A heuristic survival signature based approach for reliability-redundancy allocation

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Abstract

In recent research, the major focus on reliability-redundancy allocation problems has been on the possibility of using more efficient and effective algorithms to improve convergence speed and solution accuracy of the optimization model. But the model of reliability-redundancy allocation itself has not been investigated further. In this paper, we try to simplify the optimization model of the reliability-redundancy allocation problem by using the theory of survival signature. To achieve this, the information of the structure of a system is summarized by the survival signature. The reliability-redundancy allocation problem is formulated as an optimization problem with the objective of maximizing system reliability under some constraints. A new adaptive penalty function is proposed to transfer the constraint optimization problem to an unconstraint one. Then a heuristic algorithm called stochastic fractal search is applied to solve the unconstraint optimization. Moreover, the (joint) structure importance is used to measure the relative importance of components to concretely allocate the redundancy level of each component. The proposed method only needs to calculate the survival signature once, reduces the dimension of the optimization problem and provides insight into system reliability-redundancy allocation.

Keywords: System reliability, Survival signature, Reliability optimization, Redundancy allocation.

1. Introduction

Rapid progress in science and technology in recent years has made today's engineering systems more and more powerful and complex. However, uncertainty and risk of system failure in high-tech industrial processes have increased due to high operating speeds, large loads and severe working conditions. Furthermore, system failures (e.g. breakdown of a nuclear power plant, malfunction of an air traffic control system or miscommunication in internet systems) are resulting in greater economic losses and more significant effects on society than ever before. As a consequence, it becomes even more important to perform

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system reliability optimization in order to ensure safe and reliable operation of the system to get a good balance between costs and risks in engineering practice.

Generally, there are two approaches that can be used to optimize system reliability. The first way is increasing component reliability (reliability allocation), the second way is using redundant components in parallel (redundancy allocation). Unfortunately, these two approaches do not always yield competitive results. For example, reliability allocation may incur large costs to improve the system reliability only a little because of difficulties in design, verification, and production. As for redundancy allocation, it not only increases costs, but also adds undesirable extra volume and weight to the system. To overcome these problems, the reliability-redundancy allocation problem (RRAP) has been considered [1, 2].

The RRAP is usually formulated as a non-linear optimization problem, which determines the reliabilities and the redundancy levels of components to maximize system reliability under design constraints on, for example cost, volume, or weight. RRAP presents a powerful and attractive method for system reliability optimization, however at the same time, it is known as one of the most challenging problems in the area of reliability optimization due to its high dimension and complexity. Numerous techniques, especially intelligent optimization algorithms, have been suggested to solve the optimization model arising in RRAP in recent years. For example, artificial bee colony algorithms [3–5], cuckoo search algorithms [6, 7], article swarm optimization methods [8–11], genetic algorithms [2, 12–14] and simulation optimization methods [15] were subsequently reported in the literature. In addition, Ha and Kuo [16] presented a branch-and-bound approach to solve RRAP based on a search space elimination of disjoint sets in a solution space. Caserta and Voß [17] proposed a new solution approach by transforming RRAP into a multiple-choice knapsack problem. Muhuri et al. [18] proposed a novel formulation of RRAP with fuzzy uncertainty. In [19–21], cold-standby strategies for redundant components are used to model the RRAP. Chatwattanasiri et al. [22] studied RRAP with uncertain stress-based component reliability. Feizabadi and Jahromi [23] proposed a new model for reliability optimization of series-parallel systems with nonhomogeneous components.

As is clear from the literature, the major focus of recent research has been on the development of more efficient and effective algorithms for solving the constraint optimization problem for RRAPs. However, the model of RRAP itself has not been simplified or improved further. The aim of the present work is to develop a new and efficient approach for RRAP using the survival signature [24, 25]. The rest of this paper is organized as follows. Section 2 gives a brief description of the theory of survival signature for reliability analysis of systems with redundant components in parallel, followed by the formulation of a constraint optimization model for RRAP in Section 3. In Section 4, a new adaptive penalty function is proposed to transfer the constraint optimization to an unconstraint one and stochastic fractal search (SFS) is applied to solve the unconstraint optimization problem. The (joint) structural importance is proposed to concretely allocate the redundancy level of each component in Section 5 and the validity and effectiveness of the proposed approaches are illustrated in Section 6 by two numerical examples. Finally, Section 7 presents the conclusions of the paper and some ideas for related future work.

2. System reliability analysis using the survival signature

Consider a system with $K \geq 2$ types of components, with m_k components of type k, for $k = 1, 2, \ldots, K$, and $\sum_{k=1}^{K} m_k = m$. Assume that the random failure times of components of different types are fully independent, while components of the same type have exchangeable failure times. The probability that the system functions given that precisely $l_k \in \{0, 1, \ldots, m_k\}$ of its type k components function, for $k = 1, 2, \ldots, K$, is [24, 25]:

$$\Phi(l_1, l_2, \ldots, l_K) = \left[\prod_{k=1}^{K} \binom{m_k}{l_k}^{-1} \right] \times \sum_{\mathbf{X} \in S} \phi(\mathbf{X}) \tag{1}$$

where vector $\mathbf{X} = [X_1, \ldots, X_m]^T$ represents the states of all components and $\phi(\mathbf{X})$ is called the structure function of the system; S denotes the set of all possible state vectors for which precisely l_k components of type k function; $\Phi(l_1, l_2, \ldots, l_K)$ is called survival signature.

The probability that the system functions at time t is

$$R_s(r_1, \ldots, r_K) = Pr(T_s > t) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_K=0}^{m_K} \left[\Phi(l_1, \ldots, l_K) \prod_{k=1}^{K} \binom{m_k}{l_k} F_k^{m_k - l_k}(t)(1 - F_k(t))^{l_k} \right] \tag{2}$$

where $r_k = 1 - F_k(t)$ is the reliability of components of type k at time t for $k = 1, 2, \ldots, K$; $F_k(t)$ is the cumulative distribution function for the failure time of components of type k.

Reliability of a system can be significantly improved by adding the same type of components as redundancy to each type of components. Therefore, redundancy allocation is a direct way of enhancing system reliability. In this case, the system reliability is

$$R_s(r_1, \ldots, r_K, n_1, \ldots, n_K) = \sum_{l_1=0}^{m_1} \cdots \sum_{l_K=0}^{m_K} \left[\Phi(l_1, \ldots, l_K) \prod_{k=1}^{K} \binom{m_k}{l_k} F_k^{n_k(m_k - l_k)}(t)(1 - F_k^{n_k}(t))^{l_k} \right] \tag{3}$$

where n_k is the redundancy level of components of type k. For example, if we add a parallel compnent to each component of type k, the parallel level of type k components would become $n_k = 2$.

From Equation (3), we see that the survival signature does not change with redundancy of components if the same level of redundancy is applied to all the components of the same type; this is a key aspect of the two-step heuristic procedure presented in this paper. The survival signature only needs to be calculated once which makes it efficient and easy to implement in RRAP.

3. Optimization model for RRAP

The aim of RRAP is to maximize the reliability of a system during its life cycle $t \in [0,\ T]$, where T is assumed to be fixed, through component reliability improvement and component redundancy under constraints of component cost, weight, volume, etc. Since the reliability of a system is decreasing as time t increases to T, the optimization model of the RRAP can be represented as follows:

$$
\begin{aligned}
&\min f(\mathbf{u}) = -R_s(r_1, \ldots, r_K, n_1, \ldots, n_K) \\
&\text{s.t. } g_1(\mathbf{u}) = C_s(r_1, \ldots, r_K, n_1, \ldots, n_K) - C \leq 0 \\
&\quad g_2(\mathbf{u}) = V_s(n_1, \ldots, n_K) - V \leq 0 \\
&\quad g_3(\mathbf{u}) = W_s(n_1, \ldots, n_K) - W \leq 0 \\
&\quad\quad 1 \leq n_k \leq n_{k,\max},\ k = 1, 2, \ldots, K
\end{aligned} \tag{4}
$$

where $\mathbf{u} = [r_1, \ldots, r_K, n_1, \ldots, n_K]^T$ is the vector of design variables; K is the number of types of components in the system; r_k is the reliability of components of type k at time $t = T$; n_k is the number of redundant components of type k in parallel; $n_{k,max}$ is the maximum number of components of type k which can be in parallel; the objective function $R_s(\cdot)$ is the reliability of the system at time $t = T$ as shown in Equation (3); $C_s(\cdot)$, $V_s(\cdot)$ and $W_s(\cdot)$ are the cost, volume and weight functions of the system; C, V and W are the upper limits on the cost, volume and weight of the system, respectively.

We assume that the cost of a component of type k is a decreasing function of the failure rate λ_k of the component [3–10]

$$c(\lambda_k) = \alpha_k \lambda_k^{-\beta_k} \tag{5}$$

where $\alpha_k > 0$ and $0 < \beta_k < 1$ are constants representing the inherent characteristics of the component. We assume that the values of α_k and β_k would be provided by manufacturers. If the failure time of the component follows the exponential distribution, then cost of the component can be written as:

$$c(r_k) = \alpha_k \left(-\frac{T}{\ln(r_k)} \right)^{\beta_k} \tag{6}$$

where T is the recognized operating time of the system and r_k is the reliability of components of type k at time T. Using the above equations, the total cost function of the system can be determined as

$$C_s(r_1, \ldots, r_K, n_1, \ldots, n_K) = \sum_{k=1}^{K} c(r_k)(m_k n_k + \exp(m_k n_k/4)) = \sum_{k=1}^{K} \left[\alpha_k \left(-\frac{T}{\ln(r_k)} \right)^{\beta_k} (m_k n_k + \exp(m_k n_k/4)) \right] \tag{7}$$

where m_k is the number of components of type k in the original system; n_k is the redundancy level of components of type k; $\exp(m_k n_k/4)$ is a term reflecting the cost of interconnecting parallel elements.

Additionally, let v_k and w_k be the total volume and weight of a component (including the hardware for interconnecting it) of type k. Then the volume and weight of the system can be evaluated using Equations (8) and (9), respectively

$$V_s(n_1, \ldots, n_K) = \sum_{k=1}^{K} m_k v_k n_k \tag{8}$$

$$W_s(n_1,\dots,n_K)=\sum_{k=1}^{K} m_k w_k n_k \tag{9}$$

We have assumed here that the volumes and weights are not functions of the component reliabilities.

4. Adaptive penalty function-based solution of RRAP

Generally, a traditional numerical algorithm like exhaustive search is rarely expected to yield the optimal solution to a RRAP due to its complex constraints. Moreover, since the size of the search space increases while solving high-dimensional optimization problems, classical optimization algorithms may encounter serious speed deficiencies. Therefore, heuristic algorithms (e.g. genetic algorithm, bee colony algorithm or cuckoo search) are often used to solve RRAPs.

One of the most popular and intuitive constraint handling techniques is the penalty function method [26, 27]. In this method, a penalty function is used to penalize infeasible solutions, while feasible solutions are simply evaluated based on the original objective function. In order to do that, the optimization problem defined in Equation (4) can be rewritten as an object function of the constraint, as follows

$$f_p(\mathbf{u})=f(\mathbf{u})+\sum_{i=1}^{p} h_i g_i(\mathbf{u}) \tag{10}$$

here $p=3$ is the number of constraint functions, $\mathbf{u}$ is a set of design variables, $f_p(\mathbf{u})$ is the penalized objective function, $f(\mathbf{u})$ is the original objective function as shown in Equation (4), $g_i(\mathbf{u})$ is the ith constraint function, h_i is the ith penalty parameter.

Obviously, the penalty parameters in Equation (10) have strong influence on the solution of the optimization model because they determine the levels of the penalties added to the objective function. For example, if penalty parameters are too large, the penalty function becomes 'ill-conditioned', making the optimization goal difficult to achieve. However, if the penalty parameters are too small, the penalty function does not have significant effects on the solution. That is to say the constraint violation does not contribute a high cost to the penalty function. Therefore, a key point for a penalty function method for solving constraint optimization is the selection of proper penalty parameters. Unfortunately, it is difficult because a suitable value of the penalty parameter depends on the solution of the problem.

In this section, an adaptive penalty function strategy is proposed to solve constrained optimization problems using heuristic algorithms. In the proposed adaptive penalty approach, the solution group is separated into feasible and infeasible sets. In the feasible set, all of the constraints are satisfied, whereas infeasible points fail to meet at least one of the constraints. A penalty function is used to penalize the infeasible solutions, while feasible solutions are simply evaluated based on the original objective function

$$f_p(\mathbf{u})=h_f f(\mathbf{u})+(1-h_f)\frac{1}{p}\sum_{i=1}^{p} y_i(\mathbf{u}) \tag{11}$$

where $\mathbf{u} = [r_1, \ldots, r_K, n_1, \ldots, n_K]$ is the vector of design variables, $f(\mathbf{u})$ is the original object function of the constraint optimization, h_f is the ratio between the feasible constraints and the total number of constraints. $y_i(\mathbf{u})$, $i = 1, 2, \ldots, p$, is the normalized constraint function which is defined as

$$y_i(\mathbf{u}) = \begin{cases} 0, & \text{if } g_i(\mathbf{u}) \leq 0 \\ |g_i(\mathbf{u})|, & \text{otherwise} \end{cases} \tag{12}$$

where $g_i(\cdot)$ is the ith constraint.

Once the penalty function of the RRAP is established, any efficient heuristic algorithm can be easily applied to solve the optimization problem. In this paper, the stochastic fractal search (SFS), which was recently proposed by Salimi [27], is used to solve the reliability optimization problem. The basic concept of SFS is founded on the property of an entity (object or quantity) for exploring an area. The fractal theories define the mathematical model of diffusing a particle and self-similarity of the patterns in nature, such as trees, snowflakes, animal coloration patterns, crystals, Romanesco broccoli, lungs, river networks, blood vessels and DNA. As shown in Figure 1, the diffusing process and the updating process are the two main processes in the SFS and the basic principles of the proposed method are as follows: (1) Randomly generate a certain number of initial solutions. (2) Evaluate the values of objective and constraint functions and establish the penalized objective function according to Equations (11) and (12). (3) Gaussian walk is applied to let all the points roam around their current positions to exploit the problem search space in the diffusion process. (4) Change each point to a better position through two updating procedures: the first procedure is performed on each individual point and the second procedure considers the position of other components in the group. (5) Repeat the procedures until the stopping criterion is satisfied. For more information about SFS, we refer to [26, 27].

We can get the optimal reliability and optimal redundancy levels for each type of components by solving the constraint optimization model for RRAP. Then the number of redundant components should be added to components of type k in parallel can be obtained as follows

$$n_{a,k} = (n_k - 1)m_k \tag{13}$$

where m_k is the number of components of type k in the original system; n_k is the redundancy level of components of type k. During the evolution process, the integer variable n_k is treated as a real variable. Therefore, $n_{a,k}$ should be transformed to the nearest integer value. Moreover, if $n_{a,k}$ is an integer multiple of m_k, every component of type k can actually get $n_k - 1$ components in parallel. Otherwise, extra components can be added to the components with higher structural importance to improve the system reliability as efficiently as possible. For example, if there are three components of type k and four components should be added to this type of components, every component of type k could add one component in parallel, and the extra one could be further added to the component with the highest

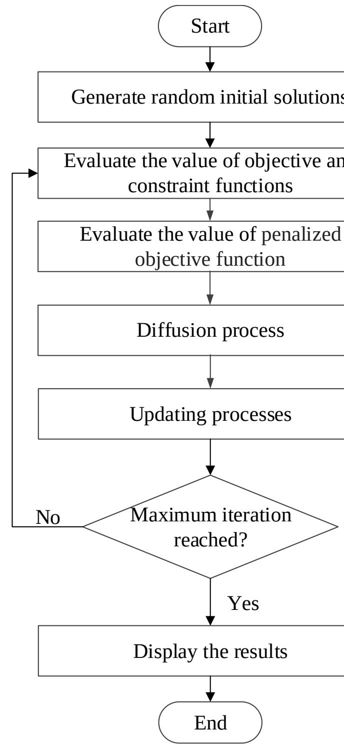

Figure 1: Procedure for optimization for RRAP

structural importance. This procedure may not provide the actual optimal solution, but we propose it as a sensible heuristic solution which can be applied to ease system due to its relatively easy computational requirements.

5. Determine the redundant level of each component

Reliability importance can be used to prioritize components in a system by quantitively measuring their importance level in contributing to system reliability [28–30]. In this section, the reliability importance is chosen as the criterion to determine the redundancy level of each component in the system, given that the redundancy level of each type of components has been determined by solving the optimization problem in Equation (4). Generally, importance measures can be divided into three classes, which are structural importance, reliability importance and lifetime importance. Structural importance measures the relative importance of components with respect to their positions. Reliability importance and lifetime importance depend on both the structure of the system and the reliabilities of the components. Since the redundancy level of each type of components has been determined in Section 4, structural importance is reasonable to allocate the redundancy of components of the same type.

The structural importance of a component depends on the number of states in which the component is critical, The structural importance of component i, denoted by I_i^S, can be defined as[31]:

$$I_i^S = E\left[\phi(1_i, \mathbf{x}^i) - \phi(0_i, \mathbf{x}^i)\right] = \frac{1}{2^{m-1}} \sum_{\Omega} \left[\phi(1_i, \mathbf{x}^i) - \phi(0_i, \mathbf{x}^i)\right] \tag{14}$$

where $\phi(\cdot)$ is the structure function of the system as shown in Section 2; Ω is the ensemble of all the possible combinations of up and down states of the m components, whose cardinality is 2^{m-1}; $(\mathbf{1}_i, \mathbf{x}^i)$ and $(\mathbf{0}_i, \mathbf{x}^i)$ represent the component vector when component i is in state 1 and 0, respectively.

The marginal structural importance (MSI), defined in Equation (14), can be used to measure the effect of a single component on the system's performance. However, MSI does not provide all information on how components affect the system reliability. In particular, MSI gives very little information about how the component reliabilities affect each other. Therefore, if more than one components have the same MSI, the joint reliability importance which measures how multiple components in the system interact in contribution to the system performance can be used to find the best combination. The joint structural importance of n components is similarly defined as [32–34]:

$$I_{i_1,\ldots,i_n}^S = \frac{1}{2^m} \sum_{\Omega} \left\{ \phi(\mathbf{1}_{i_1,\ldots,i_n}, \mathbf{x}^{i_1,\ldots,i_n}) + I_1(n)\phi(\mathbf{0}_{i_1,\ldots,i_n}, \mathbf{x}^{i_1,\ldots,i_n}) + \sum_{\Omega} \left[I_2(\mathbf{v}_{i_1,\ldots,i_n})\phi(\mathbf{v}_{i_1,\ldots,i_n}, \mathbf{x}^{i_1,\ldots,i_n}) \right] \right\} \tag{15}$$

where

$$I_1(n) = \begin{cases} 1, & \text{if } n \text{ is even} \\ -1, & \text{if } n \text{ is odd} \end{cases} \tag{16}$$

$$I_2(\mathbf{v}_{i_1,\ldots,i_n}) = \begin{cases} 1, & \text{if } n \text{ and the sum of } x^{i_1,\ldots,i_n} \text{ have the same parity} \\ -1, & \text{otherwise} \end{cases} \tag{17}$$

$\mathbf{1}_{i_1,\ldots,i_n} = \{1_{i_1}, \ldots, 1_{i_n}\}$ and $\mathbf{0}_{i_1,\ldots,i_n} = \{0_{i_1}, \ldots, 0_{i_n}\}$ represent the states that components $i_1, i_2, \ldots, i_n$ are working and failed, respectively. $\mathbf{v}_{i_1,\ldots,i_n}$ is used to enumerate the possible states of the components $i_1, i_2, \ldots, i_n$, except for the cases that all of them are working or failed. Such as, for $n = 2$, $\mathbf{v}_{i_1,i_2}$ specifies $\{1_{i_1}, 0_{i_2}\}$, and $\{0_{i_1}, 1_{i_2}\}$. $(\mathbf{1}_{i_1,\ldots,i_n}, \mathbf{x}^{i_1,\ldots,i_n})$, $(\mathbf{0}_{i_1,\ldots,i_n}, \mathbf{x}^{i_1,\ldots,i_n})$ and $(\mathbf{v}_{i_1,\ldots,i_n}, \mathbf{x}^{i_1,\ldots,i_n})$ represent the component vectors when component $i_1, i_2, \ldots, i_n$ is in state $\mathbf{1}_{i_1,\ldots,i_n}$, $\mathbf{0}_{i_1,\ldots,i_n}$ and $\mathbf{v}_{i_1,\ldots,i_n}$, respectively.

As a specific example, the joint structural importance of two components is

$$I_{i_1,i_2}^S = \frac{1}{2^m} \sum_{\Omega} \left\{ \phi(1_{i_1}, 1_{i_2}, \mathbf{x}^{i_1,i_2}) + \phi(0_{i_1}, 0_{i_2}, \mathbf{x}^{i_1,i_2}) - \phi(0_{i_1}, 1_{i_2}, \mathbf{x}^{i_1,i_2}) - \phi(1_{i_1}, 0_{i_2}, \mathbf{x}^{i_1,i_2}) \right\} \tag{18}$$

where $(1_{i_1}, 1_{i_2}, \mathbf{x}^{i_1,i_2})$ represents that components i_1 and i_2 are in working states and the meanings for $(0_{i_1}, 0_{i_2}, \mathbf{x}^{i_1,i_2})$, $(0_{i_1}, 1_{i_2}, \mathbf{x}^{i_1,i_2})$ and $(1_{i_1}, 0_{i_2}, \mathbf{x}^{i_1,i_2})$ are as described in the previous paragraph.

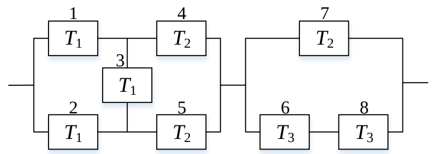

Figure 2: 8-unit structure system

6. Numerical example

Example 1. Consider the system shown in Figure 2. This system consistes of 8 components which can be divided into 3 types, namely T_1, T_2 and T_3. The life cycle of the system is $T = 1500$ hours. It is assumed that the failure time of all the components follows an exponential distribution, and the reliabilities of the components at time $t = 1500$ hour, are given in Table 1. The cost, weight and volume parameters of the components are listed in Table 2. The cost, weight and volume of the system can be obtained from the parameters shown in Table 2, and the values are 910, 72 and 79, respectively. If the weight and volume of the system could be increased to 100 and 120 (the cost cannot be increased,the proposed approach can be applied to improve the reliability of the system.

Table 1: Distribution information of components

Component Type	Component No.	r_k
1	1, 2, 3	0.9241
2	4,5,7	0.8725
3	6,8	0.9105

Table 2: Constraint parameters for the 8-unit system

Type	α_i	β_i	w_i	v_i
1	6×10^{-5}	1.5	4	3
2	8×10^{-5}	1.5	8	10
3	4×10^{-5}	1.5	18	20

The survival signature of the system can be obtained using Equation (1). All the results are shown in Table 3 and rows with $\Phi(l_1, l_2, l_3) = 0$ are omitted since these factors do not have any impact on the reliability of the system. Then we can obtain the reliability of the system at $t \in [0, 1500]$ using Equation (2) and the distribution information of the components listed in Table 1. The reliability of the system at time $t \in [0, 1500]$ is depicted as the black solid line (R_{Sys}) in Figure 4, the reliability of the system at the end of its life cycle ($t = 1500$) is 0.9556.

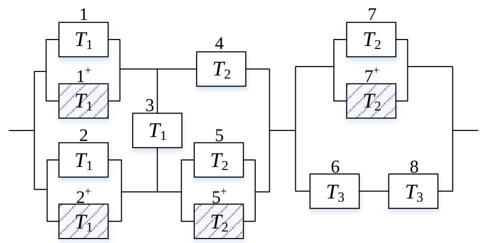

Figure 3: Reliability block diagram of the optimized system in example 1

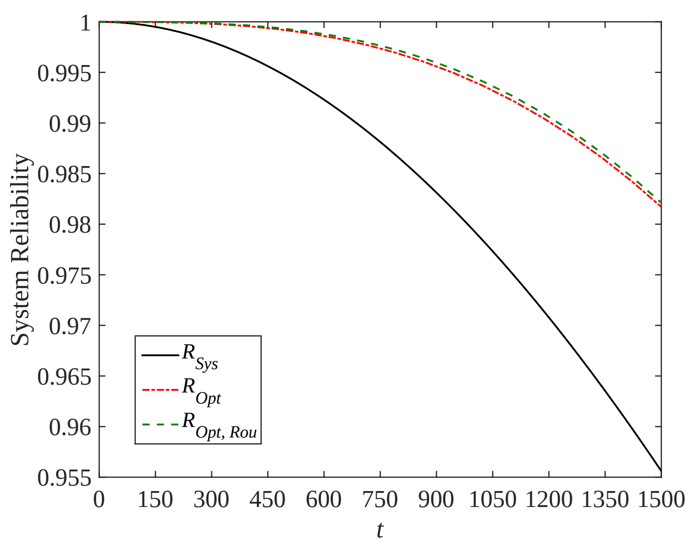

Figure 4: Reliability of the 8-unit system

Table 3: Survival signature of the 8-unit system

l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$	l_1	l_2	l_3	$\Phi(l_1, l_2, l_3)$
1	1	2	2/9	2	3	0	1
1	2	0	2/9	2	3	1	1
1	2	1	2/9	2	3	2	1
1	2	2	4/9	3	1	2	2/3
1	3	0	2/3	3	2	0	2/3
1	3	1	2/3	3	2	1	2/3
1	3	2	2/3	3	2	2	1
2	1	2	2/3	3	3	0	1
2	2	0	2/3	3	3	1	1
2	2	1	2/3	3	3	2	1
2	2	2	1				

The proposed adaptive penalty approach is applied to solve the RRAP. 253 iterations are needed to get a converged result. The result shows that in order to obtain the maximum system reliability under the constraints, the reliability of components at time $t = 1500$ needs to be changed to 0.8047, 0.8363 and 0.8913, and the redundancy level of each type of component should to be 1.77, 1.78 and 1, respectively. The reliability of the system under these conditions is shown as the dash-dotted line (R_{Opt}) in Figure 4 .

Equation (13) is used to calculate the number of components that should be added to each type of components ($n_{a,k}$). Since the integer variable n_k is treated as a real variable, $n_{a,k}$ should be rounded to the nearest integer value. And the results show that 2, 2 and 0 components need to be added to the first, second and third types of components, respectively. Finally, as recommended in Section 5, the redundancy level of each component could be determined by the importance level of it in contributing to system reliability.

Structural importance analysis of the system is performed to prioritize components in the system and the results are shown in Table 4. From the results we can learn that, for the first type of component (T_1), the structural importance of component 1 is equal to that of component 2, and both of them are greater than the structural importance of component 3 ($I_1^S = I_2^S > I_3^S$). Therefore, we may be wise to add a component to component 1 and to component 2 in parallel.

Table 4: structural importance of the 8-unit system

I_1^S	I_2^S	I_3^S	I_4^S	I_5^S	I_6^S	I_7^S	I_8^S	$I_{4,7}^S$	$I_{5,7}^S$
0.2344	0.2344	0.0781	0.2344	0.2344	0.3750	0.1250	0.1250	0.2813	0.2813

For the second type of component (T_2), the structural importance of component 4 is equal to that of component 5, and both of them are smaller than the structural importance of component 7 ($I_7^S > I_4^S = I_5^S$). So we could add a component to component 7 and the other component may need to be added to component 4 or 5. Since $I_4^S = I_5^S$, whether it has to be added to component 4 or 5 is determined by the joint structural importance $I_{4,7}^S$ and $I_{5,7}^S$. As shown in Table 4, since $I_{4,7}^S = I_{5,7}^S$, we can add a component to any of them.

Table 5: System reliability and constraint values of the optimized system in example 1

	Reliability	Cost	Weight	Volume
Optimized	0.9870	910	96	105
Rounding	0.9821	859	96	105

The reliability block diagram of the optimized system is illustrated in Figure 3, components 1^+, 2^+, 5^+ and 7^+ are the newly added components. The reliability, cost, volume and weight of the optimized system are calculated. All the results are compared in Table 5. In this case, the reliability of the system is shown as the dashed line ($R_{Opt,Rou}$) in Figure 4. The reliability of the system at time t=1500 is improved to 0.9821 and all constraints are satisfied. From the results, we can learn that the proposed approach has been successfully implemented.

For comparison, the traditional reliability-redundancy allocation model, which is a 16-dimension optimization problem for this example, is applied to allocate the reliability of the system. If the SFS approach and the proposed adaptive penalty method are also used to solve the optimization model, 299 iterations are needed for convergence of the optimization algorithm. Therefore, the proposed method model requires fewer iterations. It shows that the proposed procedure effectively reduces the computational complexity of RRAP.

Because the components of the same type follow the same distribution, their reliability is set to be equal during the optimization process. Then the dimension of the RRAP is reduced to 11. The optimum reliability of these three types of components is obtained as follows: r_1=0.7696, r_2=0.8530, r_3=0.7823. The redundancy levels of components 1-8 are 3, 1, 1, 2, 1, 1, 2 and 1, respectively. The reliability of the system at time t=1500 is 0.9844 which is a little higher than the result obtained by the proposed method. However the cost of this plan is 913 which is slightly higher than the allowable value (910).

Example 2. In this example, a 15-unit system as shown in Figure 5 is used to demonstrate the application of the proposed method for system reliability optimization for a complex structure system with different types of components. The failure times of all the components are considered to follow exponential distributions, and the types of the components are listed in Table 6. Moreover, the cost, weight and volume parameters of the components are listed in Table 7. The life cycle of the system is designed to

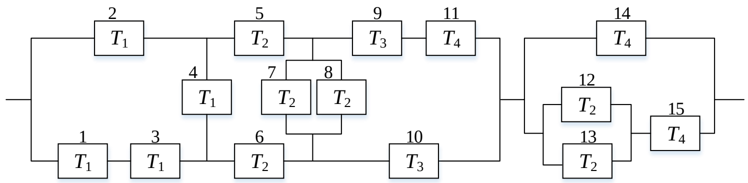

Figure 5: 15-unit structure system

be $T = 2500$ hours. The upper limits on the cost, volume and weight of the system are 400, 100 and 120, respectively.

Table 6: Types of components

Type	1	2	3	4
Component No.	1, 2, 3, 4	5, 6, 7, 8, 12, 13	9, 10	11, 14, 15

Table 7: Constraint parameters for the 15-unit system

Type	α_i	β_i	w_i	v_i
1	1.2×10^{-5}	1.3	10	9
2	1×10^{-5}	1.3	3	2
3	6×10^{-6}	1.3	8	7
4	3×10^{-6}	1.3	4	2

The proposed approach is applied to allocate the reliability and redundancy levels of the components. Convergence is reached after 272 iterations. For maximum benefit, the reliabilities of the four types of components need to be 0.9560, 0.9247, 0.9642 and 0.9666 at time $t = 2500$, respectively. Moreover, 1, 3, 1 and 1 components should be added to the four types of components in parallel, respectively.

Structural importance analysis is performed to determine the redundancy level of each component and the results are shown in Table 8. As can be seen from the results, for the first type of components (T_1), the structural importance of component 2 is the highest. It is therefore advisable to add a component to component 2 in parallel. For the second type of components (T_2), three components are needed to be added. Since $I_5^S > I_6^S > I_7^S = I_8^S = I_{12}^S = I_{13}^S$, one component should be added to each of components 5 and 6 in parallel, respectively. The only component left is supposed to be added to component 7, 8, 12 or 13 according to their joint structural importance with components 7 and 8. As shown in Table 8, since $I_{5,6,12}^S = I_{5,6,13}^S > I_{5,6,7}^S = I_{5,6,8}^S$, so we add a component to either component 12 or 13. By using the same method, we can obtain that for the third and fourth types of components, one component would be added to components 10 and 14, respectively. The reliability, cost, volume and weight of the system are calculated. The reliability of the optimized system at time t=2500 is 0.9997 and all constraints are satisfied. The reliability of the optimized system at time t, for $t \in [0, 2500]$, is depicted in Figure 6.

Table 8: structural importance of the 15-unit system

I_1^S	I_2^S	I_3^S	I_4^S	I_5^S	I_6^S	I_7^S	I_8^S	I_{12}^S	I_{13}^S
0.084	0.187	0.076	0.054	0.143	0.112	0.029	0.029	0.029	0.029
I_9^S	I_{10}^S	I_{11}^S	I_{14}^S	I_{15}^S	$I_{5,6,7}^S$	$I_{5,6,8}^S$	$I_{5,6,12}^S$	$I_{5,6,13}^S$	
0.070	0.193	0.070	0.144	0.087	-0.062	-0.062	-0.023	-0.023	

Table 9: System reliability and constraint values of the optimized system in example 2

	System Reliability	Cost	Weight	Volume
Optimized	0.9989	400	120	94
Rounding	0.9997	384	117	92

If the traditional reliability-redundancy allocation model is applied to allocate the reliability of the system, a 30-dimension optimization problem should be solved for this example, and 1121 iterations are needed to obtain the optimal result. This number of iterations is much higher than that is required in the proposed method. It shows that the proposed procedure effectively reduce the computational complexity of RRAP.

The reliability of the same type of components is set to be equal during the optimization process. Then the dimension of the RRAP is reduced to 19. The optimum reliability of the four types of components is obtained as follows: r_1=0.9692, r_2=0.7031, r_3=0.9759, r_4=0.9312. The redundancy levels of components 1-15 are 1, 1, 1, 1, 5, 4, 1, 1, 1, 1, 2, 1, 1, 3 and 1, respectively. The reliability of the system at time t=2500 is 0.9973 which is lower than the result obtained by the proposed method. This example demonstrates that the proposed approach, combining RRAP and structural importance by using survival signature and adaptive penalty guided heuristic optimization, is effective and of practical benefit.

7. Discussion

Recently, the theory of survival signature has attracted increasing attention for performing reliability analysis of larger systems due to its high efficiency and low complexity [35–39]. Specially, in order to further promote the application of survival signature on system reliability analysis, Reed [40] proposed an efficient method to compute survival signatures and Aslett [41] created a software package. In this paper, an efficient method is proposed by using the theory of survival signature to open a new way for

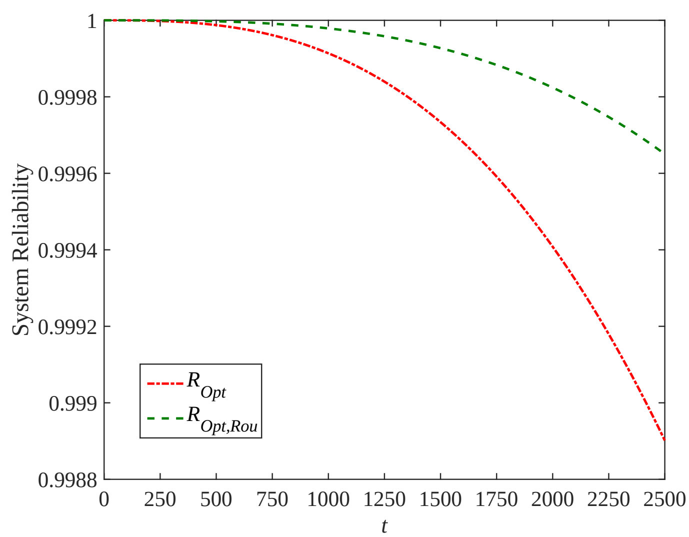

Figure 6: Reliability of the 15-unit system

reliability-redundancy allocation of systems. Once the survival signature of a system is obtained, the reliability-redundancy allocation of the system could be performed by using the proposed procedure.

Compaired with the traditional method, the dimension of the RRAP is effectively reduced according to the number of components of the same type in the proposed method. This improvement can effectively reduce the computational complexity of RRAP. Moreover, a new adaptive penalty function is proposed to solve the constrained optimization model for RRAP. Survival signature separates the system structure from its failure distributions, and it only needs to be calculated once. Therefore, as shown in the numerical examples, the proposed approach is easy to implement in practice and has high computational efficiency.

It should be noted that during the optimization process, because the components of the same type follow the same distribution, the (joint) structure importance is used to concretely allocate the redundancy level of each component. Although this approach is reasonable and simple, other importance measures (e.g. Birnbaum importance, FV reliability importance or global sensitivity measures) and allocation plans could be explored to see if these may lead to better results. This is a topic of ongoing research of the authors. In general, however, this paper presents a new and practical method for the RRAP by integrating the theories of survival signature and (joint) structure importance.

Acknowledgement

This research was performed while Xianzhen Huang visited Durham University. The authors gratefully acknowledge the support of National Natural Science Foundation of China (51575094, U1708254, 51875094). Moreover, the authors would like to give their thanks to the anonymous referees and the editor for their valuable comments and suggestions leading to an improvement of the paper.

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