Reliability sensitivity analysis of coherent systems based on survival signature

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Abstract: The reliability sensitivity can be used to rank distribution parameters of system components concerning their impacts on the system's reliability. Such information is essential to purposes such as component prioritization, reliability improvement, and risk reduction of a system. In this paper, we present an efficient method for reliability sensitivity analysis of coherent systems using survival signature. The survival signature is applied to calculate the reliability of coherent systems. The reliability importance of components is derived analytically to evaluate the relative importance of the component with respect to the overall reliability of the system. The closed-form formula for the reliability sensitivity of the system with respect to component's distribution parameters is derived from the derivative of lifetime distribution of a component to further investigate the impacts of the distribution parameters on the system's reliability. The effectiveness and feasibility of the proposed approaches are demonstrated with two numerical examples.

Keywords: Reliability sensitivity; Survival signature; System reliability; Component importance; Structure function

Introduction

In modern industry, the widespread of automatic and intelligent equipment contributes to more and more powerful and complex manufacturing systems which can greatly improve production efficiency. However, the risk of failure of manufacturing systems and potential losses from failure are also greatly increased simultaneously. Therefore, performing system reliability analysis and taking steps to eliminate weaknesses to ensure safe and reliable operation of the system is now even more important[1-3].

Many methods exist to improve system reliability, such as using high-quality components, increasing redundancy and implementing better maintenance. Generally, it is hard to determine whether among them an optimal method exists. What is certain, however, is that it would be the most cost-effective if we could find critical (important) components and propose improvement plans for them. Importance analysis, as one of such tools, can be used to prioritize components in a system by mathematically measuring the importance level of each component[4-7].

In 1960s Birnbaum[8] first introduced an importance measure to characterize rate at which system's reliability changes with respect to changes in the reliability of a given component. During the following decades, because of the easy implementation and understanding, reliability importance was widely used and rapidly developed[9,10]. Natvig et al.[11] extended the Birnbaum importance to the time-dependent lifetime using component lifetime distributions. Dutuit et al.[12] studied the potential extensions of Birnbaum importance to complex components whose failures are modeled by a gate rather than just a basic event.

Kamalja et al.[13] and Shen et al.[14] studied the reliability importance of consecutive systems, whereas Borgonovo et al.[15], Vaurio et al.[16] and Aliee et al.[17] extended reliability importance to non-coherent systems through Boolean expression. Li et al.[18], Lisnianski et al.[19] and Kvassay et al.[20] proposed methods for computation of reliability importance for multi-state systems. Moreover, Eryilmaz et al.[21] and Zhu et al.[22] focused on the interaction effect between the components on system reliability to evaluated joint reliability importance of two components. Baraldi et al.[23] and Li et al.[24] developed methods for quantifying epistemic and aleatory uncertainties in Birnbaum importance. Wu et al.[25] and Dui et al.[26] proposed cost-based importance measures by considering the joint effect of component maintenance cost and time on system reliability.

The above methods lay the foundation for reliability importance analysis, and the underlying principle is fairly well established. However, the existing importance measures pay little attention to probabilistic characteristics of system components while ranking them. In this paper, a general method for reliability sensitivity analysis of coherent systems with respect to the distribution parameters of components is studied using survival signature. The paper is organized as follows: Section 2 describes reliability, reliability importance, and reliability sensitivity analysis based on structure function. Section 3 gives a brief description of the survival signature, followed by reliability importance and sensitivity analysis with respect to the distribution parameters of system components. Numerical examples in Section 4 illustrate the application of the proposed method. Finally, Section 5 draws conclusions.

Reliability sensitivity analysis using structure function

System reliability analysis is concerned with estimating the lifetime of complex systems subject to several uncertainties, such as type, working time, and failure rate. Theory of system reliability has been established over many decades and led to an extensive literature[27] whose heart is the ‘structure function’ defined as

$$\varphi\left(\boldsymbol{X}(t)\right)=\varphi\left(X_1(t), X_2(t), ..., X_n(t)\right) \tag{1}$$

where n is number of components, $X_i(t)$ is state of component i at time t being i=1, 2, …, n; $X_i(t)$=1 if the ith component functions at time t, and $X_i(t)$=0 if component i fails at time t. Structure function $\varphi(\boldsymbol{X}(t))$ is state of the system; $\varphi(\boldsymbol{X}(t))$=1 if the system functions at time t for state vector $\boldsymbol{X}(t)$, $\varphi(\boldsymbol{X}(t))$=0 if not.

The system at time t is

$$R_s(t)=\Pr\left\{\varphi(\boldsymbol{X}(t))=1\right\}=\mathrm{E}\left(\varphi(\boldsymbol{X}(t))\right) \tag{2}$$

If components are independent, the system reliability $R_s(t)$ is

$$R_s(t)=h\left(R_1(t), R_2(t), \ldots, R_n(t)\right)=h\left(\boldsymbol{R}(t)\right) \tag{3}$$

where $R_i(t)$=1-$F_i(t)$ is reliability function of component i at time t being i=1, 2, …, n; $F_i(t)$ is lifetime distribution of component i.

The sensitivity of the reliability $R_s(t)$ with respect to the distribution parameters of lifetime distribution of component i, $\theta_i^{(l)}$, being l=1,2, …, (e.g. mean μ_i or standard deviation, σ_i) can be derived from Eq. (3)

$$\frac{\partial R_s(t)}{\partial\theta_i^{(l)}}=\frac{\partial h(\boldsymbol{R}(t))}{\partial R_i(t)}\frac{\partial R_i(t)}{\partial\theta_i^{(l)}}=h\left(1_i,\boldsymbol{R}(t)\right)-h\left(0_i,\boldsymbol{R}(t)\right)\frac{\partial R_i(t)}{\partial\theta_i^{(l)}}, i=1,...,n \tag{4}$$

where $\partial h(\boldsymbol{R}(t))/\partial R_i(t)$ is Birnbaum importance of component i, $h(\cdot i, \boldsymbol{X}(t))= h(X_1(t), \ldots, X_{i-1}(t), \cdot, X_{i+1}(t), \ldots, X_n(t))$, $\partial R_i(t)/\partial \theta_i^{(l)}$ can be derived analytically from lifetime distribution of type i components. Examples for common distributions are given in the Appendix.

From Eq.(3) and Eq.(4), the reliability and reliability sensitivity of a system can be directly obtained from its structure function. However, there are generalizations of the structure function to multi-state scenarios. Already in a simplest form, it requires a specification for 2^n inputs for a system consisting of n components. Practical systems of interest may include hundreds or thousands of components; therefore, it is almost impossible to specify its structure function.

Reliability sensitivity analysis using survival signature

Samaniego[27] separated the structure of the system from system reliability analysis to avoid the task of specifying the structure function and proposed the system signature to quantify the reliability of systems consisting of independent and identically distributed or exchangeable components. However, the use of the system signature is associated with the assumption that all components in the system are of the same type, which in the engineering practice is almost impossible. To overcome this limitation, Coolen et al.[28,29] proposed the use of survival signature for analyzing complex systems consisting of more than one single component type increasing the applicability of the signature approach to characterize complex systems.

Consider a coherent system with $K \geq 2$ types of components, with n_k components of type $k \in \{1, 2, \ldots, K\}$ and $\sum_{k=1}^{K} n_k = n$. Components of the same type can be grouped together leading to a state vector that can be written as $\boldsymbol{x}=(\boldsymbol{x}^1, \boldsymbol{x}^2, \ldots, \boldsymbol{x}^k, \ldots, \boldsymbol{x}^K)$ with $\boldsymbol{x}^k = (x_1^k,\ x_2^k, \ldots, x_{n_k}^k)$ the sub-vector representing the states of the components of type k. Assume that the random failure times of components of the different types are fully independent and the components are exchangeable within the same component types, the survival signature of the system can be rewritten as [28, 29]:

$$\varphi_s(l_1, l_2, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}^{-1} \right] \times \sum_{\boldsymbol{x} \in S_{l_1,\ldots,l_K}} \varphi(\boldsymbol{x}) \tag{5}$$

where $S_{l_1,\ldots,l_K}$ is set of all state vectors for the whole system. $\varphi_s(l_1, l_2, \ldots, l_K)$ with $l_k=0, 1, \ldots, n_k$ for $k=1, \ldots, K$, is the probability that the system functions given that precisely l_k of its n_k components of type k function, for each $k \in \{1, 2, \ldots, K\}$. Although the survival signature sets out a significantly reduced representation of the system structure, its derivation may still be complicated. Aslett[30] provided a package to compute the survival signature. Recently, Reed[31] proposed a more efficient method to calculate the survival signature based on binary decision diagrams and dynamic programming.

Let $C_k(t) \in \{0, 1, \ldots, n_k\}$ denote the number of k components working at time t. If the failure times of components of different types are independent, the probability that the system functions at time t is

$$\begin{aligned} R_s(t) &= \Pr(T_s > t) = \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_K} \left(\varphi_s(l_1, \ldots, l_K) \Pr\left(\bigcap_{k=1}^{K} \{C_k(t) = l_k\} \right) \right) \\ &= \sum_{l_1=0}^{n_1} \cdots \sum_{l_K=0}^{n_K} \left[\varphi_s(l_1, \ldots, l_K) \prod_{k=1}^{K} \left(\binom{n_k}{l_k} R_k^{l_k}(t) (1 - R_k(t))^{n_k - l_k} \right) \right] \end{aligned} \tag{6}$$

where $R_k(t)=1-F_k(t)$, is the reliability function of component k at time t being $k=1, 2, \ldots, n$; $F_k(t)$ is lifetime distribution of component k.

Reliability importance and reliability sensitivity play an important role in security assessment and risk management of an industry system. For example, the results of reliability importance and sensitivity analysis may be very useful to the designer, who can know how, and to what extent, the reliability of the system changes with perturbations of the reliability of the components and their distribution parameters. In addition, the maintenance technician can allocate resources for inspection, maintenance, and repair activities in an optimal manner over the lifetime of a system.

The sensitivity of the reliability $R_s(t)$, with respect to the distribution parameters of each type of components $\theta_i^{(l)}$, being $l=1, 2, \ldots$, and $i= 1, 2, \ldots, K$, can be derived from Eq. (6)

$$\frac{\partial R_s(t)}{\partial \theta_i^{(l)}}=\frac{\partial R_s(t)}{\partial R_i(t)}\frac{\partial R_i(t)}{\partial \theta_i^{(h)}}, i=1,\ldots,K \tag{7}$$

where

$$\frac{\partial R_s(t)}{\partial R_i(t)}=\sum_{l_1=0}^{n_1}\cdots\sum_{l_K=0}^{n_K}\left[\frac{l_i-n_iR_i(t)}{R_i(t)\left(1-R_i(t)\right)}\varphi_s\left(l_1,\ldots,l_K\right)\prod_{k=1}^{K}\left(\binom{n_k}{l_k}\left(R_k\right)^{l_k}(t)\left(1-R_k(t)\right)^{n_k-l_k}\right)\right], 0<R_i(t)<1 \tag{8}$$

is reliability importance of each type of component, and $\partial R_i(t)/\partial\theta_i^{(l)}=-\partial F_i(t)/\partial\theta_i^{(l)}$ can be derived analytically from lifetime distribution of type i components.

In a given system, different components play distinct roles being some components more important than others. Therefore, we may want to know the reliability importance and the reliability sensitivity with respect to distribution parameters of each component. Note that if no components are of the same type, then the survival signature just equals the structure function. In this case, we can obtain the reliability importance and sensitivity of system reliability with the distribution parameters of each components from Eqs. (7) and (8). If there exist the same type of components, the relative sensitivity of the reliability $R_s(t)$ can be expressed as follows:

$$\frac{\partial R_s(t)}{\partial \theta_i^{(l)}}=I_B\left(i;\boldsymbol{R}(t)\right)\frac{\partial R_i(t)}{\partial \theta_i^{(l)}}, i=1,\ldots,n \tag{9}$$

where

$$I_B(i;\boldsymbol{R}(t))=R_s\left(1_i,\boldsymbol{R}(t)\right)-R_s\left(0_i,\boldsymbol{R}(t)\right) \tag{10}$$

is reliability importance of component i, $R_s(i, \boldsymbol{X}(t))= R_s(X_1(t), \ldots, X_{i-1}(t), \cdot, X_{i+1}(t), \ldots, X_n(t))$, $R_s(1_i, \boldsymbol{R}(t))$ and $R_s(0_i, \boldsymbol{R}(t))$ which can be derived from Eq.(6) are the reliability of the system given the ith component functions or fails.

Numerical example

Example 1 (A 'bridge' system) Consider the 'bridge' system most widely found in the literature, including Coolen et al.[28] and Walter et al.[32]. As Fig.1 depicts, the 'bridge' system consist of two types of components, namely T1 and T2. The failure time of Type 1 components follows an exponential distribution, and their expected value is 1. The failure time of Type 2 components has a Weibull distribution with scale parameter a=1 and shape parameter b=2.

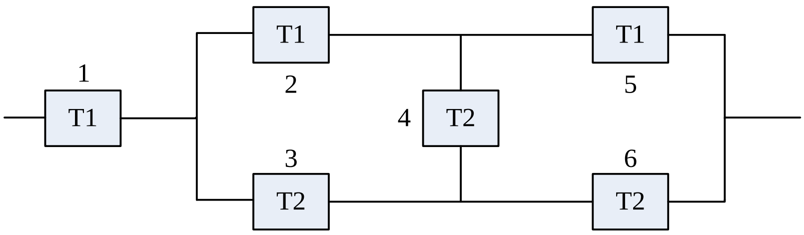


Fig.1 Reliability block diagram of a 'bridge' system

Table 1 Survival signature of the 'bridge' system

l_1	l_2	$\varphi_S(l_1,l_2)$
0	[0, 1, 2, 3]	0
[1, 2]	[0, 1]	0
1	2	1/9
1	3	1/3
2	2	4/9
2	3	2/3
3	[0, 1, 2, 3]	1

First, calculate the survival signature of the system (see Table 1). Then the reliability and reliability importance of the 'bridge' system can be obtained using Eqs. (6) and (8). The results of the studies on reliability and reliability importance ($\partial R_S(t)/\partial R_i^{(T)}(t)$ i=1, 2) using survival signature are reported in Fig. 2 and Fig. 3, respectively. The results suggest that T1 components are important than T2, so the engineer should allocate more resources to the T1 components while designing or maintaining the system. However, reliability importance analysis did not consider the probabilistic characteristics of components while ranking them. Some different conclusions can be drawn from reliability sensitivity analysis.

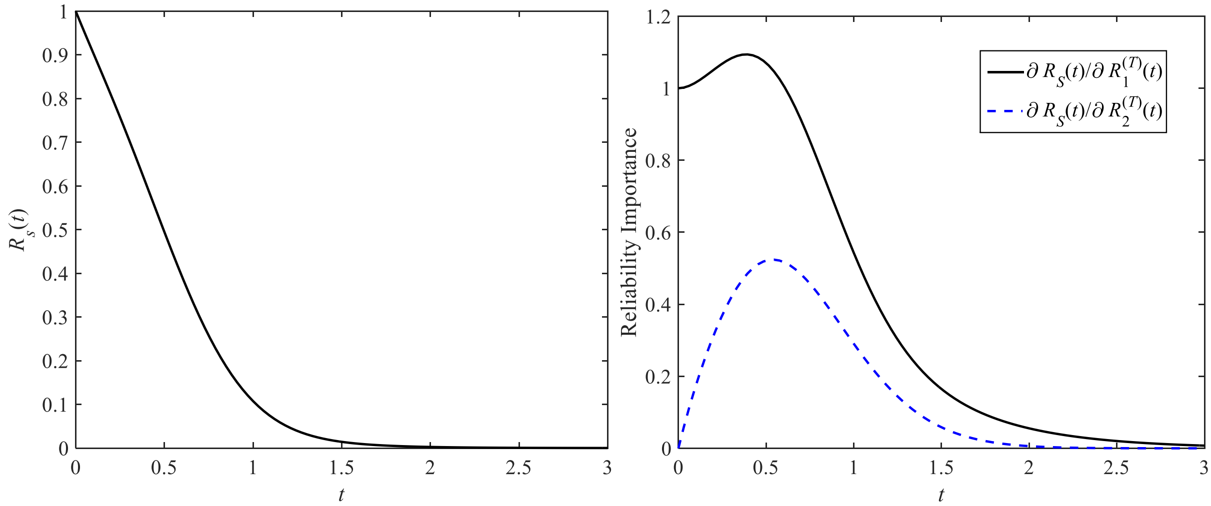
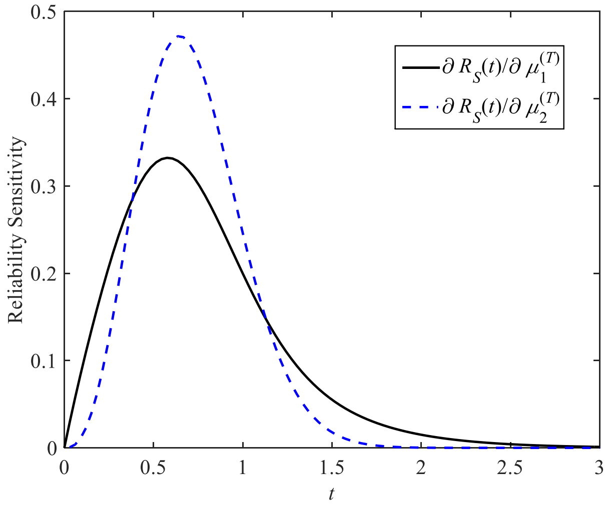


Fig2. Reliability of the 'bridge' system

Fig3. Reliability importance of the 'bridge' system

Reliability sensitivity analysis of the 'bridge' system is conducted using Eqs. (7) and (8). The results of the studies on reliability sensitivity ($\partial R_S(t)/\partial\mu_i^{(T)}$) and $\partial R_S(t)/\partial\sigma_i^{(T)}$) , i=1, 2) using survival signature are depicted in Fig. 4 and Fig. 5, respectively. The reliability sensitivity of the system with respect to mean life of each component is positive, that is to say, an increase in the mean life of each component increases the overall reliability. Since reliability sensitivity of the system with respect to the standard deviation of T2 is negative, the reliability of the system can be improved by decreasing the standard deviation of the component. Moreover, the reliability sensitivity with respect to the mean life of T2 is larger than that of T1 while 0.38<t<1.12. Within this time, priority should be given to T2 to ensure safe and reliable operation of the system.



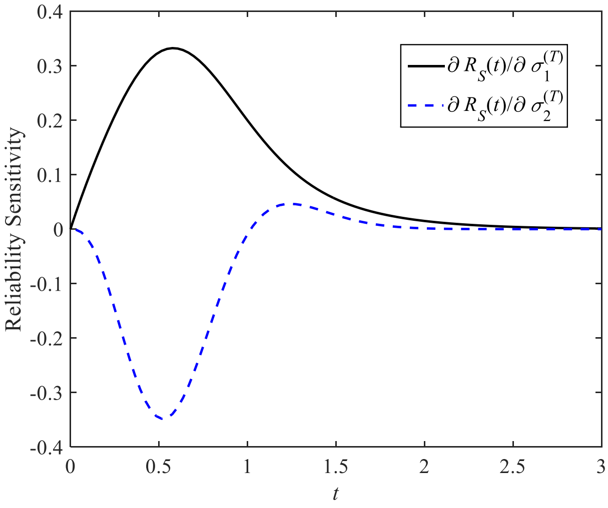
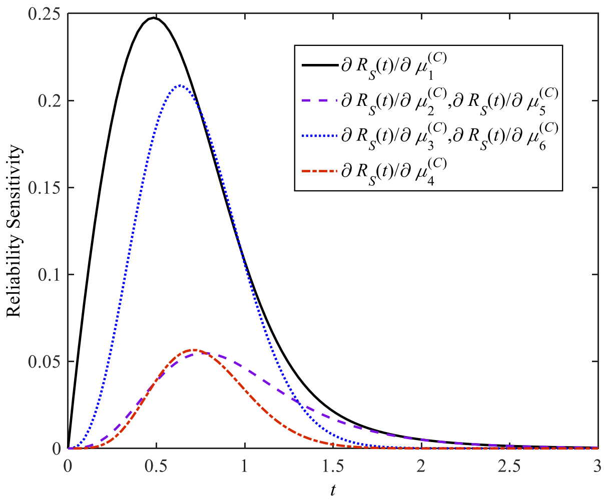


Fig4. Reliability sensitivity with respect to means **Fig5. Reliability sensitivity with respect to standard deviations**

The reliability sensitivity with respect to distribution parameters of each component is calculated to measure the importance of each component. The results of the studies on reliability sensitivity ($\partial R_S(t)/\partial\mu_j^{(C)}$) and $\partial R_S(t)/\partial\sigma_j^{(C)}$) , j=1, …, 6) using survival signature are depicted in Fig. 6 and Fig. 7, respectively. The three most significant components are components 1, 3, and 6.



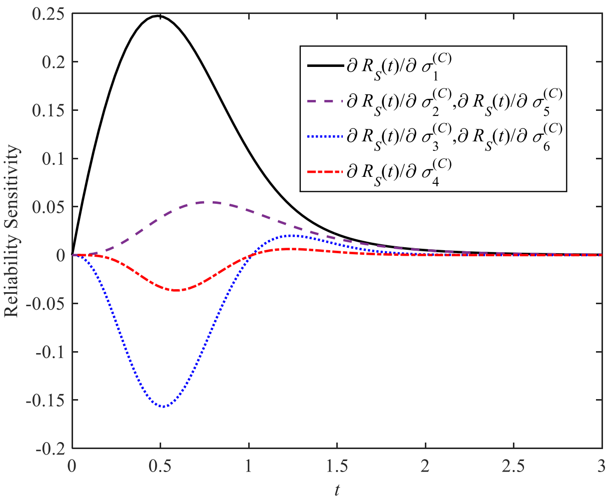


Fig 6. Reliability sensitivity with respect to means **Fig 7. Reliability sensitivity with respect to standard deviations**

Example 2 (A wind turbine). In this example, reliability and reliability sensitivity analysis of a wind turbine are studied based on survival signature. Typically, a wind turbine is composed of two bearings, a main shaft, a gear box, and a generator. As Wu et al.[25] suggests, the reliability block diagram is a serial and parallel structure (see Fig.8); Table 2 summarizes the distribution information of each component.

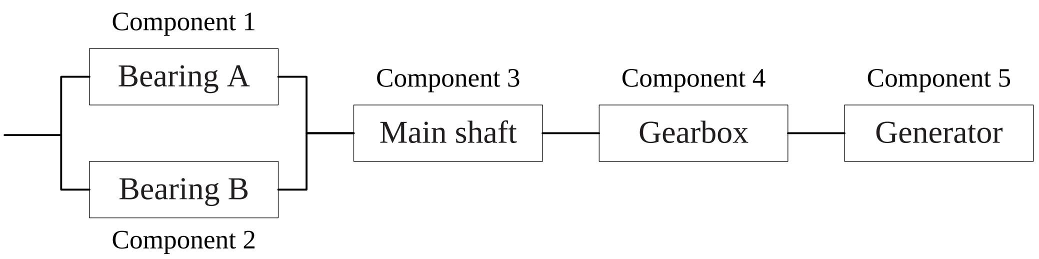


Fig. 8. Reliability block diagram of a wind turbine

Table 2　Distribution information of components

No.	Component name	Distribution type	Scale parameter	Shape parameter
1	Bearing A and B	Weibull	3835	1.09
2	Main shaft	Weibull	6389	1.43
3	Gear box	Weibull	29051	1.05
4	Generator	Weibull	17541	1.11

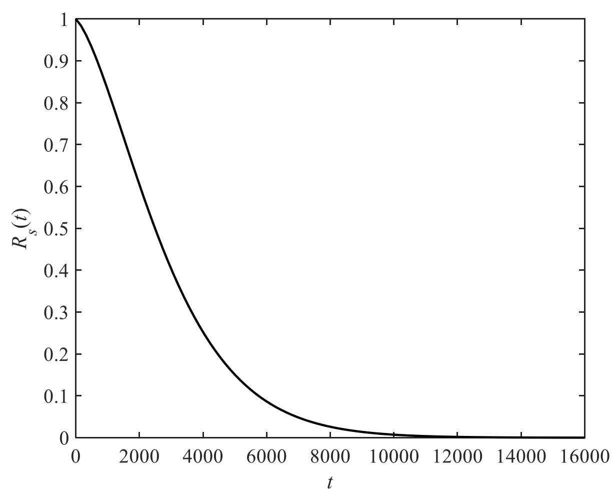


Fig9. Reliability of the system

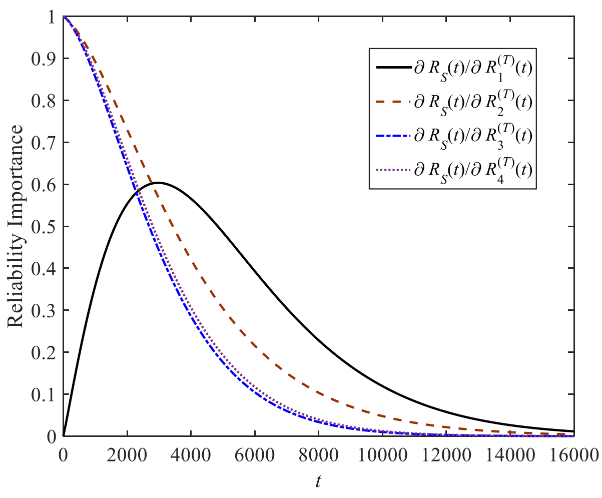


Fig10. Reliability importance of the system

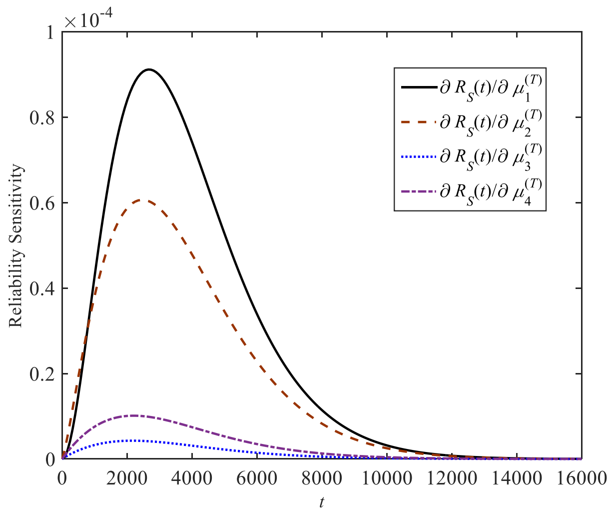
The survival signature of the wind turbine is summarized in Table 3. For the sake of brevity, rows with $\varphi_s(l_1,l_2,l_3,l_4)$=0 are omitted. Based on survival signature, the reliability, reliability importance, and reliability sensitivity of the system for each point at time t can be obtained from Eqs. (6)-(8). The results are shown in

Figs. 9-12.

Table 3 Survival signature of the wind turbine; rows with $\varphi_s(l_1,l_2,l_3,l_4)$=0 are omitted

l_1	L_2	l_3	l_4	$\varphi_s(l_1,l_2,l_3,l_4,)$
1	1	1	1	1
2	1	1	1	1

According to the results of reliability importance analysis, the most significant component is the main shaft while 0<t<2800, and the bearings are the most significant components while t>2800. Results of reliability sensitivity analysis indicate that the bearings and main shaft are the most significant components. A comparison of sensitivities with respect to means reveals that the influence of the small changes in the mean values on the reliability of the wind turbine is decline in the order, $\partial R_s(t)/\partial\mu_1 > \partial R_s(t)/\partial\mu_2 > \partial R_s(t)/\partial\mu_4 > \partial R_s(t)/\partial\mu_3$ for most of time. Based on the sensitivities with respect to standard deviations, the uncertainties in the life course of the main shaft and bearings have more influence on the reliability of the system than those in others. Reliability sensitivities of the wind turbine change rapidly as the time passes by. With the help of the information of reliability sensitivity, engineers could design different maintenance strategies at distinct stages to reduce the risk to the lower extent.



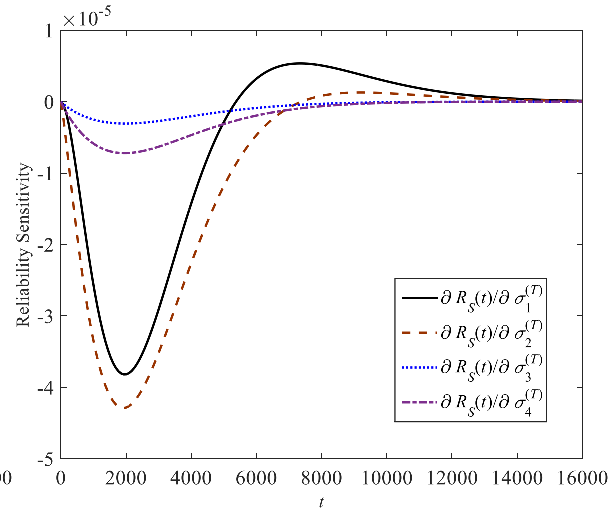


Fig11. Reliability sensitivity with respect to mean **Fig12. Reliability sensitivity with respect to standard deviation**

Conclusion

The reliability sensitivity quantifies the degree of the influence of the components and their distribution parameters on the reliability of the system. For example, the higher the value of reliability sensitivity the greater is the influence of the parameter on the system reliability at a given time t, and vice versa. At each point in time, the largest reliability sensitivity show the most "critical" factor, thus helping the allocation of resources for inspection, maintenance and repair activities in an optimal manner over the lifetime of a system.

In this paper, an efficient approach for reliability important analysis of coherent systems is proposed based on survival signature. Survival signature has been proven to be an efficient method for estimating the reliability of systems with multiple component types[33-36]. Reliability analysis of a system using survival signature could separate the system structure from the component probabilistic failure distribution. Therefore, calculation of all the cut-sets, which is a cumbersome and error prone task can be bypassed. Therefore, the proposed approach is easy to be implemented in practice and has high computational efficiency.

Reliability importance measures have not considered the probabilistic characteristics of components while ranking them. Along with reliability importance measure, this paper studies the method for reliability sensitivity analysis of coherent systems with respect to distribution parameters of components. The information of reliability sensitivity can be used to rank distribution parameters of components with respect to their impacts on the system's reliability. Therefore, reliability sensitivity analysis is often critical towards understanding the industrial systems underlying failure and provides more useful information for reliability improvement and risk reduction.

It should be noted that the reliability sensitivity discussed in this paper is proposed based on derivatives. Sensitivity obtained in this study is local sensitivity which is valid when the parameter is changed by a small amount. If one has the opportunity to improve reliability of components by changing the parameters more than just a small amount, then of course the picture may be very different. Moreover, costs incurred by maintaining a system and its components is not consider in this paper. In practice there may perhaps be some indication that some improvement of a specific parameter may be possible at a specific cost. And cost-based global reliability sensitivity analysis of industry systems is the subject of current research by the authors. In general, however, this paper presents a practical method for reliability sensitivity analysis of coherent systems using survival signature.

Declaration of Conflicting Interests

The authors declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

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Appendix

1. The Exponential distribution

The exponential distribution is defined by a constant failure rate. The CDF of an exponential random variable *T* is given by

$$F(t\,|\,a) = 1 - \exp\left(-\frac{t}{a}\right), \quad t, a > 0 \tag{A.1}$$

The partial derivative of the CDF with respect to *a* is

$$\frac{\partial F(t)}{\partial a} = -\frac{t}{a^2}\exp\left(-\frac{t}{a}\right) \tag{A.2}$$

The mean and standard deviation of *T* are *a.* Therefore, The partial derivatives of the CDF with respect to the mean and standard deviation of *T*, $\partial F(t)/\partial\mu$ and $\partial F(t)/\partial\sigma$, are equal to $\partial F(t)/\partial a$.

2. The Weibull distribution

The Weibull distribution is by far the most popular life distribution used in reliability engineering due to its variety of shapes and generalization or approximation of many other distributions. The CDF of a Weibull distribution random variable *T* is

$$F(t\,|\,a, b) = 1 - \exp\left[-\left(\frac{t}{a}\right)^b\right], \quad t, a, b > 0 \tag{A.3}$$

where *a* is the scale parameter and *b* is the shape parameter.

The partial derivatives of the CDF with respect to *a* and *b* as

$$\frac{\partial F(t)}{\partial a} = -\frac{b}{a}\exp\left[-\left(\frac{t}{a}\right)^b\right]\left(\frac{t}{a}\right)^b \tag{A.4}$$

$$\frac{\partial F(t)}{\partial b} = \ln\left(\frac{t}{a}\right)\exp\left(-\left(\frac{t}{a}\right)^b\right)\left(\frac{t}{a}\right)^b \tag{A.5}$$

The mean and standard deviation of *T* are

$$\begin{cases} \mu = a\Gamma\left(\dfrac{1}{b}+1\right) \\ \sigma = a\sqrt{\Gamma\left(\dfrac{2}{b}+1\right) - \Gamma^2\left(\dfrac{1}{b}+1\right)} \end{cases} \tag{A.6}$$

It is noted that $\Gamma(\cdot)$ is the gamma function.

The Jacobian matrix of the mapping from *a* and *b* to *μ* and *σ* is

$$\mathbf{J} = \begin{bmatrix} \partial\mu/\partial a & \partial\mu/\partial b \\ \partial\sigma/\partial a & \partial\sigma/\partial b \end{bmatrix} = \begin{bmatrix} \Gamma(1/b+1) & -a/b^2\Gamma(1/b+1)\psi(1/b+1) \\ \dfrac{\sigma}{a} & \dfrac{\Gamma^2(1/b+1)\psi(1/b+1) - a^2\Gamma(2/b+1)\psi(2/b+1)}{b^2\sigma} \end{bmatrix} \tag{A.7}$$

where $\psi(\cdot) = \Gamma'(\cdot)/\Gamma(\cdot)$ is the psi function.

$\partial F(t)/\partial\mu$ and $\partial F(t)/\partial\sigma$, are calculated as

$$\begin{cases} \dfrac{\partial F(t)}{\partial \mu} = \dfrac{\partial F(t)}{\partial a}\dfrac{\partial a}{\partial \mu} + \dfrac{\partial F(t)}{\partial b}\dfrac{\partial b}{\partial \mu} \\ \dfrac{\partial F(t)}{\partial \sigma} = \dfrac{\partial F(t)}{\partial a}\dfrac{\partial a}{\partial \sigma} + \dfrac{\partial F(t)}{\partial b}\dfrac{\partial b}{\partial \sigma} \end{cases} \tag{A.8}$$

The partial derivatives of the CDF with respect to the mean and standard deviation, $\partial F(t)/\partial \mu$ and $\partial F(t)/\partial \sigma$, can be calculated as

$$\begin{bmatrix} \dfrac{\partial F(t)}{\partial \mu} \\ \dfrac{\partial F(t)}{\partial \sigma} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial F(t)}{\partial a} & \dfrac{\partial F(t)}{\partial b} \end{bmatrix} \begin{bmatrix} \dfrac{\partial a}{\partial \mu} & \dfrac{\partial a}{\partial \sigma} \\ \dfrac{\partial b}{\partial \mu} & \dfrac{\partial b}{\partial \sigma} \end{bmatrix} = \begin{bmatrix} \dfrac{\partial F(t)}{\partial a} & \dfrac{\partial F(t)}{\partial b} \end{bmatrix} \boldsymbol{J}^{-1} \tag{A.9}$$

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