Clear and Measurable Signature of Modified Gravity in the Galaxy Velocity Field

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The velocity field of dark matter and galaxies reflects the continued action of gravity throughout cosmic history. We show that the low-order moments of the pairwise velocity distribution v_{12} are a powerful diagnostic of the laws of gravity on cosmological scales. In particular, the projected line-of-sight galaxy pairwise velocity dispersion $\sigma_{12}(r)$ is very sensitive to the presence of modified gravity. Using a set of high-resolution N-body simulations, we compute the pairwise velocity distribution and its projected line-of-sight dispersion for a class of modified gravity theories: the chameleon $f(R)$ gravity and Galileon gravity (cubic and quartic). The velocities of dark matter halos with a wide range of masses would exhibit deviations from general relativity at the (5–10)σ level. We examine strategies for detecting these deviations in galaxy redshift and peculiar velocity surveys. If detected, this signature would be a "smoking gun" for modified gravity.

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Introduction.—Measurements of temperature anisotropies in the microwave background radiation and of the large-scale distribution of galaxies in the local universe have established "lambda cold dark matter," or ΛCDM, as the standard model of cosmology. This model is based on Einstein's theory of general relativity (GR) and has several parameters that have been determined experimentally to high precision, e.g., Refs. [1–7]. One of these parameters is the cosmological constant Λ, which is responsible for the accelerating expansion of the Universe but has no known physical basis within GR. Modifications of GR, generically known as "modified gravity" (MG), could, in principle, provide an explanation (see e.g., Ref. [8] for a comprehensive review). In this case, gravity deviates from GR on sufficiently large scales so as to give rise to the observed accelerated expansion, but on small scales such deviations are suppressed by dynamical screening mechanisms that are required for these theories to remain compatible with the stringent tests of gravity in the Solar System [9].

Significant progress has been achieved in recent years in designing observational tests of gravity on cosmological scales that might reveal the presence of MG, e.g., Refs. [10–12]. Most viable MG theories predict changes in the clustering pattern on nonlinear and weakly nonlinear scales, on galaxy and halo dynamics, e.g., Refs. [13–19], on weak gravitational lensing signals, and on the integrated Sachs-Wolfe effect, e.g., Refs. [20,21]. However, a common feature of these observational probes is that they typically rely on quantities for which we have limited model-independent information due, in part, to various degeneracies, many related to poorly understood baryonic processes associated with galaxy formation [22–24]. These processes can introduce further degeneracies in the case of MG cosmology [25]. In addition, there are numerous statistical and systematic uncertainties in the observational data whose size can be comparable to the expected deviations from GR.

In this Letter, we introduce the use of the low-order moments of the distribution of galaxy pairwise velocities as a probe of GR and MG on cosmological scales. We illustrate the salient physics by reference to two classes of currently popular MG models. The first is the $f(R)$ family of gravity models [26–28], in which the Einstein-Hilbert action is augmented by an arbitrary and intrinsically nonlinear function of the Ricci scalar R. These models include the environment-dependent "chameleon" screening mechanism. The second class is *Galileon* gravity [29,30], in which the modifications to gravity arise through nonlinear derivative self-couplings of a Galilean-invariant scalar field. These models restore standard gravity on small scales through the Vainshtein effect [31].

Our analysis is based on the high-resolution N-body simulations of Ref. [15], for the Hu-Sawicki $f(R)$ model [32], and of Refs. [14,33], for Galileon gravity [30,34].

These consider three flavors of $f(R)$ gravity corresponding to different values of the parameter $|f_{R0}|$ (10^{-4}, 10^{-5}, 10^{-6}), which determine the degree of deviation from standard GR [32]. We refer to these as $F4$, $F5$, and $F6$, respectively. For Galileon gravity, we study the so-called cubic ^3G and quartic ^4G models, which are characterized by the order at which the scalar field enters into the Lagrangian [29].

Pairwise velocities.—The mean pairwise relative velocity of galaxies (or pairwise streaming velocity) v_{12} reflects the "mean tendency of well-separated galaxies to approach each other" [35]. This statistic was introduced by Davis and Peebels [36] in the context of the kinetic BBGKY theory [37–40], which describes the dynamical evolution of a system of particles interacting through gravity. In the fluid limit, its equivalent is the pair density-weighted relative velocity,

$$\mathbf{v}_{12}(r) = \langle \mathbf{v}_1 - \mathbf{v}_2 \rangle_\rho = \frac{\langle (\mathbf{v}_1 - \mathbf{v}_2)(1+\delta_1)(1+\delta_2) \rangle}{1+\xi(r)}, \quad (1)$$

where $\mathbf{v}_1$ and $\delta_1 = \rho_1/\langle\rho\rangle - 1$ denote the peculiar velocity and fractional matter density contrast at position $\mathbf{r}_1$, $r = |\mathbf{r}_1 - \mathbf{r}_2|$, and $\xi(r) = \langle \delta_1 \delta_2 \rangle$ is the 2-point density correlation function. The expression $\langle \cdots \rangle_\rho$ denotes a pair-weighted average, which differs from the usual spatial averaging by the weighting factor $\mathcal{W} = \rho_1\rho_2/\langle\rho_1\rho_2\rangle$. Note that $\mathcal{W}$ is proportional to the number density of pairs.

Gravitational instability theory predicts that the amplitude of $v_{12}(r)$ is determined by the 2-point correlation function $\xi(r)$ and the growth rate of matter density perturbations $g \equiv d\ln D_+/d\ln a$ [where $D_+(a)$ is the linear growing mode solution and a is the cosmological scale factor] through the pair conservation equation [35]. Juszkiewicz *et al.* [41] provided an analytic expression for Eq. (1) that is a good approximation to the solution of the pair conservation equation for universes with Gaussian initial conditions: $v_{12} = -\frac{2}{3}H_0 r g \bar{\xi}(r)[1+\alpha\bar{\xi}(r)]$, where $\bar{\xi}(r) = (3/r^3)\int_0^r \xi(x)x^2dx \equiv \xi(r)[1+\xi(r)]$. Here, α is a parameter that depends on the logarithmic slope of $\xi(r)$ and $H_0 = 100h$ km s^{-1} Mpc^{-1} is the present-day value of the Hubble constant. It is clear that $v_{12}(r)$ is a strong function of $\xi(r)$ and g, both of which will differ in MG theories from the GR values. This dependency motivates the use of the low-order moments of the pairwise velocity distribution as tracers of MG and of the fifth force it induces on galaxies and dark matter halos. Specifically, we will consider the following quantities: the mean radial pairwise velocity v_{12}, the dispersion (not centered) of the (radial) pairwise velocities $\sigma_\parallel = \langle v_{12}^2 \rangle^{1/2}$, the mean transverse velocity of pairs $v_\perp$, and the dispersion of the transverse velocity of pairs $\sigma_\perp = \langle v_\perp^2 \rangle^{1/2}$.

Since none of these quantities is directly observable, following Ref. [42] we also consider the centered line-of-sight pairwise velocity dispersion $\sigma_{12}^2(r) = \int \xi(R)\sigma_p^2(R)dl / \int \xi(R)dl$. Here, r is the projected galaxy separation, $R = \sqrt{r^2 + l^2}$, and the integration is taken along the line of sight within $l \pm 25h^{-1}$ Mpc. The quantity σ_p^2 is the line-of-sight centered pairwise dispersion, defined as in Ref. [42],

$$\sigma_p^2 = \frac{r^2\sigma_\perp^2/2 + l^2(\sigma_\parallel^2 - v_{12}^2)}{r^2 + l^2}. \quad (2)$$

Figure 1 shows the scale dependence of the lower-order moments of the pairwise velocities measured in our N-body simulations in the GR case (black lines and symbols) and in the $F4$ model (red lines and symbols). We choose the $F4$ model for illustration because this model is the one for which the chameleon screening mechanism is the least effective [20].

For the purposes of this comparison and to allow for a better connection to observations, we construct mock galaxy catalogs for these two models by performing a halo occupation distribution (HOD) analysis [43]. Our HOD catalogs are tuned to resemble a sample of luminous red galaxies with a satellite fraction of ~7% and a total galaxy number density of $4 \times 10^{-5}(h/\mathrm{Mpc})^3$. This number density is roughly consistent with that of the SDSS DR7 sample presented in Ref. [44]. We do this by following a procedure similar to that described in Refs. [45,46]. The shaded region in the figure shows an illustrative error that reflects the accuracy of σ_{12} measurements from galaxy redshift surveys as in Refs. [3] and [4]. First, we note that the stable clustering regime [35] (the scales over which the mean infall velocity exceeds the Hubble expansion $-v_{12} > Hr$) extends to larger separations for the $F4$ model than for the GR case. However, v_{12} in $F4$ differs significantly from that in GR only in the mildly nonlinear regime,

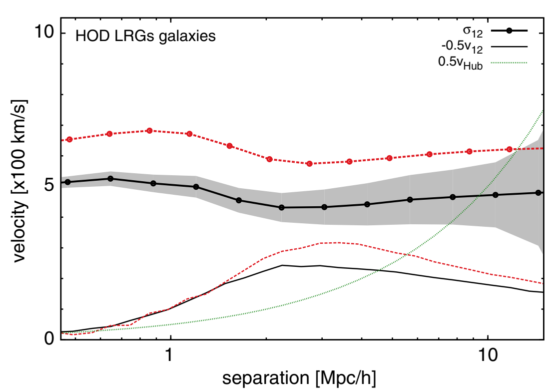

FIG. 1 (color online). The scale dependence of the pairwise velocity moments extracted from HOD mock galaxy catalogs. The black solid lines show the GR case, whereas the red dashed lines show the $F4$ model. The thin red and black lines show minus the mean streaming velocity $-v_{12}(r)$, scaled down by factor of 2 for clarity; the lines with filled circles show the dispersion $\sigma_{12}(r)$. The shaded region represents an illustrative error as in Refs. [3] and [4]. The dotted green line shows the Hubble velocity $H_0 r$, also scaled down for comparison.

$2 \lesssim r \lesssim 10h^{-1}$ Mpc. The maximum difference between the two models occurs at $r \sim 3.5h^{-1}$ Mpc and is $\sim$30%. The situation is quite different when we consider σ_{12}. While the $F4$ values are also roughly 30–35% greater than those in GR, the signal now is noticeable on all scales plotted. Now, if we compare σ_{12} for $F4$ with the GR case with errors obtained as in Refs. [3,4], we can see that the amplitude of this statistics in $F4$ is $(2-4)\sigma$ away from the GR case.

The differences between $F4$ and GR are driven by the fact that the distribution of v_{12} never reaches the Gaussian limit, even at large separations. This is because at a given separation r the velocity difference between a galaxy pair does not have a net contribution from modes with wavelengths greater than the pair separation since those modes make the same contribution to the velocities of both galaxies. Hence, on the scale of the typical interhalo separation (at which the galaxies in a pair inhabit different halos), the distribution of v_{12} factorizes into two individual peculiar velocity distributions, one for each galaxy or halo, and these are always sensitive to nonlinearities driven by virial motions within the galaxy host halo (see Ref. [47] for more details). In most MG theories, the effects of the fifth force on the dynamics are only significant on small nonlinear or mildly nonlinear scales ($\lesssim 10h^{-1}$ Mpc), which are probed by the pairwise velocity dispersion. Because of this, the amplitude of σ_{12} is potentially a powerful diagnostic of MG.

The effect of the fifth force on σ_{12} is illustrated in Fig. 2 where we plot $\xi(r) \equiv \langle \delta_1 \delta_2 \rangle$, v_{12}, $\sigma_{\parallel}$, and σ_{12} as a function of M_{200} [48] for the MG models we consider. Results are shown at pair separations $r = 1h^{-1}$ Mpc and $5h^{-1}$ Mpc. Here, the error bars show the variance estimated from the ensemble average of simulations from different phase realizations of the initial conditions. We also plot the relative deviation $\Delta X = X_{\rm MG}/X_{\rm GR} - 1$ from a fiducial model that has the same expansion history but includes a fifth force. This helps identify changes driven by the modified force law rather than by the modified expansion dynamics. For clarity, we only show results for the Galileon model in the relative difference panels. In the ^4G model, although gravity is enhanced in low-density regions, it is suppressed in the high-density regions of interest because the Vainshtein mechanism does not fully screen out all of the modifications to gravity [33,49]. This is the reason the results for this model point in the direction opposite to those for the other models ($F4$, $F5$, $F6$, and ^3G), for which gravity can only be enhanced by a positive fifth force. For models other than ^4G, Fig. 2 shows positive enhancements relative to GR in v_{12}, $\sigma_{\parallel}$, and σ_{12} but a small reduction in the amplitude of ξ_2. Furthermore, the size of the MG effect in both $\sigma_{\parallel}$ and σ_{12} is approximately independent of halo mass, although there is a weak trend in σ_{12} for the most massive halos ($M_{200} \gtrsim 10^{13} M_{\odot}/h$).

The most striking result of this Letter is the amplitude of the halo mass-binned σ_{12} both at $r = 1h^{-1}$ Mpc and $5h^{-1}$ Mpc. Relative to GR, the deviations in the $F4$ model range from 30% to 75%. For the $F5$ and ^3G models, the deviation is smaller but still visible at the $\Delta\sigma_{12} \sim 0.25$ level.

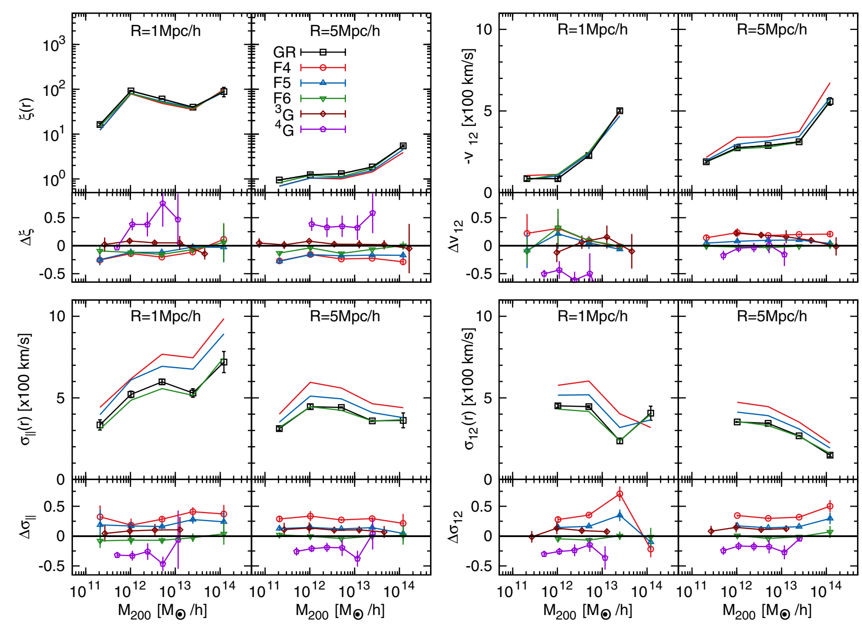

FIG. 2 (color online). Comparison of absolute values (top panel in each pair) and the relative deviation from the GR case (bottom panel in each pair) of the 2-point correlation function $\xi_2(r)$ (top-left panels); minus the mean streaming velocity $-v_{12}(r)$ (top-right panels); the pairwise velocity dispersion $\sigma_{\parallel}(r)$ (bottom-left panels); and the projected pairwise velocity dispersion $\sigma_{12}(r)$ (bottom-right panels). The data are binned in halo mass M_{200} and shown at two different pair separations: 1 and $5h^{-1}$ Mpc. The legend in the panel for $\xi_2(5h^{-1}$ Mpc) gives the colors and symbols that we use to distinguish the different models. Top panels show only the LCDM and $f(R)$ cases; the QCDM and Galileon cases were omitted for clarity.

The strong signal in the amplitude of σ_{12} is a combination of the contributions from Δv_{12}, $\Delta\sigma_{\parallel}$, and $\Delta\sigma_{\perp}$ that are incorporated in σ_p as shown in Eq. (2) and from $\Delta\xi_2$, which appears in the line-of-sight integrals for σ_{12}. Together, their combined effect results in a prominent fifth-force-like signature. The amplitude of σ_{12} is the strongest observable deviation from GR on cosmological scales so far identified, a potential "smoking gun" for MG. This signal, however, is not entirely generic. For example, the $F6$ model is virtually indistinguishable from GR: the fifth force in this flavor of $f(R)$ gravity is much too weak to produce a detectable effect in the dynamics of galaxies and halos.

Summary.—Using dark matter halo catalogs extracted from high-resolution N-body simulations of the formation of cosmic structure in two representative classes of modified gravity theories, we have computed the mean pairwise streaming velocity and its dispersion (radial and projected along the line of sight). Our simulations show that there is a strong MG signal contained in the line-of-sight projected pairwise velocity dispersion. For the $F5$, ^3G, and ^4G models, deviations from GR are at the $> 5\sigma$ level for all masses. The deviation is even more pronounced for the $F4$ model, where it is at the $> 10\sigma$ level and higher. This is the clearest footprint of modified gravity found to date in quantities that are, in principle, observable. Nonetheless, in a realistic observational situation, one can expect the significance of the MG signal to be reduced due to ambiguities related to galaxy formation and observational errors, as illustrated by our HOD analysis. However, the quality of the data as used by Refs. [3,4] would already be enough to distinguish between GR and $F4$, $F5$, and ^3G at the 2σ level, and these are relatively older data sets. With current and future surveys such as SDSS-II, BOSS, and Pan-STARRS1, e.g., Refs. [50–54], one can hope to do better, since the new data already provide $\sim 30\%$ improved accuracy.

The remaining important question is whether the MG footprint we have identified is actually observable in the real Universe. As mentioned above, the $\sigma_{12}(r)$ value can be estimated from galaxy redshift survey data but only in a model-dependent way. Specifically, one can obtain the line-of-sight dispersion by fitting the 2D galaxy redshift space correlation function to a model $\xi^s(r_p, \pi) = \int \xi'(r_p, \pi - v/H_0) h(v_{12}) dv$, where ξ' is the linear theory model prediction (which depends on coherent infall velocities) and the convolution is made with the assumed distribution of pairwise velocities $h(v_{12})$ [35,47,55,56]. Alternatively, one can use the redshift space power spectrum of the galaxy distribution to derive a quantity in Fourier space $\sigma_{12}(k)$, which is not an exact equivalent of the configuration space dispersion but is closely related to it [57–59]. To apply either of these methods, one needs a self-consistent model of the redshift-space clustering expected in a given MG theory. In particular, such a model needs to describe the linear galaxy bias parameter b, the linear growth rate of matter g, and the pairwise velocity distribution in configuration space $h(v_{12})$ or, equivalently, the damping function in Fourier space $D[k\mu\sigma_{12}(k)]$. Fortunately, all these quantities can be derived self-consistently for MG theories using linear perturbation theory complemented with N-body simulations. Such a program is currently being developed.

Instead of using redshift data, it is possible, in principle, to estimate v_{12} and σ_{12} directly from measurements of galaxy peculiar velocities. The advantage of this approach is that it is model independent. The disadvantage is that peculiar velocities can only be measured with sufficient accuracy for a small sample of local galaxies ($z < 0.05$), and even then there are potentially large systematic errors in the estimates of redshift-independent distance indicators [60,61]. A further complication is that only the radial component of a galaxy peculiar velocity is observable (but see Ref. [62]), so it is necessary to construct special estimators for pairwise velocities such as those proposed in Refs. [63–66].

There is already a large body of velocity data of potentially sufficient quality for the test we propose (cf. the size of the velocity error bars in Fig. 23 of Ref. [3]). Further theoretical work is required to refine the redshift-space probes, and further observational work is needed to exploit direct peculiar velocity measurements. It is to be hoped that the presence of a fifth force, if it exists, will be revealed in measurements of the galaxy velocity field.

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[1] M. Colless, G. Dalton, S. Maddox, W. Sutherland, P. Norberg, S. Cole, J. Bland-Hawthorn, T. Bridges, R. Cannon, C. Collins *et al.*, Mon. Not. R. Astron. Soc. **328**, 1039 (2001).
[2] M. Tegmark, M. A. Strauss, M. R. Blanton, K. Abazajian, S. Dodelson, H. Sandvik, X. Wang, D. H. Weinberg, I. Zehavi, N. A. Bahcall *et al.*, Phys. Rev. D **69**, 103501 (2004).
[3] E. Hawkins, S. Maddox, S. Cole, O. Lahav, D. S. Madgwick, P. Norberg, J. A. Peacock, I. K. Baldry, C. M. Baugh, J. Bland-Hawthorn *et al.*, Mon. Not. R. Astron. Soc. **346**, 78 (2003).
[4] I. Zehavi, M. R. Blanton, J. A. Frieman, D. H. Weinberg, H. J. Mo, M. A. Strauss, S. F. Anderson, J. Annis, N. A. Bahcall, M. Bernardi *et al.*, Astrophys. J. **571**, 172 (2002).
[5] S. Cole, W. J. Percival, J. A. Peacock, P. Norberg, C. M. Baugh, C. S. Frenk, I. Baldry, J. Bland-Hawthorn, T. Bridges, R. Cannon *et al.*, Mon. Not. R. Astron. Soc. **362**, 505 (2005).
[6] D. J. Eisenstein, I. Zehavi, D. W. Hogg, R. Scoccimarro, M. R. Blanton, R. C. Nichol, R. Scranton, H.-J. Seo, M. Tegmark, Z. Zheng *et al.*, Astrophys. J. **633**, 560 (2005).
[7] P. A. R. Ade, N. Aghanim, C. Armitage-Caplan, M. Arnaud, M. Ashdown, F. Atrio-Barandela, J. Aumont, C. Baccigalupi, A. J. Banday *et al.* (The Planck Collaboration), arXiv:1303.5076.
[8] T. Clifton, P. G. Ferreira, A. Padilla, and C. Skordis, Phys. Rep. **513**, 1 (2012).
[9] C. M. Will, Living Rev. Relativity **9**, 3 (2006).
[10] L. Guzzo, M. Pierleoni, B. Meneux, E. Branchini, O. Le Fèvre, C. Marinoni, B. Garilli, J. Blaizot, G. De Lucia, A. Pollo *et al.*, Nature (London) **451**, 541 (2008).
[11] D. H. Weinberg, M. J. Mortonson, D. J. Eisenstein, C. Hirata, A. G. Riess, and E. Rozo, Phys. Rep. **530**, 87 (2013).
[12] G.-B. Zhao, B. Li, and K. Koyama, Phys. Rev. Lett. **107**, 071303 (2011).
[13] W. A. Hellwing, B. Li, C. S. Frenk, and S. Cole, Mon. Not. R. Astron. Soc. **435**, 2806 (2013).
[14] A. Barreira, B. Li, W. A. Hellwing, C. M. Baugh, and S. Pascoli, J. Cosmol. Astropart. Phys. 10 (2013) 027.
[15] B. Li, W. A. Hellwing, K. Koyama, G.-B. Zhao, E. Jennings, and C. M. Baugh, Mon. Not. R. Astron. Soc. **428**, 743 (2013).
[16] W. A. Hellwing and R. Juszkiewicz, Phys. Rev. D **80**, 083522 (2009).
[17] Y. Zu, D. H. Weinberg, E. Jennings, B. Li, and M. Wyman, arXiv:1310.6768.
[18] E. Jennings, C. M. Baugh, B. Li, G.-B. Zhao, and K. Koyama, Mon. Not. R. Astron. Soc. **425**, 2128 (2012).
[19] R. Wojtak, S. H. Hansen, and J. Hjorth, Nature (London) **477**, 567 (2011).
[20] B. Li, G.-B. Zhao, and K. Koyama, Mon. Not. R. Astron. Soc. **421**, 3481 (2012).
[21] Y.-C. Cai, B. Li, S. Cole, C. S. Frenk, and M. Neyrinck, Mon. Not. R. Astron. Soc. **439**, 2978 (2014).
[22] S. D. M. White and M. J. Rees, Mon. Not. R. Astron. Soc. **183**, 341 (1978).
[23] D. J. Croton, V. Springel, S. D. M. White, G. De Lucia, C. S. Frenk, L. Gao, A. Jenkins, G. Kauffmann, J. F. Navarro, and N. Yoshida, Mon. Not. R. Astron. Soc. **365**, 11 (2006).
[24] S. D. M. White and C. S. Frenk, Astrophys. J. **379**, 52 (1991).
[25] E. Puchwein, M. Baldi, and V. Springel, Mon. Not. R. Astron. Soc. **436**, 348 (2013).
[26] S. M. Carroll, V. Duvvuri, M. Trodden, and M. S. Turner, Phys. Rev. D **70**, 043528 (2004).
[27] T. Chiba, Phys. Lett. B **575**, 1 (2003).
[28] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, 451 (2010).
[29] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. D **79**, 064036 (2009).
[30] C. Deffayet, G. Esposito-Farèse, and A. Vikman, Phys. Rev. D **79**, 084003 (2009).
[31] A. I. Vainshtein, Phys. Lett. **39B**, 393 (1972).
[32] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).
[33] B. Li, A. Barreira, C. M. Baugh, W. A. Hellwing, K. Koyama, S. Pascoli, and G.-B. Zhao, J. Cosmol. Astropart. Phys. 11 (2013) 012.
[34] C. Deffayet, S. Deser, and G. Esposito-Farèse, Phys. Rev. D **80**, 064015 (2009).
[35] P. J. E. Peebles, *The Large-Scale Structure of the Universe* (Princeton University Press, Princeton, NJ, 1980).
[36] M. Davis and P. J. E. Peebles, Astrophys. J. Suppl. Ser. **34**, 425 (1977).
[37] N. N. Bogoliubov, J. Phys. USSR **10**, 265 (1946).
[38] M. Born and H. S. Green, Proc. R. Soc. A **188**, 10 (1946).
[39] J. G. Kirkwood, J. Chem. Phys. **14**, 180 (1946).
[40] J. Yvon, *La Théorie Statistique des Fluides et l'Équation d'État* (Hermann & Cie, Paris, France, 1935), Vol. 203.
[41] R. Juszkiewicz, V. Springel, and R. Durrer, Astrophys. J. Lett. **518**, L25 (1999).
[42] A. Jenkins, C. S. Frenk, F. R. Pearce, P. A. Thomas, J. M. Colberg, S. D. M. White, H. M. P. Couchman, J. A. Peacock, G. Efstathiou, and A. H. Nelson, Astrophys. J. **499**, 20 (1998).
[43] A. V. Kravtsov, A. A. Berlind, R. H. Wechsler, A. A. Klypin, S. Gottlöber, B. Allgood, and J. R. Primack, Astrophys. J. **609**, 35 (2004).
[44] S. Roychowdhury, J. N. Chengalur, A. Begum, and I. D. Karachentsev, Mon. Not. R. Astron. Soc. **404**, L60 (2010).
[45] D. A. Wake, R. K. Sheth, R. C. Nichol, C. M. Baugh, J. Bland-Hawthorn, M. Colless, W. J. Couch, S. M. Croom, R. de Propris, M. J. Drinkwater *et al.*, Mon. Not. R. Astron. Soc. **387**, 1045 (2008).
[46] A. Barreira, B. Li, W. A. Hellwing, L. Lombriser, C. M. Baugh, and S. Pascoli, J. Cosmol. Astropart. Phys. 04 (2014) 029.
[47] R. Scoccimarro, Phys. Rev. D **70**, 083007 (2004).
[48] $M_{200} \equiv 4/3\pi R_{200}^3 200 \times \rho_m \Omega_m^{-1}$ is the halo virial mass.
[49] A. Barreira, B. Li, C. M. Baugh, and S. Pascoli, J. Cosmol. Astropart. Phys. 11 (2013) 056.
[50] H. Aihara, C. Allende Prieto, D. An, S. F. Anderson, É. Aubourg, E. Balbinot, T. C. Beers, A. A. Berlind, S. J. Bickerton, D. Bizyaev *et al.*, Astrophys. J. Suppl. Ser. **193**, 29 (2011).
[51] E. A. Kazin, A. G. Sánchez, A. J. Cuesta, F. Beutler, C.-H. Chuang, D. J. Eisenstein, M. Manera, N. Padmanabhan, W. J. Percival, F. Prada *et al.*, Mon. Not. R. Astron. Soc. **435**, 64 (2013).
[52] L. Anderson, E. Aubourg, S. Bailey, F. Beutler, A. S. Bolton, J. Brinkmann, J. R. Brownstein, C.-H. Chuang,

A. J. Cuesta, K. S. Dawson *et al.*, Mon. Not. R. Astron. Soc. **439**, 83 (2014).

[53] N. Metcalfe, D. J. Farrow, S. Cole, P. W. Draper, P. Norberg, W. S. Burgett, K. C. Chambers, L. Denneau, H. Flewelling, N. Kaiser *et al.*, Mon. Not. R. Astron. Soc. **435**, 1825 (2013).

[54] D. J. Farrow, S. Cole, N. Metcalfe, P. W. Draper, P. Norberg, S. Foucaud, W. S. Burgett, K. C. Chambers, N. Kaiser, R. P. Kudritzki *et al.*, Mon. Not. R. Astron. Soc. **437**, 748 (2014).

[55] M. Davis and P. J. E. Peebles, Astrophys. J. **267**, 465 (1983).

[56] A. J. Benson, C. M. Baugh, S. Cole, C. S. Frenk, and C. G. Lacey, Mon. Not. R. Astron. Soc. **316**, 107 (2000).

[57] C. Li, Y. P. Jing, G. Kauffmann, G. Börner, S. D. M. White, and F. Z. Cheng, Mon. Not. R. Astron. Soc. **368**, 37 (2006).

[58] J. L. Tinker, P. Norberg, D. H. Weinberg, and M. S. Warren, Astrophys. J. **659**, 877 (2007).

[59] S. de la Torre and L. Guzzo, Mon. Not. R. Astron. Soc. **427**, 327 (2012).

[60] M. A. Strauss and J. A. Willick, Phys. Rep. **261**, 271 (1995).

[61] R. B. Tully, H. M. Courtois, A. E. Dolphin, J. R. Fisher, P. Héraudeau, B. A. Jacobs, I. D. Karachentsev, D. Makarov, L. Makarova, S. Mitronova *et al.*, Astron. J. **146**, 86 (2013).

[62] A. Nusser, E. Branchini, and M. Davis, Astrophys. J. **755**, 58 (2012).

[63] K. M. Gorski, M. Davis, M. A. Strauss, S. D. M. White, and A. Yahil, Astrophys. J. **344**, 1 (1989).

[64] R. Juszkiewicz, P. G. Ferreira, H. A. Feldman, A. H. Jaffe, and M. Davis, Science **287**, 109 (2000).

[65] H. Feldman, R. Juszkiewicz, P. Ferreira, M. Davis, E. Gaztañaga, J. Fry, A. Jaffe, S. Chambers, L. da Costa, M. Bernardi *et al.*, Astrophys. J. **596**, L131 (2003).

[66] P. G. Ferreira, R. Juszkiewicz, H. A. Feldman, M. Davis, and A. H. Jaffe, Astrophys. J. **515**, L1 (1999).

[67] www.dirac.ac.uk.