# Logic Differential Calculus for Reliability Analysis Based on Survival Signature

Patrik Rusnak, *Member, IEEE*, Elena Zaitseva, *Member, IEEE*, Frank P.A. Coolen, Miroslav Kvassay, *Member, IEEE*, Vitaly Levashenko, *Member, IEEE*

***Abstract*— The structure function is an often-used mathematical representation of the investigated system in reliability analysis. It is a binary function that models system state according to states of its components. The size of the structure function depends on the number of components and can be enormous for systems with many components. Therefore, the system reliability analysis based on the structure function needs special methods to decrease this dimension and to measure the system reliability. The concept of survival signature provides a useful transformation of the structure function to simplify reliability assessment for systems with many components of specified types. The survival signature is a complete probabilistic description of the system. The new methods and algorithms of system reliability analysis based on this mathematical representation should be developed. The Direct Partial Logic Derivative is one of approaches that are effective in system reliability evaluation based on the structure function. This approach is used to determine different aspects of system failure depending on system components breakdowns. The development of this derivative for survival signature permits to obtain the new method for the reliability analysis of system failure caused by system component breakdown depending on components types.**

***Index Terms*—direct partial logic derivative, logic differential calculus, structure function, survival signature, system reliability**

## I. Introduction

The reliability analysis allows investigating and evaluating specifics of the system behavior at the different phases of its life cycle and can be used to find system's weaknesses, which can cause, in worst case, catastrophic effects. The important step in this analysis is the mathematical representation of system. There are different types of system representations as structure function, Markov models, Bayesian networks and other [1], [2]. The structure function is used for the mathematical representation of system in this study because an important advantage of this representation is the possibility of application for systems of any structure complexity. The structure function maps all of the possible combinations of states of system components to one of possible system states at time *t*.

Let us consider a system of $n$ components for which the $i$-th component is denoted as $x_i$ $(i = 1, \dots, n)$ where $x_i = 1$ if this component is in a working state and $x_i = 0$ if not. The system state is defined based on the components states and is mathematically described by the structure function [3], [4]:

$$\phi(x_1, \dots, x_n) = \phi(\boldsymbol{x}): \ \{0,1\}^n \to \{0,1\}, \quad (1)$$

where $\boldsymbol{x} = (x_1, \dots, x_n)$ is a vector of the system components states (state vector).

In this paper, the coherent systems are considered. In these systems, any system component must be relevant and a failure of any component cannot cause improvement of the system. According to these conditions, the structure function is monotonically non-decreasing [3], [5], [6]: $\phi(1_i, \boldsymbol{x}) \geq \phi(0_i, \boldsymbol{x})$ for any component and $\phi(1_i, \boldsymbol{x}) \neq \phi(0_i, \boldsymbol{x})$ for some state vectors (where $\phi(1_i, \boldsymbol{x}) = \phi(x_1, \dots, x_{i-1}, 1, x_{i+1}, \dots, x_n)$ and $\phi(0_i, \boldsymbol{x}) = \phi(x_1, \dots, x_{i-1}, 0, x_{i+1}, \dots, x_n)$.

One of the principal disadvantages of the structure function is computational complexity of its analysis with is correlated with the structure function size. It is defined as $2^n$ [3], [4] and grows exponentially with the number of components. There are some approaches to solve this problem. One of them is transformation of the structure function (1) to its survival signature, which was introduced in [7]. The survival signature is effective for the system consisting of several types of components if number of these types is less than number of components.

Let us consider a system that has $K \geq 2$ types of components and $n_k$ represents the number of components of type $k = 1, \dots, K$ where $\sum_{k=1}^{K} n_k = n$. We will also consider that the failure time of components of the same type are independently and identically distributed or exchangeable. It is possible to group together the components of the same type, thanks to the random ordering of the state vector's components. This allows us to use $\boldsymbol{x} = (\boldsymbol{x}^1, \dots, \boldsymbol{x}^K)$ as a state vector, where $\boldsymbol{x}^k = (x_1^k, \dots, x_{n_k}^k)$ represents the states of the components of type $k$.

This paragraph of the first footnote will contain the date on which you submitted your paper for review.

This work was supported under Grant APVV 18-0027 of the Slovak Research and Development Agency.

Patrik Rusnak, Elena Zaitseva, Miroslav Kvassay and Vitaly Levashenko are with the Department of Informatics, University of Žilina, Univerzitna 8215/1, 01026, Žilina, Slovakia (e-mails: {patrik.rusnak, elena.zaitseva, miroslav.kvassay, vitaly.levashenko }@fri.uniza.sk).

Frank P.A. Coolen is with the Department of Mathematical Sciences, Durham University, DH1 3LE Durham, U.K. (e-mail: frank.coolen@durham.ac.uk).

Survival signature $\Phi(l_1, \dots, l_K)$ is the probability that the structure function has value 1, given that precisely $l_1, \dots, l_K$ of its components of types $1, \dots, K$ are functioning and is defined as follows [7]:

$$\Phi(l_1, \dots, l_K) = \left[\prod_{k=1}^{K} \binom{n_k}{l_k}^{-1}\right] * \sum_{x \in S_{l_1,\dots,l_K}} \phi(\boldsymbol{x}), \quad (2)$$

where $S_{l_1,\dots,l_K}$ is a set of all state vectors for the whole system at which $\sum_{i=1}^{n_k} x_i = l_k$, for $k = 1, \dots, K$, and $\boldsymbol{x} = (x_1, \dots, x_n)$ is a binary state vector defining states of the system components.

According to study in [8], the survival signature computed from the structure function restricts the dimension of system mathematical representation. The dimension of mathematical representation based on survival signature is depended on *K* instead *n* (*K* ≤ *n*). However, application of this new representation requires development of new methods and algorithms for reliability analysis and evaluation. One of often considered problem in reliability analysis is identification and evaluation of the system critical states. Typically, the critical system state is considered as a situation, in which a change of state of one of system components result in the change of the system state. This problem is well investigated for the system represented by the structure function as part of importance analysis [5]. Methods for identification of those states have been developed based on different mathematical approaches among which the Logic Differential Calculus will be considered in this paper. This approach has been proposed in [9] for Boolean algebra and in [10] for application in logic design. The structure function (1) is a Boolean function. It means that methods based on Boolean algebra can be used in reliability analysis and evaluation of the considered system.

The central term of Logic Differential Calculus is logic derivative. *Direct Partial Logic Derivative* (DPLD) was proposed for the reliability analysis and evaluation in [11], [12]. This type of direct derivative is used to analyze how a specific change of component state (from $s$ to $\bar{s}$) affects the system functionality (from $j$ to $\bar{j}$). The DPLD with respect to variable $x_i$ is defined as follows [13]:

$$\frac{\partial\phi(j \to \bar{j})}{\partial x_i(s \to \bar{s})} = \begin{cases} 1, & \phi(s_i, \boldsymbol{x}) = j \text{ and } \phi(\bar{s_i}, \boldsymbol{x}) = \bar{j} \\ 0, & \text{otherwise} \end{cases}, \quad (3)$$

where $s, j \in \{0,1\}$, $\bar{s}$ and $\bar{j}$ are negations of values $s$ and $j$ respectively.

Study [14] presented that there are 4 DPLDs (3) that are: $\partial\phi(1 \to 0)/\partial x_i(1 \to 0)$, $\partial\phi(0 \to 1)/\partial x_i(0 \to 1)$, $\partial\phi(1 \to 0)/\partial x_i(0 \to 1)$, $\partial\phi(0 \to 1)/\partial x_i(1 \to 0)$. For monotonically non-decreasing functions two last are equal zero and the other two are equal to each other and can be calculated based on Boolean expression:

$$\frac{\partial\phi(1 \to 0)}{\partial x_i(1 \to 0)} = \frac{\partial\phi(0 \to 1)}{\partial x_i(0 \to 1)} = \overline{\phi(0_i, \boldsymbol{x})} \wedge \phi(1_i, \boldsymbol{x}), \quad (4)$$

where ∧ denotes Boolean operation AND and $\overline{\phi(0_i, \boldsymbol{x})}$ negation of function $\phi(0_i, \boldsymbol{x})$.

According to (4), state vectors are indicated as non-zero elements of the derivative if the change of the *i*-th variable from value 1 to value 0 results in the change of the structure function value from 1 to 0. It means that the DPLD indicates the state vectors for which the breakdown of the *i*-th component causes the system failure. The derivative (4) in context of reliability engineering allows investigating the influence of change of component state on the system state and identifying the critical system states [15], minimal cut/path sets [12], or to calculate importance measures [11]. In this paper, we propose to generalize the DPLD for the analysis of the survival signature of the structure function to analyze the system critical states.

The paper is organized as follows: the state of the art of paper's subject is considered in section II. The hand calculated example for the illustration of the proposed method is introduced in section III. The key definitions and concepts as system structure function, DPLDs, and survival signature for this example are illustrated in this section too. Section IV introduces three new types of logic derivatives for the analysis of system critical states based on the system survival signature. Section V focuses on specifics of the proposed approach using two case studies that illustrate computational aspects of the new DPLDs and their evaluation.

## II. The Problem State Analysis

The development of the mathematical representation of the system is an important step in the reliability analysis. There are different types of the system mathematical representation. Each of them has specific application in reliability analysis and its usability depends on system type, goal of the analysis, or application problems. For example, Markov models are useful for analysis of dynamic properties of the system reliability [16], [17]. Bayesian networks are recommended for the analysis of system reliability based on uncertain initial data [2], [18]. The Universal Generating Function is an effective mathematical representation for calculation of system reliability, which allows mapping relation between the working states of the system or the components and the corresponding state probabilities [19]. The structure function is used for the mathematical representation of the system if the topology or topological properties are well indicated [3], [7], [20]. It is needed to point out that the structure function can be formed simply for system with any structure complexity. However, at the same time the structure function size dramatically increases with increasing number of system components. This means that the large size of real-life systems complicates the reliability analysis based on the structure function. Therefore, the methods for the size reduction or/and methods for large size system evaluation have been developed for the structure function.

One of often used approaches for the development of methods for large dimensional system evaluation is Binary Decision Diagram (BDD). A BDD was introduced in Boolean algebra by Akers [21] for the analysis of dimensionally large Boolean functions. Effective applications of this approach in reliability analysis have been considered in many investigations, such as calculation of system reliability and availability [11], [22], fault tree construction and analysis [15], [23], or importance analysis [4], [24].

In addition to the methods based on BDDs, there are methods that allow reducing the dimension/size of the structure function. Such methods can be based on the decomposition of the structure function [25], or they can be based on survival signature approach that also reduces the dimension of the structure function [26]. This approach is based on the concept of survival signature introduced in [7]. The concept of the survival signature is an improvement of the system signature concept defined in [27]. In reliability analysis the survival signature has been recognized as an important tool to quantify the reliability of systems. This approach has been used for reliability analysis of systems with different types of components (which can have different life time distributions) [28], [29], for system importance analysis [30]–[32], for evaluation of phased mission systems [33]. The review of survival signature based methods in studies [7], [26], [30]–[33] shows that the development of methods for importance analysis based on survival signature is relevant problem in system reliability analysis.

The importance analysis [5] is an important part of reliability analysis, which has been intensively studied and successfully used in many industrial applications. This analysis is implemented for different system types and conditions and includes time-independent analysis [6], [34], dynamic importance analysis [5], [32], [35], or non-coherent system evaluation [3], [35]. The principal purpose of the importance analysis is to qualify and quantify how change of component state influences the system functioning state. This analysis is carried out by evaluating the critical states of the system. Therefore, the identification of the system critical states is a necessary step in importance analysis [34].

There are different mathematical approaches used in the development of methods for identification of critical system states and in the evaluation of their importance for system failure. One such effective approach used in time-independent analysis is based on the Logic Differential Calculus. The Logic Differential Calculus was introduced by Akers [9] and Talantsev [10] in Boolean algebra. This approach found its application also in reliability analysis as is shown in studies [4], [12], [20].

In general, the Logic Differential Calculus has been proposed for analysis of dynamic properties of logic functions by logic derivatives. The DPLD (4) is one type of derivatives in Logic Differential Calculus. The DPLD allows investigating the influence of change of variable value on the function value [9-10]. If the system components states are interpreted as values of the structure function variables and the system state as a value of the structure function, then the DPLD permits analyzing influence of changes of the components states on change of the system state. This derivative has been used in topological analysis of system reliability [36], in finding system minimal cut/path sets [12], in identification of critical system states [34], or in importance analysis [4], [37]. All of these methods based on the DPLD have been developed for the structure function (1). In this paper, we propose the new DPLD based method for reliability evaluation of the system defined by the survival signature. In particular, we consider the method for identification of the critical system states. This method combines the advantages of (a) survival signature in decreasing the dimensionality of the system representation and (b) efficiency of the system analysis based on the DPLD.

One more advantage of the proposed method is a possibility to quantify influence of a specific type of system components on the system operation from topological point of view. Similar evaluation based on the structure function of system (1) with the use of derivative (3) requires additional effort and transformation. DPLDs (3) for such analysis based on structure function (1) must be calculated for every of system component of the indicated type and transformed to define the quantification of the components type. The proposed method focuses on the analysis of influence of a components type and doesn't need additional transformation in this analysis, which is important in the system maintenance.

## III. Hand Calculation Example

Let us consider a simple example of the storage system with three components ($n$ = 3) to illustrate the concept of the structure function, the analysis of the system based on logic derivatives, and system representation based on the survival signature. This system has two main modules, in which the same data is stored. In the first module, two Hard Drive Disks (HDDs) HDD 1 of type 1 and HDD 2 of type 2 are organized in Redundant Array of Independent Disks (RAID) 0. In RAID 0, the capacity of the unit is equal to the sum of capacities of the used drives, which implies no redundancy of data. Therefore, failure of one drive means that the entire RAID 0 is lost. In the second module, the single HDD of type 1 marked as HDD 3 is used to store data. At least one module must be in working state to write and read data successfully.

The reliability block diagram of this system is in Fig. 1, and its mathematical representation by structure function is:

$$\phi(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee x_3, \tag{5}$$

where the operator $\wedge$ is the Boolean operation AND, operator $\vee$ is the Boolean operation OR and state of each HDD is represented by the corresponding Boolean variable $x_i$, for $i = 1,2,3$.

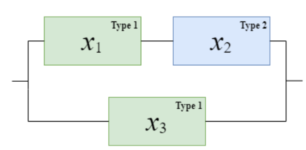


Fig. 1. Reliability block diagram of the storage system.

Let us consider the application of the DPLD (4) to find critical states of data storage system (Fig. 1). From the structure function (5) it is possible to compute the DPLDs to indicate the critical states of this system. The DPLDs calculation according to (5) allows us to obtain the symbolic form of every DPLD:

$$\frac{\partial\phi(1 \to 0)}{\partial x_1(1 \to 0)} = \overline{(0 \wedge x_2 \vee x_3)} \wedge (1 \wedge x_2 \vee x_3) = x_2 \wedge \overline{x_3};$$

$$\frac{\partial\phi(1\to 0)}{\partial x_2(1\to 0)} = \overline{(x_1 \wedge 0 \vee x_3)} \wedge (x_1 \wedge 1 \vee x_3) = x_1 \wedge \overline{x_3};$$

$$\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)} = \overline{(x_1 \wedge x_2 \vee 0)} \wedge (x_1 \wedge x_2 \vee 1) = \overline{x_1} \vee \overline{x_2}.$$

These DPLDs can be expressed by truth tables shown in Table I. It is possible to see from Table I, that HDD 1 or HDD 2 have influence on the system performance only in one situation, in which HDD 2 or HDD 1 are working and HDD 3 is failed, i.e., state vectors $(x_1\ x_2\ x_3) = (*\ 1\ 0)$ for $\phi(1\to 0)/\partial x_1(1\to 0)$ and $(x_1\ x_2\ x_3) = (1 *\ 0)$ for $\phi(1\to 0)/\partial x_2(1\to 0)$ for these situations have non-zero values in Table I. The derivative $\phi(1\to 0)/\partial x_3(1\to 0)$ has only one zero value for state vector $(x_1\ x_2\ x_3) = (1\ 1\ *)$. Therefore, the failure of HDD 3 does not influence the system performance if HDD 1 and HDD 2 are in working state. In other cases, the failure of HDD 3 leads to the system failure.

TABLE I
DPLDS FOR DATA STORAGE SYSTEM

| $x_1$ | $x_2$ | $x_3$ | $\phi(x_1,x_2,x_3)$ | $\frac{\partial\phi(1\to 0)}{\partial x_1(1\to 0)}$ | $\frac{\partial\phi(1\to 0)}{\partial x_2(1\to 0)}$ | $\frac{\partial\phi(1\to 0)}{\partial x_3(1\to 0)}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | - | - | - |
| 0 | 0 | 1 | 1 | - | - | 1 |
| 0 | 1 | 0 | 0 | - | 0 | - |
| 0 | 1 | 1 | 1 | - | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | - | - |
| 1 | 0 | 1 | 1 | 0 | - | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | - |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |

DPLDs in Table I allow us to obtain the system critical states. However, they tell us almost nothing about the importance of individual types of the components. Furthermore, these DPLDs are calculated based on structure function, which has large size if the system consists of a large number of components.

The considered storage system consists of 3 components of 2 types: 2 HDDs, namely HDD 1 and HDD 3, have same type that will be marked as type 1, and HDD 2 has type 2. Therefore, this system can be presented by the survival signature (2). We will compute the survival signature for the data storage system with structure function (5) for each $l_1 \in \{0,1,2\}$ and $l_2 \in \{0,1\}$. For example, in case of $l_1 = 1$ and $l_2 = 0$, there are two state vectors (1,0,0) and (0,0,1) that represent the situation with exactly one working component of type 1 and zero working components of type 2. In these cases, the system is in working state only for one state vector (0,0,1), therefore, $\Phi(1,0) = 0.5$. Computed values of survival signature for storage system can be seen in Table II.

TABLE II
SURVIVAL SIGNATURE OF THE DATA STORAGE SYSTEM

| **Number $l_1$ of working components of type 1** | **Number $l_2$ of working components of type 2** | $\Phi(l_1, l_2)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0.5 |
| 1 | 1 | 1 |
| 2 | 0 | 1 |
| 2 | 1 | 1 |

## IV. NEW DPLDS FOR SURVIVAL SIGNATURE

The DPLD for Boolean function with respect to variable $x_i$ according to (3) allows us to indicate state vector $(x_1, \dots, x_n)$ of the function for which the specified change of the variable $x_i$ results in the specified change of the function. In terms of reliability analysis, this derivative allows indicating critical states of system that agree with state vectors of components at which the breakdown of the $i$-th component causes the failure of the system. We suppose that the approach of logic derivatives can also be developed for the system represented by survival signature (2). In this chapter, possible interpretations of the DPLD for analysis of system by the survival signature are introduced. These derivatives can be used for analysis of the critical system states based on its mathematical representation by survival signature. The considered derivatives are focused on the case of the component breakdown and system degradation for coherent systems.

As the first step, the conception of the critical system state for the survival signature should be considered and defined. Since the survival signature determines the dependency of the state of the system on the number of functioning elements of a certain type, the critical state for the survival signature can be defined as such state for which the failure of one of components of the certain type leads to a change in the state of the system as a whole. In other words, the fact (possibility) of the system state change should be fixed if it is caused by a component of a certain type fails, provided that other components of other types remain unchanged. This change can be identified by comparing two values of the survival signature which one is the system states under the condition of $a$ working components of certain type $k$ and other is the system state for $(a-1)$ working components of type $k$ (one component of the type $k$ is failed) and the unchanged states of components of other types.

Firstly, the critical system states for the fixed number of components of type $k$ are considered. Let the number of working components of the system for type $k$ be equal to $a$. It is necessary to determine under what conditions failure of one component of this type will lead to failure or degradation of the system. In this case, the numbers of working components of other types are considered as conditions for the critical system states. We propose to identify these states based on the first DPLD for survival signature.

**The first DPLD for survival signature** with respect to the variable $l_k$, the value of which changes from $a$ to $(a-1)$, permits to determine the values of survival signature and the corresponding values of the variables for which the specified change of the variable value causes a change in the value of survival signature:

$$\frac{\partial\Phi(l_1,\dots,l_K)\downarrow}{\partial l_k(a_k \to a_k - 1)} = \begin{cases} 1, \Phi(l_1,\dots,a_k,\dots,l_K) > \Phi(l_1,\dots,a_k-1,\dots,l_K) \\ 0, \text{otherwise} \end{cases} \quad (6)$$

where $a_k \in \{1,\dots,n_k\}$ is a number of working components of type $k \in \{1,\dots,K\}$.

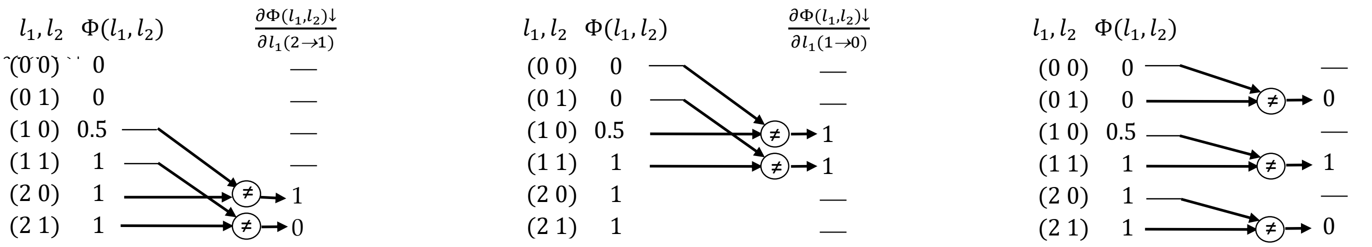


Fig. 2. Flow diagram for the first DPLD for survival signature (Table II).

This derivative in terms of the critical system states allows defining the system states for which the change of the failure of one of $a$ working components of type $k \in \{1, \ldots, K\}$ results in the system degradation or failure. These states correspond to the non-zero values of the DPLD (6).

Let us illustrate the use of the first DPLD for survival signature (Table II) in the analysis of the storage system from section III (Fig. 1). For this example, three DPLDs exist. There are two derivatives for the components of the first type, i.e., $(\partial\Phi(l_1, l_2)\downarrow/\partial l_1(2\to1))$ and $(\partial\Phi(l_1, l_2)\downarrow/\partial l_1(1\to0))$. The derivative for type 2 of components allows analyzing failure of one component $(\partial\Phi(l_1, l_2)\downarrow/\partial l_2(1\to0))$ and its influence to the system state. Calculation of these derivatives is illustrated by the flow diagram in Fig. 2.

According to the first DPLD for the considered system, we can conclude that the value of DPLD $\partial\Phi(l_1, l_2)\downarrow/\partial l_1(2\to1)$ has value 1 in situation, where only two components of type 1 are working because $(\Phi(2{,}0) = 1) > (\Phi(1{,}0) = 0.5)$ and has value 0 in situation when all system components are working, because $(\Phi(2{,}1) = 1) \ngtr (\Phi(1{,}1) = 1)$. Values of all DPLDs can be seen in Table III. It is possible to see that the most crucial change is change of type 1 from one working component to zero because it will always result in decrease of the system signature's value. On the other hand, the change of type 2 from one working component to zero is the least crucial change because it is significant only in one of the three possible situations.

TABLE III
THE FIRST DPLDS OF THE DATA STORAGE SYSTEM

| Type 1 ($l_1$) | Type 2 ($l_2$) | $\Phi(l_1, l_2)$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | - | - | - |
| 0 | 1 | 0 | - | - | 0 |
| 1 | 0 | 0.5 | - | 1 | - |
| 1 | 1 | 1 | - | 1 | 1 |
| 2 | 0 | 1 | 1 | - | - |
| 2 | 1 | 1 | 0 | - | 0 |

Thus, the first DPLD (6) for the survival signature allows the determination of critical system states for a specified type of component and for an exactly specified number of working components of this type. But in some tasks, critical states of the system should be defined as situations in which the failure of one of the components of a certain type causes degradation or failure of the system, regardless of the number of operating components of this type. Such critical system states can be identified using the following DPLD type for the survival signature.

**The second DPLD for survival signature** with respect to the variable $l_k$ permits determining the values of survival signature and the corresponding values of the variables for which any decrease of specified variable by value one causes a change in the value of the survival signature:

$$\frac{\partial\Phi(l_1, \ldots, l_K)\downarrow}{\partial l_k\downarrow} = \begin{cases} 1, \Phi(l_1, \ldots, b_k, \ldots, l_K) > \Phi(l_1, \ldots, b_k - 1, \ldots, l_K) \\ 0, \text{otherwise} \end{cases} \quad (7)$$

for all $b_k = 1, \ldots, n_k$.

Derivative (7) is similar to derivative (6). The difference of these derivatives is number of working components which are analyzed. The derivative (6) allows analyzing exactly defined number of working components. The derivative (7) analyses all possible numbers of working components and provides evaluation of system failure if one working component breakdowns, or one of two working components breakdowns and so on until breakdown of one of all possible working components of fixed type. Therefore, the DPLD (7) can be considered as the union of all possible DPLDs (6) for different values of parameter $a_k$:

$$\frac{\partial\Phi(l_1, \ldots, l_K)\downarrow}{\partial l_k\downarrow} = \bigcup_{a_k=1}^{n_k} \frac{\partial\Phi(l_1, \ldots, l_K)\downarrow}{\partial l_k(a_k\to a_k - 1)} \quad (8)$$

The second DPLD for survival signature allows indicating the critical system states for which the failure of one component of specified type results in the system degradation or failure regardless of the number of working components of this type. This derivative can be calculated according to definition (7) or by unification of the all possible DPLDs (6) for fixed type of components according to (8).

Let's us to continue the analysis of the storage system in Fig.1. There are just two DPLDs of the second type (7) for survival signature of the data storage system (Fig. 1). These DPLDs, namely $\partial\Phi(l_1, l_2)\downarrow/\partial l_1\downarrow$ and $\partial\Phi(l_1, l_2)\downarrow/\partial l_2\downarrow$ are shown in Table IV, and their calculation is illustrated by flow diagram in Fig. 3. The calculation of these derivatives is similar to the calculation of the first DPLD for survival signature (6), which can be seen from the comparison of the flow diagrams in Fig. 2 of the first DPLDs and flow diagrams in Fig. 3 of the second DPLDs for survival signature.

It is possible to see that the most critical type is type 1 because the value of the second DPLD for survival signature is 0 only in one situation out of four, while in case of type 2, there is only one situation out of three, in which the system signature's value will degrade with failure of a component of such a type.

TABLE IV
THE SECOND DPLDS OF THE DATA STORAGE SYSTEM

| Type 1 ($l_1$) | Type 2 ($l_2$) | $\Phi(l_1,l_2)$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | - | - |
| 0 | 1 | 0 | - | 0 |
| 1 | 0 | 0.5 | 1 | - |
| 1 | 1 | 1 | 1 | 1 |
| 2 | 0 | 1 | 1 | - |
| 2 | 1 | 1 | 0 | 0 |

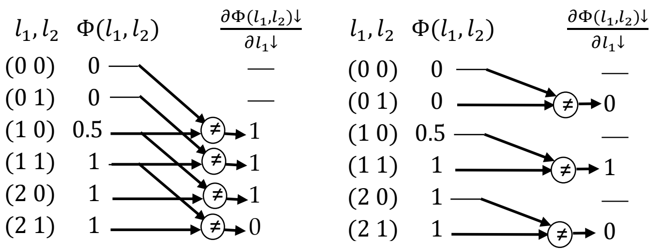


Fig. 3. Flow diagram for the second DPLD for survival signature (Table II).

The first (6) and second (8) DPLDs for survival signature are useful to analyze influence of failure of one component of specified type on a change of system states and can be used in qualitative analysis of the system. However, these derivatives do not quantify the indicated critical system states. The quantitative assessment of the critical system states can be performed if not only the values of the survival signature are compared but also the differences are calculated. In this case, the value of the derivative is calculated not just as a comparison but as the difference in the probability of the system functioning under the condition of the functioning of some components of the analyzed type of system and the probability of the system functioning if one of them fails.

**The third DPLD for survival signature** with respect to the variable $l_k$ permits determining the quantitative assessment of values of survival signature and the corresponding values of the variables for which any decrease by value one of specified variable causes a change in the value of survival signature:

$$\frac{\partial\Phi(l_1,\dots,l_K)\Downarrow}{\partial l_k\downarrow} = \begin{cases}\xi, & \Phi(l_1,\dots,b_k,\dots,l_K) > \Phi(l_1,\dots,b_k-1,\dots,l_K)\\ 0, & \text{otherwise}\end{cases} \quad (9)$$

for all $b_k = 1,\dots,n_k$ and where $\xi = \Phi(l_1,\dots,b_k,\dots,l_K) - \Phi(l_1,\dots,b_k-1,\dots,l_K)$.

The third DPLD for survival signature allows defining the critical system states which correspond to the non-zero values of the derivative. Unlike other DPLDs for survival signature, the non-zero values of the derivative (9) quantify the level of corresponding system state for the system failure: the larger this value, the more likely the effect of component failure of the considered type on the system. As can be seen from the flow diagram in Fig. 4, the computation of the derivative (9) is similar to the computation of the second derivative for survival signature (8).

The third DPLDs, $\partial\Phi(l_1,l_2)\Downarrow/\partial l_1\downarrow$ and $\partial\Phi(l_1,l_2)\Downarrow/\partial l_2\downarrow$ for the survival signature (Table II) of data storage system (Fig. 1) are presented in Table V. These derivatives are calculated for all values of the survival signature.

TABLE V
THE THIRD DPLDS OF THE DATA STORAGE SYSTEM

| Type 1 ($l_1$) | Type 2 ($l_2$) | $\Phi(l_1,l_2)$ | $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ | $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_2\downarrow}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | - | - |
| 0 | 1 | 0 | - | 0 |
| 1 | 0 | 0.5 | 0.5 | - |
| 1 | 1 | 1 | 1 | 0.5 |
| 2 | 0 | 1 | 0.5 | - |
| 2 | 1 | 1 | 0 | 0 |

For example, for the survival signature values $l_1 = 1, l_2 = 0$, the DPLD $\partial\Phi(l_1,l_2)\Downarrow/\partial l_1\downarrow$ has value $(\Phi(1,0) = 0.5) - (\Phi(0,0) = 0) = 0.5$ and, for values $l_1 = 0, l_2 = 1$, the DPLD $\partial\Phi(l_1,l_2)\Downarrow/\partial l_2\downarrow$ is 0, because $(\Phi(0,1) = 0) \ngtr (\Phi(0,0) = 0)$. From Table V, it is possible to see, that the most crucial type is type 1 as it was in case of second DPLD. However, from the third DPLD we can clearly see, that the most critical situation is when one component of type 1 fails for $l_1 = 1, l_2 = 1$ because the system will surely fail (value of the DPLD $\partial\Phi(l_1,l_2)\Downarrow/\partial l_1\downarrow$ is 1) while other non-zero values of the third DPLD for type 1 has value 0.5.

There is another possibility to calculate the third DPLD. It can be calculated based on DPLDs (3) of the structure function by the transformation of the DPLDs (3) calculated for the components of the corresponding type according to rules of survival signature:

$$\frac{\partial\Phi(l_1,\dots,l_K)\Downarrow}{\partial l_k\downarrow} = (n_k)^{-1}\cdot\sum_{x_i\in N_k}\Phi\left(\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}\right) \quad (10)$$

where $\Phi\left(\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}\right)$ is transformation of each DPLD $\frac{\partial\phi(1\to0)}{\partial x_i(1\to0)}$ based on the rules of the survival signature (2) and $N_k$ is a set of all components of type $k$.

Let us suppose that the calculation of the DPLD is denoted as transformation Đ and the forming of the survival signature is denoted as the transformation ϑ. Then the calculation of the third DPLD according to (9) is represented as $\partial\Phi(l_1,\dots,l_K)\Downarrow/\partial l_k\downarrow = Đ(\vartheta(\phi(\boldsymbol{x})))$. The calculation of the third DPLD according to (10) is represented as $\partial\Phi(l_1,\dots,l_K)\Downarrow/\partial l_k\downarrow = \vartheta(Đ(\phi(\boldsymbol{x})))$. Therefore, calculation of this derivative is possible according to (9) and (10).

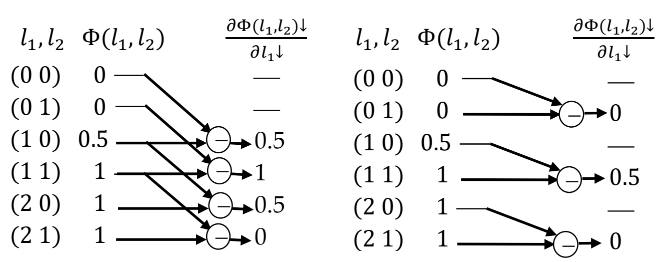


Fig. 4. Flow diagram for the third DPLD for survival signature (Table II).

The comparison of the results for all DPLDs for survival signature shows that all of these derivatives identify similar critical situation for the considered system, but each of the proposed derivatives has a specific purpose. The first DPLD for survival signature identifies the critical states for the system failure depending on the breakdown of one component of specified type for exact number of working components of this type. The second DPLD for survival signature allows us to find the critical system states for every component type and does not depend on the number of working components of this type. This DPLD identifies the critical situation without its quantification. The third DPLD for survival signature has a similar context as the second DPLD but indicates the probability of the critical system state depending on the failure of one of the components of this type.

It is needed to point out that the DPLDs for survival signature (6) – (10) will identify system critical states depending only on the component type, but they do not allow us to find which component has such influence. This information can be found by using the DPLD for structure function (3), which allows finding critical system states depending on specific component. For example, the failure of one of the components of type 1 (components 1 and 3) of data storage system in Fig. 1 leads to the system failure in all cases except situation when all components are initially functional according to DPLDs for the survival signature. According to the third DPLD (10), the system failure will occur with the probability 1 when one component of type 1 and the component of type 2 are working, and one component of type 1 fails. But we cannot identify which of two component of type 1 has greater impact. The analysis of this system based on the DPLDs $\phi(1 \to 0)/\partial x_3(1 \to 0)$ and $\phi(1 \to 0)/\partial x_1(1 \to 0)$ for the structure function (Table I) reveals that there are three critical states for the component 3 and one critical state for the component 1. Therefore, the component with the greater influence is component 3.

## V. Evaluation of DPLDs for Survival Signature

### A. *Case Study I*

In this section, the usage of Logic Differential Calculus for survival signature on series system with bridge topology from studies [7], [26] is demonstrated (Fig. 5). This system is composed of six components and their states are represented by Boolean variables $x_1, x_2, x_3, x_4, x_5, x_6$. There are two different types of system components. Components represented by variables $x_1, x_2, x_3$ have one type (type 1) and $x_4, x_5, x_6$ have another type (type 2). This is shown by label in the top right corners of the blocks and by different colors in reliability block diagram in Fig. 5 (green – type 1, blue – type 2). The structure function of this system has the following form:

$$\phi(\boldsymbol{x}) = x_1(x_2x_3 \vee x_2x_4x_6 \vee x_5x_6 \vee x_5x_4x_3). \qquad (11)$$

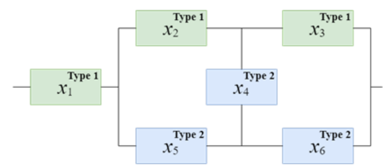


Fig. 5. Reliability block diagram of the system with bridge topology.

The survival signature for this system was computed according to (2) and its values can be seen on the left side in each DPLD table for this system (for example, Table VI).

The derivatives (6) allow us to indicate system states for which the breakdown of one of components of fixed type results in the system failure for indicated numbers of working components of each type. Since we have three components from both types, we have to consider three derivatives for type 1 and three for type 2 (Table VI).

TABLE VI
THE FIRST DPLDS OF THE ANALYZED SYSTEM REPRESENTED BY (11)

| Type 1 ($l_1$) | Type 2 ($l_2$) | $\Phi(l_1,l_2)$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(3\to2)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(2\to1)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1(1\to0)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(3\to2)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(2\to1)}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2(1\to0)}$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | - | - | - | - | - | - |
| 0 | 1 | 0 | - | - | - | - | - | 0 |
| 0 | 2 | 0 | - | - | - | - | 0 | - |
| 0 | 3 | 0 | - | - | - | 0 | - | - |
| 1 | 0 | 0 | - | - | 0 | - | - | - |
| 1 | 1 | 0 | - | - | 0 | - | - | 0 |
| 1 | 2 | 0.111 | - | - | 1 | - | 1 | - |
| 1 | 3 | 0.333 | - | - | 1 | 1 | - | - |
| 2 | 0 | 0 | - | 0 | - | - | - | - |
| 2 | 1 | 0 | - | 0 | - | - | - | 0 |
| 2 | 2 | 0.444 | - | 1 | - | - | 1 | - |
| 2 | 3 | 0.667 | - | 1 | - | 1 | - | - |
| 3 | 0 | 1 | 1 | - | - | - | - | - |
| 3 | 1 | 1 | 1 | - | - | - | - | 0 |
| 3 | 2 | 1 | 1 | - | - | - | 0 | - |
| 3 | 3 | 1 | 1 | - | - | 0 | - | - |

It is needed to point out that these derivatives indicate the possibility of the system failure (the system can fail) because the system can be in failed state before the specified component breakdown. The value 1 for the derivatives of this type means the system failure is possible, but it is not required. There is not influence of failure of one component from indicated number of working components of specified type if derivative value is 0. For example, the derivative $\partial\Phi(l_1,l_2)\downarrow/\partial l_1(3\rightarrow2)$ has all values non-zero, which means a failure of one of three components of type 1 can result in a failure of the system regardless of the number of working components of type 2. The failure of one of two working components of this type (the derivative $\partial\Phi(l_1,l_2)\downarrow/\partial l_1(2\rightarrow1)$) can result in the system failure if there are two or three working components of type 2. The similar influence on the system failure has breakdown of one working component of type 1 (the derivative $\partial\Phi(l_1,l_2)\downarrow/\partial l_1(1\rightarrow0)$). On the other hand, all values of derivative $\partial\Phi(l_1,l_2)\downarrow/\partial l_2(1\rightarrow0)$ are equal to zero, which indicates that a failure of only one working component of type 2 has no influence on the system. However, this does not exclude the fact that the system has already failed earlier.

The second DPLD for survival signature (8) is generalization of the first DPLD (6) and allows us to define conditions under which the breakdown of one of components of specified type causes the system failure. There are two derivatives for this system (Table VII), which can be calculated according to (7) or as the union of derivatives from Table VI for every variable according to (8).

TABLE VII
THE SECOND DPLDS OF THE ANALYZED SYSTEM REPRESENTED BY (11)

| Type 1 ($l_1$) | Type 2 ($l_2$) | $\Phi(l_1,l_2)$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_1\downarrow}$ | $\frac{\partial\Phi(l_1,l_2)\downarrow}{\partial l_2\downarrow}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | - | - |
| 0 | 1 | 0 | - | 0 |
| 0 | 2 | 0 | - | 0 |
| 0 | 3 | 0 | - | 0 |
| 1 | 0 | 0 | 0 | - |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 2 | 0.111 | 1 | 1 |
| 1 | 3 | 0.333 | 1 | 1 |
| 2 | 0 | 0 | 0 | - |
| 2 | 1 | 0 | 0 | 0 |
| 2 | 2 | 0.444 | 1 | 1 |
| 2 | 3 | 0.667 | 1 | 1 |
| 3 | 0 | 1 | 1 | - |
| 3 | 1 | 1 | 1 | 0 |
| 3 | 2 | 1 | 1 | 0 |
| 3 | 3 | 1 | 1 | 0 |

According to the non-zero values of $\partial\Phi(l_1,l_2)\downarrow/\partial l_1\downarrow$, we may consider next scenarios of the system failure:

- the breakdown of one component out of one or two working components of type 1 can cause the system failure if two or three components of type 2 are working;
- the breakdown of one component out of three working components of type 1 can cause the system failure regardless of the number of working components of type 2.

The non-zero values of the derivative $\partial\Phi(l_1,l_2)\downarrow/\partial l_2\downarrow$ reveals the system failure possibility if:

- there is one working component of type 1 and one of two or three working components of type 2 fails;
- there are two working components of type 1 and one of two or three working components of type 2 fails.

The third DPLD for survival signature allows us to quantify the system failure depending on the breakdown of one of components of the specified type (Table VIII).

TABLE VIII
THE THIRD DPLDS OF THE ANALYZED SYSTEM REPRESENTED BY (11)

| Type 1 ($l_1$) | Type 2 ($l_2$) | $\Phi(l_1,l_2)$ | $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_1\downarrow}$ | $\frac{\partial\Phi(l_1,l_2)\Downarrow}{\partial l_2\downarrow}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | - | - |
| 0 | 1 | 0 | - | 0 |
| 0 | 2 | 0 | - | 0 |
| 0 | 3 | 0 | - | 0 |
| 1 | 0 | 0 | 0 | - |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 2 | 0.111 | 0.111 | 0.111 |
| 1 | 3 | 0.333 | 0.333 | 0.222 |
| 2 | 0 | 0 | 0 | - |
| 2 | 1 | 0 | 0 | 0 |
| 2 | 2 | 0.444 | 0.333 | 0.444 |
| 2 | 3 | 0.667 | 0.333 | 0.222 |
| 3 | 0 | 1 | 1 | - |
| 3 | 1 | 1 | 1 | 0 |
| 3 | 2 | 1 | 0.556 | 0 |
| 3 | 3 | 1 | 0.333 | 0 |

The derivative $\partial\Phi(l_1,l_2)\Downarrow/\partial l_1\downarrow$ has 8 non-zero values. The maximum value of two of them is 1. From those values it is possible to see, that the most critical failure of component of type 1 is when there are 3 working components of type 1 and one or none working component of type 2. In this situation, if any component of type 1 fails, then the system will surely fail. The derivative $\partial\Phi(l_1,l_2)\Downarrow/\partial l_2\downarrow$ has 4 non-zero values, which are generally less than non-zero values of the derivative for the type 1. Therefore, the components of the type 1 should be evaluated in more details based on the DPLD for the structure function (4). The derivatives $\partial\phi(1\rightarrow0)/\partial x_1(1\rightarrow0)$, $\partial\phi(1\rightarrow0)/\partial x_2(1\rightarrow0)$ and $\partial\phi(1\rightarrow0)/\partial x_3(1\rightarrow0)$ allows us to find exact vector states for which the failure of the first, second or third component results in the system failure.

The third DPLD for survival signature can be presented as a color matrix where the rows agree with the number of working components of investigated type, columns with the number of working components of other types, and color saturation in cells depends on the value of the derivative. Such matrices for the considered system with bridge topology are shown in Fig. 2.

Based on the previous, we can conclude the following interpretation of the third DPLD for survival signature:

- this derivative shows proportion of the system states among of all possible states of fixed numbers of functioning components of every type for which the breakdown of one specified component causes the system failure;
- this derivative indicate the probability of the system failure caused by breakdown of one component of fixed type for system states for which specify numbers of functioning components of each type.

| | | Number of working components of type 2 | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| Number of working components of type 1 | 1 | 0 | 0 | 0.111 | 0.333 |
| | 2 | 0 | 0 | 0.333 | 0.333 |
| | 3 | 1 | 1 | 0.556 | 0.333 |

| | | Number of working components of type 1 | | | |
|---|---|---|---|---|---|
| | | 0 | 1 | 2 | 3 |
| Number of working components of type 2 | 1 | 0 | 0 | 0 | 0 |
| | 2 | 0 | 0.111 | 0.444 | 0 |
| | 3 | 0 | 0.222 | 0.222 | 0 |

Fig. 2. Color matrices of the third DPLDs for the system in Fig. 2.

## B. Case Study II

Next, we analyse the hydro power plant that is presented in [31]. We will be focusing on the inside mechanism of the hydro power plant, which can be seen in a form of reliability block diagram in Fig. 6. Firstly, the water comes from the reservoir through the gate (component $x_1$ of type 1) that controls the flow of the water to the two butterfly valves (components $x_2$ and $x_6$ of type 2). Then the water flows to the two turbines (components $x_3$ and $x_7$ of type 3) in which the kinetic energy of the water flow is used to move the turbine and to produce alternating current in the two generators (components $x_4$ and $x_8$ of type 4). Finally, there are three circuit breakers that protects the hydro power plant system (components $x_5$, $x_9$ and $x_{10}$ of type 5) and two transformers (components $x_{11}$ and $x_{12}$ of type 6) used to obtain a higher voltage for the output electricity. The structure function representing this system has following form:

$$\phi(\boldsymbol{x}) = x_1(x_2x_3x_4x_5 \vee x_6x_7x_8x_9)x_{10}(x_{11} \vee x_{12}). \qquad (12)$$

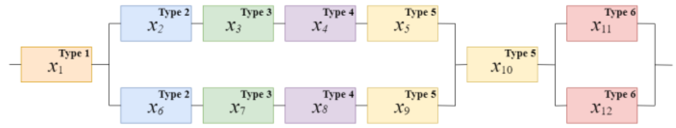


Fig. 6. Reliability block diagram for the hydro power plant.

The survival signature for this system is computed according to (2). The critical system states according to the third DPLD for this system are shown in form of diagram in Fig. 7. From the third DPLD in Fig. 7 we can recognize that the most important type is type 1, which is understandable because this type has only one component and it is the first component in series topology. Next, let us consider the derivative $\partial\Phi(l_1,l_2,l_3,l_4,l_5,l_6)\Downarrow/\partial l_5\downarrow$ for type 5. This type is quite interesting because, according to the reliability block diagram in Fig. 6, the functionality of component $x_{10}$ of type 5 is crucial for the system functionality. This can be seen in situations, when only two components of type 5 are functioning and the system is functioning. In these situations, if one component of type 5 fails, then the system will surely fail.

## C. Evaluation

The complexity of the approach proposed in this work depends on two main calculations, namely the calculation of the survival signature, if the analysis is based on a structural function and it is necessary to obtain a survival signature, and the calculation of the derivatives. As for the first part, in the article [8] the authors described the complexity of the survival signature calculation directly from the structural function, while the calculation according to (2) is suitable for systems with a size of about 20 components, then the complexity of calculations begins to increase rapidly. The article also states that an improved method of calculating survival signature using binary decision diagrams has been developed. The authors also present other approaches to calculating survival signature that do not require knowledge of the whole structural function, and these approaches are applicable to the efficient calculation of survival signature for large systems.

The important advantages of DPLDs for survival signature is possibility to decrease the dimensional of the structure function (1) of investigated system that determines the computational complexity of the DPLD calculation. For example, the number of values of the structure function for 10 variables are 1,024.

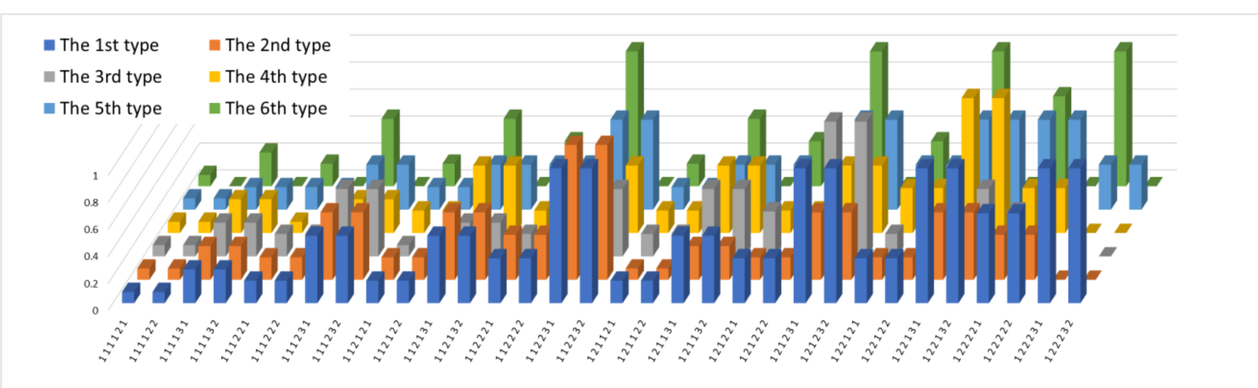


Fig. 7. The critical system states of the hydro power plant (Fig. 4).

At the same time, for example, number of values of survival signature for $K = 2$ and $n_1 = 7, n_2 = 3$ is 32; for $K = 3$ and $n_1 = 2, n_2 = 3, n_3 = 5$ is 72; for $K = 5$ and $n_1 = n_2 = n_3 = n_4 = n_5 = 2$ is 243. According to the study in [8], increase in the number of components types results in the increase in the dimension of survival signature for the same number of system components. This fact allows stating that the decrease in the dimension of the structure function by its transformation to the survival signature results in the decrease in the computational complexity of the critical system states for every type of the system components. However, the increase in the number of components types brings the dimension of the survival signature to the dimension of the structure function. To prove this, we conduct experiments studying the computational complexity of the approach for various randomly generated structure functions. The functions represent systems of $n$ components ($n$ is changed from 15 to 20). The number of generated structure functions is 100. For each of these structure functions, the number of components type $K$ is changed from 1 to $n$ (if $K = n$, the survival signature agrees with the structure function (1)). According to this analysis, we can see that the time for the survival signature computation exponentially depends on the number of system components $n$ (Fig. 8). This is confirmed in [8] and corresponds to recommendation to use survival signature for the system with number of components less than 20.

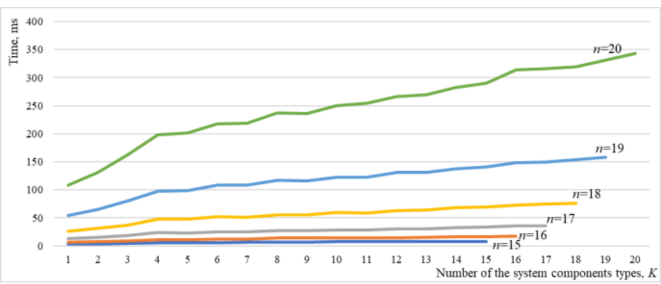


Fig. 8. The analysis of the time for the survival signature calculation depending on the number of the system components and the number of types of the components.

The comparison of the times for the calculation of the DPLD (3) for the structure function and the third DPLD (9) for the survival signature is shown in the diagram in Fig. 9. In this investigation the set of one hundred random generated structure function. According to this diagram, the survival signature is efficient in terms of computational time for the analysis of system with the lesser number of components types. At the same time, we can see that the number of components types $K$ does not influences the time for the calculation and analysis of the system based on DPLD (3) where the system is represented by the structure function.

As for the calculation of the presented DPLDs, if the survival signature is known, then, for the third DPLD, it is necessary to calculate the difference in the value of the survival signature for each change in the value of the number of functional elements of a given type. The second DPLD can be obtained from the already calculated third DPLD so that if its value is 0, the second DPLD will also have a value of 0, if its value is greater than 0, then the second DPLD will have value 1. The first DPLD can then be calculated from the second DPLD since the value of the second DPLD is retained for the first DPLD if the number of functional elements has a given value, otherwise it is undefined. The second and first DPLDs can also be calculated without knowledge of other DPLDs only on the basis of survival signature. The change compared to the computation of the third DPLD is that for the second DPLD the difference is replaced by a value of comparison of survival signature values. As for the first DPLD, the difference is replaced by a value of comparison of survival signature values, in which the survival signature has a value of the number of functional elements of the type equal to the required values specified in the first DPLD. Times needed for all these types of calculations are shown in Fig. 10 for the systems of eighteen system components ($n$=18). The shown tendencies of calculation time for different DPLDs in Fig.10 are similar for other number of system components. The time for the derivatives calculation increases depending on the number of system components increasing.

All investigations have been implemented on a computer with Intel Core I7 330 CPU, 16GB of RAM, and Windows 10 operating system.

The implemented analysis of the computational complexity of the proposed DPLDs for the survival signature shows that they are efficient if the system consists of lesser number of components types. Approximately, this number can be evaluated as half of number of system components ($K < n/2$).

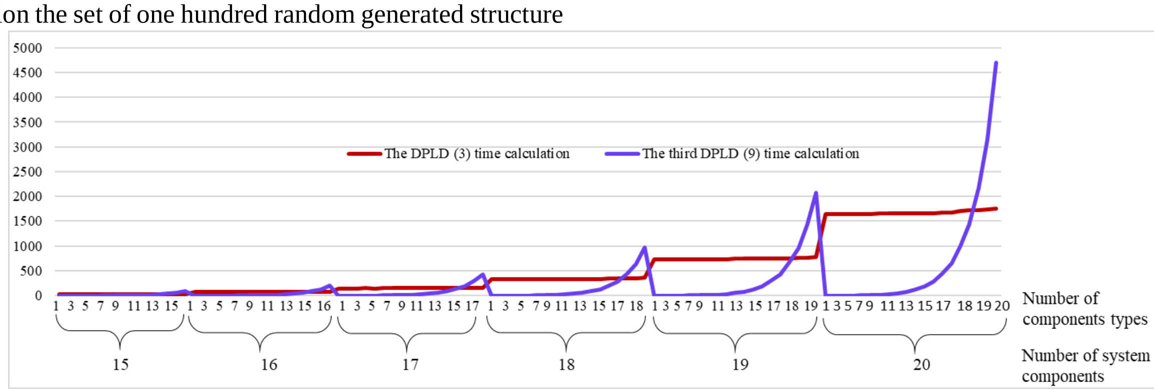


Fig. 9. The analysis of the time for the calculation of DPLD (3) and the third DPLD (9).

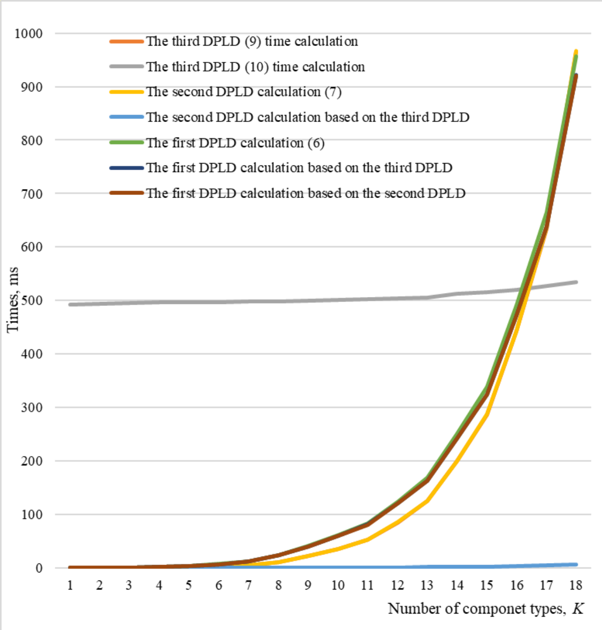


Fig. 10. The analysis of the time needed for calculation of different types of DPLDs based on the structure function survival signature for $n$=18.

## VI. Discussion

In this paper, three new types of DPLD for survival signature are introduced. They can be used to analyze the influence of failure of specified type of components on the system failure. The first and second DPLDs for survival signature determine the fact of such influence. The first DPLD for survival signature analyses the effect of failure of one component of specified type and number of working components of this type on the system failure. State vectors of survival signature are determined for which the breakdown of one component can affect the system failure. Important condition for this derivative is number of working components of investigated type. According to this derivative, we can choose the type of components and indicate number of working components and find scenarios for which the breakdown of one component influences the system failure. The second DPLD for survival signature is generalization of the first DPLD and allows us to investigate the influence of one component failure of specified type regardless of number of working components of this type. However, this derivative does not rate the probability of the system failure depending on the breakdown of one component. It only indicates that it is possible. The third DPLD for survival signature of structure function allows computing the probability of the system failure depending on the breakdown of one component of a given type.

The considered application of DPLDs for the analysis of survival signature of structure function can be used in qualitative and quantitative topological analysis. In qualitative analysis the DPLD of survival signature can be used to find scenarios of the system failure depending on the breakdown of one of components of a fixed type. These scenarios are used in the development of maintenance strategy of a complex system [2], [29], [41], [42]. In this paper, we consider only the influence of one component type. Unlike the analysis of system based on structure function by DPLD, the analysis based on survival signature allows examining the influence of a specified type of system components rather than one component. The influence of the specified type is considered in case of system failure depending on failure of one component of this type. In further research, we will develop the DPLDs that will take into account the breakdown of several components not only of one type but also of different types of components. In quantitative analysis, the DPLD of survival signature can be used in the importance analysis for the calculation of importance measures [30], [39].

The future study, in particular, will be focused on the development of the Importance analysis for the system represented by the survival signature that will be based on the. The Importance analysis of the survival signature can be based on the results of DPLD investigation, which were published in [36-37]. Another way of the investigation of survival signature can be the adaptation of mathematical approach of the multi-state system for its representation and analysis. In particular, the multi-valued decision diagram can be used for survival signature representation and analysis [40].

## References


[1] V. V. Rykov, N. Balakrishnan, and M. S. Nikulin, Eds., *Mathematical and Statistical Models and Methods in Reliability: Applications to Medicine, Finance, and Quality Control*. Boston, MA: Birkhäuser Boston, 2010.

[2] T. Aven and E. Zio, Eds., *Knowledge in Risk Assessment and Management*. Chichester, UK: John Wiley & Sons, Ltd, 2018.

[3] H. Aliee, E. Borgonovo, M. Glaß, and J. Teich, "On the Boolean extension of the Birnbaum importance to non-coherent systems," *Reliab. Eng. Syst. Saf.*, vol. 160, pp. 191–200, 2017, doi: 10.1016/j.ress.2016.12.013.

[4] E. Zaitseva, V. Levashenko, and J. Kostolny, "Importance analysis based on logical differential calculus and Binary Decision Diagram," *Reliab. Eng. Syst. Saf.*, vol. 138, pp. 135–144, Jun. 2015, doi: 10.1016/J.RESS.2015.01.009.

[5] W. Kuo and X. Zhu, *Importance Measures in Reliability, Risk, and Optimization: Principles and Applications*. John Wiley and Sons, 2012.

[6] E. Zaitseva and V. Levashenko, "Reliability analysis of multi-state system with application of multiple-valued logic," *Int. J. Qual. Reliab. Manag.*, vol. 34, no. 6, pp. 862–878, 2017, doi: 10.1108/IJQRM-06-2016-0081.

[7] F. P. A. Coolen and T. Coolen-Maturi, "Generalizing the signature to systems with multiple types of components," *Adv. Intell. Soft Comput.*, vol. 170 AISC, pp. 115–130, 2012, doi: 10.1007/978-3-642-30662-4-8.

[8] F. P. A. Coolen, T. Coolen-Maturi, A. H. Al-Nefaiee "Nonparametric predictive inferencefor system reliability using the survivalsignature," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 228(5), pp. 437-448, 2014, doi:10.1177/1748006X14526390

[9] S. B. Akers, Jr., "On a Theory of Boolean Functions," *J. Soc. Ind. Appl. Math.*, vol. 7, no. 4, pp. 487–498, Dec. 1959, doi: 10.1137/0107041.

[10] A. D. Talantsev, "Analysis and synthesis of certain electric circuits by means of special logical operators," *Autom. Remote Control*, vol. 20, no. 9, pp. 874–883, 1959, [Online]. Available: http://mi.mathnet.ru/at12783.

[11] X. Zang, H. Sun, and K. S. Trivedi, "A BDD-based algorithm for reliability analysis of phased-mission systems," *IEEE Trans. Reliab.*, vol. 48, no. 1, pp. 50–60, 1999, doi: 10.1109/24.765927.

[12] M. Kvassay, V. Levashenko, and E. Zaitseva, "Analysis of minimal cut and path sets based on direct partial Boolean derivatives," *Proc. Inst. Mech. Eng. Part O J. Risk Reliab.*, vol. 230, no. 2, pp. 147–161, 2016, doi: 10.1177/1748006X15598722.

[13] S. N. Yanushkevich, D. Michael Miller, V. P. Shmerko, and R. S. Stanković, *Decision diagram techniques for micro- and nanoelectronic design: Handbook*. CRC Press, 2005.

[14] B. Steinbach and C. Posthoff, "Boolean Differential Calculus", *Synthesis Lectures on Digital Circuits and Systems*, vol. 12, no. 1, pp. 1-215, Jun. 2017, doi: 10.2200/S00766ED1V01Y201704DCS052

[15] J. D. Andrews and S. J. Dunnett, "Event-tree analysis using binary decision diagrams," *IEEE Trans. Reliab.*, vol. 49, no. 2, pp. 230–238,

2000, doi: 10.1109/24.877343.

[16] M. Alam and U. M. Al-Saggaf, “Quantitative Reliability Evaluation of Repairable Phased-Mission Systems Using Markov Approach,” *IEEE Trans. Reliab.*, vol. 35, no. 5, pp. 498–503, 1986, doi: 10.1109/TR.1986.4335529.

[17] E. Mingcheng, B. Li, Z. Jiang, and Q. Li, “An optimal reprofiling policy for high-speed train wheels subject to wear and external shocks using a semi-Markov decision process,” *IEEE Trans. Reliab.*, vol. 67, no. 4, pp. 1468–1481, 2018, doi: 10.1109/TR.2018.2870276.

[18] T. Ye, Y. Zhou, A. Chen, L. Liu, and S. Liu, “Extend GO Methodology to Support Common-Cause Failures Modeling Explicitly by Means of Bayesian Networks,” *IEEE Trans. Reliab.*, vol. 69, no. 2, pp. 471–483, 2020, doi: 10.1109/TR.2019.2917752.

[19] G.Levitin, *The Universal Generating Function in Reliability Analysis and Optimization*. Springer, 2005

[20] W. G. Schneeweiss, “A short Boolean derivation of mean failure frequency for any (also non-coherent) system,” *Reliab. Eng. Syst. Saf.*, vol. 94, no. 8, pp. 1363–1367, Aug. 2009, doi: 10.1016/j.ress.2008.12.001.

[21] S. B. Akers, “Binary Decision Diagrams,” *IEEE Trans. Comput.*, vol. C–27, no. 6, pp. 509–516, 1978, doi: 10.1109/TC.1978.1675141.

[22] I. Antoniano-Villalobos, E. Borgonovo, and S. Siriwardena, “Which Parameters Are Important? Differential Importance Under Uncertainty,” *Risk Anal.*, vol. 38, no. 11, pp. 2459–2477, 2018, doi: 10.1111/risa.13125.

[23] Z. Li, Y. Ren, L. Liu, and Z. Wang, “Improving parallel modularization algorithm of large complex fault trees,” in *Proceedings - Annual Reliability and Maintainability Symposium*, 2016, pp. 1–6, doi: 10.1109/RAMS.2016.7447964.

[24] A. Imakhlaf and M. Sallak, “Birnbaum’s Measure Extension for Non-Coherent Systems Based on BDD,” in *Proceedings - Annual Reliability and Maintainability Symposium*, 2018, pp. 1–7, doi: 10.1109/RAM.2018.8463138.

[25] P. Lollini, A. Bondavalli, and F. di Giandomenico, “A Decomposition-Based Modeling Framework for Complex Systems,” *IEEE Trans. Reliab.*, vol. 58, no. 1, pp. 20–33, Mar. 2009, doi: 10.1109/TR.2008.2011663.

[26] F. P. A. Coolen and T. Coolen-Maturi, “The structure function for system reliability as predictive (imprecise) probability,” *Reliab. Eng. Syst. Saf.*, vol. 154, pp. 180–187, Oct. 2016, doi: 10.1016/J.RESS.2016.06.008.

[27] F. J. Samaniego, *System Signatures and their Applications in Engineering Reliability*, vol. 110. 2007.

[28] S. Eryilmaz, “Review of recent advances in reliability of consecutive k-out-of-n and related systems,” *Proc. Inst. Mech. Eng. Part O J. Risk Reliab.*, vol. 224, no. 3, pp. 225–237, 2010, doi: 10.1243/1748006XJRR332.

[29] R. Briš, P. Byczanski, R. Goňo, and S. Rusek, “Discrete maintenance optimization of complex multi-component systems,” *Reliab. Eng. Syst. Saf.*, vol. 168, pp. 80–89, 2017, doi: 10.1016/j.ress.2017.04.008.

[30] S. Eryilmaz, F. P. A. Coolen, and T. Coolen-Maturi, “Marginal and joint reliability importance based on survival signature,” *Reliab. Eng. Syst. Saf.*, vol. 172, pp. 118–128, 2018, doi: 10.1016/j.ress.2017.12.002.

[31] G. Feng, E. Patelli, M. Beer, and F. P. A. Coolen, “Imprecise system reliability and component importance based on survival signature,” *Reliab. Eng. Syst. Saf.*, vol. 150, pp. 116–125, Jun. 2016, doi: 10.1016/J.RESS.2016.01.019.

[32] J. Mi, M. Beer, Y. F. Li, M. Broggi, and Y. Cheng, “Reliability and importance analysis of uncertain system with common cause failures based on survival signature,” *Reliab. Eng. Syst. Saf.*, vol. 201, 2020, doi: 10.1016/j.ress.2020.106988.

[33] X. Huang, F. P. A. Coolen, T. Coolen-Maturi, and Y. Zhang, “A New Study on Reliability Importance Analysis of Phased Mission Systems,” *IEEE Trans. Reliab.*, vol. 69, no. 2, pp. 522–532, Jun. 2020, doi: 10.1109/TR.2019.2923695.

[34] E. Zaitseva and V. Levashenko, “Multiple-Valued Logic mathematical approaches for multi-state system reliability analysis,” *J. Appl. Log.*, vol. 11, no. 3, pp. 350–362, Sep. 2013, doi: 10.1016/j.jal.2013.05.005.

[35] P. Miziula and J. Navarro, “Birnbaum Importance Measure for Reliability Systems With Dependent Components,” *IEEE Trans. Reliab.*, vol. 68, no. 2, pp. 439–450, Jun. 2019, doi: 10.1109/TR.2019.2895400.

[36] M. Kvassay and E. Zaitseva, “Topological analysis of multi-state systems based on direct partial logic derivatives,” in *Springer Series in Reliability Engineering*, 2018, pp. 265–281.

[37] M. Kvassay, E. Zaitseva, and V. Levashenko, “Importance analysis of multi-state systems based on tools of logical differential calculus,” *Reliab. Eng. Syst. Saf.*, vol. 165, pp. 302–316, Sep. 2017, doi: 10.1016/j.ress.2017.03.021.

[38] J. Huang, X. Qin, F. Zhang, W. Ku and C. Xie, "MFTS: A Multi-Level Fault-Tolerant Archiving Storage with Optimized Maintenance Bandwidth," in *IEEE Transactions on Dependable and Secure Computing*, vol. 11, no. 6, pp. 524-537, Nov.-Dec. 2014, doi: 10.1109/TDSC.2014.2304296.

[39] P. Rusnak, M. Kvassay, A. Forgac and E. Zaitseva, "Logic differential calculus in time-dependent analysis of a pair of system components," *2018 IEEE 9th International Conference on Dependable Systems, Services and Technologies (DESSERT)*, Kyiv, UKraine, 2018, pp. 442-447, doi: 10.1109/DESSERT.2018.8409174.

[40] E. Zaitseva, V. Levashenko, J. Kostolny and M. Kvassay, "A multi-valued decision diagram for estimation of multi-state system," *Eurocon 2013*, Zagreb, Croatia, 2013, pp. 645-650, doi: 10.1109/EUROCON.2013.6625049.

[41] M. Calder and M. Sevegnani, "Stochastic Model Checking for Predicting Component Failures and Service Availability," in *IEEE Transactions on Dependable and Secure Computing*, vol. 16, no. 1, pp. 174-187, 1 Jan.-Feb. 2019, doi: 10.1109/TDSC.2017.2650901.

[42] I. Kaitovic and M. Malek, "Impact of Failure Prediction on Availability: Modeling and Comparative Analysis of Predictive and Reactive Methods," in *IEEE Transactions on Dependable and Secure Computing*, vol. 17, no. 3, pp. 493-505, 1 May-June 2020, doi: 10.1109/TDSC.2018.2806448.

**Patrik Rusnak** received the M.S. degree in information systems and the Ph.D. degree in applied informatics from the University of Žilina, Žilina, Slovakia, in 2017 and 2020, respectively. He has been with the Department of Informatics, University of Žilina, since 2015.

**Elena Zaitseva** received the M.S. and Ph.D. degrees from the Belarus State University of Informatics and Radioelectronics, Minsk, Belarus, in 1989 and 1994, respectively, both in computer science. From 1996 to 2004, she was with the Belarus State Economic University. Since 2004, she has been with the Department of Informatics, University of Žilina, Slovakia.

**Frank P.A. Coolen** received the M.S. and Ph.D. degrees in mathematics with specialization in statistics and operations research from the Eindhoven University of Technology, Eindhoven, the Netherlands, in 1989 and 1994, respectively.
He is a Full Professor with the Department of Mathematical Sciences, Durham University, Durham, U.K. He has authored and coauthored more than 300 papers. His main research interests include nonparametric predictive inference, reliability and survival analysis, and imprecise probability.

**Miroslav Kvassay** received the M.S. degree in information systems and the Ph.D. degree in applied informatics from the University of Žilina, Žilina, Slovakia, in 2012 and 2015, respectively. He has been with the Department of Informatics, University of Žilina, since 2015.

**Vitaly Levashenko** received the M.S. and Ph.D. degrees from the Belarus State University of Informatics and Radioelectronics, Minsk, Belarus, in 1991 and 1997, respectively, both in computer science. From 1998 to 2004, he was with the Belarus State Economic University, Minsk. Since 2004, he has been with the Department of Informatics, University of Žilina, Slovakia