Can subsidies rather than pollution taxes break the trade-off between economic output and environmental protection?

by Thomas I. Renström*, Luca Spataro‡, Laura Marsiliani**

Abstract

We build a general equilibrium dynamic model in which individual investors are endowed with "warm-glow" preferences a la Andreoni (1990) so that they feel partly responsible for the pollution content of their portfolio. Through investors' portfolio choice, firms are induced to engage in costly abatement activities, given that higher pollution also implies a higher cost of capital. In this scenario, we characterize the equilibrium of the economy and investigate, through a fiscal reform analysis, the effects of such tax instruments on the equilibrium scale of the economy, per-capita consumption, pollution abatement and "pollution premium". We show that an increase of the pollution tax, while reducing pollution, also depresses consumption, the scale of the economy and the pollution premium. On the contrary, an increase of subsidies on abatement activity increases the scale of the economy and can also decrease pollution and the pollution premium and increase per-capita consumption. All our results have relevant testable implications, which we leave for future empirical research.

JEL Classification: D21, D53, G11, H21, H23, M14, Q58.

Keywords: Socially responsible investment, corporate social responsibility,

pollution, fiscal policies.

* Durham University Business School (UK). Email: t.i.renstrom@durham.ac.uk.

‡ Dipartimento di Economia e Management, University of Pisa (Italy). Email: luca.spataro@unipi.it

** Durham University Business School, and Durham Energy Institute (UK). Email:

laura.marsiliani@durham.ac.uk

1. Introduction

In this paper, we analyse the effects of fiscal policies aimed at reducing pollution in the context of financial markets populated by socially responsible investors. According to Eurosif, socially responsible investing (SRI) "is a long-term oriented investment approach, which integrates ESG [i.e. Environmental, Social, Governance] factors in the research, analysis and selection process of securities within an investment portfolio. It combines fundamental analysis and engagement with an evaluation of ESG factors in order to better capture long term returns for investors, and to benefit society by influencing the behaviour of companies." (Eurosif 2016, p. 9).[1] As a consequence, SRI has been argued to be a possible instrument to improve environmental quality through a market mechanism.

On the other hand, under the impulse of several international summits (from Kyoto in 1997 to Paris 2015), many public institutions have been implementing or proposing fiscal and regulatory policies aimed at contrasting the worsening of environmental quality and at increasing the ESG-practices (as for the recent initiatives, see OECD 2017 and, for the EU, see Eurosif 2018) For example, in 1995 the Dutch government has launched the Green Funds Scheme, a tax incentive scheme for investors into green initiatives, while in the U.S. there are examples of tax-credit bonds, (bond investors receive tax credits instead of interest payments so issuers do not have to pay interest on their green bond issuances) or tax-exempt bonds. In 2018, total environmental tax revenue in the EU amounted to €324.6 billion, about 2.4% of EU GDP and 6.0% of total EU government revenue from taxes and social contributions. Taxes on energy accounted for the largest share (77.7%) of total revenues from environmental

[1] Hence, SRI is a process of identifying and investing in companies that meet certain standards of Corporate Social Responsibility (CSR) through such activities and strategies as positive or negative screening, shareholder advocacy, impact and community investing (for more details see GSIA, 2016). For a recent review of economic literature on CSR, see Brekke and Pekovic (2018).

taxes, followed by taxes on transport (19.1%) and pollution and resources taxes (3.3%) (Eurostat 2020).

As far as the welfare implications of such policies are concerned, it is well known that market-based instruments like taxes and subsidies are superior, compared to other tools, in terms of economic efficiency if they are aimed at addressing environmental externalities and in the presence of perfectly competitive markets (Baumol and Oates 1988, Dröge and Schröder 2005, Renström et al. 2019).[2]

Given that the positive implications of such policies have been not fully explored, and they are still embryonic in the case of economies that are populated by socially responsible investors, in this paper we aim to fill this gap. More precisely, in a continuous-time model populated by perfectly competitive polluting firms and socially responsible investors, we aim to shed new light on the effectiveness of fiscal policies in enhancing environmental quality and to provide the conditions under which they can also improve the economic performance of the economy, increase per-capita consumption and reduce the pollution premium. For doing so, under the assumption of balanced public policies, we compare the effects of two fiscal instruments, a tax on firms' pollution flow and a subsidy on firms' abatement activities and we show that, while both may succeed in reducing pollution, their effects on the performance of the economy can be quite different: in fact, while the pollution tax always reduces per capita consumption and the capital installed in the economy, the subsidy can increase both.

The economic rationale behind our results is that the tax on pollution corresponds to a tax on profits, and thus, by reducing the marginal productivity of capital, induces firms to reduce the capital invested and thus to reduce pollution flows and the pollution premium.

[2] In fact, it has been shown that the effectiveness of such policy instruments typically decreases in the presence monopoly power of producers (see Buchanan 1969).

Moreover, given that per-capita consumption is proportional to net-of-abatement production, it will be reduced by an increase of a pollution tax.

On the contrary, the subsidy on abatement reduces firms' production costs, so that it will provide an incentive to increase the scale of the firm, thus exerting two opposite effects: on one hand, by increasing the installed capital, it also tends to increase production, per capita consumption and pollution (which is a by-product of production); on the other hand, it tends to increase the resources that the firms devote to abatement, thus reducing pollution and increasing per-capita consumption. While the final result depends on the relative strength of each effect, we show that, under fairly general assumptions on the production and abatement technology and preferences, an increase of the subsidy for pollution abatement generates a reduction of both pollution and pollution premium, while it increases per capita consumption.

Finally, we show that, when the warm-glow parameter increases, other things being equal, investors penalize firms by asking a higher pollution premium, which, in turn, reduces the scale of the economy, pollution, total production and may reduce per-capita consumption.

The paper is organized as follows: in section 2 we discuss the related literature, in sections 3 and 4 we specify the model and we characterize the equilibrium and its stability, respectively; in section 5 we carry out a tax-reform analysis and discuss the results. Section 6 concludes.

2. Related literature

Although several scholars have analysed the issue of environmental quality and fiscal incentives from an economic perspective[3], the economic literature on SRI[4] and on its consequences on taxation is still embryonic and results are mixed.

For example, Heinkel et al. (2001), adopt a one-period model to show that negative screening on polluting firms by fund managers can induce firms to adopt cleaner technologies in order to avoid higher costs of capital. The positive effects of financial markets on environmental quality are also stressed by Dam (2011), who argues that SRI creates a role for the stock market to deal with intergenerational environmental externalities. The author shows that, although socially responsible investors are short-lived, the forward-looking nature of stock prices, reflecting the warm-glow motive, can help to mitigate the conflict between current and future generations. Dam and Scholtens (2015) develop a model that links SRI and CSR, showing that responsible firms display higher returns on assets, although the overall effect on stock market returns depends on the relative strength of supply and demand side effects.

On the other hand, Barnea et al. (2005) argue that negative screening reduces the incentives of polluting firms to invest so that also the total level of investment in the economy decreases. Canwalleghem (2017) argues that SRI may have a mixed effect on firms' incentives to remove negative externalities. In fact, whereas SRI screening incentivizes the removal of externalities (as predicted by Heinkel et al. 2001 and confirmed by the empirical work of Hong and Kacperczyk 2009), SRI trading can disincentivise it when traders disagree on the externality removal's cash flow effects.

---

[3] The seminal work is Sandmo (1975). See also Cremer et al. (2001) and the survey by Bovenberg and Goulder (2002). More recent works on this subject are Bontems and Bourgeon (2005), Goulder and Parry (2008), Gahvari (2014), Jacobs and De Mooij (2015), Kampas and Horan (2016), Belfiori (2018), Pizer and Sexton (2019).

[4] For a survey on the topic, see Renneboog et al. (2008).

Finally, also Graff Zivin and Small (2005) and Baron (2007) also focus on socially responsible firms and financial markets. However, these are partial-equilibrium and static models, in which social responsibility is concerned with charitable giving and not with abatement of externalities or public bads.

While providing interesting results, the above-mentioned literature has not analysed the effects of fiscal instruments in presence of SRI, which is the focus of the present paper.[5] In a recent paper, Renström et al. (2019) present a normative analysis of second-best taxes in presence of socially responsible investors, although disregarding subsidies to abatement activities, and show that SRI is not sufficient to fully restore economic efficiency, so that even in this environment public corrective intervention is called for.

Fiscal policies for pollution abatement are also discussed by Poyago-Theotoky, (2007, 2010), Ouchida and Goto (2014, 2016), Lambertini et al. (2017), although in partial equilibrium static models with imperfect competition. We depart from the latter contributions by assuming perfect competition and adopting a dynamic general equilibrium framework.

Finally, Dam and Heijdra (2011) analyse the effects of SRI and public abatement on environmental quality in a growth model with socially responsible investors and show that SRI behaviour by households partially offsets the positive effects on environmental quality of public abatement policies. However, differently from our paper, the latter contribution does not introduce any abatement technology for firms and adopts lump-sum taxes to finance the government's pollution abatement.

3. The model setup

[5] Martín-Herrán, Rubio (2018) carry out an analysis of second-best taxation for a polluting monopoly.

We specify a continuous-time model, where pollution is a by-product of production activity of profit-maximizing firms, but the latter can engage in (costly) abatement activity, reducing net pollution. We model investors' social-responsibility objective through a warm-glow mechanism as in Andreoni (1990) and Dam (2011)[6], so that they feel partly responsible for the pollution content of their portfolio and, at the equilibrium, ask for a "pollution premium" in order to hold "dirty assets". Through investors' portfolio choice, firms are induced to engage in socially responsible activities (abatement), given that higher pollution also implies a higher cost of capital on the capital markets. Finally, the Government levies a tax on firm's pollution flow and a subsidy on its abatement activity.

In this scenario, we carry out a tax-reform analysis to evaluate the effects of fiscal instruments (i.e. taxes on pollution and subsidies to abatement activities) on the economy scale of production, on pollution, on consumption and on the "pollution premium".

More precisely, we assume an infinite horizon economy, populated by H identical households and J identical firms. At date t, individuals' utility, $u(c(t), p(t))$, is an increasing function of consumption, $c(t)$, and decreasing function of the perceived pollution content, $p(t)$, of their portfolio holdings. The idea is that an individual (as an investor) feels responsibility for the pollution caused by firms it their portfolio (*warm-glow* objective). At each date an individual chooses consumption, portfolio allocation, and savings. We treat government bonds, $b(t)$, as the only clean (pollution free) asset. Firms operate on perfectly competitive markets under constant returns technologies, both in production and abatement activity. They face taxes on pollution flow and subsidies on the amount spent on abatement.

[6] While there is increasing evidence of the very existence of warm-glow preferences (see Andreoni et al. 2017), the exact shape is far from being clear. However, some recent works have produced axiomatizations of the warm glow that can help to characterize its shape (see Evren and Minardi 2017 and the literature therein). In this work, we follow Bernehim and Rangel (2005) when stating that "one can interpret it [the warm-glow] as a reduced form for a variety of mechanisms with starkly differing welfare implications" (p. 63).

3.1. Individuals

An individual household's life-time utility, at date 0, is:

$$U(0) = \int_0^\infty e^{-\rho t} u(c(t), p(t))dt \tag{1}$$

with $\rho > 0$ the intertemporal discount rate, and $u_c > 0, u_p < 0, u_{cc}, u_{pp} < 0$ a. For simplicity we assume that u is additively separable. Labour supply is exogenously given at unity, so total labour supply equals population size, H, (assumed constant). Let $e^j(t)$ denote the number of shares of firm j owned by the individual, $\bar{E}^j$ the total number of shares of firm j, and $\bar{p}^j(t)$ the "pollution content" of firm j (as perceived by the individual), then, as in Dam and Scholtens (2015), the portfolio perceived pollution index $p(t)$ is:

$$p(t) \equiv \sum_{j=1}^{J} \frac{e^j(t)}{\bar{E}^j} \bar{p}^j(t) \tag{2}$$

We also follow previous literature (e.g. Dam 2011 and Dam and Heijdra 2011), in assuming $\bar{p}^j$ is linear in x^j (as any non-linearity can be captured by u):

$$\bar{p}^j(t) = \gamma \cdot x^j(t) \tag{3}$$

where $x^j(t)$ is the flow of pollution produced by the j^{th} firm.[7] Notice that $x^j(t)$ is chosen by firm j, through its production and abatement decision, taking into account it can affect its "cleanness rating." $X(t) = \sum_{j=1}^{J} x^j(t)$ is the aggregate flow of pollution. At date t, the

[7] We assume that pollution is observable by investors, so we ignore any monitoring issues. In this respect see Aldashev et al. (2015).

individual's wealth is

$$a(t) \equiv b(t) + \sum_{j=1}^{J} e^j(t)\, P_e^j(t) \tag{4}$$

where $P_e^j(t)$ the stock-market price of share *j*. It is convenient to define

$$\omega^j(t) \equiv \frac{e^j(t) P_e^j(t)}{a(t)} \tag{5}$$

as the portfolio share invested in firm *j*, and $V^j(t) \equiv \bar{E}^j P_e^j(t)$ as the stock market value of firm *j*. Then the portfolio pollution content is:

$$p(t) \equiv \sum_{j=1}^{J} \frac{\omega^j(t) a(t)}{V^j(t)} \bar{p}^j(t) \tag{6}$$

Denoting $r_e^j(t)$ as the return on share *j*, $r(t)$ as the interest rate on public debt, and $w(t)$ as the wage rate, the individual's budget constraint is[8]:

$$\dot{a}(t) = \sum_{j=1}^{J} \omega^j(t) r_e^j(t)\, a(t) + \left[1 - \sum_{j=1}^{J} \omega^j(t)\right] r(t) a(t) + w(t) - c(t) - z(t) \tag{7}$$

where $z(t)$ a lump sum tax. Finally, the returns on shares of firm *j* are:

$$r_e^j(t) \equiv \frac{\dot{V}^j(t)}{V^j(t)} e^j(t) + \frac{d^j(t)}{V^j(t)} \tag{8}$$

[8] We follow Merton (1971).

where $\frac{d^j(t)}{V^j(t)}$ is the dividend pay-out ratio and $d^j(t)$ total dividend payments by firm *j*.

The individual maximizes (1) w.r.t. $c(t)$ and $\omega^j(t)$ subject to (4) and (7). The current value Hamiltonian reads:

$$\Lambda(t) = u(t) + q(t)\dot{a}(t) \tag{9}$$

where $q(t)$ is the shadow price of wealth. The first-order conditions are:

$$u_c(t) - q(t) = 0 \tag{10}$$

$$u_p(t)\frac{\bar{p}^j(t)}{V^j(t)}a(t) + q(t)a(t)[r_e^j(t) - r(t)] = 0 \tag{11}$$

$$q(t)\left[\sum_{j=1}^{J}\omega^j(t)r_e^j + \left(1 - \sum_{j=1}^{J}\omega^j(t)\right)r(t)\right] + u_p(t)\sum_{j=1}^{J}\frac{\omega^j(t)\bar{p}^j(t)}{V^j(t)} = \rho q(t) - \dot{q}(t) \tag{12}$$

Combining eq. (10) and eq. (11) we have:

$$\frac{u_p(t)}{u_c(t)}\bar{p}^j(t) + V^j(t)[r_e^j(t) - r(t)] = 0 \tag{13}$$

Equation (10) is the usual consumption-optimality consumption, while (13) is the optimal portfolio-choice condition. We notice that there is a "pollution premium" (the difference between the return on assets and the return on government bonds), which proportional to the pollution content by the firm. This is the compensation required by the household for holding "dirty assets". Moreover, the equilibrium pollution premium is independent of firm's policy concerning distributed profits and capital appreciation (see eq. 8). In this sense, distributed and undistributed profits are equivalent for the equilibrium and thus, in our model the consumer "pierces the corporate veil" (see Poterba et al. 1987). Combining (8) and (13) we have:

$$\frac{u_p(t)}{u_c(t)}\bar{p}^j(t) + \dot{V}^j(t) + d^j(t) - r(t)V^j(t) = 0 \qquad (14)$$

Next we pre-multiply (11) by $\omega^j(t)$ and sum from j=1 to J and use (12) to obtain the simplified the law of motion for the co-state:

$$q(t)r(t) = \rho q(t) - \dot{q}(t). \qquad (15)$$

By time-differentiating eq. (10) and exploiting (15), we get the usual consumption-Euler equation:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(t)}(r(t) - \rho). \qquad (15')$$

with $\sigma(c) \equiv -\frac{u_{cc}(t)c(t)}{u_c(t)}$.

3.2. Firms

Firms operate on perfectly competitive markets, producing a homogenous good under identical constant-returns to scale production technologies, using capital and labour. We will then be able to aggregate the firms to obtain a representative firm. We denote firm *j*'s production function as:

$$y^j(t) = f^j\left(k^j(t), l^j(t)\right) \qquad (16)$$

where $k^j(t)$ and $l^j(t)$ are physical capital- and labour-inputs, respectively. Following Brock and Taylor (2005), we assume that, at date t, every unit of output generates ε units of pollution, and that pollution can be reduced by abatement activity, $\alpha(t)$. Abatement is a constant returns-to-scale function, increasing in the total scale of firm activity $f(t)$ and of the firm's efforts at abatement, $f^{\alpha}(t)$. Let abatement level $\alpha(t)$ remove $\varepsilon \cdot \alpha(t)$ units of pollution, we can write total pollution by firm *j* as:

$$x^j(t) = \varepsilon \cdot f^j(t) - \varepsilon \cdot \alpha\left(f^j(t), f^{\alpha j}(t)\right) \qquad (17)$$

It is convenient to define $\psi^j(t) \equiv \frac{f^{\alpha j}(t)}{f^j(t)}$ as the fraction of output devoted to abatement. Then by exploiting constant returns-to-scale, we obtain:

$$\frac{x^j(t)}{f^j(t)} = \varepsilon \cdot \left[1 - \alpha\left(1, \psi^j(t)\right)\right] = \varepsilon \cdot \left[1 - \alpha\left(\psi^j(t)\right)\right] \qquad (18)$$

with α increasing in ψ^j and, thus, eq. (18) gives $\psi^j(t) = \Psi\left(\frac{x^j(t}{f^j(t)}\right)$ [9], with $\Psi' < 0, \Psi'' > 0$.[10]

As the government levies taxes, $\tau^x(t)$, on pollution and subsidizes abatement spending $s(t)\Psi f^j = S(t)$, the gross operating profits of firm *j* are:

$$\pi^j(t) \equiv \left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\left(k^j(t), l^j(t)\right) - w(t)l^j(t) - \tau^x(t)x^j(t) \qquad (19)$$

We abstract from corporate bonds[11] and assume that the total number of shares remains constant, then new investments, $i^j(t)$, can only by financed via retained earnings, $Re(t)$, i.e. $\pi^j(t) = d^j(t) + Re^j(t)$. The firm's capital accumulation is then:

---

[9] From (18), $\alpha(\psi^j(t)) = 1 - \frac{1}{\varepsilon} \cdot \frac{x^j(t}{f^j(t)}$ gives $\psi^j(t) = \alpha^{-1}\left(1 - \frac{1}{\varepsilon} \cdot \frac{x^j(t}{f^j(t)}\right) \equiv \Psi\left(\frac{x^j(t}{f^j(t)}\right)$.

[10] For example, assuming the following form for the abatement technology: $\alpha(f, f^{\alpha}) = f^{(1-\xi)} f^{\alpha \xi} = \psi^{\xi} f$, then $\frac{x}{f} = \varepsilon \cdot \left[1 - \psi^{\xi}\right]$ and $\psi = \left(1 - \frac{x}{f}\frac{1}{\varepsilon}\right)^{\frac{1}{\xi}}$.

$$\dot{k}^j(t) = i^j(t) - \delta k^j(t) \tag{20}$$

where δ is the (constant) instantaneous depreciation rate. Then we have:

$$\dot{k}^j(t) = \pi^j(t) - d^j(t) - \delta k^j(t) \tag{21}$$

which together with (19) and (21) becomes:

$$\dot{k}^j(t) = \left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\big(k^j(t), l^j(t)\big) - w(t)l^j(t) - \tau^x(t)x^j(t) - d^j(t) - \delta k^j(t) \tag{22}$$

We integrate (14) to obtain:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds}\left[d^j(t) + \frac{u_p(t)}{u_c(t)}\bar{p}^j(t)\right]dt \tag{23}$$

which is the firm value at date 0. Finally, using (22) to substitute for $d^j(t)$ in (23), we have:

$$V^j(0) = \int_0^\infty e^{-\int_0^t r(s)ds}\left\{\left[1 - (1 - s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)\right] f^j\big(k^j(t), l^j(t)\big) - w(t)l^j(t) - \tau^x(t)x^j(t) - \delta k^j(t) + \right.$$
$$\left. + \frac{u_p(t)}{u_c(t)}\bar{p}^j(t) - \dot{k}^j(t)\right\}dt \tag{24}$$

Firm j maximizes its value (24) w.r.t. $l^j(t)$ and $x^j(t)$, yielding the first-order conditions:

---

[11] Corporate bonds would be equivalent to shares in our model, as they would also carry the same pollution premium as shares.

$$\left[1-(1-s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)+(1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right]f_l^j(t)-w(t)=0 \qquad (25)$$

$$\frac{u_p(t)}{u_c(t)}\frac{\partial\bar{p}^j(t)}{\partial x^j(t)}-(1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)-\tau^x(t)=0 \qquad (26)$$

As for the optimality condition for $k^j(t)$, $\frac{dV^j(0)}{dk^j(t)}=\frac{d}{dt}\frac{dV^j(0)}{dk^j(t)}$, classical calculus of variation yields:

$$\int_0^\infty e^{-\int_0^t r(s)ds}\left\{\left[1-(1-s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)+(1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right]f_k^j(t)-\delta\right\}dt=\frac{d}{dt}\int_0^\infty e^{-\int_0^t r(s)ds}\,dt\Rightarrow$$

$$\left[1-(1-s(t))\Psi\left(\frac{x^j(t}{f^j(t)}\right)+(1-s(t))\Psi'\left(\frac{x^j(t}{f^j(t)}\right)\frac{x^j(t)}{f^j(t)}\right]f_k^j(t)-\delta=r(t) \qquad (27)$$

It can be shown that, by substituting (25)-(27) into (24), and exploiting CRS in $f^j(t)$, the maximized firm value is $maxV^j(0)\equiv\bar{E}^jP_e{}^j(0)=k^j(0)$.

4. Equilibrium and stability

We now characterize the equilibrium of the economy. At each date t, given that all firms are equal, plugging eq. (27) into (15)' yields:

$$\frac{\dot{c}(t)}{c(t)}=\frac{1}{\sigma(c)}\left\{\left[1-(1-s(t))\Psi\left(\frac{x(t)}{f(t)}\right)+(1-s(t))\Psi'\left(\frac{x(t)}{f(t)}\right)\frac{x(t)}{f(t)}\right]F_K(t)-\delta-\rho\right\}. \qquad (28)$$

Moreover, under the assumption that firms are equal, the feasibility constraint stating that private plus investment be equal to aggregate output (recall that we assume a balanced Government budget), reads as.[12]

[12] In fact, aggregating over firms we get:

$$\sum_{j=1}^{J}\left[1-\left(1-s(t)\right)\Psi^j\left(\frac{x^j(t}{f^j(t)}\right)\right]f^j\left(k^j(t),l^j(t)\right)=$$

$$\dot{K}(t) = \left[1 - \Psi\left(\frac{X(t)}{F(t)}\right)\right] F(K(t), H) - c(t)H - \delta K(t) \qquad (29)$$

We notice that in equilibrium $\omega^j(t) \equiv \frac{V^j(t)}{H \cdot a(t)}$; which by using (4), yields the perceived pollution function (warm-glow):

$$p(t) \equiv \sum_{j=1}^{J} \frac{\omega^j(t)a(t)}{V^j(t)} \bar{p}^j(t) = \sum_{j=1}^{J} \frac{\bar{p}^j\left(x^j(t)\right)}{H} = \frac{\bar{p}\left(X(t)\right)}{H} = \gamma \cdot \frac{X(t)}{H}$$

where the second equality follows the linearity of $\bar{p}^j(t)$ in $x^j(t)$, and, hence, $\frac{\partial \bar{p}}{\partial X} = \gamma$. Finally, in equilibrium, eq. (26) becomes:

$$\frac{u_p(t)}{u_c(t)} \frac{\partial \bar{p}(t)}{\partial X(t)} - (1 - s(t))\Psi'\left(\frac{X(t)}{F(t)}\right) - \tau^x(t) = 0 \qquad (30)$$

Eqs. (28), (29) and (30) characterise the equilibrium at each date for $(K(t), c(t), x(t))$. From (30) we obtain:

$$X(t) = X(c(t), s(t), \tau^x(t), K(t)) \qquad (31)$$

Total differentiation of (31) yields:

$$-\left(R\eta + T\frac{F}{K}\right)\frac{dX}{X} + \left(T\frac{F}{K}\theta\right)\frac{dK}{K} - (R\sigma)\frac{dc}{c} - \left(\frac{X}{K}\right)d\tau^x + \left(\Psi'\frac{X}{K}\right)ds = 0 \qquad (31')$$

$= \sum_{j=1}^{J}\left[1 - (1 - s(t))\Psi\left(\frac{x(t)}{f(t)}\right)\right] f(k(t), l(t)) = \left[1 - (1 - s(t))\Psi\left(\frac{X(t)}{F(t)}\right)\right] F(K(t), L(t))$, with $L(t) = H$.

where

$$T \equiv \left(\frac{X}{F}\right)^2 (1-s)\Psi'' > 0, R \equiv -\frac{u_p}{u_c}\frac{\partial \bar{p}}{\partial X}\frac{X}{K} > 0, \eta \equiv \frac{u_{pp}}{u_p}p > 0, \theta \equiv \frac{F_K}{F}K > 0$$

Hence, from (31') we get:

$$\frac{\partial X}{\partial K} = \frac{X}{K}\frac{T\frac{F}{K}\theta}{R\eta+T\frac{F}{K}} > 0, \frac{\partial X}{\partial c} = -\frac{X}{c}\frac{R\sigma}{R\eta+T\frac{F}{K}} < 0, \frac{\partial X}{\partial s} = \frac{\Psi'\frac{X}{K}X}{R\eta+T\frac{F}{K}} < 0, \frac{\partial X}{\partial \tau^x} = -\frac{\frac{X}{K}X}{R\eta+T\frac{F}{K}} < 0 \qquad (31'')$$

By substituting (31) into (28) and (29), the dynamic system describing the equilibrium path of the economy boils down to the following two equations in $[c(t), K(t)]$:

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma(c)}\left\{\left[1-(1-s(t))\Psi\left(\frac{X(t)}{F(t)}\right) + (1-s(t))\Psi'\left(\frac{X(t)}{F(t)}\right)\frac{X(t)}{F(t)}\right]F_K(t) - \delta - \rho\right\}. \qquad (32)$$

$$\dot{K}(t) = \left[1-\Psi\left(\frac{X(t)}{F(t)}\right)\right]F(K(t), H) - c(t)H - \delta K(t). \qquad (33)$$

By recognizing that

$$\frac{d\left(\frac{X}{F}\right)}{dK} = \frac{1}{F}\frac{dX}{dK} - \frac{X}{F}\frac{F_K}{F} = \frac{X}{K}\frac{T\theta\frac{1}{K}}{R\eta+T\frac{F}{K}} - \frac{X}{F}\frac{\theta}{K} = -\frac{X}{F}\frac{\theta}{K}\left(\frac{R\eta}{R\eta+T\frac{F}{K}}\right) < 0$$

$$\frac{d\left(\frac{X}{F}\right)}{dc} = \frac{1}{F}\frac{dX}{dc} = -\frac{X}{F}\frac{1}{c}\left(\frac{R\sigma}{R\eta+T\frac{F}{K}}\right) < 0$$

and defining $\beta \equiv -\frac{F_{KK}}{F_K}K$, the Jacobian matrix of system (32)-(33) can be written as:

$$J = \begin{bmatrix} -\frac{F_K TR}{R\eta+T\frac{F}{K}} & -\frac{c}{\sigma}\frac{F_K}{K}\left[T\theta\frac{R\eta}{R\eta+T\frac{F}{K}} + \left(1-(1-s)\Psi + (1-s)\Psi'\frac{X}{F}\right)\beta\right] \\ \Psi'\frac{X}{c}\frac{R\sigma}{R\eta+T\frac{F}{K}} - H & \Psi'\theta\frac{X}{K}\frac{R\eta}{R\eta+T\frac{F}{K}} + (1-\Psi)F_K - \delta \end{bmatrix}$$

The following Proposition contains sufficient conditions for this economy to display saddle-path stability.

Proposition 1: *Sufficient for saddle path stability of the economic system is:*

$$F_K\left(1-\Psi+\Psi'\frac{X}{F}\right)-\delta>0$$

Proof: See Appendix A. □

Notice that the sufficient condition above is equivalent to the corresponding dynamic efficiency condition in standard in Ramsey-Cass-Koopmans models.

5. Tax reforms

In this section we carry out a comparative statics analysis to verify the effects of the tax instruments on the endogenous variables of the model, i.e. the scale of the economy, per-capita consumption, pollution and the pollution premium. In this exercise we assume that the reforms are carried out by keeping the public budget balanced, i.e. any tax change is financed by a corresponding change in individuals' lump sum tax z(t), so that public debt remains constant.[13]

Total differentiation of system (31'), (32) and (33) provides:

$$\begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}\begin{bmatrix} dc \\ dK \end{bmatrix}=\begin{bmatrix} -\frac{c}{\sigma}F_K\left[\Psi-\Psi'\frac{X}{F}\frac{R\eta}{R\eta+T\frac{F}{K}}\right]ds+\frac{c}{\sigma}F_K\frac{X}{K}\frac{T}{R\eta+T\frac{F}{K}}d\tau^X \\ \frac{(\Psi')^2}{K}\frac{X^2}{R\eta+T\frac{F}{K}}ds-\frac{\Psi'}{K}\frac{X^2}{R\eta+T\frac{F}{K}}d\tau^X \end{bmatrix}$$

Hence, using Cramer's rule we get the following results:

---

[13] The public budget equation reads as: $\dot{B}(t)=r(t)B(t)+s(t)\Psi\left(\frac{X(t)}{F(t)}\right)F(K(t),H)-\tau^X(t)X(t)-z(t)H$. We notice that the exact time path of $z(t)$ does not matter for the equilibrium, as $B(t)$ would have to adjust accordingly so as to maintain intertemporal budget balance. Thus, with respect to $z(t)$there is Ricardian equivalence. Of course, a different time path of $z(t)$, for given τ^X and s, would produce a different portfolio, as $B(t)$ would be different, but the equilibrium in terms of capital, consumption, pollution, prices, etc. remains the same.

Proposition 2: *An increase of the tax on pollution reduces the capital installed in the economy, while an increase of the subsidy on abatement activity increases the capital installed*.

Proof: See Appendix B. □

The economic rationale behind the results can be stated as follows: the tax on pollution corresponds to a tax on profits, and thus, by reducing the marginal productivity of capital, induces firms to reduce the capital invested. On the contrary, the subsidy on abatement reduces firms' production costs, so that it will provide an incentive to increase the scale of the firm.

One could argue that the result that an increase of the tax on pollution always reduces the scale of the economy is a direct consequence of our assumptions, in particular of pollution being proportional to the scale of the economy, so that reducing pollution would necessarily imply reducing the capital installed in the economy[14]. In fact, we can show that the result is quite general and holds true also in the case in which pollution is the consequence of the production "process", i.e. of the adoption of a "dirty" input, z (e.g. fuel). In other words, we can show that the two economies are equivalent, so that a tax on z will produce a reduction of the scale of the economy and production too (the proof is provided in Appendix C).[15]

---

[14] We are grateful to a referee for pointing us to this issue.

[15] Any environmental concerns (either through increased cost of capital or through government taxation) will cause the firm to substitute away from the polluting input, and possibly increase the demand for other production factors. Thus, in general one cannot rule out that production would increase with an increase in the price of a factor input. However, that case would not only require the factors to be gross substitutes, in the sense that the marginal product of the alternative factor would fall with an increase in z (negative cross partial derivative), but also in our case that the cost minimising level of z is declining in output (i.e. a negative output elasticity of demand for factor z). Our homogeneity assumption on the production function in Appendix C

For illustrative purposes, we present Figure 1, providing a graphical representation of the content of Proposition 2, where we have assumed CIES utility functions with $\eta = \sigma = 1$, Cobb-Douglas production function $F = AK^{\alpha}L^{1-\alpha}$ and abatement technology as the one already presented in section 2.2 and parameters specified below.

Figure 1: Steady state aggregate capital K as a function of the fiscal instruments and warm-glow parameter γ .

automatically implies that k and z are gross complements. This is why production falls with an increase in the price of z. On this issue, see Rader (1968) and Silberberg (1974).

Figure 1a

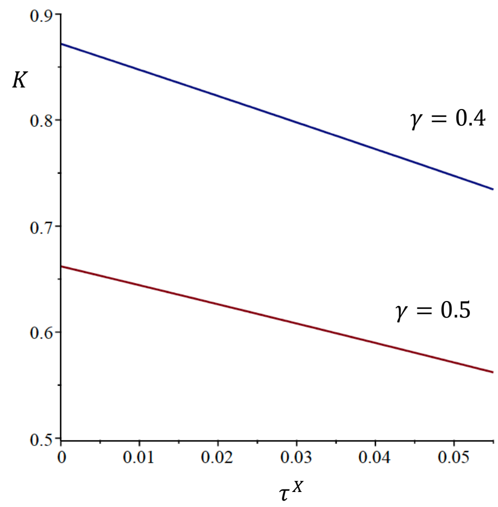
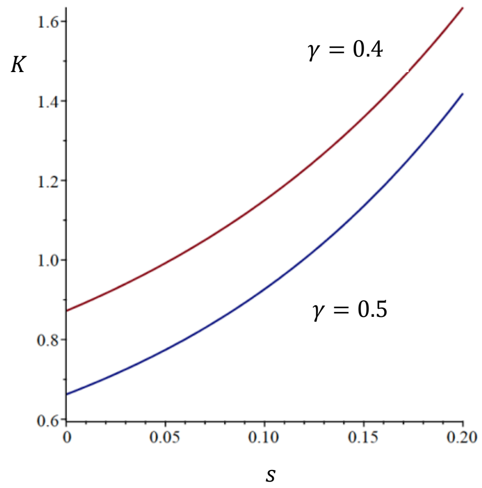


Figure 1b



Parameters: CIES utility function with parameters $\eta = \sigma = 1$, $F = AK^{\alpha}L^{1-\alpha}$, $a(f, f^{\alpha}) = f^{(1-\xi)}f^{\alpha\xi} = \psi^{\xi}f$; $A = 1, H = 1, \alpha = 0.5, \xi = 0.5, \epsilon = 5, \rho = 0.1, \tau^{x} = 0, \delta = 0.01$

Note that the warm-glow parameter γ, other things being equal, exerts a negative effect on the equilibrium level of capital, in that investors, as γ increases, penalize firms by asking for a higher pollution premium.

Finally, by eq. (5) and the result $\bar{E}^j {P_e}^j(0) = k^j(0)$, where $\bar{E}^j = e^j H$, it follows that the portfolio weight for stocks of firm j at the equilibrium is equal to $\omega^j \equiv \frac{e^j(t)P_e^j(t)}{a(t)} = \frac{He^jP_e^j}{aH} = \frac{k^j}{A}$, so that the total portfolio weight for stocks is equal to $\sum_{j=1}^{J} \omega^j = \sum_{j=1}^{J} \frac{k^j}{A} = \frac{K}{K+B}$, where we have exploited the equilibrium relation $A = K + B$. Hence, given that B remains constant, from the content of Proposition 2 it follows that an increase of the pollution tax reduces the portfolio weight for stocks, while an increase of the subsidy on abatement activity increases it.

As for the effects on pollution, we can summarize the results in the following proposition:

Proposition 3: *An increase of the tax on pollution reduces pollution, while an increase of the subsidy on abatement activity can either increase or reduce pollution. However, when* $\tau^X = 0$, *sufficient for* $\frac{dX}{ds} < 0$ *is*:

$$s \leq \frac{\rho - \frac{(1-\xi)}{\sigma}}{\phi}.$$

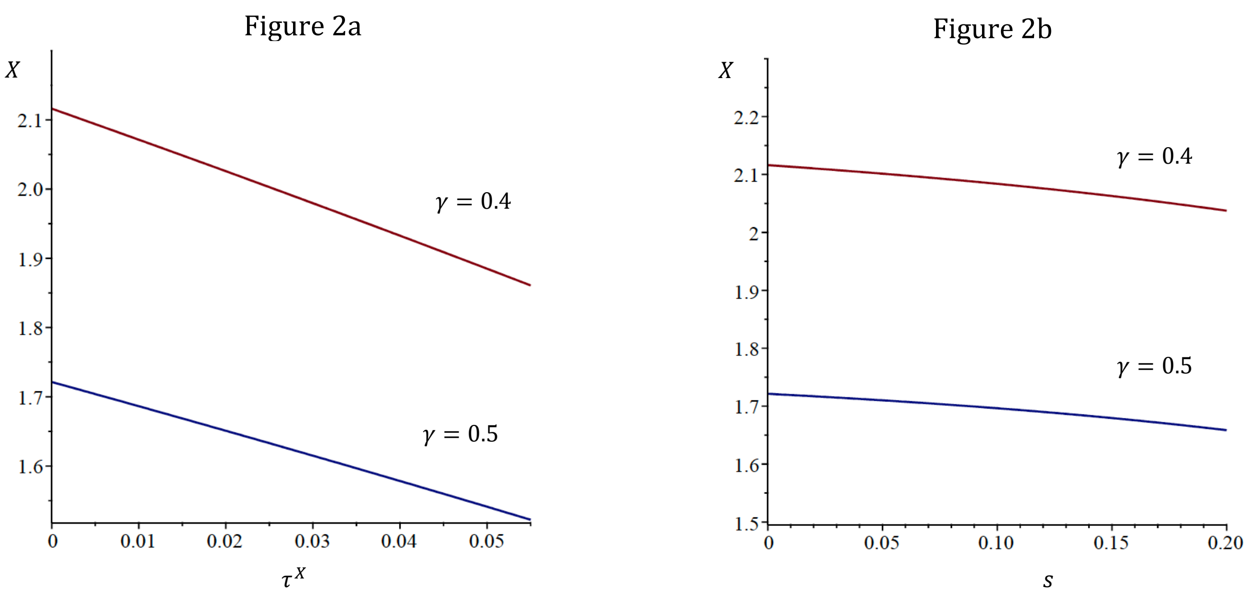
with $\phi \equiv F_K(s = 0)$, i.e. marginal productivity of capital for $s = 0$.

Proof: See Appendix D. □

The economic rationale of the results is the following: as for the pollution tax, as expected, an increase of the latter will reduce pollution. As for the subsidy, the latter exerts two opposite effects on pollution: on one hand, by increasing the installed capital, it also tends to increase production and, thus, pollution (which is a by-product of production); on the other hand, it tends to increase the resources that the firms devote to abatement, thus reducing pollution. The final result depends on the relative strength of each effect.

Results of Proposition 3 are summarized in Figure 2, where we assumed the same parameters' specification as the one used in Figure 1.

Figure 2: Steady state pollution flow X as a function of the fiscal instruments and the warm-glow parameter γ.



Parameters: same as Figure 1

In this case, as expected, a higher warm-glow parameter, other things being equal, is associated with a lower flow of pollution X at the steady state, due to the fact that investors, as γ increases, penalize firms by asking a higher pollution premium, which, in turn, reduces the scale of the economy, production and, consequently, X.

We now focus on the effects of fiscal instruments on steady state per-capita consumption. Proposition 4 summarizes our results:

Proposition 4: *An increase of the tax on pollution reduces per-capita consumption, while an increase of the subsidy on abatement activity can either increase or reduce per-capita consumption. However, sufficient for* $\frac{dc}{ds} > 0$ *is* $s \leq 1 - \frac{\frac{\beta}{\theta}\left(\frac{\rho+\delta}{\rho}\right)}{(\eta+1-\xi)}$ *where* $0 < \xi < 1$ *is the elasticity of the abatement technology* $\alpha(f, f^{\alpha})$ *with respect to* Ψ *(i.e. the fraction of total output devoted to abatement activity).*

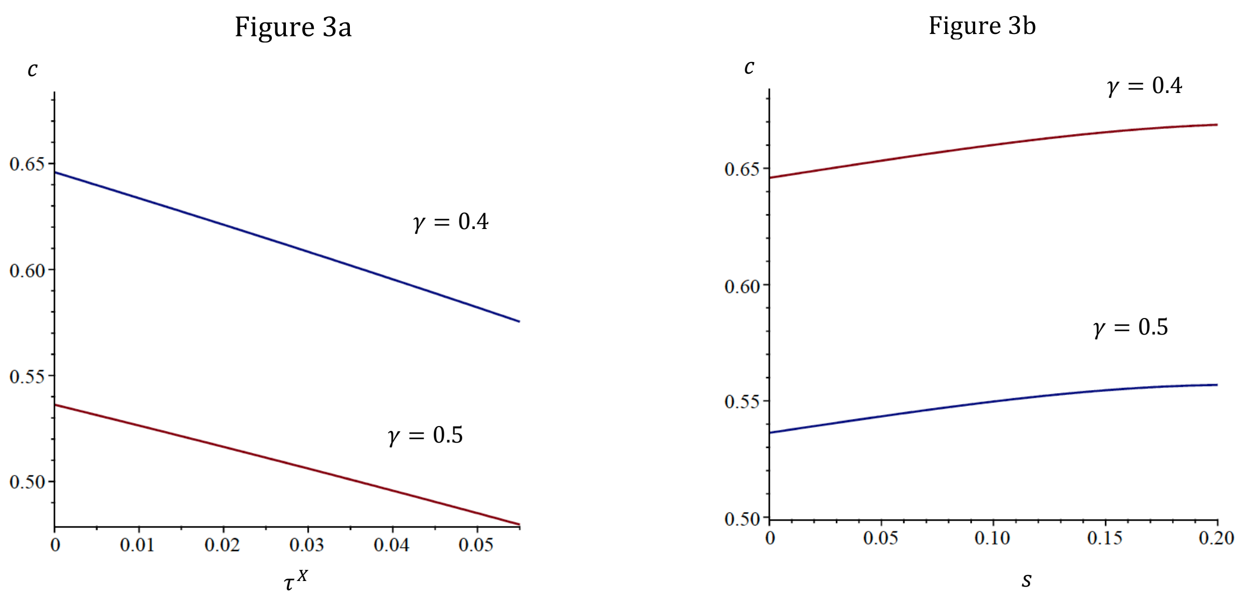
Proof: See Appendix E. □

The economic intuition of the result can be stated as follows: preliminarily, recall that steady state consumption, by eq. (33), is proportional to net-of-abatement product. Given that an increase of the tax on pollution reduces net-of-abatement production, it follows that also consumption decreases. On the other hand, the subsidy can either increase or decrease net-of-abatement production, given that it exerts two opposite effects: on one hand, by increasing the installed capital, it will also increase production and, thus, consumption; on the other hand, it tends to increase the resources that firms devote to abatement, thus reducing net-of-production and, consequently, consumption. The final result depends on the relative strength of each effect.

Figure 3 provides a graphical representation of the content of Proposition 4, where we assumed the same parameters' specification as the one used in Figure 1.

In this case, a higher warm-glow parameter, other things being equal, is associated with a lower per-capita consumption c at the steady state, due to the fact that investors, as γ increases, penalize firms by asking a higher pollution premium, which, in turn, reduces the scale of the economy, total production and, consequently, per-capita consumption.

Figure 3: Steady state per capita consumption c as a function of the fiscal instruments and the warm-glow parameter γ.



Parameters: same as Figure 1

More in general, the sufficient conditions provided in Propositions 3-4 can be further summarised through the following interval for *s:*

$$0 \leq s \leq min\left\{\frac{\rho - \frac{(1-\xi)}{\sigma}}{\phi}, 1 - \frac{\frac{\beta}{\theta}\left(\frac{\rho+\delta}{\rho}\right)}{(\eta + 1 - \xi)}\right\}$$

within which an increase of such a subsidy produces not only an increase of the installed capital, but also an increase of per-capita consumption and a reduction of pollution.

We now turn to the effects of the fiscal instruments on the pollution premium, which, by eq. (13), at the steady state, is:

$$R \equiv [r_e^j - r] = \frac{u_p}{u_c}\gamma\frac{X}{K} \tag{34}$$

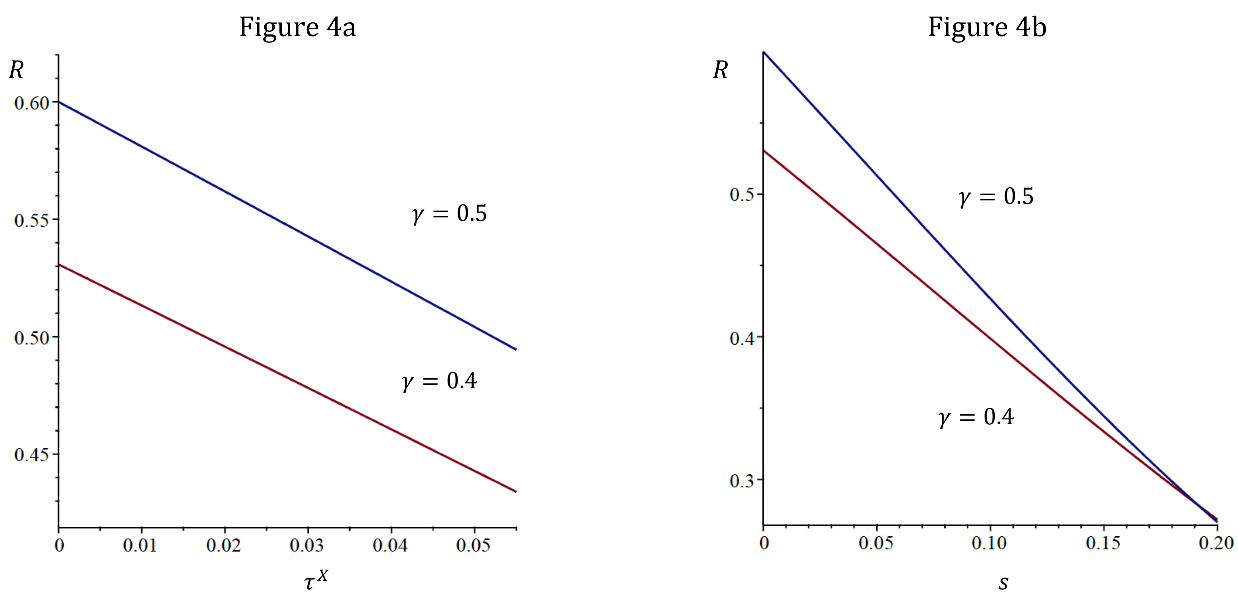
The following Proposition summarizes the results:

Proposition 5: *An increase of the tax on pollution reduces the pollution premium, while an increase of the subsidy on abatement activity can increase or decrease the pollution premium. when* $\tau^X = 0$*, sufficient for sufficient* $\frac{dR}{ds} < 0$ *is* $(1+\eta)\theta - 1 \leq 0$*.*

Proof: See Appendix F. □

Notice that the sufficient condition above holds for values of η (elasticity of marginal (dis)utility of the warm-glow) sufficiently close to 1 and values of θ (the elasticity of output with respect to capital) sufficiently smaller that ½.

Figure 4: Steady state pollution premium R as a function of the fiscal instruments and the warm-glow parameter γ*.*



Parameters: same as Figure 1

Figure 4 provides a graphical representation of the content of Proposition 5 concerning the effect of both τ^X and s on the pollution premium R, for different values of the warm-glow parameter γ.

Notice that a higher warm-glow parameter, for any level of τ^X, implies a higher pollution premium *R,* due to the fact that investors, as γ increases, penalize polluting firms by asking for a higher renumeration for holding "dirty assets". However, the role of γ on the pollution premium can change as s increases*.* In fact, as shown by Figure 4b, stronger warm glow motive reinforces the effect of s. At s=0, an economy with stronger warm glow will have a higher pollution premium. As s increases the pollution premium reduces more rapidly for high-worm glow economies, meaning that eventually for large enough s, the R curves will cross. In any case, even though the pollution premium is reduced by increases in s (public subsidy crowding out worm glow), marginal worm glow is still present, rendering subsidy an effective instrument in reducing X.

To sum up, our analysis shows that the mechanisms of s and τ^X are different. Given that the firm will equate the marginal cost of abatement to the sum of the pollution tax and the marginal pollution premium (marginal cost of capital in production units), an increase in τ^X directly affects the firms' incentive to abate (see eq. 26). Hence, an increase in τ^X (for every given level of marginal pollution premium) will call for an increase in the marginal abatement cost (i.e. an increase in abatement) and, given that pollution declines, the marginal pollution premium will also decline (but not enough to offset the first effect). Moreover, τ^X acts as a tax on economic activity, calling for an equilibrium reduction in K, while the worm glow motive (higher value of γ) will tend to reduce pollution levels at each level of τ^X, although $\frac{dX}{d\tau^X}$ quantitatively remains roughly the same.

On the other hand, an abatement subsidy hinges on the pollution premium. If there is no warm glow, the firm will always have higher profits by doing no abatement (unless it is subsided more than 100%), thus, s is not effective here. In the presence of warm glow, the firms equate the (net after subsidy) marginal cost of abatement to the marginal pollution premium, so that, for given marginal pollution premium, an increase in s lowers the marginal abatement cost, calling for an increase in abatement. This will also lower the marginal pollution premium, but under certain conditions, will not offset the first positive effect on abatement. At the same time, s acts as a production subsidy calling for an increase in K in equilibrium and this effect, in turn, reduces the effect of s on X, so that, coeteris paribus, quantitatively X responds less to changes in s (than it does to changes in τ^X).

Finally, in general the two fiscal instruments will produce different effects on total tax revenues: on one hand, when $s = 0$, given that the shape of the (τ^X, X) locus depicted in Fig.2 is broadly linear and decreasing, the Revenue function $\tau^X X$ displays a hump shape ("Laffer curve"); hence, although we cannot exclude that total revenues eventually decrease as the pollution tax increases, we can say that for sufficiently low levels of τ^X the relationship between τ^X and total revenue is positive (in our numerical example tax revenues start decreasing for values of τ^X beyond 20%). On the other hand, when $\tau^X = 0$ and for sufficiently low levels of the subsidy to abatement activities (i.e. sufficient condition of Proposition 3 is satisfied), an increase of s reduces fiscal revenues (which amount to $-s\Psi\left(\frac{X}{F}\right)F(K,H)$): indeed, when s increases, X decreases and K increases, so that both Ψ and F increase (recall that $\Psi' < 0$).

6. Conclusions

In this paper we analysed the effects that fiscal instruments, aimed at reducing pollution, can exert on the scale of the economy, on pollution and on the pollution premium. In particular, we compared two different instruments: a tax on pollution and a subsidy on abatement activity.

We found that the former, besides reducing pollution, depresses also per capita consumption and the capital installed in the economy. As for the subsidy, rather interestingly, we found that, under fairly general assumptions, it decreases pollution and increases per-capita consumption. As for the pollution premium, we also provided very general conditions ensuring that an increase of both pollution taxes and subsidies for pollution abatement generates a reduction of the pollution premium. Finally, given that the portfolio weight for stocks turns out to be an increasing function of the capital installed in the economy, it follows that an increase of the pollution tax reduces the portfolio weight for stocks, while an increase of the subsidy on abatement activity increases it.

Some policy implications follow: in an economy populated by socially responsible investors, pollution abatement, a goal which is on the political agenda of most developed countries and international organizations is not necessarily at odds with economic performance. In fact, while the subsidy to abatement has smaller quantitative effects on pollution, relative to the tax on pollution, other things equal, it increases steady state consumption and capital. For these reasons the former fiscal instrument may be politically more feasible than latter, especially in economies characterised by investors with stronger social responsibility motives (warm glow).

Finally, we notice that our results have clear testable implications, in that one could empirically verify whether and to what extent differences in green fiscal policies and individual preferences can explain the differences and correlations between economic performance (i.e. dimension of the economy, per-capita consumption or pollution) and environmental outcomes (i.e. pollution flows) displayed by different groups of countries. We leave this empirical analysis for future research.

References

Aldashev, G., Limardi, M., & Verdier, T. (2015). Watchdogs of the invisible hand: Ngo monitoring and industry equilibrium. *Journal of Development Economics*, *116*, 28-42.

Andreoni, J. (1990). Impure altruism and donations to public goods: A theory of warm-glow giving. *The Economic Journal*, *100*(401), 464-477.

Andreoni, J., Rao, J. M., Trachtman, H. (2017). Avoiding the ask: A field experiment on altruism, empathy, and charitable giving. *Journal of Political Economy*, *125*(3), 625-653.

Barnea, A., Heinkel, R., Krause, A., (2005). Green investors and corporate investment. *Structural Change and Economic Dynamics* 16 (3), 332–346.

Baron, D. P. (2007). Corporate social responsibility and social entrepreneurship. *Journal of Economics & Management Strategy*, *16*(3), 683-717.

Baumol, W.J., Oates, W.E. (1988). *The Theory of Environmental Policy*, 2nd Edition. New York: Cambridge University Press.

Belfiori, M. E. (2018). Climate change and intergenerational equity: Revisiting the uniform taxation principle on carbon energy inputs. *Energy Policy*, *121*, 292-299.

Bontems, P., Bourgeon, J. M. (2005). Optimal environmental taxation and enforcement policy. *European Economic Review*, *49*(2), 409-435.

Bovenberg, A.L., Goulder, L.H. (2002). Environmental taxation and regulation. In: Auerbach, A.J., Feldstein, M.S. (eds) *Handbook of public economics*, vol 3. North-Holland, Amsterdam, 1471–1545.

Brekke, K. A., Pekovic, S. (2018). Why Are Firms Environmentally Responsible? A Review and Assessment of the Main Mechanisms. *International Review of Environmental and Resource Economics*, *12*(4), 355-398.

Brock, W. A., Taylor, M. S. (2005). Economic Growth and the Environment: A Review of Theory and Empirics, in: Philippe Aghion & Steven Durlauf (ed.), *Handbook of Economic Growth*, edition 1, volume 1, chapter 28, pages 1749-1821 Elsevier.

Buchanan, J. M. (1969). External Diseconomies, Corrective Taxes and Market Structure. *American Economic Review* 59: 174–177.

Cremer, H., Gahvari, F., Ladoux, N. (2001). Second-Best Pollution Taxes and the Structure of Preferences. *Southern Economic Journal, 68*(2), 258-280.

Dam, L. (2011). Socially responsible investment in an environmental overlapping generations model. *Resource and Energy Economics*, *33*(4), 1015-1027.

Dam, L., Heijdra, B. J. (2011). The environmental and macroeconomic effects of socially responsible investment. *Journal of Economic Dynamics and Control*, *35*(9), 1424-1434.

Dam, L., Scholtens, B. (2015). Toward a theory of responsible investing: On the economic foundations of corporate social responsibility. *Resource and Energy Economics*, *41*, 103-121.

Dröge, S., Schröder, P. J. (2005). How to turn an industry green: Taxes versus subsidies. *Journal of Regulatory Economics*, *27*(2), 177-202.

Eurosif (2018). EUROPEAN SRI STUDY 2018.

Eurostat (2020): Environmental tax statistics. (https://ec.europa.eu/eurostat/statistics-explained/index.php/Environmental_tax_statistics#Environmental_taxes_in_the_EU)

Evren, Ö., Minardi, S. (2017). Warm-glow Giving and Freedom to be Selfish. *The Economic Journal*, *127*(603), 1381-1409.

Gahvari, F. (2014). Second-best pigouvian taxation: a clarification. *Environmental and Resource Economics*, *59*(4), 525-535.

Graff Zivin, J., Small, A. (2005). A Modigliani-Miller theory of altruistic corporate social responsibility. *The BE Journal of Economic Analysis & Policy*, *5*(1).

Goulder, L. H., Parry, I. W. (2008). Instrument choice in environmental policy. *Review of environmental economics and policy*, *2*(2), 152-174.

GSIA (2016): *Global Sustainable Investment Review.* Available at: http://www.gsi-alliance.org/wp-content/uploads/2017/03/GSIR_Review2016.F.pdf

Heinkel, R., Kraus, A., Zechner, J. (2001). The effect of green investment on corporate behavior. *Journal of financial and quantitative analysis*, *36*(4), 431-449.

Hong, H., Kacperczyk, M. (2009). The price of sin: The effects of social norms on markets. *Journal of Financial Economics*, *93*(1), 15-36.

Jacobs, B., & De Mooij, R. A. (2015). Pigou meets Mirrlees: On the irrelevance of tax distortions for the second-best Pigouvian tax. *Journal of Environmental Economics and Management*, *71*, 90-108.

Kampas, A., Horan, R. (2016). Second-best pollution taxes: revisited and revised. *Environmental Economics and Policy Studies*, *18*(4), 577-597.

Lambertini, L., Poyago-Theotoky, J., Tampieri, A. (2017). Cournot competition and “green” innovation: An inverted-U relationship. *Energy Economics*, *68*, 116-123.

Martín-Herrán, G., & Rubio, S. J. (2018). Second-best taxation for a polluting monopoly with abatement investment. *Energy Economics*, *73*, 178-193.

Merton, R. C. (1971). Optimum Consumption and Portfolio-Rules in a Continuous-Time Framework. *Journal of Economic Theory*, 3, 373-413.

OECD (2017), Investment governance and the integration of environmental, social and governance factors.

Ouchida, Y., Goto, D. (2014). Do emission subsidies reduce emission? In the context of environmental R&D organization. *Economic Modelling*, *36*, 511-516.

Ouchida, Y., Goto, D. (2016). Environmental research joint ventures and time-consistent emission tax: Endogenous choice of R&D formation. *Economic Modelling*, *55*, 179-188.

Pizer, W. A., Sexton, S. (2019). Distributional impacts of energy taxes. *Review of Environmental Economics and Policy*, rey021, https://doi.org/10.1093/reep/rey021.

Poterba, J.M., Hall, R.E., Hubbard, R.G. (1987). Tax policy and corporate saving. *Brookings Papers on Economic Activity*, *1987*(2), 455-515.

Poyago-Theotoky, J. A. (2007). The organization of R&D and environmental policy. *Journal of Economic Behavior & Organization*, *62*(1), 63-75.

Poyago-Theotoky, J. A. (2010). Corrigendum to" The organization of R&D and environmental policy"[j. econ. behav. org. 62 (2007) 63-75]. *Journal of Economic Behavior & Organization*, *76*(2), 449-449.

Rader, T. (1968). Normally Factor Inputs Are Never Gross Substitutes. *Journal of Political Economy*, 76(1): 38-43.

Renneboog, L., Ter Horst, J., Zhang, C. (2008). Socially responsible investments: Institutional aspects, performance, and investor behavior. *Journal of Banking & Finance*, *32*(9), 1723-1742.

Renström, T.I., Spataro, L. Marsiliani, L. (2019): Optimal Taxation, Environment Quality, Socially Responsible Firms and Investors, *International Review of Environmental and Resource Economics*: Vol. 13: No. 3-4, pp 339-373. http://dx.doi.org/10.1561/101.00000112.

Sandmo, A. (1975). Optimal taxation in the presence of externalities. *The Swedish Journal of Economics*, 86-98.

Silberberg, E. (1974). The Theory of the Firm in "Long-Run Equilibrium". *The American Economic Review*, 64(4): 734-741.

Vanwalleghem, D. (2017). The real effects of sustainable & responsible investing?. *Economics Letters*, *156*, 10-14.

Appendix A. Proof of Proposition 1

We recall that saddle-path stability requires $Tr(J) > 0$ and $Det(J) < 0$.

The trace of the Jacobian matrix can be written as:

$$Tr(J) = F_K\left[-\frac{T}{R\eta+T\frac{F}{K}}\left(R+\Psi'\frac{X}{K}\right)+\left(1-\Psi+\Psi'\frac{X}{F}\right)\right]-\delta$$

By eq. (30), $R+\Psi'\frac{X}{K} = -\frac{X}{K}(-s\Psi'+\tau^X)$, which is non-positive if $s, \tau^X \geq 0$. Hence, if $F_K\left(1-\Psi-\Psi'\frac{X}{F}\right)-\delta > 0$, then $Tr(J) \geq F_K\left(1-\Psi+\Psi'\frac{X}{F}\right)-\delta > 0$.Next, given that $Tr(J) = J_{11}+J_{22}$ and that $J_{11} < 0, Tr(J) > 0$ implies $J_{22} > 0$. Moreover, since $Det(J) = J_{11}J_{22}-J_{12}J_{21}$ and $J_{12} < 0, J_{21} < 0$ and $J_{22} > 0$, then the result $Det(J) < 0$ follows. □

Appendix B. Proof of Proposition 2

As for the effect of the tax on pollution, by defining $A \equiv \frac{c}{\sigma}F_K\frac{X}{K}\frac{T}{R\eta+T\frac{F}{K}} > 0, B \equiv -\frac{\Psi'}{K}\frac{X^2}{R\eta+T\frac{F}{K}} > 0$

and given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following:

$|J|\frac{dK}{d\tau^X} = \begin{vmatrix} J_{11} & A \\ J_{21} & B \end{vmatrix} = J_{11}B - AJ_{21} > 0$. Given that $|J| < 0$, it follows that $\frac{dK}{d\tau^X} < 0$.

As for the effect of the subsidy on abatement activity, by defining

$a \equiv -\frac{c}{\sigma} F_K \left[\Psi - \Psi' \frac{X}{F} \frac{R\eta}{R\eta + T\frac{F}{K}} \right] < 0, b \equiv \frac{(\Psi')^2}{K} \frac{X^2}{R\eta + T\frac{F}{K}} > 0$

and given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following: $|J| \frac{dK}{ds} = \begin{vmatrix} J_{11} & a \\ J_{21} & b \end{vmatrix} =$

$J_{11} b - a J_{21} < 0$. Given that $|J| < 0$, it follows that $\frac{dK}{ds} > 0$.

Appendix C. Introducing a polluting input

In this Appendix we show the equivalence of our model with the case in which pollution is not proportional to the scale of production but is the direct effect of a "dirty" input, z (i.e. fuel). More precisely, we will prove that this extended model is equivalent to ours if the flow of pollution x for a representative firm is either equal to z ($x = z$) or an increasing function of z ($x = \phi(z), \phi' > 0$). As a consequence, the effects a tax on z will be qualitatively the same as those produced by τ^x and shown in section 5 of the paper.

Suppose that each firm has the following production function: $G(t) = G(k(t), l(t), z(t))$, with G displaying CRS in (k, l, z) and homogeneity of degree m in (k, l), i.e. $G(\lambda k, \lambda l, z) = \lambda^m G(k, l, z)$. Let p_z denote the price of input z and let us define $F \equiv G(k, l, z) - p_z z$ as the net production value and $\tilde{F} \equiv \max_z G(k, l, z) - p_z z = G(k, l, \tilde{z}) - p_z \tilde{z}$ as potential output. Note that $\tilde{F}$ is CRS in (k, l): in fact, from FOC w.r.t. z: $G_z(k, l, \tilde{z}) = p_z$, we can obtain the demand function for $z : \tilde{z} \equiv \tilde{Z}(k, l; p_z)$, so that $\tilde{F} = G\left(k, l, \tilde{Z}(k, l; p_z)\right) - p_z \tilde{Z}(k, l; p_z)$. Partial derivatives of $\tilde{F}$ (and Envelope theorem) read as $\tilde{F}_k = G_k$, $\tilde{F}_l = G_l$ and hence, $\tilde{F}_k k + \tilde{F}_l l = G_k k + G_k l = G - G_z \tilde{z} = \tilde{F}$ (the second equality follows from CRS of G).

Next, let ψ be defined as: $F = (1 - \psi)\tilde{F}$, that is the proportion by which potential output is reduced due to environmental concerns (e.g. equivalent to abatement of pollution in our original model), so that:

$$(1 - \psi) = \frac{F}{\tilde{F}} = \frac{G(k, l, z) - p_z z}{\tilde{F}} = \frac{z}{\tilde{F}} \left[\frac{G(k, l, z)}{z} - p_z \right]. \quad \text{(A.1)}$$

In the next steps of the proof we will show that $\frac{G(k,l,z)}{z}$ can be written as a function of $\frac{z}{\tilde{F}}$, so that, from (A.1), $\psi = \psi\left(\frac{z}{\tilde{F}}\right)$, i.e. ψ is a function of the polluting input divided by potential output.

By CRS of G and given $G_z(k,l,\tilde{z}) = p_z$ can write:

$$\tilde{F} = G(k,l,\tilde{z}) - p_z\tilde{z} = G_k(k,l,\tilde{z})k + G_l(k,l,\tilde{z})l = G_k\left(\frac{k}{\tilde{z}},\frac{l}{\tilde{z}},1\right)k + G_l\left(\frac{k}{\tilde{z}},\frac{l}{\tilde{z}},1\right)l \qquad \text{(A.2)}$$

From homogeneity of degree m of G in (k,l), we can write $G_i\left(\frac{k}{\tilde{z}},\frac{l}{\tilde{z}},1\right) = \left(\frac{l}{\tilde{z}}\right)^{m-1} G_i\left(\frac{k}{l},1{,}1\right)$, $i = k,l$ and substituting into (A.2) it follows:

$$\tilde{F} = \left(\frac{l}{\tilde{z}}\right)^{m-1}\left[G_k\left(\frac{k}{l},1{,}1\right)k + G_l\left(\frac{k}{l},1{,}1\right)l\right] \qquad \text{(A.3)}$$

From homogeneity of degree m of G in (k,l) we get also $\left(\frac{l}{z}\right)^{1-m} G_i\left(\frac{k}{z},\frac{l}{z},1\right) = G_i\left(\frac{k}{l},1{,}1\right)$, $i = k,l$ and substituting into (A.3) we get:

$$\tilde{F} = \left(\frac{z}{\tilde{z}}\right)^{m-1}\left[G_k\left(\frac{k}{z},\frac{l}{z},1\right)k + G_l\left(\frac{k}{z},\frac{l}{z},1\right)l\right]. \qquad \text{(A.4)}$$

From homogeneity of degree m in (k,l) of G it follows that:

$$m \cdot G = G_k k + G_l l \qquad \text{(A.5)}$$

and substituting for the RHS of (A.5) into (A.4) yields:

$$\tilde{F} = \left(\frac{z}{\tilde{z}}\right)^{m-1} \cdot m \cdot G(k,l,z) \qquad \text{(A.6)}$$

Under CRS of G:

$$G = G_k k + G_l l + G_z z \qquad \text{(A.7)}$$

and subtracting (A.5) from (A.7) we get $(1-m)\cdot G = G_z z$, or, $(1-m)\cdot G(k,l,\tilde{z}) = p_z\tilde{z}$; substituting for G from the latter equation into $\tilde{F} = G(k,l,\tilde{z}) - p_z\tilde{z}$ we get $\tilde{F} = \frac{p_z\tilde{z}}{(1-m)} - p_z\tilde{z} = \frac{mp_z\tilde{z}}{(1-m)}$, from which we obtain $\tilde{z} = \frac{(1-m)}{p_z m}\tilde{F}$. Substituting for $\tilde{z}$ into [A.6] we get: $\tilde{F} = \left(\frac{z}{\frac{(1-m)}{p_z m}\tilde{F}}\right)^{m-1} \cdot m \cdot G(k,l,z)$ or

$$\frac{G}{z} = \left(\frac{p_z}{1-m}\right)^{1-m} m^{-m} \left(\frac{z}{\tilde{F}}\right)^{-m}$$

that is, $\frac{G}{z}$ is a function of $\frac{z}{\tilde{F}}$, and thus, from (A.1), $\psi = \psi\left(\frac{z}{\tilde{F}}\right)$.

Finally, at firm level, net output minus wage costs, taxes, capital cost, and investment (which enters in into equation (24)) is $G(k,l,z) - p_z z - \tau^z z - \delta k - wl + \frac{u_p}{u_c} p(z) - \dot{k} = F(k,l,z) - \tau^z z - \delta k - wl + \frac{u_p}{u_c} p(z) - \dot{k}$, that is

$$\left[1 - \psi\left(\frac{z}{\tilde{F}}\right)\right] \tilde{F}(k,l;p_z) - \tau^z z - \delta k - wl + \frac{u_p}{u_c} p(z) - \dot{k} \tag{A.8}$$

Notice that equation (A.8) is precisely what enters into equation (24) for our original model, with either $p(z)$ and $x = z$ or $p(\phi(z))$and $x = \phi(z)$. Also, the first and second derivatives of $\psi(\cdot)$ have the same signs, so that the effects of τ^z will be qualitatively the same as those of τ^x. This implies that an economy with a polluting factor is equivalent to our economy with pollution proportional to output where the firm has an abatement technology.

Appendix D. Proof of Proposition 3

As for the effect of τ^X on pollution, by eq. (31') it follows that $\frac{dX}{d\tau^X} = \frac{X}{K} \frac{1}{R\eta + T\frac{F}{K}} \left(T \frac{F}{K} \theta \frac{dK}{d\tau^X} - R \frac{\sigma}{c} K \frac{dc}{d\tau^X} - X\right)$

and, by Cramer's rule $|J| \frac{dX}{d\tau^X} = \frac{X}{K} \frac{1}{R\eta + T\frac{F}{K}} \left[T \frac{F}{K} \theta (J_{11}B - AJ_{21}) - R \frac{\sigma}{c} K (J_{22}A - BJ_{12}) - X (J_{11}J_{22} - J_{12}J_{21})\right]$

By expanding J_{11}, J_{21}, A, B and J_{22} and collecting terms we get:

$|J| \frac{dX}{d\tau^X} = \frac{X}{K} \frac{1}{R\eta + T\frac{F}{K}} \left[T \frac{F}{K} \theta H \frac{\sigma}{c} F_K \frac{X}{K} \frac{T}{R\eta + T\frac{F}{K}} - XHJ_{12}\right] > 0$, so that $\frac{dX}{d\tau^X} < 0$.

As for the effect of s on pollution, by exploiting eq. (31) it follows that $|J| \frac{dX}{ds} = |J| \frac{\partial X}{\partial K} \frac{dK}{ds} + |J| \frac{\partial X}{\partial c} \frac{dc}{ds} + |J| \frac{\partial X}{\partial s}$. Applying (31''), the expression above can be written as:

$\left(R\eta + T\frac{F}{K}\right)|J|\frac{dX}{ds} = T\frac{X}{K}\frac{F\theta}{K}|J|\frac{dK}{ds} - \frac{X}{c}R\sigma|J|\frac{dc}{ds} + \Psi'\frac{X^2}{K}|J|$. Using Cramer's rule for $|J|\frac{dK}{ds}$ and $|J|\frac{dc}{ds}$, we obtain: $\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = \frac{T}{K}\frac{F\theta}{K}(J_{11}b - aJ_{21}) - \frac{R\sigma}{c}(aJ_{22} - bJ_{12}) + \Psi'\frac{X}{K}(J_{11}J_{22} - J_{12}J_{21})$

After some manipulation and collecting terms the above equation can be written as:

$\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = \frac{T}{K}\frac{F\theta}{K}(J_{11}b - aJ_{21}) + J_{22}RF_K\left(\Psi - \Psi'\frac{X}{F}\right) + H\Psi'\frac{X}{K}J_{12}$. The term $(J_{11}b - aJ_{21})$ is equal to $F_K\left(\Psi - \Psi'\frac{X}{F}\right)\Psi' X\frac{R}{R\eta + T\frac{F}{K}} - HF_K\frac{c}{\sigma}\left(\Psi - \Psi'\frac{X}{F}\frac{R\eta}{R\eta + T\frac{F}{K}}\right)$. Morover, recognizing that, by eqs. (27) and (28), $J_{12} = -\frac{c}{\sigma}\frac{F_K}{K}\left[T\theta\frac{R\eta}{R\eta + T\frac{F}{K}} + \frac{\rho+\delta}{F_K}\beta\right]$ the equation above is:

$\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = J_{22}RF_K\left(\Psi - \Psi'\frac{X}{F}\right) - \frac{c}{\sigma}H\Psi'\frac{X}{K}\frac{F_K}{K}\frac{\rho+\delta}{F_K}\beta + \left[\left(\Psi - \Psi'\frac{X}{F}\right)\Psi'\frac{X}{c}\frac{R\sigma}{R\eta + T\frac{F}{K}} - H\Psi\right]\frac{T}{K}\frac{F\theta F_K}{K}\frac{c}{\sigma}$.

Exploiting the expression for J_{22} and collecting terms the latter equation reads as

$\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = RF_K\left(\Psi - \Psi'\frac{X}{F}\right)\left[F_K\left(1 - \Psi + \Psi'\frac{X}{F}\right) - \delta\right] + \frac{c}{\sigma}\frac{H}{K}\left[-\Psi'\frac{X}{F}\frac{\rho+\delta}{F_K}\beta - TF_K\Psi\right]$

Next, exploiting the steady state relationships $F_K\left(1 - \Psi + \Psi'\frac{X}{F}\right) - \delta = \rho - s\left(\Psi - \Psi'\frac{X}{F}\right)F_K > 0$ and $c\frac{H}{K} = (1 - \Psi)\frac{F}{K} - \delta$ and collecting terms we obtain:

$\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = RF_K\left(\Psi - \Psi'\frac{X}{F}\right)\left[\rho - s\left(\Psi - \Psi'\frac{X}{F}\right)F_K\right] + \frac{1}{\sigma}\left[(1 - \Psi)\frac{F}{K} - \delta\right]\left[-\Psi'\frac{X}{F}\frac{\rho+\delta}{F_K}\beta - TF_K\Psi\right]$

that is: $\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = \Theta + RF_K\left(-\Psi'\frac{X}{F}\right)\left[\rho - s\left(\Psi - \Psi'\frac{X}{F}\right)F_K\right] - \frac{1}{\sigma}\left[(1 - \Psi)\frac{F}{K}\right]TF_K\Psi$

with $\Theta \equiv RF_K\Psi\left[\rho - s\left(\Psi - \Psi'\frac{X}{F}\right)F_K\right] + \frac{1}{\sigma}\left[(1 - \Psi)\frac{F}{K} - \delta\right]\left[-\Psi'\frac{X}{F}\frac{\rho+\delta}{F_K}\beta\right] + \frac{\delta}{\sigma}TF_K\Psi > 0$.

Exploiting the expressions $R = -(1 - s)\Psi'\frac{X}{K}$, $T \equiv \left(\frac{X}{F}\right)^2(1 - s)\Psi''$ and the relationship $\frac{\Psi''}{\Psi'} = (1 - \xi)\frac{\Psi'}{\Psi}$, the above equation becomes:

$\left(R\eta + T\frac{F}{K}\right)\frac{|J|}{X}\frac{dX}{ds} = \Theta + (1 - s)(\Psi')^2\frac{X}{K}\frac{X}{F}F_K\left[\rho - s\left(\Psi - \Psi'\frac{X}{F}\right)F_K - \frac{(1-\xi)}{\sigma}(1 - \Psi)\right]$

Sufficient for the RHS to be positive (i.e. for $\frac{dX}{ds} < 0$) is: $\rho - s\left(\Psi - \Psi' \frac{X}{F}\right) F_K - \frac{(1-\xi)}{\sigma}(1-\Psi) \geq 0$.

Given that $0 < \left(\Psi - \Psi' \frac{X}{F}\right) < 1$ and $0 < (1-\Psi) < 1$, sufficient for the above inequality to hold is $\frac{\rho - \frac{(1-\xi)}{\sigma}}{F_K} \leq s$. Given that $\frac{dK}{ds} > 0$ and $\frac{dF_K}{ds} < 0$, sufficient for the above inequality to hold is $\frac{\rho - \frac{(1-\xi)}{\sigma}}{\phi} \leq s$, with $\phi \equiv F_K(s=0)$, i.e. marginal productivity of capital for $s = 0$. □

Appendix E. Proof of Proposition 4

As for the effect of the tax on pollution, given that $A, B, J_{22}, > 0$ and $J_{12} < 0$, Cramer's rule yields:

$$|J| \frac{dc}{d\tau^X} = \begin{vmatrix} A & J_{12} \\ B & J_{22} \end{vmatrix} = AJ_{22} - BJ_{12} > 0$$

Given that $|J| < 0$, it follows that $\frac{dc}{d\tau^X} < 0$.

As for the effect of the subsidy on abatement activity, by defining given that $J_{11} < 0$ and $J_{21} < 0$, Cramer's rule provides the following:

$$|J| \frac{dc}{ds} = \begin{vmatrix} a & J_{12} \\ b & J_{22} \end{vmatrix} = aJ_{22} - bJ_{12}$$

whose sign is ambiguous. However, by exploiting the definitions of a, J_{22}, b, J_{12} , the above equation can be written as:

$$|J| \frac{dc}{ds} = -\frac{c}{\sigma} \frac{F_K}{R\eta + T\frac{F}{K}} \left\{ R\eta \left(\Psi - \Psi' \frac{X}{F}\right) \left[\left(1 - \Psi + \Psi' \frac{X}{F}\right) F_K - \delta\right] + \Psi T \frac{F}{K} [(1-\Psi)F_K - \delta] - \left(\Psi' \frac{X}{K}\right)^2 \beta \left[1 - (1-s)\Psi + (1-\mathrm{s})\Psi' \frac{X}{F}\right] \right\}.$$

Recognizing that $\frac{\Psi''}{\Psi'} = (1-\xi)\frac{\Psi'}{\Psi}$, so that $T = \left(\frac{X}{F}\right)^2 (1-s)(1-\xi) \frac{(\Psi')^2}{\Psi}$ and $T \frac{F}{K} \Psi = \frac{F}{K} \frac{X}{F} (1 - s)(1-\xi)(\Psi')^2$ and $\frac{F}{K} = \frac{F_K}{\theta}$, $\left[1 - (1-s)\Psi + (1-\mathrm{s})\Psi' \frac{X}{F}\right] F_K - \delta = \rho$, the above expression takes the form:

$$|J|\frac{dc}{ds}=\frac{c}{\sigma}\frac{F_K(\Psi')^2\frac{FX}{KF}}{R\eta+T\frac{F}{K}}\left\{-\eta(1-s)\frac{\left(\Psi-\Psi'\frac{X}{F}\right)}{-\Psi'\frac{X}{F}}\left[\left(1-\Psi+\Psi'\frac{X}{F}\right)F_K-\delta\right]-(1-s)(1-\xi)[(1-\Psi)F_K-\delta]+\frac{\beta}{\theta}(\rho+\delta)\right\}.$$

Given that $|J|<0$, $-(1-s)(1-\xi)[(1-\Psi)F_K-\delta]<0$, and $0<\frac{\left(\Psi-\Psi'\frac{X}{F}\right)}{-\Psi'\frac{X}{F}}<1$, sufficient for $\frac{dc}{ds}>0$ is $-\eta(1-s)\left[\left(1-\Psi+\Psi'\frac{X}{F}\right)F_K-\delta\right]+\frac{\beta}{\theta}(\rho+\delta)<0$. Given that $\left(1-\Psi+\Psi'\frac{X}{F}\right)F_K-\delta=\frac{\rho-s(F_K-\delta)}{1-s}>0$, the latter inequality reads as $-\eta\left[\frac{\rho-s(F_K-\delta)}{1-s}\right]+\frac{\beta}{\theta}(\rho+\delta)<0$, that is $0\leq s\leq\frac{(\eta+1-\xi)-\frac{\beta}{\theta}\left(\frac{\rho+\delta}{\rho}\right)}{(\eta+1-\xi)\left(\frac{F_K-\delta}{\rho}\right)}$. Finally, recognizing that, by eq. (32) evaluated at steady state, $1-\frac{\rho+\delta}{F_K}=(1-s)\left[\Psi-\Psi'\frac{X}{F}\right]>0$ implies $F_K>\delta+\rho$, the above restrictions can be written as:

$$s\leq 1-\frac{\frac{\beta}{\theta}\left(\frac{\rho+\delta}{\rho}\right)}{(\eta+1-\xi)}$$ □

Appendix F. Proof of Proposition 5

As for the effect of the tax on the pollution premium R, by total differentiation of logs of (34) we get:

$$\frac{dR}{R}=p\,\frac{u_{pp}}{u_p}\frac{dp}{p}-\frac{u_{cc}}{u_c}c\frac{dc}{c}+\frac{dX}{X}-\frac{dK}{K}=(1+\eta)\frac{dX}{X}+\sigma\frac{dc}{c}-\frac{dK}{K} \tag{D.1}$$

Taking (31’) and collecting terms we get:

$$\frac{\left(R\eta+T\frac{F}{K}\right)}{(1+\eta)}\frac{dR}{R}=\left(T\frac{F}{K}\frac{\theta}{K}\right)dK+\left(T\frac{F}{K}-R\right)\frac{\sigma}{c(1+\eta)}\,dc-\left(\frac{X}{K}\right)d\tau^x+\left(\Psi'\frac{X}{K}\right)ds=0 \tag{D.2}$$

Focusing on τ^x, exploiting the results of previous Propositions, we can write:

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{(1+\eta)R}\frac{dR}{d\tau^x}$$

$$=\left(T\frac{F}{K}\frac{\theta}{K}\right)(J_{11}B-AJ_{21})+\left(T\frac{F}{K}-R\right)\frac{\sigma}{c(1+\eta)}(AJ_{22}-BJ_{12})$$

$$-\frac{X}{K}(J_{11}J_{22}-J_{12}J_{21})$$

After some manipulation and collecting terms we can write

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{(1+\eta)R}\frac{dR}{d\tau^x}=\left(T\frac{F}{K}\frac{\theta}{K}\right)H\frac{c}{\sigma}F_K\frac{X}{K}\frac{T}{R\eta+T\frac{F}{K}}+\frac{T}{1+\eta}\frac{X}{K}F_KJ_{22}+\frac{\Psi'X}{(1+\eta)}\frac{\sigma}{c}\frac{X}{K}J_{12}-\frac{X}{K}HJ_{12}$$

$$>0$$

and thus, given that $|J|<0, \frac{dR}{d\tau^x}<0$.

As for the subsidy on pollution abatement, from (D.1) and (31') we know that

$$\frac{dR}{R}=(1+\eta)\frac{dX}{X}+\sigma\frac{dc}{c}-\frac{dK}{K}=\left[\frac{(1+\eta)}{X}\frac{\partial X}{\partial K}-\frac{1}{K}\right]dK+\left[\frac{(1+\eta)}{X}\frac{\partial X}{\partial c}+\frac{\sigma}{c}\right]dc+\frac{(1+\eta)}{X}\frac{\partial X}{\partial s}ds \qquad \text{(D.3)}$$

Substituting from (31'') and rearranging terms we get $\frac{1}{R}\frac{dR}{ds}=\frac{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta}{K\left(R\eta+T\frac{F}{K}\right)}\frac{dK}{ds}+\frac{\sigma}{c}\frac{T\frac{F}{K}-R\eta}{R\eta+T\frac{F}{K}}\frac{dc}{ds}+\frac{(1+\eta)\Psi'}{R\eta+T\frac{F}{K}}\frac{X}{K}$.

By exploiting previous results on $|J|\frac{dK}{ds}$ and $|J|\frac{dc}{ds}$ we can write the following

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}\frac{1}{K}(J_{11}b-J_{21}a)+\frac{\sigma}{c}\left(T\frac{F}{K}-R\eta\right)(aJ_{22}-bJ_{12})+(1+\eta)\Psi'\frac{X}{K}(J_{11}J_{22}-J_{12}J_{21})$$

which can also be written as:

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}$$
$$=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}\frac{1}{K}(J_{11}b-J_{21}a)+\left[a\frac{\sigma}{c}T\frac{F}{K}-\frac{\sigma}{c}R\eta a+(1+\eta)\Psi'\frac{X}{K}J_{11}\right]J_{22}$$
$$+\left[-\frac{\sigma}{c}\left(T\frac{F}{K}-R\eta\right)b-(1+\eta)\Psi'\frac{X}{K}J_{21}\right]J_{12}$$

Recognizing that $\Psi'\frac{X}{K}J_{11}-\frac{\sigma}{c}Ra=RF_K\left(\Psi-\Psi'\frac{X}{F}\right)$ and that $\frac{\sigma}{c}b=\Psi'\frac{X}{K}J_{21}+\frac{H}{R}\Psi'\frac{X}{K}$, the above equation becomes:

$$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}$$

$$=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}\frac{1}{K}(J_{11}b-J_{21}a)+\left(T\frac{F}{K}+R\eta\right)\frac{\Psi'}{R}\frac{X}{K}J_{11}J_{22}$$

$$-\left(T\frac{F}{K}+R\eta\right)\frac{\Psi'}{R}\frac{X}{K}J_{21}J_{12}-\left(T\frac{F}{K}-R\right)F_K\left(\Psi-\Psi'\frac{X}{F}\right)J_{22}-\frac{\Psi'}{R}\frac{X}{K}H\left(T\frac{F}{K}-R\right)J_{12}$$

that is, collecting J_{12} and J_{22}:

$|J|\frac{\left(R\eta+T\frac{F}{K}\right)}{R}\frac{dR}{ds}=\left\{[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\right\}\frac{1}{K}(J_{11}b-J_{21}a)+F_KJ_{22}\left[R\left(\Psi-\Psi'\frac{X}{F}\right)-T\frac{F}{K}\Psi\right]+$ $\Psi'\frac{X}{K}J_{12}\left[(1+\eta)H-\Psi'X\frac{\sigma}{c}\right]$

Notice that $F_KJ_{22}\left[R\left(\Psi-\Psi'\frac{X}{F}\right)-T\frac{F}{K}\Psi\right]=F_KJ_{22}\left\{-\frac{X}{K}(1-s)\Psi'\left[\Psi-\Psi'\frac{X}{F}+(1-\xi)\Psi'\frac{X}{\mathrm{F}}\right]-\frac{X}{\mathrm{K}}\tau^X\left(\Psi-\Psi'\frac{X}{F}\right)\right\}$, which is positive if τ^X=0, given that $J_{22}>0$. Moreover, $\Psi'\frac{X}{K}J_{12}\left[(1+\eta)H-\Psi'X\frac{\sigma}{c}\right]>0$, given that $J_{12}<0$. Hence, given that $(J_{11}b-J_{21}a)<0$, sufficient for the RHS to be positive (i.e. $\frac{dR}{ds}<0$) is $T\frac{F}{K}-R\leq 0$ and $[(1+\eta)\theta-1]T\frac{F}{K}-R\eta\leq 0$.

If $\tau^X=0$, the latter inequality reads as

$[(1+\eta)\theta-1]\frac{X}{F}(1-\xi)\frac{(\Psi')^2}{\Psi}+\Psi'\eta\leq 0$ or $\frac{[(1+\eta)\theta-1]}{\eta}(1-\xi)\frac{X}{F}\Psi'+\Psi\geq 0$. Sufficient for the latter inequality to hold true is $(1+\eta)\theta-1\leq 0$. □