

Market Thickness in Online Food Delivery Platforms: The Impact of Food Processing Times

Yanlu Zhao

Durham University Business School, yanlu.zhao@durham.ac.uk

Felix Papier

ESSEC Business School, papier@essec.edu

Chung-Piaw Teo

NUS Business School, National University of Singapore, bizteocp@nus.edu.sg

Problem definition: Online Food Delivery (OFD) platforms have witnessed rapid global expansion, partly driven by shifts in consumer behavior during the COVID-19 pandemic. These platforms enable customers to order food conveniently from a diverse array of restaurants through their mobile phones. A core functionality of these platforms is the algorithmic matching of drivers to food orders, which is the focus of our study as we aim to optimize this driver-order matching process.

Methodology/results: We formulate real-time matching algorithms that take into account uncertain food processing times to strategically ‘delay’ the assignment of drivers to orders. This intentional delay is designed to create a ‘thicker’ marketplace, increasing the availability of both drivers and orders. Our algorithms utilize machine learning techniques to predict food processing times, and the dispatching of drivers is subsequently determined by balancing costs for idle driver waiting and for late deliveries. In scenarios with a single order in isolation, we show that the optimal policy adopts a threshold structure. Building on this insight, we propose a new k -level thickening policy with driving time limits for the general case of multiple orders. This policy postpones the assignment of drivers until a maximum of k suitable matching options are available. We evaluate our policy using a simplified model and identify several analytical properties, including the quasi-convexity of total costs in relation to market thickness, indicating the optimality of an intermediate level of market thickness. Numerical experiments with real data from Meituan show that our policy can yield a 54% reduction in total costs compared to existing policies.

Managerial implications: Our study reveals that incorporating food processing times into the dispatch algorithm remarkably improves the efficacy of driver assignment. Our policy enables the platform to control two vital market parameters of real-time matching decisions: the number of drivers available to pick up and deliver an order promptly, and their proximity to the restaurant. Based on these two parameters, our algorithm matches drivers with orders in real-time, offering significant managerial implications.

Key words: Online food delivery, Market thickness, Dynamic matching, Bipartite min-cost matching

History:

1. Introduction

The online food delivery (OFD) market has been expanding at an annual rate of 25% over the past five years (Curry 2021). This growth has led to the rise of OFD platforms such as Meituan, UberEats, and GrabFood (Chen et al. 2020). The COVID-19 pandemic in 2020 further accelerated this trend as social distancing measures forced many restaurants to close their in-person dining services, leaving them reliant on takeaway and delivery options. Many restaurants worldwide were able to survive solely because of the OFD service (Chen and Hu 2023). Consequently, the top four OFD platforms in the U.S. – DoorDash, UberEats, Grubhub, and Postmates – experienced a revenue increase of USD 3 billion in the first half of 2020 (Levi Sumagaysay 2020).

OFD platforms operate in a highly competitive landscape where operating and marketing costs often exceed sustainable revenues. A significant contributor to these high operating costs is the strict service requirements, particularly the consumer expectation for reliable and quick deliveries, often within minutes of placing an order. According to a large-scale survey conducted by McKinsey (Hirschberg et al. 2016), more than 60% of customers indicated that timely deliveries and meeting delivery targets are crucial factors in OFD services. Failure to meet these targets can result in customers abandoning the platform, leading to a significant loss of revenue (Mao et al. 2019, Hasija et al. 2020). However, meeting delivery targets necessitates a high availability of drivers, thereby elevating operating costs. For example, Meituan’s financial report revealed that the costs of their OFD business grew by 36% in 2019, primarily due to increased expenses related to delivery riders (Meituan 2020).

There are also various other operational challenges in addition to the financial pressures. For instance, the stock price of the OFD platform Deliveroo plummeted by as much as 31% within minutes of its London debut in March 2021. One contributing factor was the scrutiny the company faced regarding its treatment of food delivery drivers (Gemmell 2021). There is a growing body of literature that discusses how food delivery platforms exert control over their drivers, often more so than other types of delivery services. Today, millions of delivery drivers are effectively ‘managed’ by algorithms. They are continuously monitored through customer ratings and are influenced by various behavioral nudges, leading to new societal challenges (Sun 2019). These conditions have prompted drivers to evaluate whether they should disclose their genuine preferences and to develop strategies for selecting jobs, aiming to optimize their earnings (see, e.g., temporal pooling in Chen and Hu 2023).

The task of assigning orders to drivers is arguably the most crucial aspect of platform operations. Effective matching seeks to balance the costly idle time for drivers against the erosion of customer goodwill due to late deliveries. While the importance of effective matching extends across various sectors – from kidney exchanges and job searches to online dating (Roth 2015) – what sets OFD platforms apart are the immediate nature of the assignments, the significant costs and goodwill losses associated with delayed

deliveries, and the substantial variability in food processing times. Our paper aims to contribute to this field by specifically examining the role of food processing time in the order assignment process. We aim to address the following research questions: (a) How can matching policies for OFD platforms effectively incorporate food processing times to delay matching decisions? (b) What impact does this delayed matching have on total costs and other performance metrics for the platform?

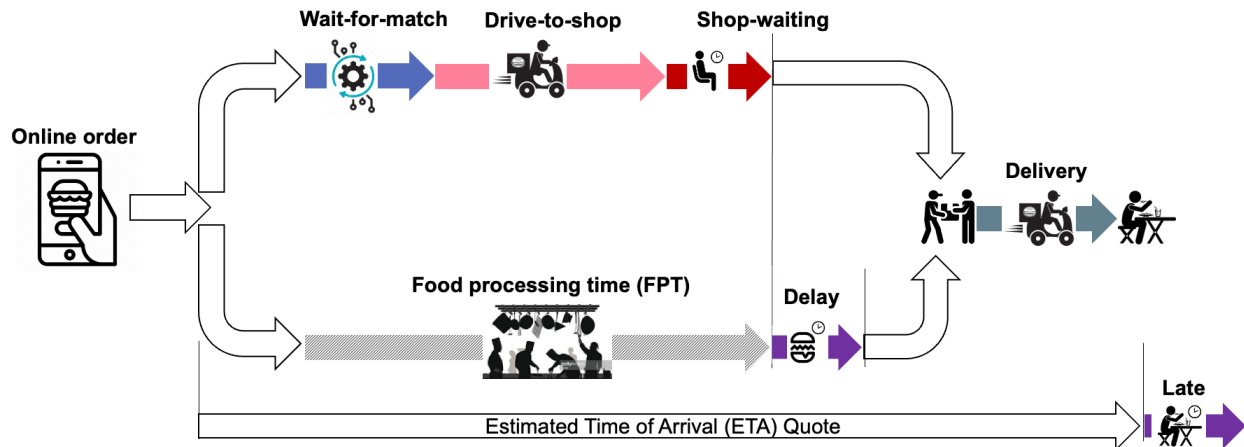


Figure 1 Order matching and food dispatching process in online food delivery platform.

OFD Process: Our research is motivated by the OFD operations (Figure 1) at Meituan, which involve two main activities: *order matching* and *food dispatching*. When a *customer* decides to place a meal order, the platform’s web portal shows a selection of nearby restaurants. The customer then chooses her preferred dish from a specific restaurant and places the order. Upon receiving the order, the platform notifies the *provider*, such as a restaurant or shop, to prepare the food as requested. The *service time* for the platform is defined as the total duration from the moment an order is placed until the food is delivered to the customer. This duration is subject to a service time target set by the platform. A *driver* is assigned to pick up and deliver each order. Food can only be picked up from the restaurant when it is ready. In this study, we do not focus on the dispatching process after an order is picked up, as it does not directly influence the OFD matching problem. If a driver arrives at the restaurant before the order is ready, she must wait. Conversely, if she arrives after the food is ready (delayed pick up), it results in a late delivery, which can lead to decreased food quality and lower customer satisfaction. The platform must strike a balance between *driver waiting time* and the *lateness of deliveries* (Ryan et al. 2018). *Food Processing Time* (FPT) is defined as the time it takes for the restaurant to prepare the food once an order is received. This duration is typically unknown to the platform. Some platforms provide customers with a guaranteed *Estimated Time of Arrival* (ETA). In these cases, FPT in our model can be replaced by the time difference between the promised delivery time, offset by the travel time from the restaurant to the customer, and the time the request arrived at the system.

At Meituan, the average time to fulfill a food order ranges from 32 to 38 minutes. This duration includes the time spent waiting for an order to be matched, driving to the shop, waiting at the shop, and delivering the food to the customer. *Shop-waiting time* refers to the period between the driver’s arrival at the restaurant and the moment the food is ready for pickup. Based on our data (Figure 2), shop-waiting time accounts for 35% of the total service time for an order, which is nearly as long as the time for delivery to the customer (41%). According to a study by Meituan (2021), delays in food preparation are a major cause of tension between drivers and restaurants, because these delays result in significant waiting time at shops. Similar issues related to shop-waiting time have been reported by UberEats and Deliveroo drivers (Ryan et al. 2018, Bosredon 2021). Feedback from drivers indicates that there are some flaws in the current delivery process, stating “The pickup is sometimes not ready at all... sometimes [it takes] even 20 minutes.”(Ryan et al. 2018)

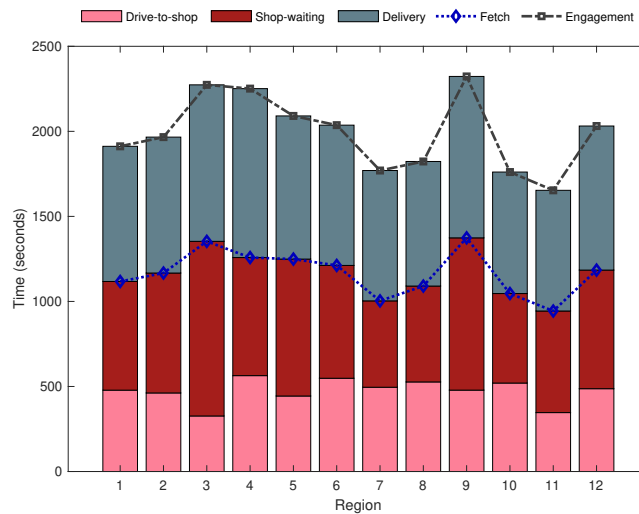


Figure 2 Processing times in different stages of OFD in different regions in which the company operates.

Most previous research has primarily focused on the question of ‘*Whom to match?*’ in order to find the closest driver for an order. This is typically achieved with routing optimization and other techniques. However, few studies have addressed the issue of minimizing the shop-waiting time, a period for which the OFD platform must pay the driver. This shop-waiting time is as important as the drive-to-shop time. Ryan et al. (2018) developed a trip-inference model and a machine learning optimization tool specifically aimed at reducing the average time drivers spend waiting, walking, and parking at individual restaurants. However, they did not propose a general framework for making system-wide matching decisions. Recently, Mao et al. (2022) analyzed an OFD platform’s data set and found that drivers typically wait almost six minutes on average at restaurants. They asked the same question we examine in this paper: how can drivers be better matched to minimize time spent waiting for orders? Note that the opportunity costs of

‘waiting for matching’ and ‘shop-waiting’ differ: In the former situation, drivers have the option to accept other jobs, while in the latter, they are unable to take on other work. Therefore, minimizing shop-waiting time is crucial for the platform.

Food processing begins as soon as an order is placed on the platform (this is a conservative way to describe the food processing activities as it is challenging for platforms to track when meal preparation actually begins in the kitchen, see Mao et al. (2022)), even before driver matching takes place. In our data set, the FPT is not known in advance and is highly variable, with an average of 23 minutes and a coefficient of variation of 0.52. Despite this variability, advanced prediction techniques can forecast FPT using data from past orders. Several studies have already incorporated ETA values into research related to on-demand platforms (Liu et al. 2021b, Zehtabian et al. 2022, Cohen et al. 2022, Tao et al. 2022). In this paper, we develop matching algorithms designed to minimize total costs per order by taking into account both driving time and shop-waiting time. Specifically, we leverage the FPT to answer the question of ‘*When to match?*’. Our approach falls under the umbrella of real-time supply chain analytics, as we use extensive real-time operational data from various sources to enhance visibility and improve decision-making (Gupta 2022).

Specifically, we investigate whether intentionally delaying the matching of an order to a driver offers advantages in finding better matches. This concept, known as *market thickening*, was introduced by Akbarpour et al. (2020) in the context of online gaming and kidney exchanges. The FPT in OFD gives the platform the leeway to delay matching without compromising food quality (because the driver has to wait until the food being ready). However, excessive delays can result in higher costs due to late deliveries. The platform must therefore strike a careful balance between waiting to improve market thickness with better, shorter drive-to-shop matches and minimizing the costs of delivery delays. The complexity of this decision problem is compounded by the high variability of the FPT and the real-time influx of orders across the entire delivery area. Many OFD platforms use a batching policy to build market thickness, where orders are accumulated and drivers are assigned at regular intervals. In this paper, we demonstrate that our method outperforms the batching policy even when batch intervals are optimally chosen. The underlying algorithmic idea of our approach is straightforward: monitor the number of drivers who can fulfill an order within a specific range and assign the order only if the number of available drivers falls below a certain threshold, denoted by k . One of the challenges is to identify the optimal value for k . Our computational results indicate that performance is best when k is set at a moderate level - neither too high nor too low.

The order-driver matching problem discussed above can be categorized as a *Bipartite Min-cost Matching* (BMM) problem. We explore the properties of an optimal policy, set forth upper and lower bounds, and introduce two new policies: a k -level thickening (KT) policy for scenarios with known FPT and a buffered k -level thickening (BKT) policy for cases with unknown FPT. Our paper makes three key contributions:

- To the best of our knowledge, this is the first study to investigate the impact of FPT in the OFD sector on the resulting matching cost and driver waiting time.
- We formulate the original BMM problem as a dynamic matching model. In the specific case of a single order in isolation, we analytically demonstrate that the optimal matching decisions adhere to a state-dependent threshold policy. Inspired by this threshold structure in the single-order scenario, we develop the driving-time-constrained KT policy to address the general problem.
- Through numerical experiments using real-world data, we show that our BKT policy decreases the total costs per order by 54% compared to existing real-case matching policies.

2. Literature Review

Our research is situated at the intersection of literature on dynamic matching, market thickness, and online food delivery. In this section, we outline the main contributions from each domain and elaborate on how we build upon them.

Research on dynamic matching: In recent years, there has been a surge in studies related to dynamic matching, particularly due to the emergence of the *on-demand economy* (Chen et al. 2020). Hu and Zhou (2022) investigated stochastic and dynamic matching control within a two-sided discrete-time system. Özkan and Ward (2020) introduced dynamic matching and pricing policies derived from a continuous linear program. To minimize unmatched requests, Banerjee et al. (2016) developed state-dependent matching policies that take into account spatial demand heterogeneity. Varma et al. (2022) explored a two-sided queuing system, focusing on joint pricing and matching controls.

Most of the aforementioned studies have considered matching policies in a myopic manner; that is, they instantly match demand with supply. However, this approach is unlikely to be optimal, as it might be more advantageous to reserve potential demand or supply for future periods. Akbarpour et al. (2020) explored this trade-off between matching frequency and market thickness, developing the so-called *greedy* and *patient* matching mechanisms. These mechanisms were calibrated by Liu et al. (2019) using a data set from DiDi. Ashlagi et al. (2018) demonstrated the superiority of batching policies with a data set of New York City yellow cabs, while Yan et al. (2020) from Uber developed a *dynamic waiting* strategy to enhance matching probability and supply efficiency in ride-hailing practices. Castro et al. (2021) analyzed a matching system featuring both flexible and specialized agents. Manshadi et al. (2022) examined matching on online volunteering platforms. Aouad and Saritac (2022) delved into the dynamic, stochastic matching problem with limited times, developing matching algorithms that provide worst-case performance guarantees. We draw inspiration from the idea of market thickening and extend it to a setting with heterogeneous agents and variable edge weights that change in real-time.

Research on market thickness: Market thickness is defined as the number of participants in a market (McLaren 2003). In matching markets, a long-standing discussion revolves around whether thickness

positively or negatively affects the effectiveness of matching. Gan and Li (2016) demonstrated that the probabilities of matches in a thin market are significantly lower than those in a thick market. Akbarpour et al. (2020) analyzed the impact of search friction on market thickness in online markets. Li and Netessine (2020) discovered that a higher market thickness does not necessarily correlate with higher matching rates, attributing this to the presence of abundant search friction. Liu et al. (2019) indicated that the incremental value of market thickness, brought about by strategic waiting, might lead to fewer matches. Chen (2019) employed an $M/M/\infty$ queue to investigate mismatch performance. Arnosti et al. (2021) illustrated that straightforward operational interventions could alleviate congestion in matching markets effectively. Cui et al. (2022) examined the influence of market thickness on delivery efficiency. Xie et al. (2022) demonstrated that a little delay yields benefits in real-time online decision-making. They provided evidence that the difference between the proposed online policy with a delay and an offline hindsight policy diminishes at an exponential rate as the length of delay increases. We contribute to this ongoing debate by showing that the benefits derived from market thickening exhibit a quasi-convex pattern, suggesting the existence of an optimal thickening level that balances the positive and negative effects.

Research on online food delivery: Our work also contributes to the OFD literature, which has garnered significant attention in recent years (Reyes et al. 2018, Mao et al. 2019, Liu et al. 2021b, Chen et al. 2022). The OFD routing problem poses particular challenges in last-mile delivery due to the stringent service level targets. Although it bears similarities to the same-day delivery problem (Voccia et al. 2019), the need for quick-response decisions to adhere to short delivery times distinguishes it fundamentally from same-day delivery. Mao et al. (2022) conducted a comprehensive analysis of OFD operations using an industrial data set, and our current study aligns with the future directions mentioned in their work. Furthermore, Mao et al. (2019) empirically demonstrated that delivery performance is vital for an OFD platform to retain customers, and Liu et al. (2021b) proposed a data-driven framework to model the OFD problem and optimize assignment decisions. Ulmer et al. (2021) addressed a stochastic dynamic pickup and delivery problem, considering random FPT and deadlines. Chen and Hu (2023) compared temporal pooling and dedicated delivery in an on-demand delivery dispatching system. Distinctly, our work diverges from existing research on OFD routing by incorporating intentional delay to foster a thicker market, thereby enhancing compatibility between orders and drivers and increasing matching opportunities.

3. OFD Model Description

In this section, we introduce several key components to characterize the dynamic matching model in the OFD business. Orders and drivers arrive at the platform according to Poisson processes with rates λ_o and λ_d , respectively. We assume that $\lambda_o < \lambda_d$ and that drivers abandon the system at exponential rate $\lambda_a > 0$ for system stability (see also, e.g., Aouad and Saritac 2022).

Orders: An order is described by tuple $x_i = (\tau_i, l_i, \xi_i)$, where τ_i is the arrival time, l_i is the restaurant location, and ξ_i is the FPT. Throughout the paper, we use index i for orders. Restaurant locations l_i are independently distributed random variables with probability density function (pdf) $f_r(l_i)$. FPTs ξ_i are independently distributed random variables with pdf $f_\xi(\xi_i)$, which includes the duration of all activities between order placement and the earliest time it can be fetched. We assume that the FPT is independent of the matching policy and the state of the system (see Appendix 7.4.1 for an extension to a model in which the FPT is affected by the level of order congestion in the restaurant). We also assume that the system planner can only observe the realization of the FPT ξ_i at the end of the food processing. However, the planner predicts the FPT at order arrival (see Section 5 for details about FPT prediction). We denote the predicted value by $\hat{\xi}_i$, and assume that this prediction is unbiased, i.e., $\hat{\xi}_i = \mathbb{E}[\xi_i]$. Let the variance of the prediction error $\hat{\xi}_i - \xi_i$ denoted by σ_ξ^2 . We refer to the special case of exact predictions ($\sigma_\xi^2 = 0$) as *known FPT*. Orders can only leave the platform when they are matched with a driver. Let \mathcal{I}_t represent the set of orders that arrived until time t .

Drivers: A driver is described by tuple $y_j = (\tau_j, l_j, t_j^a)$ with arrival time τ_j , driver location l_j , and driver abandonment time $t_j^a > \tau_j$. Throughout the paper, we use index j for drivers. Driver locations are independently distributed random variables with pdf $f_d(l_j)$. Drivers leave the platform either when they are matched with an order or when they abandon. Abandonment times t_j^a are not known to the planner before the abandonment. We assume that a driver can only deliver a single order at a time; otherwise our problem would integrate a routing problem, which is not in the scope of this research. Let \mathcal{J}_t denote the set of drivers that arrived and have not abandoned by time t .

Driving times: For picking up orders at the restaurant, let $d_{ij}(t)$ denote the deterministic driving time from the location l_j of driver j to the location l_i of the restaurant of order i , if the driver begins the drive at time t , i.e., we allow driving times to depend, for example, on time of day (our model can be extended to integrate real-time estimation of non-deterministic driving times in which $d_{ij}(t)$ is determined at the time of matching). We only make the assumptions that $t + d_{ij}(t) < t' + d_{ij}(t')$ for $t < t'$, i.e., that a driver cannot arrive earlier by waiting, and that $d_{ij}(t) = d_{ij}(t + \delta)$ for some δ and any i, j, t , i.e., that there is periodicity (of length δ , e.g., daily, weekly) in the driving times. For the driving time from restaurant to customer, we assume that it is independent of the matching decision, and we therefore treat it as a constant value, denoted by $\bar{\psi}$.

Order-Driver-Matching: Based on the state of the system, the planner has to decide at what time to match an order with a driver. For a given sample path, and policy Ω (i.e., the rule with which to make matches), let \mathcal{M}_t^Ω be the set of order-driver pairs *already matched* by time t :

$$\mathcal{M}_t^\Omega = \{(i, j, t_{ij}) : \text{order } i \text{ and driver } j \text{ matched at time } t_{ij}, i \in \mathcal{I}_t, j \in \mathcal{J}_t, \max(\tau_i, \tau_j) \leq t_{ij} \leq \min(t, t_j^a)\}. \quad (1)$$

We consider a pair (i, j) between an order i and a driver j to be *compatible* at time t if the assignment does not result in a delay in delivery, i.e., if $t + d_{ij}(t) \leq \tau_i + \xi_i$. Otherwise, we refer to the edge (i, j) as *expired*. We refer to the time epoch t_{ij}^c that solves equation $t_{ij}^c + d_{ij}(t_{ij}^c) = \tau_i + \xi_i$, i.e., the exact time at which driver j has to drive towards the restaurant and arrive at the end of the food processing to pick up order i , as the *edge expiration time*. Assumption $t + d_{ij}(t) < t' + d_{ij}(t')$ for $t < t'$, guarantees that there is a unique edge expiration time per order-driver pair. Note that the exact edge expiration time is known in advance to the planner only in the case of known FPT.

Cost model: The cost of an order incurs at the time when the order is matched or at the end of the time horizon if it is unmatched. It comprises two types of costs: First, the *fetching cost*, which reflects the time a driver spends on fetching the order, consisting of the drive-to-shop time, $d_{ij}(t_{ij})$, and the potential shop-waiting time at the restaurant (given by the positive difference between the arrival time of the driver at the restaurant and the time of order completion by the restaurant): $(\tau_i + \xi_i - t_{ij} - d_{ij}(t_{ij}))^+$. Second, the potential *cost of lateness*, which represents the loss in customer satisfaction and which is proportional to the delivery delay, $(t_{ij} + d_{ij}(t_{ij}) - \tau_i - \xi_i)^+$. For a given matching policy Ω , we write the expected total costs per order, W^Ω , as

$$W^\Omega = \lim_{t \rightarrow \infty} \mathbb{E} \left\{ \frac{1}{|\mathcal{I}_t|} \sum_{(i,j,t_{ij}) \in \mathcal{M}_t^\Omega} [d_{ij}(t_{ij}) + (\tau_i + \xi_i - t_{ij} - d_{ij}(t_{ij}))^+ + c \cdot (t_{ij} + d_{ij}(t_{ij}) - \tau_i - \xi_i)^+] \right\}, \quad (2)$$

where c is the per-time-unit penalty for delay in fetching. Without loss of generality, we normalize the per-time-unit fetch cost to 1. The objective of the model is to minimize the sum of fetching cost and cost of lateness. We denote the optimal policy by Ω^* , i.e., the policy that achieves the *minimal* expected costs per order ($W^* = W^{\Omega^*}$).

In the above model with known FPT, order and driver arrivals and driver abandonment are Markovian, but the food processing times are not Markovian. Hence, the model belongs to the class of *Semi-Markov Decision Processes (SMDP)*, which we formally establish in Appendix 7.2.

In our analysis, we also report on performance indicators beyond total costs. These indicators include average *drive-to-shop* time (V_p^Ω), average *in-shop waiting* time (V_w^Ω), average *delay per order* (V_d^Ω), *service level* (V_l^Ω), average *number of drivers* available (V_n^Ω), average time that drivers are *engaged* (V_e^Ω), average time that drivers are *idle* waiting for jobs (V_q^Ω), as well as average *net income per driver* per time unit (V_s^Ω); see Appendix 7.1 for detailed definitions.

4. Policy Development and Analysis

In this section, we analyze the dynamic matching problem defined in Section 3. We commence by establishing a property of the optimal policy and illustrating the value of waiting in matching. Subsequently, we define three benchmark policies that are used in practice and literature, and present a lower bound

on the total costs (Subsection 4.1). We then derive the optimal matching policy for the special case of a single order in isolation (Subsection 4.2) and, building on this result, develop a heuristic matching policy for multiple orders (Subsection 4.3).

In classical matching models, agents arrive randomly to a bipartite graph. Each edge has a fixed weight representing the matching cost, with the objective of minimizing the sum of the weights of all selected edges. A more generalized model is the dynamic matching problem with time windows, a topic rarely investigated in existing research; see, for example, Cao et al. (2020) or Aouad and Saritac (2022). In these models, the introduction of time windows significantly complicates the analysis. Our model is even more complex, incorporating both time windows and variable edge weights that change over time as an edge expires. To facilitate our analysis, we first examine the properties of the cost function, which we rely on in the remainder of our study. The following condition is necessary for the optimal matching time.

PROPOSITION 1. *In the case that the FPT is known at order arrival ($\sigma_\xi^2 = 0$), it is never optimal to match order i with a driver j at a time t if the edge is not yet expired (i.e., $t + d_{ij}(t) < \tau_i + \xi_i$).*

All proofs can be found in Appendix 7.3. The concept underlying Proposition 1 is straightforward: the decision-maker is always better off waiting until the edge expires than matching when the edge is not expired. Nonetheless, the selection of the edge is not *a priori* apparent, a point illustrated in settings such as the Uber Marketplace (<https://www.uber.com/us/en/marketplace/matching/>), and further demonstrated in the subsequent example for our context.

EXAMPLE 1. (Edge selection and value of waiting) Let us illustrate the potential benefits of waiting through a simple example. In Figure 3, we compare *myopic* order-driver matching (left-hand side of the figure) with *optimal* order-driver matching (right-hand side of the figure). The horizontal axis of each side represents the geographical location, denoted by l , while the vertical axis represents the travel time along the geographical line, denoted by t . At time $t = 0$, order 1 arrives to the system at $l_1 = 3$ with FPT $\xi_1 = 3$ time units. A second order 2 arrives at $t = 2$ at location $l_2 = 0$ with FPT $\xi_2 = 3$ time units. A driver A is available at $t = 0$ and $l_A = 1$, and a second driver B arrives at $t = 1$ and $l_B = 4$. Let us analyze the performance of the two policies:

- *Myopic matching:* As depicted on the left-hand side of Figure 3, if the planner matches the order 1 with driver A *myopically* at time $t = 0$, then the planner must match order 2 with driver B at time $t = 2$. The first matching incurs a cost of 3 (two units for driving and one for waiting until the food is ready); the second matching incurs a cost of $4 + c(4 - 3) = 4 + c$. The total costs per order are consequently $(3 + 4 + c)/2 = 3.5 + 0.5 \cdot c$.
- *Optimal matching:* As depicted on the right-hand side of Figure 3, if the planner waits until time $t = 2$ to match the order 1 with driver B and until $t = 4$ to match the order 2 with driver A , the

first matching incurs a cost of 1, and the second matching incurs a cost of 1, leading to total costs per order as $(1 + 1)/2 = 1$.

The cost difference between the two policies is $2.5 + 0.5 \cdot c$, reflecting the value of waiting and optimal edge selection. The challenge, therefore, lies in determining the optimal matching combinations.

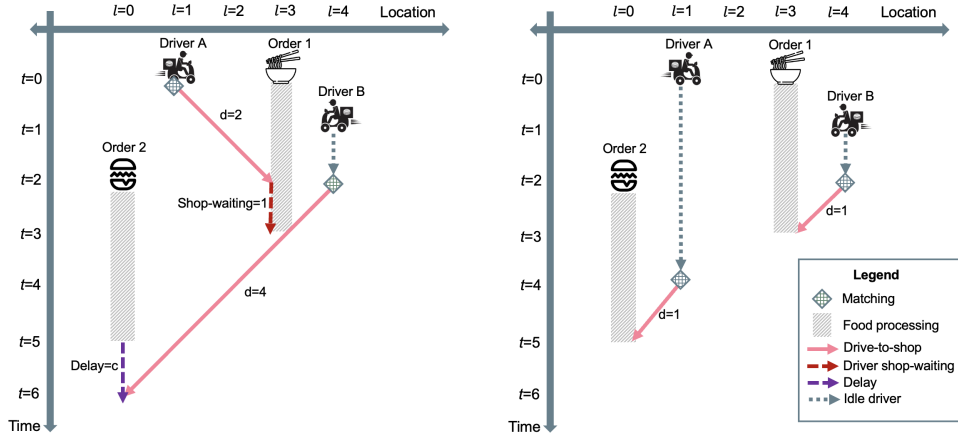


Figure 3 Example of myopic (left-hand side) and optimal (right-hand side) matching of two orders to two drivers.

4.1. Benchmark policies as lower and upper bounds

In this subsection, we introduce benchmark policies that act as lower and upper bounds for the performance of the optimal policy, as defined in the model presented in Section 3. Firstly, we describe the *Greedy policy*, denoted by Ω^g , which emulates the ‘match-as-you-go’ policies (Akbarpour et al. 2020) extensively employed in OFD platforms in practice, including platforms like Meituan.

DEFINITION 1. (Greedy Policy Ω^g) *If an order i arrives at the platform at time t , then match it with the closest driver in \mathcal{J}_t whenever $\mathcal{J}_t \neq \emptyset$. If a driver j arrives at the platform at time t , then match her with the closest order in \mathcal{I}_t whenever $\mathcal{I}_t \neq \emptyset$.*

The immediate greedy matching policy exposes the benefits achievable through delaying the matching decision. It assigns drivers to potentially non-ideal orders, resulting in extended travel distances. Recognizing this, several OFD platforms are exploring alternative matching policies to enhance the efficiency of the dispatch process. For instance, Meituan, in a pilot project, has installed terminals (*ChuCanBao*) in restaurants, requesting chefs to update the FPT in real-time (Meituan 2021). Once the meal is ready, the planner greedily allocates the order to a specific driver. We summarize this policy as follows.

DEFINITION 2. (Match-after-ready (MAR) Policy Ω^r) *When an order $i \in \mathcal{I}_t$ finishes processing at time t , then match it with the closest driver in \mathcal{J}_t whenever $\mathcal{J}_t \neq \emptyset$. If a new driver j enters the market at time t , then match her with the closest, ready order in \mathcal{I}_t .*

Policy Ω^r emulates a *delayed* greedy policy, as it defers the myopic matching until the order is ready. Intuitively, this policy eliminates the drivers' waiting time at the restaurant. However, this improvement comes with a trade-off in food quality and customer satisfaction, as the prepared food must now await the arrival of the drivers.

We also evaluate the performance of the well-known *Batching* policy, denoted by Ω^b , that is employed by several online platforms (Yan et al. 2020). This policy groups jobs and drivers in intervals of ω time units (e.g., one minute) before executing a linear program to optimally match the available orders with drivers. Any unmatched orders and drivers are retained for the subsequent batch. In our simulations, we determine the optimal batching period ω through back-testing (see Appendix 7.6 for a detailed formulation of the procedure).

Next, we define the optimal policy for a *full-information* case where FPTs, arrival and abandonment times, and order and driver locations are known in advance. We determine the optimal policy Ω^T over a finite horizon \mathcal{T} by solving the following linear program.

$$\begin{aligned} \widetilde{W}^{\Omega^T} = \min \frac{1}{|\mathcal{I}_{\mathcal{T}}|} & \left\{ \sum_{i \in \mathcal{I}_{\mathcal{T}}} \sum_{j \in \mathcal{J}_{\mathcal{T}}} \sum_{t_{ij} \in \mathcal{T}} x_{ijt_{ij}} [d_{ij}(t_{ij}) + (\tau_i + \xi_i - t_{ij} - d_{ij}(t_{ij}))^+ + c \cdot (t_{ij} + d_{ij}(t_{ij}) - \tau_i - \xi_i)^+] \right. \\ & \left. + c \cdot \sum_{i \in \mathcal{I}_{\mathcal{T}}} (\mathcal{T} - \tau_i - \xi_i) \cdot (1 - \sum_{j \in \mathcal{J}_{\mathcal{T}}} \sum_{t_{ij} \in \mathcal{T}} x_{ijt_{ij}}) \right\} \end{aligned} \quad (3)$$

subject to

$$\sum_{j \in \mathcal{J}_{\mathcal{T}}} \sum_{t_{ij} \in \mathcal{T}} x_{ijt_{ij}} \leq 1, \forall i \in \mathcal{I}_{\mathcal{T}}; \sum_{i \in \mathcal{I}_{\mathcal{T}}} \sum_{t_{ij} \in \mathcal{T}} x_{ijt_{ij}} \leq 1, \forall j \in \mathcal{J}_{\mathcal{T}} \quad (4)$$

$$x_{ijt_{ij}} \in \{0, 1\}, \forall i \in \mathcal{I}_{\mathcal{T}}, t_{ij} \in \mathcal{T}, j \in \mathcal{J}_{\mathcal{T}}; x_{ijt_{ij}} = 0, \forall t_{ij} < \max(\tau_i, \tau_j) \text{ and } \forall t_{ij} > t_j^a. \quad (5)$$

The decision variables $x_{ijt_{ij}}$ indicate that an order i is matched with a driver j at time t_{ij} . Constraint (4) ensures each order is matched with at most one driver and vice versa. Constraint (5) ensures that only available orders and drivers are considered for matching. Since the scheduling horizon \mathcal{T} is finite, we have to account for delay costs of orders that are not matched at the end of the horizon, which is represented by the last term in (3). We approximate the policy for the full-information case, denoted by Ω^f , by taking the limit of \mathcal{T} as it approaches infinity, i.e., $W^{\Omega^f} = \lim_{\mathcal{T} \rightarrow \infty} \widetilde{W}^{\Omega^T}$. It is evident that the benchmark policies and the full-information case provide upper and lower bounds to the optimal costs, a fact we establish in the following lemma (without proof).

LEMMA 1. $\min\{W^{\Omega^g}, W^{\Omega^r}, W^{\Omega^b}\} \geq W^* \geq W^{\Omega^f}$.

4.2. Analysis of dynamic matching with a single order

In this section, we focus on the case of a single order as described in Section 3, without any competing orders for drivers. We aim to analytically determine the optimal policy for this case to gain a better understanding of the broader matching problem's core mechanisms.

We assume in this subsection that the FPT is known, i.e., $\sigma_\xi^2 = 0$, and that the driving time is constant and does not vary with the time of the day, i.e., $d_{ij}(t) \equiv d_{ij}$. We also assume that driver abandonment is minimal due to the short matching time frame, which implies that it has little or no effect on the matching decision. We denote the inter-arrival time between driver j and the previous driver by \hat{t}_j . Without loss of generality, we set the order's arrival time to $\tau_i = 0$. We denote the *remaining* food processing duration at any given time t as ξ_i^r , which is given by $\xi_i^r = \tau_i + \xi_i - t$. For simplicity, we omit the order index i in our notation. For instance, d_j (instead of d_{ij}) represents the driving time of driver j to the restaurant. Our analysis starts by identifying the optimal policy's potential matching times. Generally, if there is any active non-expired edge, it is preferable to wait until the last edge expires. This idea is formally established in the following proposition:

PROPOSITION 2. *For a single order scenario, it is never ideal to match a driver with the order while any edges remain unexpired.*

Considering the moment when the last edge expires (i.e., $d_j = \xi^r$), the decision-maker has two options:

- Instantly match driver j to the order, incurring a cost of d_j or
- Wait for a driver with shorter driving time.

If d_j is significant, waiting might be the better option. Our next theorem formalizes this intuition. The decision-maker will choose to match only if driver j 's driving time is below a certain threshold. This threshold depends only on the remaining FPT and not on prior driver arrivals.

THEOREM 1. *In a single order scenario:*

- (a1) *At the moment the last edge with driver j expires ($d_j = \xi^r$), the optimal decision is to match the order if and only if:*

$$d_j \leq \tilde{d}(\xi^r) = \tilde{d}(d_j), \quad (6)$$

where $\tilde{d}(\xi^r)$ is a threshold function dependent solely on the remaining FPT, ξ^r .

- (a2) *If no drivers are available, upon the arrival of a new driver j with travel time and remaining FPT such that $d_j > \xi^r$ (i.e., the edge of the driver is already expired), the order is matched if and only if:*

$$d_j \leq \tilde{d}(\xi^r). \quad (7)$$

- (b) *The threshold function, $\tilde{d}(\xi^r)$, is defined as:*

$$\tilde{d}(\xi^r) = \frac{W^+(\infty, \xi^r) + c\xi^r}{c+1}, \quad (8)$$

with $W^+(\infty, \xi^r)$ representing the expected future cost after the last non-expired edge's expiration when the remaining FPT is ξ^r . The function $\tilde{d}(\xi^r)$ (non-strictly) increases with ξ^r .

Theorem 1(a) suggests that the optimal policy follows a threshold structure, wherein immediate matching is favorable only when the travel time of the last remaining driver falls below a certain threshold. Theorem 1(b) further establishes that for drivers arriving after their edges have expired, this threshold increases with the remaining FPT, ξ^r . Equivalently, the threshold decreases over time, indicating that the later the match occurs, the lower the likelihood of accepting a driver with a given travel time. This aligns with intuition: a driver arriving after the expiration of her edge is generally less preferred than one arriving prior to expiration.

To develop the aforementioned theorem, we show that, at any given moment, the driver possessing the shortest driving time to the order dominates all other drivers in the eligible driver pool. Consequently, the system exclusively relies on the driver with the minimum driving time. Subsequently, by employing the Markov property of the arrival stream, we ascertain that the net value of matching a particular driver at her edge expiration time dominates the net value of matching that same driver at any later moment. Therefore, if the net value of matching a driver is negative at the edge expiration time, it will always remain negative in the future, making it consistently advantageous to await the arrival of a more suitable driver. These two observations imply the threshold structure of the optimal policy, as established in Theorem 1.

Interestingly, an implication of Theorem 1 is that drivers satisfying the condition $d_j > \tilde{d}(\xi^r = d_j)$ will never be matched, as the system always opts to hold out for a more suitable match. However, fulfilling the condition $d_j \leq \tilde{d}(\xi^r = d_j)$ upon arrival does not ensure a match at the time of edge expiration, as a more close-by driver might arrive in the meantime.

The future expected cost, $W^+(\infty, \xi^r)$, ensuing after the expiration of the last edge, when no non-expired edges remain in the system (denoted by ∞ in the first argument), can be deduced from the function $W^+(d, \xi^r)$. This function represents the minimum future expected cost when the last edge to expire exhibits a driving time d and the remaining FPT is ξ^r . The function $W^+(d, \xi^r)$ always has a solution and it can be evaluated numerically based on the feasible state transitions: (a) Expiry of the last driver's edge, (b) arrival of new driver with non-expired edge (and the edge of the last driver is not expired), (c) arrival of a new driver with expired edge (and the edge of the last driver is expired), (d) arrival of a new driver with non-expired edge (and the edge of the last driver is expired). These state transitions can be formalized as follows:

$$W^+(d, \xi^r) = \mathbb{E}_{d_j'} \mathbb{E}_{\hat{t}_{j'}} \begin{cases} \min \{d, W^+(\infty, \xi^r - d)\}, & \text{if } (\xi^r - d) \in [0, \hat{t}_{j'}], \\ W^+(\min\{d, d_{j'}\}, \xi^r - \hat{t}_{j'}), & \text{if } (\xi^r - d) > \hat{t}_{j'}, \\ \min \{d_{j'} + c[d_{j'} - \xi^r]^+, W^+(\infty, \xi^r - \hat{t}_{j'})\}, & \text{if } (\xi^r - d) < 0 \text{ and } d_{j'} > (\xi^r - \hat{t}_{j'}), \\ W^+(d_{j'}, \xi^r - \hat{t}_{j'}), & \text{if } (\xi^r - d) < 0 \text{ and } d_{j'} \leq (\xi^r - \hat{t}_{j'}). \end{cases} \quad (9)$$

and $W^+(d, \xi^r) = W^+(\infty, \xi^r)$ for any $d > \xi^r$, where j' is the next driver arrival with driving time $d_{j'}$ and inter-arrival time $\hat{t}_{j'}$. (See also the detailed discussions in the proof of Theorem 1 and in Appendix 7.7).

We present an example to illustrate the application of the threshold policy. We select a single restaurant and ten driver locations from the data set introduced in Section 5. The locations are chosen to ensure that drivers have an equal probability of being in any of the ten locations, i.e., $f_d(l_j) \approx 1/10$ for $j = 1, \dots, 10$. The average FPT is $\xi = 23$ minutes, and the arrival rate is $\lambda_d = 0.4$. The results are depicted in Figure 4. On the left-hand side, the blue dashed line represents the threshold $\tilde{d}(\xi^r)$ as a function of the remaining FPT ξ^r . If the driving time at the last-edge expiration, d_j , (the red solid line) is below the threshold function (blue dashed line), the driver will be matched with the order. On the right-hand side of the figure, we display the driving radius around the restaurant corresponding to the threshold at varying values of ξ^r . Drivers with a driving time less than the time at which the blue dashed line and the red solid line on the left-hand side of Figure 4 intersect ($d_j = 13.02$ minutes), i.e., drivers located within the shaded circle area on the right-hand side, are potential candidates for matching at edge expiration. Those with a greater driving time, residing outside the shaded area (with gray, dashed circles), will never be matched. For instance, drivers with $d_j = 1$ arriving before $\xi^r = 1$ (i.e., before the edge expires) could be matched as $d_j = 1 < 10 = \tilde{d}(1)$; this also applies to drivers with $d_j = 11$. Conversely, drivers with $d_j = 17$ or $d_j = 20$ are excluded from matching. This observation elucidates that for drivers arriving before the edge expiration, a *single threshold value* determines whether the last non-expired driver is matched upon edge expiration.

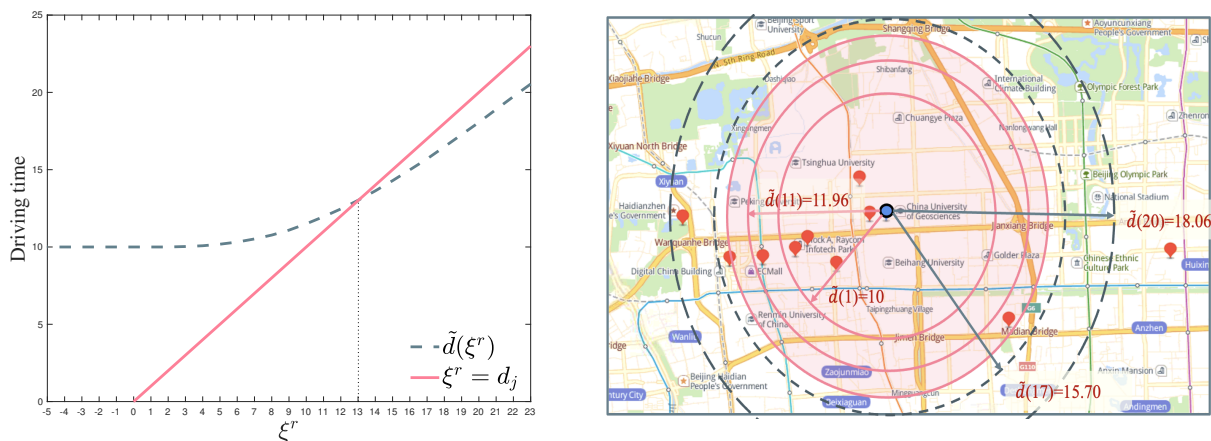


Figure 4 Threshold function $\tilde{d}(\xi^r)$ as a function of remaining FPT ξ^r (left) and ‘threshold’ circles around the restaurant location for different values of ξ^r (right). (The red-shaded area on the right-hand side corresponds to a maximum driving time of $d_{ij} = 13.02$ minutes).

In most real-world scenarios, orders do not occur in isolation, and drivers have the potential to be matched with other orders. However, the fundamental decision problem remains consistent in the general model: the immediate cost of matching a driver to an order, dependent on the driving time, must be

weighed against the expected future cost of waiting. In the general model, the future cost of waiting likely depends not only on the remaining FPT but also on additional factors such as the presence of other outstanding drivers and orders. This renders the ‘threshold’ for acceptance versus waiting more complex than in the scenario of an isolated, single order. We leverage this insight in the following subsection to develop a matching policy for the general case, where a driving-time threshold is complemented by a thickening level to regulate competition between orders for the best drivers.

4.3. KT Policy Development and Analysis

In this section, we develop the KT policy for the model in Section 3, based on the insights that we have gained from our prior analysis. We describe the policy, analyze its properties, and develop a simple, closed-form approximation of its performance.

KT Policy with Known FPT. Compared to the single-order case, in the presence of multiple orders, it is not always optimal to wait until the last edge expires. This is because the corresponding driver might not be available anymore at this time; it may have already been matched with another order. This risk of drivers ‘dropping out’ from the pool of eligible drivers for a given order renders it analytically infeasible to establish a result equivalent to Theorem 1 for the general case. We, therefore, develop a heuristic policy for the general case, incorporating a threshold structure with additional mechanisms to match earlier when necessary to mitigate the risk of drop-outs. We refer to this policy as the *Driving-time-constrained k-level Thickening (KT) Policy*.

The ‘design’ of the KT policy comprises three ‘mechanisms’, each grounded in our prior analysis. First, the policy constrains the neighborhood through a threshold function on the driving time, denoted by $\check{d}(\xi^r)$, which is optimal in the single-order scenario (Theorem 1). Second, in contrast to the optimal strategy in the single-order case of awaiting the expiration of the last driver (see Proposition 2), the KT policy regulates the level of *market thickness* (refer to the discussion in Section 1) through a threshold parameter k . This parameter guards orders against the risk of a driver, for which the order is waiting, to ‘drop out’, that is, being matched with a different order. A larger thickening level k results in reduced waiting times, albeit potentially leading to less desirable driving times; a smaller k extends the waiting period but could yield shorter driving times. Third, the KT policy also matches orders to the nearest available driver when *all* compatible drivers have dropped out, preventing unnecessary delays due to extended time for waiting for incoming drivers, especially in situations with low driver arrival rate.

Before introducing the policy, let us define the *driving-time constrained neighborhood* $\mathcal{N}_t(i, d)$ of an order i at time t as the set of *compatible* drivers that can reach the restaurant of order i within driving time d , i.e., available drivers j for which $d_{ij}(t) \leq d$.

DEFINITION 3. (*Driving-time-constrained KT Policy with known FPT, $\Omega^{k, \check{d}(\cdot)}$*) *Let the parameter k represent the level of market thickness and $\check{d}(\cdot)$ a threshold function. In the case of multiple orders,*

- (a1) If, at the edge expiration time t_{ij}^c for an order i and driver j , driver j is within the driving-time constrained range of order i , i.e., $d_{ij}(t_{ij}^c) \leq \check{d}(d_{ij}(t_{ij}^c)) = \check{d}(\xi_i^r)$, and if the number of drivers in this neighborhood is less than or equal to k , i.e., $|\mathcal{N}_{t_{ij}^c}(i, \check{d}(\xi_i^r))| \leq k$, then order i is immediately matched with driver j . Moreover, for any other order $i' \neq i$ for which driver j was the last non-expired driver in its neighborhood, i.e., $|\mathcal{N}_{(t_{ij}^c)^+}(i', \check{d}(\xi_{i'}^r))| = 0$, where $(t_{ij}^c)^+$ is the time epoch immediately following the matching (i, j) , order i' is immediately matched with the nearest available driver, i.e., driver $\arg \min_{j'} d_{i'j'}(t_{ij}^c)$.
- (a2) If, upon the arrival of a driver j , the edge expiration time t_{ij}^c with an order i has already passed, i.e., $t_{ij}^c < \tau_j$, and if driver j is within the driving-time-constrained range of order i , i.e., $d_{ij}(\tau_j) \leq \check{d}(\xi_i^r)$, and the neighborhood of this order is empty, i.e., $|\mathcal{N}_{\tau_j}(i, \check{d}(\xi_i^r))| = 0$, then order i is immediately matched with driver j . If multiple orders fulfill these conditions, the order with the greatest lateness cost is selected for matching, i.e., the order i' where $i' = \arg \max_i (\tau_j + d_{ij}(\tau_j) - \tau_i - \xi_i)^+$.

Statement (a1) of the KT policy prescribes an action for order-driver pairs that reach their edge expiration time. If the number of compatible drivers of the same order in the neighborhood falls below the threshold, the pair is matched; otherwise, it is not. Statement (a1) also concerns a second action: if the expiring pair is matched and if the matching leaves another order without any compatible drivers in the neighborhood, it is matched with the closest driver. This additional action avoids situations in which orders are ‘starved’ and have to wait for the arrival of new drivers. The statement (a2) of the KT policy is similar to the statement (a1) but concerns order-driver-pairs that are already expired at driver arrival. In such a situation, the pair is matched if and only if the new driver is positioned within the order’s driving-time constrained neighborhood and if this neighborhood is empty.

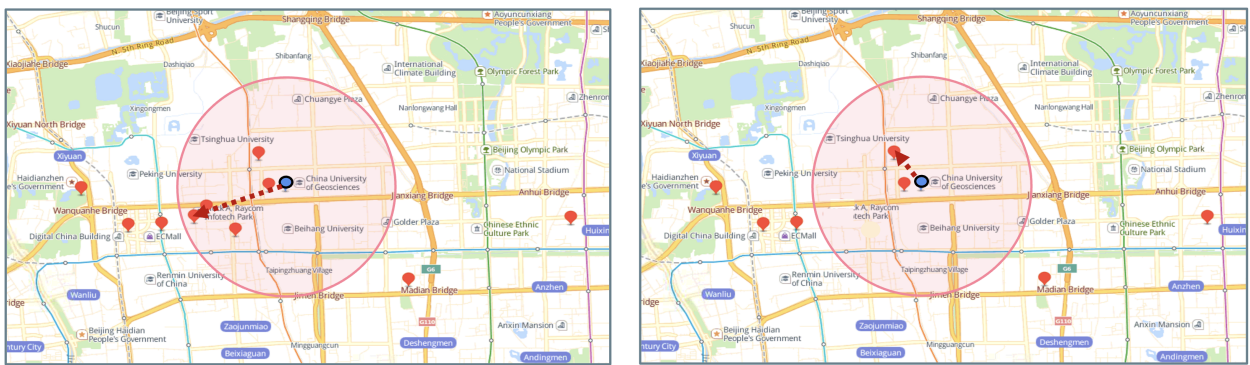


Figure 5 Illustration of the KT policy ($k = 2$ and $\check{d}(\cdot) = 10$) with an example at an early time (left) and another example at a later time (right).

Figure 5 provides a simple example of the functioning of the KT policy with parameters $k = 2$ and $\check{d}(\cdot) = 10$ based on two decision epochs, where the earlier point in time is on the left-hand side and the later point in time on the right-hand side. The blue dot in the center of the circle represents the order’s restaurant location, while red dots represent idle drivers. The red circle represents the neighborhood

defined by the policy parameter $\check{d}(\cdot) = 10$. The dashed arrow indicates the edge between the order and a driver that is about to expire. In the figure on the left-hand side, five idle drivers are located within the neighborhood. Because $k = 2 < 5$, the driver of the expiring edge is not matched with the order. However, as time progresses, we come to the time epoch that is illustrated in the figure on the right-hand side. At this time, only two idle drivers are located in the neighborhood, which equals parameter $k = 2$, such that we immediately match the driver of the expiring edge with the order.

For subsequent analysis, let k^* denote the optimal thickening level. It is evident that the KT policy with $k = 1$ aligns with the well-known *Patient* policy proposed by Akbarpour et al. (2020). Moreover, the KT policy, with a sufficiently large value of k , is tantamount to matching when the first edge with an eligible driver expires. In the presence of a single order, the KT policy converges to the optimal policy, which we formally show in the following corollary.

COROLLARY 1. *In the case of a single order (see Section 4.2), the KT policy $\Omega^{k, \check{d}(\cdot)}$ with $k = 1$ and $\check{d}(\cdot) \equiv \tilde{d}(\cdot)$ coincides with the optimal policy.*

The result of Corollary 1 holds, because for $k = 1$ the policy always waits to match until the last edge expires, a strategy that is optimal in the single-order case (Proposition 2). Furthermore, the threshold policy, characterized by function $\tilde{d}(\xi^r)$, results in optimal matching, as shown in Theorem 1. In Appendix 7.5, we discuss with the case of Example 1 how the KT policy can coincide with the optimal policy.

KT Policy with Unknown FPT. In the case of unknown FPT ($\sigma_\xi > 0$), the FPT value manifests as a random variable, only revealed to the decision-maker once the food is prepared. Consequently, upon the arrival of an order, the decision-maker can only determine an edge expiration time based on the *predicted* FPT, $\hat{\xi}_i$. This scenario introduces the possibility that an order may expire earlier or later than anticipated. To address this, we extend our KT policy to the *Buffered KT (BKT) Policy*, which protects against FPT prediction errors by adding a buffer time $b_{\sigma_\xi}^{\hat{P}}$ to the predicted edge expiration time. The buffer $b_{\sigma_\xi}^{\hat{P}}$ is set such that the probability $P[\xi_i \leq \hat{\xi}_i + b_{\sigma_\xi}^{\hat{P}}] = \hat{P}$ is satisfied, where policy parameter \hat{P} is the *protection level*. If the prediction error adheres to a Normal distribution, the buffer is assigned as $b_{\sigma_\xi}^{\hat{P}} = z_{\hat{P}} \cdot \sigma_\xi$, with $z_{\hat{P}}$ representing the z -score corresponding to \hat{P} .

We define the ‘*predicted*’, ‘*protected*’ expiration time of an edge (i, j) with unknown FPT, denoted by t_{ij}^{bc} , to be the value satisfying the equation $t_{ij}^{bc} + d_{ij}(t_{ij}^{bc}) = \tau_i + \hat{\xi}_i + b_{\sigma_\xi}^{\hat{P}}$. This value is known to the planner upon the arrival of the order. However, it is possible that food preparation finishes even before the time t_{ij}^{bc} . Consequently, under the BKT policy, decisions have to be made at the earlier of the two events, referred to as the ‘critical decision’ time, given by $\min(t_{ij}^{bc}, \tau_i + \xi_i)$, a value unknown to the planner until either of the two events occurs.

DEFINITION 4. *(BKT Policy with unknown FPT, $\Omega^{k, \check{d}(\cdot), \hat{P}}$) In the case of multiple orders,*

(a1) If, at the ‘critical decision’ time $\min(t_{ij}^{bc}, \tau_i + \xi_i)$ of an order i and driver j , driver j is within the driving-time constrained range of order i , i.e., $d_{ij}(t_{ij}^{bc}) \leq \check{d}(d_{ij}(t_{ij}^{bc}))$, and if k or fewer drivers are in the neighborhood, i.e., $|\mathcal{N}_{t_{ij}^{bc}}(i, \check{d}(d_{ij}(t_{ij}^{bc})))| \leq k$, then order i is immediately matched with driver j . Furthermore, upon such a matching, for any other order $i' \neq i$ for which driver j was the last non-expired driver in the neighborhood of order i' , i.e., $|\mathcal{N}_{(t_{ij}^{bc})^+}(i', \check{d}(\tau_{i'} + \hat{\xi}_{i'} + b_{\sigma_\xi}^{\hat{P}} - t_{ij}^{bc}))| = 0$, where $(t_{ij}^{bc})^+$ is the time epoch right after the matching (i, j) , order i' is immediately matched with the nearest driver, i.e., driver $\arg \min_{j'} d_{i'j'}(t_{ij}^{bc})$.

(a2) If, upon the arrival of a driver j , the ‘critical decision’ time $\min(t_{ij}^{bc}, \tau_i + \xi_i)$ with an order i has already passed, i.e., $\min(t_{ij}^{bc}, \tau_i + \xi_i) < \tau_j$, and driver j is within the driving-time-constrained range of order i , i.e.,

$$d_{ij}(\tau_j) \leq \begin{cases} \check{d}(\tau_i + \hat{\xi}_i + b_{\sigma_\xi}^{\hat{P}} - \tau_j), & \tau_j \leq \tau_i + \xi_i, \\ \check{d}(\tau_i + \xi_i - \tau_j), & \tau_j > \tau_i + \xi_i, \end{cases} \quad (10)$$

and if this neighborhood is empty, i.e.,

$$\begin{cases} |\mathcal{N}_{\tau_j}(i, \check{d}(\tau_i + \hat{\xi}_i + b_{\sigma_\xi}^{\hat{P}} - \tau_j))| = 0, & \tau_j \leq \tau_i + \xi_i, \\ |\mathcal{N}_{\tau_j}(i, \check{d}(\tau_i + \xi_i - \tau_j))| = 0, & \tau_j > \tau_i + \xi_i, \end{cases} \quad (11)$$

then order i is immediately matched with driver j . If multiple orders fulfill these conditions, the order with the greatest ‘predicted’ lateness cost is selected for matching, i.e., the order i' with

$$i' = \arg \max_i \begin{cases} (\tau_j + d_{ij}(\tau_j) - \tau_i - \hat{\xi}_i - b_{\sigma_\xi}^{\hat{P}})^+, & \tau_j \leq \tau_i + \xi_i, \\ (\tau_j + d_{ij}(\tau_j) - \tau_i - \xi_i)^+, & \tau_j > \tau_i + \xi_i. \end{cases} \quad (12)$$

In Appendix 7.4, we show how to adapt the BKT policy to different model extensions.

Simplified Performance Analysis of the KT Policy. Next, we explore the sensitivity of the performance of the KT policy with known FPT with respect to the level of market thickness k . As the original system is intractable, we perform this investigation on a simpler context, for which we can derive a closed-form expression for the total costs. For this simplified model, we assume that:

- The driving time to the restaurant and FPT are constant values, denoted by \bar{d} and $\bar{\xi}$, respectively. \bar{d} represents the *non-compressible* driving time, i.e., the average driving time under maximum waiting (i.e., with $k = 1$); $\bar{\xi}$ represents the average FPT.
- The number of compatible drivers for an order at its arrival is a constant value denoted by \tilde{k} (see also Banerjee et al. 2018, who use a similar assumption).
- The times between edge expiration events follow an exponential distribution with rate θ , and we assume drop-outs and new arrivals of drivers to be balanced during the time an order waits to be

matched. The rate θ is determined by average driving times, arrival rates of orders and drivers, and other factors. As a consequence, the matching duration $t_{ij} - \tau_i$ (i.e., the time between arrival of an order and its matching) corresponds to the sum of $\tilde{k} - k + 1$ exponentially distributed delays with rates $\mu_j = j\theta$. We denote this random variable by $X(k)$. It follows a hypo-exponential distribution with mean $\mu(k) = \sum_{j=k}^{\tilde{k}} \mu_j^{-1}$ and pdf $\tilde{f}_k(x)$.

The values \bar{d} , \tilde{k} , and θ can be derived from the neighborhood definition, i.e., from the threshold function $\check{d}(\cdot)$ of the KT policy. Based on the above simplifications, we obtain from Equation (2) the following expression for the total matching costs per order of the platform:

PROPOSITION 3. *Expected total costs per order in the simplified system are given by*

$$\begin{aligned} \widetilde{W}^{\Omega^k, \check{d}(\cdot)}(k) &= \bar{d} + \mathbb{E}[(\bar{\xi} - \bar{d}) - X(k)]^+ + c \cdot \mathbb{E}[X(k) - (\bar{\xi} - \bar{d})]^+ \\ &= \bar{d} + \int_0^{\bar{\xi} - \bar{d}} (\bar{\xi} - x - \bar{d}) \tilde{f}_k(x) dx + c \cdot \int_{\bar{\xi} - \bar{d}}^{\infty} (x + \bar{d} - \bar{\xi}) \tilde{f}_k(x) dx \\ &= \bar{\xi} - \mu(k) + (1 + c) \cdot p(k), \end{aligned} \tag{13}$$

with

$$p(k) = \mathbb{E}[X(k) - (\bar{\xi} - \bar{d})]^+ = \frac{e^{-k\theta(\bar{\xi} - \bar{d})}}{k\theta} \cdot \binom{\tilde{k}}{k} \cdot \sum_{m=0}^{\tilde{k}-k} (-1)^m \cdot \frac{\binom{\tilde{k}-k}{m}}{e^{m\theta(\bar{\xi} - \bar{d})} \left(1 + \frac{m}{k}\right)^2}. \tag{14}$$

The delay penalty is represented by the term $c \cdot p(k)$, where $p(k)$ represents the expected delay. The fetch cost is given by $\bar{\xi} - \mu(k) + p(k)$, representing the *effective* driving time under thickening level k as the sum of the non-compressible part of the driving time (for $k = 1$) plus the additional driving time from ‘earlier’ matching (for $k > 1$).

Proposition 3 provides insights into the structure of the total costs of the thickening policy. We can interpret $\mu(k)$ as the average duration that orders remain on the platform before they are matched. Since $\mu(k)$ is decreasing in k , the smaller the k , the longer the order stays in the system to allow the market to thicken. Fetch costs are intuitively decreasing with waiting (i.e., the smaller the k), while the penalty cost is intuitively increasing with the matching delay (i.e., with smaller k), which is evident from the structure of the term for expected delay, $\mathbb{E}[X(k) - (\bar{\xi} - \bar{d})]^+$. As fetch costs and penalty costs are influenced by the thickening level in opposite directions, the optimal thickening level k^* is likely the one that balances both types of costs. We formally establish that the expected cost per order is quasi-convex in the thickening level k and sub-modular in k and c in the following proposition.

PROPOSITION 4. *For given $(\bar{d}, \tilde{k}, \theta)$, total costs per order in the simplified system, $\widetilde{W}^{\Omega^k, \check{d}(\cdot)}$, are*

- (a) *Quasi-convex in the thickening level k .*
- (b) *Sub-modular in parameters k and c , such that $k^*(c)$ is (non-strictly) increasing in c .*

Part (a) of Proposition 4 implies that the total costs initially decrease and then increase with the thickening level k , illustrating the fundamental trade-off of thickening in matching markets (see, e.g., Chen 2019). This result facilitates a straightforward search for the optimal k^* . The value derived from our simplified model can serve as a starting point in the search for the optimal policy parameters, a process we undertake in Section 5. Part (b) of Proposition 4 indicates that the parameters k and c are strategically complementary in the marginal cost (Topkis 1978). Consequently, the optimal thickening level $k^*(c)$ is monotone increasing in the penalty unit cost c . The Greedy policy (Subsection 4.1) incurs total costs of $\bar{\xi}$ under the assumptions of our simplified model, meaning that the performance difference between the Greedy policy and the KT policy can be conveniently expressed as $W^{\Omega^g} - \widetilde{W}^{\Omega^k, \check{d}(\cdot)} = \mu(k) - (1+c)p(k)$. We observe that, for some values of k , the Greedy policy may outperform the KT policy; however, with large k , the KT policy coincides with the Greedy policy. Hence, it is always possible to find an appropriate k with which the KT policy (weakly) dominates the Greedy policy.

5. Numerical Experiments

In this section, we use the data set from the OFD platform *Meituan* in China to test the performance of different policies. After a description of the data set, we analyze the performance of different policies under the assumption that the FPT is known. Then, we develop a prediction model for the FPT and apply the prediction model and the different policies to the case in which the FPT is unknown upon order arrival.

5.1. Data Description and Preliminary Analysis

Meituan is the largest OFD platform in China. We have access to one month of data in July 2017. This data set includes more than 1 million detailed orders, drivers, and weather information collected from eleven regions with an area of $\sim 60km^2$ per region. Order information contains, among others, order ID, order arrival time, region, food provider ID, food value, locations of the food provider and the customer, arrival time of driver at the shop, and delivery begin and end time. The orders cover 2 447 food providers and 153 096 customers. Driver information contains minute-wise numbers of waiting and busy drivers in the network.

In our data set, the platform experiences an average number of 3 200 orders per day per region. Demand, however, varies considerably across time periods and in its spatial density. The temporal analysis of arrival times shows that nearly 40% of orders are made for lunch time (10:00-14:00) and 30% for dinner time (18:00-20:00). To simplify our experiments, we define three scenarios based on the arrival time of the order: *no-rush*, *rush*, and *overall*. The *no-rush* scenario is comprised of all orders that arrive in the period of 7:00-10:00, the *rush* scenario comprises all orders that arrive during the period 10:00-14:00, and the *overall* scenario includes all orders during a day (7:00-23:00). The order arrival rate in the *overall*

scenario is $\lambda_o = 4.6$ per minute ($\lambda_o = 6.5$ per minute in the *rush* scenario and $\lambda_o = 1.1$ per minute in the *no-rush* scenario). The average FPT values of the three scenarios in the data set are 14'28 (14 minutes and 28 seconds) for the no-rush-scenario, 25'42 for the rush-scenario, and 23'13 for the overall scenario. The standard deviation in the overall scenario is 11'94 minutes. More than 70% of FPT values are less than or equal to 30 minutes. We provide a detailed description of the data set in Appendix 7.8.

The non-uniformity of the demand over the time of the day implies, however, that the demand for driver capacity also varies significantly during the day. Relying only on a fixed number of inhouse drivers would render it difficult to maintain a consistent service level during rush hours. Therefore, Meituan uses crowdsourced, on-demand drivers to satisfy demand variation over the course of the day. Inhouse drivers are paid a fixed salary (3000 yuan per month) plus compensation for each delivered order. Crowdsourced drivers, on the other hand, only receive compensation for each delivered order, but this compensation is higher than the variable compensation of inhouse drivers. For ease of exposition, we assume that the unit fetch cost paid to both types of drivers are standardized to one (~ 20 yuan/hour). See Appendix 7.4.2 for an extension in which drivers have type-specific compensation. We did not observe driver abandonment during the day in our data set, and therefore use a negligible abandonment rate ($\lambda_a \rightarrow 0$). However, the impact of λ_a is further tested and presented in Appendix 7.4.6.

In our simulation, to characterize the one-to-one delivery process of our model, we assume that a driver has an average speed of $v = 18km/h$ and becomes available instantly after she finishes a delivery. We also assume that the previous matching neither affects the driver location after delivery nor the restaurant location of new demands. This assumption is reasonable, because when drivers are on time, the driving time from restaurant to customer is fixed. It allows us to mimic the driver arrivals in the data set since drivers appear as new arrivals upon service completion.

To determine the unit penalty cost c , the platform's marketing department has to assess the deterioration in customer experience per time unit of delay and the resulting loss in future revenues. In our numerical experiments, we use a base value of $c = 6$, which is six times the cost of a driver, i.e., $20 \times 6 = 120$ yuan per hour of delay; a value similar to the approx. USD 10-20 per order used in prior research (see Mao et al. 2019).

For each analysis, we numerically determine the optimal policy parameters (k^* and $\check{d}^*(\cdot)$ for the KT policy, and \hat{P}^* with the other parameters for the BKT policy). We use a constant threshold value, \check{d} , instead of a threshold function, because estimating a function for each restaurant would be too complex. We use the simplified model (see Subsection 4.3) for an initial calibration of the search for the policy parameters. This leads to substantial run time savings in the implementation of our policy in practice. After the first calibration, we vary each parameter successively and compute the total costs with simulation, until a local optimum is found. (Details about the implementation of the simulation procedure can be obtained from the authors upon request.)

5.2. Performance Analysis with Known Food Processing Time

In this subsection, we assume that the exact FPT is known to the planner at the arrival time of the order. We perform a simulation study to compare the performance of different policies under this assumption. Note that the exact FPT of each order is included in the order history of our data set, so that we can feed it into our simulation.

Impact of driving-time-constraint $\check{d}(\cdot)$. We analyze how the driving-time constraint \check{d} impacts the KT policy. We simulate our policy with values $\check{d} \in \{100, 200, 300, 400, 600, \infty\}$ seconds (which leads to driving distances of $\{0.5, 1, 1.5, 2, 3, \infty\}$ km with the average driver speed), and numerically optimize thickness level k^* for each value of \check{d} . Note that the case of $\check{d} = \infty$ refers to the case without a limit on driving time. In Table 1, we find that the total costs $W^{\Omega^{k^*, \check{d}}}$ are at first strongly decreasing in \check{d} , then slightly increasing, with $\check{d}^* = 300$ being the optimal value ($W^{\Omega^{k^*, \check{d}^*}} = 104$). The larger the neighborhood, the more opportunities for the order to be matched with a ‘good’ driver, thereby incurring less overtime penalty and reducing total costs. The protection against the assignment of far-away drivers is partially achieved by the market thickening mechanism. However, an oversized neighborhood can also lead to a slight increase in the costs due to matching with faraway drivers. This is consistent with the insights of Li and Netessine (2020): Range \check{d} helps the planner reduce unnecessary search friction and avoid certain costly drivers.

Table 1 Results of the KT policy $\Omega^{k^*, \check{d}}$ as a function of driving-time range \check{d} (for the overall scenario).

\check{d} (sec)	100	200	300	400	600	∞
Opt. level k^*	4	4	5	5	5	5
Total Costs $W^{\Omega^{k^*, \check{d}}}$	17827	520	104	108	110	110
Service Level $V_l^\Omega(\%)$	32.96	91.57	98.64	99.32	99.72	99.72
Drive-to-Shop time V_p^Ω (sec)	68	98	100	105	108	108

Accuracy of the simplified model. Next, we analyze the accuracy of the simplified model (Subsection 4.3). Given \check{d} as the driving-time limit, we set $\bar{\xi}$ to the average FPT and \bar{d} to the non-compressible driving time computed from the KT policy with $k = 1$. Value \tilde{k} is determined numerically with simulation. To determine value θ , we utilize the equation $\theta = \ln \tilde{k} / (\bar{\xi} - \bar{d})$ to reflect the relationship $(\bar{\xi} - \bar{d}) \approx \sum_{j=1}^{\tilde{k}} \frac{1}{\mu_j}$, and then we further fine-tuned numerically parameter θ for better accuracy. Figure 6 shows the total costs obtained from the simulation of the KT policy and from the simplified model for all three scenarios (no-rush, rush, overall) and as a function of thickening level k . The average approximation gap in Figure 6, i.e., the mean absolute percentage error $\epsilon = |\widetilde{W}^{\Omega^{k, \check{d}}} - W^{\Omega^{k, \check{d}}}| / W^{\Omega^{k, \check{d}}}$, over all three scenarios is 5.17%. We conclude that our simplified model approximates the actual performance of the KT policy fairly accurately. Given Figure 6, we also conclude that the approximation model exhibits the same structural behavior as the KT policy, such as the quasi-convexity in k (see Proposition 4), and that it has similar optimal values k^* , indicating that the simplified model can be used to optimize parameter k .

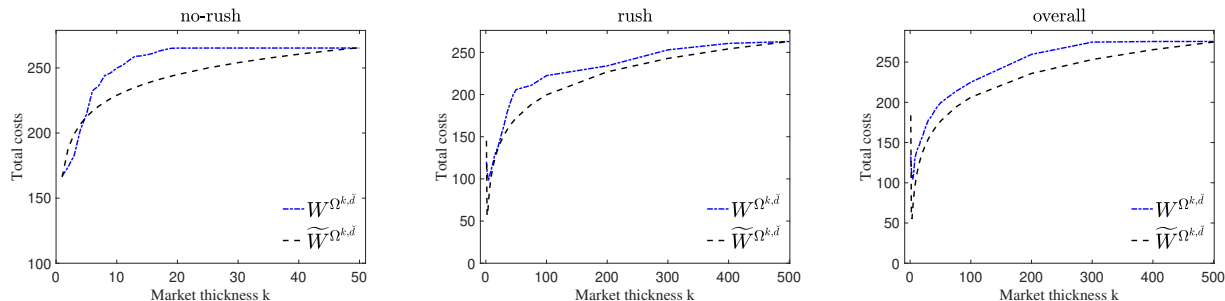


Figure 6 Total costs of the KT policy and the simplified model for the three scenarios.

Table 2 Comparison of total costs of different matching policies.

	Greedy	Batching	MAR	KT	Lower bound
Scenarios	W^{Ω^g}	W^{Ω^b}	W^{Ω^r}	$W^{\Omega^{k^*,\check{d}^*}}$	W^{Ω^f}
<i>no-rush</i>	872	626	969	187	135
<i>rush</i>	1542	1198	813	98	66
<i>overall</i>	1346	1017	833	104	69

Performance comparison of different policies and lower bound. Table 2 shows the total costs obtained from the simulation of the different policies that we analyze. For the KT policy, we numerically optimized the thickness level k and the driving time constraint \check{d} , and obtained $k^* = 4$ and $\check{d}^* = 300$ for the overall scenario. In comparison to the benchmark policies, in the *overall* scenario, the KT policy has 92% lower total costs than the Greedy policy, 90% lower total costs than the Batching policy, and 88% lower costs than the MAR policy. The results indicate that the KT policy significantly outperforms the benchmark policies, which confirms the value of market thickening. In addition, we find that the KT policy has a performance close to the lower bound, W^{Ω^f} of the full-information case, indicating the effectiveness of the policy.

Table 3 Performance indicators of different matching policies in the overall scenario.

Policies	Drive-to-Shop	Delay	Service Level	Engagement Time	Engaged Drivers	Waiting Time	Idle Time	Net-income
	V_p^Ω	V_d^Ω	V_l^Ω (%)	V_e^Ω	V_n^Ω	V_w^Ω	V_q^Ω	V_s^Ω
Ω^g	127	2	99.18	2135	164	1209	1675	11.1
Ω^b	274	3	90.39	1708	131	634	3668	6.3
Ω^r	119	119	0	919	70	0	2068	6.0
Ω^{k^*,\check{d}^*}	100	1	98.64	900	69	0	1596	7.1
Ω^f	69	0	100	869	67	0	1620	6.9

All time values in seconds.

Table 3 shows performance indicators other than the total costs of the policies under investigation. For ease of exposition, we only provide the performance indicators for the *overall* scenario. The average drive-to-shop time V_p^Ω of the KT policy is comparatively shorter than those of the other policies such as Greedy and Batching, indicating the benefits of market thickening. In addition, we observe that the engagement time V_e^Ω per order of a driver is shorter under the KT policy than that under other policies, in particular compared to the Greedy and the Batching policies. As a result, the platform can considerably

increase the delivery capacity (V_n^Ω under the KT policy is approx. half than those of the Greedy and Batching policies). Furthermore, from V_w^Ω , we observe that the KT policy eliminates shop-waiting time, indicating the significance of finding an appropriate matching time to reduce waiting. The MAR policy also eliminates shop-waiting, but creates serious delay that has significant impact on the service level. The social welfare of drivers, represented by net income V_s^Ω and expected idle time V_q^Ω in Table 3 is highest under the Greedy policy, as the policy compensates drivers even if they are waiting. However, it achieves this outcome with high costs for the platform for waiting at restaurants. The KT policy, on the contrary, achieves comparably high net-income for drivers at competitive platform costs, leading to higher overall efficiency of the ecosystem than the other policies. The discussion shows that a matching policy has to trade-off the triangular objectives of three parties: platform (low cost), drivers (high income), and customers (high service level). The company has to define its balance to these parties. In Appendix 7.4.5, we propose a socially friendly KT policy that pays and releases drivers after a cut-off limit on idle time.

5.3. Performance Analysis with Unknown FPT

In Subsection 5.2, we considered the case where the platform knows the exact FPT at the time of assigning a driver to an order. This is, however, rarely the case in practice. In this subsection, we analyze how FPT uncertainty affects the different policies. First, we present a machine learning model to predict FPT values in real time. Then, we analyze the policy performance. We focus on the *overall* case in this section.

5.3.1. Prediction model for food processing times In recent years, the prediction of delivery time for online food delivery has received much attention (e.g., Gao et al. 2021, Liu et al. 2021b) but we are unaware of any work to predict FPT. For our problem, however, an accurate prediction of the FPT facilitates the matching between order and drivers, reducing ‘shop-waiting’ of drivers and delivery delays.

We propose a *causal* forecasting model, i.e., a model that exploits the characteristics of an order to predict its FPT. We compare several machine learning models to calibrate the prediction. To feed the FPT prediction procedure, we take the $\sim 120,000$ orders of a complete month in the target region in the *overall* scenario. We use 80% of the order data in-sample to train the model, and the remaining 20% out-of-sample to test the performance of the prediction model.

First of all, we have to identify the characteristics that influence the FPT most. A technical report (Hao 2017) from scientists at Meituan indicate the following factors as key determinants for the FPT: Order arrival time (weekday, hour, minute of a day), food value, packaged box value and number of dishes (food number), order congestion situation of the food provider (number of waiting orders), food provider ID, and weather condition (rainy vs. sunny). To align the scale of features, we normalize the continuous features and convert the categorical features by the *one-hot encoding* technique (Brownlee 2018). Then, we test a wide range of machine learning methods including *eXtreme Gradient Boosting (XGBoost)*,

k-Nearest Neighborhood (*k*NN), Lasso, Deep Neural Network (DNN), Random Forest (*RndForest*) and Ridge Regression (*Ridge*), see e.g., Hastie et al. (2009). We feed each method with the training data and conduct 10-fold cross-validation to select the best hyper-parameters for the models, for example, the learning rate, the shrinkage parameters and the number of trees. We implemented the training and testing procedures with *scikit-learn* package in Python.

Table 4 Average predicted FPT and FPT prediction errors of different machine learning methods (in minutes)

Method		XGBoost	kNN	Lasso	DNN	RndForest	Ridge
In-Sample	$\bar{\xi}$	19.57	19.51	19.55	19.50	19.55	19.55
	RMSE	7.25	8.19	8.88	9.12	7.73	7.89
	MAE	5.52	6.17	6.85	6.98	5.95	5.99
Out-of-Sample	$\bar{\xi}$	21.52	21.67	20.17	23.55	20.51	21.47
	RMSE	8.62	8.98	11.08	11.76	9.65	10.08
	MAE	6.58	6.84	8.41	9.35	7.35	7.76

The out-of-sample results of different measurements and methods are summarized in Table 4. We report the average FPT $\bar{\xi}$, the square root of the mean squared error (RMSE) and the mean absolute error (MAE) for each method. The results indicate that XGBoost produces the smallest out-of-sample RMSE, 8.62, and hence, we adopt XGBoost for our numerical experiments. The four main factors that affect the predicted FPT by XGBoost are order arrival time (F-score 6,281; where the F-score is the number of times that a feature is used to split the data across the prediction process, indicating its relative importance), food value (F-score 5,322), the congestion situation/waiting number (F-score 1,854), and the packaged box value (F-score 959). Our predictors explain 49% of the total FPT variance ($R^2 = 0.49$). Note that several of the above factors (e.g., arrival time, food value, and order waiting number) may be proxies for the difficult-to-observe congestion level at the restaurant.

5.3.2. Performance analysis with predicted FPT. First, we analyze the additional costs from uncertainty in the FPT. Figure 7 shows the differences in total costs between the KT policy applied to the case of perfect FPT (see Subsection 5.2) and the BKT policy applied to the case of unknown FPT, under different protection levels \hat{P} . Thickness level k^* and \check{d}^* are numerically optimized for each protection level. From Figure 7, we observe that the total costs in the case of unknown FPT decrease and then increase with the protection level (with $\hat{P}^* = 0.4$), indicating that it can even be advantageous to include a negative buffer (i.e., $\hat{P} < 0.5$). The results also indicate that ignoring the uncertainty leads to a considerable cost increase, sixfold for $\hat{P} = 0.5$.

Table 5 shows total costs and other indicators for the BKT, Greedy, MAR, and Batching policies as well as for the real decisions that we observed in the data set from Meituan. The Meituan data set does not contain information about the delay in fetching. Because we know that the company uses a policy

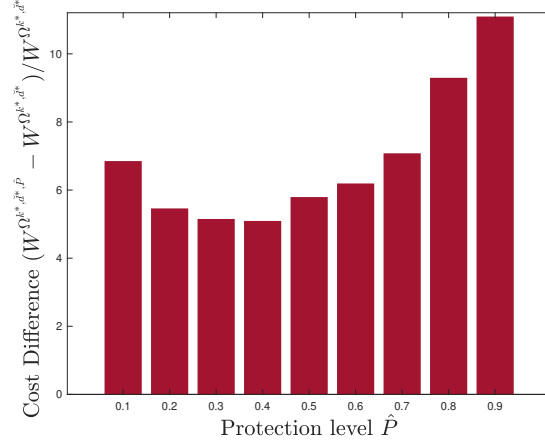


Figure 7 Percentage difference in total costs between the KT policy with known FPT ($\Omega^{k^*, \check{d}^*}$) and the BKT with unknown FPT ($\Omega^{k^*, \check{d}^*, \hat{P}}$) for different protection levels \hat{P} .

Table 5 Performance comparison for different policies and real-case (with unknown FPT for overall scenario).

Policies	Total Costs	Drive-to-Shop	Delay	Service Level	Engagement Time	Engaged Drivers	Waiting Time	Idle Time	Net-income
	W^Ω	V_p^Ω	V_d^Ω	V_l^Ω (%)	V_e^Ω	V_n^Ω	V_w^Ω	V_q^Ω	V_s^Ω
Ω^g	1346	127	2	99.18	2135	164	1209	1675	11.1
Ω^b	1180	287	41	81.19	1731	133	645	3730	6.3
Ω^r	833	119	119	0	919	70	0	2068	6.0
$\Omega^{k^*, \check{d}^*, \hat{P}^*}$	634	110	39	64.42	1201	92	291	1522	8.7
real-case	1383	472	0	100	2183	167	693	–	19.8

All time values in seconds.

similar to the Greedy policy, we assume that the delay is close to zero. The BKT policy with numerically optimized parameters $k^* = 5, \check{d}^* = 300, \hat{P}^* = 0.4$ reduces total costs by 54% compared to the real decisions (and to the Greedy policy), by 24% compared to the MAR policy, and by 46% compared to the Batching policy. The BKT policy also dominates other policies for the drive-to-shop time V_p^Ω . The impact of the policies on driver net income is similar as the one we have observed for known FPT.

In Table 5, we have numerically searched for the optimal parameters $k^*, \check{d}^*, \hat{P}^*$ for the BKT policy. To provide some intuition on how total costs are affected by the parameters, Figure 8 plots total costs by thickness level k and protection level \hat{P} for given $\check{d}^* = 300$ seconds. We do not only observe that total costs are quasi-convex in the thickening level k , confirming the result of Proposition 4, we also observe that total costs are quasi-convex in protection level \hat{P} for a given thickening level k .

Impact of prediction error σ_ξ . Next, we analyze the sensitivity of the BKT policy to the prediction error σ_ξ . In the simulation, we replace the FPT prediction with a weighted combination between the FPT prediction with XGBoost and the real FPT value from the data set (based on the *overall* scenario). Changing the weight of the exact FPT value in the weighted combination allows us to construct FPT predictions with varying degrees of σ_ξ . The corresponding results are shown in Figure 9. With given $\check{d}^* = 300$, the results indicate that the BKT policy with numerically optimized parameters k^* and \hat{P}^* dominates the Batching policy for all values of prediction error. Furthermore, the performance advantage

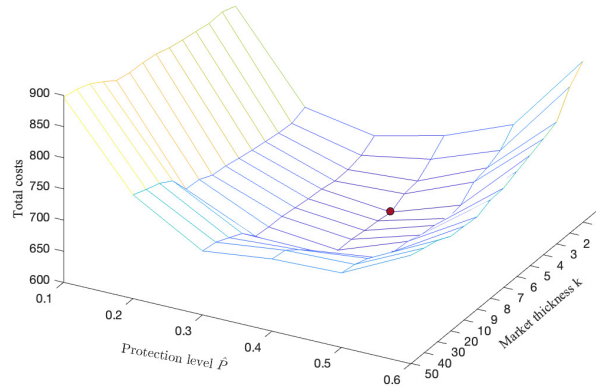


Figure 8 Total costs under BKT policy as a function of thickening and protection levels for given $\check{d}^* = 300$.

of the BKT policy over the Batching policy increases significantly when the prediction error decreases (i.e., when the weight of the exact FPT in the prediction increases). This observation demonstrates the importance of accurate FPT prediction. In practice, OFD platforms can invest in better FPT prediction, for example, by collecting more information about food orders, such as food details and the number of employees currently on shift, or by providing incentives to restaurants to reduce the variability of the FPT. The food platform may provide revenue sharing or rewards to restaurants for limiting the FPT to a predefined range. Similar measures are used by online ride-hailing platforms to reduce variability of driving times. For example, drivers at platform Uber have to follow GPS-calculated routes to avoid low ratings and even a penalty (Liu et al. 2021a).

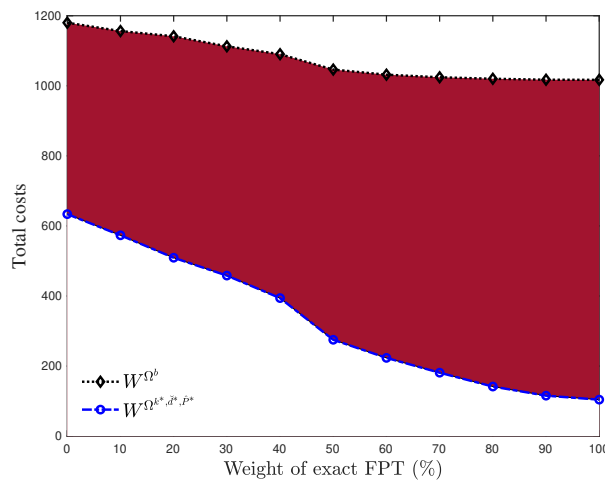


Figure 9 Total costs of the BKT policy and the Batching policy as a function of the precision of FPT prediction for given $\check{d}^* = 300$.

In Appendix 7.4, we analyze the relaxation of certain model assumptions, including scenarios with endogenous FPT, driver-type-specific compensation schemes, variable driver speed, regional clustering, driver abandonment, and a socially friendly adaptation of the BKT policy.

6. Conclusion

In this paper, we have explored market thickening and dynamic order-driver matching on OFD platforms, drawing motivation from the operations of the Meituan platform in China. We have expanded upon existing matching solutions by incorporating uncertain food processing times into the matching decisions. We have formally defined a dynamic matching model as an SMDP and derived bounds on the performance based on different information schemes (known FPT vs. unknown FPT). In the case of a single order in isolation, we demonstrated that the optimal policy follows a threshold structure. Based on this result, for the general matching system, we have formulated a driving-time-constrained k -level thickening (KT) policy for real-time order and driver matching, and extended this policy to the Buffered k -level thickening (BKT) policy, which includes a risk buffer to safeguard against the prediction error of the FPT. We have developed a closed-form approximation of the KT policy's performance and derived several analytical properties of the approximation, including the quasi-convexity of the total costs with respect to the thickening level. Through extensive numerical experiments using a real data set from Meituan, we observed that the BKT policy reduces the drive-to-shop time by 77% and shop-waiting time by 58% compared to the real-case benchmark, effectively doubling driver efficiency. Similarly, under the BKT policy, the number of required drivers decreased from 167 to 92. In conclusion, the BKT policy achieved a total cost reduction of 54% compared to the benchmark.

Our research yields several noteworthy insights. First, we ascertain that delaying the matching of orders during food processing to cultivate a thicker marketplace is advantageous. Strategies such as greedy matching, matching after the food is ready, and short-interval batching are significantly outperformed by the KT policy, which dynamically adapts the market thickness. Importantly, we note that the FPT is pivotal information for making matching decisions. Despite utilizing an advanced prediction model, a degree of prediction error persists. This error contributes to a substantial increase in costs when the policy is not adapted to accommodate the uncertainty of the FPT, even though, the cost increase can be significantly mitigated through the BKT policy. This observation suggests that OFD platforms benefit from investing in enhancing FPT prediction accuracy, possibly by gathering more information from restaurants or by offering incentives to restaurants to reduce FPT variability.

The solutions and insights derived from our analysis are not confined to OFD platforms but extend to any setting where dispatching and driving run concurrently with processing. This is observable, for instance, in one-to-one delivery platforms with time-sensitive targets, such as *Shansong* (ishansong.com)

and *Lalamove* (huolala.cn). These platforms necessitate the processing of goods (e.g., packaging or loading) before they are ready for shipping. Container-trucking platforms like *Duckbill* (en.duckbillscm.com) present another analogous scenario. These platforms coordinate the delivery of containers to ports for international transportation, where the uncertain container loading times at the factories resemble the FPT in our study. The platform must judiciously assign containers to available trucks to ensure that neither the truck waits unnecessarily at the factory nor the container misses its scheduled customs clearance time at the port.

Furthermore, our research opens several avenues for future exploration. In this paper, we have solely focused on one-to-one matching. Future research may consider the in-vehicle consolidation of multiple orders for drivers. This practice allows drivers to collect orders for multiple customers in a single pickup activity, implying that a matched driver might remain on the platform post-assignment. Another promising avenue is real-time forecasting of driver speed, potentially employing machine learning techniques as suggested by Tao et al. (2022), and incorporating this information dynamically into the matching policy. Lastly, exploring the coordination between the restaurant's operational decisions and the platform's matching/routing decisions also offers a compelling direction to further enhance efficiency.

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