Annals of Operations Research
https://doi.org/10.1007/s10479-025-06501-2

ORIGINAL RESEARCH

Do green bonds provide diversification benefits? The need for tax incentives

Ellie Papavassiliou[1] · Nikolas Topaloglou[1,2] · Stavros A. Zenios[3,4,5,6]

Received: 2 February 2024 / Accepted: 15 January 2025

Abstract
We examine whether, or under what conditions, green bonds offer diversification benefits when included in broad investor portfolios alongside assets from other financial markets. Using stochastic spanning non-parametric tests without any distributional assumptions on returns, we find that US and European investors benefit from adding green bonds to a diversified portfolio. However, the benefits are nuanced and non-robust out-of-sample in the global markets, and we evaluate whether tax incentives can make green bonds uniformly attractive for international investors. We find that with a somewhat lower tax rate and tax credits in case of losses, green assets offer consistent diversification benefits. Tax credits during market downturns preserve the diversification benefits even with higher tax rates.

Keywords Finance · Stochastic spanning · Green bonds · Risk premium · Diversification

JEL Classification C02 · C12 · C14 · C44 · C58 · G11 · G18

This article is dedicated to Professor William T. Ziemba, who was intrigued over his long career by perceived market anomalies. He researched a wide range of markets and instruments, but his efforts did not stop in mapping anomalies; he always went further to design strategies for exploiting them and managing the inherent risks. Novel challenges, such as those arising from climate change risks or heightened political uncertainty, pave the way for the identification of new risk factors and the design of new instruments, and the research initiated by Professor Ziemba continues to grow and finds numerous new applications.

✉ Nikolas Topaloglou
nikolas@aueb.gr

1 Department of International and European Economics Studies, Athens University of Economics and Business, Athens, Greece

2 IPAG Business School, Paris, France

3 Durham University, Durham, UK

4 University of Cyprus, Nicosia, Cyprus

5 Cyprus Academy of Sciences, Letters, and Arts, Nicosia, Cyprus

6 Bruegel, Brussels, Belgium

Published online: 12 March 2025

1 Introduction

The Paris Agreement of 2015 asked to align financial flows with the pathway towards low greenhouse gas emissions in support of climate policies.[1] Green bonds are debt securities used to finance investments with environmental or climate-related objectives issued for the first time in 2007 by the European Investment Bank, and their issuance by governments, quasi-governments entities, or corporations has been accelerating (Cheong & Choi, 2020).[2] On the asset demand side, the last decades have witnessed investors' growing interest in environmentally friendly business strategies and socially responsible investing (Pedersen, Fitzgibbons, and Pomorski, 2021). Moreover, green assets delivered high returns that were unexpected over the past few years, driven more by significant increases in news about environmental concerns than by high expected returns (Pástor, Stambaugh, and Taylor, 2022). Green bonds also provide investors with opportunities to take a stand on environmental projects, with the Paris Agreement stimulating this interest in spite of increasing concerns about greenwashing (Gorovaia & Makrominas, 2024). In this paper, we take at the investors' perspective.

We set aside investors' non-pecuniary interests and ask whether, or under what conditions, green bonds offer diversification benefits when included in broad portfolios alongside assets from other financial markets. We conduct a non-parametric analysis and ask if investors should hold green bonds to improve their efficiency set and not only on the argument that their investment satisfies some social common good. If the answer is affirmative, then no government intervention is required to develop this market. However, investors must be incentivized for the common good if the answer is negative. Our analysis shows that the answer is nuanced, and we identify two incentives in the form of tax reduction in good times or tax credits in bad times.

Recent literature regarding potential diversification benefits from green bonds across asset classes is ambiguous.[3] Correlations analyses and studies of spillover effects between green bonds and other financial instruments suggest that green bonds are not well integrated with the stock, commodity, and energy markets, so they could potentially provide diversification benefits to investors in these markets. However, the opposite is true for investors in conventional bonds due to high correlations. Only Hammoudeh et al. (2020) support that green bonds provide diversification benefits when added to portfolios of corporate and sovereign bonds. However, what is lacking is an analysis of the diversification benefits for investors with broad portfolios of stocks and corporate and government bonds; such analysis would cover a wide set of institutional investors within an optimal portfolio selection framework.

A significant step forward was taken by Han and Li (2022), who compared the performance of mean-variance efficient portfolios of stocks, commodities, real estate, and green bonds with that of portfolios with conventional *instead of* green bonds in the US and European markets. They find that the portfolio with green bonds achieves higher risk-adjusted returns than conventional ones. However, these tests leave open the more difficult question of whether the inclusion of green bonds enhances the performance of portfolios that already include traditional bonds. The same conclusion, with the same limitation, was reached by Han et

[1] See article 2.1(c) at https://unfccc.int/sites/default/files/english_paris_agreement.pdf.

[2] See, e.g., https://www.eib.org/en/press/all/2022-308-15-years-of-eib-green-bonds-leading-sustainable-investment-from-niche-to-mainstream. At a Jan. 2024 World Bank workshop of finance ministries and academics working with ministries, globally, on climate risk to debt sustainability, two-thirds of the attendees indicated that they are working with green bonds.

[3] See, among others, Pham (2016), Reboredo (2018), Reboredo and Ugolini (2020), Reboredo et al. (2020), Nguyen et al. (2021), Pham and Nguyen (2021),Hammoudeh et al. (2020).

al. (2022) and Bai et al. (2023) when adding green bonds to energy stock portfolios, both for the Chinese market. Akhtaruzzaman et al. (2023) establish diversification benefits from clean energy funds (that are not green bonds only) when added to oil, gold and US treasuries (thus missing corporate and long-term government bonds) using a variety of mean-variance based portfolio selection methods. Fender et al. (2019) use an illustrative asset allocation example for US dollar- and euro-denominated portfolios to indicate that adding both green and conventional bonds can provide diversification benefits. These works rely on parametric methods.

In summary, results are conclusive that green bonds have a low or negative correlation with other asset classes, such as equities, commodities, real estate, and energy markets, albeit the evidence in the literature is that investments in green bonds offer diversification benefits with respect to somewhat narrow asset classes. The choice of portfolio selection models with underlying assumptions about quadratic investor preferences and normally distributed return distributions casts a shadow that the conclusions of these earlier studies may not be valid for more general non-quadratic utility functions.

In this paper, we revisit the open question on the diversification benefits of green bonds, expanding the investigation towards broader asset classes and overcoming the limitations of parametric tests. First, we deviate from the previous literature by constructing optimal portfolios and assessing their performance in a non-parametric way using stochastic spanning (Arvanitis et al., 2019). That is, we examine whether optimal portfolios augmented by green bonds outperform portfolios constructed from conventional bonds, stocks, and other assets without any assumptions on the distributions of returns and for quite general utility functions. Second, we conduct the tests with a broader set of financial markets and consider the incremental benefits of green bonds only. Third, we conduct both in-sample and out-of-sample tests and find that the diversification benefits of green bonds are nuanced. Green bonds appear to be unequivocally beneficial in-sample but not out-of-sample. This raises a question about the potential of governments providing incentives for adopting green bonds. Our fourth contribution is to identify government interventions that render green bonds uniformly attractive. To the best of our knowledge, this is the most comprehensive empirical investigation of the relative efficiency of the green bond markets from the perspective of investors and the first non-parametric analysis.

We start our analysis with the largest and most active US and European green bond markets, as in Han and Li (2022) before moving to the global markets. We add green bonds to conventional (corporate and government) bonds and then expand further to the equities, real estate, commodities, and energy markets. Thus, we investigate whether including green bonds in portfolios alongside all other asset classes makes the internationally diversified USD risk-averse investor better off. We further study the impact of favorable tax rates on green bond positive returns (gains) compared to other financial assets. Specifically, we fix the tax rate for the financial assets (bonds, equities, energy, commodities, real estate indices) at a typical 20% while lowering the tax rate for green assets to find if there is a threshold that makes them attractive to investors during the good times. We also analyze the impact of tax credits on negative returns (losses) as an incentive during bad times. We find that reasonable tax treatments can strongly incentivize investors towards green bonds. Our analysis provides empirical validation to the opinion expressed in the survey by Stroebel and Wurgler (2021) that government subsidies can be an essential factor in inducing corporations and investors to switch towards more environmentally friendly behaviors.

We proceed in two steps. First, using stochastic spanning (Arvanitis et al., 2019), we test in-sample whether portfolios of conventional bonds and other financial assets do not span optimal portfolios augmented with green bonds. Then, using stochastic bounding (Arvanitis et al., 2021), we compare the out-of-sample realized performance of optimal portfolios with and without green bonds to assess any benefits for investors.

We find that green bonds provide clear diversification benefits in the US and European markets both in and out-of-sample, in line with Han and Li (2022), but obtain mixed results in the global markets. We find that green bonds are not spanned by conventional corporate and government bond indices as well as by representative global indices of the other financial markets. Thus, including them in the conventional asset universe will increase the portfolio's expected return per unit of risk. We also carry out this test considering a potential regime shift after the Paris Agreement entered into force on 4 November 2016 to test if the alignment of finance with climate goals stipulated in the Agreement affects the benefits of green bond diversification. We find benefits from the in-sample test for the whole period, as well as after the Paris Agreement.

However, the out-of-sample empirical results show that the augmented portfolios perform worse than portfolios constructed from investment sets without the green indices. This is especially true after the crash due to COVID-19 in March 2020. No benefits were identified when conducting the backtesting experiments one year after the Paris Agreement when the green bond market was expected to thrive. These results are corroborated by several commonly used parametric performance measures in addition to our non-parametric tests. Specifically, we use the Sharpe ratio (Sharpe, 1994), downside Sharpe ratio (Ziemba, 2005), upside potential and downside risk ratio (UP) (Sortino & Van Der Meer, 1991), and the opportunity cost (Simaan, 1993); see Appendix A for the definitions of these measures. The out-of-sample results cast doubts on the diversification benefits of green bonds documented in-sample. They are also at odds with earlier literature; we attribute this difference to our use of a non-parametric assumption-free methodology and of a broader set of global financial instruments. We consider our findings on the lack of consistent benefits from green bonds for global investors more reliable.

Given the negative out-of-sample findings, we consider potential incentives. Following Agliardi and Agliardi (2021), who find that the tax regime strongly directs investors' preferences on returns, we look at favorable tax rates that policymakers may adopt to scale up the green bond markets. Several incentives can be put in place.[4] We first consider reduced tax rates on the gains of green bonds to identify the lowest tax rate that will incentivize investors to add green bonds to their portfolios. We also consider tax credits to enhance the appeal of green bonds during market downturns. Our findings, especially during the significant market downturn of the COVID-19 pandemic, indicate that preferential tax rates or tax credits are needed to effectively incentivize investors to add green bonds to their portfolios.

[4] Incentives can be provided to the investors or the issuer, such as tax credit bonds (TCBs), direct subsidy bonds, and tax-exempt bonds. Investors receive tax credits instead of interest payments for the tax credit bonds. There are several types of TCBs, most of which are provided for a specific purpose, location, or type of project. An example is the US Federal Government Clean Renewable Energy Bonds and Qualified Energy Conservation Bonds program. The same programs are used for direct bond subsidy, with the issuers receiving cash rebates from the government to subsidize their net interest payments. Finally, for tax-exempt bonds, investors do not have to pay income tax on interest from the green bonds, and this type of tax incentive is typically applied to municipal bonds in the US market. For a discussion of such tax incentives see https://www.climatebonds.net/policy/policy-areas/tax-incentives.

1.1 Literature review

There is a growing literature examining the green bonds' connectedness with other financial classes and whether these relatively new instruments can provide diversification benefits for investors.

The earlier studies focused on price correlations and spillover effects between green bonds and other asset classes. Pham (2016) used a multivariate GARCH model to show a positive correlation between green and conventional bond markets, recommending a combination of both to enhance portfolio performance. Likewise, Reboredo (2018) showed that the green bond market is highly integrated with corporate and treasury bond markets and underscores the benefits of diversification in stock and energy markets. Reboredo and Ugolini (2020) used a structural VAR model to show that green bonds behave similarly to treasury bonds and USD exchange rates but have weak connections with high-yield corporate bonds, stocks, and the energy market. Reboredo et al. (2020) employed a wavelet-based model to show that green bonds provide hedging and diversification opportunities across various investment horizons for stocks, high-yield corporate bonds, and energy markets in the EU and US markets. Nguyen et al. (2021) examined correlations between green bonds and various assets, finding low or negative correlations with stocks and commodities, suggesting green bonds as potential diversifiers across all investment horizons. Pham and Nguyen (2021) analyzed cross-quantile tail-dependence between green bonds and other assets, identifying conditions where green bonds are less effective diversifiers. Hammoudeh et al. (2020) using time-varying Granger causality, showed green bonds offer diversification benefits for corporate and government bonds, which is not in full agreement with Reboredo (2018).

In a nutshell, the above studies suggest that green bonds may provide diversification benefits for investors in stocks, commodities, and the energy markets where they have weak linkage, but not for the conventional bond market, where correlations are very high. However, Hammoudeh et al. (2020) support that green bonds can be added to diversified portfolios that include commercial and sovereign bonds.

To establish diversification benefits one needs to construct portfolios, and there are only a few studies analyze the diversification benefits of green bonds withing a portfolio optimization framework. Fender et al. (2019) use an illustrative asset allocation example for USD- and Euro-denominated portfolios, albeit without optimal portfolio composition, to illustrate that adding both green and conventional bonds can provide diversification benefits. However, an illustrative example does not provide conclusive evidence since the portfolio without green bonds could be sub-optimal.

The papers closer to ours are Han et al. (2022), Han and Li (2022), Akhtaruzzaman et al. (2023). Han et al. (2022) and Han and Li (2022) study the investment value of the green bond market by comparing the performance of portfolios including green bonds with that of portfolios including conventional bonds in the US and European markets using the dynamic R-vine copula-based mean-CVaR model and in the Chinese market using the mean-variance model, respectively. Their results reveal that the portfolio with green bonds leads to higher risk-adjusted returns than the portfolio with conventional bonds across the different cases. Akhtaruzzaman et al. (2023) found that portfolios including a clean energy equity index as a proxy for a green asset, except for gold, crude oil, USD, and 3 M T-bill, outperformed the S&P 500 for short horizons using GARCH-EVT-copula-VaR and CVaR models with four portfolio strategies.

However, the implicit assumptions in these earlier works are known not to hold in the international markets (Christoffersen et al., 2012; You and Daigler, 2010), and green (and conventional) bonds have negative skewness and high positive kurtosis that deviate from normality. Hence, the conclusions of these earlier studies may not be valid for more general non-quadratic utility functions. To these works we add non-parametric tests without implicit assumptions of normality of returns or investor utility functions. We also perform the empirical tests with broader asset classes.

The paper is organized as follows. We describe our computational strategies for stochastic spanning and bounding tests in Sect. 2. In Sect. 3, we conduct the in- and out-of-sample tests for the US and European markets. In Sect. 4, we conduct the tests for the global markets and study the tax incentives. Section 5 concludes.

2 Stochastic dominance

Second-order stochastic dominance (SSD) ranks investments based on conditions that characterize decision-making under uncertainty with respect to the class of utilities that exhibit non-satiation and risk aversion. It is a model-free alternative to mean-variance (MV) dominance.[5] SSD is represented by sets of conditions in the form of lower partial moment inequalities between the compared distributions, defined by mild non-parametric restrictions on the distributions. The non-parametric nature of SSD makes it suitable for comparing investment strategies of securities with asymmetric, non-normal risk profiles.

Stochastic spanning (Arvanitis et al., 2019), is a model-free alternative to MV spanning (Huberman and Kandel, 1987; De Roon et al., 2003). Spanning occurs if introducing new securities (or relaxing investment constraints) does not improve the investment possibility set over a given class of investor preferences. Hence, stochastic spanning is suitable for checking whether portfolios augmented with green bonds dominate a broad market benchmark. If we were to add green bonds to a portfolio of conventional bonds or other financial assets and fail to reject the spanning hypothesis, then the additional bonds would be redundant for any risk-averse investor. We employ stochastic spanning to test empirically the null hypothesis H_0 vis-á-vis the alternative H_1:

H_0: Green bonds are spanned by a set of conventional financial assets.
H_1: There exist some portfolios augmented with green bonds that are not spanned by any set of conventional financial assets.

2.1 Preliminaries and definitions

We work with a portfolio space defined as the set of positive convex combinations of N benchmark assets represented by the set $\left\{\boldsymbol{\lambda} \in \mathbb{R}_+^N : \boldsymbol{\lambda}'\mathbf{1}_N = 1\right\}$. The benchmark assets are the vertices of the portfolio space. The returns of the benchmark assets form the random vector $X := (x_1, \dots, x_N)$, and we assume that its support is bounded by $\mathcal{X}^N := \left[\underline{x}, \overline{x}\right]^N$, $-\infty < \underline{x} < \overline{x} < +\infty$.

F denotes the continuous CDF of X, and $F(y, \boldsymbol{\lambda}) := \int 1(X^{\mathrm{T}}\boldsymbol{\lambda} \leq y)dF(X)$ the marginal CDF for portfolio $\boldsymbol{\lambda}$. The CDF integral $L(x, \boldsymbol{\lambda}; F) := \int_{\underline{x}}^{x} F(y, \boldsymbol{\lambda})dy$ equals the first-order lower-partial moment (LPM), or expected shortfall $\int_{\underline{x}}^{x}(x - y)dF(y, \boldsymbol{\lambda})$, for each return threshold $x \in \mathcal{X}$ (Bawa, 1975). Let $D(x, \boldsymbol{\lambda}, \boldsymbol{\kappa}; F) := L(x, \boldsymbol{\lambda}; F) - L(x, \boldsymbol{\kappa}; F)$, denotes the LPM spread between portfolios $\boldsymbol{\lambda}$ and $\boldsymbol{\kappa}$. Then, $\boldsymbol{\lambda}$ stochastically dominates $\boldsymbol{\kappa}$ by SSD, or

[5] See the surveys by Levy (2016), Whang (2019), Perrakis (2019) and representative financial applications, such as, Constantinides et al. (2009), Constantinides et al. (2011), Hodder et al. (2015).

$\boldsymbol{\lambda} \succeq_F \boldsymbol{\kappa}$, iff $D(x, \boldsymbol{\lambda}, \boldsymbol{\kappa}; F) \leq 0,\ \forall x \in \mathcal{X}^{\mathbb{N}}$. SSD implies that $\boldsymbol{\lambda} \succeq_F \boldsymbol{\kappa}$ iff $\boldsymbol{\lambda}$ achieves a higher expected utility than $\boldsymbol{\kappa}$ for every increasing and concave utility function.

We consider the nested sets of assets $\mathrm{K} \subset \Lambda$, where K is the convex hull of the benchmark assets and Λ is the convex hull of K augmented with green bonds. We use stochastic spanning to compare optimal portfolios from these sets to test the effect of augmenting the set of benchmark assets with green bonds,

2.2 Stochastic spanning

Definition 1 (Stochastic spanning) K spans Λ by SSD iff for every portfolio $\boldsymbol{\lambda} \in \Lambda$ there exists an SSD portfolio $\boldsymbol{\kappa} \in \mathrm{K}$. That is, $\forall \boldsymbol{\lambda} \in \Lambda, \exists \boldsymbol{\kappa} \in \mathrm{K} : \forall x \in \mathcal{X}^{\mathbb{N}},\ D(x, \boldsymbol{\kappa}, \boldsymbol{\lambda}; F) \leq 0$.

Using the continuity properties of $D(\cdot, \cdot, \cdot; F)$ and the compactness of sets $\Lambda,\ K,\ \mathcal{X}$, it is easy to characterize spanning by the scalar-valued functional of F,

$$\eta(F) := \sup_{\boldsymbol{\lambda} \in \Lambda} \inf_{\mathrm{k} \in \mathrm{K}} \sup_{\mathcal{X}} D(x, \boldsymbol{\kappa}, \boldsymbol{\lambda}; F). \tag{1}$$

Spanning occurs iff $\eta(F) = 0$, while some $\boldsymbol{\lambda} \in \Lambda$ exist that are not stochastically dominated by any portfolio $\boldsymbol{\kappa} \in K$ by SSD (i.e., no spanning occurs), iff $\eta(F) > 0$.

To perform hypothesis testing, we note that F is latent so $\eta(F)$ is unknown. The analyst has access to a time series sample of realized returns $(X_t)_{t=1}^T,\ X_t \in \mathcal{X}$, t = 1, ..., T, for the benchmark assets. Assuming stationarity and mixing for the benchmark asset return process, an empirical analogue of $\eta(F)$ scaled by $\sqrt{T}$ is used as a Kolmogorov-Smirnov type test statistic for the null

$$\eta_T := \sqrt{T} \sup_{\boldsymbol{\lambda} \in \Lambda} \inf_{\mathrm{k} \in \mathrm{K}} \sup_{\mathcal{X}} D(x, \boldsymbol{\kappa}, \boldsymbol{\lambda}; F_T), \tag{2}$$

where F_T denotes the empirical CDF associated with the sample.

The asymptotic decision rule is to reject H_0 in favor of H_1 iff $\eta_T > q(\eta_\infty, 1-\alpha)$, the $(1-\alpha)$ quantile of the distribution of η_∞, for significance level $\alpha \in]0, 1[$. Because the distribution of $q(\eta_\infty, 1-\alpha)$ depends on the underlying distribution, we use the subsampling procedure of Arvanitis et al. (2019) to approximate it by feasible decision rules. Specifically, given the choice of the subsampling rate $1 \leq b_T < T$, we generate the maximally overlapping subsamples $(X_s)_{s=t}^{t+b_T-1}, t = 1, \cdots, T - b_T + 1$, evaluate the test statistic on each subsample, thereby obtaining $\eta_{b_T;T,t}$ for $t = 1, \cdots, T - b_T + 1$, and evaluate $q_{T,b_T}(1-\alpha)$, the $(1-\alpha)$ quantile of the empirical distribution of $\eta_{b_T;T,t}$ across the subsamples. The implementable decision rule is to reject H_0 in favor of H_1 iff $\eta_T > q_{T,b_T}(1-\alpha)$.

2.3 Stochastic bounding

We also test out-of-sample the performance of optimal benchmark and augmented portfolios using *stochastic bounding*. The stochastic bounding portfolio is the non-spanned portfolio that spans any portfolio constructed from the set of given assets. The stochastic bounding portfolio dominates any portfolio that can be constructed with respect to the SSD criterion. We get the stochastic bounding portfolio from the benchmark and the augmented set to draw inferences about any diversification benefits from the augmentation.

In this case, we set $\Lambda = \mathrm{K}$ and search for a portfolio $\lambda \in \Lambda$ that stochastically dominates every other portfolio in Λ. We use the method of Arvanitis et al. (2021) to get the portfolio

λ that stochastically bounds portfolio set Λ. We assume Λ to be a convex polytope to allow for linear programming formulations.

2.4 Computational strategies

We now give the computational strategies for our two tests.

2.4.1 Spanning

The test statistic η can be represented in terms of expected utility as:

$$\eta(F) := \sup_{\boldsymbol{\lambda}\in\Lambda; u\in\mathcal{U}} \inf_{\boldsymbol{\kappa}\in\mathrm{K}} \mathbb{E}_F\left[u\left(X^{\mathrm{T}}\boldsymbol{\lambda}\right) - u\left(X^{\mathrm{T}}\boldsymbol{\kappa}\right)\right] \tag{3}$$

$$\mathcal{U} := \left\{u \in \mathcal{C}^0 : u(y) = \int_{\underline{x}}^{\overline{x}} v(x)r(y;x)dx \ v \in \mathcal{V}\right\} \tag{4}$$

$$\mathcal{V} := \left\{v : \mathcal{X} \to \mathbb{R}_+ : \int_{\mathcal{X}} v\left(x\right) = 1\right\} \tag{5}$$

$$r(y;x) := (y-x)1(y \leq x), \ (x,y) \in \mathcal{X}^2. \tag{6}$$

$\mathcal{U}$ is comprised of normalized, increasing and concave utility functions that are constructed as convex mixtures of elementary Russell and Seo (1989) ramp functions $r(y;x),\ x \in \mathcal{X}$.[6] This implies that K spans Λ, iff for any $\boldsymbol{\lambda} \in \Lambda$ there exists some $\boldsymbol{\kappa} \in \mathrm{K}$, weakly preferred to the former, by every utility in $\mathcal{U}$. Equivalently, spanning occurs iff no risk averter in $\mathcal{U}$ loses expected utility from the excision $\Lambda - \mathrm{K}$. This representation can be used for the numerical implementation of the testing procedure.

The test statistic can be expressed as

$$\eta_T := \sqrt{T}\sup_{u\in\mathcal{U}}\left(\sup_{\boldsymbol{\lambda}\in\Lambda}\mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\lambda}\right)\right] - \sup_{\boldsymbol{\kappa}\in\mathrm{K}}\mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\kappa}\right)\right]\right). \tag{7}$$

The computational complexity of evaluating η_T stems from the complexity of $\mathcal{U}$. Following Arvanitis et al. (2019) we approximate every element of $\mathcal{U}$ with arbitrary prescribed accuracy using a finite set of increasing and concave piecewise-linear functions. Let $N_1, N_2 \geq 2$ denote integers. We partition $\mathcal{X}$ into N_1 equally spaced values as $\underline{x} = z_1 < \cdots < z_{N_1} = \overline{x}$, where $z_n := \underline{x} + \frac{n-1}{N_1-1}(\overline{x}-\underline{x}), n = 1, \cdots, N_1$, and partition $[0,1]$ as $0 < \frac{1}{N_2-1} < \cdots < \frac{N_2-2}{N_2-1} < 1$. Using those partitions, consider

$$\underline{\eta_T} := \sqrt{T}\sup_{u\in\underline{\mathcal{U}}}\left(\sup_{\boldsymbol{\lambda}\in\Lambda}\mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\lambda}\right)\right] - \sup_{\boldsymbol{\kappa}\in\mathrm{K}}\mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\kappa}\right)\right]\right) \tag{8}$$

$$\underline{\mathcal{U}} := \left\{u \in \mathcal{C}^0 : u(y) = \sum_{n=1}^{N_1} v_n r(y;z_n) \ v{\in}V\right\} \tag{9}$$

$$V := \left\{v \in \left\{0, \frac{1}{N_2-1}, \cdots, \frac{N_2-2}{N_2-1}, 1\right\}^{N_1} : \sum_{n=1}^{N_1} v_n = 1\right\}. \tag{10}$$

[6] This simplifies the portfolio selection problem since using the Russel and Seo utility functions, investor choices are not wealth-dependent.

Every $u \in \underline{\mathcal{U}}$ consists of at most N_2 linear segments with endpoints at N_1 possible outcome levels. Furthermore $\underline{\mathcal{U}} \subset \mathcal{U}$, it is finite as it has $N_3 := \frac{1}{(N_1-1)!}\prod_{i=1}^{N_1-1}(N_2+i-1)$ elements, and $\underline{\eta_T}$ approximates η_T from below as the partitioning scheme is refined. Then for every $u \in \underline{\mathcal{U}}$, the two embedded maximization problems in (8) can be solved using linear programming. Consider

$$c_{0,n} := \sum_{m=n}^{N_1} \left(c_{1,m+1} - c_{1,m}\right) z_m \tag{11}$$

$$c_{1,n} := \sum_{m=n}^{N_1} w_m \tag{12}$$

$$\mathcal{N} := \{n = 1, \cdots, N_1 : v_n > 0\} \bigcup \{N_1\}. \tag{13}$$

For any $u \in \underline{\mathcal{U}}$, $\sup_{\boldsymbol{\lambda} \in \Lambda} \mathbb{E}_{F_T}\left[u\left(X^{\mathrm{T}}\boldsymbol{\lambda}\right)\right]$ is the optimal objective function value of the linear program

$$\begin{aligned}
\max T^{-1} \sum_{t=1}^{T} y_t \\
\text{s.t. } y_t - c_{1,n} X_t^{\mathrm{T}} \boldsymbol{\lambda} \leq c_{0,n},\ t = 1, \cdots, T; n \in \mathcal{N} \\
\sum_{i=1}^{M} \lambda_i = 1 \\
\lambda_i \geq 0,\ i = 1, \cdots, M \\
y_t \text{ free},\ t = 1, \cdots, T.
\end{aligned} \tag{14}$$

This linear program always has a feasible solution and is tractable for typical data dimensions.

For our empirical work, we use the entire history of available quarterly investment returns of a large set of standard benchmark assets. For example, for the US data we have $M = 5$, $T = 1862$, with $N_1 = 10$ and $N_2 = 5$. This gives $N_3 = \frac{1}{9!}\prod_{i=1}^{9}(4+i) = 715$ distinct utility functions with 1,430 small linear programming problems. The number of assets varies up to 16, depending on the markets we are testing The total run time for our tests is a few working days on a desktop PC with a 2.93 GHz quad-core Intel i7 processor and 16GB of RAM, using MATLAB and GAMS with the Gurobi solver.

2.4.2 Bounding

The bounding methodology considers all portfolios in Λ and the joint empirical support generally consists of infinitely many points, introducing the need for discretization.

Let $\hat{\mathcal{X}}_\Lambda := \left[\hat{A}_\Lambda, \hat{B}_\Lambda\right]$, $\hat{A}_\Lambda := \min_{\Lambda,\lambda} \boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\lambda}$ and $\hat{B}_\Lambda := \max_{\Lambda,t} \boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\lambda}$. We partition $\hat{\mathcal{X}}_\Lambda$ using J equally spaced grid points $\hat{x}_j := \hat{A}_\Lambda + (j-1)\left(\hat{B}_\Lambda - \hat{A}_\Lambda\right)(J-1)^{-1}$, $j = 1, \cdots, J$. For every grid point, let $\hat{L}^*_{\Lambda,j} := \min_\Lambda L(\boldsymbol{\lambda}, \hat{x}_j, \hat{F})$, $j = 1, \cdots, J$, and we use the approximation

$\xi(\Lambda, \Lambda, \sqrt{T}\hat{F}) \approx \sqrt{T}\chi_J(\Lambda, \Lambda, \hat{F})$, where

$$\begin{aligned}\chi_J(\Lambda, \Lambda, \hat{F}) &:= \min_{\boldsymbol{\lambda}\in\Lambda} \max_{\boldsymbol{\lambda}\in\Lambda, j} D(\boldsymbol{\lambda}, \boldsymbol{\lambda}, \hat{x}_j, \hat{F}) \\ &= \min_{\boldsymbol{\lambda}\in\Lambda} \max_{j} \left(L(\boldsymbol{\lambda}, \hat{x}_j, \hat{F}) - \hat{L}^*_{\Lambda,j}\right) \\ &= \min_{\boldsymbol{\lambda}\in\Lambda,\sigma} \left(\sigma : L(\boldsymbol{\lambda}, \hat{x}_j, \hat{F}) - \sigma \le \hat{L}^*_{\Lambda,j},\ j = 1, \cdots, J\right). \end{aligned} \tag{15}$$

The number of grid points $J = 100$ balances accuracy with solution time and is highly accurate for typical applications.

The approximate $\chi_J(\Lambda, \Lambda, \hat{F})$ can be computed by solving a series of linear programs using linear relaxations (Rockafellar et al., 2000). Each value $\hat{L}^*_{\Lambda,j}$, $j = 1, \cdots, J$, can be computed as the optimal value of the objective function of the following program

$$\begin{aligned} &\min_{\boldsymbol{\lambda}\in\Lambda} T^{-1}\sum_{t=1}^{T}\eta_{j,t} \\ -\eta_{j,t} - \boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\lambda} &\le -\hat{x}_j,\ t = 1, \cdots, T \\ \eta_{j,t} &\ge 0,\ t = 1, \cdots, T \\ \boldsymbol{\lambda} &\in \Lambda. \end{aligned} \tag{16}$$

Given the solutions to the J problems, $\chi_J(\Lambda, \Lambda, \hat{F})$ can be computed by solving

$$\begin{aligned} &\min_{\boldsymbol{\lambda}\in\Lambda} \sigma \\ T^{-1}\sum_{t=1}^{T}\theta_{j,t} - \sigma &\le \hat{L}^*_{\Lambda,j},\ j = 1, \cdots, J \\ -\theta_{j,t} - \boldsymbol{x}_t^{\mathrm{T}}\boldsymbol{\lambda} &\le -\hat{x}_j,\ j = 1, \cdots, J;\ t = 1, \cdots, T \\ \theta_{j,t} &\ge 0,\ j = 1, \cdots, J;\ t = 1, \cdots, T \\ \boldsymbol{\lambda} &\in \Lambda \\ \sigma &\text{ free.} \end{aligned} \tag{17}$$

The optimal solution identifies the portfolio that stochastically spans but is not spanned by any portfolio in $\lambda \in \Lambda$.

3 US and European markets

We start our analysis with the US and European markets that have the most active green bond markets in recent years; existing literature mainly focuses on them. We test whether a set of benchmark assets spans the set augmented with green bonds. If green bonds are spanned, including them in the benchmark asset universe will not increase the portfolio's expected return per unit of risk, and they do not provide diversification benefits.

3.1 Data

We use the data of Han and Li (2022) for benchmark assets comprising stocks, commodities, real estate, and bonds. Daily closing prices in USD are sourced from Thomson Reuters Eikon and Bloomberg, spanning Dec. 31, 2013, to March 11, 2021.

The benchmark dataset consists of the US and European conventional bond markets represented by the BBG Barclays US Aggregate Total Return Index Value Unhedged USD and BBG Barclays Euro Aggregate Total Return Index Value Unhedged USD, respectively. Additionally, it comprises equities (MSCI USA Index and MSCI Europe Index, respectively), commodities (S&P GSCI, S&P WCI Europe Index), and real estate (Dow Jones U.S. Real Estate Index, Dow Jones Europe Select Real Estate Securities Index). For the green bond markets, we use the BBG Barclays MSCI US Green Bond Total Return Index Unhedged USD and BBG Barclays MSCI Euro Green Bond Total Return Index Unhedged USD, for the two markets, respectively. There are Exchange Traded Funds (ETFS) that track all the indices we use in the analysis. Retail investors are able to directly invest in these ETFs..

Summary statistics for all input time series are given in Appendix C. The Jarque-Berra tests indicate that asset returns are not normally distributed, justifying our choice of stochastic dominance and questioning the findings from tests that rely on mean-variance analysis.

3.2 In-sample testing

We test in-sample the null hypothesis that green bonds are spanned by the benchmark set, using stochastic spanning. To obtain a powerful and efficient test for our small sample size, we use a bias correction procedure for the quantile estimates $q_{T,b_T}(1-\alpha)$ to mitigate their sensitivity on the choice of b_T in finite samples. We choose sample sizes $b_T = \lfloor T^c \rfloor$, with c ranging from 0.6 to 0.9 (Arvanitis et al., 2019). Using OLS regression on the empirical quantiles $q_{T,b_T}(1-\alpha)$ for significance level $\alpha = 0.05$, we get the estimate q_T^{BC} for the critical value. We reject spanning if the test statistic η_T is higher than q_T^{BC}.

Table 1 reports the test statistics η_T, and the regression estimates q_T^{BC} for the full sample. Panel A displays the test results for the US market, and Panel B for the European. Spanning is rejected in both cases.

Our finding aligns and significantly strengthens Han and Li (2022). They consider either green bonds or conventional bonds in the benchmark portfolio of stocks, commodities, and real estate separately and find that portfolios with green bonds outperform conventional bonds. Our test goes further to show that the inclusion of green bonds in benchmark portfolios that already include conventional bonds provides diversification benefits, i.e., we do not restrict investors' choice between green or conventional bonds.

3.3 Out-of-sample testing

We carry out rolling window backtesting experiments to form optimal benchmarks and augmented portfolios to test if the latter outperforms the former out-of-sample. This test assesses the diversification benefits of green bonds in a more realistic setting by mimicking a real-time investor. Following Han and Li (2022), we run the model for a 60-day horizon using the previous 1000 days for calibration and repeat this process by moving the sample period 60 days forward. We start with the period Jan. 2, 2014 to Nov. 9, 2017 for the first calibration and repeat the rolling window test fourteen times until Feb. 9, 2021.

Table 1 In-sample spanning tests for the US and European market.

Period	Test statistic η_T	Regression estimates q_T^{BC}	Result
(a) US market			
Full sample	0.0019	0.0014	Reject spanning
(b) European market			
Full sample	0.0024	0.0013	Reject spanning

Stochastic Spanning tests for the US (Panel A) and European (Panel B) markets. Tests are conducted for the full sample from 31/12/2013 to 11/3/2021. The dataset spans for a total of 1862 daily returns. Entries report the test statistics η_T as well as the regression estimates q_T^{BC} to test in-sample the null hypothesis

Figure 1 illustrates the out-of-sample cumulative returns of the optimal benchmark and augmented portfolios for the US (Panel A) and European (Panel B) markets. The figures suggest that the augmented portfolio outperforms the benchmark, in line with the in-sample spanning tests.

We compute several performance measures to compare the results from the figures. Specifically, we report the mean return, volatility, Sharpe ratio, downside Sharpe ratio, UP ratio, and opportunity cost. The downside Sharpe and UP ratios are more appropriate measures of performance than Sharpe, given the asymmetric return distributions.

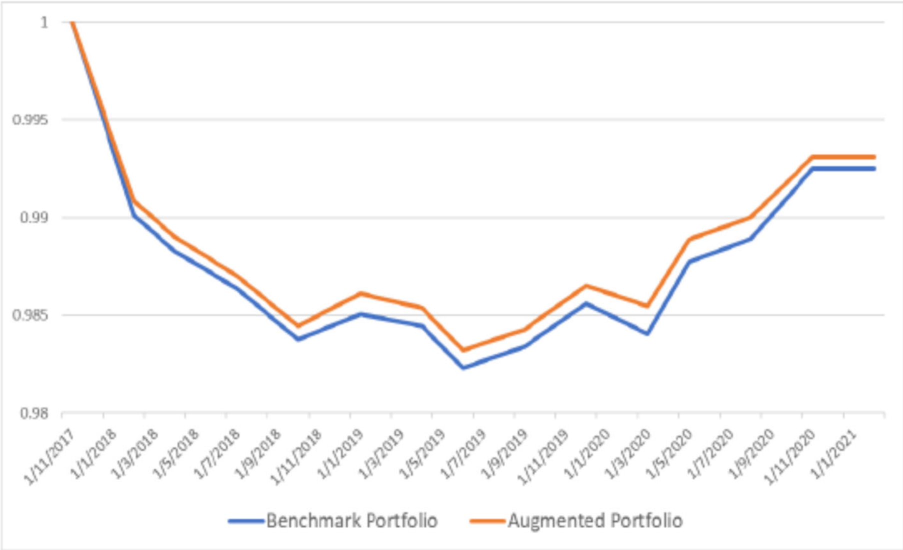
Table 2 reports the results for both portfolios. We observe that the augmented portfolio exhibits higher returns and lower risk, with higher Sharpe, downside Sharpe, and UP ratios. The opportunity cost θ, computed with different utility functions and different levels of risk aversion, is also positive, implying that we should add a positive return to the benchmark portfolio to have the same expected utility as the augmented. Hence, the portfolio with green bonds outperforms the benchmark, as suggested by the figure.

We finally conduct various robustness checks to evaluate the sensitivity of our results to different estimation windows and rebalancing frequencies, as in Han and Li (2022), and report the results in Appendix B. Our findings are robust with all performance measures and for all robustness test specifications.

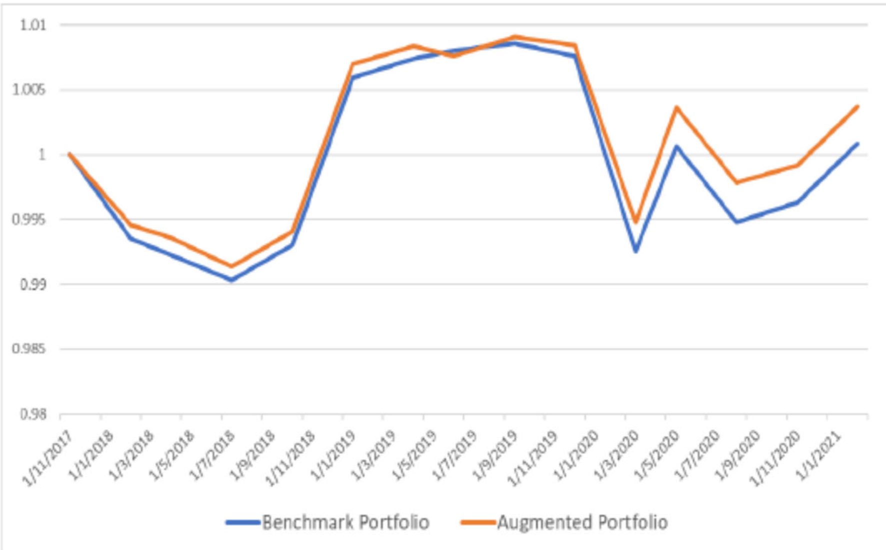
In conclusion, a portfolio augmented with green bonds consistently outperforms the benchmark. This out-of-sample result is consistent with our in-line test above. We document in and out-of-sample performance gains when including green bonds, in addition to conventional bonds, in benchmark portfolios, thereby also extending Han and Li (2022). This is a strong result, albeit it holds only in the local US and European markets.

4 Global markets

We take a step further to ask whether the spanning of green bonds is also rejected in the global markets. We use the available global green bond indices to augment a broad benchmark set of global indices of government and corporate bonds, equities, real estate, commodities, and energy markets. We test whether the benchmark assets span the augmented set both in- and out-of-sample. To the best of our knowledge, this is the most comprehensive empirical investigation of the relative efficiency of the green bond markets.



(a) US market



(b) European market

Fig. 1 Out-of-sample testing in the US and European market. Cumulative performance of the benchmark optimal portfolio and the optimal augmented portfolio with green bonds. Panel A is the case of the US market, and Panel B is for the European market. The out-of-sample test is for the full sample with 1000-day estimation windows and 60-day rebalancing frequency

Table 2 Out-of-sample portfolio performance in the US and European market

	(a) US market		(b) European market	
Performance measures	Benchmark	Augmented	Benchmark	Augmented
Mean	−12.52	−11.60	2.033	7.359
Volatility	5.427	5.078	10.53	10.17
Sharpe Ratio	−2.509	−2.504	0.065	0.585
Downside_SR	−2.010	−2.006	0.063	0.599
UPratio	0.315	0.323	0.489	0.550
Opportunity Cost				
exponential utility				
ARA=2		1.089		5.301
ARA=4		1.115		5.381
ARA=6		1.140		5.434
power utility				
RRA=2		1.089		5.301
RRA=4		1.115		5.381
RRA=6		1.140		5.461

Entries report the parametric performance measures (annualized mean and volatility, annualized Sharpe ratio, annualized downside Sharpe ratio, UP ratio, and annualized opportunity cost) for the benchmark and the augmented portfolio with green bonds. Panel A is for the US market and Panel B is for the European market. The dataset spans the whole period from 31/12/2013 to 11/3/2021, for a total of 1862 daily returns. The out-of-sample test is for the full sample with 1000-day estimation windows and 60-day rebalancing frequency. Mean, volatility, and opportunity cost (θ) are in %. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) for the exponential utility function and different degrees of relative risk aversion (RRA=2,4,6) for the power utility function

4.1 Data

We consider incrementally three benchmark portfolios. First, a benchmark consisting of corporate bond indices only, using the S&P IG Corp BI, S&P INT Corp BI, ICE BofA Global Corp BI, FTSE World Corp BI, and BBG Barclays Global Agg Corp BI indices. Second. we add government bonds (ICE BofA Global Gov BI, FTSE World Gov BI, BBG Barclay Global Agg Gov BI, and 5Yr, 10Yr, 30Yr US Benchmark Gov BI). Finally, we consider a broad benchmark of global indices of five financial markets. Namely, (i) stocks (MSCI World Index), (ii) real estate (DJ Global Select RESI), (iii) commodities (S&P GSCI Commodity Index), (iv) energy markets (S& GSCI Energy Spot Index), and (v) the global bond market (BBG Barclays Global Agg Index). Thus, we test successively if green bonds offer diversification benefits to investors in the corporate bond market, corporate and government bond markets, and a broad universe of financial markets.

The green bond market is represented by five global indices (S&P Green Bond Index, S&P Green Bond Select Index, ICE BofA Green Bond Index, Bloomberg Barclays MSCI Green Bond Index, and Solactive Green Bond Index). We employ multiple indices to cover the broadest set of issued green bonds since each index uses a different methodology and criteria for including bonds. The indices are not perfectly correlated, with correlation coefficients ranging from 0.43 to 0.99.

We use data on daily closing USD prices obtained from Thomson Reuters Eikon and Bloomberg. The dataset spans the period from Dec. 2, 2014, when all green bond indices

Table 3 Spanning tests for the global green bond indices. Stochastic Spanning tests for the five global green bond indices

Period	Test statistic η_T	Regression estimates q_T^{BC}	Result
(a) Corporate bonds			
Whole	0.0061	0.0060	Reject spanning
12 m after	0.0031	0.0024	Reject spanning
(b) Corporate and government bonds			
Whole	0.0030	0.0028	Reject spanning
12 m after	0.0829	0.0554	Reject spanning
(c) Five markets			
Whole	0.0168	0.0157	Reject spanning
12 m after	0.0064	0.0057	Reject spanning

Panel A is for the case where the benchmark Portfolio includes only Corporate bond indices, Panel B is for the case where the benchmark Portfolio includes corporate and government bond indices, Panel C is for the five different markets (stock, commodity, real estate, energy and bond market), one global index from each market, and Panel D is for the five different markets (stock, commodity, real estate, energy and bond) and Corporate with Government bond indices from the bond market. Tests are conducted for the whole period and one year after the Paris Agreement that entered into force on 4 November 2016. Entries report the test statistics η_T as well as the regression estimates q_T^{BC} in order to test in-sample the null hypothesis. The dataset spans the whole period from 2/12/2014 to 17/9/2021, for a total of 1767 daily returns

were introduced, to Sept. 17, 2021, for a total of 1767 daily observations. Appendix C provides summary statistics of all market indices over the sample period and their correlation coefficients. Jarque-Berra tests indicate that asset returns are not normally distributed, justifying our use of non-parametric stochastic spanning tests instead of the parametric models used in earlier literature.

4.2 In-sample testing

We repeat the in-sample test of Sect. 3.2 for the global markets and report the results in Table 3 for the three different benchmark assets. We report results for the whole period, (from 2/12/2014 to 17/9/2021), as well as the period starting one year after the Paris Agreement that entered into force on November 4, 2016 and provided a boost for the green bonds market.[7] Panel A exhibits results for the benchmark portfolio of global corporate bond indices, Panel B for the benchmark with corporate and government bonds, and Panel C for the broader benchmark set.

We observe that we reject the null hypothesis of spanning in favor of the alternative for all three benchmark test assets and both periods. The in-sample tests indicate that green bonds could provide diversification benefits to USD investors in the global markets, even with respect to a broad benchmark portfolio of five asset classes.

[7] We also tested for different subperiods before and after the Paris Agreement as well as for the whole period, and in all cases we rejected the null hypothesis of spanning. The findings of this section are robust to the use of different periods.

4.3 Out-of-sample testing

We finally conducted out-of-sample backtesting. We form optimal portfolios from the benchmark and the augmented asset sets and carry out weekly rolling window backtesting experiments. The rolling window covers the period from Dec. 12, 2014 to Sept. 17, 2021, for a total of 354 weekly returns. The first calibration period is until Nov. 3, 2017, and each week, we use data from the preceding 152 weeks for calibration. We obtain stochastic bounding benchmark and augmented portfolios, advance the clock for a week, and calculate the ex-post realized returns. This procedure is repeated 202 times until Sept. 17, 2021. We then compare the ex-post realized returns using the parametric performance measures of both portfolios over the entire period and also perform a non-parametric pairwise stochastic non-dominance test (Anyfantaki, Maasoumi, Ren, and Topaloglou, 2021).

Definition 2 **(Stochastic non-dominance)** *The augmented portfolio λ does not strictly second order stochastically dominate the benchmark portfolio κ* iff

$$\exists z \in \mathcal{Z} : \ D(z, \lambda, \kappa, F) > 0, \text{ or } \forall z \in \mathcal{Z} : \ D(z, \lambda, \kappa, F) = 0.$$

Strict second-order stochastic non-dominance holds iff κ achieves a higher expected utility for some non-decreasing and concave utility function or achieves the same expected utility for every non-decreasing and concave utility function. Equivalently, strict stochastic non-dominance holds iff κ is strictly preferred to λ by some risk averter, or every risk averter is indifferent between them.

We test the null hypothesis H_0' vis-á-vis the alternative H_1':

- $\mathbf{H_0'}$: The augmented portfolio λ does not strictly second order stochastically dominate the benchmark portfolio κ,
- $\mathbf{H_1'}$: The augmented portfolio λ stochastically dominates the benchmark portfolio κ,

The test statistic for the pairwise comparison of two portfolios is

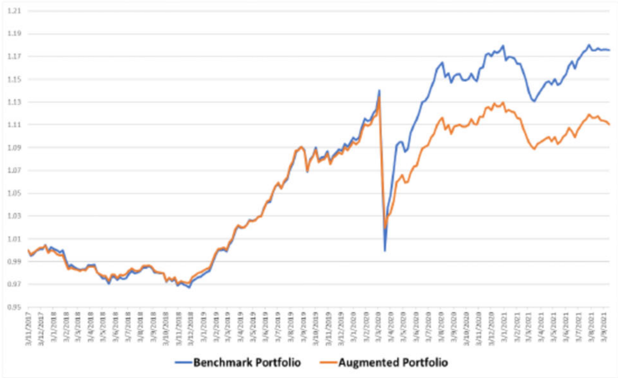
$$\xi_T = \sup_{z \in \mathcal{Z}} D(z, \kappa, \lambda, F). \tag{18}$$

To calculate the p value, we use block-bootstrapping. The p value is approximated by $\tilde{p}_j = \frac{1}{R}\sum_{r=1}^{R}\{\xi^{\star}_{T,r} > \xi_T\}$, where ξ_T is the test statistic, $\xi^{\star}_{T,r}$ is the bootstrap test statistic, averaging over $R = 1000$ replications. We reject the null if the p value is less than 0.05.

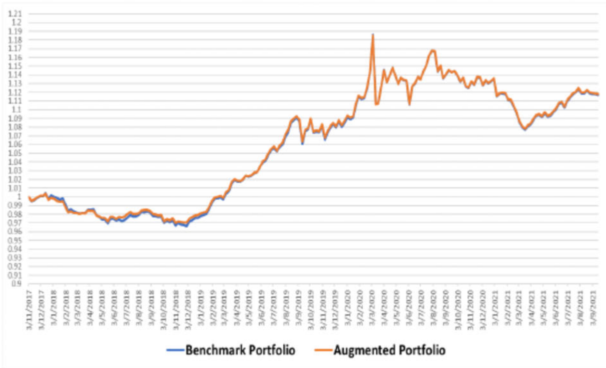
We additionally test for the non-dominance of the benchmark portfolio over the augmented using the Davidson and Duclos (2013) stochastic non-dominance test, as a robustness check. We do that for two reasons. First, the test allows for correlated samples. Second, the Davidson and Duclos (2013) test has also as null hypothesis that one portfolio does not stochastically dominate another, i.e., the non-dominance. The test is discribed in the Appendix D.

4.3.1 Uniform taxation on green and conventional assets

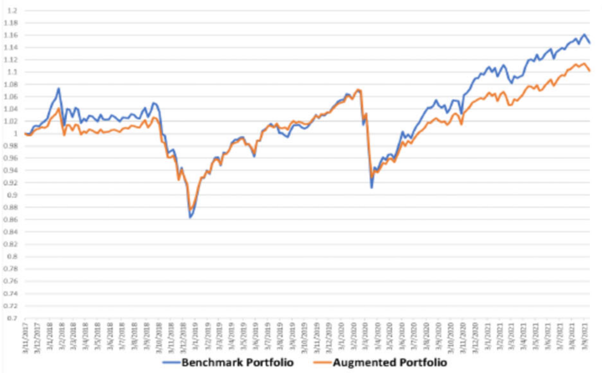
We first perform the test using the same tax rate on realized returns for all asset classes. Like in our earlier tests, we assume zero taxes without loss of generality. Figure 2 illustrates the out-of-sample cumulative returns of the optimal benchmark and augmented portfolios. Panel A shows the results for the benchmark portfolio with only corporate bonds, and we observe that the augmented portfolio has lower performance after the COVID-19 crash. Panel B is for the benchmark portfolio that also includes government bonds, and we observe almost



(a) Corporate bonds



(b) Corporate and government bonds



(c) Five markets

Fig. 2 Out-of-sample testing without taxes. Cumulative performance of the benchmark optimal portfolio and the optimal augmented portfolio with green bond indices for each case. The dataset spans the period 12/12/2014–17/9/2021, with out-of-sample testing conducted over the period 3/11/2017–17/9/2021, 12 months after the Paris Agreement identical performance with the augmented. In Panel C, the benchmark invests in five different markets with the augmented portfolio underperforming.

Overall, the figure suggests that green bonds do not improve portfolio performance. This conclusion is supported by comparing in Table 4, Panel A, the parametric performance measures of all time series of Fig. 2. We observe that the inclusion of green bonds does not improve the performance of portfolios of corporate bonds or the five financial markets. The benchmark portfolios have higher annualized Sharpe and downside Sharpe ratios. The opportunity cost is negative, i.e., we need to subtract a return from the benchmark portfolio to have equal expected utility with the augmented. The augmented portfolio performs only marginally better than the benchmark of corporate and government bonds. The pair-wise non-dominance hypothesis testing results in Panel B of Table 4 show that the null of non-dominance cannot be rejected for all benchmark portfolios.

In conclusion, international portfolios that include green bonds perform no better than the benchmarks out-of-sample. This contrasts our findings for the US and European markets. The global green bond indices exhibit higher correlations with other global indices compared to US and European assets, making it harder to achieve diversification benefits in the global market. Additionally, in contrast to the US and European markets, global green bonds have less favourable performance (with lower volatility, their return is the lowest resulting in low Shape ratios) when compared to global corporate bonds and five markets, making them less preferred in portfolio optimization. Moreover, the green bond indices differ significantly from other global indices in their currency composition, which also impacts returns (Ehlers & Packer, 2017). Hence, while green bonds may offer diversification benefits in narrow markets, they are not attractive investments for internationally diversified investors. This suggests a need for incentives to make them universally attractive and spur their further adoption based on portfolio selection criteria. We turn to such potential incentives next.

4.3.2 Preferential taxation

We consider the preferential tax treatment of green bonds. Specifically, we impose the standard tax rate of 20% on benchmark asset returns and gradually decrease the tax rates for green bonds until they become beneficial for global investors.

Figure 3 illustrates the out-of-sample cumulative returns of the optimal benchmark and augmented portfolios with a tax rate of 15 or 5% on the green bond indices. We observe (Panels A–D) that the green bonds improve the performance of the benchmark portfolios with conventional corporate and government bonds only when the tax rate is decreased to 5%. However, even a modest tax reduction from 20 to 15% makes green bonds attractive for investors in the five global financial assets (Panels E–F).

We report the parametric and non-parametric performance measures for all time series of this figure in Table 5, including results with an intermediate green bond tax rate of 10%. Panel A reports the results with the parametric performance measures, and Panel B with the non-parametric stochastic dominance test. Sub-panels (i), (ii), and (iii) are for benchmark portfolios with corporate bonds, corporate and government bonds, and all five markets, respectively. A tax rate of 5% makes green bonds uniformly attractive against all benchmarks and with all performance criteria (all panels). With tax rate of 10% they remain marginally attractive (*p* value 0.10) with respect to the five markets, Panel B (iii). In conclusion, international investors must be incentivized with substantial tax reductions to include green bonds in their portfolios.

Table 4 Out-of-sample portfolio performance without taxes. In Panel A, entries report the parametric performance measures (annualized mean and volatility, annualized Sharpe ratio, annualized downside Sharpe ratio, UP ratio, portfolio turnover, annualized return-loss and annualized opportunity cost) for the benchmark and the augmented portfolio with green bond indices without taxes in each case

	(a) Parametric performance measures					
	(i) Corporate bonds		(ii) Corporate and government bonds		(iii) Five markets	
	Benchmark	Augmented	Benchmark	Augmented	Benchmark	Augmented
Mean	4.458	2.856	3.074	3.088	4.062	2.823
Volatility	6.152	4.877	5.918	5.806	9.514	7.522
Sharpe Ratio	0.529	0.342	0.318	0.327	0.300	0.217
Downside_SR	0.457	0.282	0.288	0.294	0.267	0.186
UPratio	0.363	0.348	0.405	0.399	0.444	0.404
Portfolio Turnover	8.9337	14.790	16.908	19.448	6.106	13.540
Return-Loss		−1.574		−0.157		−1.372
Opportunity Cost						
exponential utility						
ARA=2		−1.394		0.026		−0.854
ARA=4		−1.246		0.042		−0.503
ARA=6		−1.086		0.052		−0.140
power utility						
RRA=2		−1.394		0.026		−0.849
RRA=4		−1.240		0.042		−0.498
RRA=6		−1.081		0.052		−0.130
	(b) Non parametric tests					
Test statistic	0.0024		−0.0012		0.0032	
p value	(48.83)		(34.92)		(41.17)	

Panel I is for the corporate bond market only, Panel II is for the corporate and government bond market, and Panel III is for the five different markets (stock, commodity, real estate, energy and bond market).
Mean, volatility, turnover, return-loss, and opportunity cost (θ) are in %. The transaction cost is 15bp for all assets. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) for the exponential utility function and different degrees of relative risk aversion (RRA=2,4,6) for the power utility function. In Panel B, entries report test statistics and *p* values for stochastic non-dominance tests of the augmented portfolios with respect to the benchmark portfolio in each case. *p* values are in %. The dataset spans the period 12/12/2014–17/9/2021, with out-of-sample testing conducted over the period 3/11/2017–17/9/2021, 12 months after the Paris Agreement

4.3.3 Preferential taxation and tax credits

We finally consider tax credits in case of losses in addition to preferential tax rates on gains. That is, if the returns of green bonds are negative, all losses are carried forward and netted from the positive returns of subsequent periods. Taxes are charged on the cumulative net returns when positive. The results are, pictorially, very similar to those of Fig. 3, with the performance of the augmented portfolios vis-á-vis the benchmark somewhat improved compared to the case of only tax incentives, as expected. We compute the parametric and non-parametric performance measures for all benchmark and augmented portfolios and different tax rates and report the results in Table 6.

Table 5 Out-of-sample portfolio performance with taxes. In Panel A, entries report the parametric performance measures (annualized mean and volatility, annualized Sharpe ratio, annualized downside Sharpe ratio, UP ratio, portfolio turnover, annualized return-loss and annualized opportunity cost) for the benchmark and the augmented portfolio with green bond indices with taxes in each case

	(a) Parametric performance measures											
	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
Mean	1.829	0.981	1.294	1.781	0.328	−0.565	−0.161	0.852	−0.787	−0.195	0.030	0.452
Volatility	5.693	4.104	3.868	3.764	4.265	4.429	4.461	3.826	6.254	4.205	4.045	3.977
Sharpe Ratio	0.114	−0.046	0.032	0.161	−0.196	−0.389	−0.296	−0.083	−0.309	−0.321	−0.279	−0.179
Downside_SR	0.093	−0.037	0.026	0.138	−0.155	−0.303	−0.234	−0.069	−0.265	−0.262	−0.231	−0.150
UPratio	0.285	0.305	0.351	0.389	0.269	0.244	0.266	0.345	0.363	0.329	0.342	0.355
Portfolio Turnover	4.243	12.007	12.403	9.935	10.132	17.473	21.492	15.870	4.347	6.989	10.143	12.217
Return-Loss		−1.356	−0.906	0.198		−1.370	−1.263	0.095		0.045	0.021	0.421
Opportunity Cost												
exponential utility												
ARA=2		−0.674	−0.348	0.140		−0.906	−0.503	0.563		0.814	1.056	1.488
ARA=4		−0.503	−0.151	0.344		−0.921	−0.524	0.605		1.035	1.293	1.731

Table 5 continued

	(a) Parametric performance measures											
	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
ARA=6		−0.317	0.057	0.568		−0.937	−0.539	0.657		1.261	1.530	1.979
power utility												
RRA=2		−0.674	−0.343	0.146		−0.906	−0.503	0.563		0.814	1.056	1.488
RRA=4		−0.498	−0.145	0.354		−0.921	−0.524	0.610		1.035	1.293	1.731
RRA=6		−0.306	0.073	0.584		−0.937	−0.539	0.663		1.266	1.535	1.979
	(b) Non parametric tests											
Test statistic		0.0036	0.0026	−0.0031		0.0057	0.0024	−0.0010		0.0021	−0.0013	−0.0045
p value		(20.43)	(24.56)	(4.96)**		(32.69)	(18.43)	(4.99)**		(45.63)	(6.12)*	(4.96)**

Panel I is for the corporate bond market only, Panel II is for corporate and government bond market, and Panel III is for the five different markets (stock, commodity, real estate, energy and bond market) The financial assets of the benchmark portfolio have 20% tax on positive returns, and the green assets of the augmented portfolio have lower taxes on positive returns. The numbers in parentheses denote the tax rate.

Mean, volatility, turnover, return-loss and opportunity cost (θ) are in %. The transaction cost is 15bp for all assets. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) for the exponential utility function and different degrees of relative risk aversion (RRA=2,4,6) for the power utility function. In Panel B, entries report test statistics and p values for stochastic non-dominance test of the augmented portfolios with respect to the benchmark portfolio in each case with taxes. p values are in % and **, * asterisks indicate that the null hypothesis of the non-dominance is rejected at 5% and 10% significance level, respectively. The dataset spans the period 12/12/2014–17/9/2021, with out-of-sample testing conducted over the period 3/11/2017–17/9/2021, 12 months after the Paris Agreement

Table 6 Out-of-sample portfolio performance with taxes and tax credits

	(a) Parametric performance measures											
	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
Mean	1.829	1.382	1.515	1.905	0.328	−0.150	0.581	1.166	−0.787	0.022	0.333	0.722
Volatility	5.693	3.880	3.929	3.806	4.265	4.434	3.865	3.886	6.254	4.032	3.998	3.965
Sharpe Ratio	0.114	0.054	0.087	0.191	−0.196	−0.296	−0.152	−0.001	−0.309	−0.282	−0.207	−0.112
Downside_SR	0.093	0.045	0.073	0.165	−0.155	−0.233	−0.126	−0.001	−0.265	−0.233	−0.173	−0.095
UPratio	0.285	0.354	0.361	0.393	0.269	0.265	0.331	0.351	0.363	0.339	0.350	0.364
Portfolio Turnover	4.243	10.014	10.292	10.333	10.132	18.565	16.502	14.961	4.347	12.725	12.459	13.745
Return-Loss		−0.485	−0.290	0.340		−1.051	−0.250	0.525		−0.339	0.213	0.672
Opportunity Cost												
exponential utility												
ARA=2		−0.260	−0.135	0.260		−0.493	0.286	0.867		1.045	1.366	1.762
ARA=4		−0.068	0.057	0.459		−0.508	0.328	0.909		1.282	1.604	2.006
ARA=6		0.140	0.260	0.678		−0.519	0.375	0.956		1.524	1.852	2.255
power utility												

Table 6 continued

	(a) Parametric performance measures											
	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
RRA=2		−0.254	−0.130	0.266		−0.493	0.286	0.872		1.051	1.366	1.762
RRA=4		−0.057	0.062	0.469		−0.508	0.333	0.914		1.287	1.609	2.006
RRA=6		0.156	0.276	0.694		−0.519	0.380	0.961		1.530	1.858	2.261
	(b) Non parametric tests											
Test statistic		0.0031	−0.0011	−0.0019		0.0015	−0.0016	−0.0024		−0.0019	−0.0027	−0.0041
p value		(28.44)	(8.74)*	(4.23)**		(18.38)	(4.85)**	(4.11)**		(4.28)**	(3.89)**	(4.03)**

In Panel A, entries report the parametric performance measures (annualized mean and volatility, annualized Sharpe ratio, annualized downside Sharpe ratio, UP ratio, portfolio turnover, annualized return-loss and annualized opportunity cost) for the benchmark and the augmented portfolio with green bond indices with taxes in each case. Panel I is for the corporate bond market only, Panel II is for the corporate and government bond market, and Panel III is for the five different markets (stock, commodity, real estate, energy and bond market) The financial assets of the benchmark portfolio have 20% tax on positive returns, and the green assets of the augmented portfolio have credit and lower taxes on positive returns. The numbers in parentheses denote the tax rate.

Mean, volatility, turnover, return-loss and opportunity cost (θ) are in %. The transaction cost is 15bp for all assets. The results for the opportunity cost are reported for different degrees of absolute risk aversion (ARA=2,4,6) for the exponential utility function and different degrees of relative risk aversion (RRA=2,4,6) for the power utility function.

In Panel B, entries report test statistics and p values for stochastic non-dominance test of the augmented portfolios with respect to the benchmark portfolio in each case with tax and credit. p values are in %, and ** asterisks indicate that the null hypothesis of the non-dominance is rejected at a 5% significance level. The dataset spans the period 12/12/2014–17/9/2021, with out-of-sample testing conducted over the period 3/11/2017–17/9/2021, 12 months after the Paris Agreement

Table 7 Non parametric tests of Davidson and Duclos (2013). Entries report quartile rejection rates from the distribution rejection rates across out-of-sample periods under the null hypothesis that the augmented portfolio does not second order stochastically dominate the benchmark portfolio using the Davidson and Duclos (2013) test statistic, over the period from 12/12/2014 to 17/9/2021

(a) Out-of-sample portfolio performance without taxes	(i) Corporate bonds		(ii) Corporate and Government bonds		(iii) Five markets	
	Benchmark	Augmented	Benchmark	Augmented	Benchmark	Augmented
Quartile						
25% Rejection rate		3.87%		3.45%		4.12%
50% Rejection rate		3.88%		3.99%		4.32%
75% Rejection rate		4.93%		4.12%		4.78%

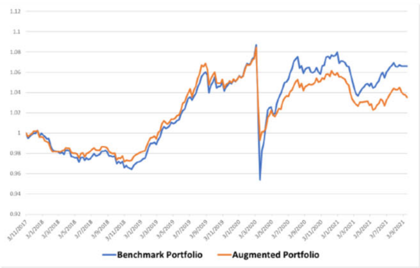
(b) Out-of-sample portfolio performance with taxes	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
Quartile												
25% Rejection rate		2.56%	3.77%	24.55%		3.56%	3.99%	27.45%		2.88%	7.44%	34.40%
50% Rejection rate		2.97%	4.12%	31.44%		4.23%	4.34%	42.87%		3.12%	8.12%	57.70%
75% Rejection rate		3.91%	4.45%	42.88%		4.42%	4.89%	59.33%		4.19%	10.45%	71.50%

(c) Out-of-sample portfolio performance with taxes and tax credits	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
Quartile												
25% Rejection rate		3.12%	5.44%	22.44%		4.47%	4.88%	19.23%		12.27%	27.44%	41.22%

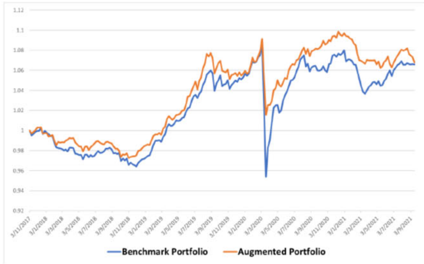
Table 7 continued

	(c) Out-of-sample portfolio performance with taxes and tax credits											
	(i) Corporate bonds				(ii) Corporate and Government bonds				(iii) Five markets			
	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)	Benchmark (20%)	Augmented (15%)	Augmented (10%)	Augmented (5%)
50% Rejection rate		3.88%	6.17%	34.55%		5.12%	5.03%	32.66%		13.56%	43.12%	58.32%
75% Rejection rate		4.11%	8.16%	59.43%		6.03%	7.12%	44.11%		18.45%	54.12%	69.12%

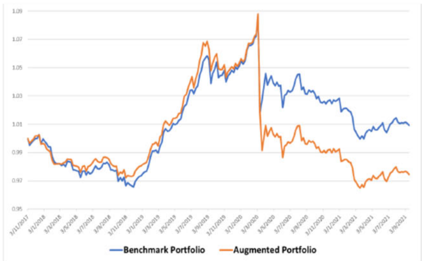
We use T-1 overlapping periods of 100 weekly returns for the in-sample fitting of the two portfolios with corresponding out-of-sample comparisons. Panel A is the out-of-sample portfolio performance without taxes, Panel B is with taxes and Panel C is with taxes and tax credits on green bonds. Panel I is for the corporate bond market only, Panel II is for corporate and government bond market, and Panel III is for the five different markets (stock, commodity, real estate, energy and bond market) The financial assets of the benchmark portfolio have 20% tax on positive returns, and the green assets of the augmented portfolio have lower taxes on positive returns. The numbers in parentheses denote the tax rate



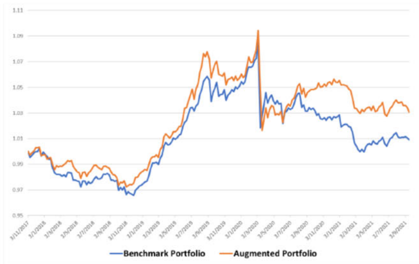
(a) Corporate bonds, green tax 15%



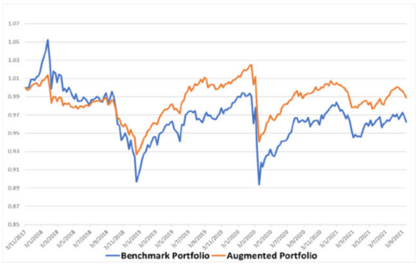
(b) Corporate bonds, green tax 5%



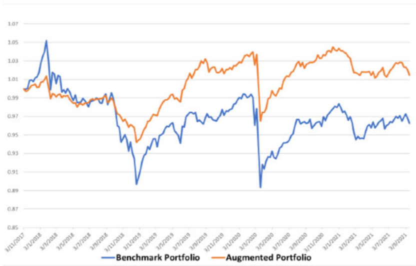
(c) Corporate and government bonds, green tax 15%



(d) Corporate and government bonds, green tax 5%



(e) Five markets, green tax 15%



(f) Five markets, green tax 5%

Fig. 3 Out-of-sample testing with taxes. Cumulative performance of the benchmark optimal portfolio and the optimal augmented portfolio with green bond indices for the three different cases with taxes. Panels A and B are the cases where the benchmark portfolio includes only corporate bond indices, Panels C and D are the cases where the benchmark portfolio includes corporate and government bond indices, and Panels E and F are the cases of five markets (stock, commodity, real estate, energy, and bond market). The benchmark assets have 20% tax on positive returns, and the green assets have 15 and 5% tax on positive returns, as indicated. The dataset spans the period 12/12/2014–17/9/2021, with out-of-sample testing conducted over the period 3/11/2017–17/9/2021, 12 months after the Paris Agreement

From Panel A (ii) and (iii), we observe that adding tax credits makes the 10% tax rate sufficient for the augmented portfolios to outperform the benchmark. However, from Panel A (i), we observe that although the augmented portfolios exhibit improved performance with a tax credit, it remains essential to offer the preferential tax rate of 5% to clearly outperform the benchmark portfolios with corporate bonds.

The non-parametric test gives a crisper picture. Panel B shows that the null hypothesis is rejected when the tax rate is 10% in all sub-panels (i)–(iii). Consistent with the parametric test results, for the case of the five markets, the null hypothesis is rejected even when the tax rate is 15% at the 0.05 level. We also tested the case of a uniform 20% tax rate on all assets with tax credit on green bonds. For the broadest case of five markets, we find that the null hypothesis is rejected at the 0.10 significance level, and the augmented portfolio outperforms the benchmark for most performance measures.

Finally, Table 7 reports the quartile p values from the distribution of weekly portfolio returns, under the null hypothesis that the augmented portfolio does not dominate the benchmark one in all different cases, with and without taxes. Panel A is the out-of-sample portfolio performance without taxes, Panel B is with taxes and Panel C is with taxes and tax credits on green bonds. Panel I is for the corporate bond market only, Panel II is for corporate and government bond market, and Panel III is for the five different markets (stock, commodity, real estate, energy and bond market) The financial assets of the benchmark portfolio have 20% tax on positive returns, and the green assets of the augmented portfolio have lower taxes on positive returns. The numbers in parentheses denote the tax rate. We observe that the Davidson and Duclos (2013) test confirm the results of the non-parametric pairwise stochastic non-dominance test of Anyfantaki et al. (2021).

In conclusion, investors could find green bonds appealing when tax credits are available for losses and somewhat preferential tax rates for gains.

5 Conclusions

Financial markets could contribute in addressing climate change challenges by financing the growing demand for low-carbon projects around the world. Green bonds are issued to channel more capital to such projects, but it is unclear whether or under what conditions they are attractive to investors.

In this paper, we address the question of potential diversification benefits to investors who include green bonds in their portfolios. We employ a stochastic spanning methodology to answer this question both in- and out-of-sample. We construct and compare optimal portfolios from a benchmark set of equities, commodities, energy, real estate and bond assets, against portfolios augmented with green bonds. We find clear diversification benefits to investors in the US and European markets who invest in green bonds. This result holds both in- and out-of-sample.

However, the answer to our question is not affirmative in the global markets. Although in-sample green bonds appear to be beneficial, out of sample they do not improve the benchmark portfolio performance. Hence, we take a step further to analyze the impact of tax incentives. We find that significantly preferential tax rates (5% compared to 20%) are needed for green assets to provide diversification benefits. We also consider tax credits on losses and find the augmented portfolio to be more attractive to investors for modestly preferential tax rates (10–15%).

Consequently, an appropriate taxation policy can significantly modify investor allocation strategies and guide the market toward environmentally friendly investments. Governments could provide incentives by offering credits when green bonds suffer losses and collect taxes when they have gains. This approach can entice investors to incorporate green investments into their conventional portfolios. Overall, our findings support the need for tax interventions if green bonds are to become consistently appealing to investors on purely financial performance grounds.

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s10479-025-06501-2.

References

Agliardi, E., & Agliardi, R. (2021). Pricing climate-related risks in the bond market. *Journal of Financial Stability, 54*, 100868.

Akhtaruzzaman, M., Banerjee, A. K., Boubaker, S., & Moussa, F. (2023). Does green improve portfolio optimisation? *Energy Economics, 124*, 106831.

Anyfantaki, S., Maasoumi, E., Ren, J., & Topaloglou, N. (2021). Evidence of uniform inefficiency in market portfolios based on dominance tests. *Journal of Business & Economic Statistics*, 1–13.

Arvanitis, S., Post, T., & Topaloglou, N. (2021). Stochastic bounds for reference sets in portfolio analysis. *Forthcoming in Management Science*, available on https://doi.org/10.1287/mnsc.2020.3838.

Arvanitis, S., Hallam, M., Post, T., & Topaloglou, N. (2019). Stochastic spanning. *Journal of Business & Economic Statistics, 37*, 573–585.

Bai, L., Wei, Y., Zhang, J., Wang, Y., & Lucey, B. M. (2023). Diversification effects of China's carbon neutral bond on renewable energy stock markets: A minimum connectedness portfolio approach. *Energy Economics, 123*, 106727.

Bawa, V. S. (1975). Optimal rules for ordering uncertain prospects. *Journal of Financial Economics, 2*, 95–121.

Cheong, C., & Choi, J. (2020). Green bonds: A survey. *Journal of Derivatives and Quantitative Studies, 28*, 175–189.

Christoffersen, P., Errunza, V., Jacobs, K., & Langlois, H. (2012). Is the potential for international diversification disappearing? A dynamic copula approach. *The Review of Financial Studies, 25*, 3711–3751.

Constantinides, G. M., Czerwonko, M., Carsten Jackwerth, J., & Perrakis, S. (2011). Are options on index futures profitable for risk-averse investors? Empirical evidence. *The Journal of Finance, 66*, 1407–1437.

Constantinides, G. M., Jackwerth, J. C., & Perrakis, S. (2009). Mispricing of S&P 500 Index Options. *Review of Financial Studies, 22*, 1247–1277.

Davidson, R., & Duclos, J.-Y. (2000). Statistical inference for stochastic dominance and for the measurement of poverty and inequality. *Econometrica, 68*, 1435–1464.

Davidson, R., & Duclos, J.-Y. (2013). Testing for restricted stochastic dominance. *Econometric Reviews, 32*, 84–125.

De Roon, F. A., Nijman, T. E., & Werker, B. J. (2003). Currency hedging for international stock portfolios: The usefulness of mean-variance analysis. *Journal of Banking & Finance, 27*, 327–349.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus Naive diversification: How inefficient is the 1/N portfolio strategy? *The Review of Financial Studies, 22*, 1915–1953.

Ehlers, T., & Packer, F. (2017). Green bond finance and certification. *BIS Quarterly Review*.

Fender, I., McMorrow, M., Sahakyan, V., & Zulaica, O. (2019). Green bonds: The reserve management perspective. *BIS Quarterly Review*.

Gorovaia, N., & Makrominas, M. (2024). Identifying greenwashing in corporate-social responsibility reports using natural-language processing. *European Financial Management*. https://doi.org/10.1111/eufm.12509

Hammoudeh, S., Ajmi, A. N., & Mokni, K. (2020). Relationship between green bonds and financial and environmental variables: A novel time-varying causality. *Energy Economics, 92*, 104941.

Han, Y., Li, P., & Wu, S. (2022). Does green bond improve portfolio diversification? Evidence from China. *Evidence From China (January 9, 2022)*.

Han, Y., & Li, J. (2022). Should investors include green bonds in their portfolios? Evidence for the USA and Europe. *International Review of Financial Analysis, 80*, 101998.

Hodder, J. E., Jackwerth, J. C., & Kolokolova, O. (2015). Improved portfolio choice using second-order stochastic dominance. *Review of Finance, 19*, 1623–1647.

Huberman, G., & Kandel, S. (1987). Mean-variance spanning. *The Journal of Finance, 42*, 873–888.

Levy, H. (2016). *Stochastic dominance: Investment decision making under uncertainty* (3rd ed.). Springer.

Nguyen, T. T. H., Naeem, M. A., Balli, F., Balli, H. O., & Vo, X. V. (2021). Time-frequency comovement among green bonds, stocks, commodities, clean energy, and conventional bonds. *Finance Research Letters, 40*, 101739.

Pástor, L., Stambaugh, R. F., & Taylor, L. A. (2022). Dissecting green returns. *Journal of Financial Economics, 146*, 403–424.

Pedersen, L. H., Fitzgibbons, S., & Pomorski, L. (2021). Responsible investing: The ESG-efficient frontier. *Journal of Financial Economics, 142*, 572–597.

Perrakis, S. (2019). *Stochastic dominance option pricing: An alternative approach to option market research*. Springer.

Pham, L. (2016). Is it risky to go green? A volatility analysis of the green bond market. *Journal of Sustainable Finance & Investment, 6*, 263–291.

Pham, L., & Nguyen, C. P. (2021). Asymmetric tail dependence between green bonds and other asset classes. *Global Finance Journal, 50*, 100669.

Politis, D. N., Romano, J. P., Wolf, M., Politis, D. N., Romano, J. P., & Wolf, M. (1999). *Subsampling in the IID Case*. Springer.

Reboredo, J. C. (2018). Green bond and financial markets: Co-movement, diversification and price spillover effects. *Energy Economics, 74*, 38–50.

Reboredo, J. C., & Ugolini, A. (2020). Price connectedness between green bond and financial markets. *Economic Modelling, 88*, 25–38.

Reboredo, J. C., Ugolini, A., & Aiube, F. A. L. (2020). Network connectedness of green bonds and asset classes. *Energy Economics, 86*, 104629.

Rockafellar, R. T., Uryasev, S., et al. (2000). Optimization of conditional value-at-risk. *Journal of Risk, 2*, 21–42.

Russell, W. R., & Seo, T. K. (1989). Representative sets for stochastic dominance rules. In *Studies in the economics of uncertainty: In Honor of Josef Hadar* (pp. 59–76). Springer.

Sharpe, W. (1994). The sharpe ratio. *The Journal of Portfolio Management, 21*, 49–58.

Simaan, Y. (1993). What is the opportunity cost of mean-variance investment strategies? *Management Science, 39*, 578–587.

Sortino, F. A., & Van Der Meer, R. (1991). Downside risk. *Journal of Portfolio Management, 17*, 27.

Stroebel, J., & Wurgler, J. (2021). What do you think about climate finance? *Journal of Financial Economics, 142*, 487–498.

Whang, Y.-J. (2019). *Econometric analysis of stochastic dominance: Concepts, methods, tools, and applications*. Cambridge University Press.

You, L., & Daigler, R. T. (2010). Is international diversification really beneficial? *Journal of Banking & Finance, 34*, 163–173.

Ziemba, W. T. (2005). The symmetric downside-risk Sharpe ratio. *The Journal of Portfolio Management, 32*, 108–122.