Social Networks and Occupational Choice: The Endogenous Formation of Attitudes and Beliefs about Tax Compliance*

Nigar Hashimzade
University of Reading

Gareth D. Myles†
University of Exeter and Institute for Fiscal Studies

Frank Page
Indiana University

Matthew D. Rablen
Brunel University

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Abstract

The paper analyses the emergence of group-specific attitudes and beliefs about tax compliance when individuals interact in a social network. It develops a model in which taxpayers possess a range of individual characteristics – including attitude to risk, potential for success in self-employment, and the weight attached to the social custom for honesty – and make an occupational choice based on these characteristics. Occupations differ in the possibility for evading tax. The social network determines which taxpayers are linked, and information about auditing and compliance is transmitted at meetings between linked taxpayers. Using agent-based simulations, the analysis demonstrates how attitudes and beliefs endogenously emerge that differ across sub-groups of the population. Compliance behaviour is different across occupational groups, and this is reinforced by the development of group-specific attitudes and beliefs. Taxpayers self-select into occupations according to the degree of risk aversion, the subjective probability of audit is sustained above the objective probability, and the weight attached to the social custom differs across occupations. These factors combine to lead to compliance levels that differ across occupations.
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†__Correspondence__ Gareth D. Myles, Department of Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, United Kingdom. email: g.d.myles@ex.ac.uk.

1 Introduction

Tax evasion is the illegal concealment of a taxable activity. Measuring how much economic activity is concealed will always be difficult since those who engage in evasion have every motivation to hide their activities. Even so, the estimates that are available from official sources (such as H. M. Revenue and Customs, 2010) and from academic researchers (Schneider and Enste, 2000) are in agreement that evasion is an economically significant activity. This emphasizes the importance of understanding the decision process of a taxpayer when choosing whether to comply with tax law or to engage in evasion. A good theory of the compliance decision is essential for designing an audit policy that deters evasion.

The initial analysis of the compliance decision by Allingham and Sandmo (1972) models the taxpayer as facing a decision under risk, with the extent of evasion chosen to maximise expected utility. The model provides precise comparative statics predictions, but these are not always in accord with data (Clotfelter, 1983; Crane & Nourzad, 1986) or intuition. In particular, when evaluated using levels of the audit probability and the fine rate close to those observed in practice, the model predicts that all taxpayers should engage in evasion. This has motivated numerous extensions of the standard model – surveyed by Pyle (1991) and Sandmo (2005) – but these extensions do not address its fundamental limitations.

Two sets of issues have to be addressed in constructing an improved

model. First, behavioural economics has demonstrated that individuals generally do not evaluate risky prospects using the objective probabilities of events. In practice, decisions are made using subjective probabilities that can differ significantly from the objective probabilities (Kahneman & Tversky, 1979). Contributions drawing on these ideas are made by al-Nowaihi and Dhami (2007), Arcand and Graziosi (2005), and Snow and Warren (2005). Second, there is now compelling empirical (Spicer & Lundstedt, 1976) and experimental (Baldry, 1986) evidence that the tax evasion decision is not simply an individualistic gamble. Instead, a wide range of social and psychological factors enter the compliance decision. Gordon (1989) analyses a psychic cost of evasion, Myles and Naylor (1996) introduce a social custom into compliance, and the concept of tax morale (Torgler, 2002) subsumes a range of social and equity factors. For reasonable parameter values these recent models can predict the levels of evasion that are consistent with empirical data.

An issue that has not been given much attention is the processes through which attitudes towards compliance are formed. Attitudes and beliefs are not exogenous but must result from interaction with other taxpayers and with the tax authority. The appropriate method for modelling such interaction is a social network that allows endogenous evolution of attitudes and beliefs. Placing appropriate structure on the social network can also permit investigation of how the degree of separation determines the divergence of attitudes and beliefs that can emerge among distinctive social groups in a

heterogeneous society.

Attitudes and beliefs are not only important for how a taxpayer will act when confronted with an evasion opportunity: they are equally important in determining whether a taxpayer has such an opportunity in the first place. This is because opportunities for evasion are very limited in most forms of employment. The deduction of income tax at source and third-party reporting make evasion of employment income very difficult, if not impossible. In contrast, income earned in self-employment (or entrepreneurship) is not taxed at source and can have limited third-party reporting. This makes it possible to undertake evasion when in self-employment. When an individual makes a choice of occupation the possibility of evasion in self-employment must be taken into account, which makes occupational choice partly dependent on the perceived benefit of evasion.

This paper seeks to combine these ideas and to explore the endogenous emergence of group-specific attitudes and beliefs. We embed a behavioural model of the evasion decision into a social network, through which information on the activities of the revenue service is transmitted. Individual taxpayers are heterogenous in several dimensions, such as skill in employment, attitude to risk, and success in self-employment. We also include a choice between occupations that differ in the riskiness of reward and the opportunity to engage in evasion. The model is analysed by simulation, which permits us to trace the tax evasion dynamics that emerge from repeated taxpayer interaction within the network. The endogenous separation of tax-

payers into occupations with differing evasion opportunities creates different behavioural types who comply to different degrees and can develop differing attitudes and beliefs.

The central result that emerges from the analysis is that risk aversion, occupational choice, compliance, and attitude to evasion are inter-related and mutually reinforcing. The true probability of audit is unknown, so taxpayers learn a subjective probability through information transmission via social interaction. The simulations show, first, that interaction can sustain a subjective probability of audit that exceeds the objective probability. Thus, the model provides an illustration of how beliefs can be formed in subjective expected utility theory. Second, they demonstrate how taxpayers self-select into occupations in a way that maximises the amount of tax evasion. Self-employment is risky, so it is chosen by those with lower degrees of risk aversion. But it also opens the opportunity to evade, and those with a low risk aversion will evade the most when given the opportunity. Hence, we see a process of self-selection of those who will evade the most into a situation where they can evade. The self-selection is reflected in the levels of compliance of the different occupational groups: compliance in the most risky occupation is lowest. The endogenous choice of occupation results in different groups having markedly different rates of compliance. The attitude toward compliance also differs across the occupational groups. The information exchange between people in the same occupation reinforces groups beliefs, and sustains different social customs across groups. Taxpayers in the riskier oc-

cupation comply less, and this is mutually reinforcing with a lower value of the social custom. In this sense the network can permit group-specific social attitudes to develop.

The paper is structured as follows. Section 2 describes the separate components that are built into the model. Section 3 provides analytical details on how these components are implemented. Section 4 describes the simulation results. Section 5 concludes.

2 Modelling

This section describes the separate elements that constitute the model. The intention is to place them into the context of the literature.

2.1 Subjective beliefs

The analysis of the evasion decision by Allingham and Sandmo (1972) applied expected utility theory. The standard interpretation is that the expectation is taken using the objective probability of an audit. One criticism of the model is that it over-predicts the extent of evasion when evaluated using the objective probability. This has motivated the application of different forms of non-expected utility theory to the evasion decision. These are surveyed in Hashimzade, Myles, and Tran-Nam (2012).

The situation is referred to as one of risk when the decision maker knows the probabilities of events. However, these probabilities can be distorted into "decisions weights" to form the expected payoff. Rank dependent ex-

pected utility (Quiggin, 1981, 1982; Quiggin & Wakker, 1994) uses a particular weighting scheme to transform the objective probability of events into subjective probabilities and has been applied to the evasion decision by Arcand and Graziosi (2005), Bernasconi (1998) and Eide (2001). Prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) also uses a weighting scheme but payoffs are determined by gains and losses relative to a reference point. Applications to compliance include al-Nowaihi and Dhami (2007), Bernasconi and Zanardi (2004), Rablen (2010), and Yaniv (1999).

Uncertainty occurs when the decision maker does not know the probabilities. It has been modelled by assuming the decision maker forms a probability distribution over possible probabilities of outcomes. This gives rise to the concept of "ambiguity" which has been surveyed in Camerer and Weber (1992) and applied by Snow and Warren (2005).

Non-expected utility model can predict the correct level of evasion for reasonable parameter values. This is because they permit the subjective probability of audit (the weighting on the payoff when audited) to be greater than the objective probability. They also open the possibility of designing compliance policy to manipulate the subjective nature of the decision (Elffers & Hessing, 1997). We incorporate these ideas into the analysis by assuming the probability of audit is subjective, and by providing an explicit process through which the subjective belief is formed. The model therefore provides an endogenous explanation of subjective probabilities that are systematically different from the objective probabilities.

2.2 Social customs

The experiments of Baldry (1986) suggest that the evasion decision is systematically different from a straightforward gamble. One explanation for this finding is that choosing to evade results in costs being incurred. For instance, Bayer (2006) assumes that concealing income has a direct financial cost. A related interpretation is provided by Lee (2001), who assumes a taxpayer can reduce the assessed income after audit by paying an additional cost (such as a reduction in return from using concealed investments or the cost of professional advice for securing income in non-taxable forms).

A different approach is to view these costs as psychic, rather than financial. Such psychic costs might arise through the fear of detection, or the shame of being exposed. Their magnitude may, therefore, reflect an individual's underlying attitudes towards compliance – an important feature of psychological theories of tax evasion (e.g., Kirchler, Hoelzl, & Wahl, 2008; Weigel, Hessing, & Elffers, 1987). Gordon (1989) formally incorporates psychic costs into the evasion decision. He interprets such psychic costs as arising from the social setting in which the taxpayer operates, so are a result of the loss of social prestige or reputation. An alternative to the psychic cost is the "conscience parameter" of Eisenhauer (2006, 2008). In this formulation of the compliance decision an individual recognises that evading tax results in free-riding on the taxes paid by compliant taxpayers. This generates a sense of guilt for the tax evader. The guilt is represented by discounting the untaxed income by the moral equivalent of a tax rate.

An alternative interpretation of the psychic cost is that it represents the loss of the payoff from a social norm to honest tax payment. Under this interpretation it becomes natural to assume that the additional cost is generated by explicit social interaction, with the cost an increasing function of the proportion of taxpayers who do not evade. This formulation captures the idea that more social prestige will be lost the more out of step the taxpayer is with the remainder of society. This approach has been developed by Davis, Hecht, and Perkins (2003), Fortin, Lacroix, and Villeval (2007), Kim (2003), Myles and Naylor (1996), and Traxler (2010) to show that reputation effects can lead to multiple equilibria and epidemics of evasion.

The existence of additional costs seems essential to explain some features of the tax evasion decision. To capture underlying attitudes towards compliance, we choose to include in the model a social custom of honest tax payment, so there is a utility gain from following the social custom. The social custom is influenced by interaction in the social network, which emphasizes the importance of the links between individuals.

2.3 Occupational choice

There are two reasons why occupational choice is important in the context of evasion. Firstly, there are differences in the possibility of concealing income in different occupations. This is partly due to the operation of the tax system. For example, the UK employs the Pay-As-You-Earn system in which income tax is deducted directly from the salaries of employees. This

prevents any opportunity for evasion (except in collaboration with the employer). Also, the nature of the occupation can explain whether there is a tradition of payment in cash, and different occupations can support different social customs (Ashby, Webley, & Haslam, 2009). It is therefore important that the choice of occupation be built into the model. Occupational choice is not an issue that has featured prominently in the literature on tax evasion, although Cowell (1981), Isachsen and Strøm (1980) and Trandel and Snow (1999) consider the choice between work in the regular and informal economy.

The choice of occupation also has another aspect that is of interest from a theoretical perspective. It is standard to assume that occupations differ in the combination of risk and reward that they offer. For instance, employment can be taken as the least risky occupation with all forms of self-employment having greater risk. Individuals allocate to occupations on the basis of their ability at that occupation and their attitude to risk. This is the basis of the analyses of Kanbur (1981) and Black and de Meza (1997), which address whether aggregate risk-taking is socially efficient. In particular, they are concerned with whether an inefficiently low proportion of individuals enter risky occupations and, if so, whether tax policy can be used to raise welfare.

Evasion has been incorporated into this model of occupational choice by Pestieau and Possen (1981). The possibility of evasion in the risky occupation has an interesting implication: if there is too little risk-taking without tax evasion then the possibility of evading encourages risk-taking. In this case,

setting policy to reduce evasion will drive risk-taking further from the social optimum. At the same time, a more relaxed tax enforcement would serve as an indirect subsidy and may, therefore, improve welfare. The converse of this argument is that taxation has a variance-reducing effect on earnings from self-employment (government engages in risk-sharing), and, therefore, encourages self-employment. Evasion has the opposite effect and raises the variance again. So, from this argument, policy should try to reduce evasion.

2.4 Social network

Tax evasion is an illegal act which has to be concealed from public view. Similarly, revenue services do not normally reveal their audit strategies. Together, these imply that taxpayers are not fully informed and can gain from obtaining additional information. It seems natural to assume that information will not be publicly traded, but will be passed between taxpayers who are in a position of mutual trust. It is this situation that is modelled by the social network.

The importance of social contacts is supported by empirical evidence. There is a positive connection between the number of tax evaders known to a taxpayer and the level of that taxpayer's own evasion (De Juan, Lasheras, & Mayo, 1994; Geeroms & Wilmots, 1985; Spicer & Lundstedt, 1976; Wallschutzky, 1984; Webley, Robben, & Morris, 1988). This suggests that the compliance decision is not made in isolation by each taxpayer but is made with reference to the norms and observed behaviour of the general society of the taxpayer.

This social interaction is captured through the application of network theory, as described in Goyal (2009) and Jackson (2004). In particular, we wish to apply recent advances in the endogenous formation of networks (Page & Wooders, 2009) to track change in the network over time, especially changes produced by switches in occupational choice. There is some existing work using networks in evasion analysis. Korobow, Johnson, and Axtell (2007) considers agent-based simulations in a simple network. Franklin (2009) looks at more complex networks but with a simpler model of the compliance decision. Networks have also been applied to the analysis of crime more generally (e.g., Glaeser, Sacerdote, & Scheinkman, 1996).

The social network plays two roles. First, it transmits the social custom from one person to another. If two non-evaders meet then the importance of social custom of honest payment is increased for both, but if a non-evader meets an evader then it is reduced for the non-evader and increased for the evader. Second, the audit policy of the revenue service is not public information. Individuals infer its policy partly from their own experience and partly by receiving information about the experiences of others. The simulation approach we employ can be seen as an application of agent-based modelling (Bloomquist, 2004; Tesfatsion, 2006) with agent interaction controlled by network structure.

3 A Model

In this section we model the formation of attitudes and beliefs as the outcome of social interaction, and opportunities as the outcome of occupational choice. This is achieved by applying the theory of network formation to track the links between taxpayers and the transmission of attitudes and beliefs, and combining this with agent-based modelling that incorporates a behavioural approach to describe individual choices.

There are n individuals interacting repeatedly; time is discrete. In every period individuals choose a preferred occupation and an optimal level of evasion. Each individual is characterised by a vector of parameters $\{w, \rho, q^1, q^2, z; p, \chi\}$, where each parameter is independently distributed across taxpayers. The first five parameters are assigned to the taxpayer at the outset of the analysis and remain constant. These parameters are:

$w \equiv$ wage in employment;

$\rho \equiv$ coefficient of relative risk aversion;

$q^i \equiv$ probability of success in self-employed occupation i, $i = 1, 2$;

$z \equiv$ payoff from the social custom.

The remaining two parameters are updated through interaction in the social network. They are:

$p \equiv$ perceived (subjective) probability of audit;

$\chi \equiv$ weight attached to payoff from following the social custom.

We now describe how these variables enter into the choice problem of a

taxpayer and how the subjective probability and weight attached to social custom are updated.

At the start of every period an individual has a choice between employment or entering one of two self-employment occupations. If employment is chosen the wage, w, is obtained with certainty. The self-employment opportunities are represented as risky "projects". Project i ($i = 1, 2$) has a probability of success, q^i, a pay-off of π_s^i, if successful, and a pay-off π_u^i if unsuccessful, where $\pi_s^i > \pi_u^i > 0$. Note that the probability of success is specific to an individual, whereas the payoffs are specific to the projects and, therefore, are the same for all individuals undertaking a given project. That is, the mean and variance of the pay-off from project i, conditional on the realisation of q^i for individual h, are given by

$$\begin{aligned} E\left[\pi^i \middle| q_h^i\right] &= q_h^i \pi_s^i + \left(1 - q_h^i\right) \pi_u^i, \\ Var\left(\pi^i \middle| q_h^i\right) &= q_h^i \left(1 - q_h^i\right) \left(\pi_s^i - \pi_u^i\right)^2. \end{aligned}$$

The unconditional mean pay-off of project i is the expected value taken at the societal level:

$$E\left[\pi^i\right] = E\left[E\left[\pi^i \middle| q_h^i\right]\right] = E\left[q_h^i \pi_s^i + \left(1 - q_h^i\right) \pi_u^i\right] = \left(\pi_s^i - \pi_u^i\right) E\left[q^i\right] + \pi_u^i.$$

For example, if q^i is uniformly distributed between 0 and 1 across taxpayers, then $E\left[\pi^i\right] = \left(\pi_s^i + \pi_u^i\right)/2$. The unconditional variance (at the societal level) of project i is

$$\begin{aligned} Var\left(\pi^i\right) &= E\left[Var\left(\pi^i \middle| q_h^i\right)\right] + Var\left(E\left[\pi^i \middle| q_h^i\right]\right) \\ &= \left(\pi_s^i - \pi_u^i\right)^2 E\left[q^i\right] E\left[1 - q^i\right] + 2\pi_s^i \pi_u^i Var\left(q^i\right) \end{aligned}$$

Project 1 is termed riskier than project 2 in the sense that $Var(\pi^1) > Var(\pi^2)$ and $E[\pi^1] \geq E[\pi^2]$. A strict inequality appears to be a more interesting situation than a mean-preserving spread, because of the trade-off between the mean and the variance at the societal level. This can be obtained by imposing various conditions on the distribution of π^i and q^i.[1] In particular, assuming $\pi_u^1 < \pi_u^2$, $\pi_s^1 > \pi_s^2$, $\pi_s^1 \pi_u^1 = \pi_s^2 \pi_u^2$, and identical distributions for q^i, results in $Var(\pi^1) > Var(\pi^2)$ and $E[\pi^1] > E[\pi^2]$. Such a parameterisation is employed further in the numerical simulations of the model. For individual agents, however, the expected pay-offs and the variances differ, according to their individual probabilities of success.

Both projects are riskier than employment, in the sense that for each agent the wage in employment is certain. In the simulations we also assume that, at the societal level, the mean gain from self-employment is larger than the mean gain from employment, $E[\pi^1] > E[\pi^2] > E[w]$. This assumption appears realistic, and it adds to the trade-off between the mean and the variance in the pay-offs across occupations at the societal level.

It is not possible to evade tax in employment: the possibility arises only when self-employment is chosen. In this case, the taxpayer has a belief, p, over the probability of audit. The value of the perceived probability of detection is updated through the experience of the taxpayer with audits and through exchange of information when meeting other taxpayers. The choice of occupation and the choice of evasion level involve risk. Each taxpayer has

[1] We are grateful to the anonymous referees for helping to clarify this point.

a (constant) degree of relative risk aversion measured by the risk aversion parameter, ρ. Taxpayers behave as if they maximise subjective expected utility and, for analytical tractability, we assume throughout a CRRA form for utility:

$$U(Y) = \frac{Y^{1-\rho} - 1}{1-\rho}. \tag{1}$$

There is a social custom that rewards honest tax payment. The payoff from the social custom is given by z and the individual weight, or the importance, assigned to this payoff by the taxpayer is determined by χ. Hence, compliance generates an additional utility from the social custom of χz. At each point in time the payoff is a fixed parameter for each taxpayer, but the weight changes over time through interaction with other taxpayers in the network.

In employment there is no opportunity for evasion. The taxpayer obtains a payoff given by

$$V_0 = \frac{[(1-\tau)w]^{1-\rho} - 1}{1-\rho} + \chi z,$$

where τ is the constant marginal tax rate. The possibility of tax evasion makes the choice of self-employment a compound lottery: the outcome of the project is random, as is the outcome of choosing to evade. If self-employment occupation i is chosen the outcome π_s^i or π_u^i is randomly realised, with probabilities q^i and $1 - q^i$, respectively. Those in self-employment occupation i are then audited, according to a fixed probability. If evasion is discovered then unpaid tax is reclaimed and a fine is imposed on unpaid tax. We may therefore define the expected payoff from the optimal choice of evasion in

self-employment occupation i in state v, $(v = s, u)$ as

$$V_v^i = \max_{E_v^i \in [0, \pi_v^i]} \left\{ p \frac{\left[(1-\tau)\, \pi_v^i - f \tau E_v^i \right]^{1-\rho} - 1}{1-\rho} \right.$$
$$\left. + (1-p) \frac{\left[(1-\tau)\, \pi_v^i + \tau p E_v^i \right]^{1-\rho} - 1}{1-\rho} + \chi z \mathbf{1}_{[E_v^i = 0]} \right\},$$

where E_v^i is the amount of evasion in state v for occupation i, and $f > 1$ is the fine rate. The term $\mathbf{1}_{[A]}$ is an indicator function that takes the value one if A is true and zero otherwise: the payoff from the social custom is obtained if there is no evasion. The expected payoff from the compound lottery describing occupation i is then

$$V_e^i = q^i V_s^i + \left(1 - q^i\right) V_u^i. \tag{2}$$

The choice of occupation is made by comparing the utility levels from employment and from self-employment. Hence, the chosen occupation is given by the maximum of $\{V_0, V_e^1, V_e^2\}$.

A network is modelled as a set of bidirectional links. An example of a network is illustrated in Figure 1. In this example individual 1 is linked to individual 2; individual 2 is linked to individuals 1 and 3; individual 3 is linked to 2 and 4; individual 4 is linked to 3.

<Figure 1 here>

Figure 1: Representation of network

A network can be described by an $n \times n$ symmetric matrix of zeros and

ones. For example, the network shown in Figure 1 is described by matrix A:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

In matrix A the first row, representing the links of individual 1, has a single 1 in column 2; the second row, representing the links of individual 2, has 1s in columns 1 and 3, etc. That is, the element in row i and column j of matrix A is defined as

$$A_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are linked in the network,} \\ 0 & \text{otherwise.} \end{cases}$$

In the simulation, the network is created at the outset and does not change.[2] The network determines who may meet whom to exchange information. In each period a random selection of meetings occur; this is described by a matrix C of zeros and ones which is randomly selected every period. Individuals i and j meet during a period if $A_{ij}C_{ij} = 1$. At a meeting of i and j there is a probability that information is exchanged. When information is exchanged it consists of three elements: the taxpayer's subjective probability of audit, whether or not the taxpayer was compliant in that period, and whether or not the agent was audited. The probability of information exchange depends on the occupational groups to which i and j belong; the probability is highest when they are in like occupations. The probabilities of information exchange occurring at a meeting are given by p_{ij} where $i, j = e, 1, 2$.

[2] Another possibility would be to have the network revised as a consequence of chosen actions, i.e. employed and self-employed belonging to different social networks.

We consider two different processes for the formation of subjective beliefs. As studies have reliably demonstrated important deviations from Bayesian inference (e.g. Grether, 1980), we allow for non-Bayesian updating. The first process, which is qualitatively similar to a Bayesian process, is to assume that individuals feel marked as targets if they are audited, so that one audit is believed likely to be followed by another. We term this the "target effect". In contrast, those not audited in a period believe they are less likely to be audited in the next period. Formally, if audited in period t, an individual's belief about being audited in the next period is raised to the level P, otherwise it decays. The updating rule for the subjective probability is therefore

$$\tilde{p}_{t+1} = X_t^i P + \left(1 - X_t^i\right) dp_t, \;\; d \in [0,1], \;\; P \in [0,1] \tag{3}$$

where $X_t^i = 1$ if agent i was audited in period t and $X_t^i = 0$ otherwise. Thus, the perceived probability of evasion of being detected is determined by the individual's own past experience and by the past experience of a randomly met member of that individual's social network. The process in (3) can also be written as

$$\tilde{p}_{t+1} = \begin{cases} P \in [0,1] & \text{if audited at } t, \\ dp_t, \;\; d \in [0,1] & \text{otherwise.} \end{cases} \tag{4}$$

The second process we consider captures the "bomb-crater" effect documented experimentally by Guala and Mittone (2005), Kastlunger, Kirchler, Mittone, and Pitters (2009), Maciejovsky, Kirchler, and Schwarzenberger (2007) and Mittone (2006). In this process, a taxpayer who has been audited

in one period believes that they will not be audited in the next, but the belief slowly rises over time. The process is therefore described by

$$\tilde{p}_{t+1} = \begin{cases} 0 & \text{if audited at } t, \\ p_t + \delta\left(1 - p_t\right), \ \delta \in [0,1] & \text{otherwise.} \end{cases} \tag{5}$$

The choice of occupation in period $t+1$ is made on the basis of the belief $\tilde{p}_{t+1}$. After occupational choice is made, *if an information exchange occurs* at a meeting with another individual, the subjective probability is updated according to the rule

$$p^i_{t+1} = \mu \widetilde{p}^i_{t+1} + (1-\mu)\left[X^j_t P + \left(1 - X^j_t\right)\widetilde{p}^j_{t+1}\right],$$

which can also be written as

$$p^i_{t+1} = \begin{cases} \mu \widetilde{p}^i_{t+1} + (1-\mu) P & \text{if } j \text{ audited at } t, \\ \mu \widetilde{p}^i_{t+1} + (1-\mu)\widetilde{p}^j_{t+1} & \text{otherwise.} \end{cases}$$

Note that under the target effect $\widetilde{p}^j_{t+1} = P$ if j was audited at t, and so the updating rule reduces to $p^i_{t+1} = \mu \widetilde{p}^i_{t+1} + (1-\mu)\widetilde{p}^j_{t+1}$. However, under the bomb-crater effect $\widetilde{p}^j_{t+1} = 0$ if j was audited at t, and so the updating rule reflects an assumption that agent i's perceived probability of audit may rise after information exchange with agent j if i learns that j has just been audited, even though j's own perceived probability drops to zero.[3]

The importance attributed to the social custom is also determined by interaction in the social network. Each individual is initially assigned a random level of importance, χ^i_0. This value is updated after each information

[3]We are grateful to an anonymous referee for raising this point.

exchange with another taxpayer. The updating process is described by

$$\chi^i_{t+1} = \frac{1}{X(i)+1}\left[\chi^i_t X(i) + \mathbf{1}_{[E^j_t=0]}\right], \tag{6}$$

where $X(i)$ is the number of previous meetings for i at which information was exchanged. Hence, $\chi^i_{t+1} > \chi^i_t$ if information is exchanged with an honest taxpayer, and $\chi^i_{t+1} < \chi^i_t$ if information is exchanged with an evader.

4 Simulation

The network model described above is simulated to investigate the nature of the equilibrium and the consequences of alternative updating rules on beliefs. In this context, equilibrium is a state in which aggregate behaviour is stationary around a steady-state level. Even in equilibrium, however, some amount of variation always remains because of randomness in the outcome of self-employment and the selection of taxpayers for audit.

This section summarises and discusses the most significant findings that emerge from the simulations. Results are reported for the two methods of updating beliefs in eqns. (4) and (5). In addition, for the target effect, we also consider the effect of varying the probability of exchanging information. The parameter values and the distributions for those random variables that remain constant across the simulations are given in the Appendix. As discussed previously, we set the parameter values such that, on average, the payoff from self-employment will exceed that from employment.[4]

[4]The value of the social custom, z, is measured in units of utility. Therefore, although

4.1 Target effect

In the case of the target effect the subjective probability is increased after an audit and decays when no auditing occurs. Two set of results are given for this process of belief formation, which differ in the probability of information exchange between different groups. Both sets of results are based on a true audit probability of 0.05 and a rate of decay in belief of $d = 0.75$

The first set of results emphasizes the differences that can emerge between groups by assuming that, when two taxpayers in the same occupation meet, information is exchanged always, but when two taxpayers in different occupations meet no information is exchanged. This is termed *focussed information exchange.*

The results are summarised in Figure 2. The simulation is run for 200 periods. It can be seen that the effect of the initial random assignment of values to the subjective probability and the honesty weight is negligible after about 20 periods. The proportions of the population in employment, risky self-employment, and less risky self-employment are 30 per cent, 20 per cent, and 50 per cent, respectively. The level of compliance – measured by the proportion of agents declaring true income – is lowest for occupation 1 (about 24 per cent), which is the riskier of the two forms of self-employment. Occupation 2 has a rate of compliance of about 53 per cent, about twice

z appears constrained to take very small values, these values are comensurate with the values taken by the utility function in equation (1). Thus, with given parameterisation, a true report increases the utility of an "average" individual by about 10 per cent.

that of occupation 1. The remaining three figures explain the source of this difference. What is interesting is that the difference in compliance between occupations does not come from the subjective probability. The average belief of taxpayers in both occupations is approximately 0.15. This is much higher than the true probability of 0.05, but does not explain the different compliance rates. The employed learn about audits only from self-employed who have been audited and switched into employment as a result. Hence, the subjective probability of the employed decays, on average, to just below the true value. The second driver of the difference in compliance is the weight given to the social custom. This is significantly lower in occupation 1 than in occupation 2, and close to 1 among the employed. The social custom reinforces the separation by risk aversion, and these jointly determine the compliance outcome.

The network effects can be seen to endogenously generate a culture of non-compliance that varies across groups. The non-compliance is not driven by differences in beliefs but by self-selection into occupations according to risk aversion, and is reinforced by the emergence of distinct group-specific social customs. This illustrates the process of endogenous behavioural differences among population sub-groups.

<Figure 2 here>

Figure 2: Focussed information exchange

($p_{ee} = 1$; $p_{e1} = 0$; $p_{e2} = 0$; $p_{11} = 1$; $p_{22} = 1$; $p_{12} = 0$)

The second set of results report the outcome with *diffused information transmission*. In this case there is a positive probability (0.15) that a meeting between taxpayers in dissimilar occupations results in information exchange. In addition, the probability of information exchange at meetings between members of the same occupation is reduced (compared to the results in figure 2) to 0.75. Figure 3 shows the effect that this has upon the outcome of the simulation. Compared to Figure 2 there is very little change in the rate of compliance (but it is slightly higher for self-employed occupation 1), the separation by risk aversion, or the levels of the subjective probability. The only significant difference between the two set of results is seen in the weight attached to the social custom. The diffused information transmission means that some employed taxpayers exchange information with evaders, and evaders in the self-employed occupations have an increased probability of meeting a compliant taxpayer. As a consequence, the social custom weight among the employed is reduced, while among occupation 1 it is increased. This enhanced importance of the social custom in occupation 1 explains the slightly higher compliance level of that group. The proportion of the population belonging to each occupational group remains approximately unchanged.

<Figure 3 here>

Figure 3: Diffused information exchange

($p_{ee} = 0.75$; $p_{e1} = 0.15$; $p_{e2} = 0.15$; $p_{11} = 0.75$; $p_{22} = 0.75$; $p_{12} = 0.75$)

The central message of these results is that sub-groups of the population

can endogenously form different attitudes to compliance. These differing attitudes combine with self-selection into occupations to produce significantly differing levels of compliance across occupations. The social network also results in the subjective probability of audit being above the true value for the self-employed. The self-employed groups hold similar beliefs, which are distinctly different from those of the employed. These features highlight the importance of social networking effects in explaining patterns of compliance.

4.2 Bomb-crater effect

The final set of results explore the outcome when the probability of audit is updated according to the bomb-crater process. In this case the probability is revised down after an audit, but then tends upward until the next audit occurs. The results reported in Figure 4 are for diffused information transmission. The true audit probability is again chosen to be 0.05 and the rate of increase in belief is $\delta = 0.05$.

The bomb-crater effect leads to a much higher level of compliance across occupations than the target effect. For the same values of the parameters in the simulations, about 86 per cent of taxpayers in the risky occupation and about 92 per cent in the less risky occupation report their true income. This high rate of compliance is a consequence of the infrequent audits. The infrequency of audits means that there are numerous periods in which a taxpayer is not audited and in each of these periods the subjective probability increases. This effect is especially marked for a taxpayer who always chooses

to be employed: no audit is ever undertaken so the subjective belief increases toward unity. This may seem inconsistent, but, owing to the impossibility of non-compliance in employment, an interpretation of the effect is that employed taxpayers behave "as if" p is unity.[5]

The occupational groups are still characterised by differing degrees of risk aversion, but for this simulation there is little to separate the employed and those in the less risky self-employment occupation. The major distinction between the bomb-crater effect and the target effect is that the weights attached to the social custom are fairly similar across occupations: the pattern is the same as under the target effect, in that for the individuals in risky occupation the weight, on average, is the lowest, and for the individuals in employment the weight, on average, is the highest, but the differences between groups are very small. The proportions of population in employment, risky self-employment and less risky self-employment are 36.1 per cent, 18.6 per cent, and 45.3 per cent, respectively. Comparison with the target effect scenario shows that under the bomb-crater effect more agents choose employment (the size of this group is larger by 20 per cent), and fewer agents choose self-employment (the size of the group in the risky occupation is lower by 7 per cent, and in the less risky occupation it is lower by 9.4 per cent). This is consistent with the overall higher compliance rates across groups and higher subjective probability of audit, which makes self-employment less attractive from the viewpoint of the opportunity to evade tax.

[5] We are grateful to an anonymous referee for this interpretation.

<Figure 4 here>

Figure 4: Bomb crater process for subjective probability

($p_{ee} = 0.75$; $p_{e1} = 0.15$; $p_{e2} = 0.15$; $p_{11} = 0.75$; $p_{22} = 0.75$; $p_{12} = 0.75$)

4.3 Summary statistics

A summary of the descriptive statistics for the variables in Figs. 2–4 is given in Table 1. The means and the standard deviations are calculated over the last 50 simulated periods. Table 2 shows the averages over occupational groups for the wage in employment and probabilities of success in self-employment, as well as the mean and the standard deviation of the pay-offs in self-employment.

One can clearly see the outcome of the self-selection process, in particular, for the distribution of earnings. For individuals who choose employment the probability of success in either self-employment occupation is below average, whereas their wage in employment is significantly above average. At the same time, for individuals in self-employed occupation 1 the probability of success in occupation 1 is above average, and the probability of success in occupation 2 is below average; the converse holds for individuals in self-employed occupation 2. The wage in employment is below the average pay-off to both self-employment occupations. Individuals who self-select in occupation 1 earn, on average, a higher payoff but with a higher variance than they would in occupation 2. At the same time, individuals who self-select in occupation 2 earn, on average, a higher payoff with a lower variance than they

	Target effect						**Bomb-crater effect**		
	Focussed exchange			Diffused exchange			Diffused exchange		
	SE1	SE2	EM	SE1	SE2	EM	SE1	SE2	EM
Risk	2.665	5.325	6.092	2.913	5.195	6.205	2.940	5.438	5.529
aversion	(.034)	(.016)	(.050)	(.027)	(.020)	(.053)	(.009)	(.001)	(.000)
Honesty	.293	.722	.966	.506	.689	.818	.939	.940	.972
weight	(.006)	(.001)	(.004)	(.003)	(.001)	(.002)	(.000)	(.000)	(.000)
Compliance	.240	.533	1	.322	.532	1	.864	.920	1
	(.026)	(.014)	–	(.020)	(.015)	–	(.026)	(.012)	–
Beliefs	.150	.155	.030	.158	.155	.038	.616	.612	.940
	(.022)	(.010)	(.004)	(.015)	(.010)	(.006)	(.015)	(.015)	(.002)

Table 1: Means and standard deviations (in parentheses) by occupational group

would in occupation 1. This is consistent with self-selection of individuals with lower risk aversion into occupation 1.

5 Conclusions

An understanding of the individual tax compliance decision is important for revenue services. Their aim is to design policy instruments to reduce the tax gap (the difference between anticipated and actual tax revenue). Empirical evidence demonstrates that a wide range of factors, including social groupings and network effects, may impact upon the individual compliance decision.

The research we report in this paper combines ideas from behavioural economics and social networks to model occupational choice and tax compliance in an integrated framework. The analysis is based on the consequence

	Target effect			Bomb-crater effect		
	Pr success	Exp pay-off	StDev	Pr success	Exp pay-off	StDev
SE1						
Project 1	0.819	14.192	3.849	0.824	14.247	3.802
Project 2	0.400	9.602	1.960	0.437	9.746	1.984
Employment		6.748			6.049	
SE2						
Project 1	0.410	10.106	4.920	0.416	10.159	4.929
Project 2	0.575	10.298	1.978	0.578	10.311	1.976
Employment		5.148			4.751	
EM						
Project 1	0.473	10.728	4.993	0.473	10.729	4.993
Project 2	0.472	9.886	1.997	0.467	9.869	1.996
Employment		13.046			12.586	

Table 2: Success probabilities and pay-offs by groups

of taxpayers possessing social connections through which information and attitudes relevant to the compliance decision are transmitted. The model accommodates differences in preferences, in productivity, and in opportunities for evasion. Occupational choice operates as a form of self-selection that places those who will evade into situations where evasion is possible. Social interaction results in the subjective probability of audit differing from the objective probability. Combined with a social custom that rewards compliance, this can generate relatively high levels of compliance (when compared to the "standard" model).

The simulations have considered two different processes for the formation of subjective beliefs. These are distinguished by whether an audit causes an increase in the subjective probability (the target effect) or a reduction (the bomb-crater effect). Although these processes are very different, the

important qualitative properties of the simulations are the same in both cases. First, taxpayers self-select into occupations according to the degree of risk aversion. Second, the subjective probability of audit can be sustained above the objective probability. Third, the weight attached to the social custom differs across occupations, a finding relevant to the literature on the evolution of social norms (Bendor, 2001; Boyd and Richerson, 1994). Finally, these factors combine to lead to a compliance level that is lower in the riskier occupation.

The model has also demonstrated how it is possible for attitudes and beliefs to emerge endogenously that differ across sub-groups of the population. The population is heterogenous in characteristics and chooses occupational groups on the basis of characteristics. The behaviour is different across occupational groups, and this is reinforced by the development of group-specific attitudes and beliefs.

A prominent avenue for future work is to relax the assumption of random auditing. This would allow for an analysis of the effectiveness of alternative audit strategies when taxpayers form beliefs about auditing from interaction in a social network.

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Appendix

Parameter values
Tax rate: $\tau = 0.25$
Fine rate: $f = 1.5$
Payoffs in occupation 1: $\pi_s^1 = 16$; $\pi_u^1 = 6$
Payoffs in occupation 2: $\pi_s^2 = 12$; $\pi_u^2 = 8$
Weight in information exchange: $\mu = 0.75$

Probability distributions
Wage in employment: $w \sim U[0, 16]$
Risk aversion: $\rho \sim U[0, 10]$
Success in occupation i: $q^i \sim U[0, 1]$
Initial belief on audit probability: $p_0 \sim U[0, 1]$
Importance assigned initially to social custom: $\chi_0 \sim U[0, 1]$
Value of social custom: $z \sim U[0, 3 \times 10^{-5}]$

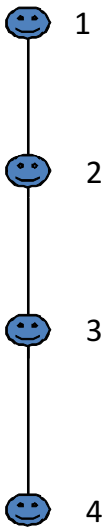
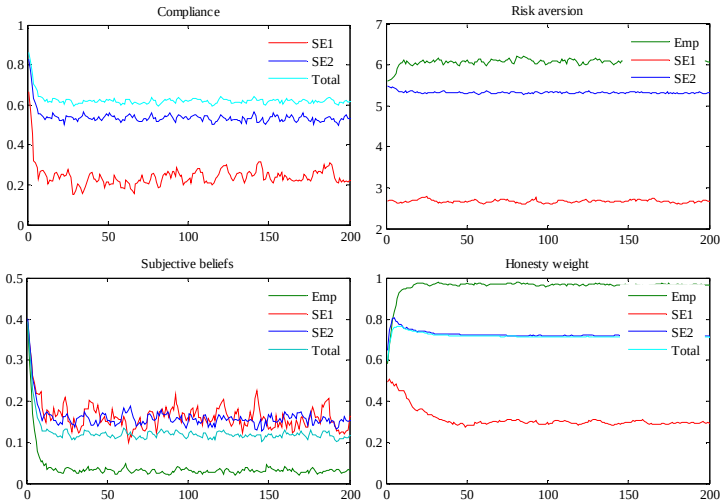
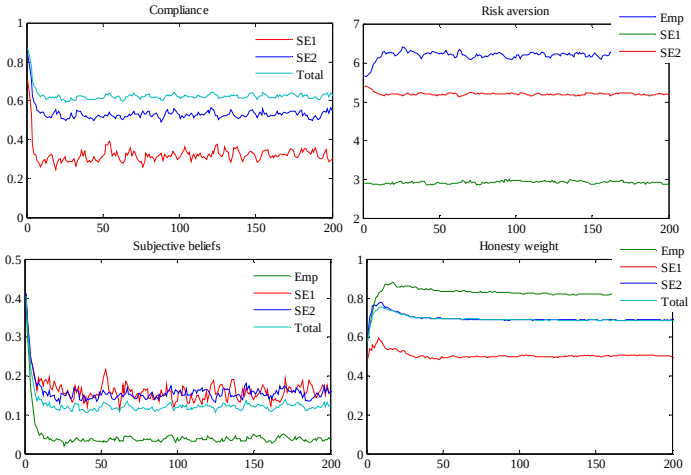


Figure 1:



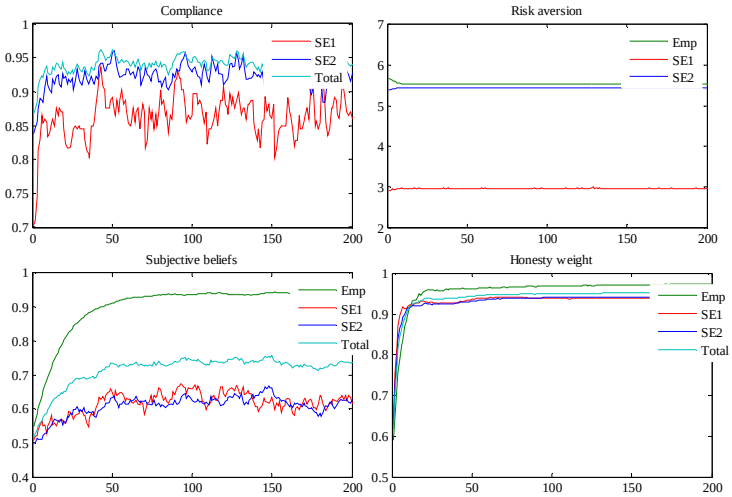
$p_{ee} = 1$; $p_{e1} = 0$; $p_{e2} = 0$; $p_{11} = 1$; $p_{22} = 1$; $p_{12} = 0$

Figure 2:



$p_{ee} = 0.75$; $p_{e1} = 0.15$; $p_{e2} = 0.15$; $p_{11} = 0.75$; $p_{22} = 0.75$; $p_{12} = 0.15$

Figure 3:



$p_{ee} = 0.75$; $p_{e1} = 0.15$; $p_{e2} = 0.15$; $p_{11} = 0.75$; $p_{22} = 0.75$; $p_{12} = 0.15$; $= 0.05$

Figure 4: