Making Sense of Mathematics Mastery: A comparative judgement approach towards pre-service teachers' understanding

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Teaching for Mastery (TfM) has been developing for over a decade in England. As a result, there is the need to assess teacher trainees' understanding of concept. We therefore report a preliminary study that tested an alternative approach, called comparative judgement, as a formative assessment method to identify teacher trainees' views of essential aspects of TfM. Twenty-seven teacher trainees from two research-intensive university PGCE Secondary Maths courses drew a mind-map to show their understanding by the end of the course. Their work was then assessed by 14 Maths course leads in teacher training programmes. We report two main findings. First, the perceived best piece and the least one of trainees' understanding reveal the range of their current understanding. Second, the comparative judgement approach to assessing pre-service teachers' work seemed to be a reliable method to utilise. The results can facilitate mentors to support noticing and implementing of mastery approaches further.

Keywords: teaching for mastery; pre-service teachers; comparative judgement; Initial Teacher Training

Introduction

Initial Teacher Training (ITT) is a professional course that requires in-depth understanding and action in integrating research into practice. Pre-service teachers are expected to combine theoretical knowledge and purposeful practice to assess what works, how, and why, in a certain classroom environment. This goes beyond simply applying or adapting current theory. In the case of mathematics education, as an example, a policy-level intervention involving all primary and secondary settings in recent years is the harnessing of TfM approaches. From a theoretical perspective, the Teaching for Mastery Framework proposed by the NCETM (2014) approaches mastery from the point of view of pedagogy. It summarises mastery pedagogy with reference to five big ideas: coherence, representation & structure, mathematical thinking, fluency, and variation. The present study sets out to explore precisely how these five big ideas have been perceived by pre-service teachers as falling under the umbrella of mastery pedagogy, and if other elements have also merged with them under this umbrella.

We adapt theory-based evaluation principles to identify core components of this pedagogy. While education policies often claim to be evidence-based or evidence-informed, this does not always translate into rigorous policy development. Through comparing pre-service teachers' understanding with the NCETM framework, this project aims to assess potential similarities and differences between their perceptions of mastery and policy intentions, highlighting the importance of scrutinising the efficacy of policy innovations.

Teaching for mastery

The term "mastery" is used in various ways, often referring to beliefs that, for instance, ability is malleable, not fixed, and that all learners have the potential to achieve mastery (Dweck, 2006), alongside aspects of pedagogy such as regular formative and high-quality corrective instruction followed by enrichment activities (Guskey, 2010). The aim is developing deep understanding and applying knowledge and skills effectively to solve mathematical problems (Drury, 2018). East-Asian-informed mastery is focused, in particular, on cultural expectations towards learning outcomes and rigorous instruction (NCETM, 2012). It is, however, crucial to acknowledge that mastery pedagogy is deeply rooted in specific cultural and systemic contexts (Simpson & Wang, 2023).

Drury (2018) posits that teaching mathematics for mastery is about setting high expectations for every learner with the aim to deepen their understanding so they can tackle mathematical problems in unfamiliar situations. Based on this definition, Drury (2018) conceptualises mastery as a state where a learner can represent a mathematical concept in multiple ways, communicate related ideas with mathematical language, and think mathematically to apply it independently to new problems in unfamiliar contexts. This is achieved through exploration, clarification, practice, and application over time. From this point of view, problem solving is the ultimate goal of teaching for mastery, thus it fits with one of the three aims for the National Curriculum for Mathematics in England (with the other two aims being fluency and reasoning). The goal is for all students to apply learned concepts and skills to a variety of problems to tackle both routine and non-routine problems, with increasing sophistication (DfE, 2014). Teaching for mastery has been interpreted in various ways, such as through a concrete-pictorial-abstract (CPA) approach (Salingay & Tan, 2018) and using conceptual variation in recognising the uniqueness of the mathematical concept at hand (Wang, Brown & Dawson, 2023). In summary, given the variety of interpretations of mastery, as well as that policy makers in the UK have promoted a particular explanation, care needs to be taken regarding how pre-service teachers construct the meaning of TfM.

Research Questions

The promotion of teaching for mastery by the NCETM (2014) has led to mastery being a central focus for ITT providers and the Maths Hub network initiative. Using a multifaceted policy initiative influenced by East Asian systems, particularly those in Shanghai and Singapore, the NCETM has established various programmes and professional development initiatives for both primary and secondary teachers of Maths, including pre-service teachers. Therefore, two research questions are put forward:

1. How do pre-service teachers understand TfM in theory?
2. Apart from the five big ideas proposed by the NCETM, what theoretical elements do these pre-service teachers value in mathematics mastery?

Methodology

Methods – comparative judgement

For the current research, we used comparative judgement to gain insight into pre-service teachers' views of mastery. This is where repeated pair-wise comparisons of responses (in this case trainees' responses) by judges (in this case ITT leads) result in a measurement scale of the quality of these responses (Pollitt, 2012). In mathematics

generally, Ian Jones and colleagues at Loughborough University have put forward comparative judgement as a way of looking at understanding (Bisson, Gilmore, Inglis & Jones, 2016) and problem solving (Jones & Inglis, 2015). In particular, comparative judgement is suitable for "assessing nebulous constructs that are deemed important but which are difficult to specify comprehensively in mark schemes" (Jones & Inglis, 2015, p. 341).

In addition to the summative potential of comparative judgement, the approach was also utilised for its formative potential in building up judges' insight and understanding into what constitutes 'quality' in trainees' views of mastery. Van Daal et al (2023, p.76) highlights how comparative judgement allows the judge to "reflect on how they conceptualise the quality of a piece of work." The act of comparing inherent in the judging "is a fundamental part of human cognition and a powerful learning mechanism. It permeates our everyday lives and has been shown to improve learning in a broad range of domains" (Rittle-Johnson & Star, 2011, p.221). We adapted the process model proposed by Alferi et al (2013) for what occurs during a comparison:

1. Prompt – judging one case to be better in some way than another;
2. Identify similarities and differences – influenced by the prompt, the judge's prior knowledge and type of case;
3. Considering the similarities and differences and making a judgement;
4. Repeated comparisons developing a conceptualisation of quality in the given context.

Inherent in the comparative judgement process, therefore, is the identification of similarities and differences or distinctions between cases. Palisse, King and MacLean (2024) and Marton and Pang (2006) highlight that as part of Variation Theory, the identification of these distinctions is a necessary part of the learning process. We therefore argue that the comparative judgement was also useful in developing the judges' understanding of what quality looks like in trainees' views of mastery.

There are three steps to triangulate trainees' understandings from a theoretical perspective:

(1) trainees producing an A3-page mind map to represent what they think mastery lessons look like;

(2) employing a comparative-judgment approach to show which descriptions are considered better by ITT Maths course leads;

(3) holding a focus group to learn about how these judges make their decisions.

Participants

In England, routes to teaching are diverse, but they show two broad directions: school-led and university-led systems. The former was established by the Coalition government in 2011 (Mutton, Burn & Menter, 2017) and views teaching as a craft. The latter, a more traditional route, approaches teaching as an occupational profession (Davies, et al., 2016). This study purposefully selected its sample from the university-led system, with trainees who are enrolled in research-intensive universities in England. Participants were all studying at postgraduate level, on PGCE Secondary Mathematics. They were in the last term of their training year (in the 23/24 academic year), which means they had all already completed their university-based sessions. During these university-based sessions, all the participants had received input framed as related to mathematics mastery. Ethics approval for this research was gained from the ethics committee in the School of Education, Durham University.

Collecting data

Twenty-seven teacher trainees drew mind maps to illustrate their understanding of TfM. Through these drawings, these pre-service teachers revealed what they perceived as the key elements. Their work was then assessed by fourteen ITT Maths course leads via a comparative judgement method provided by No More Marking. The judges' task was straightforward, simply being asked to consider two scripts and decide which one was 'better' in terms of showing understanding of teaching for mastery. After the judges had chosen the 'better' script from a pair, they were given a new pair to judge, until all 27 scripts had been scored satisfactorily.

Results

Reliability

No More Marking's system generates statistical reports automatically. The Scale separation reliability (SRR, a measure of internal consistency) was found to be high, SSR= 0.76, for 166 judgements on 27 candidates (average 6.1 judgements per candidate). The judges' decision-making process shows that the criteria were related to the NCETM's five big ideas, maths specific, and using related vocabulary.

Pre-service teachers' understandings

The analysis of their understanding identified that perceived features of TfM were related to 'fluency', 'reasoning', 'variation' with an emphasis on conceptual variation, 'deep understanding', application to 'real life', and 'bar modelling'. Compared with the five big ideas from NCETM, two features - fluency and variation, were coherent. The idea of representation and structure was expressed using bar modelling in teaching practice. Mathematical thinking was perceived as reasoning and problem solving. However, the idea of coherence did not come across strongly as a definitional feature. Compared with the three aims of the National Curriculum - fluency, reasoning and problem solving, pre-service teachers recognised these well and found that these intimately linked to TfM.

Besides these two policy documents' interpretations of TfM, pre-service teachers pointed to the essentials of learning outcomes, such as 'the success rate', '80 percents of success rate', and 'monitoring progress'. They were keen to know the effectiveness, e.g. TfM would ensure the positive outcome. In addition, checking for understanding, and checking for learning, were seen as important, using 'questioning', 'diagnostic questions', and Assessment for Learning. These features lead to the elaboration of TfM in teaching practice. Appendix A shows the best-rated piece about TfM as this piece covered most of elements.

Final marks

Through this preliminary study, overall, we found that the judging process seems to be relatively straightforward on measuring pre-service teachers' understanding of TfM. It led to good reliability. These judges used their expertise while judging, which indicates that it allows differences in conceptualising TfM. Therefore, this study laid a foundation for building a shared consensus on the meanings of TfL across judges. The next step of the research is to invite pre-service teachers themselves to be judges, to challenge their own understandings by considering the extent to which different aspects of TfM might

be valued more. This will afford an expansion of pre-service teachers' current view of a limited set of aspects of TfM. Further study could explore the relevance of their own defined criteria and holistic professional judgement to evidence their learning.

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Appendix A: The best rated piece of TfM