables)*

ALS model:

$$\begin{aligned} \ln q_{it} = & \beta_0 + \beta_Y \ln Y_{it} + \beta_P \ln P_{it} + \beta_{POP} \ln POP_{it} + \beta_{AHS} \ln AHS_{it} \\ & + \beta_{HDD} \ln HDD_{it} + \beta_{CDD} \ln CDD_{it} + \beta_{SDH} SDH_i + u_{it} + v_{it} \end{aligned} \quad (B1)$$

SC model:

$$\begin{aligned} \ln q_{it} = & \beta_0 + \beta_Y \ln Y_{it} + \beta_P \ln P_{it} + \beta_{POP} \ln POP_{it} + \beta_{AHS} \ln AHS_{it} \\ & + \beta_{HDD} \ln HDD_{it} + \beta_{CDD} \ln CDD_{it} + \beta_{SDH} SDH_i \\ & + \left(1 - \frac{e^{\gamma_0 + \gamma_{PCI} \ln(Y/POP)_{it} + \gamma_P \ln P_{it} + \gamma_{AHS} \ln AHS_{it}} - 1}{e^{\gamma_0 + \gamma_{PCI} \ln(Y/POP)_{it} + \gamma_P \ln P_{it} + \gamma_{AHS} \ln AHS_{it}}} \right) u_{it} + v_{it} \end{aligned} \quad (B2)$$

PA model:

$$\begin{aligned} \ln q_{it} = & \beta_0 + \beta_Y \ln Y_{it} + \beta_P \ln P_{it} + \beta_{POP} \ln POP_{it} + \beta_{AHS} \ln AHS_{it} \\ & + \beta_{HDD} \ln HDD_{it} + \beta_{CDD} \ln CDD_{it} + \beta_{SDH} SDH_i \\ & + \left(1 - \frac{e^{\gamma_0 + \gamma_{PCI} \ln(Y/POP)_{it} + \gamma_P \ln P_{it} + \gamma_{AHS} \ln AHS_{it}}}{1 + e^{\gamma_0 + \gamma_{PCI} \ln(Y/POP)_{it} + \gamma_P \ln P_{it} + \gamma_{AHS} \ln AHS_{it}}} \right) u_{it} + v_{it} \end{aligned} \quad (B3)$$

APPENDIX C

Testing the assumption of equivalence in responses

Let us assume that the demand function is Cobb-Douglas and we use the SC rebound-effect function. In this case, the derivative of the rebound-effect function that appears in footnote #9 with respect to the logarithm of the price variable is

$$\frac{\partial R}{\partial \ln P} = \frac{\partial \left[\frac{(e^{\gamma_P \ln P} - 1)}{e^{\gamma_P \ln P}} \right]}{\partial \ln P} = \gamma_P \left[1 - \frac{e^{\gamma_P \ln P} - 1}{e^{\gamma_P \ln P}} \right] = \gamma_P [1 - R] \quad (C1)$$

where for simplicity we have ignored here other determinants of the rebound-effect function, R . Therefore, the overall price elasticity in footnote #9 for a Cobb-Douglas energy demand function and a SC rebound-effect function can be rewritten as:

$$\varepsilon_P = \beta_P + \gamma_P [1 - R] \ln E \quad (C2)$$

where β_P is the *frontier* price elasticity and γ_P is the coefficient of $\ln P$ in the SC rebound-effect function. As $\varepsilon_E = -(1 - R)$, equation (C2) can be rewritten now as follows:

$$\varepsilon_P = \beta_P - (\gamma_P \ln E) \varepsilon_E \quad (C3)$$

In summary, equations (12) and (C3) jointly indicate that the equivalence of responses assumption will be satisfied in our model if we cannot reject the following null hypothesis:

$$H_0 : \hat{\beta}_P = \hat{\gamma}_P \ln E = -1 \quad (C4)$$

Testing this hypothesis is difficult as energy efficiency varies across states and over time. An alternative way to test the equivalence of responses assumption is to test a sufficient (but weaker) condition for the fulfilment of the above hypothesis evaluated at the estimated mean of the energy inefficiency term:

$$H_0 : \hat{\beta}_P - \hat{\gamma}_P \hat{E}(u) = 0 \quad (C5)$$

As we assume that $u = -\ln E$ follows a half-normal distribution, the expected mean in (C5) is simply a function of σ_u and hence the sufficient condition in (C5) can be finally expressed as follows:

$$H_0 : \hat{\beta}_P - \hat{\gamma}_P \sqrt{2/\pi} \hat{\sigma}_u = 0 \quad (C6)$$

If instead we use the PA rebound-effect function, the sufficient condition in (C5) becomes:

$$H_0 : \hat{\beta}_P - \hat{\gamma}_P \frac{e^{\hat{\gamma}_P z}}{1 + e^{\hat{\gamma}_P z}} \sqrt{2/\pi} \hat{\sigma}_u = 0 \quad (C7)$$