

1 **Cost-Effective Non-Metric Photogrammetry from Consumer-Grade sUAS:**
2 **Implications for Direct Georeferencing of Structure from Motion**
3 **Photogrammetry**

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10

11 **Abstract**

12 The declining costs of small Unmanned Aerial systems (sUAS), in combination with
13 Structure from Motion (SfM) photogrammetry have triggered renewed interest in
14 image-based topography reconstruction. However, the potential uptake of sUAS-
15 based topography is limited by the need for ground control acquired with expensive
16 survey equipment. Direct georeferencing (DG) is a workflow that obviates ground
17 control and uses only the camera positions to georeference the SfM results.
18 However, the absence of ground control poses significant challenges in terms of the
19 data quality of the final geospatial outputs. Here, we present an examination of DG
20 carried out with consumer-grade sUAS. We begin with a study of surface
21 deformations resulting from systematic perturbations of the radial lens distortion
22 parameters. We then test a number of flight patterns and develop a novel error
23 quantification method to assess the outcomes. Our perturbation analysis shows

1 families of predictable equifinal solutions of K_1 - K_2 which minimise doming in the
2 output model. The equifinal solutions can be expressed as $K_2 = f(K_1)$ and they have
3 been observed for both the DJI Inspire 1 and Phantom 3 sUAS platforms. This
4 equifinality relationship can be used as an external reliability check of the self-
5 calibration and allow a DG workflow to produce topography exempt of non-affine
6 deformations and with random errors of 0.1% of the flying height (e.g. $\pm 5\text{cm}$ @ 50m),
7 off-vertical tilts below 1° and easily-corrected linear offsets from 0.3m to 8m. Whilst
8 not yet of survey-grade quality, these results demonstrate that low-cost sUAS are
9 capable of producing reliable topography products without recourse to expensive
10 survey equipment and we argue that direct georeferencing and low-cost sUAS could
11 transform survey practices in both academic and commercial disciplines.

12

13 Keywords: sUAS, UAV, Structure from Motion, SfM, photogrammetry, Direct
14 Georeferencing, camera calibration, Point Clouds.

1 **Introduction**

2 The staggering uptake of small Unmanned Aerial Systems (sUAS), commonly
3 referred to as drones, has led some to coin the term 'the Drone Age' in reference to
4 our era (Economist 2015). In parallel, the integration of Structure from Motion (SfM)
5 and multi-view stereo (MVS) algorithms into the standard workflow of digital
6 photogrammetry has led to a series of software products that can reconstitute
7 topography from imagery with an unprecedented level of automation and ease
8 (Westoby *et al.* 2012; Fonstad *et al.* 2013). Several authors have already
9 demonstrated that imagery acquired from sUAS platforms can deliver high quality
10 topographic survey data (Niethammer *et al.* 2012; Hugenholtz *et al.* 2013; Bemis *et*
11 *al.* 2014; Immerzeel *et al.* 2014; Clapuyt *et al.* 2015; Eltner *et al.* 2015; Ryan *et al.*
12 2015; Turner *et al.* 2015; Woodget *et al.* 2015). The appearance of consumer-grade
13 sUAS with imaging sensors that approach the quality of professional and scientific
14 digital cameras is therefore precipitating a fundamental shift in topographic mapping
15 whereby individuals or small organisations in need of such data no longer depend on
16 national mapping agencies or geospatial/survey companies. Whilst some progress
17 has been made towards lowering the cost of terrain mapping via digital
18 photogrammetry *e.g.* (Carbonneau *et al.* 2003), the uptake has largely remained
19 contained to academic circles with the commercial topographic sector currently
20 dominated by laser-ranging technology. However, with low-cost drones and
21 affordable SfM-photogrammetry software, photogrammetry is gaining and may return
22 as the dominant method of topography production and become as common a tool as
23 standard photography.

24 Currently, the combination of a consumer-grade sUAS equipped with a high-
25 definition camera and free SfM-photogrammetry software, such as 123D Catch by

1 Autodesk or Microsoft Photosynth, can produce virtual 3D objects that are suitable
2 for visualization or 3D printing. However, the production of topographic data and the
3 associated orthoimagery for the purpose of formal mapping or any other scientific
4 application is significantly more challenging and has much more stringent data
5 quality requirements. The 3D data initially produced by SfM are not scaled and
6 oriented to any real dimensions or directions (Westoby *et al.* 2012; Fonstad *et al.*
7 2013; Javernick *et al.* 2014). Therefore, SfM photogrammetry requires generally
8 expensive survey equipment (e.g. RTK-GPS) to provide accurate (*i.e.* small mean
9 error) and precise (*i.e.* small standard deviation of error) real-world map-coordinates
10 of objects in the scene (e.g. ground control points or targets). These points are
11 used to scale, rotate, and translate the initial model to map coordinates. However,
12 errors present in the survey data propagate through the transformation and result in
13 errors of scaling, rotation, and translation as illustrated in figure 1. In addition to
14 these linear errors, detailed investigations have revealed that systematic doming
15 deformations can often be present in the final topographic outputs for both standard
16 photogrammetry (Wackrow and Chandler 2008, 2011) and SfM-Photogrammetry
17 (Carbonneau and James 2012; James and Robson 2014; Woodget *et al.* 2015;
18 Dietrich 2016). These deformations are caused by optical lens distortion (Figure 2).
19 Figure 2a, illustrates how lens distortion can warp images. Under ideal
20 circumstances, a lens would produce an image with a regular orthogonal grid pattern
21 preserves straight lines and right angles. In practice, lens design and aberration
22 effects warp the conformal image projection and result in non-right angles and
23 curved lines with either barrel or pin-cushion distortion patterns. Furthermore, many
24 compact lens systems, increasingly prevalent in small-format and mobile cameras,
25 display complex patterns where the distortion is not a monotonic function of radial

1 distance from the centre of the image. When the lens distortions from multiple
2 images are combined and allowed to propagate through the SfM-MVS process, the
3 effect on the final topographic outputs is generally seen as either a doming or dishing
4 effect (Figure 2b). Correcting the lens-distortion effects has been the focus of a very
5 large body of work in both photogrammetry and computer vision (Brown 1966, 1971;
6 Fraser 1997; Clarke and Fryer 1998; Luhmann *et al.* 2014). Use of the Brown-
7 Conrady lens distortion model (table 1) is now standard practice in all forms of
8 photogrammetry. Furthermore, the parametrisation of this model is now performed
9 by so-called, automatic-, or on-the-job self-calibration algorithms which automatically
10 parametrise the Brown-Conrady model, (*i.e* calibrating the camera/lens combination)
11 from the image dataset intended for 3D reconstruction (Fraser 1997; Luhmann *et al.*
12 2014).

13 High-quality SfM-photogrammetry, therefore, requires both a correct scaling,
14 rotation, and translation of a raw 3D point cloud and an accurate parametrisation of
15 the Brown-Conrady lens distortion model. The dominant approach for this is the
16 acquisition of centimetric quality ground control data (ground control points, GCPs)
17 with professional survey equipment (e.g. RTK-GPS or total station). The
18 photogrammetric software (standard or SfM) can then use ground control data for
19 both georegistration and camera calibration. However, the requirement for GCPs to
20 be acquired with survey-grade equipment remains the largest barrier, both in terms
21 of time and cost, to the wider uptake of sUAS-based photogrammetry. The
22 alternative is to develop so-called 'Direct Georeferencing (DG)', whereby the
23 photogrammetric solution is determined with precise and accurate knowledge of the
24 camera positions (X, Y, and Z) and orientation (pitch, roll, and yaw) at the time of
25 image acquisition (Turner *et al.* 2014). This methodology is routinely used in the

1 case of full-sized airborne photography and LiDAR surveys (without the camera
2 calibration requirement) and some authors have started reporting on its application
3 to sUAS (Turner *et al.* 2014).

4 The advantages of DG are clear: by removing the need for ground control,
5 professional survey equipment is no longer required, which could dramatically
6 reduce the cost of topographic surveys (e.g. RTK-GPS equipment costs in excess of
7 \$10 000). Furthermore, with DG, ground access to the survey area would not be
8 required, which would facilitate high-quality topographic monitoring in hazardous or
9 inaccessible areas. We therefore argue that the development and wider uptake of
10 cost-effective photogrammetry from sUAS platforms that are capable of producing
11 outputs suitable for a range of mapping and technical applications is reliant on the
12 further development of direct georeferencing. To a certain extent, this has already
13 begun. In recent years, there has been an explosion of sUAS options made
14 available to consumers and researchers, both fixed-wing and rotorcraft. They range
15 from higher priced ready-to-fly (RTF) models to lower priced do-it-yourself or
16 “scratch-built” options. There are now a wide range of options for consumer-grade
17 camera drones priced from £500 to £2500. As discussed further in this paper, these
18 drones have many desirable features for scientific mapping and they have enabled
19 an emergence in consumer-grade photogrammetry, colloquially called ‘3D mapping’.
20 However, much of this activity is not informed by photogrammetry and the specific
21 challenges of data quality and camera calibration quality in the absences of GCPs
22 have not been addressed in the consumer market and only researched by a few
23 authors in academic circles (Turner *et al.* 2014; Eling *et al.* 2015; Milik and Gabrlík
24 2015). Direct georeferencing of SfM has been demonstrated from manned aircraft
25 (Nolan *et al.* 2015). However, the reliability of the DG approach from consumer-

1 grade sUAS and its suitability as a fully-fledged topographic survey method is yet to
2 be demonstrated.

3 In this paper, we deploy two popular consumer-grade sUAS models and we examine
4 their ability to produce topography within a direct georeferencing workflow. Our
5 approach is informed by core photogrammetry concepts and particular attention was
6 paid to camera calibration issues: Perturbation analysis of lens distortion and
7 corresponding doming deformations were conducted within a space of the K_1 and K_2
8 lens distortion parameters (Table 1). Furthermore, we develop a new approach to
9 error assessment which is tailored to the SfM-photogrammetry workflow and
10 explicitly determines errors in translation, rotation, scale, and surface noise and
11 demonstrates that whilst some form of limited ground truthing is still required, survey-
12 grade ground control is not required to achieve a satisfactory data quality.

13

14 **Methods**

15 *sUAS Platforms*

16 We deployed a Phantom 3 Professional (P3P) and an Inspire 1 (I1) both
17 manufactured by DJI Inc. These popular quadcopters have many features of interest
18 for scientific applications. The P3P and the I1 both have an integrated camera
19 (Model FC300 on the P3P and Model FC350 on the I1) mounted on a three-axis
20 gimbal that stabilizes the camera, absorbs vibrations, and compensates for the
21 rotational motion of the quadcopter. The integrated cameras both use a wide-angle
22 rectilinear lens and thus avoid the heavy distortions common with the fish-eye lenses
23 employed in several drone and camera models (e.g. GoPro cameras or the DJI

1 Phantom 2-series). Both the P3P and I1 cameras can acquire 12-megapixel still
2 imagery in raw (DNG) or JPEG formats. For navigation and flight stabilisation, the
3 internal consumer-grade GPS system uses both the GNSS and GLONASS systems,
4 increasing the number of satellites used in the GPS position determination. DJI's
5 specifications report a GPS accuracy of ≈ 2.5 meters in X-Y. These positions are
6 automatically exported to the EXIF metadata for each image in WGS84 latitude and
7 longitude thus providing a location stamp (geotag) for each image. Finally, the low-
8 cost of the P3P (£1200) and the I1 (£2400) make them accessible and limits the
9 impact of a total loss in the event of a crash, allowing for flights in risky environments
10 such as volcanos or large flood-stricken areas.

11 *Camera calibration experiment*

12 The optical components of any real lens system do not transmit light rays from the
13 object scene to the imaging sensor in a perfectly linear manner (Wolf *et al.* 2014) .
14 In order for SfM, or any photogrammetric method, to recover accurate 3D data
15 defining and correcting lens distortions are a critical (Zhengyou 1996; Heikkila and
16 Silven 1997). Lens distortions are typically generalized with two components:
17 symmetric radial distortions and decentring (tangential) distortions (Förstner *et al.*
18 2013). The Brown-Conrady lens model was developed in order to correct these lens
19 distortions. In table 1, we can see the form of the model which is based on Taylor
20 expansions of radial, K_n and tangential P_n , distortion terms. The radial distortion
21 parameters (K_1 , K_2 , and K_3) are associated with barrel and pincushion distortion
22 patterns (Wolf *et al.* 2014) (Figure 2) and are combined to create a 2D polynomial
23 representation of the lens distortions. The P_n terms quantify the decentring
24 distortions, offsets of the radial distortion from the center of the image. Fraser (2013)
25 states that there is a temptation to use the full 10-parameter form of the model to

1 achieve the highest possible accuracy. However, the two most critical terms in the
2 model are the focal length (F) and K_1 , followed by K_2 and the principal point
3 coordinates (C_x, C_y) (Förstner *et al.* 2013; Wolf *et al.* 2014). For “medium accuracy”
4 applications, K_1 is sufficient, but for higher accuracy applications, especially those
5 utilizing wide-angle lenses (e.g. those used in modern, compact camera systems), K_2
6 and possibly K_3 are required (Förstner *et al.* 2013). For K_3, P_1 , and P_2 to be solved
7 accurately the image network needs to be highly redundant with a very strong,
8 convergent geometry (Wolf *et al.* 2014), which is often only possible in highly
9 controlled environments. It is however recognized that the K_n parameters can be
10 highly correlated, due to the nature of the polynomial base (Zienmann 1986; Fraser
11 1997). Fraser (1997) suggests that statistical tests can be conducted on the
12 parameters to determine if additional K_n parameters result in a statistically significant
13 difference in the radial distortion profile. However, the K_n parameters are not strongly
14 coupled to the other parameters in the model or with the external orientation
15 parameters and therefore an over-parameterization (additional K_n terms) will still
16 yield a valid distortion profile (Fraser 1997). Additionally, P_1 and P_2 are highly
17 correlated with the principle point coordinates and the errors are typically small
18 compared to the K-terms. If P_1 and P_2 are suppressed (i.e. set to zero) the errors can
19 be absorbed by C_x, C_y (Fraser 2013, Förstner *et al.* 2013). Förstner *et al.* (2013) also
20 states that the perturbations associated with P-terms are “universally ignored in
21 analytical photogrammetry”.

22 When errors associated with radial image distortion are allowed to propagate
23 through the photogrammetric process, it has been established that the final
24 topographic model can contain non-linear deformations that take form as doming or
25 dishing of the land surface (Figure 2b) (Wackrow and Chandler 2008; James and

1 Robson 2014). We therefore begin with an experiment aimed at determining the
2 optimal radial distortion parameters for the P3P and I1 lenses. We used Photoscan
3 Professional (ver. 1.1) for all the processing described in this work. However, this
4 software expresses the above distortion parameters in focal length units according
5 to:

$$6 \quad K_1(\text{focal units}) = K_1(\text{pixel units}) * f^2 \quad (1)$$

$$7 \quad K_2(\text{focal units}) = K_2(\text{pixel units}) * f^4 \quad (2)$$

$$8 \quad K_3(\text{focal units}) = K_3(\text{pixel units}) * f^6 \quad (3)$$

9 Where f is the focal length in pixel units. The expression of K_n in focal length units
10 yield easily manageable numbers which can be expressed without scientific notation,
11 but it hinders cross-comparability of calibration results with different focal lengths.
12 Therefore, from this point onwards, we will systematically report K_n values in pixel
13 units as converted from equations 1-3.

14 To identify the correct distortion parameters for our lenses, we used a systematic
15 perturbation approach with the objective of iteratively finding the optimal K_1 and K_2
16 values that minimized the distortions in a flat wall test site. The sUAS were operated
17 hand-held without propellers. By walking in front of a flat wall on the campus of
18 Durham University (for the P3P) and Dartmouth College (for the I1) at a distance of
19 roughly 10 meters, a series of 24 images were acquired as a series of 12 convergent
20 viewing pairs. For each pair, the optical axis of the camera intersects the wall at
21 $\approx \pm 45^\circ$. Our only assumption is that the wall is flat and, in keeping with the objective
22 of direct georeferencing, no other ground validation is used. Based on our other
23 research and recreational photogrammetry experiments and data collections, the

1 range of K_1 and K_2 values was set as $K_1 \in [-3.7, -1.9] \times 10^{-8}$ pixels and $K_2 \in [1.7, 7]$
2 $\times 10^{-15}$ pixels converted as per equations 1-3 with a focal length of 2320.06 pixels for
3 the P3P and 2326.07 for the I1.

4 Within Photoscan, the 24 flat wall images were processed in order to establish initial
5 camera calibration values. Using the Python scripting API (application program
6 interface) for Photoscan we were able to produce and process a total of 6232
7 photogrammetric blocks with the same imagery but with fixed calibration parameters
8 spanning the full K_1 - K_2 space while keeping the calibration values for the focal
9 length, principal point offset and K_3 as per the initial self-calibration. Tangential
10 distortion and K_4 were not used in this experiment. The resulting point clouds were
11 exported automatically as part of the Python processing script. In Matlab, each
12 model was centred and normalised to the same dimensions and a 3D second-order
13 polynomial fit in (x,y) was calculated for each point-cloud data. Given that the test
14 wall is flat, any significant doming distortions in the point clouds are detected in the
15 second order terms of the polynomial regression. In this case, the wall was longer in
16 the x direction and therefore we expect significant doming to be detected in the x^2
17 term of the polynomial. Optimal calibration parameters will be those where these
18 second order terms are minimised (≈ 0). The compiled results of the experiments
19 were plotted as a matrix to display the data as a function of K_1 and K_2 .

20 *Lens distortion modelling*

21 Our personal experience with camera calibration has shown that for a given camera,
22 the outputs of self-calibration are never identical even in cases where imagery is
23 acquired on the same day, under similar conditions and where external validation
24 indicates that the final topographic model is of high quality. In order to further

1 examine similarity in the outputs of the camera calibration as observed in the
2 perturbation experiments, we conducted additional simulations of lens perturbations.
3 We used a Monte Carlo framework (i.e. randomly generated perturbations)
4 implemented in MATLAB. This random Monte Carlo approach is less
5 computationally intensive than a full examination of the parameter space and thus
6 facilitates the exploration of this parameter space. To start, we take the optimal K_1 ,
7 K_2 and K_3 calibration parameters as determined by self-calibration in the flat wall
8 experiment described above and we calculate the associated displacement with the
9 Brown-Conrady model (table 1), in pixels, over a profile of 2000 pixels (the positive
10 horizontal axis of our photographs). Then a Monte Carlo approach is used to
11 randomly generate 1 million combinations of K_1 and K_2 , once again in the interval K_1
12 $\in [-3.7, -1.9] \times 10^{-8}$ pixels and $K_2 \in [1.7, 7] \times 10^{-15}$ pixels (as above). For each of the
13 parameter combinations the pixel displacement profile is re-calculated. We then
14 take the maximum difference, irrespective of location in the distortion profile,
15 between the simulated K_1 - K_2 combination and the optimal K_1 - K_2 - K_3 calibration.
16 Finally, the maximal difference results from the 1 million Monte Carlo samples are
17 interpolated to a regular grid for analysis and display.

18 *Airborne Surveys and validation*

19 Additional experiments were conducted with the P3P at two sites in county Durham,
20 UK. First, a fallow field in the village of Lanchester was used as the main site (Site
21 A). This triangular field is ringed by trees and it is characterised by a curved
22 topography which is sloping in an eastwards direction. The field is fallow with wild
23 grasses and flowers grown to a height of 10-40cm. Second, a sports field on the
24 grounds of Durham University was used (Site B). This area was flat and had
25 trimmed lawn. The northern limit of the area is set by a flood levee approximately 1

1 meter high. Figure 3 shows an aerial view of each site. The topography of both
2 sites was established with an RTK-GPS survey using a Leica 1200 model RTK rover
3 and base pair. For each site, RTK-GPS points were acquired with the objective of
4 establishing the overall shape of the area. Instead of working on a gridded basis,
5 care was taken to capture the outer bounds of the study sites and break-lines in the
6 local topography. The GPS base-stations were set to relative positions but post-
7 processed with added data from a permanent base station located at Newcastle
8 Airport. The results of the survey were two sparse GPS point datasets that captured
9 the shape of each study site.

10 *Flight patterns and image acquisition*

11 Based on the work of James and Robson (2014) and Fonstad *et al.* (2013) there is a
12 consensus view that image acquisition geometry has an impact on the quality of the
13 outputs with convergent views and multiple flight altitudes to be preferred. We
14 therefore designed an intuitive set of flight patterns based on reasonable
15 combinations of altitude, camera orientation, and flight direction that are expected to
16 illustrate both good and bad performance of the SfM-photogrammetry process and
17 test the potential of DG. The flights were grouped in two experiments both aiming to
18 test the effect of flight pattern and image acquisition on final model geometry
19 (detailed in Table 2). We experimented with a combination of NADIR and
20 convergent imagery (James and Robson 2014) . Four flight patterns were tested: 1)
21 nadir imagery from an altitude of 60 meters; 2) nadir imagery from an altitude of 60
22 and 80 meters; 3) nadir imagery combined with eight convergent-view images at an
23 altitude of 60 meters; and 4) nadir imagery at 60 and 80 meters combined with eight
24 convergent view images acquired at 60 meters. It was found that the P3P had
25 sufficient battery life to acquire all the imagery for these four flight patterns in one

1 flight. Flights began with the camera at nadir at two altitudes and ended with the
2 series of oblique view images. In the analysis stage, the images were grouped
3 according to the four flight patterns for both sites thus yielding eight 'flights' labelled
4 A1 – A4 and B1 – B4. Finally, at the time of our experiments, the DJI firmware for
5 both the P3P and I1 recorded the relative elevation from the launch point to the EXIF
6 data of each photo. Before every flight, we needed to establish a zero elevation
7 datum for the flight by taking a single image on the ground from the launch position
8 in order to get the GPS position corresponding to the launch point.

9

10 *Pre-Processing*

11 For all 8 flights, the raw RGB images (DNG format) were converted to 48-bit TIFF
12 imagery (16-bit per band) with Adobe Photoshop elements. No image equalisation
13 or other such adjustments were applied. We imported the initial TIFF images into
14 MATLAB, removed the geotag (latitude /longitude) from the EXIF data, converted
15 and output the coordinates into UTM (Zone 30 North, WGS84) to a text file, which
16 could be imported into Photoscan Pro. In addition, the X-Y coordinates of the launch
17 point were used to get the absolute elevation from a hydrologically corrected SRTM
18 dataset available from the United States Geological Survey (USGS). This value was
19 added to the relative elevation data recorded in the image metadata thus resulting in
20 each image defined by a 3D position in UTM coordinates with elevations above the
21 WGS 1984 ellipsoid.

22 TIFF images are then imported into Photoscan Pro in separate 'chunks' which act as
23 distinct processing blocks. The converted camera station coordinates were imported
24 providing an immediate geographic reference for each block. Based on the DJI

1 manual, which states that the on-board GPS is accurate to 2.5 meters in X-Y, the
2 camera position accuracy setting, in Photoscan, was set to a conservative value of 5
3 meters. Each image block was reconstructed using the high quality image-alignment
4 setting followed by a dense reconstruction at a low-density setting.

5

6 *Error modelling and assessment*

7 Error and quality assessment for 3D point clouds and topography data typically relies
8 on root mean square error (RMSE) values when compared to some ground truth-
9 value (ASPRS 2015; Whitehead and Hugenholtz 2015). We argue that these
10 generic metrics do not provide a full description of the possible errors associated
11 with SfM-photogrammetry. A single RMS statistic cannot explicitly identify
12 systematic patterns such as tilt or non-affine warp. Mapping the spatial distribution
13 of error at checkpoints can be useful in identifying these patterns, but often low
14 checkpoint density does not capture the full extent of any systematic distortions. In
15 the absence of a dense network of survey-grade GCPs, the georeferencing process
16 used in SfM-photogrammetry relies on a rigid 7-parameter transformation, errors in
17 the ground control or camera position can propagate as errors of position,
18 orientation, and/or scale in the model (Figure 1). Each of these errors has impacts
19 with respect to specific applications. Position/translation errors can affect change
20 detection studies by creating false horizontal or vertical offsets. Orientation errors,
21 especially off-vertical rotations, will affect gravity-dependent models (i.e. flow
22 models) by producing incorrect slopes and flow directions. Scale errors will also
23 influence change detection and volume calculations, again by creating false
24 horizontal or vertical offsets. A single RMSE statistic, even if accompanied by a full

1 error distribution, is therefore not a powerful diagnostic tool when deciding if a
2 topographic dataset is suitable for any given application. A more process-specific
3 error assessment approach is therefore required. We propose the following error
4 model:

$$5 \quad P_{SfM} = M_7 P_{true} + \eta \quad (4)$$

6 Each of the three terms in equation 4 is a point cloud: P_{true} is a reference point-cloud
7 giving the true (i.e. ground truth) representation of the surface. P_{SfM} is the point cloud
8 calculated by SfM-photogrammetry. The matrix factor M_7 denotes the affine rigid-
9 body 7-parameter transform needed to scale (one parameter), rotate (3 parameters:
10 R_ϕ, R_θ, R_ψ), and translate (3 parameters: T_x, T_y, T_z) P_{true} in order to match P_{SfM} . η
11 is a non-linear, non-rigid, error term (also a point cloud) which can be seen as a
12 quasi-random field produced by noise in the SfM process. We can therefore use the
13 standard deviation of η as a measure of precision (scatter) of P_{SfM} . This will be
14 reported as a single number and noted as η_p .

15 To assess errors in the SfM reconstructions we used dGPS survey data and the
16 open-source point cloud processing software Cloud Compare (Girardeau-Montaut
17 2014). The first step in the process is establishing the parameters for M_7 for all 8
18 experiments. We duplicated each model (A1 – A4 and B1 – B4) in Photoscan and
19 using the point cloud editing functions, edited each model to clear all points except
20 those around the periphery of the sites where dGPS point density was the greatest.
21 For site A, only those points located along the unvegetated and narrow footpath that
22 circumscribes the field were kept. For site B, only the narrow concrete border (3 cm
23 in width) of the paved footpath and the bottom and top of the flood defence ridge
24 were preserved. These ‘hollow’ clouds were imported into Cloud Compare along

1 with the matching dGPS data. Then, automated cloud-to-cloud registration, an
2 iterative closest point algorithm, was employed to calculate a transformation matrix
3 that could transform the dGPS data (P_{true}) to match the hollow P_{SfM} clouds. The
4 resulting transformation matrix is M_7 in equation 1. The accuracy and precision of
5 the automated cloud-to-cloud registration (i.e. co-registration) procedure were
6 evaluated separately for sites A and B and we use the residuals of points after the
7 adjustment as an indication of fit quality. After automated co-registration for site A,
8 we obtain a mean residual of 5.2cm with a standard deviation of 23.4cm. For site B,
9 we obtain a mean residual of 0.1cm and a standard deviation of 3.1 cm. The higher
10 errors for site A are to be expected due to the presence of tall grasses which will add
11 error in the comparison of a bare earth DSM derived from dGPS to a DTM derived
12 from photogrammetry.

13 Next we calculated η with the full point clouds. In practice, we calculate η as the
14 differences between P_{true} and P_{SfM} after the application of M_7 . The transformation
15 matrix derived from the first step was applied to the full dGPS dataset and we
16 constructed a Delaunay mesh of the dGPS points. Then the full SfM point cloud was
17 imported into Cloud Compare and a cloud-to-mesh distance was calculated. The
18 advantage of using the cloud-to-mesh function is that distances between the
19 transformed P_{true} and P_{SfM} are calculated along the Z-axis and the sign of the
20 difference is preserved. Positive differences are above P_{true} and points with negative
21 differences are below P_{true} . The topography at our sites was relatively flat allowing us
22 to use the simpler cloud-to-mesh distances along the Z-axis. In more complicated
23 terrain with vertical surfaces and/or overhangs, other differencing algorithms, such
24 as Multiscale Model to Model Cloud Comparison (M3C2), could be used to generate
25 the difference map (Lague *et al.* 2013) . The differences are stored as an additional

1 field in the point cloud, and we calculate precision by taking the standard deviation of
2 η . Furthermore, we extracted profiles from the difference clouds (see figure 3 for
3 profile locations) and used in conjunction with the spatial distribution of cloud-to-
4 mesh differences we were able to check for the presence of the doming deformation
5 associated with errors in the camera calibration process. Our main objective here
6 was to test if a hypothesised degradation of camera calibration parameters would
7 result in the now famous doming deformation (Wackrow and Chandler 2008, 2011;
8 James and Robson 2014; Woodget *et al.* 2015) . In this case, the reference surface
9 could not be taken as the dGPS surface. The presence of tall grasses on site A
10 hides any dome since the dGPS data represents the ground and the
11 photogrammetric point clouds represent the top of the vegetation, which is a variable
12 10 cm to 40 cm above the bare earth. Therefore, the profiles were based on what
13 was assumed as the best possible reference for the top of the vegetation: flights A4
14 and B4. In all cases, we corrected for translation, rotation, and scaling errors with
15 the application of the appropriate M_7 transformation matrix (as described above) prior
16 to differencing and profile extraction.

17

18 **Results**

19 Figure 4 examines the self-calibration results for the Phantom 3 camera. For the
20 wall experiment, the self-calibration of the image block returned a focal length of
21 2320.06 pixels with K_1 of -2.53×10^{-8} pixels⁻², K_2 of 3.96×10^{-15} pixels⁻⁴ and K_3 of -
22 1.15×10^{-22} pixels⁻⁶. In figure 4, we plot lens distortion profiles over the 2000 pixel
23 positive horizontal axis of the image for each K_n component separately. The K_1
24 component profile has a parabolic, concave-down shape. With an opposite sign, the

1 K_2 component profile has concave-up shape and, especially at the edges, a similar
2 displacement magnitude when compared to the K_1 component. Finally, the K_3
3 component profile can be seen to have a small contribution. Figure 5 presents the
4 results of the Monte Carlo lens distortion simulations for both the Phantom 3 and
5 Inspire 1 lens calibrations. These graphs display a strong degree of equifinality.
6 Along the central diagonal, it is clear that a large range of K_1 - K_2 combinations can
7 produce lens distortion profiles which match the optimal lens calibration to within
8 displacements of 10 pixels. Further, we see zones which match to within 5 and even
9 2 pixels. Crucially, these zones follow the diagonal and indicate that only certain
10 combinations of K_1 and K_2 yield equifinal solutions. Given the opposing concavity of
11 the K_1 and K_2 components seen in figure 4, this is a sensible result. Linear
12 regression indicates that solutions similar to within 2 pixels are predicted by:

13
$$P3: K_2 = -2.7364 * 10^{-7} \times K_1 + -3.4086 * 10^{-15} \quad (5)$$

14
$$I1: K_2 = -2.7335 * 10^{-7} \times K_1 + -3.5858 * 10^{-15} \quad (6)$$

15

16 Where the domain of validity for eq. (5) is $K_1 \in [-2.56, -2.35] \times 10^{-8}$ pixels and $K_2 \in$
17 $[2.97, 3.62] \times 10^{-15}$ pixels and for eq (6) is $K_1 \in [-2.46, -2.25] \times 10^{-8}$ pixels and $K_2 \in$
18 $[2.52, 3.18] \times 10^{-15}$ pixels.

19 Figure 6 shows the output of the perturbation experiments for the P3P and I1 in
20 raster format. Similar to figure 5, we see a strong degree of equifinality with a large
21 range of K_1 - K_2 combinations resulting in lens distortion profiles which match the
22 optimal lens calibration to within displacements of 10 pixels. Also, we see positive
23 curvature (pin-cushion distortion) on the top-left and negative curvature (barrel
24 distortion) on the bottom right. In the central diagonal, we have a ledge where a

1 family of solutions in the K_1 - K_2 parameter space that minimise doming deformations
2 with the following fits of $K_2 = f(K_1)$, where K_n is expressed in pixel units,

3 $P3P: K_2 = -2.392 * 10^8 \times K_1^3 - 18.264 \times K_1^2 - 6.924 * 10^{-7} \times K_1 - 5.887 * 10^{-15}$ (7)

4 $I1: K_2 = -2.781 * 10^8 \times K_1^3 + -19.604 \times K_1^2 - 6.900 * 10^{-7} \times K_1 - 5.362 * 10^{-15}$ (8)

5 Furthermore, since each point in figure 6 is the result of a least-squares surface
6 fitting, we examined the fit quality of all points. For the P3P, the mean R^2 is 0.94, the
7 5th percentile is 0.75 and the median is 0.98. For the I1 the mean R^2 is 0.99, the 5th
8 percentile is 0.99 and the median is also 0.99. Additionally, if we consider only the
9 subset of solutions along the line of equifinality, we find that for the P3P the mean R^2
10 is 0.97, the 5th percentile is 0.94 and the median is 0.98. For the I1, the results are
11 the same as above with a mean R^2 of 0.99, a 5th percentile of 0.99 and the median
12 of 0.99. The differences between the P3P results and the I1 results suggest an
13 improved matching success for the I1 which could either be due to lighting
14 conditions, surface texture or sensor quality. Overall these statistics indicate that the
15 wall remained reasonably flat in the entire K_1/K_2 parameter space, especially for the
16 subset of equifinal solutions defined by equations (7) and (8). However, based on
17 figure 5, we might expect very slight doming in the upper left and bottom right of
18 figures 6A and 6B. Woodget *et al.* (2015) observed a doming amplitude of 1.5cm
19 during calibration flights at similar distances. It can therefore not be ruled out that
20 our usage of the 2nd order polynomial coefficient as a test of flatness does not
21 capture curvature with amplitudes below $\approx 0.1\%$ to 0.15% of flying height. Figure 6
22 also displays points that correspond to calibration results (in pixel units) from the
23 flight experiments with the P3P and, for the I1, a sample of other data acquired at a
24 variety of field sites. We can see that most, but not all, self-calibration outputs plot

1 near the zero distortion line. Table 3 lists the self-calibration parameters returned for
2 the eight P3P flights. We first evaluated the self-calibration by calculating the value
3 of K_2 as per equation (7) and then calculating the error, $K_{2\epsilon}$, between the self-
4 calibrated and predicted values of K_2 . According to this metric, flights A1 and A2
5 were the furthest from the zero distortion line and flights A3, A4 and B2 the closest.
6 Flights B1, B3 and B4 seem to be in an intermediate range. Equations 5 and 7
7 suggests that doming deformations present due to a mis-calibration of K_1/K_2 can be
8 corrected with an adjustment of K_2 or K_1 . Preliminary experiments with corrections
9 derived from equation 7 were only partial successful at removing doming. We
10 systematically observed that using pre-calibrated, fixed, values lead to a degradation
11 of results, even when using optimal values derived from figures 5 or 6. However,
12 constraining the corrections to the zone in figure 5 with solutions equifinal to within
13 displacements of 2 pixels (i.e. using equation 5) was successful. When applying
14 equation (5), K_1 values self-calibrated as being outside the domain of validity were
15 forced to the nearest value within the domain and the K_2 value re-calculated
16 accordingly. Figure 7 presents both self-calibrated and adjusted deformation profiles
17 along the α to β profile in figure 3. Deformation is calculated as a given profile
18 differenced from the optimally calibrated profiles A4 and B4. Figure 7 shows that
19 self-calibration for flights A1 and B1 returned a strong doming deformation while self-
20 calibration of A2 resulted in a dishing deformation. The adjusted calibration values
21 eliminated both doming and dishing deformations. However, this has come at the
22 cost of increased surface error (random noise). This level of random noise, η in
23 equation 4, along with other errors, are presented in table 4. In table 4, we note that
24 T_z (datum shifts) are large for A1 and A2, and a re-examination of table 3 shows that
25 variations in T_z to be a part of a complex and non-intuitive response to focal length

1 calibration issues. According to single image geometry, the scale of the image is
2 directly proportional to the focal length. Therefore, if we consider the calibration
3 results for A1 and A2, we might expect that a focal length difference of 565 out of
4 2539 would result in a $\approx 22\%$ difference in scale. Table 4 shows that this is clearly
5 not the case with a scaling errors of 0.2% in the case of A1. Furthermore, a re-
6 examination of the point clouds via simultaneous display for cases A1-A2 shows that
7 these models, with focal length calibrations ranging from 1974 to 2539 have a clear
8 vertical offset but visible scale differences in the X-Y plane. This is illustrated in
9 figure 8, which shows raw elevation profiles for flights A1 and A2 taken from points
10 along the α to γ cross section in figure 3. The raw elevations in figure 8 were
11 calculated from separate DEM outputs where the exact beginning and end-points of
12 the profiles were manually chosen from accurately visible points in the respective
13 orthoimagery rather than from fixed coordinates. For this purpose, we used
14 orthoimagery with a 2.5 cm resolution and we estimate that the point selection was
15 accurate to at least 3 pixels (7.5 cm). By using recognisable conjugate points as the
16 bounds of the profiles, any scaling errors in the models will be visible as a difference
17 in profile length between points α and γ . However, figure 8 shows no visible
18 difference in profile length but a significant vertical offset of ≈ 12 m. The exact profile
19 length in the X-Y plane was 132.36 m for A1 and 132.79 m for A2. If we consider
20 our estimated measurement error, we obtain a scale discrepancy of 0.26% to 0.38%.
21 This is slightly smaller but consistent with the results in table 4, which shows scaling
22 ranging from 99.7% to 100.9%. However, closer examination does reveal errors in
23 the vertical scaling of the model. Figure 9 plots the full set of deformations (i.e.
24 residuals) vs elevation. This figure shows a strong correlation between errors and
25 elevations with large scatter capable of enveloping the profiles shown in figure 7.

1 Closer examinations of the data confirmed this and showed that the vertical scale
2 was either compressed (A2) or dilated (A1). Therefore, focal length calibration
3 remains crucial. In this regard, a re-examination of table 3 shows that imagery
4 acquired at nadir viewing angles (A1, A2, B1 and B2) leads to poor focal length
5 calibrations whilst focal lengths calibrated from convergent view acquisitions (A3, A4,
6 B3 and B4) are all consistent with the initial flat wall experiment.

7

8 Overall, the results for rotation, scale, and random error are very encouraging. In
9 terms of rotation, R_ϕ is a rotation around the vertical axis (Z) and therefore leads to
10 errors in the azimuths of the model. R_θ is rotation around the X-axis (north and
11 south tilt, after application of R_ϕ). This therefore represents a rotation away from
12 verticality and should be considered as an important source of error for surface
13 process science applications. Finally, R_ψ gives the rotation around the Y-axis (east
14 and west tilt) after the application of R_ϕ and R_θ . Values of R_θ are encouraging, most
15 values are below 1 degree and as low as -0.12 degrees. As a reference, our
16 maximum and minimum R_θ errors of 2.42 and -1.42 degrees would translate as
17 vertical errors of 4.23 m and -2.48 m (respectively) at a distance of 100 m from the
18 rotation centre. Errors in the scaling parameter are also a potentially important since
19 they will affect any measurements of distance and volume. The results are again
20 encouraging with several values being within $\pm 1\%$ of the correct scale (i.e. 99-
21 101%). However, some results are poor with the minimum value of 94.9% for B3
22 and a maximum value 104.8% for B4. Random error (η_p) values are also
23 encouraging. Given the non-uniform vegetation present at site A, we would expect
24 an precision of 0.3-0.4m when comparing the photogrammetric point clouds to the

1 dGPS bare earth surface. However, at site B the short-cropped lawn should have
2 little impact on the data. This is consistent with the data in table 4 where random
3 errors are in the area of 0.4 m for site A and as low as 0.06 m for site B. This
4 promising result of 0.06, at a flying height of 60m, m would indicate an optimal
5 precision of 0.1% (1:1000) of flying height.

6

7

1 Discussion

2 Our results demonstrate that Direct Georeferencing from low-cost aerial platforms is
3 a viable workflow for high-quality topography mapping. Predictable equifinality in the
4 K_1 - K_2 parameter space of calibration solutions provides a measure of external
5 reliability capable of assessing the quality of a camera calibration without the
6 recourse to external ground control. In optimal cases, we find that surface errors of
7 0.1% of flying height can be achieved. James and Robson (2012) also found a
8 similar performance but since these authors used a network of 45 surveyed ground
9 control points, our observation of a similar performance in a DG context is a
10 significant step forward. Our findings further confirm the recommendations of
11 James and Robson (2014) stating that convergent viewing angles optimise the
12 results of camera self-calibration. However, in addition to the knowledge that
13 convergent viewing angles are qualitatively 'good' for camera calibration, our results
14 provide a quantitative approach to calibration assessment. We recommend the
15 following steps (summarised in figure 10): The camera should be calibrated before
16 aerial acquisition, 20-30 images at convergent angles should be acquired of a flat
17 wall on a recently constructed surface with a good level of texture. In our
18 experiments, we acquired heavily overlapping (>80%) imagery with the optical axis
19 of the camera intersecting the wall surface at an angle of $\approx \pm 45^\circ$. Rather than
20 implementing the computationally onerous parameter space exploration we
21 conducted in Photoscan (leading to figure 6), we recommend that users simply self-
22 calibrate the convergent imagery in order to obtain an assumed optimal calibration.
23 As a check of this assumption, the flat wall surface point cloud should be regressed
24 in a 2nd order polynomial model. The second order terms should be negligible.
25 Once an optimal calibration value is determined, users can replace the Photoscan

1 parameter exploration with the low-impact Monte Carlo approach described here
2 (leading to figure 5) to determine equifinal solutions in the K_1 - K_2 (potentially adding
3 K_3) parameter space. Our findings indicate that solutions need to be similar to within
4 a maximum displacement of 2 pixels. However, if we consider figures 5 and 6, it
5 becomes apparent that the acceptable domain of equifinal solutions will scale with
6 flying height. In the case of the flat wall experiment where the effective flying height
7 was 10 meters (roughly 20% of the width of the wall), equifinality was observed over
8 a wide range of solutions with displacements similar to within 10 pixels. In the case
9 of real data, the flying height was 60 meters (50% of the site width) and in this case,
10 we found that displacements similar to 2 pixels were required. Users operating at
11 lower altitudes can therefore expect satisfactory results from calibrations delivering
12 lens distortions that match optimum values to displacements somewhere between 2
13 and 10 pixels. Based on our experience, it is recommended that at least 10% of an
14 image dataset be acquired at convergent angles of 20°-45° off-nadir. One possible
15 approach might be to use the 'orbit' flight function now available on most consumer-
16 grade drones. This function allows the drone to fly a fully automated orbit around
17 specific point at a user-determined height and radius. By setting the camera to 20°-
18 45° off-nadir in the direction of the centre of the orbit, a set of strongly convergent
19 images can easily be acquired to augment nadir imagery. Once real aerial survey
20 data are processed, the resulting self-calibration can then be checked against the
21 range of equifinal K_n parameters. Furthermore, the focal length calibration should
22 match to within 0.1% of the initially calibrated value. Since SfM-photogrammetry
23 packages self-calibrate the camera during the initial camera alignment stage, the
24 calibration can be checked on a field laptop without the need for the more
25 computationally demanding stage of dense topography reconstruction. If calibration

1 results are not satisfactory, we recommend adding extra convergent-view images
2 acquired from a higher altitude. At this point in our research, we do not recommend
3 adjusting calibration values to match equifinal K_n solutions and/or using a fixed
4 calibration during photogrammetric processing. Figure 7 clearly shows that whilst
5 this improves the accuracy of the model by eliminating doming and dishing, it
6 increases surface noise and thus lowers the precision of the final model.
7 Furthermore, calibrations values fixed to those of the flat wall experiment also
8 produced increased levels of noise with the associated loss of precision. This is
9 consistent with the findings of (Fraser 1997; Förstner *et al.* 2013; Fraser 2013; Wolf
10 *et al.* 2014) that recommend on-the-job self-calibration as best-practice for small
11 format, non-metric, photogrammetry. However, we note that our approach requires
12 a certain pattern of lens distortion which is based in the effect of parameter
13 correlation as discussed by Zienmann (1986) and Fraser (1997). This concept of
14 parameter correlation for the K_n components is relatively obscure but it explains our
15 observations of equifinality. Given the level of development and computational
16 expense of digital photogrammetry in the 1980s and 1990s, it is not surprising that
17 earlier authors did not examine large parameter spaces of K_n . However, in the
18 modern context of SfM where automated camera calibration is the norm,
19 practitioners of SfM-photogrammetry should be aware of parameter correlation and
20 equifinality in the calibration solutions. If we examine the Brown-Conrady lens
21 model in table 1 and the lens distortion profiles in figure 4, it can be inferred that a
22 necessary condition for equifinality and correlation in the K_1 - K_2 parameter space is
23 opposite signs for K_1 and K_2 . This is what allows for various combinations of K_1 and
24 K_2 to compensate each other and forms the basis for the parameter correlation
25 effect. In order to assess transferability of these findings, we conducted a survey of

1 a range of cameras and examined their K_n radial distortion parameters as obtained
2 by the self-calibration algorithm in Photoscan. Table 5 shows that in all cases, K_1
3 and K_2 have opposite signs. This suggests that the lens distortion profile will have
4 inflection points that require the use of at least K_1 and K_2 . Therefore, it can
5 reasonably be expected that these lenses will display some equifinality (i.e.
6 correlation between K_1 and K_2) in their own K_1 - K_2 parameter spaces.

7 Once it is confirmed that a self-calibrated block adjustment has been achieved
8 within a family of zero distortion solutions, the rigid-body 7-parameter transformation
9 that is employed to transform a point cloud from the arbitrary space of the initial
10 block adjustment to map coordinates, can successfully produce topographic models
11 to a precision of 0.1% of flying height. This translates to decimetric precisions at
12 altitudes less than 100m AGL. These findings have significant implications for
13 surface process studies where the DG workflow from low-cost drones offers the
14 potential for easy and rapid topographic surveys, which are suited to a wide range of
15 study environments including hazard-stricken and inaccessible areas. In the
16 commercial sector, the use of drones and the DG workflow could provide a low-cost
17 alternative to laser range-finding based approaches. If future research can improve
18 the quality of DG-derived topography, the DG-workflow might out-compete laser
19 scanning for certain applications with much smaller operational and capital costs.
20 We therefore argue that the combination of low-cost drones, low-cost SfM-
21 photogrammetry and a DG workflow will transform mapping by allowing both
22 specialists and non-specialists to generate topography and 3D virtual landscapes
23 with meaningful levels of accuracy and precision.

24 We will now proceed to a closer examination of the errors and likely sources that
25 affect the direct georeferencing approach with sUAS and propose a few simple steps

1 that will improve both the accuracy and precision of the resulting topographic
2 models. Generally, table 4 shows that flight patterns with convergent views at
3 multiple altitudes have the most reliable performance with self-calibrations that plot
4 along the line of zero distortion in figure 6. However, there is variability that needs
5 further consideration. Since the model is georeferenced via the image geolocations,
6 the spatial structure of the GPS error, and the size of the overall flight path envelope
7 relative to this GPS error, can have a significant impact on the outcome. In figure 3,
8 we can see that Site B is significantly narrower than Site A. This might explain why
9 the scaling errors for Site B are larger, it suggests the size and shape of the flight
10 path envelope is crucial in averaging out GPS errors. Whilst it is generally accepted
11 that photogrammetric errors should be reported as a fraction of the flying altitude, in
12 this case, this should be taken with the caveat that lower flights covering a smaller
13 area will likely have larger relative errors caused by the GPS error occupying a larger
14 percentage of the flight path envelope.

15 An examination of table 4 clearly shows that the optimal precision (η_p) does not
16 coincide with optimal scale and tilt. Corrective steps must be taken to get the best
17 results and in fact, we find that DG from consumer-grade sUAS still requires minimal
18 ground-truthing for most science applications. However, this ground-truthing does
19 not necessarily require survey-grade data or equipment. Datum shift and translation
20 errors in XY (T_x , T_y , T_z in table 4) could arguably have no impact where a single-
21 epoch DEM is acquired for purposes such as hydrologic or hydraulic modelling.
22 These errors are the easiest to correct as they require only one point with a known
23 3D position. This can be retrieved from freely available global or national datasets.
24 However, for change detection studies, correction of these linear shifts will be critical.
25 In these cases, any given point in the survey area which has remained static can be

1 used to correct translation errors. Scaling errors may have an impact on studies of
2 erosion and volumetric change. Their correction requires 2 points with a precisely
3 known relative distance. Fortunately, a single scale object with accurately known
4 dimensions in the study site is sufficient. Since the scale parameter used in the 7-
5 parameter transform is isotropic, the scale object can be in any orientation and the
6 resulting scale correction is valid for the whole model. The accuracy of this scale
7 correction will be a function of the size of the object. For corrections to within 0.1%,
8 a scale object separated by 1000 pixels is required which can be obtained from
9 larger man-made features. In certain cases, even Google Earth can provide suitable
10 scales. In the case of B4, we used Google Earth to measure the length of a pale
11 cement border along the path and used this measurement in Photoscan as a scale
12 bar. This improved model quality and resulted in a scaling parameter of 100.2 %
13 (down from 104.8%). However, readers should be cautious when using Google
14 Earth data for accurate mapping of any sort. Google Inc. acquires data from multiple
15 sources and there is no stated, universal, quality standard. In fact, care should be
16 taken with scale measurements from remotely sensed imagery, often times the
17 measurement error from certain imagery sources can be larger error than the errors
18 from the sUAS GPS. Tilt errors are significant in the context of surface processes
19 where models of sediment and/or water fluxes will be sensitive to gradient errors.
20 These errors can be corrected with two scale objects set at a right angle and levelled
21 to an accurate horizontal pitch. If using projected map coordinates, it is also
22 recommended to use a compass to align the azimuths of these two scale objects to
23 N-S and E-W directions. Alternatively, secondary elevation datasets with sub-metric
24 vertical precisions might be used (i.e. existing airborne or terrestrial LiDAR). Here
25 we recommend a simple planar detrending operation based on the difference

1 between the DG topography and the reference topographic data. With the growing
2 availability of high-resolution (metric and sub-metric) datasets, correcting for datum
3 shifts, scale and tilt is now relatively straightforward in many parts of the globe.

4 If survey-grade RTK-GPS is available, the workflow presented here still has
5 implications. Traditionally, it is recommended to have in excess of 20 ground control
6 points well distributed in the XYZ space of the study site (Carbonneau *et al.* 2003).
7 However, in combination with a DG workflow, three to four survey-grade ground
8 control points distributed along the periphery of the study site, preferably collocated
9 with pseudo-invariant features, could accurately correct for datum shifts, scale, and
10 tilt. This could allow for topographic surveys in inaccessible areas provided that the
11 edges of these sites are accessible. In particular, change detection studies with
12 such a DG workflow will require careful experimental design. Multiple static points in
13 the study site will be needed to correct for scale and datum shifts. Levelled features
14 will also be required to correct for off-vertical tilt errors. Furthermore, the error
15 tolerance of each specific study should be used to establish the sUAS flying height as
16 one thousand times the maximum error.

17 The residual error of 0.1% of flying height (here 0.06 m with flights at an altitude of
18 60 m AGL) can at best be qualified as a 'good' performance. With the use of GCPs,
19 significantly better sUAS photogrammetry precisions have been reported (Eltner *et*
20 *al.* 2015; Woodget *et al.* 2015) and therefore the next steps should be the
21 understanding of DG non-affine errors. However, it is difficult to assess this
22 explicitly. Proprietary SfM-photogrammetry software packages such as Photoscan
23 Pro protect many details in their matching processes for commercial reasons. At
24 the moment, carefully designed empirical experiments appear to be the only way
25 forward and further research on the relationship between surface noise amplitude

1 and pattern matching performance in the DG workflow will be required to achieve
2 survey-grade outputs. Additionally, improving the positional accuracy of each image
3 geotag should be a priority. (Bláha *et al.* 2012) have demonstrated that sub-metric
4 positioning of sUAS can be achieved with on-board RTK-GPS positioning.
5 Furthermore, (Chiang *et al.* 2012) and (Milik and Gabrlík 2015) have demonstrated
6 that when RTK-GPS positioning is propagated through a photogrammetric solution,
7 the position of GCPs can also be predicted with decimetric accuracies. Moreover,
8 (Eling *et al.* 2015) report predictions of GCP positions with centimetric accuracy.
9 Whilst these authors use experimental RTK-GPS equipment and high-cost platforms,
10 low-cost RTK-GPS units are appearing. Swiftnav inc. is currently marketing their
11 PIKSI GPS, which is a fully miniaturised GPS capable of real-time kinematic
12 differential corrections. At a cost of ≈ 900 USD, the units are well suited to low-cost
13 sUAS. Furthermore, RTK-enabled UAVs are appearing on the consumer market
14 with models aimed at both the scientific (.e.g the RTK-ebee fixed-wing made by
15 Sensefly inc.) and the cinema (e.g. the DJI Matrice 600 hexacopter) sectors. These
16 units will inevitably make their way into the consumer market and we expect
17 publications of results from these RTK-equipped platforms in the near future to
18 strengthen the case for the use of DG in sUAS photogrammetry. However, we note
19 that our findings re-emphasize the importance of camera calibration as a crucial
20 factor in accurate topography reconstruction. The addition of RTK-GPS to sUAS
21 might provide considerable improvements to the performance of the DG workflow,
22 but it will not obviate the need for accurate camera calibration. This point
23 emphasizes the fact that SfM-photogrammetry is in no way exempt from the
24 principles of photogrammetry. Advances in image matching algorithms from the area
25 of computer vision, integrated into the photogrammetric workflow, have clearly

1 enabled a step-change in image-based topography generation. However,
2 practitioners of SfM-photogrammetry still require traditional, 'pre-SfM' knowledge of
3 photogrammetry. The illusion that SfM is not photogrammetry must now be
4 dispelled.

5

1 **Conclusion**

2 The problem of producing topography from directly georeferenced sUAS imagery
3 poses many significant challenges and opportunities. When we compare our results
4 to those published by other authors, it is clear that the DG workflow results in the
5 expected higher levels of error. However, our results and innovative approach to
6 error characterization indicate that current consumer-grade drones along with low
7 cost SfM-photogrammetry packages and a DG workflow can produce topographic
8 data with sufficient quality for a limited number of applications. Given that the cost of
9 RTK-GPS equipment is generally above ~£10 000 and that the low-cost drones used
10 here are in the area of £1000-£2000, our DG approach offers a reduction of costs of
11 2 orders of magnitude. We therefore argue that further development and integration
12 of DG into the UAS/SfM workflow has significant implications for topographic survey
13 that justify further research and development. The facilitation of mass-production of
14 topographic data and associated sub-metric resolution imagery will have a
15 transformative impact on all Earth surface process sciences as well as the
16 topographic survey and natural disaster management industries.

17

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12

Tables and figures

Parameters	Description
F_x, F_y	Focal length of the camera in X and Y dimensions.
C_x, C_y	Principle point of the image (x and y image coordinates)
K_1	Second-order radial distortion
K_2	Fourth-order radial distortion
K_3	Sixth-order radial distortion
P_1, P_2	X (P_1) and Y (P_2) tangential distortion
Brown-Conrady distortional model(Brown 1966; Heikkila and Silven 1997):	
$\begin{bmatrix} \hat{x} \\ \hat{y} \end{bmatrix} = (1 + K_1 r^2 + K_2 r^4 + K_3 r^6) * \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2P_1 xy + P_2(r^2 + 2x^2) \\ P_1(r^2 + 2y^2) + 2P_2 xy \end{bmatrix}$ $r = \sqrt{(x - C_x)^2 + (y - C_y)^2}$	
where: x, y are the distorted pixel coordinates; \hat{x}, \hat{y} are the undistorted pixel coordinates	

Table 1: Camera calibration parameters in the Brown-Conrady model.

Site A	Site B	Flight Pattern
A1	B1	nadir imagery from an altitude of 60 meters
A2	B2	nadir imagery from an altitude of 60 and 80 meters
A3	B3	nadir imagery combined with eight convergent-view images at an altitude of 60 meters
A4	B4	nadir imagery at 60 and 80 meters combined with eight convergent view images acquired at 60 meters

Table 2: Description of the 11 experimental flights.

Site A					
Flight	F [pix]	$K_1 \times 10^{-8}$	$K_2 \times 10^{-15}$	$K_3 \times 10^{-23}$	$K_{2e} \times 10^{-15}$
A1: Nadir 1	2539	-2.48	3.86	-9.48	1.69
A2: Nadir 2	1974	-2.50	3.83	-9.97	3.89
A3: Oblique 1	2317	-2.52	3.84	-9.69	0.04
A4: Oblique 2	2320	-2.52	3.86	-10.6	0.06
Site B					
B1: Nadir 1	2304	-2.45	4.11	-11.9	0.45
B2: Nadir 2	2229	-2.58	4.12	-13.0	0.02
B3: Oblique 1	2323	-2.59	4.13	-13.2	0.18
B4: Oblique 2	2323	-2.58	4.10	-12.2	0.16

Table 3. Self-Calibration outputs for all flight experiments. K_{2e} gives the error in K_2 when calibrated values of K_2 are compared to those predicted by equation 5.

Experiment 1: Site A								
Flight	Tx [m]	Ty [m]	Tz [m]	R ϕ [°]	R θ [°]	R ψ [°]	S [%]	η_p [m]
A1: Nadir 1	4.02	1.32	-2.48	-0.93	-0.38	0.40	99.7	0.46
A2: Nadir 2	3.31	-1.79	8.88	-0.60	-1.42	-0.34	100.9	0.36
A3: Oblique 1	0.65	0.35	3.49	-1.34	-0.44	-0.02	99.5	0.55
A4: Oblique 2	3.04	1.63	0.36	-0.62	-0.78	-0.18	100.4	0.32
Co-registration residual [m]: 0.05 \pm 0.23								
Experiment 1: Site B								
B1: Nadir 1	0.82	6.38	-5.73	0.86	-1.20	1.11	98.4	0.12
B2: Nadir 2	-0.46	4.20	5.19	0.09	0.33	-0.06	95.8	0.24
B3: Oblique 1	0.79	5.85	1.17	-0.08	-1.05	0.66	94.9	0.15
B4: Oblique 2	-3.31	2.43	-0.49	0.68	0.23	0.02	104.6	0.06
Co-registration residual [m]: 0.00 \pm 0.03								

Table 4. Error parameters for all flight experiments. $T_{x,y,z}$ give translations (systematic offsets), $R_{\phi,\theta,\psi}$ give Euler rotation angles with R_{θ} being giving off-vertical tilt angles. S is the scaling error with 100% meaning that the model scale is identical to the actual scale. η_p is the precision of the final SfM point clouds, calculated as the standard deviation of the quasi-random field η . For each site, we give the overall co-registration residual estimate for the cloud alignments as accuracy \pm precision where the accuracy is the mean residual and precision is one standard deviation of the residuals.

Camera	Focal [pix]	K ₁ [f]	K ₂ [f]	K ₃ [f]
Canon EOS REBEL T2i (18-135mm Zoom @ 18 mm)	4411.34	-0.169	0.144	0.026
Canon PowerShot A3300 IS	3827.51	-0.040	0.045	-0.033
Canon PowerShot A4400 IS **	3801.63	-0.082	0.070	-0.003
Canon PowerShot S110	2930.36	-0.04	0.01	-0.01
Canon PowerShot SD1000 (IXUS 70)	1822.4	-0.17	0.25	-0.07
Canon PowerShot SX170 IS **	3823.95	-0.091	0.432	-0.877
LG Nexus 5 **	3019.65	-0.215	0.385	-0.078
Microdrone 2.0 HD	1466.43	0.20	-1.40	3.00
Nikon D1X 35mm lens*	4535.33	-0.10	0.16	-0.12
Nikon D5200 (18-55mm Zoom @ 18mm) **	4635.72	-0.102	0.065	-0.079
Sony Nex-7, 20mm lens	5231.44	-0.15	0.13	0.03

* Traditional airborne survey pattern, acquired at nadir with 2 altitudes.

** Parameters derived from the camera calibration operations in OpenCV (opencv.org)

Table 5: Calibrated focal lengths and K_n parameters for a selection of cameras. Most of the calibration data was extracted from real surveys processed with Photoscan. Four of the cameras were calibrated with the camera calibration operations in the OpenCV software libraries. All of the cameras tested have opposite signed K₁ and K₂ values.

SfM model errors

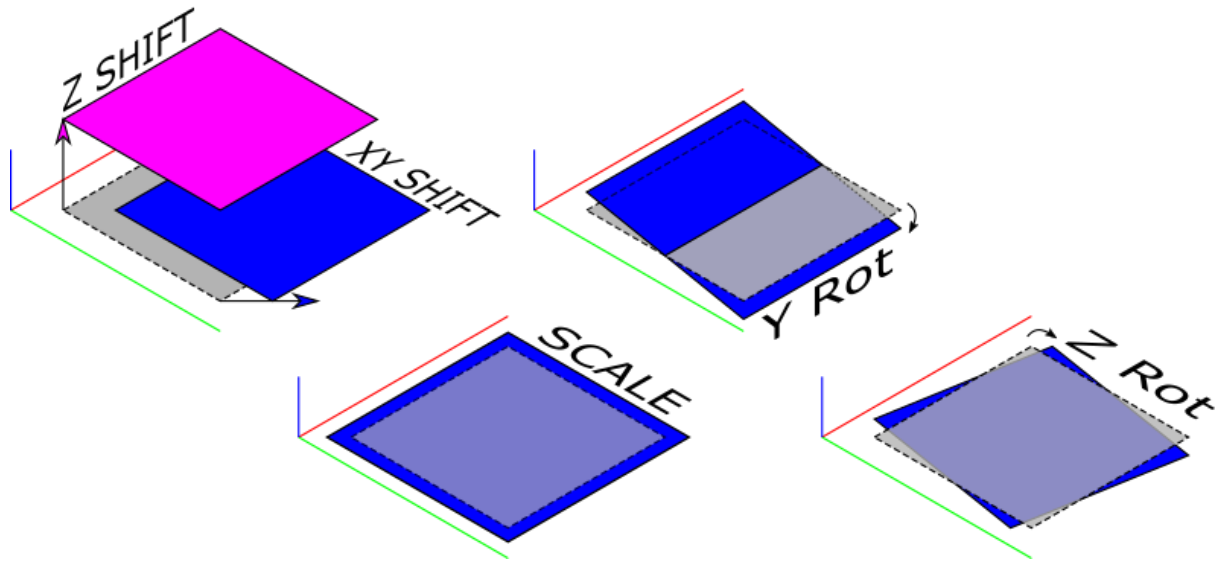
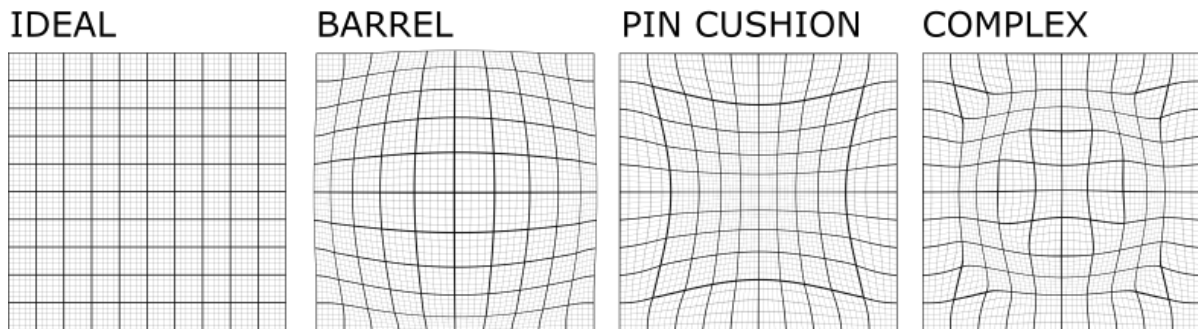


Figure 1: Potential affine model errors in SfM models. The grey squares represent the ideal orientation; the blue and magenta represent the expression of the different affine errors types. The axes in the illustration are 3D (green = x, red = y, blue = z).

a) Camera Lens distortion examples



b) SfM surface model distortions

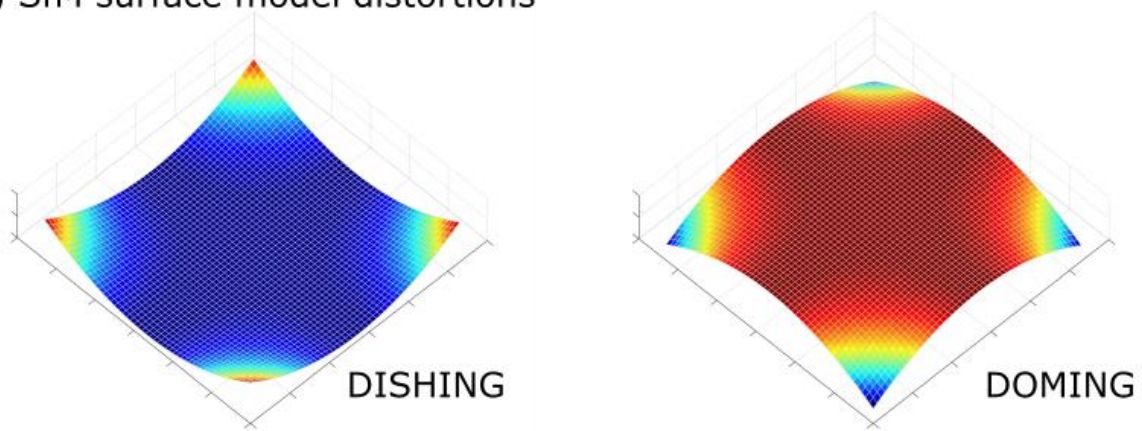


Figure 2. (a) Lens distortion examples, the ideal distortion pattern represents a lens with no distortion. The barrel and pin cushion distortions can be characterised by either a positive or negative K_1 parameter. Most modern lens systems in compact cameras have complex lens distortions that require more than a single K_1 parameter, and often need a K_2 and K_3 . (b) Illustration of systematic doming and dishing distortions that can present in SfM models. These errors are often attributed to incorrect survey patterns or incorrect lens distortion corrections (the colour ramp is relative, red = higher elevation, blue = lower elevations).

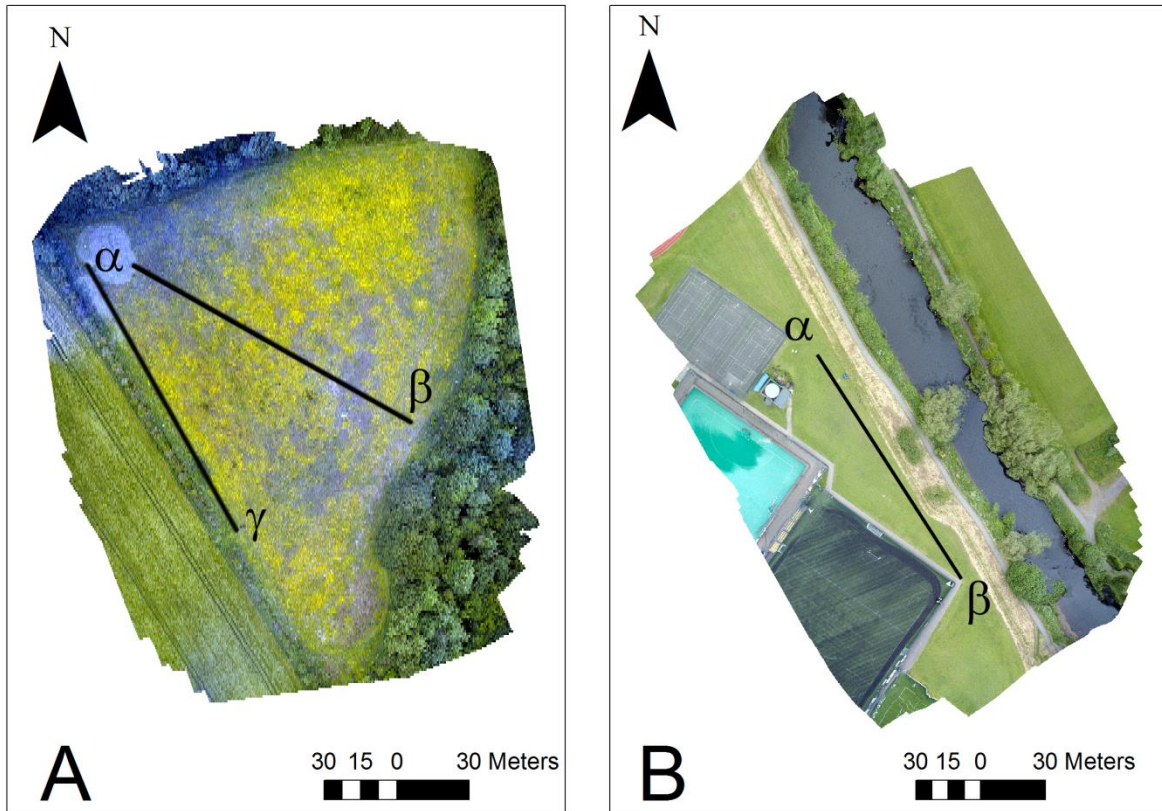


Figure 3. Study sites A and B near Durham UK, with profiles locations used in figures 7 and 8. The colour difference in the site A image (around α) was due to blending errors in orthophotograph for this shadowed area.

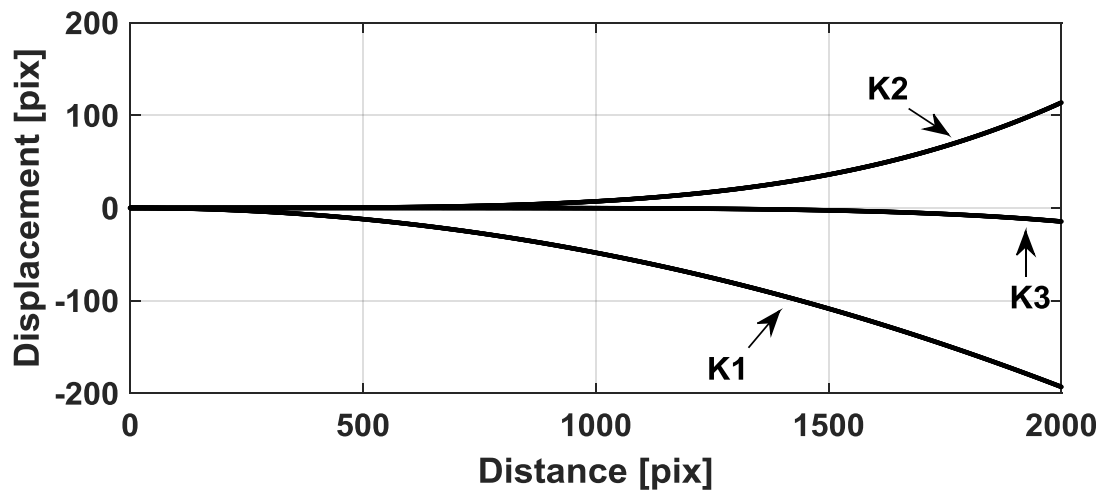


Figure 4. Lens distortion profiles for the P3P. K_1 , K_2 and K_3 components of the optimal lens distortion profile for the Phantom 3 as derived from the flat wall experiment. Note the comparable magnitudes of the K_1 and K_2 contributions in the outer portion (>1500 pixels) of the profile.

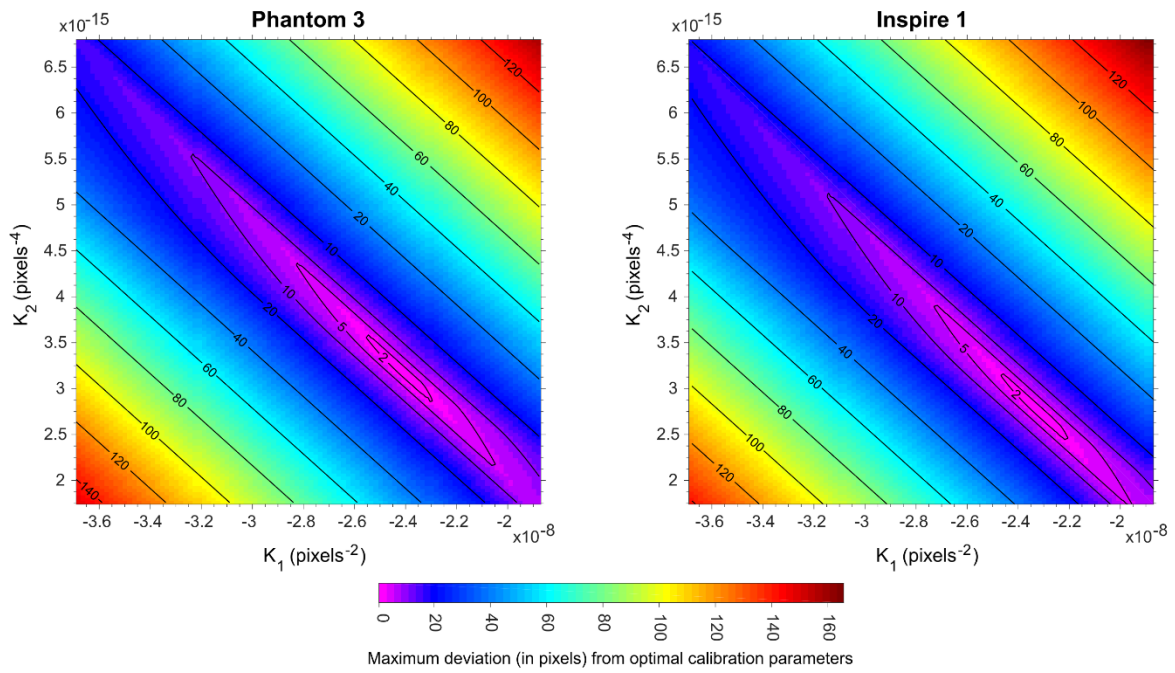


Figure 5. Equifinality in K_1 - K_2 space for the Phantom 3 and Inspire 1 lenses. Colour scales and contours represent the maximum deviation, in pixels, for a given set of (K_1, K_2) values when compared to the optimal calibration parameters. We note a wide range of solutions equifinal to within 0.01 pixels.

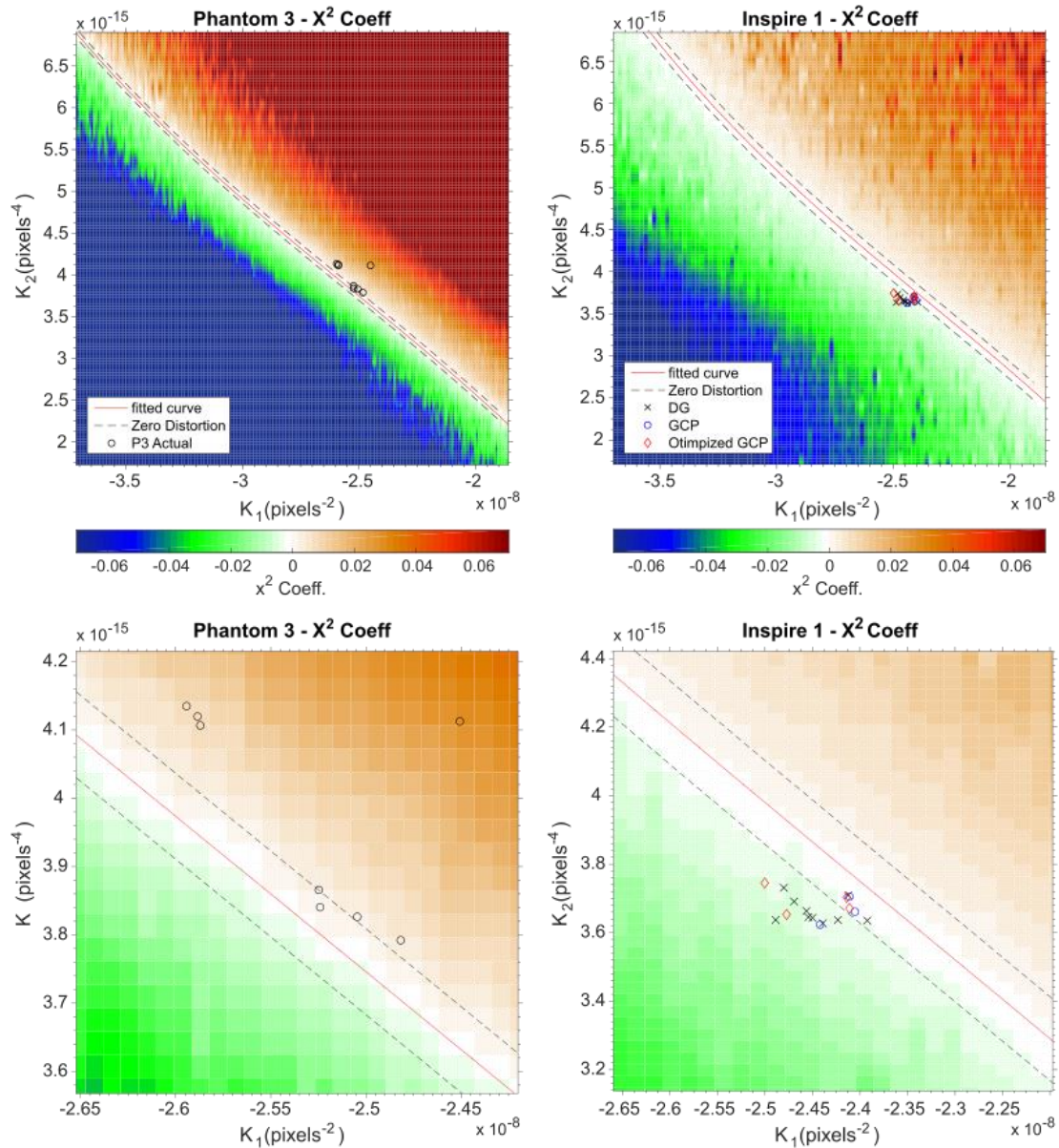


Figure 6. Perturbation outputs of surface doming as a function of K_1 , K_2 for the Phantom 3 and the Inspire 1. The color ramps represent the magnitude of x^2 coefficients from the polynomial fits. The central diagonal bands with null x^2 coefficients are interpreted as families of K_1 - K_2 solutions that successfully model lens distortion and eliminate parabolic doming. Points added to each surface represent calibrated (K_1, K_2) values for real data. In the case of the Phantom 3 surface, these are the outputs for all 11 direct georeferencing (DG) flight experiments. In the case of the Inspire 1, these represent other data not discussed in this paper and they use a DG, traditional GCP and software optimisation (a Photoscan function).

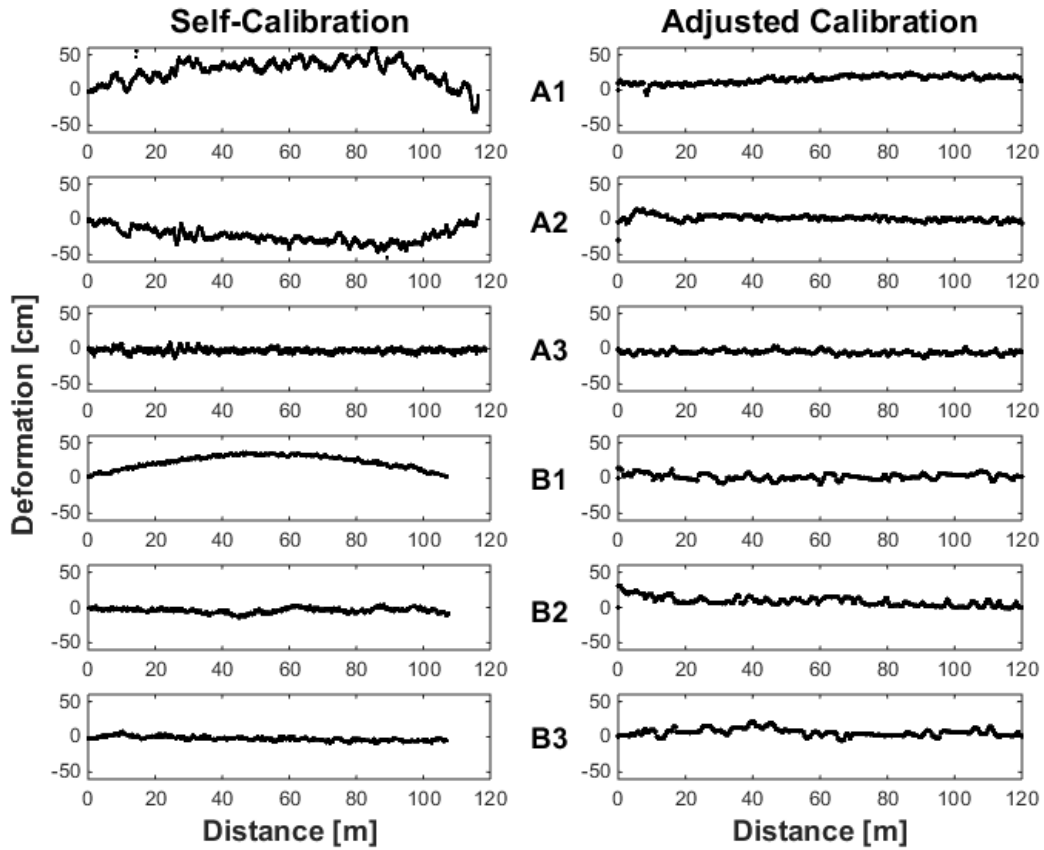


Figure 7. Linear profiles from α to β with self-calibrated and adjusted calibration values. Deformation amplitude is calculated by comparing sites A1-A3 to the optimal A4 and B1-B3 to the optimal B4. We note that whilst doming is eliminated, the adjustment results in increased surface noise.

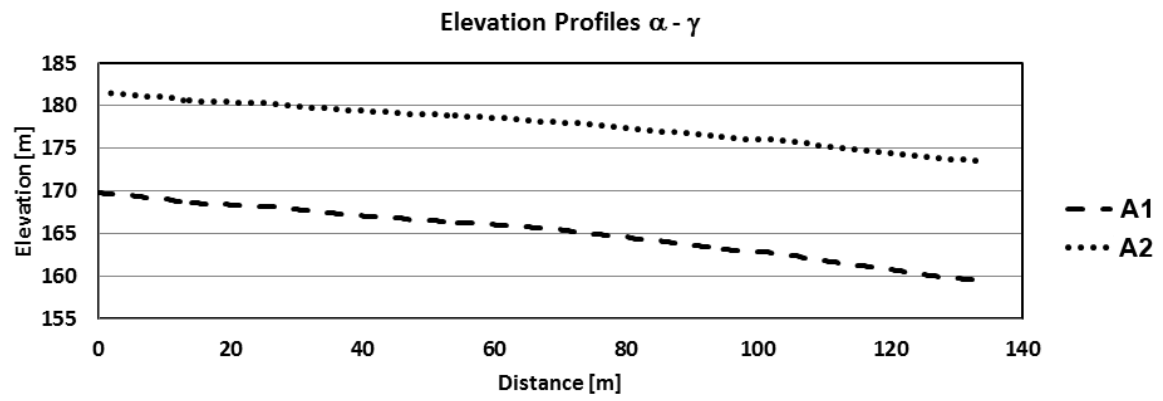


Figure 8. Effects of focal length calibration errors on datum along $\alpha-\gamma$. Note that profile length in XY is not affected.

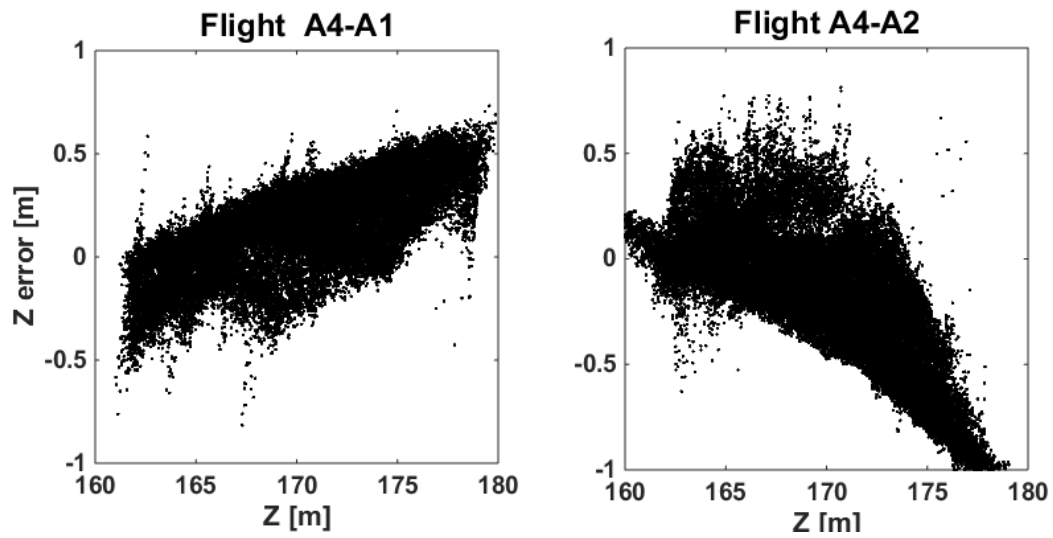


Figure 9. Altitude dependent errors. Here we show the correlation between absolute altitude and error for flights A1 and A2 which had the poorest calibration of focal length.

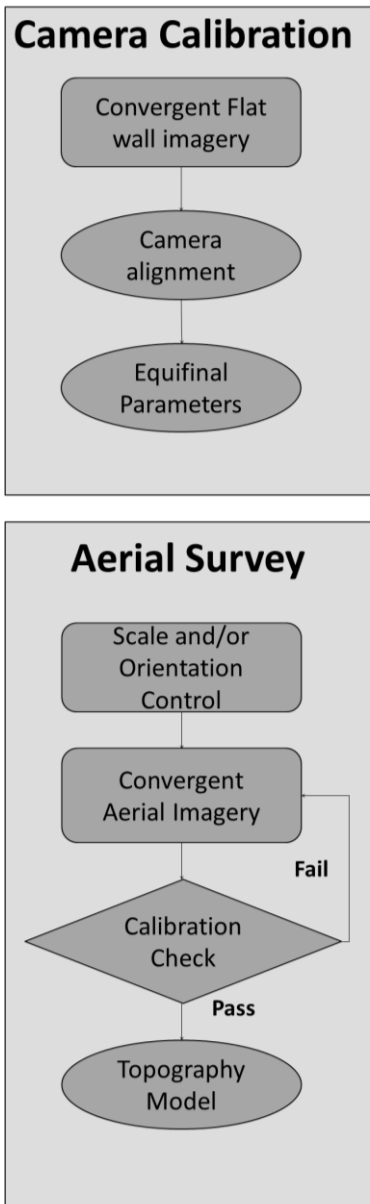


Figure 10. Summarised DG workflow. The two stages of the proposed workflow. The camera calibration stage aims to identify the focal length and the set of equifinal solutions along with their domain of applicability. This information can then be used in the calibration check, potentially carried out in the field, that allows for a reliable topographic model to be produced with a direct georeferencing.

