

Generalized cold-atom simulators for vacuum decay

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Cold-atom analog experiments are a promising new tool for studying relativistic vacuum decay, enabling one to empirically probe early-Universe theories in the laboratory. However, existing proposals place stringent requirements on the atomic scattering lengths that are challenging to realize experimentally. Here we eliminate these restrictions and show that *any* stable mixture between two states of a bosonic isotope can be used as a faithful relativistic analog. This greatly expands the landscape of suitable experiments, and will expedite efforts to study vacuum decay with cold atoms.

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Introduction. Quantum fields can escape metastable “false vacuum” states by nucleating “true vacuum” bubbles [1–5]. This process of *vacuum decay* plays a pivotal role in many aspects of cosmology, from the Universe’s inflationary beginnings [6–10] to the present-day (meta)stability of the Higgs field [11–13]. The standard treatment of this problem relies on Euclidean (imaginary-time) calculations, leaving many key questions unanswered. In particular, how does vacuum decay proceed in real time? And what happens in situations where the symmetries of the Euclidean solutions are broken, e.g., nucleation of multiple bubbles [14]?

There has been a recent surge of interest in tackling these questions using quantum analog experiments that simulate metastable relativistic fields [15–26]. In this Letter, we present the first such proposal that is both viable with current experiments and rigorously, quantitatively analogous to relativistic vacuum decay. Our proposal uses ultracold-atomic condensates to enable controlled real-time tests of early-Universe theories on a tabletop. Similar technologies have been successfully used to study discontinuous transitions in nonrelativistic quantum fields [27–32], including nonrelativistic thermal vacuum decay [33].

As we demonstrate below, our proposed analog system satisfies four requirements that are essential for delivering new cosmological insights:¹

(1) The system possesses a degree of freedom with the same *equations of motion* as a relativistic field.

(2) The *quantum fluctuation statistics* in this degree of freedom match those of the same relativistic field.

(3) The system can be made *homogeneous* in one or more dimensions, replicating the translation invariance of a relativistic theory.

(4) The effective relativistic field can be put into a *metastable state*, leading to bubble nucleation.

In mixtures between two atomic hyperfine states, this level of analogy has previously only been established under “symmetric” conditions, with equal density and *s*-wave scattering length in each state, and zero scattering between atoms in different states [16–18,20,21,23,26]. These conditions are challenging to realize in practice. While the scattering can be tuned using Feshbach resonances in an applied magnetic field [34], there is insufficient freedom to simultaneously satisfy all the symmetric conditions. As a result, previous studies have found only two discrete points in parameter space where these conditions hold (both in ⁴¹K [16,17,26]), leaving no flexibility to accommodate other experimental requirements.

Here we eliminate these restrictions, and show that *any* stable mixture between two states of a bosonic isotope can be made to simulate relativistic vacuum decay by tuning their number densities. While Ref. [17] first studied such

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¹With deviations that can be made perturbatively small by tuning the experimental parameters.

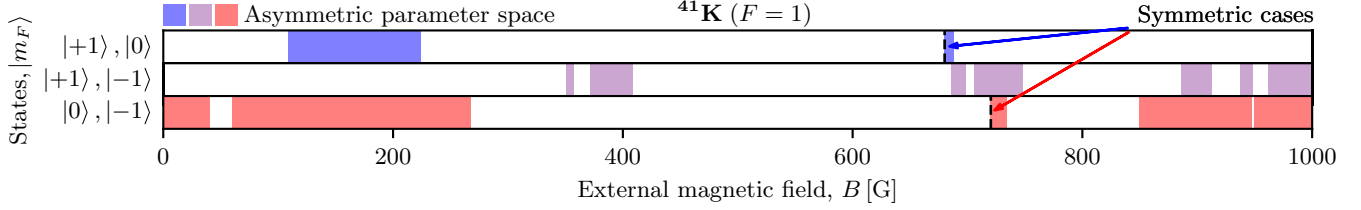


FIG. 1. Landscape of vacuum decay analogs in the $F = 1$ hyperfine manifold of ^{41}K . The horizontal bars correspond to pairs of states $|m_F\rangle \in \{|+1\rangle, |0\rangle, |-1\rangle\}$. Shading indicates the viable parameter space for our asymmetric proposal (identified using scattering lengths from Ref. [35]), while dashed black lines show two points where the symmetric conditions hold [16,17,26]. (These lines are $\sim 1000\times$ thicker than the actual symmetric regions.) As well as greatly expanding the parameter space for ^{41}K , our results enable analogs in isotopes with no symmetric options.

“asymmetric” systems in this context, there has until now been no theoretical justification for using them as vacuum decay analogs. We establish this analogy rigorously, showing that asymmetric condensates can be made to exhibit all four conditions listed above, just as in the symmetric case. By greatly expanding the landscape of experimental possibilities (see Fig. 1), our results bring robust vacuum decay analogs within reach of current technologies.

Analog system. We consider a gas of two hyperfine states (\downarrow and \uparrow) of some bosonic isotope. At low temperatures, each forms a condensate described by a nonrelativistic quantum field $\hat{\psi}_i(\mathbf{x}) = \sqrt{\hat{n}_i(\mathbf{x})} \exp[i\hat{\phi}_i(\mathbf{x})]$ ($i = \downarrow, \uparrow$), whose modulus $\hat{n}_i(\mathbf{x})$ measures the density of atoms in state i , and whose phase $\hat{\phi}_i(\mathbf{x})$ captures coherent behavior. We define the mean density $n = (n_\downarrow + n_\uparrow)/2$ and population imbalance $z = (n_\downarrow - n_\uparrow)/(n_\downarrow + n_\uparrow)$, which we treat as homogeneous. In practice, the potential that traps the atoms generates an inhomogeneous density profile; however, using an optical box trap [36,37] ensures homogeneity everywhere except a small boundary region.

We couple the species with an electromagnetic field of frequency $\omega = \omega_0 + \delta$, where $\omega_0 = (E_\uparrow - E_\downarrow)/\hbar$ is the resonant frequency (with E_i the energies of the states), and δ is the detuning away from resonance. This causes interstate transitions at a variable rate $\Omega(t)$, set by the coupling amplitude. The Hamiltonian is

$$\begin{aligned} \hat{H}(t) = \int_V d\mathbf{x} \left\{ -\hat{\psi}_\downarrow^\dagger \frac{\hbar^2 \nabla^2}{2m} \hat{\psi}_\downarrow - \hat{\psi}_\uparrow^\dagger \frac{\hbar^2 \nabla^2}{2m} \hat{\psi}_\uparrow \right. \\ \left. - \mu(\hat{\psi}_\downarrow^\dagger \hat{\psi}_\downarrow + \hat{\psi}_\uparrow^\dagger \hat{\psi}_\uparrow) - \frac{\hbar \Omega(t)}{2} (\hat{\psi}_\downarrow^\dagger \hat{\psi}_\uparrow + \hat{\psi}_\uparrow^\dagger \hat{\psi}_\downarrow) \right. \\ \left. - \frac{\hbar \delta}{2} (\hat{\psi}_\downarrow^\dagger \hat{\psi}_\downarrow - \hat{\psi}_\uparrow^\dagger \hat{\psi}_\uparrow) + \sum_{i,j} g_{ij} \hat{\psi}_i^\dagger \hat{\psi}_j^\dagger \hat{\psi}_i \hat{\psi}_j \right\}, \quad (1) \end{aligned}$$

with m the atomic mass, μ the chemical potential,² and g_{ij} describing contact interactions between species i and j , which we parametrize as

$$\begin{aligned} g &= \frac{g_{\downarrow\downarrow} + g_{\uparrow\uparrow}}{2}, \quad \Delta = \frac{g_{\uparrow\uparrow} - g_{\downarrow\downarrow}}{2}, \\ \kappa &= \frac{g_{\downarrow\downarrow} + g_{\uparrow\uparrow} - 2g_{\downarrow\uparrow}}{2}. \end{aligned} \quad (2)$$

²This is the chemical potential for $N = N_\downarrow + N_\uparrow$, while $\mu_\downarrow, \mu_\uparrow$ are associated with N_\downarrow and N_\uparrow individually.

We require repulsive self-interactions $g_{\downarrow\downarrow}, g_{\uparrow\uparrow} > 0$, while $g_{\downarrow\uparrow} = g_{\uparrow\downarrow}$ can have either sign. The condition for miscibility (i.e., a stable homogeneous mean field) is $g_{\downarrow\downarrow}g_{\uparrow\uparrow} > g_{\downarrow\uparrow}^2$ [38] or, equivalently, $g^2 - \Delta^2 > (g - \kappa)^2$.

The relative phase in this system, $\phi_\downarrow - \phi_\uparrow$, behaves like a relativistic scalar field, with a potential determined by the coupling $\Omega(t)$. To recover a false-vacuum potential, this coupling must contain a constant piece and an oscillating piece. The constant piece generates a cosine potential for the relative phase, due to the cost of putting the species out of phase with each other. This does not allow vacuum decay, as the antiphase “false vacuum” is then unstable rather than metastable. As first pointed out by Fialko *et al.* [16], however, this state can be made metastable by modulating the coupling. This generates an effective potential barrier, allowing for discontinuous transitions out of the false vacuum (as illustrated in Fig. 2).

We therefore set $\hbar\Omega(t) = 2\epsilon n\sqrt{\kappa^2 - \Delta^2} + \lambda\hbar v\sqrt{2\epsilon} \cos vt$, where $v \gg \kappa n/\hbar$ is the modulation frequency, and ϵ, λ control the amplitudes of the constant and oscillatory

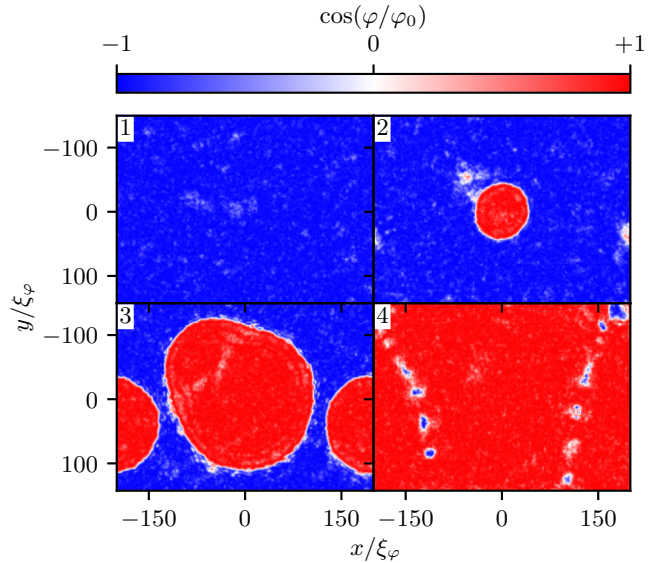


FIG. 2. Lattice simulation of a ^{39}K analog, with time increasing from left to right and top to bottom. The metastable false vacuum (blue) spontaneously decays via expanding bubbles of true vacuum (red). We show a 2D analog here for illustration, while our main numerical results are in 1D.

terms.³ Inserting this into Eq. (1), we use the formalism of Ref. [39] to calculate an effective Hamiltonian, valid for $\epsilon \ll 1$ and on timescales $\gg 2\pi/\nu$ [40]. The false-vacuum barrier stems from quadratic terms in the modulation, which are $O(\epsilon)$. The constant part of the coupling is also $O(\epsilon)$, to ensure the energy difference between vacua is comparable to the barrier height; the exact relationship depends on the $O(1)$ parameter λ , where $\lambda \geq 1$ is required for metastability, and larger values give longer-lived false vacua.

Effective relativistic theory. Our key result concerns the phase fields $\hat{\phi}_i(\mathbf{x})$, which we rewrite as

$$\begin{aligned}\hat{\vartheta}(\mathbf{x}) &= \sqrt{\frac{\hbar^2 n}{2m}} [(1+z)\hat{\phi}_\downarrow(\mathbf{x}) + (1-z)\hat{\phi}_\uparrow(\mathbf{x})], \\ \hat{\phi}(\mathbf{x}) &= \sqrt{\frac{\hbar^2 n}{2m}} \sqrt{1-z^2} [\hat{\phi}_\downarrow(\mathbf{x}) - \hat{\phi}_\uparrow(\mathbf{x})].\end{aligned}\quad (3)$$

In situations where the detuning δ , density fluctuations $n_i - \langle n_i \rangle$, and field gradients $\nabla^2 \vartheta$, $\nabla^2 \varphi$ are all $O(\epsilon)$, one can eliminate the densities to obtain coupled equations for ϑ , φ [40]. For the appropriate population imbalance,

$$z = \frac{\Delta}{\kappa} + O(\epsilon), \quad (4)$$

the equations decouple to give

$$\begin{aligned}0 &= (c_\vartheta^{-2} \partial_i^2 - \nabla^2) \vartheta, \\ 0 &= (c_\varphi^{-2} \partial_i^2 - \nabla^2) \varphi + U'(\varphi).\end{aligned}\quad (5)$$

These are *relativistic* equations of motion for a massless scalar ϑ and a self-interacting scalar φ with potential

$$U(\varphi) = m_0^2 \varphi_0^2 \frac{c_\varphi^2}{\hbar^2} \left[1 - \cos(\varphi/\varphi_0) + \frac{\lambda^2}{2} \sin^2(\varphi/\varphi_0) \right], \quad (6)$$

where we define $m_0 = m\sqrt{4\epsilon\kappa^2/(\kappa^2 - \Delta^2)}$ and $\varphi_0 = \sqrt{(\kappa^2 - \Delta^2)\hbar^2 n/(2m\kappa^2)}$. This potential contains true vacua at $\varphi_{\text{tv}}/\varphi_0 = 0 \pmod{2\pi}$, and, for $\lambda > 1$, false vacua at $\varphi_{\text{fv}}/\varphi_0 = \pi \pmod{2\pi}$, allowing one to observe vacuum decay by preparing the system in a false vacuum. The decoupling between ϑ and φ is crucial for simulating a relativistic system, as the general coupled equations cannot be derived from any relativistic Lagrangian.

The fields (3) each live on a flat Minkowski space-time with its own ‘‘speed of light’’ (i.e., phonon sound speed),

$$c_\vartheta^2 = \frac{n}{m} \left(2g - \kappa - \frac{\Delta^2}{\kappa} \right), \quad c_\varphi^2 = \frac{n}{m} \left(\kappa - \frac{\Delta^2}{\kappa} \right), \quad (7)$$

where miscibility implies $c_\vartheta^2 \geq c_\varphi^2 > 0$. These define two healing lengths $\xi_i = \hbar/(\sqrt{2mc_i})$, below which each field becomes nonrelativistic, thus determining the regime of validity of the analogy.

References [41–44] previously investigated this ‘‘bimetric’’ structure in Bose-Bose mixtures, deriving a condition analogous to Eq. (4) for linear perturbations around a fixed

background when $g_{\downarrow\downarrow} = g_{\uparrow\uparrow}$ and/or $g_{\downarrow\uparrow} = 0$. Here we generalize to any g_{ij} , greatly expanding the parameter space, and derive a fully nonlinear relativistic theory for the phase fields, which is crucial for simulating nonperturbative phenomena like vacuum decay.

Our decoupling condition (4) is equivalent to matching the chemical potentials of the two species, $\mu_\downarrow = \mu_\uparrow$. Intuitively, this is because each chemical potential sets the mean velocity of the corresponding phase, so Eq. (4) ensures that $\langle \dot{\varphi} \rangle = 0$ in the false vacuum. This also suppresses density fluctuations, as required for pseudorelativistic phase dynamics.

As well as the equations of motion (5), we have verified that the analogy holds in terms of the vacuum fluctuations [40], which is crucial for faithfully simulating the decay rate [26]. Expanding the effective Hamiltonian to quadratic order in the fluctuations, we find that it decouples into two independent sectors $\hat{H}_{\text{eff}} \simeq \hat{H}_\vartheta + \hat{H}_\varphi$, if and only if Eq. (4) holds. Diagonalizing each sector we find that, on scales much larger than their respective healing lengths ($\xi_i k \ll 1$, with k the wave number), the dispersion relations match those of a massless relativistic scalar ϑ and a self-interacting scalar φ with potential given in Eq. (6), with the sound speeds given by Eq. (7):

$$\omega_\vartheta(k) \simeq c_\vartheta k, \quad \omega_\varphi(k) \simeq \sqrt{c_\varphi^2 k^2 + \frac{m_{\text{fv}}^2 c_\varphi^4}{\hbar^2}}, \quad (8)$$

where $\omega_\vartheta(k)$, $\omega_\varphi(k)$ are the frequencies of modes with wave number k , and $m_{\text{fv}} = \sqrt{\lambda^2 - 1} m_0$. This nontrivial check confirms all the key features of the theory.

Experimental proposal. Our results open an extensive new landscape of experimental possibilities. As well as greatly expanding the ⁴¹K parameter space beyond the two symmetric cases considered in Refs. [16,17,26] (see Fig. 1), we can now consider other isotopes, where no symmetric cases exist. As an illustrative example, we present a feasible set of parameters for ³⁹K. We consider the $F = 1$ hyperfine states $|\downarrow\rangle \equiv |m_F = 0\rangle$ and $|\uparrow\rangle \equiv |m_F = -1\rangle$ (with F, m_F the total and projected angular momenta). These are miscible in a magnetic field $B \approx 57\text{--}59$ G, and have been used to study ‘‘quantum droplets’’ just outside the miscible regime [45–48].

While the asymmetric analogy is valid in any number of dimensions, we focus here on the one-dimensional (1D) case, with the atoms tightly confined in the transverse directions. This offers several practical advantages, including the option of running arrays of traps in parallel, or enforcing periodicity using a ring trap. Bubble nucleation is parametrically faster in 1D [40], allowing one to probe a broader range of decay rates within each run. A two-dimensional (2D) setup is also possible (shown in Fig. 2 for illustration), and would allow one to investigate phenomena such as domain walls between distinct true vacua. A three-dimensional (3D) setup would carry technical complications associated with levitating both species against gravity [37].

We summarize our proposed parameters in Table I. We choose these to maximize the energy scale mc_φ^2 , thereby suppressing the influence of thermal fluctuations. This results in a population imbalance $z \approx 0.7$, which differs strongly from the symmetric case $z = 0$. To scan over decay rates, we vary the number of atoms inversely to the strength of

³Note that $\Omega(t)$ can change sign, modulating the phase as well as the amplitude of the coupling.

TABLE I. Fiducial 1D experimental parameters. Here $u = 1.661 \times 10^{-27}$ kg and $a_0 = 5.292 \times 10^{-2}$ nm. The dimensionless density $\bar{n}_\varphi = \xi_\varphi n$ is set by varying the number of atoms N and the transverse trapping frequency ω_\perp .

Parameter	Value
Atomic isotope	potassium-39 (^{39}K)
Atomic mass	$m = 38.96 u = 6.470 \times 10^{-26}$ kg
Hyperfine states	$ \downarrow\rangle \equiv F=1, m_F=0\rangle$ $ \uparrow\rangle \equiv F=1, m_F=-1\rangle$
Magnetic field	$B = 58.50$ G
Scattering lengths (3D)	$a_{\downarrow\downarrow} = 31.85 a_0 = 1.686$ nm $a_{\uparrow\uparrow} = 446.2 a_0 = 23.61$ nm $a_{\downarrow\uparrow} = -51.84 a_0 = -2.743$ nm
Population imbalance	$z = 0.7159$
Healing lengths	$\xi_\vartheta = 1.797 \times 10^4 a_0 = 1.141$ μm $\xi_\varphi = 9.449 \times 10^3 a_0 = 0.600$ μm
Sound speeds	$c_\vartheta = 1.010$ mm s $^{-1}$ $c_\varphi = 1.921$ mm s $^{-1}$
Energy scales	$mc_\vartheta^2 = 0.412$ peV = $4.78 k_B$ nK $mc_\varphi^2 = 1.490$ peV = $17.29 k_B$ nK
Box trap length	$L = 400 \xi_\varphi = 240$ μm
Sound-crossing time	$L/c_\varphi = 124.9$ ms
Total number of atoms	$8000 \leq N \leq 32000$
Density per species	$16.67 \mu\text{m}^{-1} \leq n \leq 66.67 \mu\text{m}^{-1}$
Dimensionless density	$10 \leq \bar{n}_\varphi \leq 40$
Transverse trap frequency	0.356 kHz $\leq \omega_\perp/2\pi \leq 1.43$ kHz
Mean Rabi frequency	$\Omega_0 = 16.04$ Hz
Rabi coupling parameter	$\epsilon = 2.5 \times 10^{-3}$
Modulation amplitude	$\lambda = \sqrt{2}$
False vacuum mass	$m_{\text{fv}} = \sqrt{\lambda^2 - 1} m_0 = 0.1425$ m

transverse confinement, holding mc_φ^2 constant. This lets one vary the dimensionless density $\bar{n}_\varphi = \xi_\varphi n$, which controls the fluctuation amplitudes, while leaving all other parameters of the relativistic theory fixed [26]. Since the rate scales like $\log \Gamma \sim -\bar{n}_\varphi$, even a small range of \bar{n}_φ is sufficient to probe a broad range of scenarios.

One can initialize the false vacuum using a strong, constant coupling $\Omega \gg mc_\varphi^2/\hbar$, so the Hamiltonian becomes

$$\hat{H} \simeq -\frac{\hbar}{2} (|\downarrow\rangle \quad |\uparrow\rangle) \begin{pmatrix} \delta & \Omega \\ \Omega & -\delta \end{pmatrix} \begin{pmatrix} \langle\downarrow| \\ \langle\uparrow| \end{pmatrix} \quad (9)$$

(temporarily ignoring inhomogeneous modes). The resulting energy eigenstates have the species either in phase or in antiphase, corresponding to the true and false vacua, with the population imbalance in the latter given by

$$z_{\text{fv}} = \frac{\langle\text{fv}|\downarrow\rangle - \langle\text{fv}|\uparrow\rangle}{\langle\text{fv}|\downarrow\rangle + \langle\text{fv}|\uparrow\rangle} = -\frac{\delta}{\sqrt{\delta^2 + \Omega^2}}. \quad (10)$$

Starting from a pure- $|\uparrow\rangle$ condensate with large positive detuning (or pure- $|\downarrow\rangle$ with large negative detuning), one can thus initialize a false vacuum with arbitrary z by adiabatically varying δ . We then replace the strong, constant coupling with the weak, modulated coupling required for pseudorelativistic

dynamics. After allowing the system to evolve for some time, φ is converted via a radio-frequency pulse into a density contrast between true- and false-vacuum regions, and imaged with an efficiency of $\sqrt{1-z^2} \approx 70\%$. Analyzing snapshots from many runs then allows one to extract quantities like the decay rate.

Lattice simulations. We test our predictions with semiclassical lattice simulations, using the time-dependent Hamiltonian (1) to evolve $\psi_\downarrow, \psi_\uparrow$. We model the quantum nature of the system by including vacuum fluctuations in the initial state, which we evolve in real time by numerically integrating the classical equations of motion. Repeating this for an ensemble of initial states gives a semiclassical approximation of the system's behavior, known as the ‘‘truncated Wigner approximation’’ [49,50].

These simulations provide detailed dynamical information, complementing insights from Euclidean calculations, and revealing a wealth of new observables such as bubble clustering [14], precursors [51], and time-dependent decay rates [52–54] that are otherwise inaccessible, but could have important theoretical and observational implications. Both the real-time (simulation-based) and imaginary-time (Euclidean) approaches make assumptions and approximations [1,2,49] that ultimately must be empirically tested and calibrated against experiments. This comparison between simulations, Euclidean methods, and our proposed analog experiments has the potential to advance our understanding of vacuum decay, with wide-ranging impact across cosmology and high-energy physics.

We use these simulations here to test the effective relativistic theory described above. We simulate a ^{39}K mixture in a periodic 1D box, with parameters from Table I, using an eighth-order symplectic pseudospectral code. (See Ref. [26] for details and convergence tests; we obtain the same level of convergence here.) In each simulation we set z by minimizing μ in the homogeneous false vacuum, with $\delta = 0$. This corresponds to the decoupling value (4) with an $O(\epsilon)$ correction. We then draw random fluctuations in ϑ, φ and their conjugate momenta, and use our chosen z to convert these into fluctuations in $\psi_\downarrow, \psi_\uparrow$.

We perform several tests, all confirming that ϑ, φ behave like decoupled relativistic fields if and only if Eq. (4) holds:

(1) We test for decoupling by initializing φ without fluctuations, while initializing ϑ as normal; given perfect decoupling, φ should remain constant. For $z \approx 0.7$, fluctuations in φ remain small, reaching a maximum of $\approx 0.1\%$ of the mean-field after one sound-crossing time. We interpret this as a weak coupling due to, e.g., neglected $O(\epsilon^2)$ terms. For $z = 0$ the φ fluctuations grow rapidly to equilibrate with those of ϑ , demonstrating strong coupling.

(2) We test the dynamics of fluctuations (e.g., the sound speeds and false vacuum mass) by computing dispersion relations for ϑ, φ . For $z \approx 0.7$ we find excellent agreement with our predictions (see Fig. 3). For $z = 0$ we find two branches in the dispersion relation of each field, reflecting the coupling between them, as well as modified sound speeds (as expected).

(3) We test whether the nonlinear dynamics of φ are effectively relativistic by computing the Noether charges of the relativistic theory [26], with any time-variation interpreted as departures from relativistic behavior. We expect some varia-

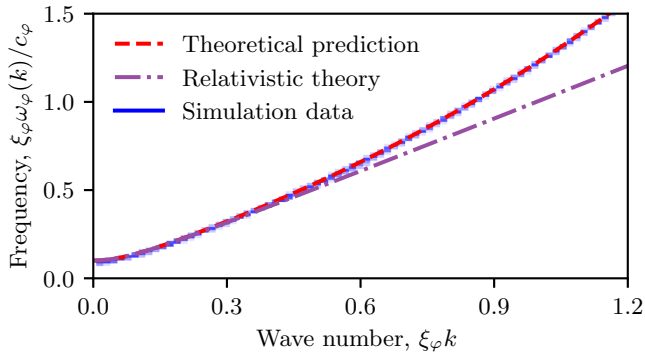


FIG. 3. Dispersion relationship for φ in the false vacuum. The blue heat map shows the average of 32 simulations with parameters from Table I (with $\bar{n}_\varphi = 100$, so no bubbles nucleate). This dispersion relation matches that of a relativistic scalar of mass m_{iv} (purple) on large scales but becomes nonrelativistic on small scales, agreeing with our theoretical prediction (red).

tion due to nonrelativistic effects on small scales, as well as neglected renormalization corrections [55]. However, we find that the charges are conserved just as well as in the symmetric case [26], with variation converging to zero as $\bar{n}_\varphi \rightarrow \infty$. This demonstrates a well-defined relativistic regime for the asymmetric analog.

We perform these tests initially with large densities $\bar{n}_\varphi \geq 100$. This suppresses the vacuum fluctuations, so that our perturbative calculations [e.g., treating density fluctuations as $O(\epsilon)$] give a good approximation. For experimentally accessible densities $\bar{n}_\varphi \lesssim 40$, we expect backreaction from fluctuations to become significant, leading to deviations from our predictions. In particular, this should modify the critical z , so that Eq. (4) no longer gives a long-lived metastable state. We verify this numerically, finding that simulations with $\bar{n}_\varphi \lesssim 40$ decay rapidly due to a homogeneous initial velocity in φ that carries it over the barrier.

In principle one could account for backreaction analytically, calculating the renormalized critical value of z . We postpone this for future work. A simpler solution is to introduce a small detuning δ that compensates for the error in z , such that the initial mean velocity of φ vanishes. Adopting this prescription, our simulations exhibit precisely the expected behavior, passing all the tests described above. This could be implemented experimentally using ensembles of runs with different detunings, selecting the ensemble with the longest-lived metastable state.

Summary and outlook. Cold-atom analog experiments raise the possibility of direct empirical tests of relativistic vacuum decay. We have extended this relativistic analogy to a much larger class of cold-atom systems, representing a proposal for analog vacuum decay that is both rigorously analogous to early-Universe theories and experimentally viable with current capabilities. These experiments have enormous discovery potential, allowing one to test decay rate predictions (potentially resolving or confirming the discrepancy between Euclidean calculations and lattice simulations [50,55]), as well as probing new phenomena, such as bubble clustering [14] and dynamical precursors [51], with far-reaching potential implications for our understanding of the early Universe.

The data that support the findings of this study are available from the corresponding author, A.C.J., under reasonable request.

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Based on the CRediT (Contribution Roles Taxonomy) system, the authors contributed as follows. A.C.J.: methodology; software; formal analysis; investigation; data curation; interpretation and validation; visualization; writing (original draft). I.G.M.: methodology; formal analysis; investigation; interpretation and validation; writing (review). T.P.B.: methodology; interpretation and validation; writing (review). Z.H.: conceptualization; interpretation and validation; writing (review). H.V.P.: interpretation and validation; writing (review); project administration. A.P.: interpretation and validation; writing (review).

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