Letter

Hidden conformal symmetry in $AdS_2 \times S^2$ beyond tree level

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Correlators of $\mathcal{N}=2$ hypermultiplets with two-derivative interactions in $AdS_2 \times S^2$ exhibit a hidden four-dimensional conformal symmetry which allows one to repackage all tree-level four-point correlators into a single four-dimensional object corresponding to a contact diagram arising from a massless ϕ^4 theory in $AdS_2 \times S^2$. This theory serves as a toy model for IIB string theory in $AdS_5 \times S^5$ and is interesting in its own right because $AdS_2 \times S^2$ describes the near-horizon limit of extremal black holes in four dimensions. We argue that, after acting with an $SU(1,1) \times SU(2)$ Casimir, all one-loop correlators can similarly be encoded by a four-dimensional function which arises from a one-loop scalar bubble diagram in $AdS_2 \times S^2$, explaining how the hidden conformal symmetry extends beyond tree level. Finally, we conjecture a scalar effective field theory with a two-derivative interaction in $AdS_2 \times S^2$ whose Witten diagrams should directly reproduce four-point correlators to all loops without acting with Casimirs.

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Introduction. The celebrated AdS/CFT correspondence [1] relates quantum gravity in anti–de Sitter (AdS) background to a nongravitational conformal field theory (CFT) living in the boundary. The basic observables of a CFT are its correlation functions, which encode the scattering of gravitons and other degrees of freedom in AdS and can be computed perturbatively using Witten diagrams. For certain backgrounds of the form $AdS \times S$, where S is a sphere, simple formula packaging tree-level correlation functions of all the modes on the sphere into a single generating function have been obtained, originating from a hidden higher dimensional conformal symmetry. From the point of view of the dual CFT, these correspond to correlation functions of an infinite number of certain protected operators known as $1/2$-BPS operators.

Hidden conformal symmetry was first discovered in four-point tree-level amplitudes of type IIB supergravity in $AdS_5 \times S^5$ [2], which is dual to maximally supersymmetric Yang-Mills theory in four dimensions ($\mathcal{N}=4$ SYM) at strong coupling, and the symmetry has also been observed at weak coupling [3–5]. If one includes higher-derivative corrections to supergravity, the higher dimensional conformal symmetry gets broken but there are still remnants of a higher dimensional organizing principle [6–9]. Indeed it was discovered that the four-point correlators of $\mathcal{N}=4$ SYM theory arise from massless scalar contact Witten diagrams on $AdS_5 \times S^5$ [10].

This higher dimensional symmetry was also found in $AdS_3 \times S^3$ [11,12], $AdS_5 \times S^3$ [13], and $AdS_2 \times S^2$ [14] backgrounds. The last case is of particular interest because the geometry arises in the near-horizon limit of four-dimensional extremal black holes [15,16] and may therefore have actual relevance for quantum gravity in the real world. The four-point tree-level amplitudes of 4d $\mathcal{N}=2$ hypermultiplets interacting via two-derivative interactions in this background [which arises from reducing $\mathcal{N}=8$ supergravity on $AdS_2 \times S^2$ [17,18] and can be described as correlators of a 1d CFT with $SU(1,1|2)$ superconformal symmetry] can be encoded by a massless ϕ^4 contact Witten diagram in the bulk [14] very similarly to the IIB/$\mathcal{N}=4$ SYM case. Note that a straightforward generalization of this describes four-point open string scattering in IIB string theory on $AdS_5 \times S^5/\mathbb{Z}_2$ [19]. Hence, the scalar theory in $AdS_2 \times S^2$ can be thought of as a toy model for IIB string theory on $AdS_5 \times S^5$.

The purpose of this work is to go beyond the classical theory and investigate this at loop level. In particular, we consider four-point functions at one-loop order and show that they can be obtained by acting with Casimir operators on a 4d generating function. We then show that, quite remarkably, this generating function can be derived from a one-loop bubble diagram of the ϕ^4 theory. We observe that the infinite sum over modes of the two-sphere which flow through each edge of a Feynman diagram can be resummed into a single bulk-to-bulk propagator in $AdS_2 \times S^2$ which takes the same form as a propagator in four-dimensional flat space.

Finally, we conjecture a full effective field theory in $\mathrm{AdS}_2 \times \mathrm{S}^2$ whose Feynman rules should directly compute four-point correlators in the two-derivative sector to all loop orders without acting with Casimirs. The key insight that leads us to this action is that acting with a boundary Casimir on a contact diagram is equivalent to acting with a Laplacian in the bulk. We verify this proposal for free-theory and tree-level correlators and write down a one-loop formula whose explicit evaluation we leave for future work.

Basics of $SU(1,1|2)$ correlators. The purpose of this section is to review what is currently known about four-point functions of half-BPS operators in the $SU(1,1|2)$ superconformal field theory. We follow the conventions of [14] and denote the 1/2-BPS operators by $\mathcal{O}_p(x,y)$, where x is the spacetime coordinate and y an analogous internal coordinate dealing with the $SU(2)$ R symmetry. These operators have protected dimension, and $SU(2)$ charge $p+\frac{1}{2}$ with $p=0,1,\ldots$.

$\mathcal{O}_p$ is the primary operator of the superfield O_p:

$$O_p = \mathcal{O}_p + \theta\mathcal{L}_p + \bar{\theta}\bar{\mathcal{L}}_p + \cdots \tag{1}$$

and so the descendant $\mathcal{L}_p$ has dimension $p+1$ and $SU(2)$ charge p. It is extremely useful to combine operators for all p into single operators $\boldsymbol{\mathcal{O}}$, $\boldsymbol{\mathcal{L}}$, $\bar{\boldsymbol{\mathcal{L}}}$:

$$\boldsymbol{\mathcal{O}} = \sum_{p=0}^{\infty} \mathcal{O}_p, \qquad \boldsymbol{\mathcal{L}} = \sum_{p=0}^{\infty} \mathcal{L}_p, \qquad \bar{\boldsymbol{\mathcal{L}}} = \sum_{p=0}^{\infty} \bar{\mathcal{L}}_p. \tag{2}$$

Correlators of these objects will be referred to as "master correlators" and they act as generating functions in the sense that correlators with given external charges p_i are obtained simply by Taylor expanding in the y variables and projecting onto the component with the correct conformal weight.

We will be interested in the four-point function of both the primaries and their descendants and we will expand all correlators in a large central charge expansion, e.g.,

$$\langle \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{L}}\bar{\boldsymbol{\mathcal{L}}}\,\bar{\boldsymbol{\mathcal{L}}}\rangle = \sum_{m=0}^{\infty} \frac{1}{c^m} \langle \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{L}}\bar{\boldsymbol{\mathcal{L}}}\,\bar{\boldsymbol{\mathcal{L}}}\rangle^{(m)}, \tag{3}$$

where the first term corresponds to free theory, the second to tree-level amplitudes in the bulk, and so on. The goal of this work is to compute the one-loop contribution $\langle \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{L}}\bar{\boldsymbol{\mathcal{L}}}\,\bar{\boldsymbol{\mathcal{L}}}\rangle^{(2)}$ and understand its structure.

Superconformal symmetry implies that the four-point function of primaries and descendants is closely related, taking the form [14]

$$\begin{aligned}
\langle \boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}\rangle &= \langle \boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}\boldsymbol{\mathcal{O}}\rangle^{(0)} + I \times \mathbf{G}(x_i, y_i; c),\\
\langle \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{L}}\bar{\boldsymbol{\mathcal{L}}}\,\bar{\boldsymbol{\mathcal{L}}}\rangle &= \langle \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{L}}\bar{\boldsymbol{\mathcal{L}}}\,\bar{\boldsymbol{\mathcal{L}}}\rangle^{(0)} + \mathcal{C}_{12}\mathbf{G}(x_i, y_i; c).
\end{aligned} \tag{4}$$

Here I is a certain kinematic factor given in (A4) of the Supplemental Material [20], and $\mathcal{C}_{12}$ is the quadratic Casimir of $SU(1,1)\times SU(2)$ arising from the superconformal $SU(1,1|2)$ Casimir [14] applied at points 1 and 2:

$$\begin{aligned}
\mathcal{C}_{12} &= \mathcal{C}^x_{12} - \mathcal{C}^y_{12},\\
\mathcal{C}^x_{12} &= \frac{1}{2}(D_{x_1} + D_{x_2})_{AB}(D_{x_1} + D_{x_2})^{AB},
\end{aligned} \tag{5}$$

where $D_{xAB} = x_A\partial/\partial x^B - x_B\partial/\partial x^A$ is a generator of the conformal or $SU(2)$ R-symmetry group. Note that we will mainly use three-component embedding space coordinates x^A, y^A throughout the paper. For more details on our conventions see Appendix A and Eq. (D5) of the Supplemental Material [20].

All of the correlators are functions of x_i, y_i, but if the hidden four-dimensional symmetry is present then the master correlators should be expressible in terms of functions of 4d distances, obtained by summing conformal and internal distances $\mathbf{x}^2_{ij} = x^2_{ij} + y^2_{ij}$, with the (boldface) 4d cross ratios defined accordingly:

$$\mathbf{u} = \frac{\mathbf{x}^2_{12}\mathbf{x}^2_{34}}{\mathbf{x}^2_{13}\mathbf{x}^2_{24}} = \mathbf{z}\bar{\mathbf{z}}, \qquad \mathbf{v} = \frac{\mathbf{x}^2_{14}\mathbf{x}^2_{23}}{\mathbf{x}^2_{13}\mathbf{x}^2_{24}} = (1-\mathbf{z})(1-\bar{\mathbf{z}}). \tag{6}$$

Now we will illustrate how these variables play a role in free theory and at tree level and then we will find that also loop level results can be written in terms of these variables albeit in a more nontrivial manner.

Free theory: The 4d conformal symmetry in free theory is realized at the level of correlators of descendants which can be written entirely in terms of 4d distances [14]:

$$\langle \boldsymbol{\mathcal{L}}\boldsymbol{\mathcal{L}}\bar{\boldsymbol{\mathcal{L}}}\,\bar{\boldsymbol{\mathcal{L}}}\rangle^{(0)} = \frac{1}{\mathbf{x}^2_{14}\mathbf{x}^2_{23}} + \frac{1}{\mathbf{x}^2_{13}\mathbf{x}^2_{24}}. \tag{7}$$

Tree level: At tree level on the other hand, the 4d conformal symmetry is directly realized on the correlators of primaries and we have $\mathbf{G}^{(1)} = -D_{1111}(\mathbf{x}^2_{ij})$,, where D_{1111} is the (normalized) one-loop box diagram:

$$\begin{aligned}
D_{1111}(\mathbf{x}^2_{ij}) &= \frac{1}{\mathbf{x}^2_{13}\mathbf{x}^2_{24}} \frac{\phi^{(1)}(\mathbf{z},\bar{\mathbf{z}})}{\mathbf{z}-\bar{\mathbf{z}}}\\
&= \frac{1}{\mathbf{x}^2_{13}\mathbf{x}^2_{24}} \frac{2\Big(\mathrm{Li}_2(\mathbf{z}) - \mathrm{Li}_2(\bar{\mathbf{z}})\Big) + \log\mathbf{u}\log(\frac{1-\mathbf{z}}{1-\bar{\mathbf{z}}})}{\mathbf{z}-\bar{\mathbf{z}}}.
\end{aligned} \tag{8}$$

One-loop correlators. Let us now consider one-loop correlators. We will focus on the trascendental weight-2 part of the one-loop correlators, which can be uniquely fixed by requiring consistency of the double discontinuity with the operator product expansion (OPE) and crossing symmetry. Now, the double discontinuity is simply related to the tree-level discontinuity via the action of the Casimir,

$$\mathbf{G}^{(2)}|_{\log^2 u} = -\frac{1}{2}\mathcal{C}_{12}(D_{1111}|_{\log \mathbf{x}_{12}^2}), \tag{9}$$

where

$$(D_{1111})|_{\log \mathbf{x}_{12}^2} = \frac{1}{\mathbf{x}_{13}^2\mathbf{x}_{24}^2(\mathbf{z}-\bar{\mathbf{z}})}\log\left(\frac{1-\mathbf{z}}{1-\bar{\mathbf{z}}}\right). \tag{10}$$

The simplicity of this relation [21] is really a consequence of the fact that the leading log is controlled by tree-level CFT data, which, in theories with hidden higher dimensional conformal symmetries, depend on the higher dimensional spin [2,14,23–26]. In this case the latter is truncated to spin 0 so there is only a single higher dimensional operator appearing in the OPE.

The crossing completion of the above double discontinuity then automatically yields the complete result for the maximal transcendental weight part of the one-loop correlator:

$$\begin{aligned}\mathbf{G}^{(2)} = -\frac{1}{2}\Big(&\mathcal{C}_{12}\Big((D_{1111})|_{\log \mathbf{x}_{12}^2}\Big)\log u \log\frac{u}{v}\\ &+\mathcal{C}_{23}\Big((D_{1111})|_{\log \mathbf{x}_{23}^2}\Big)\log v \log\frac{v}{u}\\ &+\mathcal{C}_{13}\Big((D_{1111})|_{\log \mathbf{x}_{13}^2}\Big)\log u \log v\Big),\end{aligned} \tag{11}$$

where $\mathcal{C}_{12}$ is given in (5) and $\mathcal{C}_{23}$ and $\mathcal{C}_{13}$ are its crossing versions. Note that the solution dictated by OPE and crossing symmetry carves out a three-dimensional space in the four-dimensional space of harmonic polylogarithms of weight 2, given by the three orientations of the logs. In particular, note the absence of Li_2.

A remarkable 4d uplift: Despite being quite simple, the one-loop formula presented above is still not very satisfactory from the point of view of the higher dimensional symmetry, because it is *not* a function of 4d distances. In fact, it turns out that there exists a function $\mathfrak{f}_s(\mathbf{z},\bar{\mathbf{z}})$ of four-dimensional distances *only* such that

$$\mathbf{G}^{(2)} = -\frac{1}{2}\left(\mathcal{C}_{12}\frac{\mathfrak{f}_s(\mathbf{z},\bar{\mathbf{z}})}{\mathbf{x}_{13}^2\mathbf{x}_{24}^2}+\mathcal{C}_{23}\frac{\mathfrak{f}_t(\mathbf{z},\bar{\mathbf{z}})}{\mathbf{x}_{13}^2\mathbf{x}_{24}^2}+\mathcal{C}_{13}\frac{\mathfrak{f}_u(\mathbf{z},\bar{\mathbf{z}})}{\mathbf{x}_{13}^2\mathbf{x}_{24}^2}\right), \tag{12}$$

where

$$\mathfrak{f}_s(\mathbf{z},\bar{\mathbf{z}}) = \frac{1}{3}\frac{1}{\mathbf{z}-\bar{\mathbf{z}}}\left(f^{(3)}(\mathbf{z},\bar{\mathbf{z}})+\left(\log\mathbf{u}+\log\frac{\mathbf{u}}{\mathbf{v}}\right)\phi^{(1)}(\mathbf{z},\bar{\mathbf{z}})\right), \tag{13}$$

and

$$\mathfrak{f}_t(\mathbf{z},\bar{\mathbf{z}}) = \mathfrak{f}_s(1-\mathbf{z},1-\bar{\mathbf{z}}), \qquad \mathfrak{f}_u(\mathbf{z},\bar{\mathbf{z}}) = \mathfrak{f}_s\left(\frac{1}{\mathbf{z}},\frac{1}{\bar{z}}\right). \tag{14}$$

Here $f^{(3)}$ is a weight-3 single-valued antisymmetric function which, in AdS context, recently made its first appearance in [27].

The function $\mathfrak{f}_s(\mathbf{z},\bar{\mathbf{z}})$ transforms correctly under $1 \leftrightarrow 2$ crossing, and is a weight-3 combination of single-valued multiple polylogarithms (SVMPLs) [28]. This is the expected weight for one-loop amplitudes in $d > 1$ originating from a four-point contact interaction in the bulk [26,27,29–31]. In fact, $\mathfrak{f}_s$ is the *unique* linear combination of SVMPLs built out of the alphabet $\{\mathbf{z}, 1-\mathbf{z}, \bar{\mathbf{z}}, 1-\bar{\mathbf{z}}, \mathbf{z}-\bar{\mathbf{z}}\}$ such that it correctly reproduces the double discontinuity and, crucially, its crossing versions $\mathfrak{f}_t$, $\mathfrak{f}_u$ *do not* contain $\log^2\mathbf{u}$ terms. We refer to Appendix B of the Supplemental Material [20] for a more precise definition and a list of some useful properties.

We emphasize that the only assumption that went into the derivation of the one-loop formula for all higher charge correlators in (11) was the existence of higher dimensional conformal symmetry at tree level. The fact that it can be written in the form (12) is a very nontrivial consequence of this assumption.

Generalized Witten diagrams. One of the consequences of the existence of a hidden conformal symmetry is that tree-level correlators can be obtained from a massless ϕ^4 contact Witten diagram in $\mathrm{AdS}_2 \times \mathrm{S}^2$ [14]. The purpose of the remaining part of the paper is to test the applicability of such an approach beyond tree level. Before doing so, let us first recall how to obtain the tree-level result from a ϕ^4 generalized Witten diagram.

Points in AdS and S are parametrized in $(d+2)$-dimensional embedding space as $-\hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = 1$, where we set the radii of AdS and S to 1. Analogously, boundary coordinates, denoted by unhatted letters, satisfy $x\cdot x = y\cdot y = 0$.

Tree level: The following interaction term in the bulk effective action,

$$S_{\mathrm{int}} = \frac{1}{4!}\int_{\mathrm{AdS}_2\times\mathrm{S}^2} d^2\hat{x}_0 d^2\hat{y}_0 \phi(\hat{x}_0,\hat{y}_0)^4, \tag{15}$$

computes all tree-level correlators of the supersymmetric CFT using generalized Witten diagrams which treat AdS and S on equal footing. In particular, all half-BPS correlators are generated by the following $\mathrm{AdS}_2 \times \mathrm{S}^2$ integral, which is consistent with the bootstrapped result (10):

$$\mathbf{G}^{(1)} = -\frac{1}{\pi^2}\int\frac{d^2\hat{x}_0 d^2\hat{y}_0}{\mathbf{x}_{10}^2\mathbf{x}_{20}^2\mathbf{x}_{30}^2\mathbf{x}_{40}^2} = -D_{1111}(\mathbf{x}_{ij}^2), \tag{16}$$

where $\mathbf{x}_{i0}^2 = -2x_i\cdot\hat{x}_0 - 2y_i\cdot\hat{y}_0$ is the (inverse of the) $\mathrm{AdS}\times\mathrm{S}$ bulk-to-boundary propagator, which satisfies

$$\nabla^2 \frac{1}{\mathbf{x}_{i0}^2} = (\nabla^2_{\hat{x}_0} + \nabla^2_{\hat{y}_0}) \frac{1}{\mathbf{x}_{i0}^2} = 0, \tag{17}$$

where $\nabla^2_{\hat{x}_0}$ ($\nabla^2_{\hat{y}_0}$) is the AdS (S) Laplacian, which is given in Eq. (23) below. We refer to [14] for a detailed derivation of (16).

One loop: In order to test whether the effective action extends beyond tree level, we first need to find an expression for the $\mathrm{AdS}_2 \times \mathrm{S}^2$ bulk-to-bulk propagator. Intriguingly, it turns out that the bulk-to-bulk propagator in the $\mathrm{AdS}_{d+1} \times \mathrm{S}^{d+1}$ product geometry takes exactly the same functional form as the bulk-to-boundary propagator:

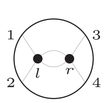
$$\frac{1}{\hat{\mathbf{x}}_{lr}^{2d}} = \frac{1}{(-2\hat{x}_l \cdot \hat{x}_r - 2\hat{y}_l \cdot \hat{y}_r)^d}. \tag{18}$$

In fact, in AdS × S this simple propagator satisfies (17). This is very different from pure AdS space where bulk-to-bulk propagators are hypergeometric functions. Remarkably, this propagator also encodes an infinite sum over the Kaluza-Klein modes of the sphere propagating in AdS:

$$\begin{aligned}\frac{1}{\hat{\mathbf{x}}_{lr}^{2d}} = \frac{(-2)^{-d}}{(d-1)!}\sum_{p=0}^{\infty}\Bigg[&(-1)^{\tilde{p}}(p-d+1)_{d-1}\\ &\times \frac{1}{(\hat{x}_l\cdot\hat{x}_r)^p}\,{}_2F_1\left(\frac{p+1}{2},\frac{p}{2},p+1-\frac{d}{2};\frac{1}{(\hat{x}_l\cdot\hat{x}_r)^2}\right)\\ &\times \frac{1}{(\hat{y}_l\cdot\hat{y}_r)^{\tilde{p}}}\,{}_2F_1\left(\frac{\tilde{p}+1}{2},\frac{\tilde{p}}{2},\tilde{p}+1-\frac{d}{2};\frac{1}{(\hat{y}_l\cdot\hat{y}_r)^2}\right)\Bigg],\end{aligned} \tag{19}$$

where $\tilde{p} = d - p$. Similar structures were derived for IIB supergravity in $\mathrm{AdS}_5 \times \mathrm{S}^5$ in [32].

With the bulk-to-bulk propagator at hand, we can then compute a one-loop bubble diagram in $\mathrm{AdS}_{d+1} \times \mathrm{S}^{d+1}$ which in the s-channel reads (up to an unimportant overall constant)



$$\propto \mathcal{K}_s \equiv \int \frac{d^{d+1}\hat{x}_l d^{d+1}\hat{x}_r d^{d+1}\hat{y}_l d^{d+1}\hat{y}_r}{\mathbf{x}_{1l}^{2d}\mathbf{x}_{2l}^{2d}\mathbf{x}_{3r}^{2d}\mathbf{x}_{4r}^{2d}\hat{\mathbf{x}}_{lr}^{4d}}, \tag{20}$$

with a similar formula for the t and u channels.

The double integral over the sphere can be evaluated combinatorially for arbitrary d and p_i, see Appendix C of the Supplemental Material [20] for more details. Here, we will focus on $d = 1$, $p_i = 0$, for which the integral reduces to

$$\mathcal{K}_s\big|_{\substack{d=1\\ y_i=0}} = \int \frac{d^2\hat{x}_l d^2\hat{x}_r \times 2^{-6}}{(x_1\cdot\hat{x}_l)(x_2\cdot\hat{x}_l)(x_3\cdot\hat{x}_r)(x_4\cdot\hat{x}_r)((\hat{x}_l\cdot\hat{x}_r)^2-1)}. \tag{21}$$

The remaining integration over AdS produces divergences and needs to be regularized. This can be done, for example, following the procedure outlined in [31]. In short, the idea is that the integral can be mapped to a flat-space integral so that one can take advantage of standard flat space techniques, such as dimensional regularization, to isolate the divergence and extract the finite part. We refer to Appendix D of the Supplemental Material [20] for details on the computation. Here we just quote the result for the finite part, which reads

$$\mathcal{K}_s^{(\mathrm{fin})}\big|_{\substack{d=1\\ y_i=0}} \propto \frac{\mathfrak{f}_s(\mathbf{z},\bar{\mathbf{z}})}{\mathbf{x}_{13}^2\mathbf{x}_{24}^2}\bigg|_{y_i=0} \tag{22}$$

and analogously for t and u channels. Quite remarkably, this is precisely the function in Eq. (12) appearing in the master correlator of the $SU(1,1|2)$ CFT. The computation of the bubble diagram beyond $y_i = 0$ is much more complicated and we leave it to the future but it is natural to expect that the relation (22) will extend beyond the $y_i = 0$ case.

At this point, it is important to remark on the difference compared to tree level: the bubble diagram at one loop reproduces the correlator only upon acting with a Casimir in each crossing orientation (12). This leads to a natural question: is there a way to explain the presence of the Casimir from a bulk perspective and make a more precise statement? We will address this in the next section.

An all-loops conjecture. The key observation we will make use of is that acting with a boundary Casimir on a pair or bulk-to-boundary propagators is equivalent to acting with the AdS × S Laplacian in the bulk. This is a generalization of similar observations in AdS [33,34]. The AdS and S Laplacians are equal (up to an important sign) to the $SO(d,2)$ or $SO(d+2)$ Casimir operators acting on bulk coordinates:

$$\nabla^2_{\hat{x}} = -\frac{1}{2}D_{\hat{x}AB}D^{AB}_{\hat{x}} \qquad \nabla^2_{\hat{y}} = +\frac{1}{2}D_{\hat{y}AB}D^{AB}_{\hat{y}}, \tag{23}$$

where $D_{\hat{x}AB}$ is a generator, identical in form to the conformal generator on the boundary (6). When the bulk Laplacian acts on a pair of bulk-to-boundary propagators meeting at a common point in the bulk, it can be traded with a Casimir acting on the boundary points:

$$\nabla^2\left(\frac{1}{\mathbf{x}_{10}^2\mathbf{x}_{20}^2}\right) = -\mathcal{C}_{12}\left(\frac{1}{\mathbf{x}_{10}^2\mathbf{x}_{20}^2}\right), \tag{24}$$

where $\nabla^2 = \nabla^2_{\hat{x}} + \nabla^2_{\hat{y}}$ is the AdS × S Laplacian, and $\mathcal{C}_{12} = \mathcal{C}^x_{12} - \mathcal{C}^y_{12}$ is the $\mathrm{AdS}_2 \times \mathrm{S}^2$ Casimir (5).

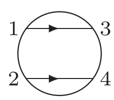
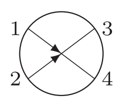
Now note that the master correlator of descendants $\langle \mathcal{L}\mathcal{L}\bar{\mathcal{L}}\bar{\mathcal{L}}\rangle$ enjoys the following properties: in free theory the higher dimensional symmetry is manifest (7); at tree level it takes the form of the Casimir $\mathcal{C}_{12}$ acting on an object with higher dimensional symmetry [(4) and (10)]; at one loop it can be written as the Casimir $\mathcal{C}_{12}$ acting on further Casimirs which themselves act on functions with higher dimensional symmetry [(4) and (12)]. Along with the observation that external Casimirs can arise from a bulk Laplacian operator, this suggests the following effective action AdS × S space (including kinetic terms):

$$S' = \int_{\mathrm{AdS}_2\times \mathrm{S}^2} d^4\hat{\mathbf{x}}_0 \left(\frac{1}{2}\bar{\phi}\nabla^2\phi - \frac{G_N}{4}\bar{\phi}^2\nabla^2\phi^2\right), \tag{25}$$

where we identify the *complex* field ϕ as the bulk field coupled to the descendant operator $\mathcal{L}$. Here the Newton's constant is proportional to the inverse of the central charge, $G_N \propto 1/c$ and we employed the notation $d^4\hat{\mathbf{x}}_0 \equiv d^2\hat{x}_0 d^2\hat{y}_0$.

The effective action (25) is conjectured to describe all-loop four-point correlators of $\mathcal{N}=2$ hypermultiplets which arise from dimensional reduction of $\mathcal{N}=8$ supergravity on $\mathrm{AdS}_2\times\mathrm{S}^2$. The effective action is therefore nonrenormalizable. Let us now work out the predictions for four-point correlators arising from this action.

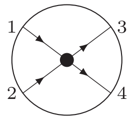
Free theory: By performing Wick contraction, the free-theory four-point function reads

$\langle \mathcal{L}\mathcal{L}\bar{\mathcal{L}}\bar{\mathcal{L}}\rangle^{(0)} =$  $+$ 

$$= \frac{1}{\mathbf{x}_{13}^2\mathbf{x}_{24}^2} + \frac{1}{\mathbf{x}_{14}^2\mathbf{x}_{23}^2}, \tag{26}$$

which indeed reproduces the result for the free theory correlator [14], quoted in (7).

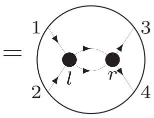
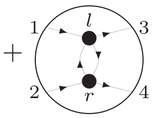
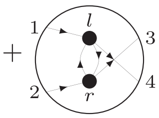
Tree level: Following the standard AdS/CFT procedure, we can readily find the tree-level four-point function, which reads

$\langle \mathcal{L}\mathcal{L}\bar{\mathcal{L}}\bar{\mathcal{L}}\rangle^{(1)} =$ 

$$= \frac{1}{\pi^2}\int d^4\hat{\mathbf{x}}_0 \nabla^2\left(\frac{1}{\mathbf{x}_{10}^2\mathbf{x}_{20}^2}\right)\frac{1}{\mathbf{x}_{30}^2\mathbf{x}_{40}^2}$$
$$= \frac{-1}{\pi^2}\mathcal{C}_{12}\int \frac{d^4\hat{\mathbf{x}}_0}{\mathbf{x}_{10}^2\mathbf{x}_{20}^2\mathbf{x}_{30}^2\mathbf{x}_{40}^2} = -\mathcal{C}_{12}D_{1111}(\mathbf{x}_{ij}^2). \tag{27}$$

The arrows on the vertex distinguish the pairs of legs on which the Laplacian acts. This indeed reproduces the result for the master correlator [(4) and (10)].

One loop: Finally, let us consider the one-loop amplitude. Here we have to consider three orientations of the bubble diagram:

$\langle \mathcal{L}\mathcal{L}\bar{\mathcal{L}}\bar{\mathcal{L}}\rangle^{(2)} =$  $+$  $+$  .

These yield the corresponding expressions

$$\int d^4\hat{\mathbf{x}}_l d^4\hat{\mathbf{x}}_r \nabla_l^2\left(\frac{1}{\mathbf{x}_{1l}^2}\frac{1}{\mathbf{x}_{2l}^2}\right)\frac{1}{\hat{\mathbf{x}}_{lr}^4}\nabla_r^2\left(\frac{1}{\mathbf{x}_{3r}^2}\frac{1}{\mathbf{x}_{4r}^2}\right),$$
$$\int d^4\hat{\mathbf{x}}_l d^4\hat{\mathbf{x}}_r \nabla_l^2\left(\frac{1}{\mathbf{x}_{1l}^2}\frac{1}{\hat{\mathbf{x}}_{lr}^2}\right)\frac{1}{\mathbf{x}_{3l}^2\mathbf{x}_{4r}^2}\nabla_r^2\left(\frac{1}{\mathbf{x}_{2r}^2\hat{\mathbf{x}}_{lr}^2}\right), \tag{28}$$

with the u-channel expression obtained by swapping $(x_3, y_3) \leftrightarrow (x_4, y_4)$ in the t-channel expression.

The first expression is directly related [via (24)] to the bubble integral evaluated in the previous section:

$$\int d^4\hat{\mathbf{x}}_l d^4\hat{\mathbf{x}}_r \nabla_l^2\left(\frac{1}{\mathbf{x}_{1l}^2}\frac{1}{\mathbf{x}_{2l}^2}\right)\frac{1}{\hat{\mathbf{x}}_{lr}^4}\nabla_r^2\left(\frac{1}{\mathbf{x}_{3r}^2}\frac{1}{\mathbf{x}_{4r}^2}\right) = \mathcal{C}_{12}^2\mathcal{K}_s|_{d=1}, \tag{29}$$

and thus it correctly yields the s-channel part of the correlator [(4) and (12)] (at least for the $y_i = 0$ component). The evaluation of the integral in the other two channels is unfortunately less trivial and we will reserve it for future studies.

Conclusion. The study of quantum gravity in curved background is both technically and conceptually very challenging, but great progress has been made in the AdS background because in this case there is a dual description in terms of a conformal field theory in the boundary whose correlation functions are strongly constrained by conformal and crossing symmetry. In recent years a new symmetry principle has emerged for certain AdS × S backgrounds known as hidden conformal symmetry. In this paper, we explored the implications of this symmetry for four-point correlators of $\mathcal{N}=2$ hypermultiplets arising from dimensional reduction of $\mathcal{N}=8$ supergravity on $\mathrm{AdS}_2\times\mathrm{S}^2$ at the quantum level.

This paper has two main results: first, we found that the weight-2 transcendental sector at one loop can be organized into a 4d master correlator, which precisely coincides with the $y_i = 0$ component of the one-loop bubble diagram of a massless ϕ^4 theory in $\mathrm{AdS}_2\times\mathrm{S}^2$, upon acting with a Casimir in each orientation of the bubble. Moreover, we conjectured a scalar effective field theory in $\mathrm{AdS}_2\times\mathrm{S}^2$ which gives rise to the aforementioned Casimir via a two-derivative interaction in the bulk, and should produce all four-point correlators to all loops in the two-derivative sector. It would be interesting to include higher derivative interactions to the effective action in (25), which would arise from the low energy expansion of a generic UV completion and work out their implications for loop-level correlators.

In this paper we have only dealt with the maximal transcendental weight part of the correlators. It would be interesting to see whether the generating function (12) automatically incorporates the full structure dictated by the OPE. Moreover, it would be very useful to test our conjectured action in (25) for correlators with arbitrary charges as well as for higher loops, and to understand how the structures we found in this paper generalize to higher-dimensional theories. For example, does a one-loop massless scalar bubble diagram in $\mathrm{AdS}_5 \times \mathrm{S}^5$ have anything to do with correlators in $\mathcal{N} = 4$ SYM? In fact, while a (Mellin) formula for arbitrary charges at order $\lambda^{-\frac{3}{2}}$ is known [30], it is still not clear if and how this can be written in terms of a generating function, which will most likely involve the sixth order differential operator found in [14].

While the correlators we considered have superconformal symmetry, the bulk effective theories we constructed are nonsupersymmetric because it is possible to factor out a polynomial which encodes all the supersymmetry from the four-point correlators, so it is conceivable that the these effective theories can be adapted to describe less idealized situations.

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[1] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. **2**, 231 (1998).

[2] S. Caron-Huot and A. K. Trinh, All tree-level correlators in $\mathrm{AdS}_5 \times \mathrm{S}_5$ supergravity: Hidden ten-dimensional conformal symmetry, J. High Energy Phys. 01 (2019) 196.

[3] S. Caron-Huot and F. Coronado, Ten dimensional symmetry of $\mathcal{N} = 4$ SYM correlators, J. High Energy Phys. 03 (2022) 151.

[4] S. Caron-Huot, F. Coronado, and B. Mühlmann, Determinants in self-dual $\mathcal{N} = 4$ SYM and twistor space, J. High Energy Phys. 08 (2023) 008.

[5] P. Heslop, The SAGEX review on scattering amplitudes, Chapter 8: Half BPS correlators, J. Phys. A **55**, 443009 (2022).

[6] J. M. Drummond, D. Nandan, H. Paul, and K. S. Rigatos, String corrections to AdS amplitudes and the double-trace spectrum of $\mathcal{N} = 4$ SYM, J. High Energy Phys. 12 (2019) 173.

[7] J. M. Drummond, H. Paul, and M. Santagata, Bootstrapping string theory on $\mathrm{AdS}_5 \times \mathrm{S}_5$, Phys. Rev. D **108**, 026020 (2023).

[8] F. Aprile and P. Vieira, Large p explorations. From SUGRA to big STRINGS in Mellin space, J. High Energy Phys. 12 (2020) 206.

[9] F. Aprile, J. M. Drummond, H. Paul, and M. Santagata, The Virasoro-Shapiro amplitude in $\mathrm{AdS}_5 \times \mathrm{S}^5$ and level splitting of 10d conformal symmetry, J. High Energy Phys. 11 (2021) 109.

[10] T. Abl, P. Heslop, and A. E. Lipstein, Towards the Virasoro-Shapiro amplitude in $\mathrm{AdS}_5 \times \mathrm{S}^5$, J. High Energy Phys. 04 (2021) 237.

[11] L. Rastelli, K. Roumpedakis, and X. Zhou, $\mathrm{AdS}_3 \times \mathrm{S}^3$ tree-level correlators: Hidden six-dimensional conformal symmetry, J. High Energy Phys. 10 (2019) 140.

[12] S. Giusto, R. Russo, A. Tyukov, and C. Wen, The CFT_6 origin of all tree-level 4-point correlators in $\mathrm{AdS}_3 \times \mathrm{S}^3$, Eur. Phys. J. C **80**, 736 (2020).

[13] L. F. Alday, C. Behan, P. Ferrero, and X. Zhou, Gluon scattering in AdS from CFT, J. High Energy Phys. 06 (2021) 020.

[14] T. Abl, P. Heslop, and A. E. Lipstein, Higher-dimensional symmetry of $\mathrm{AdS}_2 \times \mathrm{S}^2$ correlators, J. High Energy Phys. 03 (2022) 076.

[15] B. Bertotti, Uniform electromagnetic field in the theory of general relativity, Phys. Rev. **116**, 1331 (1959).

[16] I. Robinson, A solution of the Maxwell-Einstein equations, Bull. Acad. Pol. Sci. Ser. Sci. Math. Astron. Phys. **7**, 351 (1959).

[17] J. Michelson and M. Spradlin, Supergravity spectrum on $\mathrm{AdS}_2 \times \mathrm{S}^2$, J. High Energy Phys. 09 (1999) 029.

[18] J. Lee and S. Lee, Mass spectrum of $D = 11$ supergravity on $\mathrm{AdS}_2 \times \mathrm{S}^2 \times \mathrm{T}^7$, Nucl. Phys. **B563**, 125 (1999).

[19] R. Glew and M. Santagata, The Veneziano amplitude in $\mathrm{AdS}_5 \times \mathrm{S}^3$ from an 8-dimensional effective action, J. High Energy Phys. 08 (2023) 010.

[20] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevD.110.L061902 for conventions and other technical details.

[21] In fact, this relation directly generalizes to all loop orders $\mathbf{G}^{(l)}|_{\log^l u} = -\frac{1}{l!}\mathcal{C}_{12}^{l-1}(D_{1111}|_{\log \mathbf{x}_{12}^2})$. An analogous statement was also recently noted in a similar context [22].

[22] P. Ferrero and C. Meneghelli, Unmixing the Wilson line defect CFT. Part II: Analytic bootstrap, J. High Energy Phys. 06 (2024) 010.

[23] F. Aprile, J. Drummond, P. Heslop, and H. Paul, Double-trace spectrum of $N = 4$ supersymmetric Yang-Mills theory at strong coupling, Phys. Rev. D **98**, 126008 (2018).

[24] F. Aprile and M. Santagata, Two particle spectrum of tensor multiplets coupled to $\mathrm{AdS}_3 \times \mathrm{S}^3$ gravity, Phys. Rev. D **104**, 126022 (2021).

[25] J. M. Drummond, R. Glew, and M. Santagata, Bern-Carrasco-Johansson relations in $\mathrm{AdS}_5 \times \mathrm{S}^3$ and the double-trace spectrum of super gluons, Phys. Rev. D **107**, L081901 (2023).

[26] H. Paul and M. Santagata, Genus-one open string amplitudes on $AdS_5 \times S^3$ from CFT, J. High Energy Phys. 12 (2023) 057.
[27] J. M. Drummond and H. Paul, One-loop string corrections to AdS amplitudes from CFT, J. High Energy Phys. 03 (2021) 038.
[28] O. Schnetz, Generalized single-valued hyperlogarithms, arXiv:2111.11246.
[29] J. M. Drummond, R. Glew, and H. Paul, One-loop string corrections for AdS Kaluza-Klein amplitudes, J. High Energy Phys. 12 (2021) 072.
[30] F. Aprile, J. M. Drummond, R. Glew, and M. Santagata, One-loop string amplitudes in $AdS_5 \times S^5$: Mellin space and sphere splitting, J. High Energy Phys. 02 (2023) 087.
[31] T. Heckelbacher, I. Sachs, E. Skvortsov, and P. Vanhove, Analytical evaluation of AdS_4 Witten diagrams as flat space multi-loop Feynman integrals, J. High Energy Phys. 08 (2022) 052.
[32] P. Dai, R. N. Huang, and W. Siegel, Covariant propagator in $AdS_5 \times S^5$ superspace, J. High Energy Phys. 03 (2010) 001.
[33] L. Eberhardt, S. Komatsu, and S. Mizera, Scattering equations in AdS: Scalar correlators in arbitrary dimensions, J. High Energy Phys. 11 (2020) 158.
[34] A. Herderschee, R. Roiban, and F. Teng, On the differential representation and color-kinematics duality of AdS boundary correlators, J. High Energy Phys. 05 (2022) 026.
[35] M. Dodelson and A. Zhiboedov, Gravitational orbits, double-twist mirage, and many-body scars, J. High Energy Phys. 12 (2022) 163.