

# Joint Source–Relay Optimization for Fixed Receivers in Multi–Antenna Multi–Relay Networks

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**Abstract**—We jointly optimize the source and relay precoders for multi–antenna multi–relay networks employing a prefixed receiver. Prefixed receivers are of practical interest since they enable low complexity at the end–user’s receiver as well as backward compatibility. To compute the source and relay precoders, we consider two different criteria. The objective of the first criterion is to maximize the worst stream signal–to–interference–plus–noise ratio (SINR) at the output of the receiver subject to source and relay transmit power constraints. Under the second criterion, we minimize the source and relay transmit powers subject to a certain quality–of–service constraint. Both optimization problems are non–convex. To solve them, we propose iterative alternating algorithms, where, in each iteration, we compute the precoders alternately, i.e., for each precoder optimization, we fix all the precoders except the one which is optimized. For both criteria, we formulate the optimization problem for the computation of the source precoder as a second order cone programming (SOCP) problem, for which the optimal solution can be found using interior point algorithms. For each relay precoder, we formulate the optimization problem as a semidefinite relaxation (SDR) problem for which ready–to–use solvers exist. If the solution to the SDR problem is not of rank one, matrix rank–one decomposition or randomization is applied. We also provide sufficient conditions for the convergence of the proposed iterative alternating algorithms to a fixed point. Simulation results show that the performance of the proposed algorithms is close to the performance achieved if the source, relay, and receiver filters are jointly optimized.

**Index Terms**—Cooperative relaying, multiple–input multiple–output, joint optimization of source and relays, prefixed receivers.

## I. INTRODUCTION

Cooperative communication has been one of the most active areas of research over the last decade. It enables reliable communication and expands the coverage of wireless networks [1]–[3]. It is expected that relaying will be a key feature of all future wireless standards. In fact, relaying is one of the key features of the LTE–advanced standard [4], [5]. Recently, multiple–input multiple–output (MIMO) relay networks have attracted a lot of interest due to their ability to significantly increase the spectral efficiency and reliability [6], [7]. The joint design of source and relay precoders has been extensively considered in the literature [7]–[14]. Different criteria, such as mutual information [7], [8] and mean square error (MSE) [11], were considered for the optimization of the source and relay precoders. For example, in [12], the authors proposed a unified framework to jointly optimize the source and relay

precoding matrices based on majorization theory for a one–way three–node amplify–and–forward relay network. In [12], a linear minimum mean square error (MMSE) equalizer was assumed at the destination. In [15], a nonlinear decision feedback equalizer (DFE) at the destination was considered. The optimization of the source precoder, relay precoder, and receiver via minimization of the sum of the source and relay powers subject to a quality–of–service (QoS) constraint was studied in [16] for single relay networks and in [17] for multi–relay networks. The case of multiuser MIMO relay networks was studied in [18], [19]. An excellent reference on precoder design for MIMO relay networks is [20]. In all the above works, an equalizer was assumed at the destination and it was jointly optimized with the source and relay precoders. In particular, if the receiver is jointly optimized with the source and relay, i.e., the receiver is a function of the source and relay precoders, it was proven that the optimal solution for the source and relay precoders diagonalizes the end–to–end channel [12], [13], which allows simplifying the matrix–valued optimization problem to a scalar power allocation optimization problem. In general, the resulting scalar power allocation problem is not convex due to the product of the power allocation parameters of the source and relay. To solve the non–convex scalar power allocation problem, alternating optimization can be used, i.e., the scalar power allocation parameters of one node are updated while fixing the power allocation parameters of the other node [12].

In this paper, we consider the practical case of multi–relay networks, where the receiver at the user side is prefixed, and jointly optimize the source and relay precoders for this scenario. The motivation behind this work, and more specifically behind the assumption of a prefixed receiver at the destination, is that the use of MIMO relays in the downlink of future wireless networks requires some modifications of the transmitter at the source (base station) and/or the receiver at the destination (end–user) compared to the current networks. In addition to the fact that modifying the transmitter at the base station is much easier and less costly than modifying the receiver at the end–user, using prefixed receivers ensures backward compatibility. Moreover, to keep the complexity at the end–user as low as possible, equalization may not be used at all [21], [22]. The design of the transmitter in the case of a prefixed receiver for MIMO point–to–point systems was studied in [22], [23]. However, to the best of our knowledge, there is no previous work that considers joint source and relay optimization for a fixed receiver in relay networks.

The main contributions of this paper are summarized as follows:

- We are the first to consider the joint design of the

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source and relay precoders for a prefixed receiver at the destination, which is a very interesting and challenging problem from both practical and theoretical points of view.

- We consider two different criteria for optimization of the source and relay precoders for a prefixed receiver. Under the first criterion, we maximize the worst stream signal-to-interference-plus-noise ratio (SINR) at the output of the receiver subject to source and relay transmit power constraints. This criterion is of interest if the system wants to maximize the worst stream SINR given strict constraints on the source and relay powers. Under the second criterion, we minimize the source-relay power subject to a certain QoS constraint. This criterion is of interest when the system tries to ensure a certain required QoS while minimizing the used power. Both criteria lead to non-convex optimization problems.
- We also discuss the feasibility of both optimization problems. In particular, we show that for the two optimization problems to be feasible the number of antennas at each node should be greater or equal to the number of signal streams.
- We propose iterative alternating algorithms [24] to solve the two non-convex optimization problems, where, in each iteration, we compute the precoders alternately, i.e., all precoders are fixed except the one which is optimized. We show that the optimization problem for the computation of the source precoder can be formulated as a second order cone programming (SOCP) problem, for which the optimal solution can be found using interior point algorithms. Similarly, for the optimization of each relay precoder, we formulate the optimization problem as a semidefinite relaxation (SDR) problem, for which ready-to-use solvers exist.
- Since the proposed algorithms are iterative, we also discuss their convergence and provide sufficient conditions for it. The feasibility study of the optimization problems and the sufficient conditions for the convergence of the proposed algorithm provide us with some insight on how to design the system.

We note that, unlike the case of joint optimization of the source and relay precoders along with the equalizer [12], [13], for a prefixed receiver, the obtained solution does not diagonalize the end-to-end channel in general.

The remainder of this paper is organized as follows. In Section II, the system model is presented. The SINR maximization problem under transmit power constraints is studied in Section III, and the power minimization problem under QoS constraints is investigated in Section IV. In Section V, numerical results are presented, and conclusions are drawn in Section VI.

*Notation:* Throughout this paper, we use small and capital boldface letters to denote vectors and matrices, respectively. The operators  $(\cdot)^*$ ,  $(\cdot)^T$ ,  $(\cdot)^H$ , and  $(\cdot)^\dagger$  denote the complex conjugate, the transpose, the Hermitian transpose, and the pseudo-inverse, respectively.  $[\mathbf{A}]_{ij}$  denotes the  $(i, j)$ th entry of matrix  $\mathbf{A}$ .  $\text{vec}(\mathbf{A})$  denotes stacking the columns of  $\mathbf{A}$  in one column vector  $\mathbf{a}$  while  $\text{vec}^{-1}(\mathbf{a})$  denotes the inverse operation.

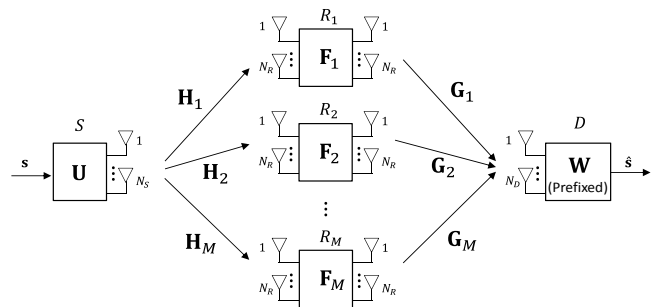


Fig. 1. Relay network with one source, one destination, and  $M$  relays. The source, relay, and destination nodes are equipped with  $N_S$ ,  $N_R$ , and  $N_D$  antennas, respectively. The receiver matrix  $\mathbf{W}$  is assumed to be prefixed.

$\otimes$  denotes the Kronecker product.  $\text{tr}(\cdot)$  and  $E[\cdot]$  denote the trace and statistical expectation operators, respectively.  $\mathbf{A} \succeq 0$  means that  $\mathbf{A}$  is a Hermitian positive semidefinite matrix.  $\mathbf{I}_K$  is the  $K \times K$  identity matrix and  $\mathbf{I}_{K \times L}$  denotes a  $K \times L$  diagonal matrix with ones on the main diagonal and zeros elsewhere.  $\|\cdot\|$  and  $|\cdot|$  denote the Euclidean norm of a vector and the absolute value of a complex scalar, respectively. Finally,  $\mathbf{e}_i$  is an all-zeros vector except for the  $i$ th position where its entry is one.

## II. SYSTEM MODEL

We consider a relay network with one source,  $S$ , one destination,  $D$ , and  $M$  relays,  $R_1, \dots, R_M$ , see Fig. 1. The source node, the destination node, and each relay are equipped with  $N_S$ ,  $N_D$ , and  $N_R$  antennas, respectively. Note that for notational convenience and without loss of generality, we assume that all relays have the same number of antennas. It is straightforward to extend the proposed schemes to the general case where the relays may have different numbers of antennas. We assume a half-duplex protocol and each transmission is organized in two time slots. In the first time slot, the source node transmits signals to all relays. In the second time slot, the relays filter the received signals and forward them to the destination node. At the destination, the signals received during the second time slot are processed and detected. We assume that there is no direct link between the source node and the destination node. We assume spatial multiplexing where the source node transmits  $L$  different signal streams. We also assume that the relays are perfectly synchronized and full channel state information (CSI) is available at the source. In practice, each relay node estimates its source-relay channel via a training sequence (known by the relays) that is sent by the source node to all relays. The destination estimates all relay-destination channels in a similar fashion. Subsequently, the destination and the relays send back the estimated channels to the source via error-free feedback channels.

The signal received at the relays during the first time slot is given by

$$\mathbf{y}_i = \mathbf{H}_i \mathbf{U} \mathbf{s} + \mathbf{n}_{ri}, \quad i \in \{1, \dots, M\}, \quad (1)$$

where  $\mathbf{s} \in \mathbb{C}^L$  is the transmit vector whose elements are independent and identically distributed (i.i.d.) and drawn from a scalar symbol alphabet  $\mathcal{A}$  such as phase-shift keying (PSK) or

quadrature amplitude modulation (QAM) with unit variance, i.e.,  $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_L$ .  $\mathbf{U} \in \mathbb{C}^{N_S \times L}$  is the precoding matrix at the source node.  $\mathbf{H}_i \in \mathbb{C}^{N_R \times N_S}$ ,  $i \in \{1, \dots, M\}$ , is the channel matrix between the source node and relay  $i$ .  $\mathbf{n}_{r_i} \in \mathbb{C}^{N_R \times 1}$  is the additive (spatially and temporally) white Gaussian noise (AWGN) vector at relay  $i$  and its elements have variance  $\sigma_{n_r}^2$ , i.e.,  $E[\mathbf{n}_{r_i}\mathbf{n}_{r_i}^H] = \sigma_{n_r}^2 \mathbf{I}_{N_R}$ .  $\mathbf{y}_i \in \mathbb{C}^{N_R \times 1}$  is the signal received at the  $i$ th relay.

The signal received at the  $i$ th relay is filtered by precoder  $\mathbf{F}_i \in \mathbb{C}^{N_R \times N_R}$ . The signal at the output of the  $i$ th relay is given by

$$\mathbf{t}_i = \mathbf{F}_i \mathbf{H}_i \mathbf{U} \mathbf{s} + \mathbf{F}_i \mathbf{n}_{r_i}, \quad i \in \{1, \dots, M\}. \quad (2)$$

The signal received at the destination during the second time slot is given by

$$\mathbf{r} = \sum_{i=1}^M \mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \mathbf{s} + \sum_{i=1}^M \mathbf{G}_i \mathbf{F}_i \mathbf{n}_{r_i} + \mathbf{n}, \quad (3)$$

where  $\mathbf{G}_i \in \mathbb{C}^{N_D \times N_R}$  is the channel matrix between relay  $i$  and the destination, and  $\mathbf{n} \in \mathbb{C}^{N_D \times 1}$  is AWGN at the destination with variance  $\sigma_n^2$ , i.e.,  $E[\mathbf{n}\mathbf{n}^H] = \sigma_n^2 \mathbf{I}_{N_D}$ . At the receiver, a linear equalizer matrix  $\mathbf{W} \in \mathbb{C}^{L \times N_D}$  is utilized to recover the transmit signal  $\mathbf{s}$ . Throughout this paper, we assume that the equalizer matrix  $\mathbf{W}$  is prefixed. We assume that the prefixed equalizer matrix  $\mathbf{W}$  is a function of the  $R$ - $D$  channel and the destination is oblivious to the existence of a precoder at the relay. For a single relay network, i.e.,  $M = 1$ , we assume that the receiver can use either no equalizer or a low-complexity equalizer (e.g., zero-forcing (ZF) or MMSE equalizer). In particular, if the receiver uses no equalizer, we have  $\mathbf{W} = \mathbf{I}_{L \times N_D}$ . However, if the receiver uses a ZF equalizer, i.e.,  $\mathbf{W} = \mathbf{G}_1^\dagger$ , or an MMSE equalizer, i.e.,  $\mathbf{W} = \mathbf{G}_1^H (\sigma_n^2 \mathbf{I}_{N_D} + \mathbf{G}_1 \mathbf{G}_1^H)^{-1}$ , we assume  $N_R = L$  and  $N_D \geq L$  so as to recover the  $L$  signal streams sent by the source at the equalizer output. For the multi-relay case, i.e.,  $M \geq 2$ , we assume that the receiver uses no equalizer, i.e.,  $\mathbf{W} = \mathbf{I}_{L \times N_D}$ , or an equalizer matrix  $\mathbf{W} \in \mathbb{C}^{L \times N_D}$ . The signal at the output of the equalizer can be written as

$$\hat{\mathbf{s}} = \mathbf{W} \mathbf{r} = \mathbf{W} \sum_{i=1}^M \mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \mathbf{s} + \mathbf{W} \sum_{i=1}^M \mathbf{G}_i \mathbf{F}_i \mathbf{n}_{r_i} + \mathbf{W} \mathbf{n}. \quad (4)$$

For future use, we compute the source and relay transmit powers. Since the transmitted symbols are i.i.d. and of unit variance, the source transmit power is given by

$$P_s = \text{tr}(\mathbf{U}\mathbf{U}^H). \quad (5)$$

From (2), it can easily be shown that the relay transmit power is given by

$$\begin{aligned} P_r &= \sum_{i=1}^M \text{tr}(\mathbf{F}_i \mathbf{H}_i \mathbf{U} \mathbf{U}^H \mathbf{H}_i^H \mathbf{F}_i^H + \sigma_{n_r}^2 \mathbf{F}_i \mathbf{F}_i^H) \\ &= \sum_{i=1}^M \text{tr}(\mathbf{F}_i (\mathbf{H}_i \mathbf{U} \mathbf{U}^H \mathbf{H}_i^H + \sigma_{n_r}^2 \mathbf{I}_{N_R}) \mathbf{F}_i^H). \end{aligned} \quad (6)$$

Hence, the total transmit power is

$$\begin{aligned} P_T &= P_s + P_r \\ &= \text{tr}(\mathbf{U}\mathbf{U}^H) + \sum_{i=1}^M \text{tr}(\mathbf{F}_i (\mathbf{H}_i \mathbf{U} \mathbf{U}^H \mathbf{H}_i^H + \sigma_{n_r}^2 \mathbf{I}_{N_R}) \mathbf{F}_i^H). \end{aligned} \quad (7)$$

As mentioned before, the computation of the precoders is done at the source node. The reason behind this assumption is to keep the complexity of the receiver at the destination as low as possible. Exploiting the full CSI of all links, the source computes all precoders and forwards the relay precoders to the relays. We note that, unlike our scheme, the schemes where the source, relay, and destination filters are jointly optimized require that the source sends the receiver filter coefficients (or the source and relay precoders coefficients if the receiver filter is computed at the destination) to the destination [12]. Hence, the signaling overhead of the scheme with a prefixed receiver in terms of forwarding the precoder coefficients is smaller than that of the schemes where the receiver filter is jointly optimized with the source and relay precoders.

In the following, we design the precoders at the source and relays according to two different criteria, namely SINR maximization subject to source and relay transmit power constraints, and source and relay transmit power minimization under QoS constraints.

### III. SINR MAXIMIZATION UNDER SOURCE AND RELAY TRANSMIT POWER CONSTRAINTS

In this section, to compute the source and relay precoders, we propose to maximize the QoS, here the worst stream SINR, which is closely related to the bit error rate (BER), subject to source and relay power constraints. Here, we assume that the source and relays are subject to separate power constraints. This assumption is more practical than a joint power constraint for the source and relays since source and relays are usually geographically separated and have their own power supplies. We first assume a joint transmit power constraint for all relays. The case of individual relay transmit power constraints will be discussed in Subsection III-C. The optimization problem for the joint relay power constraint can be formulated as

$$\begin{aligned} &\max_{\mathbf{U}, \mathbf{F}_1, \dots, \mathbf{F}_M} && \min_{j \in \{1, \dots, L\}} \text{SINR}_j \\ &\text{s. t.} && P_s \leq P_{s, \max} \\ &&& P_r \leq P_{r, \max}, \end{aligned} \quad (8)$$

where  $P_{s, \max}$  and  $P_{r, \max}$  are the maximum available transmit powers at the source and relays, respectively. The SINR of the  $j$ th signal stream at the receiver can be obtained as

$$\text{SINR}_j = \frac{\left| \left[ \sum_{i=1}^M \mathbf{W} \mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jj} \right|^2}{\sum_{k \neq j}^L \left| \left[ \sum_{i=1}^M \mathbf{W} \mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jk} \right|^2 + \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} \left| \left[ \mathbf{W} \mathbf{G}_i \mathbf{F}_i \right]_{jk} \right|^2 + \bar{\sigma}_{n, j}^2}, \quad (9)$$

where  $\bar{\sigma}_{n,j}^2 = \sigma_n^2 \sum_{k=1}^{N_D} |[\mathbf{W}]_{jk}|^2$ . Note that the max–min criterion in (8) ensures fairness among the signal streams, i.e., the optimal solution satisfies  $\text{SINR}_1 = \text{SINR}_2 = \dots = \text{SINR}_L$ .

It can easily be shown that (8) is always feasible provided that  $\min(N_S, N_R, N_D) \geq L$ . For example, one feasible suboptimal solution is

$$\mathbf{U} = \sqrt{\frac{P_{s,max}}{L}} \mathbf{I}_{N_S \times L}, \quad (10)$$

$$\mathbf{F}_j = \sqrt{\frac{P_{r,max}}{\sum_{i=1}^M \text{tr}(\mathbf{H}_i \mathbf{U} \mathbf{U}^H \mathbf{H}_i^H + \sigma_{n_r}^2 \mathbf{I}_{N_R})}} \mathbf{I}_{N_R}, \quad j = 1, \dots, M. \quad (11)$$

Optimization problem (8) is non–convex and in general NP–hard. Hence, it is very difficult, if not impossible, to solve it optimally. Notice that the non–convexity has its origin in the product of the precoders. In the following, we propose an iterative alternating algorithm to solve optimization problem (8). In particular, in each iteration of the algorithm, we compute the source and relay precoders alternately. First, we fix all the relay precoders and optimize the source precoder  $\mathbf{U}$  by solving (8). Then, we use the obtained  $\mathbf{U}$  and fix all the relay precoders except one, say  $\mathbf{F}_i$ , and optimize it by solving (8), and so on for the other relay precoders. Having the new source and relay precoders, we repeat the same procedure till convergence. The convergence of the iterative alternating algorithm will be discussed later in this section. In the following, we describe the computation of the precoders in more detail.

#### A. Computation of the Source Precoder $\mathbf{U}$

Let us assume that the relay precoders are fixed and compute the source precoder that maximizes the cost function in (8). Rewriting optimization problem (8) and after some simplifications, we obtain

$$\begin{aligned} \max_{\mathbf{U}, \lambda} \quad & \lambda \\ \text{s. t.} \quad & \frac{|[\mathbf{TU}]_{jj}|^2}{\sum_{k \neq j} |[\mathbf{TU}]_{jk}|^2 + \alpha_j} \geq \lambda, \quad j = 1, \dots, L \\ & \text{tr}(\mathbf{U} \mathbf{U}^H) \leq P_{s,max} \\ & \sum_{i=1}^M \text{tr}(\mathbf{F}_i \mathbf{H}_i \mathbf{U} \mathbf{U}^H \mathbf{H}_i^H \mathbf{F}_i^H) \leq P_1, \end{aligned} \quad (12)$$

where  $\lambda$  is a real–valued slack variable,  $\mathbf{T} = \sum_{i=1}^M \mathbf{W} \mathbf{G}_i \mathbf{F}_i \mathbf{H}_i$ ,  $\alpha_j = \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} |[\mathbf{W} \mathbf{G}_i \mathbf{F}_i]_{jk}|^2 + \bar{\sigma}_{n,j}^2$ , and  $P_1 = P_{r,max} - \sigma_{n_r}^2 \sum_{i=1}^M \text{tr}(\mathbf{F}_i \mathbf{F}_i^H)$ .

The first constraint in (12) can be rewritten as [23]

$$\begin{aligned} \frac{1}{\lambda} |[\mathbf{TU}]_{jj}|^2 & \geq \sum_{k \neq j} |[\mathbf{TU}]_{jk}|^2 + \alpha_j \\ & \geq \sum_{k=1}^L |[\mathbf{TU}]_{jk}|^2 + \alpha_j - |[\mathbf{TU}]_{jj}|^2. \end{aligned} \quad (13)$$

Then,

$$\left(1 + \frac{1}{\lambda}\right) |[\mathbf{TU}]_{jj}|^2 \geq \sum_{k=1}^L |[\mathbf{TU}]_{jk}|^2 + \alpha_j. \quad (14)$$

From (12), we can see that if  $\mathbf{U}$  is an optimal solution so is  $\mathbf{U} \text{diag}\{e^{j\theta_1}, \dots, e^{j\theta_L}\}$ , where  $\theta_i$ ,  $i = 1, \dots, L$ , are arbitrary phases. Then, without loss of generality, we can assume that  $[\mathbf{TU}]_{jj} \geq 0$ . Therefore, (14) becomes

$$\sqrt{1 + \frac{1}{\lambda}} |[\mathbf{TU}]_{jj}| \geq \left\| \frac{\mathbf{U}^H \mathbf{T}^H \mathbf{e}_j}{\sqrt{\alpha_j}} \right\|, \quad (15)$$

where  $\mathbf{e}_j$  is an all zeros vector except with a one at the  $j$ th position.

Using

$$\text{tr}(\mathbf{U}^H \mathbf{U}) = \|\text{vec}(\mathbf{U})\|^2, \quad (16)$$

the second constraint in (12) can be written as

$$\|\text{vec}(\mathbf{U})\| \leq \sqrt{P_{s,max}}. \quad (17)$$

Furthermore, the left hand side of the third constraint in (12) can be simplified as

$$\sum_{i=1}^M \text{tr}(\mathbf{F}_i \mathbf{H}_i \mathbf{U} \mathbf{U}^H \mathbf{H}_i^H \mathbf{F}_i^H) = \text{tr}\left(\mathbf{U}^H \left(\sum_{i=1}^M \mathbf{H}_i^H \mathbf{F}_i^H \mathbf{F}_i \mathbf{H}_i\right) \mathbf{U}\right). \quad (18)$$

Using Cholesky decomposition, we have

$$\sum_{i=1}^M \mathbf{H}_i^H \mathbf{F}_i^H \mathbf{F}_i \mathbf{H}_i = \mathbf{L}^H \mathbf{L}. \quad (19)$$

Combining (16), (18), and (19), the third constraint in (12) can be rewritten as

$$\|\text{vec}(\mathbf{L} \mathbf{U})\| \leq \sqrt{P_1}. \quad (20)$$

Finally, optimization problem (12) can be recast as

$$\begin{aligned} \max_{\mathbf{U}, \lambda} \quad & \lambda \\ \text{s. t.} \quad & \sqrt{1 + \frac{1}{\lambda}} |[\mathbf{TU}]_{jj}| \geq \left\| \frac{\mathbf{U}^H \mathbf{T}^H \mathbf{e}_j}{\sqrt{\alpha_j}} \right\|, \quad j = 1, \dots, L \\ & \|\text{vec}(\mathbf{U})\| \leq \sqrt{P_{s,max}} \\ & \|\text{vec}(\mathbf{L} \mathbf{U})\| \leq \sqrt{P_1}. \end{aligned} \quad (21)$$

Note that for a given  $\lambda$ , optimization problem (21) is an SOCP feasibility problem. Therefore, for a given  $\lambda$ , optimization problem (21) can be solved optimally using interior point algorithms [25], and the optimal  $\lambda$  can be found using a simple bisection search [26].

### B. Computation of Relay Precoders $\mathbf{F}_i$

Now, let us assume that all source and relay precoders are fixed except the precoder at relay  $m$ ,  $\mathbf{F}_m$ . The optimization problem for  $\mathbf{F}_m$  becomes

$$\max_{\mathbf{F}_m} \min_{j \in \{1, \dots, L\}} \frac{\left| \left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jj} \right|^2}{\sum_{k \neq j}^L \left| \left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk} \right|^2 + \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} \left| \left[ \mathbf{Q}_i \mathbf{F}_i \right]_{jk} \right|^2 + \bar{\sigma}_{n,j}^2}$$

$$\text{s. t.} \quad \text{tr}(\mathbf{F}_m (\mathbf{P}_m \mathbf{P}_m^H + \sigma_{n_r}^2 \mathbf{I}_{N_R}) \mathbf{F}_m^H) \leq P_m, \quad (22)$$

where  $\mathbf{Q}_i = \mathbf{W} \mathbf{G}_i$ ,  $\mathbf{P}_i = \mathbf{H}_i \mathbf{U}$ , and  $P_m = P_{r, \max} - \sum_{i=1}^M \text{tr}(\mathbf{F}_i (\mathbf{P}_i \mathbf{P}_i^H + \sigma_{n_r}^2 \mathbf{I}_{N_R}) \mathbf{F}_i^H)$ . Optimization problem (22) can be recast as (see Appendix for the details)

$$\max_{\mathbf{f}_m} \min_{j \in \{1, \dots, L\}} \frac{\mathbf{f}_m^H \mathbf{A}_{m,j} \mathbf{f}_m + \mathbf{f}_m^H \mathbf{a}_{m,j} + \mathbf{a}_{m,j}^H \mathbf{f}_m + a_{m,j}}{\mathbf{f}_m^H \mathbf{V}_{m,j} \mathbf{f}_m + \mathbf{f}_m^H \mathbf{v}_{m,j} + \mathbf{v}_{m,j}^H \mathbf{f}_m + v_{m,j}}$$

$$\text{s. t.} \quad \mathbf{f}_m^H \mathbf{D}_m \mathbf{f}_m \leq P_m, \quad (23)$$

where  $\mathbf{f}_m = \text{vec}(\mathbf{F}_m)$  and  $\mathbf{D}_m = (\mathbf{P}_m \mathbf{P}_m^H + \sigma_{n_r}^2 \mathbf{I}_{N_R})^T \otimes \mathbf{I}_{N_R}$ , and  $\mathbf{A}_{m,j}$ ,  $\mathbf{a}_{m,j}$ , and  $a_{m,j}$  are defined after (46), and  $\mathbf{V}_{m,j}$ ,  $\mathbf{v}_{m,j}$ , and  $v_{m,j}$  are given in (52), (53), and (54), respectively. Optimization problem (23) is a non-convex fractional quadratically constrained quadratic program (QCQP) and is NP-hard. To solve (23), we propose to relax it into a semidefinite program (SDP) referred to as semidefinite relaxation (SDR). Note that the cost function in (23) is in inhomogeneous form. Let us first write (23) as a homogeneous fractional QCQP. We define

$$\bar{\mathbf{D}}_m = \begin{bmatrix} \mathbf{D}_m & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}, \quad \mathbf{B}_{m,j} = \begin{bmatrix} \mathbf{A}_{m,j} & \mathbf{a}_{m,j} \\ \mathbf{a}_{m,j}^H & a_{m,j} \end{bmatrix},$$

$$\mathbf{C}_{m,j} = \begin{bmatrix} \mathbf{V}_{m,j} & \mathbf{v}_{m,j} \\ \mathbf{v}_{m,j}^H & v_{m,j} \end{bmatrix}, \quad \text{and } \bar{\mathbf{f}}_m = \begin{bmatrix} \mathbf{f}_m \\ t \end{bmatrix}, \quad (24)$$

where  $|t|^2 = 1$ . Using (24), optimization problem (23) can be written as a homogeneous fractional QCQP as follows

$$\max_{\bar{\mathbf{f}}_m} \min_{j \in \{1, \dots, L\}} \frac{\bar{\mathbf{f}}_m^H \mathbf{B}_{m,j} \bar{\mathbf{f}}_m}{\bar{\mathbf{f}}_m^H \mathbf{C}_{m,j} \bar{\mathbf{f}}_m}$$

$$\text{s. t.} \quad \bar{\mathbf{f}}_m^H \bar{\mathbf{D}}_m \bar{\mathbf{f}}_m \leq P_m$$

$$\bar{\mathbf{f}}_m^H \Gamma \bar{\mathbf{f}}_m = 1, \quad (25)$$

where  $\Gamma = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}$ . Letting  $\bar{\Phi}_m = \bar{\mathbf{f}}_m \bar{\mathbf{f}}_m^H$  and using the fact that  $\mathbf{x}^H \mathbf{A} \mathbf{x} = \text{tr}(\mathbf{A} \mathbf{x} \mathbf{x}^H)$ , optimization problem (25) can be written equivalently as

$$\max_{\bar{\Phi}_m} \min_{j \in \{1, \dots, L\}} \frac{\text{tr}(\mathbf{B}_{m,j} \bar{\Phi}_m)}{\text{tr}(\mathbf{C}_{m,j} \bar{\Phi}_m)}$$

$$\text{s. t.} \quad \text{tr}(\bar{\mathbf{D}}_m \bar{\Phi}_m) \leq P_m$$

$$\text{tr}(\Gamma \bar{\Phi}_m) = 1$$

$$\bar{\Phi}_m \succeq \mathbf{0}$$

$$\text{rank}(\bar{\Phi}_m) = 1. \quad (26)$$

Optimization problem (26) is still non-convex because the rank constraint is not convex. By relaxing (dropping) the rank

constraint in (26), we obtain the SDR of (26), which is a generalized quasiconvex problem and can be efficiently solved by a bisection search [25]. In particular, by introducing a new variable  $\lambda$ , the SDR of problem (26) can be written as

$$\max_{\bar{\Phi}_m, \lambda} \lambda$$

$$\text{s. t.} \quad \text{tr}(\mathbf{B}_{m,j} \bar{\Phi}_m) \geq \lambda \text{tr}(\mathbf{C}_{m,j} \bar{\Phi}_m), \quad j = 1, \dots, L$$

$$\text{tr}(\bar{\mathbf{D}}_m \bar{\Phi}_m) \leq P_m$$

$$\text{tr}(\Gamma \bar{\Phi}_m) = 1$$

$$\bar{\Phi}_m \succeq \mathbf{0}. \quad (27)$$

For the special case of one relay, i.e.  $M = 1$ , problem (23) is a homogeneous fractional QCQP since in this case we have  $\mathbf{a}_{m,j} = \mathbf{0}$ ,  $a_{m,j} = 0$ ,  $\mathbf{v}_{m,j} = \mathbf{0}$ , and  $v_{m,j} = \bar{\sigma}_{n,j}^2$ . Hence, problem (27) reduces to

$$\max_{\bar{\Phi}_m, \lambda} \lambda$$

$$\text{s. t.} \quad \text{tr}(\mathbf{A}_{m,j} \bar{\Phi}_m) \geq \lambda \text{tr}(\mathbf{V}_{m,j} \bar{\Phi}_m) + \lambda v_{m,j}, \quad j = 1, \dots, L$$

$$\text{tr}(\mathbf{D}_m \bar{\Phi}_m) \leq P_m$$

$$\bar{\Phi}_m \succeq \mathbf{0}. \quad (28)$$

where  $\bar{\Phi}_m = \mathbf{f}_m \mathbf{f}_m^H$ .

Note that for a given  $\lambda$ , both optimization problems (27) and (28) reduce to feasibility problems that can be solved efficiently using software packages like the convex optimization toolbox CVX [27]. The optimal  $\lambda$  can easily be found using a simple bisection search [25], [26]. It is worth noting that the obtained solution is not necessarily of rank one. Nevertheless, we can obtain a rank-one solution via rank reduction techniques [26], [28]. Let  $\bar{\Phi}_m^*$  denote the obtained optimal solution of problem (27). Depending on the rank of  $\bar{\Phi}_m^*$ , we have the following two cases:

**Case 1** ( $\text{rank}(\bar{\Phi}_m^*) = 1$ ): If  $\bar{\Phi}_m^*$  is of rank one, then this solution is also optimal for the rank-constrained optimization problem (26) since problems (26) and (27) are equivalent in this case. We can get  $\bar{\mathbf{f}}_m^* = [\bar{\mathbf{f}}_m^{*T}, t^*]^T$  from  $\bar{\Phi}_m^*$  by eigen-decomposition. Then, we compute  $\mathbf{f}_m^* = \bar{\mathbf{f}}_m^*/t^*$ . Finally, we get  $\mathbf{F}_m^* = \text{vec}^{-1}(\mathbf{f}_m^*)$ . For the case of a single relay, i.e.,  $M = 1$ , let  $\Phi_m^*$  denote the obtained optimal rank-one solution of problem (28). In this case,  $\mathbf{f}_m^*$  is obtained from  $\Phi_m^*$  by eigen-decomposition and  $\mathbf{F}_m^* = \text{vec}^{-1}(\mathbf{f}_m^*)$ .

**Case 2** ( $\text{rank}(\bar{\Phi}_m^*) > 1$ ): Our aim in this case is to extract an optimal rank-one solution from  $\bar{\Phi}_m^*$  if possible, and if not, a suboptimal rank-one solution. We have the following proposition.

*Proposition 1:* An optimal rank-one solution for problem (27) can always be obtained from  $\bar{\Phi}_m^*$  if either  $L = 2$  ( $\forall M$ ) or  $L \leq 3$  (if  $M = 1$ ); otherwise, only suboptimal rank-one solutions can be obtained.

*Proof:* The proof is based on the following lemma.

*Lemma 1:* The SDR of a complex-valued homogeneous QCQP with  $K$  constraints has an optimal solution with rank  $r \leq \lfloor \sqrt{K} \rfloor$ , where  $\lfloor x \rfloor$  denotes the largest integer smaller than or equal to  $x$ . Moreover, for a feasibility problem with  $K$  quadratic constraints, an optimal solution with rank  $r \leq \lfloor \sqrt{K-1} \rfloor$  for its SDR exists.

We refer the reader to [29], [30] for the proof of the first part of the lemma, and [26] for the second part.

Then, if  $L = 2$ , we have  $K = 4$  constraints (without  $\bar{\Phi}_m \succeq 0$ ). According to Lemma 1, in this case, problem (27) has an optimal solution with rank  $r \leq \lfloor \sqrt{3} \rfloor = 1$ , i.e., it has an optimal rank-one solution. The optimal rank-one solution can then be extracted from  $\bar{\Phi}_m^*$  using the rank reduction technique in [28]. Once we have the rank-one solution,  $\mathbf{F}_m^*$  can be computed in a similar way as in Case 1 above.

Now, for the special case of one relay, i.e.,  $M = 1$ , problem (28) has  $K = L + 1$  constraints. Hence, according to Lemma 1, for  $M = 1$ , problem (28) has always an optimal rank-one solution if  $L \leq 3$ . Again, the optimal rank-one solution can be extracted from  $\bar{\Phi}_m^*$  using the rank reduction technique in [28]. Once we have the rank-one solution, we can get  $\mathbf{F}_m^*$  in a similar way as in Case 1 above.

In all other cases, i.e.,  $L \geq 3$  (if  $M > 1$ ) or  $L \geq 4$  (if  $M = 1$ ), we cannot ensure the extraction of an optimal rank-one solution when the rank of the obtained optimal solution  $\bar{\Phi}_m^*$  of problem (27) (or (28) if  $M = 1$ ) is greater than one. However, we can extract a suboptimal rank-one solution using the randomization technique in [26]. This concludes the proof.  $\blacksquare$

All relay precoders are computed in the described manner. We repeat the procedure of computing the precoders alternately in each iteration till convergence.

Algorithm 1 summarizes the steps for solving optimization problem (8).

Since optimization problem (8) is non-convex, the obtained solution provided by the proposed iterative alternating algorithm is not guaranteed to be globally optimal and in general the algorithm converges to a fixed point. We have the following proposition.

*Proposition 2:* If the obtained solution for each precoder is optimal, the proposed iterative alternating algorithm converges to a fixed point. Otherwise, the convergence to a fixed point is not guaranteed.

*Proof:* We first prove the first part of the proposition. We note that the cost function is upper bounded. Also, if the obtained solution for each precoder is optimal, then the cost function in (8) is nondecreasing after each iteration. Hence, since the cost function is upper bounded and nondecreasing after each iteration, the convergence of Algorithm 1 to a fixed point is guaranteed [31]. Therefore, according to Proposition 1, a sufficient condition for convergence to a fixed point is either  $L = 2$  ( $\forall M$ ) or  $L \leq 3$  (if  $M = 1$ ). We now prove the second part of the proposition. If the obtained solution for one of the precoders is suboptimal, we cannot guarantee that the cost function in (8) is nondecreasing after each iteration. Hence, even if the cost function is upper bounded, the convergence to a fixed point is not guaranteed in this case.  $\blacksquare$

Note that according to Proposition 1, if either  $L > 2$  ( $\forall M$ ) or  $L > 3$  (if  $M = 1$ ), the iterative alternating algorithm is not guaranteed to converge to a fixed point. In this case, to get the best possible solution, we propose to keep the best value of the cost function and its corresponding precoders after updating each precoder. It is worth mentioning that the case of  $L = 2$ , which corresponds to the transmission of two signal streams, is

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**Algorithm 1** Algorithm for solving optimization problem (8)

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**Input:**  $\mathbf{H}_i, \mathbf{G}_i, i = 1, \dots, M, \sigma_{nr}^2$ , and  $\sigma_n^2$ .

**Output:** Solution:  $\mathbf{U}^*, \mathbf{F}_1^*, \dots, \mathbf{F}_M^*$ .

```

1: Set feasible  $\mathbf{U}, \mathbf{F}_1, \dots, \mathbf{F}_M$ .
2: Set the precision  $\epsilon$ , initial cost function  $CF^{(0)}$ ,  $m = 0$ .
3: repeat
4:    $m = m + 1$ .
5:   Obtain  $\mathbf{U}^*$  by solving problem (21).
6:   for  $i = 1 \rightarrow M$  do
7:     if  $M = 1$  then
8:       Obtain  $\bar{\Phi}_i^*$  by solving problem (28).
9:       if rank( $\bar{\Phi}_i^*$ ) = 1 then
10:        Set  $\Phi_i^{1,*} = \bar{\Phi}_i^*$ .
11:       else if  $L \leq 3$  then
12:        Obtain optimal rank-one solution  $\Phi_i^{1,*}$ 
from  $\bar{\Phi}_i^*$  using the rank reduction
technique in [28].
13:       else
14:        Obtain suboptimal rank-one solution
 $\Phi_i^{1,*}$  from  $\bar{\Phi}_i^*$  using the randomization
technique in [26].
15:       end if
16:       Obtain  $\mathbf{f}_i^*$  from rank-one solution  $\Phi_i^{1,*}$ 
through eigen-decomposition.
17:       else
18:        Obtain  $\bar{\Phi}_i^*$  by solving problem (27).
19:        if rank( $\bar{\Phi}_i^*$ ) = 1 then
20:         Set  $\bar{\Phi}_i^{1,*} = \bar{\Phi}_i^*$ .
21:        else if  $L = 2$  then
22:         Obtain optimal rank-one solution  $\bar{\Phi}_i^{1,*}$ 
from  $\bar{\Phi}_i^*$  using the rank reduction
technique in [28].
23:        else
24:         Obtain suboptimal rank-one solution
 $\bar{\Phi}_i^{1,*}$  from  $\bar{\Phi}_i^*$  using the randomization
technique in [26].
25:        end if
26:        Obtain  $\bar{\mathbf{f}}_i^*$  from rank-one solution  $\bar{\Phi}_i^{1,*}$ 
through eigen-decomposition.
27:        Let  $\bar{\mathbf{f}}_i^* = [\tilde{\mathbf{f}}_i^{*T}, t^*]^T$ .
28:        Evaluate  $\mathbf{f}_i^* = \tilde{\mathbf{f}}_i^*/t^*$ .
29:        end if
30:        Set  $\mathbf{F}_i^* = \text{vec}^{-1}(\mathbf{f}_i^*)$ .
31:      end for
32:      Compute the cost function in (8):  $CF^{(m)} =$ 
 $\min_{j \in \{1, \dots, L\}} \text{SINR}_j$ .
33: until  $|CF^{(m)} - CF^{(m-1)}| \leq \epsilon$ 

```

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of practical interest since at the end-user we have a constraint on power consumption and size, and hence on the number of antennas. Thus, accommodating more than two signal streams may be challenging in practice. For the case of  $L = 2$  and for any number of relays the proposed iterative alternating algorithm converges to a fixed point.

### C. Individual Relay Transmit Power Constraints

In this subsection, we discuss the practical case where the relays have individual power constraints. From (2), the  $i$ th relay transmit power is given by

$$P_{r,i} = \text{tr}(\mathbf{F}_i(\mathbf{H}_i\mathbf{U}\mathbf{U}^H\mathbf{H}_i^H + \sigma_{n_r}^2\mathbf{I}_{N_R})\mathbf{F}_i^H). \quad (29)$$

The optimization problem for individual relay transmit power constraints is the same as that in (8) except that the total relay transmit power constraint (second constraint) is replaced by individual relay transmit power constraints given by

$$P_{r,i} \leq P_{r,i,max}, \quad i = 1, \dots, M \quad (30)$$

where  $P_{r,i,max}$  is the maximum available transmit power at relay  $i$ . Following the same reasoning as in Subsection III-A, we can show that the resulting optimization problem for the source precoder  $\mathbf{U}$  when individual relay transmit power constraints are used is given by

$$\begin{aligned} \max_{\mathbf{U}, \lambda} \quad & \lambda \\ \text{s. t.} \quad & \sqrt{1 + \frac{1}{\lambda}} [\mathbf{T}\mathbf{U}]_{jj} \geq \left\| \frac{\mathbf{U}^H \mathbf{T}^H \mathbf{e}_j}{\sqrt{\alpha_j}} \right\|, \quad j = 1, \dots, L \\ & \|\text{vec}(\mathbf{U})\| \leq \sqrt{P_{s,max}} \\ & \|\text{vec}(\mathbf{F}_i \mathbf{H}_i \mathbf{U})\| \leq \sqrt{P_{1,i}}, \quad i = 1, \dots, M \end{aligned} \quad (31)$$

where  $P_{1,i} = P_{r,i,max} - \sigma_{n_r}^2 \text{tr}(\mathbf{F}_i \mathbf{F}_i^H)$ . Optimization problem (31) is an SOCP feasibility problem and therefore, similar to optimization problem (21), for a given  $\lambda$ , it can be solved optimally using interior point algorithms [25].

The optimization problem for relay precoder  $\mathbf{F}_m$  is exactly the same as that in (22) except that  $P_m$  on the right hand side of the inequality in the constraint is replaced by the maximum available transmit power at relay  $m$ ,  $P_{r,m,max}$ . It is worth noting that in the computation of relay precoder  $\mathbf{F}_m$ , the source and the other relay transmit power constraints are not active since they do not depend on  $\mathbf{F}_m$ . Therefore, the resulting optimization problem for precoder  $\mathbf{F}_m$  with individual relay transmit power constraints can be solved exactly in the same manner as problem (22) and hence Proposition 1 regarding the optimality of the solution holds. Moreover, the convergence result in Proposition 3 also holds for the maximization of the SINR under source and individual relay transmit power constraints problem. In particular, the alternating iterative algorithm converges to a fixed point if all the precoders are solved optimally in each iteration; otherwise, the convergence is not guaranteed.

## IV. SOURCE-RELAY TRANSMIT POWER MINIMIZATION UNDER QoS CONSTRAINTS

In some applications, we are interested in ensuring a given QoS with the smallest transmit power possible. The QoS metric here is the worst stream SINR which is closely related to the BER. Here, we aim at minimizing the total transmit power of the source and the relays. The optimization problem in this case can be expressed as

$$\begin{aligned} \min_{\mathbf{U}, \mathbf{F}_1, \dots, \mathbf{F}_M} \quad & P_T \\ \text{s. t.} \quad & \min_{j \in \{1, \dots, L\}} \text{SINR}_j \geq \gamma, \end{aligned} \quad (32)$$

where  $\gamma > 0$  is the given worst stream SINR.

Optimization problem (32) is non-convex and in general NP-hard. To solve it we adopt the iterative alternate procedure proposed in Section III. In particular, in each iteration, we compute the precoders alternately, i.e., in each iteration, we fix all the precoders except one and optimize the non-fixed precoder, and similarly for the other precoders.

### A. Feasibility of Problem (32)

Before describing how to optimize the source and relay precoders, we first verify the feasibility of optimization problem (32). To this end, we need to check whether for a given worst stream SINR  $\gamma_0$ , a solution exists such that [23]

$$\min_j \frac{\left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jj} \right|^2}{\sum_{k \neq j} \left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jk} \right|^2 + \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} \left| [\mathbf{W}\mathbf{G}_i \mathbf{F}_i]_{jk} \right|^2 + \bar{\sigma}_{n,j}^2} \geq \gamma_0. \quad (33)$$

Similar to [23], we consider the signal-to-interference ratio (SIR) to verify the feasibility since this is simpler compared to using the SINR<sup>1</sup>. It should be noted that the SIR is used instead of SINR only to verify the feasibility of optimization problem (32) but for the design of the precoders we always use the SINR. We have

$$\begin{aligned} & \frac{\left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jj} \right|^2}{\sum_{k \neq j} \left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jk} \right|^2 + \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} \left| [\mathbf{W}\mathbf{G}_i \mathbf{F}_i]_{jk} \right|^2 + \bar{\sigma}_{n,j}^2} \\ & < \frac{\left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jj} \right|^2}{\sum_{k \neq j} \left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jk} \right|^2}. \end{aligned} \quad (34)$$

The following proposition gives us a condition for the feasibility.

*Proposition 3:* For a given worst stream SINR  $\gamma_0$ , a solution such that

$$\min_j \frac{\left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jj} \right|^2}{\sum_{k \neq j} \left| \left[ \sum_{i=1}^M \mathbf{W}\mathbf{G}_i \mathbf{F}_i \mathbf{H}_i \mathbf{U} \right]_{jk} \right|^2} \geq \gamma_0 \quad (35)$$

exists if

$$\gamma_0 \leq \frac{1}{\frac{L}{\min\left(L, \sum_{i=1}^M \min(\text{rank}(\mathbf{H}_i), \text{rank}(\mathbf{G}_i))\right)} - 1}. \quad (36)$$

<sup>1</sup>Note that the difference between SINR and SIR can be made negligible by scaling  $\mathbf{U}$  by a large factor which corresponds to the high SNR regime. This makes the noise terms negligible compared to the interference.

*Proof:* The proof is based on the proof of Proposition 1 in [23]. From [23], we can show that

$$\gamma_0 \leq \frac{1}{\frac{L}{\text{rank}(\sum_{i=1}^M \mathbf{W}\mathbf{G}_i\mathbf{F}_i\mathbf{H}_i)} - 1}. \quad (37)$$

Moreover, using the inequalities  $\text{rank}(\mathbf{A} + \mathbf{B}) \leq \text{rank}(\mathbf{A}) + \text{rank}(\mathbf{B})$  and  $\text{rank}(\mathbf{A}\mathbf{B}) \leq \min(\text{rank}(\mathbf{A}), \text{rank}(\mathbf{B}))$  in (37), we obtain (36). This concludes the proof. ■

For example, in case of one relay, to ensure the feasibility of the problem for any  $\gamma_0$ , we need to have  $\min(\text{rank}(\mathbf{H}_1), \text{rank}(\mathbf{G}_1)) \geq L$ . Assuming random channels,  $\mathbf{H}_1$  and  $\mathbf{G}_1$  are full rank with probability one. Therefore, the feasibility condition of the problem for any  $\gamma_0$  reduces to  $\min(N_S, N_R, N_D) \geq L$ .

Assuming that problem (32) is always feasible, we now focus on the computation of the source and relay precoders.

### B. Computation of the Source Precoder $\mathbf{U}$

As in Subsection III-A, let us assume that the relay precoders are fixed and compute the source precoder that minimizes the cost function in (32). Rewriting optimization problem (32) and after some simplifications, we obtain

$$\begin{aligned} \min_{\mathbf{U}} \quad & \text{tr}(\mathbf{U}\mathbf{U}^H) + \sum_{i=1}^M \text{tr}(\mathbf{F}_i\mathbf{H}_i\mathbf{U}\mathbf{U}^H\mathbf{H}_i^H\mathbf{F}_i^H) \\ \text{s. t.} \quad & \min_{j \in \{1, \dots, L\}} \frac{|\mathbf{[TU]}_{jj}|^2}{\sum_{k \neq j} |\mathbf{[TU]}_{jk}|^2 + \alpha_j} \geq \gamma. \end{aligned} \quad (38)$$

Similar to the development in Subsection III-A, it can be shown that optimization problem (38) can be written in the form of an SOCP as follows

$$\begin{aligned} \min_{\mathbf{U}} \quad & \|\text{vec}(\mathbf{L}\mathbf{U})\| \\ \text{s. t.} \quad & \sqrt{1 + \frac{1}{\gamma}} |\mathbf{[TU]}_{jj}| \geq \left\| \frac{\mathbf{U}^H \mathbf{T}^H \mathbf{e}_j}{\sqrt{\alpha_j}} \right\|, \quad j = 1, \dots, L. \end{aligned} \quad (39)$$

This SOCP problem can be solved optimally using interior point algorithms [25].

### C. Computation of Relay Precoders $\mathbf{F}_i$

Assume now that all precoders are fixed except  $\mathbf{F}_m$ . Solving problem (32) with respect to  $\mathbf{F}_m$  is equivalent to solving the following optimization problem

$$\begin{aligned} \min_{\mathbf{F}_m} \quad & \text{tr}(\mathbf{F}_m(\mathbf{P}_m\mathbf{P}_m^H + \sigma_{n_r}^2\mathbf{I}_{N_R})\mathbf{F}_m^H) \\ \text{s. t.} \quad & \frac{\left| \left[ \sum_{i=1}^M \mathbf{Q}_i\mathbf{F}_i\mathbf{P}_i \right]_{jj} \right|^2}{\sum_{k \neq j} \left| \left[ \sum_{i=1}^M \mathbf{Q}_i\mathbf{F}_i\mathbf{P}_i \right]_{jk} \right|^2 + \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} |\mathbf{[Q}_i\mathbf{F}_i]_{jk}|^2 + \bar{\sigma}_{n_r}^2} \geq \gamma, \\ & j = 1, \dots, L. \end{aligned} \quad (40)$$

The SDR of (40) can be obtained, in a similar way as that in Subsection III-B, as

$$\begin{aligned} \min_{\bar{\Phi}_m} \quad & \text{tr}(\bar{\mathbf{D}}_m\bar{\Phi}_m) \\ \text{s. t.} \quad & \text{tr}(\mathbf{B}_{m,j}\bar{\Phi}_m) \geq \gamma \text{tr}(\mathbf{C}_{m,j}\bar{\Phi}_m), \quad j = 1, \dots, L \\ & \text{tr}(\mathbf{\Gamma}\bar{\Phi}_m) = 1 \\ & \bar{\Phi}_m \succeq 0. \end{aligned} \quad (41)$$

Problem (41) can be solved efficiently using software packages like the convex optimization toolbox CVX [27]. Note that the obtained solution is not necessarily of rank one. Nevertheless, we can obtain a rank-one solution via rank reduction techniques [26], [28]. The observations regarding how to extract the rank-one solution, its optimality, and convergence of the iterative alternating algorithm made in Subsection III-B also hold true here.

The algorithm for solving problem (32) is similar to Algorithm 1 except that in line 5,  $\mathbf{U}^*$  is obtained by solving problem (39), in line 18,  $\bar{\Phi}_i^*$  is obtained by solving problem (41), in line 32, we compute the cost function in (32) and put  $CF^{(m)} = P_T$ , and in line 8,  $\Phi_i^*$  is obtained by solving the following optimization problem

$$\begin{aligned} \min_{\Phi_m} \quad & \text{tr}(\mathbf{D}_m\Phi_m) \\ \text{s. t.} \quad & \text{tr}(\mathbf{A}_{m,j}\Phi_m) \geq \gamma \text{tr}(\mathbf{V}_{m,j}\Phi_m) + \gamma v_{m,j}, \quad j = 1, \dots, L \\ & \Phi_m \succeq 0. \end{aligned} \quad (42)$$

## V. NUMERICAL RESULTS

In this section, we assess the performance of the proposed iterative alternating algorithms. We refer to the maximization of the worst SINR and minimization of the total power schemes as “*Proposed Max-Min SINR*” and “*Proposed Min Power*”, respectively. We compare the proposed algorithms with the case where the receiver is not fixed, i.e., the receiver is equipped with an equalizer which is jointly optimized with the source and relay precoders [12], [16], [17], [32]. The comparison with the case where the equalizer at the receiver is jointly optimized with the source and relay precoders allows us to evaluate how much we loose in performance by using prefixed receivers. Ref. [12] assumed single relay networks where several cost functions were studied. Here, for the sake of comparison, we consider the cost function based on the minimization of the maximum mean square error (MSE), since it is equivalent to the maximization of the minimum SINR cost function used in our proposed scheme, and refer to it as “*Min-Max MSE with Equalizer*”. For the multiple relay case, since there are no works in the literature that consider the maximization of the minimum SINR for the joint design of source, relays, and destination, we compare with the scheme in [32] in which the precoders are optimized for minimization of the sum of MSEs of the signal streams subject to source and relay power constraints. We will refer to the scheme in [32] as “*Min Sum MSE with Equalizer*”. For the minimization of the total power consumption subject to QoS constraints, we compare our scheme with the scheme in [16] which will be referred to as “*Min Power with Equalizer*” and the scheme in [17] which will be referred to as “*Min*”



Power with Equalizer 1” for a single relay and multiple relays, respectively. Furthermore, we compare the proposed algorithms with point-to-point MIMO transmission without relaying, where the destination is also equipped with a prefixed receiver [23]. We refer to the maximization of the worst stream SINR and the minimization of the transmit power in [23] as “Max–Min SINR without Relaying” and “Min Power without Relaying”, respectively. To evaluate the BER of the schemes, we assume that the transmitted symbols are drawn from a 4-QAM constellation. The entries of the channel matrices  $\mathbf{H}_i$  and  $\mathbf{G}_i$  are modeled as i.i.d. zero mean complex Gaussian random variables. Moreover, we assume that the relays are located in the middle between the source and the destination, and the path-loss exponent is 3.5. Unless specified otherwise, we assume that the source–relay and relay–destination links have the same average SNR, i.e.,  $\text{SNR}_{SR} = \text{SNR}_{RD} = \text{SNR}$ ,  $P_{s,max} = 1$ , and  $P_{r,max} = 1$ . In case of multiple relays and individual relay transmit power constraints,  $P_{r,max}$  is divided evenly among all relays, i.e., each relay has a maximum transmit power  $P_{r,max}/M$ . For fairness reasons, we assume that the maximum available transmit power for the scheme without relaying is  $P_{max} = 2$ .

#### A. SINR Maximization Under Source and Relay Transmit Power Constraints

In Fig. 2, we investigate the convergence rate of the proposed iterative alternating optimization algorithm for the Max–Min SINR scheme. We show the instantaneous SINR vs. the number of iterations for several random realizations of the channels. We assume  $M = 1$  and  $M = 2$  relays,  $L = N_S = N_R = N_D = 2$ , and  $\text{SNR} = 25$  dB. We observe that the algorithm converges relatively quickly. It is clear that the convergence speed depends on the channel realizations. For the case of one relay, we can see that the performance gain between the first iteration and steady state is in general relatively small. However, for the case of two relays, the performance improvement over the iterations is significant.

In Fig. 3, we show the performance of the proposed Max–Min SINR scheme in terms of BER vs. SNR for three different prefixed receiver structures referred to as Max–Min SINR, Max–Min SINR ZF, and Max–Min SINR MMSE. In Max–Min SINR, we assume that  $\mathbf{W} = \mathbf{I}_{L \times N_D}$ , i.e., the prefixed receiver does not perform any equalization. In Max–Min SINR ZF and Max–Min SINR MMSE, we assume that the prefixed receivers are the linear ZF equalizer and linear MMSE equalizer for the relay–destination channel, respectively. It should be noted that the ZF and MMSE equalizers equalize only the relay–destination channel since the prefixed receiver is oblivious to the existence of the relay precoder and was designed for point-to-point systems. As a baseline scheme, we consider the Min–Max MSE with Equalizer scheme. Recall that in Min–Max MSE with Equalizer, the MMSE equalizer is jointly optimized with the source and relay precoders and it equalizes the end-to-end channel including the precoders at source and relay. In this figure, we adopt  $M = 1$  relay,  $L = 2$  signal streams and each node is equipped with two antennas, i.e.,  $N_S = N_R = N_D = 2$ . We observe that the

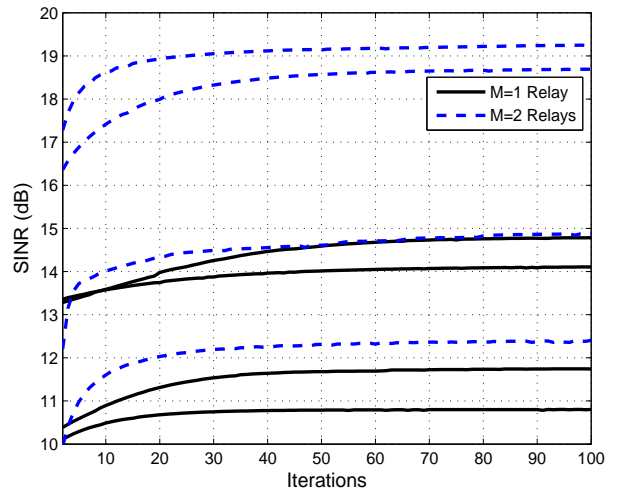


Fig. 2. SINR vs. number of iterations for several random realizations of the channels. We assume  $M = 1$  and  $M = 2$  relays,  $L = N_S = N_R = N_D = 2$ , and  $\text{SNR} = 25$  dB.

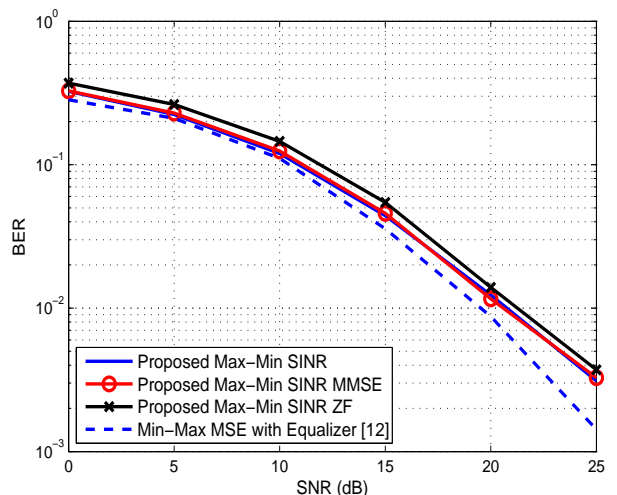


Fig. 3. BER vs. SNR for three different choices for the prefixed receiver. We assume  $M = 1$ ,  $L = N_S = N_R = N_D = 2$ .

three structures provide comparable performance. This is due to the fact that the prefixed ZF and MMSE equalizers do not consider the end-to-end channel and thus, do not offer an advantage compared to the receiver without equalization. Hence, knowing the CSI at the destination does not improve the performance in the case of prefixed receivers. As expected the Min–Max MSE scheme performs better than the schemes with prefixed receivers because the MMSE equalizer of the former is jointly optimized with the source and relay precoders. In the following simulations, we consider only the prefixed receiver without equalizer, i.e.,  $\mathbf{W} = \mathbf{I}_{L \times N_D}$ .

Fig. 4 compares the performance of the proposed Max–Min SINR scheme with the schemes in [12] and [23] in terms of BER vs. SNR for various numbers of antennas at the relay. We adopt  $M = 1$  relay,  $L = 2$  signal streams, and each node is equipped with two antennas, i.e.,  $N_S = N_R = N_D = 2$ .

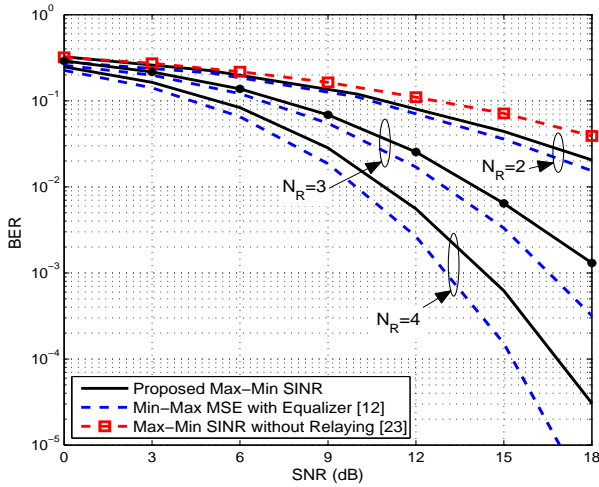


Fig. 4. BER vs. SNR for various numbers of antennas at the relay. We adopt  $M = 1$ ,  $L = N_S = N_D = 2$ .

We observe that increasing the number of antennas at the relay significantly improves the performance. In fact, increasing the number of antennas at the relay, allows a better cancellation of inter-antenna interference and hence better performance. We also notice that the Min-Max MSE with Equalizer scheme performs better than the proposed Min-Max SINR and that the gap between them slightly increases as the number of antennas at the relay increases especially in the high SNR region. This is due to the fact that the equalizer of the Min-Max MSE with Equalizer scheme is jointly optimized with the source and relay precoders contrary to the proposed scheme where a prefixed receiver is assumed. We also observe that, as expected, the scheme without relaying provides the worst performance. This figure shows that the performance loss due to using a prefixed receiver is relatively small compared with the scheme in which the equalizer is jointly optimized with the source and relay precoders.

In Fig. 5, we show BER vs. SNR for various numbers of relays. We consider  $L = N_S = N_R = N_D = 2$ . For the multiple relay case (here,  $M = 2$  and  $M = 3$ ), we compare our proposed Max-Min SINR scheme with the scheme in [32]. We note that the relay power constraint in [32] is at the output of the relay-destination channel, which may lead to a very high power at the output of the relays. We observe that, as expected, increasing the number of relays allows to considerably improve the performance. Furthermore, the Min-Max MSE with Equalizer and Min Sum MSE with Equalizer schemes perform better than the proposed Max-Min SINR scheme. This is due to the fact that the MMSE equalizers of the Min-Max MSE with Equalizer and Min Sum MSE with Equalizer schemes are jointly optimized with the source and relay precoders contrary to the proposed scheme where a prefixed receiver is assumed. We note that, contrary to the Min Sum MSE criterion where the worst signal stream dominates the performance, in our scheme the Max-Min SINR criterion ensures fairness among the signal streams. As expected, the scheme without relaying provides the worst performance.

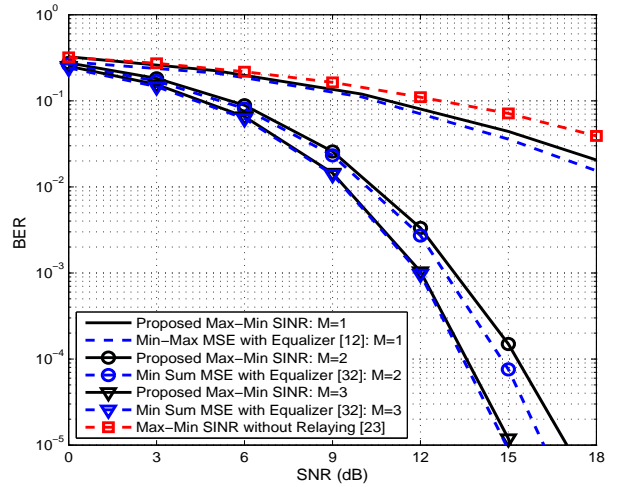


Fig. 5. BER vs. SNR for various numbers of relays. We adopt  $L = N_S = N_R = N_D = 2$ .

Therefore, Figs. 4 and 5 suggest that we can compensate for the performance loss introduced by the prefixed receiver at the destination node by increasing the number of antennas at the relay and/or the number of relays.

In Fig. 6, we evaluate the performance of the proposed algorithm in terms of BER vs. SNR in the case where convergence is not guaranteed. In particular, we consider  $M = 2$  and  $L = N_S = N_R = N_D = 3$ , for which, according to the convergence analysis in Section III, the convergence of the proposed algorithm is not guaranteed. For the sake of comparison, we also consider the case where  $M = 1$  and  $L = 3$ , for which the convergence of the proposed algorithm is guaranteed. Interestingly, we observe that the proposed Max-Min SINR scheme performs very closely to the Min-Max MSE with Equalizer for one relay and the Min Sum MSE with Equalizer scheme for two relays. It should be noted that the convergence conditions we derived are sufficient but not necessary. Moreover, we observe that increasing the number of relays results in a huge performance gain. From this figure, we can clearly see that the proposed algorithm provides satisfactory performance even in cases where its convergence cannot be guaranteed.

In Fig. 7, we investigate the effect of using individual relay transmit power constraints instead of a total relay transmit power constraint on the performance of the proposed algorithm. We show BER vs. SNR for different numbers of relays. We observe that the performance of the proposed algorithm with total relay transmit power constraint is slightly better than that with individual relay transmit power constraints. This is expected since with total relay transmit power constraint the relays can share the total available power to achieve the best performance.

### B. Source-Relay Transmit Power Minimization Under QoS Constraints

Fig. 8 compares the performance of the proposed scheme with the schemes in [16] and [23] in terms of total power

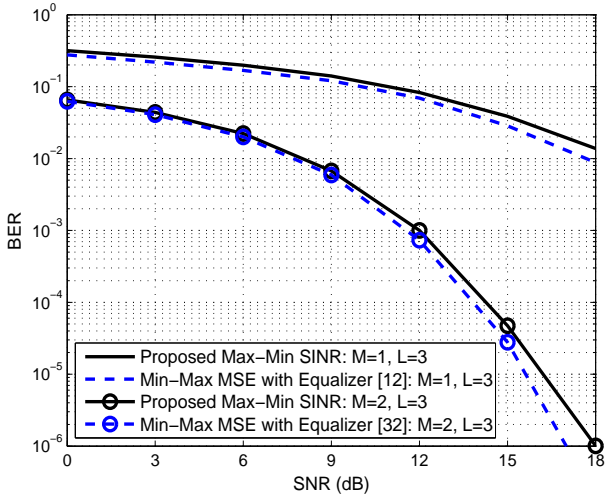


Fig. 6. BER vs. SNR for  $M = 1$  and  $M = 2$  relays and  $L = N_S = N_R = N_D = 3$ .

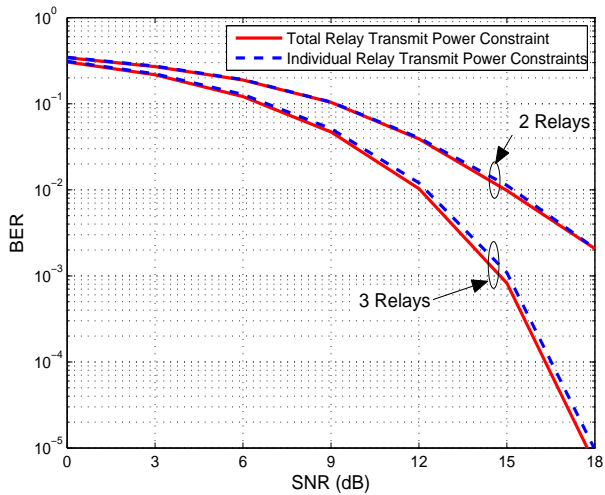


Fig. 7. BER of the proposed algorithm with total and individual relay transmit power constraints vs. SNR for  $M = 2$  and  $M = 3$  relays and  $L = N_S = N_R = N_D = 2$ .

consumption vs. required QoS  $\gamma$ . We assume  $M = 1$  relay,  $L = 2$  streams and each node is equipped with two antennas, i.e.,  $N_S = N_R = N_D = 2$ . We observe that the total power consumption increases linearly with increasing required QoS  $\gamma$ . As expected, the Min Power with Equalizer scheme outperforms the proposed scheme and the scheme without relaying.

Now, for the case of multiple relays, we compare our scheme with the Min Power with Equalizer 1 scheme [17]. In the scheme in [17], the source and relay precoders as well as the receiver are jointly optimized through the minimization of the relay power subject to QoS constraints under the assumption that the source transmit power is *constant*. To have a fair comparison between our scheme and the scheme in [17], we modify our optimization problem such that we only minimize the relay power under the assumption of a

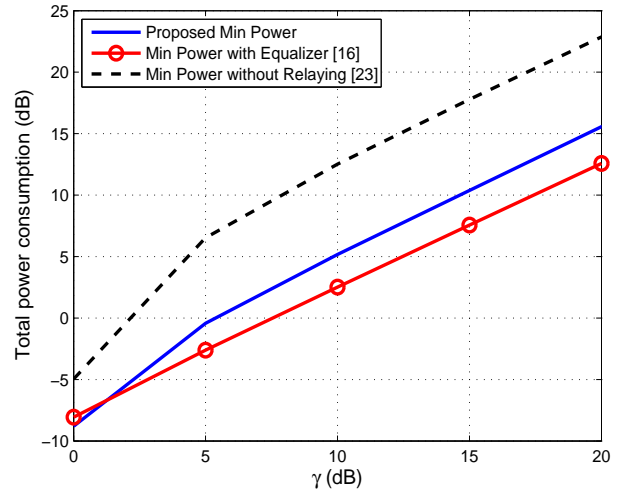


Fig. 8. Total power consumption vs.  $\gamma$ . We assume  $M = 1$ ,  $L = N_S = N_R = N_D = 2$ , and the noise variance at each receive relay and destination antenna is equal to  $-25$  dB.

constant source transmit power. We refer to our modified scheme as “*Proposed Min Power I*”. It is important to note that under the assumption of constant source transmit power both the optimization problem in [17] and our problem may be infeasible. In Fig. 9, we show the feasibility of the optimization problems (in %) for different values of source transmit powers  $P_s$ . We observe that the feasibility percentage of the Min Power with Equalizer 1 scheme is higher than that of our modified scheme. This is due to the fact that our scheme is iterative and as the required SINR  $\gamma$  increases, it becomes more challenging to find a feasible initial solution to start with. By increasing the source transmit power  $P_s$  the feasibility of both problems improves and the gap between them decreases. Note that as shown in Section IV, our considered optimization problem is always feasible if we jointly minimize the source and relay powers.

In Fig. 10, we compare our proposed Min Power 1 scheme and the Min Power with Equalizer 1 scheme [17]. We plot the total power consumption vs. required SINR  $\gamma$  for different values of source transmit power  $P_s$ . We observe that, as expected, the Min Power with Equalizer 1 scheme outperforms our scheme. This is due to the fact that the Min Power with Equalizer 1 scheme optimizes its equalizer along with the source and relay precoders and provides the optimal solution. We also notice that for small to moderate required SINR  $\gamma$ , the smaller the source transmit power the smaller the total power consumption. This is due to the fact that small source and relay powers are sufficient to satisfy small required SINRs and hence the total transmit power largely depends on the adopted constant source transmit power. We also observe that for a required SINR around 20 dB, the total power consumption for  $P_s = 1$  becomes greater than that for  $P_s = 2$ . This is due to the fact that for large required SINR, small source powers limit the performance and a large relay transmit power is required to meet the SINR constraint.

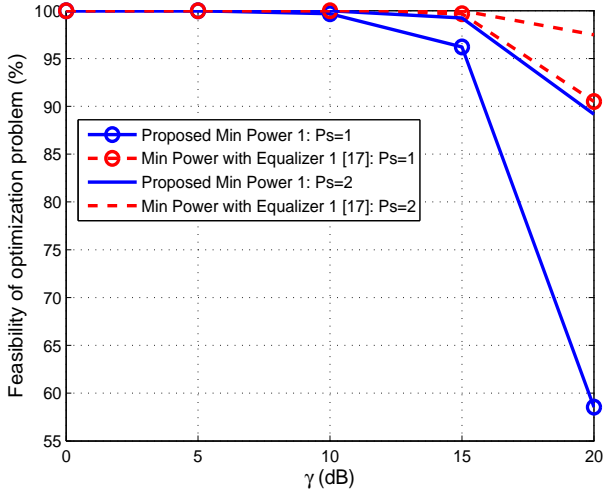


Fig. 9. Feasibility of optimization problem (in %) vs.  $\gamma$  for various different values of source transmit power  $P_s$ . We assume  $M = 2$  relays,  $L = N_S = N_D = 2$  antennas, and the noise variance at each receive relay and destination antenna is equal to  $-25$  dB.

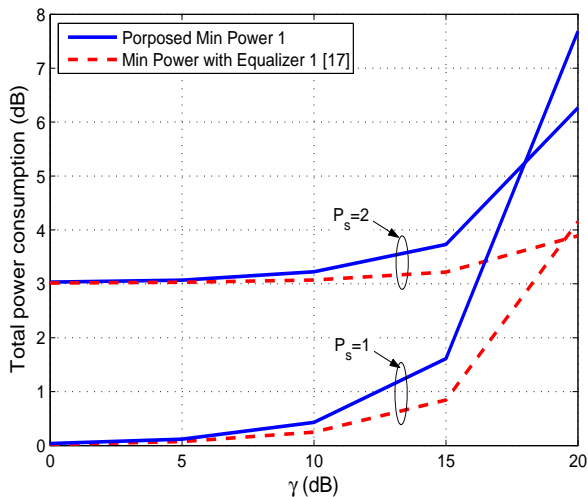


Fig. 10. Total power consumption vs.  $\gamma$ . We assume  $M = 2$  relays,  $L = N_S = N_D = 2$  antennas, and the noise variance at each receive relay and destination antenna is equal to  $-25$  dB.

## VI. CONCLUSION

In this paper, we studied the problem of joint optimization of the source and relay precoders for prefixed receivers in multi-antenna multi-relay networks. We considered two different criteria, namely the maximization of the worst stream SINR subject to source and relay power constraints, and the minimization of the joint source and relay powers while ensuring a certain QoS. Both optimization problems are non-convex and to solve them we proposed iterative alternating algorithms. For both problems, we have shown that the optimization of the source and relay precoders can be formulated as SOCP and SDR problems, respectively. Since the proposed alternating algorithms are iterative, we provided sufficient conditions under which convergence to a fixed point is guaranteed. From our

simulation results, we conclude that prefixed receivers provide low complexity at the cost of a performance loss compared to non-prefixed receivers where the equalizer is jointly designed with the source and relay precoders. The proposed design provides a good complexity and performance tradeoff and is suitable for systems where receiver complexity is an issue. However, if the receiver can afford high complexity, the joint design of the source, relays, and destination may be preferred due its superior performance.

## APPENDIX

In this appendix, we express the terms in the denominator and numerator of the cost function in (22) as functions of optimization variable  $\mathbf{f}_m = \text{vec}(\mathbf{F}_m)$ . We have

$$\begin{aligned} \left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk} &= \left[ \mathbf{Q}_m \mathbf{F}_m \mathbf{P}_m + \sum_{\substack{i=1 \\ i \neq m}}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk} \\ &= [\mathbf{Q}_m \mathbf{F}_m \mathbf{P}_m]_{jk} + \psi_{m,jk}, \end{aligned} \quad (43)$$

where  $\psi_{m,jk} = [\Psi_m]_{jk} = \left[ \sum_{\substack{i=1 \\ i \neq m}}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk}$ .

Moreover, we have

$$[\mathbf{Q}_m \mathbf{F}_m \mathbf{P}_m]_{jk} = \mathbf{f}_m^T (\mathbf{p}_{m,k} \otimes \bar{\mathbf{q}}_{m,j}) = \mathbf{f}_m^T \mathbf{b}_{m,jk}, \quad (44)$$

where  $\bar{\mathbf{q}}_{m,j}$  is the  $j$ th column vector of  $\mathbf{Q}_m^T$ ,  $\mathbf{p}_{m,k}$  is the  $k$ th column vector of  $\mathbf{P}_m$ , and  $\mathbf{b}_{m,jk} = \mathbf{p}_{m,k} \otimes \bar{\mathbf{q}}_{m,j}$ . Plugging (44) into (43) yields

$$\left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk} = \mathbf{f}_m^T \mathbf{b}_{m,jk} + \psi_{m,jk}. \quad (45)$$

Using (45), the numerator of the cost function in (22) is given by

$$\begin{aligned} \left| \left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jj} \right|^2 &= |\mathbf{f}_m^T \mathbf{b}_{m,jj} + \psi_{m,jj}|^2 \\ &= \mathbf{f}_m^H \mathbf{A}_{m,j} \mathbf{f}_m + \mathbf{f}_m^H \mathbf{a}_{m,j} + \mathbf{a}_{m,j}^H \mathbf{f}_m + a_{m,j}, \end{aligned} \quad (46)$$

where  $\mathbf{A}_{m,j} = \mathbf{b}_{m,jj}^* \mathbf{b}_{m,jj}^T$ ,  $\mathbf{a}_{m,j} = \psi_{m,jj} \mathbf{b}_{m,jj}^*$ , and  $a_{m,j} = |\psi_{m,jj}|^2$ . Again, using (45) the first term in the denominator of the cost function in (22) is given by

$$\begin{aligned} \sum_{\substack{k=1 \\ k \neq j}}^L \left| \left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk} \right|^2 &= \sum_{\substack{k=1 \\ k \neq j}}^L |\mathbf{f}_m^T \mathbf{b}_{m,jk} + \psi_{m,jk}|^2 \\ &= \mathbf{f}_m^H \left( \sum_{\substack{k=1 \\ k \neq j}}^L \mathbf{b}_{m,jk}^* \mathbf{b}_{m,jk}^T \right) \mathbf{f}_m + \mathbf{f}_m^H \left( \sum_{\substack{k=1 \\ k \neq j}}^L \psi_{m,jk} \mathbf{b}_{m,jk}^* \right) \\ &\quad + \left( \sum_{\substack{k=1 \\ k \neq j}}^L \psi_{m,jk}^* \mathbf{b}_{m,jk}^T \right) \mathbf{f}_m + \sum_{\substack{k=1 \\ k \neq j}}^L |\psi_{m,jk}|^2. \end{aligned} \quad (47)$$

Now, we compute the second term in the denominator of the cost function in (22). We have

$$\sum_{i=1}^M \sum_{k=1}^{N_R} \left| [\mathbf{Q}_i \mathbf{F}_i]_{jk} \right|^2 = \sum_{k=1}^{N_R} \left| [\mathbf{Q}_m \mathbf{F}_m]_{jk} \right|^2 + \phi_{m,j}, \quad (48)$$

$$\text{where } \phi_{m,j} = \sum_{\substack{i=1 \\ i \neq m}}^M \sum_{k=1}^{N_R} \left| [\mathbf{Q}_i \mathbf{F}_i]_{jk} \right|^2.$$

Moreover, we have

$$[\mathbf{Q}_m \mathbf{F}_m]_{jk} = \mathbf{f}_m^T (\mathbf{e}_k \otimes \bar{\mathbf{q}}_{m,j}) = \mathbf{f}_m^T \mathbf{x}_{m,jk}, \quad (49)$$

where  $\mathbf{x}_{m,jk} = \mathbf{e}_k \otimes \bar{\mathbf{q}}_{m,j}$ .

Therefore, substituting (49) into (48) yields

$$\begin{aligned} \sum_{i=1}^M \sum_{k=1}^{N_R} \left| [\mathbf{Q}_i \mathbf{F}_i]_{jk} \right|^2 &= \sum_{k=1}^{N_R} \left| \mathbf{f}_m^T \mathbf{x}_{m,jk} \right|^2 + \phi_{m,j} \\ &= \mathbf{f}_m^H \left( \sum_{k=1}^{N_R} \mathbf{x}_{m,jk}^* \mathbf{x}_{m,jk}^T \right) \mathbf{f}_m + \phi_{m,j}. \end{aligned} \quad (50)$$

Using (47) and (50), the denominator of the cost function in (22) can be written as

$$\begin{aligned} \sum_{k \neq j}^L \left| \left[ \sum_{i=1}^M \mathbf{Q}_i \mathbf{F}_i \mathbf{P}_i \right]_{jk} \right|^2 &+ \sigma_{n_r}^2 \sum_{i=1}^M \sum_{k=1}^{N_R} \left| [\mathbf{Q}_i \mathbf{F}_i]_{jk} \right|^2 + \bar{\sigma}_{n,j}^2 \\ &= \mathbf{f}_m^H \mathbf{V}_{m,j} \mathbf{f}_m + \mathbf{f}_m^H \mathbf{v}_{m,j} + \mathbf{v}_{m,j}^H \mathbf{f}_m + v_{m,j}, \end{aligned} \quad (51)$$

where

$$\mathbf{V}_{m,j} = \sum_{\substack{k=1 \\ k \neq j}}^L \mathbf{b}_{m,jk}^* \mathbf{b}_{m,jk}^T + \sigma_{n_r}^2 \sum_{k=1}^{N_R} \mathbf{x}_{m,jk}^* \mathbf{x}_{m,jk}^T, \quad (52)$$

$$\mathbf{v}_{m,j} = \lambda \sum_{\substack{k=1 \\ k \neq j}}^L \psi_{m,jk} \mathbf{b}_{m,jk}^*, \quad (53)$$

$$v_{m,j} = \bar{\sigma}_{n,j}^2 + \sum_{\substack{k=1 \\ k \neq j}}^L |\psi_{m,jk}|^2 + \sigma_{n_r}^2 \phi_{m,j}. \quad (54)$$

Furthermore, using the equality  $\text{tr}(\mathbf{A} \mathbf{B} \mathbf{A}^H) = \text{vec}(\mathbf{A})^H (\mathbf{B}^T \otimes \mathbf{I}) \text{vec}(\mathbf{A})$ , the constraint in (22) can be recast as

$$\begin{aligned} &\text{tr}(\mathbf{F}_m (\mathbf{P}_m \mathbf{P}_m^H + \sigma_{n_r}^2 \mathbf{I}_{N_R}) \mathbf{F}_m^H) \\ &= \text{vec}(\mathbf{F}_m)^H \left( (\mathbf{P}_m \mathbf{P}_m^H + \sigma_{n_r}^2 \mathbf{I}_{N_R})^T \otimes \mathbf{I}_{N_R} \right) \text{vec}(\mathbf{F}_m) \\ &= \mathbf{f}_m^H \mathbf{D}_m \mathbf{f}_m, \end{aligned} \quad (55)$$

where  $\mathbf{f}_m = \text{vec}(\mathbf{F}_m)$  and  $\mathbf{D}_m = (\mathbf{P}_m \mathbf{P}_m^H + \sigma_{n_r}^2 \mathbf{I}_{N_R})^T \otimes \mathbf{I}_{N_R}$ . Using (46), (51), and (55) in (22) results in (23).

## REFERENCES

- [1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [2] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity-Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
- [3] —, "User cooperation diversity-Part II: Implementation aspects and performance analysis," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1939–1948, Nov. 2003.
- [4] Y. Yang, H. Hu, J. Xu, and G. Mao, "Relay technologies for WiMax and LTE-advanced mobile systems," *IEEE Commun. Magazine*, vol. 47, no. 10, pp. 100–105, Oct. 2009.
- [5] C. Hoymann, W. Chen, J. Montojo, A. Golitschek, C. Koutsimanis, and X. Shen, "Relaying operation in 3GPP LTE: challenges and solutions," *IEEE Commun. Magazine*, vol. 50, no. 2, pp. 156–162, Feb. 2012.
- [6] B. Wang, J. Zhang, and A. Høst-Madsen, "On the capacity of MIMO relay channels," *IEEE Trans. Inf. Theory*, vol. 51, no. 1, pp. 29–43, Jan. 2005.
- [7] O. Muñoz-Medina, J. Vidal, and A. Agustín, "Linear transceiver design in nonregenerative relays with channel state information," *IEEE Trans. on Signal Process.*, vol. 55, no. 6, pp. 2593–2604, Jun. 2007.
- [8] X. Tang and Y. Hua, "Optimal design of non-regenerative MIMO wireless relays," *IEEE Trans. Wireless Commun.*, vol. 6, no. 4, pp. 1398–1407, Apr. 2007.
- [9] I. Hammerström and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2798–2802, Aug. 2007.
- [10] W. Guan and H. Luo, "Joint MMSE transceiver design in nonregenerative MIMO relay systems," *IEEE Commun. Lett.*, vol. 12, no. 7, pp. 517–519, Jul. 2008.
- [11] A. S. Behbahani, R. Merched, and A. M. Eltawil, "Optimizations of a MIMO relay network," *IEEE Trans. on Signal Process.*, vol. 56, no. 10, pp. 5062–5073, Oct. 2008.
- [12] Y. Rong, X. Tang, and Y. Hua, "A unified framework for optimizing linear non-regenerative multicarrier MIMO relay communication systems," *IEEE Trans. on Signal Process.*, vol. 57, no. 12, pp. 4837–4851, Dec. 2009.
- [13] Y. Rong and Y. Hua, "Optimality of diagonalization of multi-hop MIMO relays," *IEEE Trans. Wireless Commun.*, vol. 8, no. 12, pp. 6068–6077, Dec. 2009.
- [14] V. Havary-Nassab, S. Shahbazpanahi, and A. Grami, "Joint receive-transmit beamforming for multi-antenna relaying schemes," *IEEE Trans. on Signal Process.*, vol. 58, no. 9, pp. 4966–4972, Sep. 2010.
- [15] Y. Rong, "Optimal linear non-regenerative multi-hop MIMO relays with MMSE-DFE receiver at the destination," *IEEE Trans. Wireless Commun.*, vol. 9, no. 7, pp. 2268–2279, Jul. 2010.
- [16] J. Mohammadi, F. Gao, and Y. Rong, "Design of amplify and forward MIMO relay networks with QoS constraint," in *Proc. of IEEE Global Communications Conference (Globecom)*, Miami, FL, USA, Dec. 2010.
- [17] Y. Fu, L. Yang, W.-P. Zhu, and C. Liu, "Optimum linear design of two-hop MIMO relay networks with QoS requirements," *IEEE Trans. on Signal Process.*, vol. 59, no. 5, pp. 2257–2269, May 2011.
- [18] C.-B. Chae, T. Tang, R. W. H. Jr., and S. Cho, "MIMO relaying with linear processing for multiuser transmission in fixed relay networks," *IEEE Trans. on Signal Process.*, vol. 56, no. 2, pp. 727–738, Feb. 2008.
- [19] Y. Yu and Y. Hua, "Power allocation for a MIMO relay system with multiple-antenna users," *IEEE Trans. on Signal Process.*, vol. 58, no. 5, pp. 2823–2835, May. 2010.
- [20] L. Sanguinetti, A. A. D'Amico, and Y. Rong, "A tutorial on the optimization of amplify-and-forward MIMO relay systems," *IEEE J. Select. Areas Commun.*, vol. 30, no. 8, pp. 1331–1346, Sep. 2012.
- [21] M. Joham, W. Utschick, and J. A. Nossek, "Linear transmit processing in MIMO communications systems," *IEEE Trans. on Signal Process.*, vol. 53, no. 8, pp. 2700–2712, Aug. 2005.
- [22] J. Wang and D. P. Palomar, "Robust MMSE precoding in MIMO channels with pre-fixed receivers," *IEEE Trans. on Signal Process.*, vol. 58, no. 11, pp. 5802–5818, Nov. 2010.
- [23] A. Wiesel, Y. C. Eldar, and S. S. (Shitz), "Linear precoding via conic programming for fixed MIMO receivers," *IEEE Trans. on Signal Process.*, vol. 54, no. 1, pp. 161–176, Jan. 2006.
- [24] J. Bezdek and R. Hathaway, "Some notes on alternating optimization," *Advances in Soft Computing-AFSS*, pp. 187–195, 2002.
- [25] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.
- [26] D. P. Palomar and Y. C. Eldar, *Convex Optimization in Signal Processing and Communications*. Cambridge University Press, 2009.
- [27] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming," in <http://cvxr.com/cvx>, Apr. 2011.
- [28] W. Ai, Y. Huang, and S. Zhang, "New results on Hermitian matrix rank-one decomposition," *Math. Program: Ser. A*, vol. 128, no. 1-2, pp. 253–283, Jun. 2011.
- [29] Y. Huang and S. Zhang, "Complex matrix decomposition and quadratic programming," *Math. oper. res.*, vol. 32, no. 3, pp. 758–768, Aug. 2007.
- [30] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," *IEEE Trans. on Signal Process.*, vol. 58, no. 2, pp. 664–678, Feb. 2010.

- [31] S. Ponnusamy, *Foundations of Mathematical Analysis*. Birkhäuser, 2012.
- [32] A. Toding, M. R. A. Khandaker, and Y. Rong, "Optimal joint source and relay beamforming for parallel MIMO relay networks," in *Proc. of 6th Int. Conf. Wireless Commun., Networking and Mobile Computing (WiCOM'2010)*, Chengdu, China, Sep. 2010.