Redistributive Innovation Policy, Inequality and Efficiency

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Abstract

We examine the efficiency and distributional effects of regressive and progressive public R&D policies that target high-tech and low-tech sectors using a heterogenous-agent growth model with in-house R&D and incomplete capital markets. We find that such policies have important implications for efficiency and inequality. A regressive public R&D investment financed by income tax could boost growth and welfare via a positive effect on individual savings and effort. However, it could also lower growth and welfare via its effect on the efficiency–inequality trade off. Thus, the relationship between public R&D spending and welfare is hump shaped admitting an optimal degree of regressivity in public R&D spending. Using our baseline model and the US state level GDP data, we back out the degree of regressiveness of public R&D investment in US states. We find that US states are more regressive in their R&D investment than the optimal regressiveness implied by our growth model.

Key words:
Public R&D investment, inequality dynamics, growth, welfare
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1. Introduction

The role of public and private research and development (R&D) investment[1] in economic growth is a widely debated topic.[2] However, the distributional effect of public R&D investment has received little attention. In the extant literature, the focus is more on public education (e.g., Glomm and Ravikumar, 1992, 2003, Benabou, 2002) and infrastructure and taxes (e.g., Alesina and Rodrik, 1994, Garcia and Turnovsky, 2007, Getachew, 2010, Chatterjee and Turnovsky, 2012, Getachew and Turnovsky, 2015). R&D investment could have uneven impacts on the economy and through this channel, it could impact rich and poor differently. In general, most public R&D investment in developed countries are concentrated on high-tech industries such as information technology, biotechnology, communication, and environment industries. In the United States, for instance, public R&D investment in equipment and software has increased from 20% in 1980 to 50% in 2001 which contributed to rising inequality in the United States (US) in recent decades (Cozzi and Impullitti, 2010). R&D investment in South Korea is concentrated more on high-tech sectors (Kim et al., 2013). In contrast, in most of the developing world, a significant amount of public R&D investment are made in agriculture, a low-tech sector dominated by

[1] R&D expenditure includes a broad range of activities: "Research and development (R&D) comprise creative work undertaken on a systematic basis in order to increase the stock of knowledge (including knowledge of man, culture and society) and the use of this knowledge to devise new applications." (http://www.oecd.org/sdd/08_Science_and_technology.pdf).

[2] Particularly, in early 90s, there was an influx of R&D based growth theories, following the seminal works by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) that emphasize the role of R&D to economic growth, through influencing technological progress. R&D policies are also widely debated as to whether public R&D investment complements private R&D investment or crowds it out (e.g., David et al., 2000). Early work in public R&D investment includes Shell (1967).

small scale farmers. Beintema et al. (2012) report an accelerated public investment in agricultural R&D in developing countries during the period 2000 and 2008. Using provincial data in China spanning more than four decades, Zhang and Fan (2004) argue that government spending on agricultural R&D contributed to a reduction in regional inequality.

Our own calculations suggest a contrasting relationship between inequality and public R&D spending, consistent with previous studies. Figures 1 and 2 show relationships between the GINI index and R&D intensity in the US and Sub-Saharan Africa (SSA), respectively.[3] In both regions, R&D spending has sharply increased during the last two decades, but the inequality experience is opposite.[4] While there is a positive correlation between GINI and R&D investment across US states, for SSA countries the correlation is negative. Given that R&D spending in the US is more geared to high tech sector while in sub-Saharan Africa (SSA) it is focused on primarily on subsistence farming, this reversal of sign of the correlation between R&D intensity and the GINI between Figures 1 and 2 is intriguing. Inequality could

[3]The R&D intensity is measured by the ratio of total R&D spending (including private and public GDP) to state GDP. The breakdown of public and private R&D for each state is not available. The National Science Foundations sources suggest that the federal share of total R&D spending is about 11.6% over the period 2008-15 All the GINI data came from US Census Bureau. The R&D intensity data came from the Science, technology, Innovation and Entrepreneurship (SSTI) database of the US. The correlation coefficient between R&D Intensity and State GINI index is 0.18. For SSA countries the GINI and public R&D spending data came from World Bank (2015). Due to sparse nature of the data, we take the average of GINI index from 2000 onward whatever data are available. Same is done for the public R&D spending ratio. Details of all these data are available from the authors upon request.

[4]The average annual agricultural R&D spending growth in SSA countries, for instance, increased from 0.3% during 1981-1990 to 2.8% during 2000-2008 except for a small dip of .01% during 1990-2000, which is indicative of the bulk of recent R&D innovations in SSA being progressive in nature (Beintema et al., 2012).

result from public R&D due to its destination.

Figure 1: Inequality and R&D Spending in US States

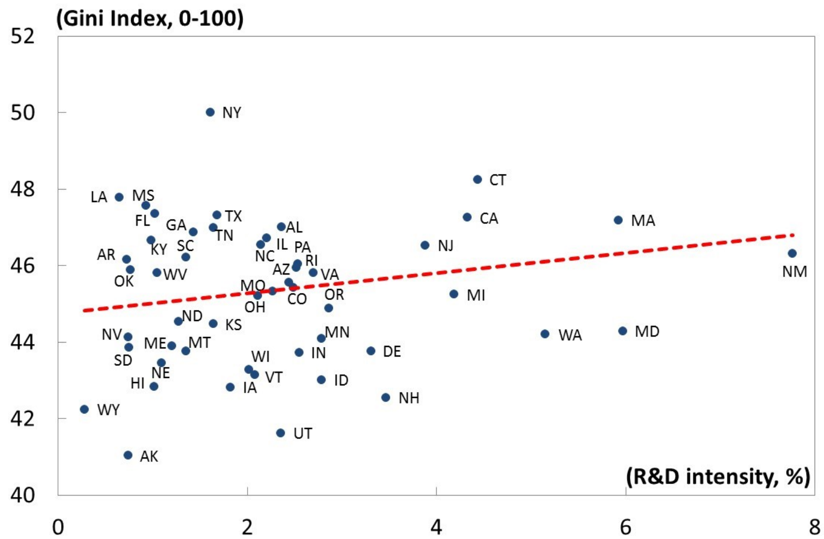
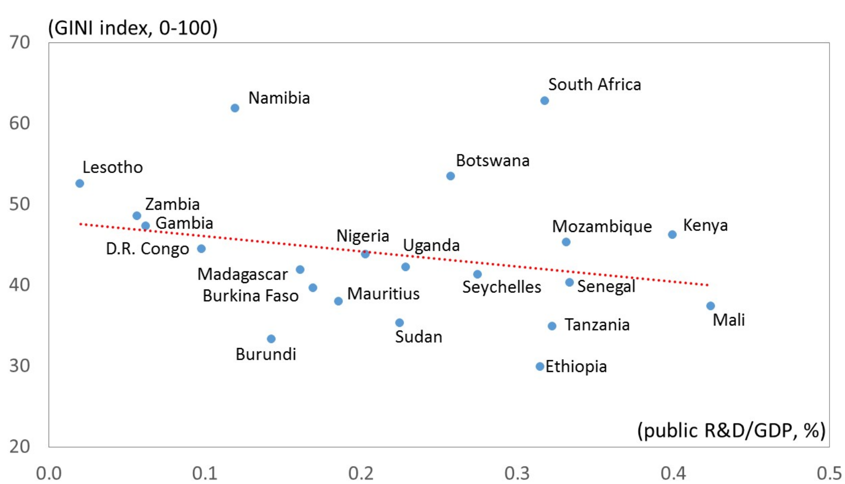


Figure 2: Inequality and PublicR &D Spending in SSA



Motivated by this, we ask our key research question in this paper whether the public R&D spending is likely to be regressive or progressive. We examine the effects of public R&D investment on efficiency and inequality using the lens of a heterogenous-agent growth model where agents are heterogenous in their initial endowments of knowledge and their ability to generate knowledge. Our model includes in-house R&D which yields monopolistic profit for the firms. Both inequality and growth are endogenously determined. The source of endogenous growth is in-house R&D investment using private and public resources. Endogenous inequality is generated due to missing credit and insurance markets, as in Loury (1981) and Benabou (2000, 2002, 2005). The dynamics of aggregate variables and inequality are jointly determined in the model that admits a closed-form analytical solution.

We do two related exercises using our growth model. First, we analyze the effects of redistributive innovation policies on steady state inequality, growth and welfare, which is the main objective of this paper.[5] Public R&D investment policies are identified as regressive and progressive, based on their disproportional impact on relatively large and small firms, respectively.[6] Regressive R&D investment policy aggravates inequality. On the other hand, regressive innovation policies have the benefit of promoting efficiency due to positive incentives on agents' savings and work effort decisions. There is a potential trade-off between growth and inequality in our model due to incomplete capital markets. This makes the relationship between R&D investment, growth as well as welfare nonlinear and hump shaped. Our calibrated

[5] Basu and Getachew (2015) study the role of redistributive policies in intergenerational mobility.
[6] We provide a more formal definition in the next section.

baseline model suggests that the positive incentive effects of a regressive innovation far outweigh the negative effects of a higher inequality on steady state welfare. This makes the optimal redistributive innovation policy regressive in our model. A sensitivity analysis suggests that the case for regressiveness of R&D policy is less in economies with higher idiosyncratic risk in productivity and private innovation because it makes the negative inequality effect of regressive R&D policy stronger. For a sufficiently high idiosyncratic risk in knowledge innovation, the R&D policy could be progressive in nature. This could explain the negative relation between inequality and R&D intensity in SSA countries, reported in Figure 2, where uninsurable risk is expected to be high. Due to lack of well developed financial markets, R&D policy could be quite progressive.

Second, motivated by the positive correlation between inequality and R&D intensity in US states, we ask the question whether the public R&D in US states are regressive in nature. Using our model and the state level GDP data for the US, we back out the degree of regressivity in each US state. We find that the estimated R&D regressiveness in all US states is higher than the socially optimal regressivity computed from our baseline model which suggests that the R&D in US states is too regressive in nature.

Our work connects to a wider literature on inequality and growth. First, it relates to the literature that analyzes growth–inequality trade off under imperfect credit markets although this literature abstracts from productive public spending feature of our model.[7] For instance, the work of Benabou (2002) focuses primarily

[7] See for example Loury, (1981), Galor and Zeira (1993), Aghion and Bolton (1997), Aghion, et

on the distributional and growth impact of progressive taxation while our focus is on the redistributive effects of productive public goods. Second, the paper is in line with the literature on public education, infrastructure and inequality.[8] With a few exceptions (e.g., Ziesemer, 1990, 1995), this literature pays scant attention to public R&D investment.[9] Third, our work complements the literature on innovation and inequality (Chu, 2010, Cozzi and Impullitti, 2010, and Aghion et al., 2015) with the following important differences. We focus on contrasting the effects of regressive and progressive innovation, assessing how regressive US innovation policy is and designing an optimal regressive and progressive R&D policy while the focus of the above studies is more on the effects of innovation or innovation policy on inequality.[10] Finally, our study accords well with a branch of literature that attributes the recent rise in inequality in many advanced economies to skill biased technical change (Acemoglu, 2002 and Aghion, 2002). This literature, however, focuses on private R&D investment or technical progress and abstracts from the optimal public

al. (1999) Benabou (2000, 2002, 2005), Bandyopadhyay and Tang (2011) and Basu and Getachew (2015).

[8]Public education is at the center of the work by Glomm and Ravikumar (1992, 2003), Saint-Paul and Verdier (1993), Sorensen (1993) and Eckstein and Zilcha (1994), among many others. In contrast, recent work by Garcia-Penalosa and Turnovsky (2007), Getachew (2010, 2012), Chatterjee and Turnovsky (2012), and Getachew and Turnovsky (2015) focus on the effects of infrastructure on inequality. In contrast, we explicitly model the intermediate goods sector as characterized by monopolistic competition, which is typical to R&D models as in Smulders and Klundert (2004).

[9]Ziesemer (1990, 1995) study the distributional conflict that arises from government involvement in knowledge creation. In his models, ability differences among individuals lead to differences in their preferred tax rates. Particularly, individuals with higher (lower) ability prefer higher (lower) technical progress which can be achieved through higher taxes than the social planner's optimal tax rate. Ziesemer, however, abstracts from the role of public policy on inequality and distributional dynamics and the impact through which it has on growth and welfare.

[10]Chu (2010) argues that strengthening patent policy increases income inequality by raising the return on assets. Whereas, Aghion et al. (2015) focus on the relationship between innovation, top income inequality and social mobility for the U.S.

R&D policy and the R&D effects on US regional inequality that we are interested in.

The paper is organized as follows: The next section develops the model. Section 3 characterizes individual and aggregate (inequality) dynamics. Section 4 discusses the effects of different R&D policies on steady-state growth and inequality while Section 5 focuses on optimality of alternative R&D policies. Section 6 computes the degree of regressivity in the US states using state level GDP data. Section 7 concludes.

2. The Model

We assume that the economy is populated with a continuum of heterogenous agents, $i \in (0,1)$. There is no population growth in the economy. The first generation of the ith agent is endowed with h_{i0} levels of knowledge.[11] Initial distribution is given and assumed to take log-normal, $\ln h_{i0} \sim N\left(\mu_0, \sigma_0^2\right)$, which evolves endogenously in equilibrium. Agents also differ in their respective productivity and creativity to generate income and knowledge, respectively, where both are assumed to be i.i.d. and log-normally distributed. Combined with labour, knowledge is used to produce intermediate goods, which are, in turn, used for production of the final goods.

There are three sectors in the economy, namely the final goods, the intermediate goods and the knowledge production sectors. Using a constant elasticity of substitution (CES) production function, a competitive firm transforms intermediate inputs into a final good. These differentiated intermediate inputs are produced by monop-

[11] There is no capital in the economy, which is standard in the R&D growth literature, without any loss of generality.

olistically competitive firms. Each firm in this sector invests in an in-house R&D, in the spirit of Smulders and Van de Klundert (1995), to expand a specialized know-how that is required to produce a specialized input. The production of knowledge requires both the use of public and private resources, and a backlog of knowledge stock. The government levies a fixed flat rate tax on the income of individual agents to finance the 'public good'. This public good is provided *disproportionately* among rich and poor agents to supplement private R&D investment.

2.1. Final goods

In the spirit of Benabou (1996), the final goods and services are produced using a continuum of differentiated intermediate inputs (x_{it}) by the representative firm in the economy:

$$y_t = a_1 \left(\int_0^1 \phi_{it} x_{it}^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)} ; \varepsilon > 1 \tag{1}$$

where x_{it} is the intermediate input supplied by the ith intermediate goods firm and a_1 is a deterministic total factor productivity (TFP) parameter; ϕ_{it} represents idiosyncratic productivity shocks, which are i.i.d. with mean one and a constant non-zero variance, attached to each intermediate input, $\ln \phi_{it} \sim N(-\varkappa^2/2, \varkappa^2)$. $\varepsilon > 1$ is the elasticity of substitution between the intermediate inputs, which determines the firms' monopoly power, in the spirit of Dixit and Stiglitz (1977).

Profit maximization by the perfectly competitive firm, given a unit price of the final goods, leads to the downward slopping input demand function:

$$x_{it} = \phi_{it}^{\varepsilon} a_1^{\varepsilon-1} y_t \left(\frac{1}{p_{it}}\right)^{\varepsilon} \tag{2}$$

where p_{it} denotes the price of the ith intermediate good and $-\varepsilon$ is the price elasticity of demand. TFP and a positive shocks shift the demand curve upward.

2.2. Intermediate goods

The differentiated intermediate goods firms are characterized with certain features. First is the presence of specialization. Knowledge is firm-specific, and hence the production of intermediate goods. Thus, each intermediate goods firm has some monopoly power over its price. Consequently, the rate of returns and earnings are different among firms in this sector. Second, a firm in this sector engages in an in-house R&D investment to expand its specialized knowledge stock. The R&D investment is the only vehicle of technical progress. Third, individuals own firms, with a one-to one correspondence between firms and individuals. This implies the capital market is missing, similar to Benabou (2000, 2002, 2005).

As in Aghion et al. (2015), the ith firm in the intermediate goods sector needs $1/h_{it}$ units of labour to produce one unit of its variety:

$$x_{it} = h_{it} l_{it} \tag{3}$$

where h_{it} represents the stock of the firm specific knowledge, generated through in-house R&D activity, which is specified below; and, l_{it} is the raw labour input. Each period, the firm's profit consists of revenue from the sale of the intermediate good, x_{it}, net of the total labor cost $(l_{it} w_{it})$ where w_{it} is the wage rate per unit of labor.

Thus, the firm has the following static optimization problem,

$$\max_{p_{it}} \pi_{it} = p_{it}\left(x_{it}, .\right) x_{it} - w_{it} l_{it}$$

subject to the demand function (2). Plugging in l_{it} from (3) and x_{it} from (2) into the above and maximizing it leads to the following pricing:[12]

$$p_{it} = \left(w_{it}/h_{it}\right) \varepsilon / \left(\varepsilon - 1\right) \tag{4}$$

While w_{it}/h_{it} is the marginal cost of producing a unit of the intermediate input, the elasticity of substitution, ε, determines the mark-up over this cost.

The ith agent income, which is the sum of wages and profit income, is given by:

$$y_{it} = p_{it} x_{it} \tag{5}$$

Using (2), (3) and (4) into (5), one obtains:

$$y_{it} = a\phi_{it} \left(l_{it} h_{it}\right)^{\alpha} y_t^{1-\alpha} \tag{6}$$

where $a \equiv a_1^{\alpha}$ and $\alpha \equiv (\varepsilon - 1)/\varepsilon$.[13]

Equation (6) is the reduced form of individual production function that matches individual income to output production, characterized by constant returns to scale

[12]Note that there is no explicit labour market in this economy as each household owns a firm and is self employed. See Angeletos and Calvet (2006) for a similar setup.

[13]To derive (6), equate (2) and (3) to solve $p_{it} = \phi_{it} a_1^{(\varepsilon-1)/\varepsilon} y_t^{(1/\varepsilon)} (h_{it} l_{it})^{-(1/\varepsilon)}$ which upon plugging this and (3) into (5) and rearranging terms yields (6).

at individual (h_{it}) and aggregate accumulative factors (h_t) in total.[14] However, there is diminishing returns to individual factor. This shows that the model is basically in the spirit of the Arrow (1962) and Romer (1986) learning-by-doing endogenous growth models.

Aggregating (6) leads to aggregate income (y_t):[15]

$$y_t = la^{1/\alpha} h_t \exp\left(d_t\right) \tag{7}$$

where d_t is a composite parameter, which captures the relationship between aggregate income and inequality:

$$d_t \equiv 0.5\alpha\left(\alpha - 1\right)\sigma_t^2$$

where σ_t^2 denotes inequality at time t. Therefore, the aggregate production has a simple Ak feature, in the spirit of Romer (1986), except that inequality drives the dynamics in aggregate output.

2.3. In-house R&D

Similar to Ziesemer (1990, 1995), private knowledge production includes productive government investment. In the spirits of Smulders and van de Klundert (2004),

[14] As we see later, y_t is a linear function of h_t and $l_{it} = l$, which is constant.

[15] Aggregating (6) from both sides gives

$$Ey_{it} = ay_t^{1-\alpha} E\left(\phi_{it}\left(l_{it}h_{it}\right)^{\alpha}\right) \Leftrightarrow y_t = ay_t^{1-\alpha} l^{\alpha} E\left(h_{it}^{\alpha}\right)$$

We used the fact that ϕ_{it} is i.i.d., $E\phi_{it} = 1$ and $l_{it} = l$ as we see later in (16b). Then, from the normal and lognormal relation, we have $E\left(h_{it}^{\alpha}\right) = h_t^{\alpha} e^{0.5\alpha(\alpha-1)\sigma_t^2}$. Substituting the latter into the above gives (7).

each intermediate goods firm invests in an in-house R&D to produce the knowhow using the following knowledge production function:

$$h_{it+1} = \zeta_{it+1} h_{it}^{\theta} s_{it}^{\nu} g_{it}^{\lambda} \tag{8}$$

Government intervenes in the R&D process by investing in public R&D input (g_{it}) that uses to complement the private sector, but with a redistributive intent. According to (8), knowledge is a product of both public and private investment (g_{it} and s_{it}, respectively), past knowledge stock of the firm (h_{it}) and idiosyncratic ability shocks (ζ_{it+1}). $\{\theta, \lambda, \nu\} \in (0, 1)$ denote knowledge elasticicities. ζ_{it+1} is i.i.d. and follows a log-normal distribution with mean one and a constant variance, $\ln \zeta_{it+1} \sim N\left(-\varrho^2/2, \varrho^2\right)$. The production function (8) exhibits constant-returns to scale ($\theta + \nu + \lambda = 1$) that makes the growth process endogenous as in any standard growth model.

2.4. Public investment in R&D

Public R&D investment is financed levying a proportional income tax (τ) on the final goods. The government balances the budget as in the growth and public investment literature (e.g., Barro, 1990):

$$g_t = \tau \int_0^1 y_{it} di = \tau y_t \tag{9}$$

where g_t denotes the total public investment in R&D and τ is the public expenditure GDP ratio. Thus, a fraction of aggregate income is used to finance the public good.

We make a critical departure from the extant literature by abstracting from a blanket public investment provision in R&D. Rather the government expenditure on

R&D has a redistributive component. Public R&D investment does not necessarily benefit individual firms proportionally. Small firms may benefit disproportionately from low-tech technologies while large firms do from high-tech technologies. For instance, an innovation of a pedal-powered tractor is more beneficial to small-scale farmers vis-à-vis a high-powered tractors benefiting large commercial farms. Formally, this can be expressed as

$$g_{it} = e^{\iota_t} \left(h_{it}/h_t \right)^{\omega} g_t \tag{10}$$

where e^{ι_t} is an adjustment factor used to make the aggregation "heterogeneity neutral" as in Benabou (2000) and given by,

$$\iota_t \equiv 0.5\omega(1-\omega)\sigma_t^2 \tag{11}$$

Aggregating through (10) leads to a balanced government budget, given by (9).[16]

The key policy parameter is ω, which brings a redistributive element in the public spending on R&D (10). The magnitude and sign of ω determines the weight and nature of this redistribution, whether public R&D investment is progressive or regressive. We see this immediately when substituting (7) and (9) into (10):

$$\ln g_{it} = \ln \Theta_t + \omega \ln h_{it} + (1-\omega) \ln h_t \tag{12}$$

[16] Note that all our results hold in the case of $e^{\iota_t} = 1$, which could happen if $\omega = 0$ or $\omega = 1$. This will be discussed later.

where

$$\Theta_t \equiv \tau l a^{1/\alpha} \exp\left(d_t + \iota_t\right)$$

Therefore, the government public expenditure is nothing but a loglinear resource redistribution.

If $\omega = 0$, then $g_{it} = g_t$ which makes the government spending on R&D a pure public good. However, if $\omega = 1$, the government expenditure is similar to a private investment subsidy. When $\omega > 0$, for a given amount of public good g_t, the public service received by the ith individual, g_{it}, is higher when h_{it}/h_t is larger. In other words, agents with above average knowledge receives higher service from the public good than the average. The reverse is true when $\omega < 0$. Based on this consideration, we refer to negative ω and positive ω as progressive and regressive public expenditure respectively in line with the literature in progressive/regressive taxation.[17]

Substituting (10) into (8), one obtains:

$$h_{it+1} = \zeta_{it+1} h_{it}^{\theta+\omega\lambda} s_{it}^{\nu} \left(e^{\iota_t} g_t / h_t^{\omega}\right)^{\lambda} \tag{13}$$

Note that in the special case, $\omega = \nu = 0$, equation (13) reduces to Ziesemer (1990, 1995). The parameters θ and λ are *ex ante* knowledge elasticities whereas $\theta + \omega\lambda$ and $\lambda - \omega\lambda$ capture *ex post* intergenerational linkages associated with firm level knowledge production. The latter account for individual and aggregate factors in

[17] In Benabou (2000, 2002), for instance, after-tax income is given by $\widehat{y}_{it} = \left(y_{it}/\widetilde{y}_t\right)^{\tau} \widetilde{y}_t$ where y_{it} and $\widetilde{y}_t$ represent before-tax and threshold incomes, respectively; τ, which has basically a similar role as ω has in our model, represents the marginal tax rate whose sign determines the progressivity/regressivity of the tax schedule. We differ from this literature, however, as we focus on the expenditure side.

the economy respectively, because from (7) and (9), g_t is a linear function of h_t. The term $\omega\lambda$ captures the redistributive nature of the public variable and its implication for individual knowledge accumulation. Redistribution thus impacts the economy via the effect on private and public knowledge elasticities. We see later individual optimal decision is crucially dependent on $\theta + \omega\lambda$, which is also the main determinant of the evolution of inequality that in turn determines other macroeconomic dynamics.[18]

2.5. Household

There is a continuum of households indexed between $(0, 1)$. Households own the firms and also work in the R&D sector.[19] Similar to Benabou (2002, 2005), the credit and insurance markets are missing. We also assume members of the households are endowed with units of labour that they supply elastically. Agents maximize their utility in accordance of the following function:

$$U_{i0} \equiv \max_{\{c_{it}, h_{it+1}, l_{it}\}_0^\infty} \mathrm{E}_0 \sum_{t=0}^{\infty} \rho^t \left(\ln c_{it} - l_{it}^{\eta}\right) \tag{14}$$

where $\eta > 1$; E_0 is an individual's expectation given information at date 0. The budget constraint is given by:

$$c_{it} + s_{it} = (1 - \tau)\, y_{it} \tag{15}$$

[18] For a given private investment (s_{it}) and public investment (g_t), the rate of growth of knowledge is not self sustained (decreases in h_{it}) as in any standard endogenous growth model. The sufficient condition to get a sustained growth is $\theta + \nu + \lambda = 1$.

[19] Other models that use similar type of individual entrepreneurship include Benabou (2000, 2002, 2005) and Angeletos and Calvet (2005, 2006).

where τ represents a flat rate income tax on both wage and profit income which is defined earlier.

Applying standard methods, individual household decision rules can be derived as follows:

$$s_{it} = b\,(1-\tau)\,y_{it} \tag{16a}$$

$$l_{it} = l = (\alpha/\,(\eta\,(1-b)))^{1/\eta} \tag{16b}$$

$$c_{it} = (1-\tau)\,(1-b)\,y_{it} \tag{16c}$$

where

$$b \equiv \rho\alpha\nu/\,(1-\rho\,(\theta+\omega\lambda)) \tag{17}$$

Equations (16) are standard forms from the view point of household optimization. Households supply constant amount of labour while saving rate is independent of rate of returns as a consequence of log utility function. Also, both the saving rate and effort are independent of the flat tax rate τ, due to the logarithmic utility and Cobb-Douglas production functions.[20] At the same time, individuals internalize the effects of their knowledge acquisition on the public service that they receive, as both saving rate and efforts depend on the public parameters ω and λ.

Individual saving rate and effort increase with the discount factor (ρ), elasticity

[20]Ziesemer (1995) and Getachew and Turnovsky (2015), by applying a more general utility and production functions, show flat rate taxes could have redistributive component.

of substitution (ε), intergenerational spillover (θ), and the elasticity of private investment (ν). But, the effect of ω and λ on saving rate and labor supply depend on the sign of ω. Both increase if the R&D program is regressive ($\omega > 0$) while they decrease if it is progressive ($\omega < 0$), the classic efficiency–equity trade off.

We thus have the following proposition:

Proposition 1. *Saving rate and labour supply increase (decrease) with regressive (progressive) public R&D investment.*

Proof. See equations (16a) and (16b). ■

Aggregate Consistency:. Note first that $c_t \equiv \int c_{it}di$, $s_t \equiv \int s_{it}di$, $y_t \equiv \int y_{it}di$, $h_t \equiv \int h_{it}di$ where the left-hand side variables represent aggregate variables. Aggregate savings and consumption are then given by, from aggregating (16a) and (16c), respectively,

$$s_t = (1-\tau)\, b y_t \tag{18}$$

$$c_t = (1-\tau)\,(1-b)\, y_t \tag{19}$$

where aggregate income is given by (7). Finally, combining (18) and (19) with (9), gives:

$$c_t + s_t + g_t = y_t \tag{20}$$

Equation (20) shows the economy-wide budget constraint at equilibrium where aggregate output is allocated between aggregate consumptions, public and private investment.

3. Dynamics

3.0.1. Optimal individual knowledge dynamics

The dynamics of optimal knowledge stock associated to the ith firm is derived from (6), (7), (9), (13) (16a) and (16b):

$$h_{it+1} = a_3 \psi \chi \zeta_{it+1} \phi_{it}^{\nu} h_{it}^{\beta} h_t^{\kappa} \exp\left(\left(\lambda + (1-\alpha)\nu\right) d_t + \lambda \iota_t\right) \tag{21}$$

where $a_3 \equiv a^{(\nu+\lambda)/\alpha} \left(\alpha/\eta\right)^{(\nu+\lambda)/\eta}$

$$\psi \equiv b^{\nu} (1-b)^{-(\nu+\lambda)/\eta} \tag{22a}$$

$$\chi \equiv \tau^{\lambda} (1-\tau)^{\nu} \tag{22b}$$

$$\beta \equiv \theta + \omega\lambda + \alpha\nu \tag{22c}$$

$$\kappa \equiv \lambda + (1-\alpha)\nu - \omega\lambda \tag{22d}$$

Equation (21) captures the optimal dynamics of knowledge at a firm level. τ and ω are policy controls while the rest are structural parameters. Such policies impact individual knowledge production function via the TFP terms ψ and χ. The dynamics of optimal individual knowledge also depends on the current individual and aggregate knowledge variables, idiosyncratic risks both in the final goods (ϕ_{it}) and R&D sectors (ζ_{it+1}) and current inequality. Risks in the final goods sectors affect individual savings and investment indirectly via individual income whereas ability shocks in the knowledge sector have a direct impact. The exponential term in (21) captures the relationship between inequality and individual knowledge dynamics.

Negative d_t reflects the negative effects of inequality on knowledge production at firm level. Through its impacts on aggregate and subsequent individual savings and investment, inequality negatively impacts individual knowledge accumulation.

3.1. Inequality dynamics

The dynamics of inequality is also derived from (21), by taking the log and variance,

$$\sigma_{t+1}^2 = \nu^2 \varkappa^2 + \varrho^2 + \beta^2 \sigma_t^2 \tag{23}$$

Given $\beta \in (0,1)$, (23) is a stable dynamics that converges to a steady state inequality. The variance of the idiosyncratic shocks ($\varkappa^2$ and ϱ^2) will determine the long-run property of the model. Volatility in the final goods sector affects inequality via its effect on individual savings while volatility in the R&D sector directly impacts inequality dynamics. The root of the dynamics of inequality is determined by β, which, in turn, is a function of policy and structural parameters, ε, λ, ω, ν and θ. Higher intergenerational linkage (higher θ) results in higher transitional inequality. Better private investment technology (higher ν) implies slower convergence in inequality. Private R&D investment elasticity (ν) also impacts inequality through individual response to luck, with a strong implication to long-run inequality.

The effect of the public variables on the dynamics of inequality rather depends on its redistributive feature (the sign of ω). If $\omega < 0$, higher elasticity of public R&D investment (higher λ) leads to faster convergence of inequality, and conversely. If $\omega = 0$, i.e. public investment in R&D is proportionally provided and the elasticity λ

has a neutral effect in inequality. We thus have the following proposition:

Proposition 2. *A regressive (progressive) R&D investment aggravates (mitigates) transitory inequality. In other words, if $\omega > 0$ ($\omega < 0$), given σ_t^2, σ_{t+1}^2 increases (decreases) in ω, and conversely.*

Proof. From (23), if $\omega < 0$, for given σ_t^2 then σ_{t+1}^2 decreases in $|\omega|$, and conversely.

■

3.2. Income and consumption inequality

Income inequality (σ_{yt}^2) is a simple transformation of knowledge inequality based on the reduced form production function (6). Since equilibrium labour supply, l_{it} is constant,

$$\sigma_{yt}^2 = \varkappa^2 + \alpha^2 \sigma_t^2 \tag{24}$$

Since private consumption (c_{it}) is proportional to private income, the consumption inequality (σ_{ct}^2) is equal to income inequality (σ_{yt}^2) in this setting. Therefore, unless idiosyncratic risks are high, consumption and income inequalities are expected to be smaller than σ_t^2.

3.3. Aggregate wealth and growth dynamics

Aggregating individual knowledge dynamics (21) gives the dynamics of aggregate knowledge:

$$h_{t+1} = a_3 \psi \chi h_t^{\theta+\nu+\lambda} \exp\left(\sigma_t^2 \left(\pi + \digamma + q\right)\right) \tag{25}$$

where

$$q \equiv 0.5\alpha \left(\alpha - 1\right) \left(\lambda + \left(1 - \alpha\right) \nu\right) < 0 \tag{26a}$$

$$\digamma \equiv 0.5\beta \left(\beta - 1\right) - \omega\lambda \left(\omega - 1\right) < 0 \text{ if } \omega < 0 \tag{26b}$$

$$\pi \equiv 0.5\nu \left(\nu - 1\right) \varkappa^2 < 0 \tag{26c}$$

Credit market imperfections, combined with diminishing returns to individual investment, are crucial for the transitional dynamics of the economy, which are driven entirely by the evolution of inequality. This contrasts sharply with Ak type endogenous growth models where the economy jumps to its long-run equilibrium in the first instance. Lack of credit availability makes initial individual productivity differences persist. With diminishing returns to investment, the poor have a higher marginal product than the rich which translates into differences in growth rates among them. However, inequality does not degenerate in equilibrium due to the presence of uninsured idiosyncratic shocks.

4. Steady state

Note that given $\beta \in (0, 1)$, which is the sufficient condition for the stability of the distributional dynamics, (23) converges to a unique inequality equilibrium. But, with constant-returns to scale in knowledge production, inequality is the only source of dynamics in the economy. As inequality converges to its equilibrium level, growth also converges to its steady-state level. In this case from (23) and (25), long-run inequality and growth are given by:[21]

$$\sigma^2 = \left(\nu^2 \varkappa^2 + \varrho^2\right) / \left(1 - \beta^2\right) \tag{27a}$$

$$1 + \gamma = a_3 \psi \chi \exp\left(\sigma^2 \left(\pi + \digamma + q\right)\right) \tag{27b}$$

where π, $\digamma$ and q are defined in (26).

γ is the steady-state growth rate of the economy. The long-run equilibrium of the economy is a balanced growth path, with a constant non-zero level of inequality (see Appendix A).

Several important results follow from (27). First, steady-state inequality increases in volatility. Second, Proposition 2 also holds in the steady state. Third, in many cases, the relationship between inequality and long-run growth is negative. If $\omega < 0$, then $\digamma < 0$, inequality has unambiguously a negative impact on long-run growth. This is easily seen as all the terms in the bracket in (27b) are negative. Also if $\omega > 0$, although we cannot rule out a positive relationship, for more plausible values of the parameters, the relationship remains negative. Finally, redistributive policy (sign of ω) impacts long-run growth directly, via its effect on agents' savings and effort, and indirectly, via the growth–inequality trade off.

[21] Appendix A provides a detail on the derivation of the balanced growth path.

5. Optimal redistributive policies

5.1. Growth maximizing policies

Should the government innovation policy be progressive or regressive to maximize growth? For a homogeneous economy (especially, $h_{i0} = h_0$ and, hence, $\sigma_0^2 = 0$), the choice of ω is straightforward. In order to maximize growth, public R&D should be regressive ($\omega > 0$) and should take the maximum attainable value because regressivity *only* promotes efficiency via agents' savings and effort decisions.

For the heterogenous case, however, this may not be necessarily true. The policy effect on growth could be rather nonlinear, as both inequality (σ^2) and ψ could increase in ω. On the one hand, a regressive R&D policy encourages growth due to its positive impact on savings and effort; on the other, it has a negative impact on growth via the inequality–growth trade off.

Proposition 3. *(i) If $\sigma_0^2 = 0$, a regressive R&D policy unambiguously promotes growth. (ii) If $\sigma_0^2 \neq 0$ and $\varkappa^2 \neq 0$ (or $\varrho^2 \neq 0$), the long-run growth effect of ω is ambiguous.*

Proof. See Appendix C. ■

We can have further insight on the nonlinearity, if we specify plausible parameter values for real economies. We fix the subjective discount factor ρ at 0.99, as in numerous macroeconomic studies. Following Benabou (2002), we set the intertemporal elasticity of substitution $\epsilon \equiv 1/(\eta - 1)$ to 0.20. The elasticity of substitution between intermediate goods is fixed at 6 (Kollmann, 2002). The initial knowledge (h_0) and the initial distribution of knowledge (σ_0^2) are normalized at unity. Getachew and Turnovsky (2015) consider a 0.4 standard deviation for the logarithm of idiosyncratic shocks. Based on this, we set $\varkappa^2 = \varrho^2 = 0.16$.

Using the World Bank (2015) database for the period 2005-2014, the average public and private R&D spending GDP ratio is computed as 2.81% for the US. To bring the R&D feature of the model closer to the data we calibrate the public R&D spending ratio (τ) at 0.548%. The parameter ν represents the elasticity of knowledge production with respect to private R&D spending. Following Jones and Williams (2000), we set ν equal to 0.5. The value for the elasticity of public knowledge widely varies among empirical estimates.[22] We use Levy's (1990) estimate for the public investment elasticity of private R&D for nine OECD countries between 1963 and 1984, which is about 0.34 and consistent with the estimate of Leyden and Link (1991).[23] This implies $\theta = 0.16$. Regarding the redistributive policy parameter, ω we set a baseline value at unity, which implies that the inequality adjustment coefficient $e^{\iota t} = 1$. It turns out that this baseline value of ω is close to the socially optimal ω reported later. Finally, we calibrate a_1 to reproduce a 2% average annual growth rates of GDP. Table 1 summarizes the benchmark parameter values.

[22] See David et al. (2000) for the survey of the literature.

[23] Leyden and Link's estimate is based on a 1987 data set of 137 R&D laboratories for the U.S. industries.

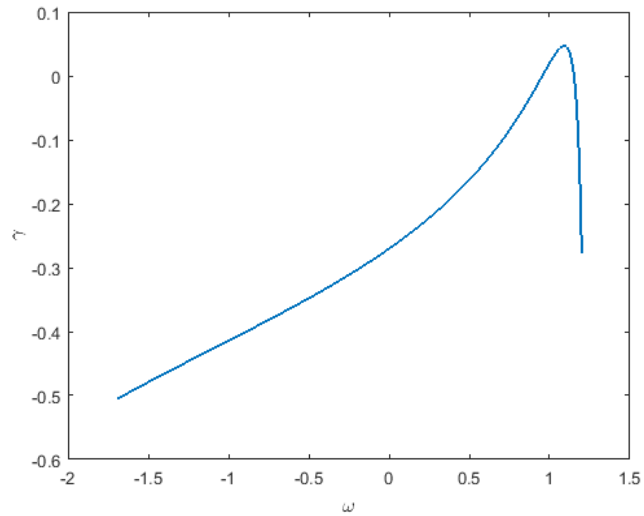
Table 1: Baseline values

ρ	0.99
λ	0.16 (Levy, 1990)
ν	0.5 (Jones and Williams, 2000)
θ	$1-\lambda-\nu$
ϵ	0.20 (Benabou, 2002)
$\varkappa^2=\varrho^2$	0.16 (Getachew and Turnovsky, 2015)
τ	0.00548 World Bank (2015)
a_1	2.845 (Reproduces 2% annual growth rate)
ε	6.00 (Kollmann, 2005)
ω	1.00

Applying these values to (27b), we find that the growth maximizing policy is quite regressive (see Figure 3). Growth is in particular maximized when $\omega = 1.01$.[24]

[24] Note that there is a restriction on ω for a stable inequality dynamics (23), $0 < \beta < 1$. Given our calibrated values, this means $-1.6961 < \omega < 1.2451$.

Figure 3: Growth Maximizing Redistributive R&D Policy



5.2. Welfare maximizing redistributive innovation

While growth and inequality are important macroeconomic variables, the economic significance of any policy should be basically judged in terms of its impacts on social welfare. Given that $V_{i0} = \max U_{i0}$ is the discounted sum of individual welfare, its aggregation across the entire population leads to the discounted sum of aggregate welfare: $W_0 = \int_0^1 V_{i0} di$. Then, the steady-state aggregate welfare is given by (see Appendix B for details):

$$W = (1-\rho)^{-1} \left(\ln c_0 - 0.5\sigma_c^2 - l^\eta \right) + \rho / (1-\rho)^2 \ln (1+\gamma) \tag{28}$$

where

$$c_0 = (1-\tau)\,(1-b)\,y_0 = (1-\tau)(1-b)la^{1/\alpha}h_0 \exp(0.5\,(\alpha-1)\,\sigma_0^2) \tag{29}$$

$$\sigma_c^2 = \varkappa^2 + \alpha^2\sigma^2 \tag{30}$$

where c_0 and σ_c^2 are initial aggregate consumption and consumption inequality, respectively.

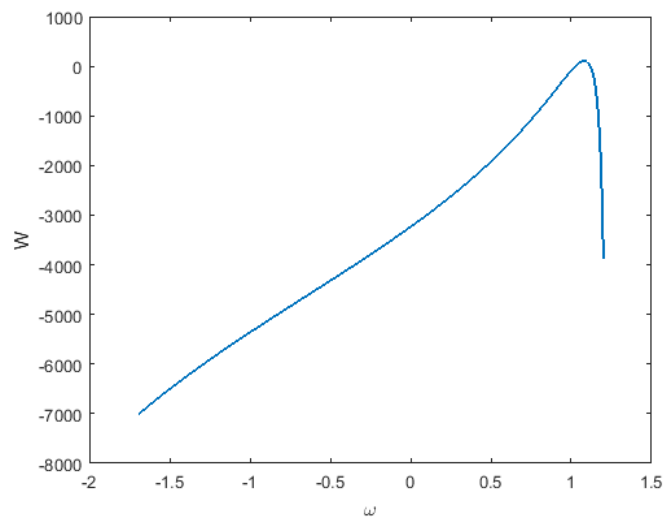
The first term in (28) captures the discounted initial aggregate welfare (at $t = 0$). Given that h_0 is predetermined, so is σ_0^2, which has a negative impact on welfare. Since individuals derive a negative utility from increased effort, the policy effects (ω) on the initial welfare is nonlinear. While a higher ω may negatively affect initial welfare via an effect on efforts and inequality, it may increase it through boosting initial income (higher y_0).

The second term in (28) comes from the economy-wide growth rate, which captures individuals' rewards for saving and investing in their future. In this case, any effect policy has on growth would directly pass to welfare. A higher ω unambiguously raises the steady state inequality, σ^2 as seen from (27a) and through this channel it lowers growth and hence welfare. On the other hand, it promotes investment and raises effort that could in turn raise growth and thus welfare.

For reasonable parameter values, welfare maximizing R&D policy is also regressive due to a strong growth effects on aggregate welfare (see Figure 4). A regressive innovation policy with positive ω clearly dominates a neutral second best innovation policy with zero value of ω. Aggregate welfare is in particular maximized when

$\omega = 0.996$, which is slightly less regressive than the case for growth due to an additional adverse effect of a regressive R&D policy on initial consumption. For our calibrated model, the optimal ω is thus close to unity.

Figure 4: Welfare Maximizing Redistributive R&D Policy



5.3. Optimal flat rate tax

Although our main focus here is on ω, note that both ω and τ are policy variables. With respect to τ, the optimal tax (the tax that maximizes welfare) is derived from (28),

$$\partial W/\partial \tau = 0 \Rightarrow \tau^* = \rho\lambda/\left(1 + \rho\left(\nu + \lambda - 1\right)\right) \tag{31}$$

which is independent of ω. It bears emphasizing that there is no distributional conflict that arises in the choice of the tax rate τ, which is similar to Glomm and Ravikumar (1992) but in contrast to Ziesemer (1990,1995).

The growth maximizing tax rate (τ_g^*) could also be easily computed from (27b):

$$\partial\gamma/\partial\tau \equiv \tau_g^* = \lambda/\left(\lambda + \nu\right) \tag{32}$$

which is also independent of redistribution, ω. τ_g^* reaches its upper bound when $\nu = 0$ – when there is, no, or, little private investment in R&D. Comparing the two,

$$\tau^* \leq \tau_g^* \Leftrightarrow \rho \leq 1$$

the welfare maximizing tax rate is higher than the growth maximizing tax rate because the individual is impatient ($\rho < 1$) and prefers to avoid lower consumption.

5.4. Public versus private R&D investment

A private subsidy environment ($\omega = 1$) uniformly dominates the pure public good environment ($\omega = 0$) both in terms of growth and welfare (see Figure 5 and 6, respectively). The optimal tax rate (τ) is independent of ω and the output elasticity of h_t, consistent with the discussion in the preceding section. This is a redeeming feature of our simple model because it implies that maintaining equity does not entail additional tax cost. Growth and welfare maximizing tax rates are 0.405 and 0.4, respectively.

Figure 5: Growth rates: $\omega = 0$ versus ω=1

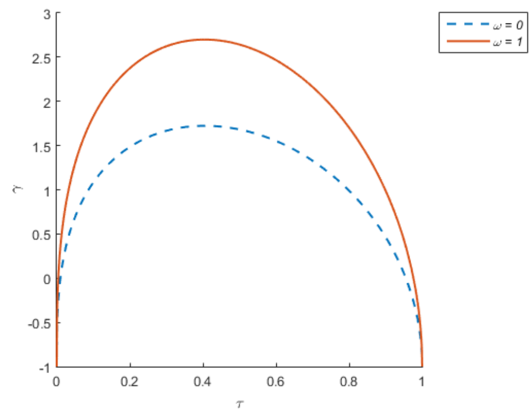
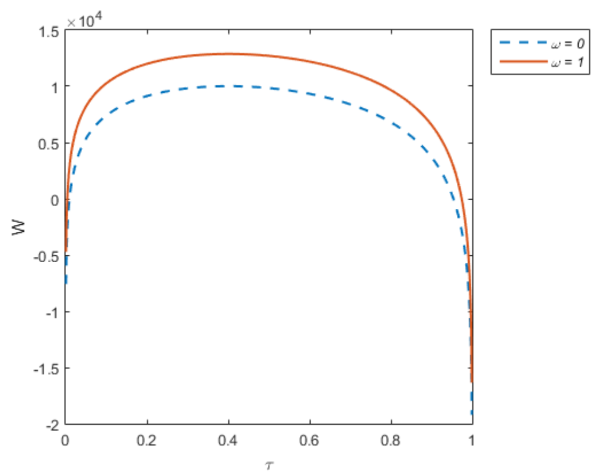


Figure 6: Welfare: $\omega = 0$ versus ω=1



5.5. Sensitivity Analysis

For our baseline calibrated model the optimal redistributive innovation is regressive. The adverse effect of regressive innovation on inequality is dominated by the positive incentive effect on saving and labour supply. A higher idiosyncratic risk either in the form a higher ϱ or higher $\varkappa$ elevates the consumption inequality σ_c^2 in the steady state welfare function (28) and thus it lowers the optimal regressiveness of R&D policy. Table 2 presents the sensitivity of the optimal ω with respect to a higher variances of idiosyncratic shocks to output and knowledge production, namely $\varkappa$ and ϱ respectively. Higher variances of these two shocks unambiguously lower the optimal ω. The decline in ω is sharper for a rise in $\varkappa$. For a sufficiently high value of the ability shock ($\varkappa$) variance of the knowledge production, the optimal ω is negative which means that the R&D policy turns progressive in nature.

Our sensitivity analysis suggests that a higher uninsurable risk either in the goods producing or knowledge producing sector lowers the optimal regressiveness of the R&D policy. This result has implications for the negative relation between inequality and R&D intensity for the SSA countries reported in Figure 2. Since the uninsurable risk in output and knowledge production technology is too high in SSA countries due to lack of well developed insurance markets, a benevolent government responds to this high uninsurable risk by making the R&D less regressive and possibly progressive to ensure that the inequality does not escalate.

Table 2: Effects of idiosyncratic risks and ability on redistributive innovation policy (ω)

$\varkappa \longrightarrow$ $\varrho \downarrow$	0.4	0.8	1.2	1.6	2.0	2.4
0.4	0.996	0.841	0.611	0.364	0.14	-0.052
0.8	0.777	0.651	0.474	0.281	0.093	-0.078
1.2	0.658	0.544	0.390	0.221	0.054	-0.102
1.6	0.599	0.490	0.344	0.185	0.028	-0.120
2.0	0.568	0.460	0.318	0.164	0.012	-0.133
2.4	0.549	0.442	0.302	0.151	0.000	-0.141

6. How regressive is R&D in US states?

In Figure 1 we presented a stylized fact using state level US data that regional inequality is higher in the US where the intensity of R&D spending is higher. In this section, we provide an estimate of regressivity of R&D (measured by ω) in each state based on our model results. We exploit two key equations of our model (namely (22c) and (27a) to back out ω from the regional GINI coefficient data for the US.

One computes ω from the model using the US state level GDP data as follows. Exploiting the lognormality property of our model, compute the mean to median ratio using the variance of $\ln y_{it}$. The steady state mean to median ratio of income (m_y) is thus given by,

$$m_y = \frac{\exp(\mu_y + 0.5\sigma_y^2)}{\exp(\mu_y)} = \exp(0.5\sigma_y^2) \tag{33}$$

which implies that $\sigma_y^2 = 2 \ln m_y$. Then, using the equation (24), rewrite σ^2 in terms of σ_y^2:

$$\sigma^2 = \frac{\sigma_y^2 - \varkappa^2}{\alpha^2} \tag{34}$$

In the next step, using the inequality equation (27a), compute β:

$$\beta = \left[1 - \frac{\nu^2 \varkappa^2 + \varrho^2}{\sigma^2}\right]^{0.5}$$

Finally, using (22c), compute ω:

$$\omega = \frac{\beta - \theta - \nu\alpha}{\lambda} \tag{35}$$

The annual median household income is available for each state, from the Bureau of Economic Analysis. However, there is no corresponding mean household income available. We, therefore, constructed a series for it by using per capita state income, population and the number of households in each state.[25] Multiplying the per capita income by population upon dividing it by the number of households, we generate a series for income per household data for each state over the sample period 2005-2015. We compute the time average of mean and median household income for computing the variance of log income using (33). Following the steps above, we compute ω for each US state. While computing ω , we fix the rest of the parameters at the baseline levels as shown in Table 1. Our finding is that estimated ω for each state is uniformly

[25] All data are available in the US Bureau of Economic Analysis website (https://www.bea.gov/) for the period 2005-2015.

higher than unity for all states. New York has the highest, at $\omega = 1.13$, while Utah has the lowest, at $\omega = 1.04$. Therefore, the regressivity of innovation policies in US states is higher than the socially optimal regressive innovation from our baseline model, which is around unity.

7. Conclusion

R&D policy has uneven impacts on the economy, a fact which has not received much attention in the literature. While regressive R&D policy could escalate economic inequality, it could promote growth through creating incentives. On the other hand, progressive R&D policy could benefit poor at the expense of long run growth. Our stylized facts suggest that in the US, R&D policy has significant regressive consequences while in SSA and other countries it is progressive in nature. In this paper, we have developed a heterogenous-agent growth model with in-house R&D to understand these broad empirical regularities. In our model, growth and inequality are endogenously determined. We show that a regressive R&D policy unambiguously escalates economic inequality while it promotes growth by incentivising private innovation and labour supply. Such impacts may partly be compromised due to the inverse inequality-growth relationship; however, for plausible parameter values, we have found that the optimal R&D policy is regressive in nature because the positive growth effect outweighs the negative inequality effect. Using US state level data, we have backed out the regressivity policy parameter from our model and have found that the R&D policy in the US is indeed regressive in nature and it is more regressive than the optimal level suggested by our baseline model. For SSA countries, our model implies that the observed negative relation between R&D intensity and inequality could reflect government's progressive redistributive stand due to high uninsurable risk in these economies, which otherwise could make the burden of inequality socially unbearable.

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Appendix

A. Aggregate wealth and growth dynamics

From (7), (9) and (19), all aggregate variables except aggregate knowledge grow at the same rate:

$$1 + \gamma_t \equiv y_{t+1}/y_t = g_{t+1}/g_t = c_{t+1}/c_t = \Omega_t h_{t+1}/h_t \tag{A.1}$$

where

$$\Omega_t \equiv \exp\left(0.5\alpha\left(1-\alpha\right)\left(\sigma_t^2 - \sigma_{t+1}^2\right)\right)$$

Note that (A.1) holds along the transitional dynamics where inequality evolves based on its own past history as in (23). In the steady state where $\sigma_{t+1}^2 = \sigma_t^2 = \sigma^2$ and $\Omega_t = 1$, the economy will be in a balanced growth path (BGP) where $\gamma_t = \gamma$.

B. Derivation of the steady state welfare

Note first that the discounted sum of individual welfare is simply $V_{i0} = \max U_{i0}$. Aggregating V_{i0} across the entire population leads to the discounted sum of aggregate welfare:

$$W_0 = \int_0^1 V_{i0} di$$

Using (14), we have

$$
\begin{aligned}
W_0 &= E_i E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln c_{it} - l_{it}^{\eta}\right) \\
&= E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln c_t - 0.5\sigma_{t,c}^2 - l^{\eta}\right)
\end{aligned}
$$

Then, the steady-state discounted expected aggregate welfare is given by,

$$
\begin{aligned}
W &= E_0 \sum_{t=0}^{\infty} \rho^t \left(\ln \left(c_0 \left(1+\gamma\right)^t\right) - 0.5\sigma_c^2 - l^{\eta}\right) \\
&= \rho / \left(1-\rho\right)^2 \ln \left(1+\gamma\right) + \left(\ln c_0 - 0.5\sigma_c^2 - l^{\eta}\right) / \left(1-\rho\right)
\end{aligned}
$$

where c_0 is given by, from (18) and (7),

$$
c_0 = (1-\tau)(1-b) l a^{1/\alpha} h_0 \exp(d_0)
$$

C. Proof for Proposition 4

(i) If $\sigma_0^2 = 0$, which implies $h_{i0} = h_0$, then, from (21), $\sigma_1^2 = \sigma^2 = \nu^2 \varkappa^2 + \varrho^2$. It is then straightforward to see γ increases in ω.

(ii) If $\sigma_0^2 \neq 0$ and $\varkappa^2 \neq 0$ (or $\varrho^2 \neq 0$), then from (21), $\sigma^2 = \nu^2 \varkappa^2 / \left(1-\beta^2\right)$ (or $\sigma^2 = \varrho^2 / \left(1-\beta^2\right)$). In this case, both σ^2 and ψ increase in ω in (27b), leading to an ambiguous effect of ω on growth.