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IDEAL DEFAULT FOR RESOLVING DISPUTES EFFICIENTLY*

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We study arbitration mechanisms where two parties to the dispute have single-peaked preferences over outcomes, represented by concave utility functions. The most preferred outcome of each party is her private information. By participating in an arbitration mechanism, the parties forfeit the default outcome, which is set without consideration of private preferences. We show that the ideal default outcome for efficient dispute resolution maximizes the sum of the reservation payoffs of the most difficult agent types to persuade to participate in the mechanism. This result is contrary to the conventional wisdom that an unattractive default could force the parties to agree.

1. INTRODUCTION

In this article, we study arbitration mechanisms, with a focus on the properties of the ideal default outcome facilitating efficient dispute resolution. The two parties to the dispute have single-peaked preferences over the set of outcomes [0, 1], represented by concave utility functions. The most preferred outcome of each party is her private information. Parties transmit messages indicating their preferences to the mechanism designer, who in turn determines the implemented outcome as well as the monetary transfers to and from the parties. The alternative to arbitration is the default outcome which is set without any consideration of the private preferences. By participating in the arbitration mechanism, the parties forfeit this default outcome.

What is surprising in our setting is that the ideal default outcome, which provides the best chances for the efficiency of arbitration, is the arrangement that *maximizes* the sum of the *reservation payoffs* of the *critical types* of the two agents. These critical types are defined as the most difficult types to persuade to opt out of this default outcome and accept arbitration instead. This result is contrary to the conventional wisdom that an unattractive default outcome could force the parties to agree. Making the default unattractive for these critical types could easily convince them to take part in dispute resolution. Under this alternative default however, the identity of the critical types would change. As we explain below, the endogeneity of the critical types is the main driver of our result.

Our analysis benefits a great deal from the rich literature on efficient mechanism design with voluntary participation. As established by this literature, implementing the efficient outcome instead of using the default regime would generate a value added for each of the two

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parties. This implementation, however, would also require that the participants share their private information with the arbitration mechanism, which would necessitate some information rent to be left for the participants above and beyond what they would get under the outside option, that is, the default outcome. It is one of the main findings of the efficient design literature that there exists an efficient mechanism if and only if the value added generated by efficiency is large enough to cover the information rent. The difference between the value added and the information rent can be interpreted as the maximized revenue of a mechanism designer constrained to offer efficient mechanisms (Krishna and Perry, 1998 and Williams, 1999).

The magnitudes of the value added from efficiency and the information rent are both influenced by the default outcome. In this article, we study the existence of default arrangements permitting efficient arbitration mechanisms. Answering this existence question involves finding the ideal default outcome that maximizes the revenue of the constrained mechanism designer introduced above. This is a similar exercise to the analyses of Che (2006), Schweizer (2006), Figueroa and Skreta (2012), Segal and Whinston (2016), Agastya and Birulin (2018), and Loertscher and Wasser (2019). In our setting, concavity of the utility functions imply that the designer's revenue is convex in the default outcome for fixed critical types. Nevertheless, once the endogeneity of the critical types is taken into consideration, the revenue turns out to be a function that attains a maximum for an interior default outcome. It follows from the envelope theorem that this latter function is maximized only if the revenue is minimized for a fixed pair of critical types.

Accordingly, the ideal default outcome is the efficient arrangement from the perspective of the types that are most obstinate to give up this default and accept arbitration. If an outside observer does not recognize the endogeneity of these obstinate types in the chosen default regime, ironically she might come to the misleading conclusion that the ideal default is in fact chosen to minimize the potential for arbitration.

One good application of our model is child-custody arbitration process, ensuing a divorce. A custody arrangement most importantly determines the proportion of the time that the child/children will spend with each parent. The preferences with regard to custody can vary a lot among individuals. More specifically, many parents have single-peaked preferences regarding time spent on child-rearing responsibilities, but their peaks may occur at different points.[1] Our modeling feature that these peaks are private information stems from parents' new and different monetary and psychological circumstances and perspectives in the postdivorce era.

Our questions and setup have a lot in common with the recent state that the legal profession has reached regarding the process of divorce and custody bargaining. Today, a vast majority of divorcing couples resolve their custody disputes outside of the court, either with the help of a mediator/arbitrator or by more informal means of bargaining.[2] The seminal judicial work by Mnookin and Kornhauser (1979) asked "how the rules/procedures used in court for adjudicating disputes would affect the bargaining process that occurs between divorcing couples *outside* the courtroom." They had very clear ideas as to what custodial arbitration would involve: Two main elements would be money and custody, which would be inextricably linked in that "over some range of alternatives, each parent may be willing to exchange custodial rights and obligations for income or wealth."

[1] Other examples to settings with single-peaked preferences, for which the mechanism design approach provides an effective framework, include bargaining over budget decisions (Dearden, 1991), discussions for location of a public facility (Lu and Yu, 2013), negotiations over timing of a delivery within a supply chain (Mishra et al., 2014), and exercising a veto over legislative bills (Ali et al., 2023). Also see Carroll (2012) for another example of single peakedness in the context of ordinal preferences.

[2] The article "Breaking up is less hard to do" in the January 22, 2022 issue of *The Economist* provides a nice overview of the recent out-of-court divorce procedures, which are deemed to be "less adversarial" and to lighten "the burden of unhappiness, especially on children caught in the middle" in divorce and custody cases through their "removal of the judicial allocation of blame" (p. 54).

Our study contributes to the debate on the assessment of the practical adjudication procedures for child-custody disputes by integrating the asymmetric information aspect into the parents' pre-trial bargaining. Our results suggest that the default rules adopted by the courts have an impact on the efficiency of the pretrial negotiation not only through the determination of the parents' reservation payoffs in case of a negotiation failure, but also through their effect on the information rent that these parents would get at the successful completion of these negotiations. Once the informational asymmetry between the bargaining parents is taken into account, we find that negotiations are more likely to result in an efficient settlement under default rules that the parents (at least the critical types of the parents) would find desirable, instead of under penalty-like defaults.

In keeping with the earlier literature on efficient mechanism design, we focus on arbitration mechanisms that aim to implement the efficient arrangement from the two disputing parties' perspective. Our analysis, however, can be extended to implementation of any outcome that is monotonic and continuous in the types of the parties. The ideal default outcome, which would facilitate implementation of such arrangements, should also maximize the reservation payoffs of the critical types.

Our framework is conducive to incorporating externalities where each party's preferences depend on both parties' types. This interdependence of payoffs is natural especially in the child-custody setting, where parents are concerned about child welfare that depends on the types of both parents. Similarly, some real-life settlements may require the consent of more parties than two (e.g., such as the children, their grandparents, grown-up siblings, and attorneys or the state representing the children in our motivating example of custody settlements). As long as all involved parties' utility functions are concave, our result that the ideal default outcome maximizes the reservation payoffs of the critical types generalizes to these interdependent-payoffs and multiple-parties settings as well.

The observation that the ideal default outcome coincides with the *efficient* outcome from the perspective of the critical types allows us to write down the maximized revenue from the constrained mechanism design problem as a function of these critical types. The properties of this function lead to novel possibility and impossibility results on the existence of efficient mechanisms.

A particular specification of our model is supported by the *quadratic disutility* function, where each party's payoff is quadratically decreasing in the *distance* between the chosen outcome and her most preferred outcome. In this setting, we show that efficient arbitration is possible only under the ideal default outcome. The ideal outcome here is identical to the ex ante efficient outcome that would have been chosen in the absence of the possibility of arbitration.[3] The efficient arbitration mechanism relies on a net monetary transfer from the less conciliatory party (whose revealed type is relatively further from the expected type of the other party) to the more conciliatory one. This result is a *general possibility result* in the sense that it holds for any continuous type distribution with full support. It is sufficient to know the expected types of the two parties in order to identify the ideal default outcome and the efficient mechanism that the designer should choose.

Dispute resolution may also involve decisions on more than one dimension: Custody settlements may concern multiple children or multiple decisions (school choice/extracurricular activities) on a single child. Our possibility result for quadratic disutility functions extends to this multidimensional environment. An efficient arbitration mechanism exists only under the ideal default outcome, which is also the ex ante efficient outcome. This possibility result holds in the multiple-parties setting as well.

We complement the possibility result with a similarly general *impossibility result* that we derive in an alternative setting. Suppose each party's (dis)utility from a chosen outcome is

[3] Ex ante efficient default rules are known to enhance ex post efficient bargaining when the negotiating parties have non-concave utility functions as well. See Che (2006) and Segal and Whinston (2016) for linear utility functions, Segal and Whinston (2011) for convex utility functions.

determined by the magnitude of the *difference* between this chosen outcome and the party's most preferred arrangement. Quadratic disutility model is a special case of this specification because *distance* is the absolute value of difference. When the second derivative of the utility function is convex and strictly monotonic (either increasing or decreasing), we show that there is no default outcome that permits efficient bargaining, regardless of the parameters of the continuous type distributions.

2. THE MODEL

Agent 1 and agent 2 are disputing over an outcome $x \in [0, 1]$. The preferred outcome from each agent's perspective is her private information. This private information is represented by agent i's type $\theta_i \in [0, 1]$. In addition to the outcome, the two parties care about the monetary transfers $t_1, t_2 \in R$ that they receive or make. The payoff function for agent i is

$$u_i(x, t_i, \theta_i) = v_i(x, \theta_i) + t_i, \tag{1}$$

where v_i is a twice continuously differentiable direct utility function of agent i with type θ_i from outcome x. Function v_i is strictly concave in x and its cross partial derivative is positive:

$$\frac{\partial^2 v_i(x, \theta_i)}{\partial x^2} < 0, \tag{2}$$

$$\frac{\partial^2 v_i(x, \theta_i)}{\partial x \partial \theta_i} > 0. \tag{3}$$

Function v_i is maximized in x when $x = \theta_i$. The types of the agents are independently distributed on $[0, 1]$. The distribution functions are continuous and they have full support.

As an example to these preferences, consider the situation where each agent's direct utility is determined by the difference between the implemented and the desired outcomes: $v_i(x - \theta_i)$, where v_i is a concave function maximized at 0. A special case for this would be the quadratic disutility function such that $v_i(x, \theta_i) = -(x - \theta_i)^2$. We will come back to these preference specifications to prove our possibility and impossibility results.

Following the notation in Segal and Whinston (2011, 2012, 2016) papers, we define the surplus generated with the outcome x as the sum of the direct utility functions of the agents:

$$s(x, \theta_1, \theta_2) = v_1(x, \theta_1) + v_2(x, \theta_2). \tag{4}$$

Strict concavity of functions v_1 and v_2 implies that function s is strictly concave in x as well, hence there is a unique outcome $x^*(\theta_1, \theta_2)$ that maximizes this surplus. We refer to this outcome as the "ex post efficient outcome." Notice that the efficient outcome x^* is strictly increasing in the types of both agents. We also define the maximized surplus as a function of the agent types:

$$S(\theta_1, \theta_2) = \max_x s(x, \theta_1, \theta_2) = s(x^*(\theta_1, \theta_2), \theta_1, \theta_2). \tag{5}$$

In this article, we are interested in arbitration mechanisms that implement the ex post efficient outcome. Invoking the revelation principle, we model arbitration as a direct revelation mechanism: The agents reveal their types θ_1 and θ_2 to the mechanism and the arbitrator sets the corresponding ex post efficient outcome $x^*(\theta_1, \theta_2)$ together with transfers $t_1(\theta_1, \theta_2)$ and $t_2(\theta_1, \theta_2)$. Each agent has the option to refuse to participate in this mechanism and opt for the default outcome $x_0 \in [0, 1]$, which does not depend on the types of the agents. We follow the normalization that the default transfer payment is zero.[4]

We say that the default outcome x_0 permits efficient arbitration if there exist transfer functions $t_1(\theta_1, \theta_2)$ and $t_2(\theta_1, \theta_2)$ which satisfy the following individual rationality, incentive compatibility, and budget balance conditions together with the ex post efficient outcome $x^*(\theta_1, \theta_2)$. An efficient arbitration mechanism is

- *individually rational* if each agent prefers arbitration to the default outcome x_0:

$$IR: \mathrm{E}_{\theta_j}[v_i(x^*(\theta_i, \theta_j), \theta_i) + t_i(\theta_i, \theta_j)] \geq v_i(x_0, \theta_i) \text{ for all } \theta_i \in [0, 1] \text{ and } i = 1, 2;$$

- *incentive compatible* if each agent prefers to reveal her type truthfully:

$$IC: \mathrm{E}_{\theta_j}[v_i(x^*(\theta_i, \theta_j), \theta_i) + t_i(\theta_i, \theta_j)] \geq \mathrm{E}_{\theta_j}[v_i(x^*(\theta_i', \theta_j), \theta_i) + t_i(\theta_i', \theta_j)]$$
$$\text{for all } \theta_i, \theta_i' \in [0, 1] \text{ and } i = 1, 2;$$

- *budget balanced* if monetary transfers add up to zero for all type pairs:

$$BB: t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2) = 0 \text{ for all } \theta_1, \theta_2 \in [0, 1];$$

  where operator E_{θ_j} refers to the expectation over the type of agent j.

The earlier literature asks analogous questions on the existence of efficient mechanisms in bilateral trade and partnership dissolution settings, where $v_i(x, \theta_i)$ is linear both in the allocation decision x and the agent type θ_i. Myerson and Satterthwaite (1983) establish that there is no efficient bilateral trade mechanism that secures allocating the good to the buyer whenever her valuation for it is higher than that of the seller. We learn from the work of Cramton et al. (1987) that this impossibility result relies on the extreme nature of the default option: The seller keeps the entire good to herself in case of a trade failure. An efficient allocation mechanism would exist as long as the default alternative to accepting the mechanism is a more equitable division. For instance, there is an efficient mechanism allocating the sole ownership of a firm to the partner who has the highest valuation for it, provided that no partner has a very large initial share in the firm.[5] These earlier results point to the importance of the default outcome x_0 for the arbitrator's ability to mediate an efficient settlement for the agents. Krishna and Perry (1998) and Williams (1999) extend the *efficient mechanism design* analysis to more general preference functions and type sets.[6] We now briefly sketch the analysis of this earlier literature, applying it to our concave utility setting. We report the main result from this analysis in Lemma 1.

As a first step to the assessment of the existence of an ex post efficient mechanism, we drop the budget balance requirement (BB). Instead, we consider the revenue maximization of a mechanism designer who maximizes $-t_1 - t_2$ subject to the individual rationality (IR) and incentive compatibility (IC) constraints, as well as the requirement that the resulting outcome is efficient $x^*(\theta_1, \theta_2)$. Because different agent types assess the default outcome differently, this is an example to a maximization problem with type-dependent reservation utility levels as in Jullien (2000). Thanks to the efficiency requirement and our differentiability assumptions on

[4] Another interpretation for t_i would be that it is the difference between the transfer from the arbitration mechanism and the transfer from the default outcome.

[5] For efficient mechanism design with linear utility, also see Che (2006), Ornelas and Turner (2007), Figueroa and Skreta (2012), Yenmez (2012), Agastya and Birulin (2018), and Loertscher and Wasser (2019).

[6] Also see Makowski and Mezzetti (1994), Neeman (1999), Schweizer (2006), and Segal and Whinston (2011, 2012, 2016).

the utility functions,[7] any incentive-compatible mechanism is an *expected externality* mechanism: The expected transfer to agent i with type θ_i is identified by the expected direct utility of the other agent up to a constant term k_i.[8]

$$\mathrm{E}_{\theta_j} t_i(\theta_i, \theta_j) = \mathrm{E}_{\theta_j} v_j(x^*(\theta_i, \theta_j), \theta_j) + k_i. \tag{6}$$

The expectation of this transfer over θ_i gives us the expected transfer $\mathrm{E}[t_i]$ to agent i as

$$\mathrm{E}[t_i] = \mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j} v_j(x^*(\theta_i, \theta_j), \theta_j) + k_i. \tag{7}$$

Consider now an agent i with type $\hat{\theta}_i$ who is contemplating to accept this arbitration mechanism. By accepting this mechanism, this type would expect to receive direct utility $\mathrm{E}_{\theta_j} v_i\left(x^*\left(\hat{\theta}_i, \theta_j\right), \hat{\theta}_i\right)$ in addition to the transfer $\mathrm{E}_{\theta_j} t_i\left(\hat{\theta}_i, \theta_j\right)$. And she would forego the reservation utility $v_i\left(x_0, \hat{\theta}_i\right)$ resulting from the default outcome x_0. Type $\hat{\theta}_i$ would be indifferent between these two options if

$$\underbrace{\mathrm{E}_{\theta_j} v_i\left(x^*\left(\hat{\theta}_i, \theta_j\right), \hat{\theta}_i\right) + \mathrm{E}_{\theta_j} v_j\left(x^*\left(\hat{\theta}_i, \theta_j\right), \theta_j\right)}_{\mathrm{E}_{\theta_j} S\left(\hat{\theta}_i, \theta_j\right)} + k_i = v_i\left(x_0, \hat{\theta}_i\right), \tag{8}$$

identifying the constant

$$k_i = v_i\left(x_0, \hat{\theta}_i\right) - \mathrm{E}_{\theta_j} S\left(\hat{\theta}_i, \theta_j\right) \tag{9}$$

as a function of type $\hat{\theta}_i$ for which the individual rationality constraint is satisfied as an equality. Once constant k_i is pinpointed as in (9), by using (7), we can write the expected transfer to agent i as

$$\mathrm{E}[t_i] = \mathrm{E}_{\theta_i}\mathrm{E}_{\theta_j} v_j(x^*(\theta_i, \theta_j), \theta_j) - \mathrm{E}_{\theta_j} S\left(\hat{\theta}_i, \theta_j\right) + v_i\left(x_0, \hat{\theta}_i\right). \tag{10}$$

Negative of the sum of these transfers for the two agents yields the expected revenue of a mechanism designer offering this efficient mechanism

$$\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right) = \mathrm{E}_{\theta_2} S\left(\hat{\theta}_1, \theta_2\right) + \mathrm{E}_{\theta_1} S\left(\theta_1, \hat{\theta}_2\right) - \mathrm{E}_{\theta_1\theta_2} S(\theta_1, \theta_2) - s\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right), \tag{11}$$

under the qualification that the individual rationality condition is barely satisfied for type $\hat{\theta}_1$ of agent 1 and type $\hat{\theta}_2$ of agent 2. For the individual rationality condition to be *globally* satisfied, *all* types of both agents must (weakly) prefer to accept the mechanism. Accordingly, the maximized expected revenue of the mechanism designer from an incentive-compatible and individually rational mechanism is

$$\bar{\pi}(x_0) = \min_{\hat{\theta}_i, \hat{\theta}_2} \pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right). \tag{12}$$

Because the types of the agents are drawn from the closed and bounded set $[0, 1]$, the above minimization problem is well-defined. The types $\hat{\theta}_1(x_0)$, $\hat{\theta}_2(x_0)$ solving this problem are called the *critical types* which are the most difficult ones to persuade to participate in this arbitration

[7] For results on implementable transfers in the absence of differentiability, see Chung and Olszewski (2007), Carbajal and Ely (2013), and Kos and Messner (2013) among others.

[8] See Arrow (1979) and d'Aspremont and Gérard-Varet (1979) for expected externality mechanisms.

mechanism. For future reference, we note the first-order necessary condition for the critical type $\hat{\theta}_i$ under default decision x_0 below:

$$\text{(13)} \qquad \mathrm{E}_{\theta_j} \frac{\partial v_i\big(x^*\big(\hat{\theta}_i, \theta_j\big), \hat{\theta}_i\big)}{\partial \theta_i} - \frac{\partial v_i\big(x_0, \hat{\theta}_i\big)}{\partial \theta_i} \begin{array}{l} \geq 0 \text{ for } \hat{\theta}_i = 0, \\ = 0 \text{ for } \hat{\theta}_i \in (0, 1), \\ \leq 0 \text{ for } \hat{\theta}_i = 1. \end{array}$$

It follows from the previous literature on efficient mechanism design that there exists a budget-balanced efficient arbitration mechanism under x_0 if and only if the maximized revenue of this constrained mechanism is nonnegative.

LEMMA 1. *Default outcome x_0 permits efficient arbitration if and only if $\bar{\pi}\ (x_0)$ is nonnegative.*

The constrained mechanism constructed above is already incentive-compatible and individually rational. As long as this constrained mechanism does not run an expected deficit, the degrees of freedom associated with Bayesian implementation can be used to balance the budget, so that the transfers sum up to zero under all possible type pairs. Here is one way to interpret this result: Implementing the efficient outcome—instead of the default outcome—generates some value added for the agents. But an efficient arbitration mechanism should leave some information rent to these agents, so that they are induced to share their private information with the mechanism designer, ensuring that the implemented outcome is indeed efficient. If the constrained-revenue-maximizing mechanism is not running a deficit, it means that the added value is high enough to cover the required information rent.

3. IDEAL DEFAULT OUTCOME

In the context of linear direct utility functions, Myerson and Satterthwaite (1983) establish that extreme default arrangements are not compatible with efficient mechanism design. With our first proposition, we confirm that their insight extends to our setting with concave direct utility functions. If the default outcome takes an extreme value, then there is no efficient arbitration mechanism.

PROPOSITION 1. *Extreme values of the default outcome ($x_0 = 0$ and $x_0 = 1$) do not permit efficient arbitration.*

Proofs of all the propositions are relegated to the Appendix. The proposition above does not rule out efficient mechanisms that can be supported by intermediate default outcomes. The same way that the default of an equitable division can sustain an efficient allocation of the assets of a dissolving firm, an intermediate default outcome could facilitate the parties' agreement on an efficient way to resolve their dispute. We would like to explore the existence of such default arrangements. More specifically, our aim is establishing possibility/ impossibility results for efficient arbitration which do not depend on the specifics of the type distributions.

We investigate the existence of default outcomes permitting efficient arbitration by examining the sign of function $\bar{\pi}\ (x_0)$ at its maximum level. We call the default outcome that maximizes $\bar{\pi}\ (x_0)$ and therefore gives the best chances for an efficient arbitration the *ideal default outcome* $\hat{x}_0$. This is analogous to the *frugal partnership* that Agastya and Birulin (2018) define as the initial ownership shares that minimizes the cost of guaranteeing the efficient dissolution of a partnership.

When the parties' payoffs are convex in the policy decisions x and linear in their private information θ_i, Schweizer (2006) establishes that the ideal default $\hat{x}_0$ constitutes a *saddle point* of the revenue function $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ together with the critical types $\hat{\theta}_1(\hat{x}_0)$ and $\hat{\theta}_2(\hat{x}_0)$ under this ideal default. That is, the same way that the critical types $\hat{\theta}_1(\hat{x}_0)$ and $\hat{\theta}_2(\hat{x}_0)$ minimize $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ for $x_0 = \hat{x}_0$, the ideal default $\hat{x}_0$ maximizes $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ when $\hat{\theta}_1$ and $\hat{\theta}_2$ are held constant at $\hat{\theta}_1(\hat{x}_0)$ and $\hat{\theta}_2(\hat{x}_0)$, respectively. Schweizer uses this saddle point property in his generalization of Cramton et al. (1987) efficient dissolution result.[9]

In our dispute resolution setting, where each agent's payoff v_i is strictly concave in decision x and nonlinear in agent's type θ_i, function $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ does not have a saddle point as above. Therefore, Schweizer's possibility result does not extend to our setup. The ideal default outcome $\hat{x}_0$ is still an extreme point for function $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ for $\hat{\theta}_1 = \hat{\theta}_1(\hat{x}_0)$ and $\hat{\theta}_2 = \hat{\theta}_2(\hat{x}_0)$. However, as we show with the following proposition, it yields a minimum for this function instead of a maximum.

Proposition 2. *The ideal default outcome* $\hat{x}_0$ *is in the interior of* $[0,1]$ *and it minimizes* $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ *for* $\hat{\theta}_1 = \hat{\theta}_1(\hat{x}_0)$ *and* $\hat{\theta}_2 = \hat{\theta}_2(\hat{x}_0)$.

We can see that the default outcome x_0 affects the constrained revenue of the mechanism designer $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ in two ways: First, x_0 enters directly in the reservation utility of the agents with the critical types $s\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$. This reservation utility has a negative sign in the constrained revenue function. Second, a variation in x_0 changes the critical types $\hat{\theta}_1(x_0)$ and $\hat{\theta}_2(x_0)$, which also enter into function π. Because $\bar{\pi}(x_0)$ is defined as the lower envelope of the $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ functions, it follows from the envelope theorem that the second effect does not have an impact on the first derivative of $\bar{\pi}(x_0)$. Accordingly, a local extreme point for $\bar{\pi}(x_0)$ is also a local extreme point for $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$, provided that $\hat{\theta}_i = \hat{\theta}_i(x_0)$.

What is surprising about the proposition is that maximization of $\bar{\pi}(x_0)$ implies minimization of $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ for $\hat{\theta}_i = \hat{\theta}_i(x_0)$. It follows from the concavity of the utility functions v_i that each $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ is a *convex* function of x_0. Yet, the lower envelope of these functions, $\bar{\pi}(x_0) = \pi\left(x_0, \hat{\theta}_1(x_0), \hat{\theta}_2(x_0)\right)$, is not convex. It has a maximum in the interior of the policy space $[0,1]$. By maximizing $\bar{\pi}(x_0)$, the ideal default outcome minimizes $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$.[10] (See Figure 1.)

This observation also sheds some light on Segal and Whinston (2012) assessment that reducing surplus from the default option for the critical types makes efficient negotiation more likely. Their assessment holds for all default arrangements other than the ideal one, because $d\bar{\pi}(x_0) = \frac{\partial \pi\left(x_0, \hat{\theta}_1(x_0), \hat{\theta}_2(x_0)\right)}{\partial x_0}$. However, under the ideal default outcome, $d\bar{\pi}(x_0)$ equals 0 and the impact of a change in the default outcome is determined by the second-order effects, which suggest different directions for $\bar{\pi}(x_0)$ and $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$.

Notice that the default outcome x_0 enters into the constrained revenue function π only at its last term $s\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ which has a negative sign. Hence, an important corollary to the above proposition is that the ideal default outcome x_0 maximizes the surplus function $s\left(x, \hat{\theta}_1, \hat{\theta}_2\right)$.

[9] A similar saddle point argument is developed by Loertscher and Wasser (2019) to find the optimal allocation maximizing a convex combination of a designer's revenue and efficiency.

[10] In partnership dissolution models with linear utility functions à la Cramton et al. (1987), the constrained revenue function that is analogous to our $\bar{\pi}(x_0)$ is concave in the initial partnership shares. On the other hand, allowing for multidimensional types determining partners' values under different ownership structures, Agastya and Birulin (2018) show that this constrained revenue may be convex in the initial shares—implying that the ideal partnership can be "skewed." In our nonlinear setting, function $\bar{\pi}(x_0)$ is neither concave nor convex, but it has a maximum in the interior of the policy space [0,1]. The ideal default outcome $\hat{x}_0$ maximizing $\bar{\pi}(x_0)$ need not be unique (Agastya and Birulin give an example for this possibility under nonconcavity). In that case, the above proposition holds for any ideal default.

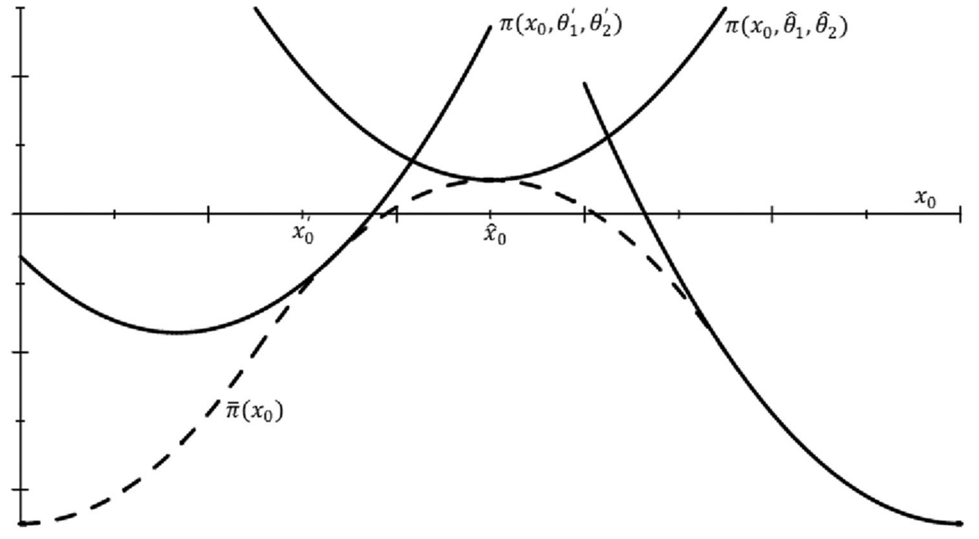


FIGURE 1

FUNCTION $\bar{\pi}(x_0)$ AS THE LOWER ENVELOPE OF $\pi(x_0, \cdot)$

COROLLARY 1. *The ideal default outcome $\hat{x}_0$ maximizes surplus $s\left(x, \hat{\theta}_1(\hat{x}_0), \hat{\theta}_2(\hat{x}_0)\right)$ for the critical agent types:*

$$\hat{x}_0 = x^*\left(\hat{\theta}_1(\hat{x}_0), \hat{\theta}_2(\hat{x}_0)\right).$$

The corollary above does not provide a closed-form solution for the ideal default outcome, because $\hat{x}_0$ appears on both sides of the equality in its statement. Instead, it yields an intriguing necessary condition for the ideal default outcome. The default outcome $\hat{x}_0$ constitutes the outside option of participation in the arbitration mechanism for the two agents. Yet, maximization of the designer's revenue—and therefore creating the value to be distributed to the agents during a budget-balanced arbitration—involves maximizing the magnitude of this outside option, for the types of the agents which are most difficult to persuade to participate in the mechanism. As alluded to in the Introduction, after identifying these critical types, a naive observer might think that the default outcome $\hat{x}_0$ is set to incite the agents' refusal of the arbitration process. This reasoning of course misses the endogeneity of the critical types on the default arrangement.

Endogeneity of the critical types is also an important feature in the earlier literature on efficient mechanism design. Equal partnerships are easier to dissolve efficiently, precisely because the critical partner types are the interior types without extreme valuations for the firm, unlike in the bilateral trade environment. What is novel in our setting is that enhancing the efficiency of the arbitration mechanism requires the provision of a default option that will be satisfying especially for the critical agent types.

To explain the idea further, we can go back to our motivating example involving child-custody allocations. Imagine that the ideal default custody rule that gives the best chances for an efficient out-of-court settlement is indeed a weighted *joint-custody* arrangement ($\hat{x}_0$ in Figure 1). Proposition 2 and its corollary imply that this arrangement is also the most efficient custody arrangement from the perspective of the critical types of the parents who are the most reluctant to give up the court-ordered custody and instead accept an ex post efficient arbitration mechanism. Now suppose that a naive policymaker shifts the default custody rule toward a *paternal-custody* standard (such as x_0' in Figure 1), with the hope that the parents would find this custody arrangement less desirable and hence they would be easier to persuade to take part in arbitration. Under this alternative default rule, the (original) critical parent types would indeed have lower reservation payoffs. However, their compliance with the efficient mechanism would require that they do not pretend to have a higher preference for paternal custody. This would entail an obligation to leave a higher information rent to these parent types during negotiations. Because the rate of change of a parent's information rent as a function of her types is identified by the efficient custody allocation x^*, the entire information rent function shifts upward uniformly under this new default custody. Accordingly, the policy shift to a less desirable default rule could put efficient arbitration in jeopardy, because the surplus generated by the ex post efficient allocation of the custody may not be sufficient to cover these higher information rents.

Proposition 2 and its corollary have the following policy implication. The intuitive understanding that an unattractive default improves efficiency of dispute resolution is wrong. In fact, if the default outcome does not coincide with the most attractive arrangement from the critical types' perspective, the potential for efficient dispute resolution can be improved by selecting a more suitable default.[11]

REMARK 1 (*LINEAR DIRECT UTILITY*). Because of the strict concavity assumption, our setting does not nest the partnership dissolution models where the partners' payoffs are linear in the shares they hold in the partnership (Cramton et al., 1987). Nevertheless, we can see that the above proposition is valid for these models as well. Our default outcome x_0 is analogous to the initial shares of the partners in the partnership setting. When critical types are held constant, function π is linear in these initial shares, with a slope equal to the derivative of the lower envelope function $\bar{\pi}$. Under the frugal partnership that maximizes function $\bar{\pi}$, function π is constant and attains its extreme value in a trivial sense.

When function π is constant, it means that the total payoff cannot be increased by shifting the firm's shares from one partner with the critical type to another partner with the critical type. This implies that under the ideal allocation of the initial shares, critical types of all the partners should be the same. This is indeed the condition identified by Che (2006) and Figueroa and Skreta (2012) for the initial allocation of shares to give the highest chances for an efficient dissolution.

REMARK 2 (*ALTERNATIVE OUTCOME FUNCTIONS*). Our focus in this article is the mechanisms implementing the ex post efficient outcome $x^*(\theta_1, \theta_2)$ from the perspective of the disputing parties. Nevertheless, it is worth noting that Proposition 2 holds for the implementation of alternative outcome functions $x'(\theta_1, \theta_2)$ that are monotonic and continuous in both dimensions.[12] The ideal default outcome $\hat{x}'_0$ that would maximize the potential rent for a mechanism implementing $x'(\theta_1, \theta_2)$ satisfies

$$\hat{x}'_0 = x^*\big(\hat{\theta}_1(\hat{x}'_0), \hat{\theta}_2(\hat{x}'_0)\big),$$

[11] This observation has an interesting implication for divorce arbitration. Starting with Mnookin and Kornhauser (1979), many legal scholars consider the joint-custody legal standard as a default arrangement that both parents would like to avoid, "very much like … Solomon's threat to cut the child in half." When comparing divorce settlements in two different states, Brinig and Alexeev (1992) observe that parents are more likely to settle out of court in Wisconsin—which followed the joint-custody standard—than in Virginia where the family courts had a tendency to give full custody to the primary caretaker before separation. Brinig (2006) interprets this finding as a confirmation that parents are unlikely to have a preference for joint custody and policymakers may use it as a penalty default to incite out-of-court settlements. Our result suggests a completely different consideration for setting legal standards. Once the informational asymmetries are taken into account, we show that encouraging parents to go through an efficient arbitration process requires a legal standard which is closer to what is desired by the parents, especially by the parent types which are more difficult to persuade to take part in arbitration.

[12] Following the arguments in Segal and Whinston (2011), a constrained revenue function $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ can be constructed for any such outcome function. As in Proposition 2, maximizing $\bar{\pi}(x_0) = \min_{\hat{\theta}_1, \hat{\theta}_2} \pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ implies minimizing $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ for fixed $\left(\hat{\theta}_1, \hat{\theta}_2\right)$.

where $\hat{\theta}_1\left(\hat{x}_0'\right)$ and $\hat{\theta}_2\left(\hat{x}_0'\right)$ are the critical types that are most difficult to persuade to accept the mechanism instead of the default $\hat{x}_0'$. Monotonicity of $x'(\theta_1, \theta_2)$ is needed for the incentive-compatibility condition to be satisfied and its continuity ensures that the envelope theorem applies. Outcome function $x'(\theta_1, \theta_2)$ here can be considered either as an exogenous function of agent types or an endogenous one that solves a maximization problem. Examples to such maximization problems are the arbitrator's revenue maximization, maximization of efficiency subject to a budget constraint (Segal and Whinston, 2016), or a combination of revenue and efficiency (Loertscher and Wasser, 2019).[13]

REMARK 3 (*EXTENALITIES*). Our analysis extends to the interdependent-value setting, where each party's value from an outcome depends on the rival's type as well as her own type: $v_i(x, \theta_i, \theta_j)$. Such direct externalities would come up naturally in a child-custody setting: Each parent may be concerned about the child's own welfare and the other parent's private information may be relevant to make an assessment on that. We can impose concavity and positive-cross-partial-derivative conditions by assuming

$$\frac{\partial^2 v_i(x, \theta_i, \theta_j)}{\partial x^2} < 0,$$

$$\frac{\partial^2 v_i(x, \theta_i, \theta_j)}{\partial x \partial \theta_i} > 0$$

for all $\theta_j \in [0, 1]$. We also maintain the single-peaked preferences by letting $v_i(x, \theta_i, \theta_j)$ be maximized in x when $x = \theta_i$, again for all θ_j.[14] The ex post efficient outcome is now defined as

$$x^*(\theta_1, \theta_2) = \arg\max_x v_1(x, \theta_1, \theta_2) + v_2(x, \theta_2, \theta_1).$$

Assuming that $x^*(\theta_1, \theta_2)$ is monotonic and continuous in both types, an analogous result to Corollary 1 holds: [15] The ideal default outcome that facilitates the implementation of the efficient outcome solves maximization problem

$$\max_{x_0} \mathrm{E}_{\theta_2} v_1\big(x, \hat{\theta}_1, \theta_2\big) + \mathrm{E}_{\theta_1} v_2\big(x, \hat{\theta}_2, \theta_1\big),$$

where $\hat{\theta}_1$ and $\hat{\theta}_2$ are the critical types under this ideal default outcome.

REMARK 4 (*MORE THAN TWO PARTIES TO DISPUTE RESOLUTION*). Some settlements require the agreement of more than two parties who may all have substantial claims. In a child-custody setting, these parties may include the children themselves, their grandparents, grown-up siblings, and attorneys or the state representing the children. The previous result generalizes to I (>2) disputants, as long as their utility functions are concave in the outcome $x \in$

[13] Loertscher and Wasser (2019) make a similar point in a model with linear payoffs and interdependent values. They show that, even though the optimal allocation rule maximizing a convex combination of revenue and efficiency is endogenous, the expected values of the critical types of the partners are "equalized" under the ideal default allocation.

[14] As in our baseline model without the externalities, it is important that the peaks of the preferences of different types cover the entire policy set [0, 1]. This ensures that function $\bar{\pi}(x_0)$ has an interior maximum.

[15] See Segal and Whinston (2011) for the construction of function $\pi(x_0, \theta_1, \theta_2)$ under interdependent values. Maximization of a similar function is also relevant for optimal auction design for bidders with interdependent values (Jehiel et al., 1999; and Figueroa and Skreta, 2009, 2011).

[0, 1]. Let θ be the profile of the types of all I agents. We can define

$$\pi\left(x_0, \hat{\theta}\right) = \sum_i \mathrm{E}_{\theta_{-i}} S\left(\hat{\theta}_i, \theta_{-i}\right) - (I-1)\mathrm{E}_\theta S(\theta) - s\left(x_0, \hat{\theta}\right),$$

$$\bar{\pi}\left(x_0\right) = \min_{\hat{\theta}} \pi\left(x_0, \hat{\theta}\right).$$

We know from the earlier literature on efficient mechanism design that an analogous result to Lemma 1 holds: Default arrangement x_0 permits efficient arbitration if and only if $\bar{\pi}\left(x_0\right)$ is nonnegative. It follows from our analysis that x_0 *maximizing* $\bar{\pi}\left(x_0\right)$ also *minimizes* $\pi\left(x_0, \hat{\theta}\right)$, and hence maximizes $s\left(x_0, \hat{\theta}\right)$, when $\hat{\theta} = \hat{\theta}\left(x_0\right)$.

REMARK 5 (*MULTIPLE DIMENSIONS OF DISPUTE*). Imagine that the two parties have to make decisions on n different dimensions of their dispute. In a child-custody setting, decisions may involve multiple children, or multiple dimensions of custody parameters for the same child such as school choices, extracurricular activities, etc. One way to model this situation is to assume that the outcome x is a vector which is an element of set $[0, 1]^n$. In this extension of our model, agent types θ_1 and θ_2 are elements of $[0, 1]^n$ as well, indicating their personal preferences on these dimensions. The payoff function of agent i is

$$u_i(x, t_i, \theta_i) = v_i(x, \theta_i) + t_i,$$

where v_i represents the direct utility of agent i with type θ_i from decision x. We maintain the assumptions that v_i is concave in vector x (i.e., $v_{i_{xx}} < 0$), its cross partial derivative is positive $\left(v_{i_{x\theta_i}} > 0\right)$, and it is maximized in x when $x = \theta_i$. The ex post efficient outcome $x^*\left(\theta_1, \theta_2\right)$ is now defined as a vector function.

We know from the earlier literature that Lemma 1 applies to this multidimensional extension of our model as well. That is, an efficient arbitration mechanism exists under default outcome $x_0 \in [0, 1]^n$ if and only if $\bar{\pi}\left(x_0\right)$ is nonnegative. However, we cannot rule out the possibility that the ideal default policy maximizing $\bar{\pi}\left(x_0\right)$ is a corner solution. Therefore, we cannot prove an analogous result to Proposition 2. The proposition would hold if the ideal default policy is in the interior of $[0, 1]^n$. We will have more remarks on this point in the context of the quadratic disutility functions.

4. CONSTRAINED REVENUE AS FUNCTION OF CRITICAL TYPES

We start this section by showing that, the same way that the ideal default outcome is in the interior of the policy space, the critical types of the two agents under this ideal default are also in the interior of the type space.

PROPOSITION 3. *Under the ideal default outcome $\hat{x}_0$, the critical types $\hat{\theta}_1\left(\hat{x}_0\right)$ and $\hat{\theta}_2\left(\hat{x}_0\right)$ are in the interior of* [0, 1].

We learn from the analysis in the previous section that, when function $\bar{\pi}\left(x_0\right)$ is maximized, its last term reduces to the maximized surplus for the critical agent types under the ideal default outcome:

$$s\left(\hat{x}_0, \hat{\theta}_1(\hat{x}_0), \hat{\theta}_2(\hat{x}_0)\right) = S\left(\hat{\theta}_1(\hat{x}_0), \hat{\theta}_2(\hat{x}_0)\right).$$

By using the above equation and (11), we can rewrite the maximized constrained revenue $\max_{x_0} \bar{\pi}(x_0)$ as a function of these critical types:

$$\bar{\pi}(\hat{x}_0) = \mathrm{E}_{\theta_2} S(\hat{\theta}_1, \theta_2) + \mathrm{E}_{\theta_1} S(\theta_1, \hat{\theta}_2) - \mathrm{E}_{\theta_1 \theta_2} S(\theta_1, \theta_2) - S(\hat{\theta}_1, \hat{\theta}_2),$$

where $\hat{\theta}_i = \hat{\theta}_i(\hat{x}_0)$. With some abuse of notation, we refer to the right-hand side of the equation above as $\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2)$. Because $x^*(\hat{\theta}_1, \hat{\theta}_2)$ maximizes function $s(x_0, \hat{\theta}_1, \hat{\theta}_2)$, function $\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2)$ defines a lower envelope of $\pi(x_0, \hat{\theta}_1, \hat{\theta}_2)$ over the $\hat{\theta}_1 \times \hat{\theta}_2$ domain:

$$\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2) = \min_{x_0} \pi(x_0, \hat{\theta}_1, \hat{\theta}_2) = \pi(x^*(\hat{\theta}_1, \hat{\theta}_2), \hat{\theta}_1, \hat{\theta}_2).$$

This function will be useful in the derivation of further results about efficient arbitration. It follows from the envelope theorem that any interior local extremum of function $\bar{\pi}(x_0)$ also constitutes an interior *candidate point* for $\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2)$ satisfying the first-order conditions for interior local extrema, where $\hat{\theta}_i = \hat{\theta}_i(x_0)$.[16]

In other words, the first-order necessary conditions for interior local extrema of $\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2)$ coincide with the first-order necessary conditions for the minimization problem defining the critical types under the ideal default outcome $\hat{x}_0$. By examining the value that function $\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2)$ takes at these candidate points, we can see whether the ideal default outcome permits efficient arbitration. This observation will be critical in the derivation of our possibility and impossibility results for particular specifications of our model in the following sections.

At this juncture, we remark that earlier possibility results on efficient mechanisms do not apply to our dispute resolution setting with concave utility functions. Cramton et al. (1987) focus exclusively on linear utility. And the generalizations by Schweizer (2006) and Segal and Whinston (2011) cover cases with payoffs that are convex in the decision variable x or with default arrangements that can be randomized.[17] Segal and Whinston's (2016) impossibility result does not apply to our setting either, since the agents do not have *efficient opt-out types* whose refusal of arbitration will result in an efficient outcome.

5. A POSSIBILITY RESULT UNDER QUADRATIC DISUTILITY

As a notable special case of our model, consider the utility function $v_i(x, \theta_i) = -(x - \theta_i)^2$. With this broadly used utility function, each agent's disutility from deviating from her personally preferred outcome is quadratically increasing in the magnitude of the deviation. Under this quadratic disutility specification, the surplus generated by outcome x_0 is $s(x_0, \theta_1, \theta_2) = -(x_0 - \theta_1)^2 - (x_0 - \theta_2)^2$ and its maximized level with the ex post efficient outcome $x^*(\theta_1, \theta_2) = \frac{\theta_1 + \theta_2}{2}$ is

$$S(\theta_1, \theta_2) = \max_x s(x, \theta_1, \theta_2) = -2\left(\frac{\theta_2 - \theta_1}{2}\right)^2.$$

PROPOSITION 4. *Under the quadratic disutility specification, the unique default outcome that permits efficient arbitration is the average of the expected types of the two agents* $\frac{E\theta_1 + E\theta_2}{2}$.

We prove this result, by showing that the critical type of each agent under the ideal default outcome is her expected type, that is, $\hat{\theta}_i(\hat{x}_0) = \mathrm{E}\theta_i$. It then follows from Corollary 1 that the ideal default outcome is the average of these critical types. Finally, we establish that function $\bar{\pi}(\hat{\theta}_1, \hat{\theta}_2)$ assumes value zero at $\hat{\theta}_i = \mathrm{E}\theta_i$. Existence of a unique default outcome permitting efficient arbitration does not depend on the specifics of the distribution functions of the agent

[16] $\bar{\pi}'(x_0) = \pi_{x_0}(x_0, \hat{\theta}_1(x_0), \hat{\theta}_2(x_0)) = 0 \Rightarrow \pi_{\hat{\theta}_i}(x^*(\hat{\theta}_1, \hat{\theta}_2), \hat{\theta}_1, \hat{\theta}_2) = 0 \Rightarrow \bar{\pi}_{\hat{\theta}_i}(\hat{\theta}_1, \hat{\theta}_2) = 0.$

[17] We will address the issue of randomized default outcome in Section 7.

types, as long as these distributions satisfy the full support and continuity conditions that we assumed from the outset. Moreover, to identify this ideal default outcome, it is sufficient to know the expectations of these distribution functions. Nevertheless, the possibility result here does not go as far as the result by Cramton et al. (1987) which identifies a positive measure of initial shares that lead to efficient dissolution of partnerships.

The ideal default outcome maximizes the surplus for the critical types of the two agents. Because the critical types are the average types under quadratic disutility, this also corresponds to maximizing the ex ante expected surplus under the prior type distributions. In other words, to ameliorate the conditions for efficient arbitration, we do not need a strategic designer with the power to commit to a suboptimal outcome in the event that arbitration fails. An uncommitted benevolent designer would choose the unique outcome that would make efficient arbitration possible, as long as she does not update her beliefs about the agent types in case of an off-path refusal to participate in arbitration.

The efficient mechanism takes the form of an expected externality mechanism à la Arrow (1979) and d'Aspremont and Gérard-Varet (1979). Therefore, it is easy to calculate the corresponding transfers that would support the efficient arrangement under the ideal default outcome:

$$t_i(\theta_i, \theta_j) = \left(\frac{\theta_j - \mathrm{E}\theta_i}{2}\right)^2 - \left(\frac{\theta_i - \mathrm{E}\theta_j}{2}\right)^2 + h_i(\theta_i, \theta_j),$$

where $h_1(\theta_1, \theta_2)$ is an arbitrary function such that its expectation is zero over either one of its arguments and $h_2 = -h_1$. Budget balance of the mechanism follows from the symmetry of the transfers for the two agents. Incentive compatibility follows from the second term in these transfers. Individual rationality is satisfied as an equality for the critical type $\mathrm{E}\theta_i$, and therefore it is satisfied as a strict inequality for all the other types. To motivate the parties reveal their preferences to the mechanism, the agent who is more conciliatory with respect to the expected position of the rival (in other words, the agent whose type is closer to the expected type of the other agent) receives a positive transfer (in expectation) from the less conciliatory one.

Remark 6 (*Ex ante efficiency*). Ex ante efficient default rules are known to enhance ex post efficient bargaining when the negotiating parties have non-strictly-concave utility functions as well. When the two bargaining parties have linear utility functions and the policymaker is constrained to choose complete ownership by one of the parties as the default rule, Segal and Whinston (2016) show that a similar ex ante efficient ownership rule is the ideal default rule that maximizes the difference between the value added from ex post efficiency and the information rent. They argue that a similar result holds when the default rule is restricted to be a liability rule where one of the parties have the option to buy at a fixed price. Also see Che (2006) on the latter result. In either case, however, even the ideal default rule does not permit an efficient bargaining mechanism.

When the bargaining parties have convex utility functions, Segal and Whinston (2011) show that the ex ante efficient default rule permits efficient bargaining, even if it is not the ideal default rule maximizing the difference between the value added of efficiency and the information rent.[18] This result applies in the absence of transferable payoffs as well.

Remark 7 (*More than two parties to dispute resolution*). Same as the result in Proposition 2, the possibility result on quadratic disutility carries on in a model with more than two

[18] Agastya and Birulin (2018) question the relevance of this result for partnership dissolution, by pointing out that the partners' valuations for the firm would depend on the final ownership structure. Also see Ornelas and Turner (2007) on this point.

disputing parties. As long as all the I parties have a quadratic disutility for deviating from their most preferred outcomes, the unique default outcome that permits efficient arbitration is the average of the expected types of all the disputants ($\sum_i \mathrm{E}\theta_i / I$) and the critical type of each disputant is her expected type.

REMARK 8 (*MULTIPLE DIMENSIONS OF DISPUTE*). If each agent's disutility is quadratic in the Euclidean distance between the chosen outcome x and her preferred outcome θ_i, Proposition 4 continues to hold in a multi-issue environment, where outcome x and the agent type θ_i are elements of the n-dimensional set $[0, 1]^n$. Suppose that each agent i's direct utility from outcome is

$$v_i(x, \theta_i) = -(dist(x, \theta_i))^2 = -(x_1 - \theta_{i1})^2 - (x_2 - \theta_{i2})^2 - \ldots - (x_N - \theta_{in})^2.$$

Because the square of the distance is separable in different dimensions of the policy space, the arbitration process can deal with each dimension of outcome separately and Proposition 4 holds for each of these dimensions, proving the result for the multidimensional extension.

6. AN IMPOSSIBILITY RESULT

It follows from our earlier analysis that the sign of function $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$ calculated at the critical types under the ideal default outcome $\hat{x}_0$ determines whether there exists an efficient arbitration mechanism. The critical types $\hat{\theta}_1(\hat{x}_0)$ and $\hat{\theta}_2(\hat{x}_0)$ under the ideal default outcome constitute an interior candidate point for this function, satisfying the first-order conditions for an extremum. By rearranging terms, we rewrite $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$ as

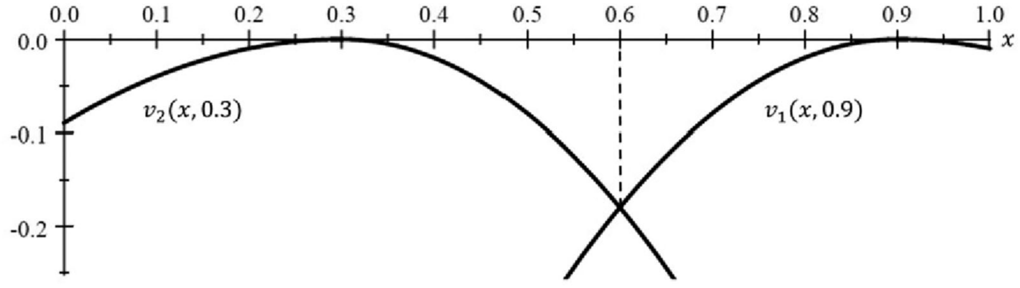
$$\left[\mathrm{E}_{\theta_2} S\left(\hat{\theta}_1, \theta_2\right) - S\left(\hat{\theta}_1, \hat{\theta}_2\right)\right] - \mathrm{E}_{\theta_1}\left[\mathrm{E}_{\theta_2} S\left(\theta_1, \theta_2\right) - S\left(\theta_1, \hat{\theta}_2\right)\right].$$

At an interior candidate point $\left(\hat{\theta}_1, \hat{\theta}_2\right)$ for an extremum, $\hat{\theta}_1$ satisfies the first-order necessary condition for minimization of the terms in the first square brackets above, given $\hat{\theta}_2$. The impossibility result that we develop in this section will rely on establishing that $\hat{\theta}_1$ is the unique solution to this minimization problem. Hence, when evaluated at this candidate point, the expression in the first square brackets would be strictly smaller than the one in the second square brackets for *almost all* values of θ_1 (when $\theta_1 = \hat{\theta}_1$, the two expressions are identical). This would imply that $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$ has a negative value for critical types $\hat{\theta}_1(\hat{x}_0)$ and $\hat{\theta}_2(\hat{x}_0)$. That is, even the ideal default arrangement $\hat{x}_0$ does not permit efficient arbitration.

To identify a sufficient condition for an efficient arbitration mechanism not to exist, we restrict attention to the following specification of our model: An agent's direct utility from the decision depends only on the difference between the implemented outcome and this agent's most desired outcome. In particular, we assume that $v_1(x, \theta_1) = v(x - \theta_1)$ and $v_2(x, \theta_2) = v(\theta_2 - x)$, where v is a strictly concave and twice continuously differentiable function.

To justify this specification, we consider again our motivating example of child-custody allocations. Under the interpretation that x is the proportion of time that the child spends with parent 1 and $1 - x$ is the proportion of time that will be spent with parent 2, these utility functions correspond to the two parents being symmetric in how they perceive custody arrangements giving them a lower (or higher) share than their preferred arrangement. See Figure 2 for an example where each parent's utility is decreasing faster for lower-than-preferred custody shares in comparison to higher-than-preferred shares. The symmetry between the parents implies that the ex post efficient custody is the average of their preferred custody levels, that is, $x^*(\theta_1, \theta_2) = \frac{\theta_1 + \theta_2}{2}$. Notice that the quadratic disutility function that we studied in the previous section is a special case for this specification.[19]

[19] Martínez-Mora and Puy (2014) explain why single-peaked preferences may exhibit *overprovision* or *shortfall* aversion, and they discuss the electoral consequences of such biases.



NOTE: Parent 1 prefers $\theta_1 = 90\%$ share of the custody. Parent 2 prefers $1 - \theta_2 = 70\%$ share. The ex post efficient custody is a 60–40% split.

FIGURE 2

PARENTS' UTILITY FUNCTIONS

PROPOSITION 5. *If the second derivative of function v is strictly monotonic and convex, then there is no default outcome permitting efficient arbitration.*

Because of the strict monotonicity condition, the hypothesis of the proposition rules out quadratic disutility functions. It identifies another class of preferences for which the disputes cannot be arbitrated efficiently. As is the case for the possibility result that we proved for quadratic disutility functions, the impossibility result holds regardless of the distribution functions of the agent types.

As an example satisfying the conditions listed in the proposition, take $v(a) = -a^2 + \varepsilon a^3$ where $0 < \varepsilon < 1/3$. The first term in the direct utility functions of the agents is representing the quadratic disutility for the deviations from their preferred outcomes. And the second term can be considered as an adjustment term for their biases. The second derivative of function v is linearly increasing in a, hence the monotonicity and convexity requirements of the proposition are satisfied. In this example, the direct utility functions of the two agents and the maximized surplus function are as below:

$$v_1(x, \theta_1) = -(x - \theta_1)^2 + \varepsilon(x - \theta_1)^3,$$
$$v_2(x, \theta_2) = -(x - \theta_2)^2 - \varepsilon(x - \theta_2)^3,$$
$$S(\theta_1, \theta_2) = -2\left(\frac{\theta_2 - \theta_1}{2}\right)^2 + 2\varepsilon\left(\frac{\theta_2 - \theta_1}{2}\right)^3.$$

7. CONCLUDING REMARKS

In this article, we study arbitration mechanisms that aim to resolve disputes in an ex post efficient manner between two agents with private preferences. Either party can refuse the arbitration process, in favor of a *default outcome*. We show that the *ideal* default outcome, which provides the best chances for efficient arbitration, is the default arrangement that maximizes the sum of the reservation payoffs of the *critical* agent types, who are most reluctant to renounce this default outcome. This result is contrary to the conventional wisdom that an unattractive default outcome could force the parties to agree. In addition to immediate implications, this observation leads to novel possibility and impossibility results on efficient arbitration. When both agents' payoffs are quadratically decreasing in the distance between their preferred outcome and the implemented arrangement, efficient arbitration is possible only under the threat of reverting to the ideal default outcome. For a more general class of preferences, which still depend on the difference between the preferred and implemented outcomes, we identify a sufficient condition under which there is no default outcome permitting efficient arbitration. Both these results are general in the sense that they do not depend on the specifics of the type distributions. We conclude the article with brief discussions of alternative modeling choices.

7.1. *Stochastic Default Outcome.* Suppose that, instead of committing to a single default outcome, the mechanism designer can support a randomization over different outcomes. Segal and Whinston (2011) show in a general setting that there exists a stochastic default arrangement which permits an efficient mechanism. [20] Following the recent literature on frugal mechanisms,[21] we can still define the ideal default arrangement as the stochastic default that would give the highest revenue to a designer constrained to offer efficient mechanisms. Given the inherent risk aversion of the agents in our environment (reflected by their concave payoff functions), the ideal default arrangement would assign positive weight only to the extreme outcomes, where x_0 equals either 0 or 1. For instance, with quadratic disutility, the ideal default arrangement would be the one that sets $x_0 = 0$ with probability $\frac{\mathrm{E}\theta_1+\mathrm{E}\theta_2+1}{4}$ and $x_0 = 1$ with the complementary probability.

7.2. *Ex Post Incentive Constraints.* Our model depicts an arbitration process where the parties forfeit the default outcome by participating in the process[22] and reveal their preferences to the arbitrator before they learn the preferences of their rivals. The incentive constraints corresponding to this scenario are the Bayesian incentive compatibility and individual rationality constraints considered in the partnership dissolution literature. A stronger implementation exercise would require replacing them with the dominant-strategy incentive-compatibility and ex post individual rationality constraints. Because the construction of our efficient arbitration mechanism is based on the Vickrey–Clarke–Groves mechanism, it is possible to modify the transfer functions to satisfy these more demanding conditions. However, this comes at the cost of replacing the ex post budget constraint $t_1(\theta_1, \theta_2) + t_2(\theta_1, \theta_2) = 0$ with the ex ante budget constraint $\mathrm{E}t_1(\theta_1, \theta_2) + \mathrm{E}t_2(\theta_1, \theta_2) = 0$.[23] This requires an enforcer of the arbitration mechanism who can act as a budget breaker with deep pockets. In real-life custody practice, there are some instances where the state jumps in to play the role of this budget breaker: In Quebec, the government takes over the child support payments when a parent defaults. In the United States, states can retain child support to reimburse welfare payments.

7.3. *Gradual Revelation Mechanisms.* Considering the sequentiality of the real-life negotiations, some readers may be uncomfortable with our modeling of the arbitration process as a direct revelation mechanism, where each party communicates with the mechanism just once and these communications take place simultaneously. A real-life arbitration can be such that first agent 1 reports her type, and agent 2 makes a report only after observing the first report. Alternatively, it could be that the agents do not report their types directly but they keep sending informative signals that lead to belief updates before the next round of communications. Such arbitration mechanisms, where information about the preferences are revealed gradually, would also involve stronger constraints than the Bayesian incentive-compatibility con-

[20] To be precise, Segal and Whinston show that, for *any* incentive-compatible allocation, there exists a stochastic default arrangement permitting implementation of that allocation.

[21] The use of a frugal default is usually justified as a part of an efficient mechanism that "requires the least amount of external financing" in the absence of a budget-balanced mechanism achieving efficiency (Agastya and Birulin, 2018). However, for a designer who has revenue maximization as a secondary objective (in addition to the main objective of sustaining efficiency), identifying the frugal default would be relevant even when a budget-balanced efficient mechanism exists.

[22] This feature of our model is consistent with the "collaborative divorce" process, where the participants to the procedure pledge not to go to court. Breaking this pledge entails important costs, because the lawyers involved in the collaborative divorce negotiations are obliged to withdraw if the case goes to court. For more on collaborative divorce, see *The Economist* piece mentioned in our Footnote 2.

[23] See Mookherjee and Reichelstein (1992) for the construction of a dominant strategy mechanism which is equivalent to a Bayesian incentive-compatible mechanism in the interim sense.

straints we study in this article. Nevertheless, it follows from Celik (2015) that, if the efficient outcome can be supported by a simultaneous revelation mechanism, then the same outcome can also be supported by a gradual revelation mechanism where the information is revealed in any arbitrary sequence. This is an implication of the monotonicity of the efficient outcome in the types of the agents.

DATA AVAILABILITY STATEMENT Data sharing is not applicable to this article as no data sets were generated or analyzed during this study.

APPENDIX A

PROOF OF PROPOSITION 1. Suppose $x_0 = 0$. To see that the critical types are also zero under this extreme default, consider the first derivative of $\pi\left(0, \hat{\theta}_1, \hat{\theta}_2\right)$ with respect to $\hat{\theta}_i$ defining the critical type $\hat{\theta}_i(0)$ in (13). It follows from the positive cross partial derivative (3) of function v_i that the left-hand side of (13) is positive for all $\hat{\theta}_i$. This observation establishes that $\hat{\theta}_i(0) = 0$ for $i = 1, 2$. When both agents have type 0, the ex post efficient outcome maximizing the sum of their utility functions is also zero, implying that $S(0,0) = s(0,0,0)$. Hence, we can write $\bar{\pi}(0) = \pi(0,0,0)$ as the following expectation:

$$\mathrm{E}_{\theta_1\theta_2}[S(0,\theta_2) + S(\theta_1, 0) - S(\theta_1,\theta_2) - S(0,0)].$$

The expression in the square brackets takes value zero at $\theta_1 = \theta_2 = 0$. To see that this expression is strictly decreasing in both θ_1 and θ_2, consider its first derivative with respect to θ_i. Because x^* is chosen optimally, it follows from the envelope theorem that this derivative is equal to

$$\frac{\partial v_i(x^*(\theta_i, 0), \theta_i)}{\partial\theta_i} - \frac{\partial v_i(x^*(\theta_i, \theta_j), \theta_i)}{\partial\theta_i}.$$

Recall that ex post efficient outcome x^* is a strictly increasing function. It follows from the positive cross-partial-derivative condition (3) on v_i that the derivative in the above display is negative for any value of θ_j other than 0. This implies that the expression in the square brackets above is negative for almost every θ_1, θ_2 pair. Therefore, its expectation is also negative and $\bar{\pi}(0) < 0$.

A similar proof can be constructed to show that $\hat{\theta}_i(1) = 1$ and $\bar{\pi}(1) < 0$. □

PROOF OF PROPOSITION 2. We first show that the ideal default outcome cannot be a boundary point of $[0,1]$. As in the proof of the earlier proposition, we start with considering the first derivative of $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ defining the critical type $\hat{\theta}_i(x_0)$ in (13). If x_0 is close enough to 0, it follows from the positive cross-partial-derivative condition (3) on function v_i that the left-hand side of (13) is positive for all $\hat{\theta}_i$. That is, for small enough values of x_0, critical types $\hat{\theta}_1(x_0)$ and $\hat{\theta}_2(x_0)$ are constant at 0. For these levels of default outcome, $\bar{\pi}(x_0)$ equals a constant minus $s(x_0, 0, 0)$. Because $s(x_0, 0, 0)$ is decreasing in x_0, function $\bar{\pi}(x_0)$ is increasing at $x_0 = 0$. Therefore, $x_0 = 0$ cannot be the ideal default outcome which is defined as the arrangement maximizing $\bar{\pi}(x_0)$. A similar argument shows that $\bar{\pi}(x_0)$ is decreasing at $x_0 = 1$ and hence $x_0 = 1$ cannot be the ideal default outcome either.

It follows from the envelope theorem that the first derivative of $\bar{\pi}(x_0)$ is equal to the first derivative of $\pi\left(x_0, \hat{\theta}_1, \hat{\theta}_2\right)$ with respect to x_0 where $\hat{\theta}_1$ and $\hat{\theta}_2$ are the critical types under x_0, because the former function is the lower envelope of the latter one:

$$\bar{\pi}'(x_0) = \pi_{x_0}\left(x_0, \hat{\theta}_1(x_0), \hat{\theta}_2(x_0)\right) = -s_{x_0}\left(x_0, \hat{\theta}_1(x_0), \hat{\theta}_2(x_0)\right).$$

For an interior extreme point of function $\bar{\pi}(x_0)$, this derivative is zero. Because s is a concave function of x_0, π is a convex function of it. Therefore, π attains a minimum for such a value of x_0. □

PROOF OF PROPOSITION 3. Suppose $\hat{\theta}_i(\hat{x}_0) = 0$. Consider again the first derivative of $\pi\left(\hat{x}_0, \hat{\theta}_i, \hat{\theta}_j\right)$ defining $\hat{\theta}_j(\hat{x}_0)$:

$$\mathrm{E}_{\theta_i} \frac{\partial v_j\big(x^*\big(\theta_i, \hat{\theta}_j\big), \hat{\theta}_j\big)}{\partial \theta_j} - \frac{\partial v_j\big(x^*\big(0, \hat{\theta}_j\big), \hat{\theta}_j\big)}{\partial \theta_j}.$$

Because x^* is an increasing function of θ_i, it follows from the cross-partial-derivative condition (3) on v_j that the difference term above is positive for all $\hat{\theta}_j$. Accordingly, $\hat{\theta}_j(\hat{x}_0) = 0$ as well. Corollary 1 implies that $\hat{x}_0 = x^*\left(\hat{\theta}_i(\hat{x}_0), \hat{\theta}_j(\hat{x}_0)\right) = 0$. This, however, is a contradiction to Proposition 2 which states $\hat{x}_0$ is in the interior of $[0, 1]$.

A similar contradiction arises for $\hat{\theta}_i(\hat{x}_0) = 1$ as well. □

PROOF OF PROPOSITION 4. We start with searching for an interior candidate for the extremum of function $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$. The derivative of this function with respect to $\hat{\theta}_i$ is

$$\begin{aligned} &\mathrm{E}_{\theta_j} \frac{\partial v_i\big(x^*\big(\hat{\theta}_i, \theta_j\big), \hat{\theta}_i\big)}{\partial \theta_i} - \frac{\partial v_i\big(x^*\big(\hat{\theta}_i, \hat{\theta}_j\big), \hat{\theta}_i\big)}{\partial \theta_i} \\ &= 2\mathrm{E}_{\theta_j} x^*\big(\hat{\theta}_i, \theta_j\big) - 2x^*\big(\hat{\theta}_i, \hat{\theta}_j\big) \\ &= \mathrm{E}\theta_j - \hat{\theta}_j. \end{aligned}$$

Therefore, the unique interior candidate point of this function, where the first-order conditions are satisfied as equalities, is $\hat{\theta}_i = \mathrm{E}\theta_i$ for $i = 1, 2$. Corollary 1 implies that the ideal default outcome is $\hat{x}_0 = \frac{\mathrm{E}\theta_1 + \mathrm{E}\theta_2}{2}$. Plugging $\hat{\theta}_i = \mathrm{E}\theta_i$ in $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$ yields zero as the maximized level of the constrained revenue. Accordingly, there exists an efficient mechanism when the default outcome is $\frac{\mathrm{E}\theta_1 + \mathrm{E}\theta_2}{2}$. But the constrained revenue is negative for any other default outcome, ruling out the possibility of efficient arbitration for these alternative default arrangements. □

PROOF OF PROPOSITION 5. Suppose v'' is increasing and convex. The first property implies that v' is strictly convex. We also know from strict concavity of the utility function that v' is decreasing.

We look for an interior candidate point for minimization of function $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$. At such a candidate point, the first-order conditions imply that the first derivative of the function with respect to $\hat{\theta}_1$ equals zero:

$$\mathrm{E}_{\theta_2} \frac{\partial v_1\big(x^*\big(\hat{\theta}_1, \theta_2\big), \hat{\theta}_1\big)}{\partial \theta_1} - \frac{\partial v_1\big(x^*\big(\hat{\theta}_1, \hat{\theta}_2\big), \hat{\theta}_1\big)}{\partial \theta_1} = -\mathrm{E}_{\theta_2} v'\left(\frac{\theta_2 - \hat{\theta}_1}{2}\right) + v'\left(\frac{\hat{\theta}_2 - \hat{\theta}_1}{2}\right) = 0.$$

It follows from strict convexity of v' and Jensen's inequality that

$$v'\left(\frac{\mathrm{E}\theta_2 - \hat{\theta}_1}{2}\right) < v'\left(\frac{\hat{\theta}_2 - \hat{\theta}_1}{2}\right)$$

for any $\hat{\theta}_1, \hat{\theta}_2$ pair satisfying this first-order equation. Because v' is decreasing, this inequality implies that $\mathrm{E}\theta_2 > \hat{\theta}_2$.

We now consider the second derivative of function $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$ with respect to $\hat{\theta}_1$ at the interior candidate point:

$$\text{(A.1)} \qquad \frac{1}{2}\mathrm{E}_{\theta_2} v''\left(\frac{\theta_2 - \hat{\theta}_1}{2}\right) - \frac{1}{2} v''\left(\frac{\hat{\theta}_2 - \hat{\theta}_1}{2}\right).$$

Because v'' is convex, Jensen's inequality implies that $\mathrm{E}_{\theta_2} v''\left(\frac{\theta_2 - \hat{\theta}_1}{2}\right) \geq v''\left(\frac{\mathrm{E}\theta_2 - \hat{\theta}_1}{2}\right)$. Moreover $v''\left(\frac{\mathrm{E}\theta_2 - \hat{\theta}_1}{2}\right) > v''\left(\frac{\hat{\theta}_2 - \hat{\theta}_1}{2}\right)$ because v'' is increasing and $\mathrm{E}\theta_2 > \hat{\theta}_2$. This establishes that the second derivative in (A.1) is positive when $\hat{\theta}_2$ is fixed at the critical type $\hat{\theta}_2\left(\hat{x}_0\right)$. Therefore, $\hat{\theta}_1\left(\hat{x}_0\right)$ uniquely satisfies the second-order sufficient conditions for minimization of $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\left(\hat{x}_0\right)\right)$ as a function of $\hat{\theta}_1$. It follows from the discussion in the text at the start of Section 6 that $\bar{\pi}\left(\hat{\theta}_1\left(\hat{x}_0\right), \hat{\theta}_2\left(\hat{x}_0\right)\right)$ takes a strictly negative value at these critical types and the ideal default outcome $\hat{x}_0$ does not permit efficient arbitration. Because there is no efficient arbitration mechanism under the ideal default outcome, there is no efficient mechanism under any default outcome.

A similar proof can be constructed when v'' is decreasing and convex. In this case, v' is decreasing and strictly concave, implying that $\hat{\theta}_2 > \mathrm{E}\theta_2$ at an interior candidate point. It again follows from monotonicity and convexity of v'' that $\hat{\theta}_1\left(\hat{x}_0\right)$ uniquely satisfies the second-order sufficient condition for minimization of $\bar{\pi}\left(\hat{\theta}_1, \hat{\theta}_2\right)$ given $\hat{\theta}_2 = \hat{\theta}_2\left(\hat{x}_0\right)$. □

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