X International Conference on Computational Plasticity
COMPLAS X
E. Oñate and D.R.J. Owen (Eds)

INFLUENCE OF LODE ANGLE DEPENDENCY ON THE CRITICAL STATE FOR ROTATIONAL PLASTICITY

WM Coombs*, **RS Crouch*** **and CE Augarde***

*School of Engineering, Durham University,
South Road, Durham. DH1 3LE. United Kingdom.
e-mail: w.m.coombs@durham.ac.uk

Key words: Computational plasticity, Constitutive modeling, Critical state, Rotational hardening, Anisotropy.

1 INTRODUCTION

The critical state in soils represents the condition whereby unbounded distortions occur with no change in volume [1]. Many constitutive models incorporate this idealised asymptotic state as a fundamental feature. It has been shown experimentally that both the critical state and yield surfaces in soils exhibit a Lode Angle, θ, Dependency (LAD) [2]. A separate feature in soils is the directional bias of the material stiffness. This has been introduced into hardening plasticity models by allowing the yield surface to rotate. Extending the classical Modified Cam-Clay (MCC) model, Dafalias proposed a rotated and distorted ellipse that maintains a constant critical state stress ratio for any degree of rotation of the yield surface [3]. He also presented the multi-axial generalised version of the yield function in which anisotropy is properly accounted for by a second order tensor $\boldsymbol{\alpha}$. A means of satisfactorily introducing LAD, together with rotational hardening has yet to be found. This paper explores the consequences of using simple approaches to overcome this omission.

2 ANISOTROPY

In-situ geotechnical materials often exhibit significant anisotropy in their stiffness due to the deposition process and the particulate nature of their fabric. Rotational hardening uses a measure of anisotropy, α, representing the degree of rotation of the yield surface from the ξ axis. α is a scalar when confined to a constant θ meridian, but becomes a second order tensor in general stress space. Dafalias [3] presented the following form of a rotated MCC yield surface

$$f = \xi^2 - \xi\xi_c + \frac{3}{2M^2}\left(3(\boldsymbol{\rho}_\alpha)^2 + (\xi_c - \xi)\xi\boldsymbol{\alpha}^2\right) = 0, \qquad (1)$$

where $\xi = \mathrm{tr}(\boldsymbol{\sigma})/\sqrt{3}$, $\boldsymbol{\rho}_\alpha = \boldsymbol{s} - (\xi/\sqrt{3})\boldsymbol{\alpha}$, $\boldsymbol{s} = \boldsymbol{\sigma} - (\xi/\sqrt{3})\mathbf{1}$. ξ_c defines the size of the yield surface and M is the gradient of the Critical State line.

Figure 1 displays the consequences of non-coincident principal directions of $\boldsymbol{\alpha}$ and $\boldsymbol{\sigma}$ when representing yield surfaces in principal stress space. The two surfaces are for the same degree of anisotropy. A represents the yield surface when it is assumed that the principal directions are coincident and B when the direction of $\boldsymbol{\alpha}$ is properly accounted for. The difference arises from the components of $(\boldsymbol{\rho}_\alpha)^2$ in (1), and although it is possible to represent the general $\boldsymbol{\alpha}$ yield surface in principal stress space, the appropriate measure of anisotropy must be used.

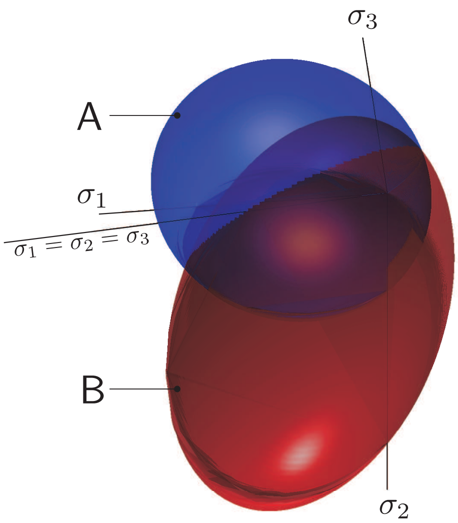


Figure 1: Implications of the principal directions of $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}$ being A coincident and B non-coincident .

3 LODE ANGLE DEPENDENCY

Incorporation of the influence of the intermediate principal stress within a yield function introduces a non-linear deviatoric section to the yield surface. Podgóski commented that the shape of the deviatoric section is important to obtain good agreement between computational and experimental results [4]. This fact led to various LADs being proposed for particulate media, some of which are given in Figure 2 ([4]–[7]). The shape functions are defined in terms of a normalised deviatoric radius $\bar{\rho}(\theta)$, using the Haigh-Westergaard co-ordinate system, where

$$\theta = \frac{1}{3}\arcsin\left(-\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}\right), \qquad -\pi/6 \le \theta \le \pi/6, \tag{2}$$

$J_2 = 1/2\text{tr}(\boldsymbol{s}^2)$ and $J_3 = 1/3\text{tr}(\boldsymbol{s}^3)$. Use of these deviatoric sections requires the identification of the normalised deviatoric yield stress under triaxial extension $\bar{\rho}_e$ $(\theta = \pi/6)$ (and also under pure shear $\bar{\rho}_s$ $(\theta = 0)$ for the Bhowmik-Long criterion [7]). The implications of using a non-circular deviatoric section can be demonstrated using a simpler mathematical form than that used by many of these shape functions, whilst still allowing control over $\bar{\rho}_e$.

The modified Reuleaux triangle is formed using three equal arcs projected from centres equidistance from the hydrostatic axis on the compression meridians, as shown in Figure 3(a). Formation requires only the normalised deviatoric yield stress under triaxial extension, $\bar{\rho}_e = \rho_e/\rho_c$, where ρ_c is the deviatoric yield stress under triaxial compression subjected to the same confining pressure. $\bar{\rho}(\theta)$, for $-\pi/6 \le \theta \le \pi/6$ and $0.5 < \bar{\rho}_e \le 1$, is defined in Figure 2. In the limits of $\bar{\rho}_e = 0.5$ and $\bar{\rho}_e = 1$, the modified Reuleaux triangle forms trianglular and circular

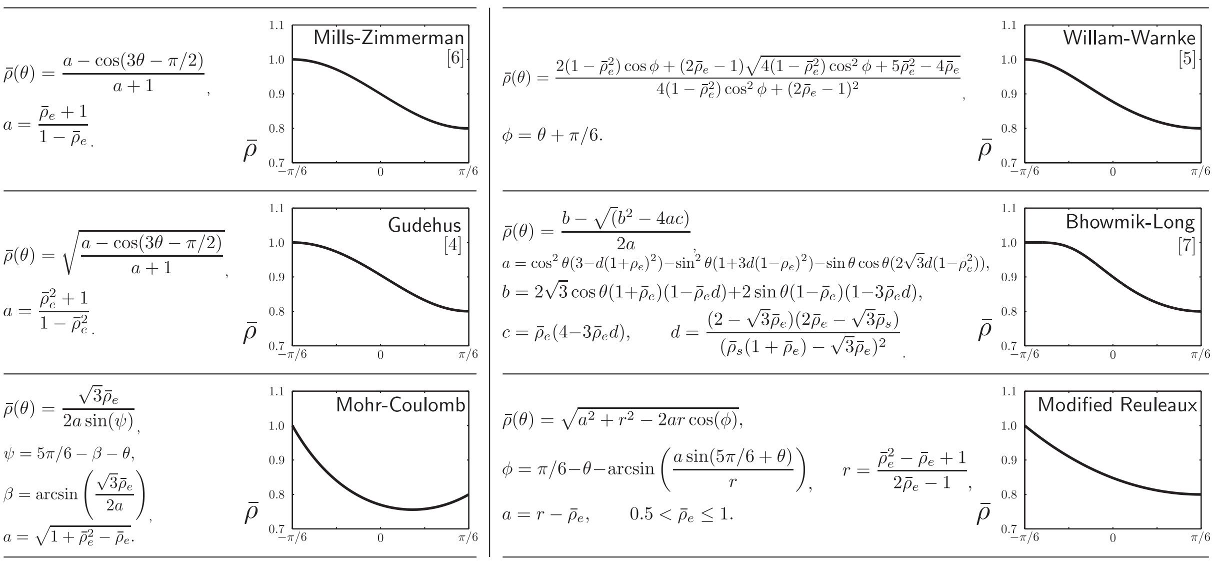
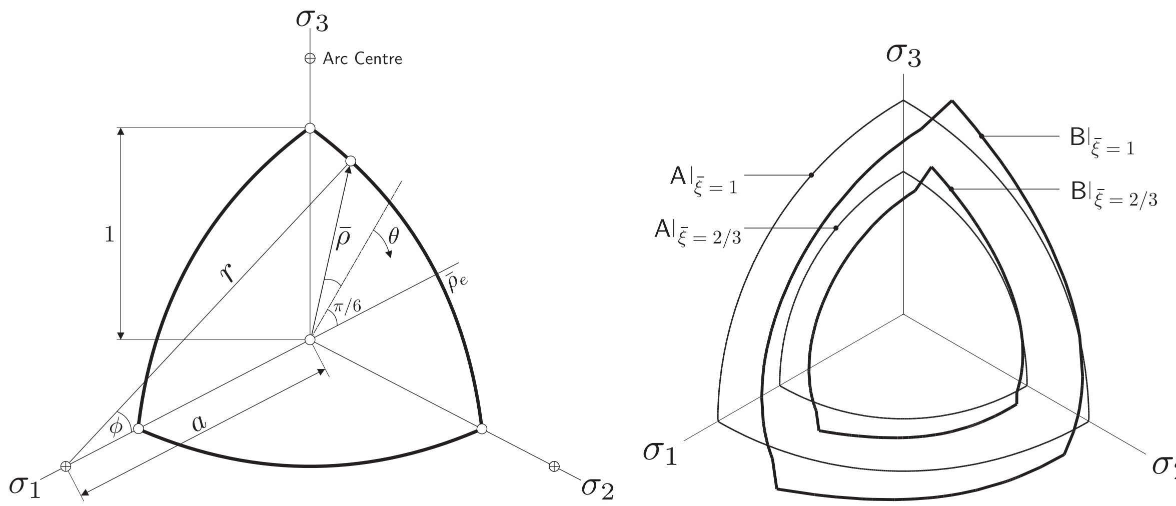


Figure 2: Lode angle shape functions for particulate media.

deviatoric sections, respectively. By setting $\bar{\rho}_e = 1/\sqrt{3}$ a Reuleaux triangle is formed where the arc centres coincide with the compression meridians apexes. Although this deviatoric shape function is not C_1 continuous at the compression meridian, it does offer a simple formulation and allows analytic backward integration for a modified Reuleaux cone.



(a) Geometric construction. (b) CSS. A $\boldsymbol{\alpha} = \mathbf{0}$. B $\boldsymbol{\alpha} \neq \mathbf{0}$.

Figure 3: Modified Reuleaux Triangle.

4 CRITICAL STATE

Figure 3(b) shows the effect on the Critical State Surface (CSS) of introducing a Reuleaux triangle deviatoric section ($\bar{\rho}_e = 1/\sqrt{3}$) to Dafalias' anisotropic MCC model. A indicates two deviatoric sections through the original CSS ($\boldsymbol{\alpha} = \mathbf{0}$) and B shows similar sections, but for $\boldsymbol{\alpha} = \{-0.0244 \quad 0.184 \quad -0.160\}^T$. Sections are located at $\bar{\xi} = 1$ and $\bar{\xi} = 2/3$, normalised with respect to the maximum ξ value. It can be seen that for $\boldsymbol{\alpha} \neq \mathbf{0}$ the uniqueness of the critical state is lost. That is, the mobilised friction angle associated with the critical state depends on the degree of anisotropy. This does not appear to be supported by experimental evidence. Furthermore, convexity of the CSS is no longer preserved. The findings here suggest that more work is required to gain a deeper understanding of the fabric condition at the critical state. Recent DEM simulations have explored how soil grading affects the critical state density, at a given mean stress [8]. Yet it is not at all clear whether that state represents an isotropic condition [9]. If this is the case, then the rotationally hardening plasticity models will require some revision as they predict residual anisotropy.

References

[1] A. Schfield and P. Wroth. *Critical State Soils Mechanics*. McGraw-Hill Pub. (1968).

[2] P.V. Lade and J.M Duncan. Cubical triaxial tests on cohensionless soil. *J. Soil Mech. Found. Div. ASCE*, **99**, 193–812, (1973).

[3] Y. Dafalias. An anisotropic critical state clay plasicity model. *Proc. 2nd Int. Conf. Const. Laws Eng. Mat.*, 513–522, (1987).

[4] J. Podgorksi. General Failure Criterion for Isotropic Media. *J. Eng. Mech. ASCE*, **111**, 188–201, (1985).

[5] K. Willam and E. Warnke. Constitutive model for the triaxial behaviour of concrete. *Seminar on Concrete Structures Subjected to Triaxial Stresses, Organised by ISMES and IABSE*, Bergamo, Italy, 1–30, (1974).

[6] L.L. Mills and R.M. Zimmerman. Compressive strength of plain concrete under multiaxial loading conditions. *J. ACI*, **67**, 59–70, (1970).

[7] S.K. Bhowmik and J.H. Long. A general formulation for the cross sections of yield surfaces in octahedral planes. *Numerical methods in Engineering: Theory and Applications*, Swansea, 795–803, (1990).

[8] D.M. Wood and K. Maeda. Changing grading of soil: effect on critical states. *Acta Geotechnica*, **3**, 3–14, (2008).

[9] D.M. Wood. *Geotechnical modelling*. Spon Press. (2004).