Bidispersive double diffusive convection

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Abstract

A model is developed for double diffusive convection in a bidisperse porous medium. Double diffusive convection is convective movement of fluid due to temperature and salt gradient effects. A bisdisperse porous medium is one where there are pores known as macropores, but the solid skeleton contains cracks or fissures which give rise to a porosity in the skeleton, known as microporosity. We concentrate on the case of a single temperature field and attenuation is focussed on the situation where the layer is heated from below and simultaneously heated from above.

1 Introduction

There is much current interest in the behaviour of double porosity, or bidispersive, porous materials. A double porosity material is one which possesses the normal pore structure, such pores being known as macropores, but, the solid skeleton has cracks or fissures in it and this gives rise to a second porosity, the smaller pores being refered to as micropores. Heat and mass transfer in a bisdispersive porous material has been of interest in the chemical engineering field for some time now, see e.g. Burghardt *et al.* [1], Szczygiel [2, 3, 4], Valus & Schneider [5]. The interest in heat and mass transfer in a bidispersive porous material is driven by the many real practical applications of these phenomena, for example, to landslides, see e.g. Montrasio *et al.* [6], Borja *et al.* [7], Borja & White [8], Pooley [9] and Scotto di Santolo & Evangelista [10]. Another important research area for heat and mass transfer in a bidisperse porous medium is in biporous wicks in heat pipes, see e.g. Taqvi *et al.* [11], Lin *et al.* [12], Mottet & Prat [13], Yeh *et al.* [14]. A further mundane area involves stockpiling coal, Hooman & Maas [15], Hooman *et al.* [16]. Relatively small pieces of coal are stockpiled but the coal itself contains small pores. The porosity of the stockpile is usually higher than that in the solid coal, and the fact that these piles can self heat makes an analysis and understanding of heat transfer vital to prevent self combustion. There are many other applications of bidispersive porous media and lots of these are mentioned in the monograph by Straughan [17].

Thermosolutal porous convection, also known as double diffusive porous convection, involves fluid movement in a porous medium in a non-isothermal situation where there is a salt dissolved in the fluid. In porous media this convection for a single porosity medium was analysed in the fundamental article of Nield [18]. Many subsequent articles have appeared dealing with linear instability, but also clever analyses of nonlinear stability, see e.g. Joseph [19, 20], Barletta & Nield [21], Mulone [22], Love *et al.* [23], Simmons *et al.* [24], Deepika & Narayana [25], Deepika [26], Straughan [27]. The nonlinear stability aspect of thermosolutal convection in a porous medium from an energy method viewponit is the focus of attention of Lombardo *et al.* [28], with further information being included in the books by Straughan [29, 30].

Fundamental theories for thermal convection in a bidisperse porous medium were developed and analysed by Nield and Kuznetsov [31, 32, 33, 34, 35] and by Nield [36]. This work is reviewed in Straughan [30]. Falsaperla *et al.* [37] and Gentile & Straughan [38] continued analysis with the Nield-Kuznetsov models but they restrict attention to the case where only one temperature is present whereas Nield and Kuznetsov [32] allowed for different temperatures in the fluid in the macro and micro pores. Further work using the single temperature model has been given by Gentile & Straughan [39] and by Franchi *et al.* [40].

The aim of this paper is to present a model for fluid flow in a bidisperse porous medium which allows for thermosolutal convection, i.e. for convective motion incorporating temperature and salt effects. We employ a single temperature and we analyse in detail the problem of determining the onset of convective movement when the layer is heated from below. Equations are derived which allow for the layer to be salted below or above, although we concentrate on the more interesting, and more complicated, case of salted from below.

We also incorporate the Soret effect, see e.g. Soret [41], Platten [42], Straughan [30], p. 40. The effect of a temperature gradient on mass transfer in a bidisperse porous medium may well be of much interest. We note that in the case of a single porosity body the problem of thermosolutal convection in a porous medium taking into account the Soret effect has been the topic of the recent article by Deepika [26]. The analysis of Deepika [26] effectively concentrates on the heated below - salted above case and so there is no overlap with the present work, which is also as far as we are aware the first analysis of double diffusive convection in a bidispersive porous medium.

2 Governing equations

Let ϕ be the porosity associated to the macropores, so that ϕ is the ratio of the volume of the macropores to the total volume of the saturated porous material. Let ϵ be the porosity associated to the micropores, i.e. ϵ is the ratio of the volume occupied by the micropores to the volume of the porous body which remains once the macropores are removed. Thus the fraction of volume occupied by the micropores is $\epsilon(1 - \phi)$.

We follow Nield and Kuznetsov [32] and use sub or superscript f and p to

denote a quantity associated to the macropores or micropores, respectively. Let V_i^f and V_i^p be the pore averaged velocities in the macro and micropores. Then the analogous seepage velocities U_i^f and U_i^p are given by

$$U_i^f = \phi V_i^f, \qquad U_i^p = \epsilon(1-\phi)V_i^p.$$

We assume the density in the buoyancy force is a linear function of the temperature, T, and salt concentration, C, and so

$$\rho = \rho_0\big[1 - \alpha(T - T_0) + \alpha_C(C - C_0)\big]$$

where α is the thermal expansion coefficient and α_C is the equivalent expression for the salt field. Then, employing a Boussinesq approximation the momentum and continuity equations in the macropores and micropores are derived as in Nield and Kuznetsov [32], Falsaperla $et\ al.$ [37] or Gentile & Straughan [38], and are

$$
\begin{aligned}
0 &= -\frac{\mu}{K_f}U_i^f - p_{,i}^f - \zeta(U_i^f - U_i^p) + g\rho_0\alpha k_i T - \alpha_C\rho_0 g k_i C, \\
U_{i,i}^f &= 0, \\
0 &= -\frac{\mu}{K_p}U_i^p - p_{,i}^p - \zeta(U_i^p - U_i^f) + g\rho_0\alpha k_i T - \alpha_C\rho_0 g k_i C, \\
U_{i,i}^p &= 0,
\end{aligned}
\tag{1}
$$

where μ is the dynamic viscosity of the fluid and ζ is an interaction coefficient, namely the coefficient for momentum transfer between the macro and micro phases, see Nield & Kuznetsov [32]. The quantities p^f and p^p are the pressures in the macro and micro phases, $\mathbf{k} = (0,0,1)$ and g is the size of the gravity vector. The terms K_f and K_p are the permeabilities in the macro and micro phases. Throughout we employ standard indicial notation.

The equation governing the energy balance, i.e. the equation for the temperature field is derived from the equations of Nield & Kuznetsov [32] as in Gentile & Straughan [38] and has the form

$$(\rho c)_m T_{,t} + (\rho c)_f (U_i^f + U_i^p)T_{,i} = \kappa_m \Delta T, \tag{2}$$

where ρ is the density, c is the specific heat at constant pressure, f denotes the macro phase and

$$
\begin{aligned}
(\rho c)_m &= (1-\epsilon)(1-\phi)(\rho c)_s + \big[\phi + \epsilon(1-\phi)\big](\rho c)_f\,, \\
\kappa_m &= (1-\epsilon)(1-\phi)\kappa_s + \big[\phi + \epsilon(1-\phi)\big]\kappa_f\,,
\end{aligned}
$$

where $(\rho c)_s, (\rho c)_f, \kappa_f$ and κ_s are the products of density and specific heat in the solid skeleton and in the fluid in the pores, respectively, and the thermal conductivity of the fluid and solid, respectively.

We need to derive and equation for the concentration. To this end we note that when the diffusion coefficient includes the Soret effect it has form

$$\mathbf{J}_C = -k_C \nabla C - k_C S_T \nabla T$$

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where $\mathbf{J}_C$ is the diffusion coefficient and S_T is a Soret coefficient. We thus write differential equations for the macro and micro phases as

$$\frac{\partial C}{\partial t} + V_i^f \frac{\partial C}{\partial x_i} = k_C^f \Delta C + k_C^f S_T^f \Delta T, \tag{3}$$

and

$$\frac{\partial C}{\partial t} + V_i^p \frac{\partial C}{\partial x_i} = k_C^p \Delta C + k_C^p S_T^p \Delta T. \tag{4}$$

Multiply (3) by ϕ and (4) by $\epsilon(1-\phi)$ and add the results. In this way we derive the following equation for the concentration field throughout the porous medium continuum,

$$\epsilon_1 \frac{\partial C}{\partial t} + (U_i^f + U_i^p)\frac{\partial C}{\partial x_i} = \epsilon_2 \Delta C + S \Delta T, \tag{5}$$

where

$$\epsilon_1 = \phi + \epsilon(1-\phi), \quad \epsilon_2 = \phi k_C^f + \epsilon(1-\phi)k_C^p, \quad S = \phi S_T^f + \epsilon(1-\phi)S_T^p.$$

Thus, the governing system of equations for double diffusion in a bidisperse porous medium consist of (1), (2) and (5) for the variables $U_i^f, U_i^p, p^f, p^p, T$ and C.

3 Basic solution and perturbation equations

We now investigate the problem of thermosolutal convection in a plane layer of bidispersive material. Let the saturated porous material occupy the horizontal layer $0 < z < d$, $\{(x,y) \in \mathbb{R}^2\}$ and satisfy equations (1), (2) and (5). The boundary conditions are,

$$\begin{aligned}
&U_i^f n_i = 0,\ U_i^p n_i = 0, \quad \text{on } z = 0, d, \\
&T = T_L,\ z = 0, \quad T = T_U,\ z = d, \\
&C = C_L,\ z = 0, \quad C = C_U,\ z = d,
\end{aligned} \tag{6}$$

where n_i is the unit outward normal to the planes $z = 0$ and $z = d$, T_L, T_U, C_L, C_U are constants with $T_L > T_U$. We derive the perturbation equations from the steady solution under the boundary conditions

$$C_L > C_U, \qquad \text{salted below}, \tag{7}$$

or

$$C_L < C_U, \qquad \text{salted above}. \tag{8}$$

The basic conduction solution is then

$$\bar{U}_i^f \equiv 0, \quad \bar{U}_i^p \equiv 0, \quad \bar{T} = T_L - \beta z, \quad \bar{C} = C_L - \beta_C z, \tag{9}$$

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where
$$\beta = \frac{T_L - T_U}{d} > 0\,, \qquad \beta_C = \frac{C_L - C_U}{d}\,. \tag{10}$$

When the layer is salted below $\beta_C > 0$ whereas when the layer is salted above $\beta_C < 0$.

Let $u_i^f, u_i^p, \pi^f, \pi^p, \theta, \gamma$ be a perturbation to the steady solution (9). The perturbations are non-dimensionalized with length scale D, time scale $\mathcal{T}$, velocity scale U, temperature scale $T^\sharp$, concentration scale $C^\sharp$, where

$$\mathcal{T} = \frac{(\rho c)_m d^2}{\kappa_m}\,, \qquad U = \frac{\kappa_m}{(\rho c)_f d}\,,$$
$$T^\sharp = \frac{\beta U (\rho c)_f d^2}{\kappa_m}\,, \qquad C^\sharp = \frac{|\beta_C| U d^2}{\epsilon_2}\,.$$

Define the quantities ξ, K_r, A and the Lewis number Le by

$$\xi = \frac{\zeta K_f}{\mu}\,, \qquad K_r = \frac{K^f}{K^p}\,, \qquad A = \frac{(\rho c)_m}{(\rho c)_f}\,, \qquad Le = \frac{\kappa_m}{(\rho c)_m \epsilon_2}\,.$$

Define the Rayleigh number, R, and the concentration Rayleigh number, $\mathcal{C}$, by

$$R = \frac{\rho_0 \alpha g \beta d^2 K_f}{\mu [\kappa_m/(\rho c)_f]}\,, \qquad \mathcal{C} = \frac{\rho_0 \alpha_C g |\beta_C| d^2 K_f}{\mu \epsilon_2}\,, \tag{11}$$

and let $\mathcal{S}$ be a non-dimensional Soret number given by

$$\mathcal{S} = \frac{S T^\sharp}{\epsilon_2 C^\sharp}\,.$$

The non-dimensional perturbation equations which arise from (1), (2) and (5) are then

$$
\begin{aligned}
&- u_i^f - \xi(u_i^f - u_i^p) - \pi_{,i}^f + R\theta k_i - \mathcal{C}\gamma k_i = 0\,,\\
&u_{i,i}^f = 0\,,\\
&- K^r u_i^p - \xi(u_i^p - u_i^f) - \pi_{,i}^p + R\theta k_i - \mathcal{C}\gamma k_i = 0\,,\\
&u_{i,i}^p = 0\,,\\
&\theta_{,t} + (u_i^f + u_i^p)\theta_{,i} = w^f + w^p + \Delta\theta\,,\\
&\epsilon_1 Le \gamma_{,t} + A Le(u_i^f + u_i^p)\gamma_{,i} = \pm(w^f + w^p) + \Delta\gamma + \mathcal{S}\Delta\theta\,.
\end{aligned} \tag{12}
$$

Equations (12) hold in the domain $\{(x,y) \in \mathbb{R}^2\} \times \{z \in (0,1)\} \times \{t > 0\}$ and with $\mathbf{u}^f = (u^f, v^f, w^f)$, $\mathbf{u}^p = (u^p, v^p, w^p)$. The boundary conditions become

$$w^f = 0,\ w^p = 0,\ \theta = 0,\ \gamma = 0, \qquad z = 0, d, \tag{13}$$

and the perturbation solution satisfies a plane tiling planform in the horizontal directions with wavenumber a.

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For comparison purpose later, we recall that the analogous system of equations to (12) but for a single porosity material have the form, cf. Straughan [29], p. 169,

$$
\begin{aligned}
& u_i = -\pi_{,i} + R\theta k_i - \mathcal{C}\phi k_i, \\
& u_{i,i} = 0, \\
& \theta_{,t} + u_i\theta_{,i} = \Delta\theta + w, \\
& \phi Le\gamma_{,t} + Leu_i\gamma_{,i} = \pm w + \Delta\gamma + \mathcal{S}\Delta\theta.
\end{aligned}
\tag{14}
$$

4 Instability

From this point we concentrate on the salted below case for which we take the positive sign in $(12)_6$. We study the linear instability of (12), (13). By following the classical method in Chandrasekhar [43] we take curl curl of $(12)_1$, $(12)_3$, and retain the w^f and w^p components of the results. Equations $(12)_5$ and $(12)_6$ are linearized and then a time dependence like $e^{\sigma t}$ is requested. This results in having to solve the eigenvalue problem for the boundary conditions (13) together with the equations

$$
\begin{aligned}
& (1+\xi)\Delta w^f - \xi\Delta w^p - R\Delta^*\theta + \mathcal{C}\Delta^*\gamma = 0, \\
& (K^r+\xi)\Delta w^p - \xi\Delta w^f - R\Delta^*\theta + \mathcal{C}\Delta^*\gamma = 0, \\
& \sigma\theta = w^f + w^p + \Delta\theta, \\
& \epsilon_1 Le\sigma\gamma = w^f + w^p + \Delta\gamma + \mathcal{S}\Delta\theta,
\end{aligned}
\tag{15}
$$

where $\Delta^* = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the horizontal Laplacian.

The instability curves are found by resolving (15) for R and minimizing in the wavenumber a. These kind of calculations are well known and we omit details. It is found that the instability boundary for stationary convection ($\sigma = 0$) is found as

$$
4\pi^2 B + \mathcal{C}(1-\mathcal{S}) = R,
\tag{16}
$$

whereas the instability boundary for oscillatory convection ($\sigma = i\sigma_1$, $\sigma_1 \in \mathbb{R}$) is

$$
4\pi^2 B\left(1 + \frac{1}{\epsilon_1 Le}\right) + \frac{\mathcal{C}}{\epsilon_1 Le} = R,
\tag{17}
$$

where the constant B is given by

$$
B = \frac{1 + \xi + \xi K_r^{-1}}{1 + 4\xi K_r^{-1} + K_r^{-1}}.
\tag{18}
$$

5 Conclusions

For most practical values $\epsilon_1 Le > 1$ and curves (16) and (17) are straight lines in the $(\mathcal{C}, R)$ plane, cf. figure 69.1 in Joseph [20], p. 46, or figure 2 in Straughan [44], p. 1615, where the single porosity case with porosity ϕ is described. In

the present case the straight lines begin on the $R-$axis at $4\pi^2 B$ and $4\pi^2 B[1 + (\epsilon_1 Le)^{-1}]$, respectively. The stationary convection curve has slope $1-\mathcal{S}$ whereas the oscillatory one has slope $1/\epsilon_1 Le$. Thus, if $1 - \mathcal{S} > 1/\epsilon_1 Le$ the lines cross as in figure 2 of Straughan [44]. Stationary convection will occur for $\mathcal{C}$ below the intersection point and oscillatory convection thereafter, for increasing $\mathcal{C}$.

The analogous lines to (16) and (17) in the single porosity case (arising from equations (14)) are

$$4\pi^2 + \mathcal{C}(1 - \mathcal{S}) = R, \tag{19}$$

and

$$4\pi^2\left(1 + \frac{1}{\phi Le}\right) + \frac{\mathcal{C}}{\phi Le} = R. \tag{20}$$

We observe that (16) and (17) reduce to (19) and (20) as the interaction parameter $\xi \to 0$ and $K^r = K^f/K^p \to \infty$. Thus, the curves for the bidispersive problem are consistent with those for the single porosity case.

While the curves for the bidispersive case display the same qualitative behaviour as those for the single porosity case the quantitative behaviour is very much influenced by the parameters ξ, K_r, ϵ_1 and Le (which is different in the bidispersive situation to the single porosity one). In particular

$$\frac{\partial B}{\partial \xi} \propto \left(1 - \frac{1}{K_r}\right)^2 > 0$$

and so for fixed K_r, the value of B increases as ξ increases. However, $\partial B/\partial K_r^{-1} < 0$ and so B decreases with increasing K_r^{-1}. Effectively, this means that for small interaction parameter ξ and $K_r > 1$ (which is likely) then $B < 1$ and the convection threshold in the bidispersive case is less than that in the single porosity case. On the other hand, if the interaction parameter ξ is sufficiently large then B could exceed the value 1. It is thus very important to have measurements of the parameters K^f and K^p and of the coefficient for momentum transfer parameter ζ.

To consider some practical values we take water as the saturating fluid and sand as the material for the porous skeleton. From Jones *et al.* [45] we find the value for the diffusion coefficient of NaCl in water to be $k_C = 2.66 \times 10^{-9}$ m^2 s^{-1}. The values of thermal conductivity, specific heat at constant pressure, and density are found from Engineeringtoolbox on the internet and we find the thermal conductivity, specific heat at constant pressure, and density for water and sand may be taken to be, respectively, 0.606, 0.25 W/m K, 4187, 830 J/kg C, and 998, 1922 kg m^{-3}. In the single porosity situation we take ϕ to have value 0.3, and we adopt the same value for the macro porosity ϕ in the bidispersive case. The micro porosity we take as 0.2. This gives rise to values of, for the single porosity case,

$$\phi Le = 9.63$$

whereas for the bidispersive case

$$\epsilon_1 Le = 55.924.$$

For the single porosity case the instability is stationary convection for $0 \leq \mathcal{C} \leq \mathcal{C}^* = 4\pi^2/\phi Le(1 - \mathcal{S} - 1/\phi Le)$ with the corresponding R^* value being $R^* = 4\pi^2 + 4\pi^2(1 - \mathcal{S})/\phi Le(1 - \mathcal{S} - 1/\phi Le)$. For $\mathcal{C} > \mathcal{C}^*$ instability is by oscillatory convection. With $\mathcal{S} = 0$ and $\phi Le = 9.63$ we find $\mathcal{C}^* = 4.5744$, $R^* = 44.0529$. Thus for $0 \leq \mathcal{C} \leq \mathcal{C}^*$ the instability boundary is a straight line connecting $(\mathcal{C}, R) = (0, 4\pi^2)$ to $(\mathcal{C}^*, R^*)$ and instability is by stationary convection. For $\mathcal{C} > \mathcal{C}^*$ the instability is by oscillatory convection and the curve is a straight line emanating from $(\mathcal{C}^*, R^*)$ with slope $1/\phi Le = 0.10384$.

The qualitaive shape for the bidisperse case is the same but $\mathcal{C}^*$ and R^* change. Also, the convection thresholds are governed by the parameter B given by (18). The constant B depends on the interactioon parameter ξ and $K_r = K_f/K_p$. For example, if $\xi = 0.1$ and $K_r = 5$ then $B = 0.875$ while for $\xi = 0.1$ and $K_r = 1.5$ one finds $B = 0.269$. For the latter value of B one finds the instability curves are

$$0.269 \times 4\pi^2 + \mathcal{C}(1 - \mathcal{S}) = R, \text{ stationary,}$$

and

$$0.269 \times 4\pi^2\left(1 + \frac{1}{9.63}\right) + \frac{\mathcal{C}}{9.63} = R, \text{ oscillatory.}$$

These may be rewritten as

$$10.6197 + \mathcal{C}(1 - \mathcal{S}) = R, \text{ stationary,}$$

and

$$11.7225 + 0.10384\mathcal{C} = R, \text{ oscillatory,}$$

respectively. Comparing this to the single porosity case we see that the instability boundary is much lower in R and the oscillatory convection curve begins at a lower value of $\mathcal{C}^*$. However, we stress that this is for $\xi = 0.1$ and $K_r = 1.5$. For other values it may be very different. We deduce that a bidisperse porous medium may yield very different instability values for double diffusive convection than those predicted using a single porosity model. There is a need for measurements of the double porosity parameters K_r and ξ.

We conclude with some remarks on nonlinear stability. If we consider the heated below - salted above case then we take the negative sign in $(12)_6$. When $\mathcal{S} = 0$ it is straightforward to show that (12) generates a symmetric linear operator and the nonlinear stability boundary is the same as the linear instability one. When $\mathcal{S} \neq 0$ this is not true and then one may proceed in an analogous manner to Deepika [26] but using equations (12). For the heated below - salted below case, one adopts the positive sign in $(12)_6$, then a nonlinear stability analysis which captures the characteristics of the linear instability threshold does not appear to be a trivial problem. It is not obvious how one could use Joseph's [19, 20] method of generalized energy since the coefficients of $\gamma_{,t}$ and of $u_i\gamma_{,i}$ in $(12)_6$ are different, unlike in the pure fluid case. This difficulty was also encountered in the analogous double diffusive problem for a single porosity medium by Lombardo *et al.* [28]. One may evidently use the generalized energy

technique of Lombardo *et al.* [28] to achieve a sharper nonlinear energy stability threshold than by simply employing the obvious L^2 theory, but at the expense of the stability being conditional upon the size of the initial data.

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