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RESEARCH ARTICLE

The relationship between phonological processing and arithmetic in children with learning disabilities

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Abstract

Phonological processing skills have not only been shown to be important for reading skills, but also for arithmetic skills. Specifically, previous research in typically developing children has suggested that phonological processing skills may be more closely related to arithmetic problems that are solved through fact retrieval (e.g., remembering the solution from memory) than procedural computation (e.g., counting). However, the relationship between phonological processing and arithmetic in children with learning disabilities (LDs) has not been investigated. Yet, understanding these relationships in children with LDs is especially important because it can help elucidate the cognitive underpinnings of math difficulties, explain why reading and math disabilities frequently co-occur, and provide information on which cognitive skills to target for interventions. In 63 children with LDs, we examined the relationship between different phonological processing skills (phonemic awareness, phonological memory, and rapid serial naming) and arithmetic. We distinguished between arithmetic problems that tend to be solved with fact retrieval versus procedural computation to determine whether phonological processing skills are differentially related to these two arithmetic processes. We found that phonemic awareness, but not phonological memory or rapid serial naming, was related to arithmetic fact retrieval. We also found no association between any phonological processing skills and procedural computation. These results converge with prior research in typically developing children and suggest that phonemic awareness is also related to arithmetic fact retrieval in children with LD. These results raise the possibility that phonemic awareness training might improve both reading and arithmetic fact retrieval skills.

KEYWORDS
arithmetic, fact retrieval, learning disabilities, phonological processing, procedural computation, reading

Research Highlights

- Relationships between phonological processing and various arithmetic skills were investigated in children with learning disabilities (LDs) for the first time.
- We found phonemic awareness was related to arithmetic involving fact retrieval, but not to arithmetic involving procedural computation in LDs.
- The results suggest that phonemic awareness is not only important to skilled reading, but also to some aspects of arithmetic.
- These results raise the question of whether intervention in phonemic awareness might improve arithmetic fact retrieval skills.

1 | INTRODUCTION

Strong reading and mathematical skills are not only critical for academic success, but individual differences in these skills also predict employability, socioeconomic success, and health (Parsons & Bynner, 2005; Purpura et al., 2019; Ritchie & Bates, 2013). Phonological processing is a language skill that reflects a person's ability to think about and manipulate phonological aspects of spoken language (Scarborough & Brady, 2002). While it is widely known that phonological processing is associated with reading ability in typically developing children (Bus & Van Ijzendoorn, 1999; Koponen et al., 2017; Swanson et al., 2003; Wagner & Torgesen, 1987), phonological processing has also been shown to be associated with math ability in typical children (Barnes et al., 2014; Durand et al., 2005; Fuchs et al., 2005, 2006; Hecht et al., 2001; Rasmussen & Bisanz, 2005; Simmons et al., 2008). The current study builds in this existing literature by examining the association between phonological processing and arithmetic ability in children with weakness in reading and/or math skills.

1.1 | Relationships between phonological processing skills, reading and mathematics

Phonological processing encompasses three main skills: (a) phonemic awareness, (b) phonological memory, and (c) rapid serial naming. *Phonemic awareness* is the ability to attend to and manipulate individual phonemes within spoken words or syllables (Scarborough & Brady, 2002). Although phonemic awareness is often referred to as phonological awareness in the literature, the more specific term phonemic awareness emphasizes processing at the level of the individual phoneme (Scarborough & Brady, 2002). Phonemic awareness is measured by tasks such as identifying sounds within words (e.g., sound elision, say "hat" without the sound /h/). *Phonological memory* (often referred to as the phonological or articulatory loop), is the ability to temporarily store phonological information (Baddeley, 2003, 1979). It is usually measured using a Digit Span task where one recalls a series of aurally-presented digits. Finally, *rapid serial naming* of phonological codes is the rate of access by which phonological representations are retrieved from long-term memory. It is usually measured with the Rapid Automatized Naming Test, which measures the time taken to name items on a page such as letters or numbers (Denckla & Rudel, 1976). In a series of landmark studies, these three measures of phonological processing were shown to make contributions to later reading outcomes (Wagner & Torgesen, 1987) as well as math growth (Hecht et al., 2001), with phonemic awareness being the most robust of the three measures in predicting reading and arithmetic performance in the later grades. Next, we consider the specific relationships between each of these three phonological processing skills and math performance in more detail.

Phonemic awareness has been shown to be important for arithmetic (for a review see Vanbinst & De Smedt, 2016). For example, Leather and Henry (1994) found that phonemic awareness was the best concurrent predictor of arithmetic skills (accounting for 31% of the variance in arithmetic problem solving), above and beyond skills such as counting span, and memory span. Relationships between phonemic awareness and math skills have also been documented longitudinally where phonemic awareness measured in 2nd grade captured roughly 10% of the variance in calculation skills in 5th grade (Hecht et al., 2001). These results demonstrate that the same phonemic awareness skills that have been shown to support reading acquisition (Ehri et al., 2001) and are weak in children with a reading disability (for review see Peterson & Pennington, 2012; Vellutino et al., 2004), also support math acquisition.

Next, phonological memory is thought to be needed in arithmetic problem solving to store, maintain, and manipulate both phonological and numerical information. For instance, a child may need to temporarily store intermediate numbers when decomposing an arithmetic problem into multiple steps, or a child may need to encode and store the phonological representations of the arithmetic problem to retrieve the arithmetic fact from memory (Geary, 1993; Hecht et al., 2001; Peng et al., 2016). A meta-analysis of 110 studies found that phonological memory was correlated with a wide variety of math tasks including single- and double-digit arithmetic (Peng et al., 2016). Together, these findings suggest phonological memory is not only related to reading skills (Peng et al., 2018) and impaired in children with dyslexia (Peng & Fuchs, 2014), but also appears to support arithmetic.

Lastly, rapid serial naming has been shown to be important for arithmetic by providing fast access to the phonological representations of

Arabic digits (Hecht et al., 2001). A recent meta-analysis of 38 studies found that rapid automatized naming (i.e., rapidly naming single letters, digits, objects, or colors) had strong relationships with single-digit and timed arithmetic measures (Koponen et al., 2017). Children with better performance on rapid serial naming tasks have also been found to be stronger at retrieving arithmetic facts from memory (Vanbinst, Ceulemans et al., 2015; Vanbinst, Ghesquière et al., 2015). Therefore, performance on rapid serial naming, which has been found to be associated with reading ability (Norton & Wolf, 2012) and impaired in children with dyslexia (Denckla & Rudel, 1976), may also play a role in arithmetic. Here we examine whether these three phonological processing skills are related to arithmetic in children with learning disabilities, and whether this relationship depends on the specific type of arithmetic task.

1.2 | The relationship between phonological processing and mathematics depends on the measures used

In addition to the different measures of phonological processing described above, researchers use various measures for math. Often it is the case that a single test of math assesses a wide variety of math skills. For example, the Woodcock-Johnson Calculation subtest (Woodcock et al., 2001) measures accuracy on problems ranging from number writing, single- and double-digit calculation, to geometry and trigonometry. The research above has documented associations between phonological processing and math skills, however, not all prior studies have found these relationships, most likely because of the multifaceted math measures used. For example, Fuchs et al. (2005) found an association between phonological skills (using a combined measure of rapid serial naming and phonemic awareness) and Addition Fact Fluency (a timed measure of addition), but not the (untimed) Woodcock-Johnson Calculation subtest. Further, Passolunghi et al. (2008) demonstrated that phonemic awareness did not predict performance on an Italian standardized measure of math achievement that measured logical reasoning, arithmetic, and geometry skills. It is likely that phonological processing may be important for some items on these tests, but not others, leading to a lack of association between phonological processing and math overall. It is therefore critical to use measures that capture specific math skills as some math skills appear to be more dependent on phonological processing than others.

Arithmetic problems can be solved in different ways. Children solve some arithmetic problems by using a verbally-mediated strategy where they retrieve a phonologically-based arithmetic fact from long-term memory (Fact Retrieval) (Barrouillet et al., 2008; Campbell & Xue, 2001). Alternatively, they may use a strategy of counting or decomposing the problem into smaller parts (Procedural Computation) (Barrouillet et al., 2008; Campbell & Xue, 2001). Arithmetic problems with small numbers (e.g., single-digit arithmetic) tend to be solved using fact retrieval (Campbell & Xue, 2001), whereas arithmetic problems with large numbers (e.g., double-digit arithmetic) tend to be solved using procedural computation (Caviola et al., 2018; Lemaire & Callies, 2009). Similarly, arithmetic operations also influence the frequency of fact retrieval and procedural computation strategies, where fact retrieval is used more on addition and multiplication problems, and procedural computation is used more on subtraction and division problems (Barrouillet et al., 2008; Campbell & Xue, 2001; Caviola et al., 2018; Imbo & Vandierendonck, 2008; Lemaire & Callies, 2009). The use of these strategies changes over development (Siegler, 2005), where fact retrieval tends to be used more with increasing age and experience (Ashcraft, 1992; Barrouillet & Fayol, 1998; Lemaire & Siegler, 1995). Taken together, problem size and operation both influence arithmetic strategy use.

A small body of literature has begun to examine the relationship between phonological processing and fact retrieval versus procedural computation strategies. Specifically, this research has manipulated factors such as problem size and operation to induce the use of these different arithmetic strategies. In typically developing children, De Smedt and colleagues used regression analyses to demonstrate that phonemic awareness (sound elision) was a strong unique predictor of performance on small arithmetic problems (i.e., likely solved using fact retrieval), but not large arithmetic problems (i.e., likely solved using procedural computation) for addition and multiplication (De Smedt et al., 2010). Dual-task studies investigating phonological memory in typically developing children have found that phonological rehearsal interferes with multiplication (more likely to rely on fact retrieval), but not subtraction (more likely to rely on procedural computation) (Lee & Kang, 2002). Together, these studies illustrate that relationships between phonological processing skills and arithmetic may be stronger for arithmetic problems solved via fact retrieval compared to procedural computation. These findings raise the question of whether children who have fragile phonological processing skills also struggle with arithmetic, especially on arithmetic problems that utilize phonological codes such as small (single-digit) problems. This unresolved question is the focus of the current investigation.

1.3 | Phonological processing skills and mathematics in children with learning disabilities (LD)

There are no investigations examining the relationships between phonological processing and arithmetic in children with learning disabilities (LDs). This is a significant oversight given the aforementioned research suggesting an association between phonological processing and arithmetic in typically developing children. Further, the high co-occurrence (30-70%) of dyscalculia (math learning disability) with dyslexia (reading learning disability) (Landerl & Moll, 2010; Lewis et al., 1994; Willcutt et al., 2013) provides another motivation to understand the nature by which phonological processing may affect not only reading, but also math skills.

Interestingly, studies conducted in dyslexia have shown deficits in arithmetic performance (Simmons & Singleton, 2008). For instance, adults with dyslexia have been found to be slower or less accurate on simple arithmetic problems compared to controls (Gobel & Snowling, 2010; Simmons & Singleton, 2006). Especially relevant to the current study, De Smedt and Boets (2010) found that adults with dyslexia used retrieval strategies less often than controls during single-digit multiplication and subtraction problems. They also found that weaker phonemic awareness skills were related to less frequent use of retrieval strategies across both groups. Children with dyslexia have also been found to be slower and less accurate when solving arithmetic problems, especially for multiplication compared to subtraction, suggesting less frequent use of fact retrieval strategies (Boets & De Smedt, 2010). However, it is not yet clear whether individual differences in phonological skills are related to arithmetic performance in children with LDs, which we examine in the present study.

Only a few studies have assessed phonological processing in children with dyscalculia. These studies did not find impairments on phonemic awareness, phonological memory, or rapid serial naming of letters or digits in children with dyscalculia (Landerl et al., 2009; Szucs et al., 2013; Willburger et al., 2008). However, they did not examine whether individual differences in phonological processing were related to retrieval- versus procedural-based arithmetic.

Taken together, while phonological processing is known to correlate with arithmetic tasks that rely on fact retrieval in typically developing children, no studies have investigated the role of phonological processing on arithmetic in children with LD.

1.4 | The present study

Previous research in typically developing children has found that phonological processing skills, widely known to be related to reading, are also related to retrieval-based arithmetic (e.g., small arithmetic problems) but not procedural arithmetic (e.g., large arithmetic problems) (De Smedt et al., 2010). This raises the possibility that poor phonological processing skills may negatively impact arithmetic problem solving through arithmetic fact retrieval in children with LD. Understanding these relationships is especially important in children with LD as it may help uncover the cognitive underpinnings of math difficulties, offer an explanation as to why reading and math disabilities frequently co-occur, and inform which cognitive skills could be targeted by behavioral interventions.

We studied a heterogeneous group of children with difficulties in reading, math, or both skills. We used a continuous approach because reading and math abilities fall on a continuum and cut-offs for dyslexia and dyscalculia are arbitrary (Branum-Martin et al., 2012; Peters & Ansari, 2019). Since different phonological processing measures are likely related to arithmetic skills in different ways, we included all three measures (phonemic awareness, phonological memory, and rapid serial naming). We examined untimed (Calculation subtest of the Woodcock-Johnson) and timed (Math Fluency subtest of the Woodcock-Johnson) measures from published tests of general mathematical processing. Because these published measures do not differentiate well between different arithmetic processes, we also employed timed measures of addition and subtraction (Math Battery by Fuchs et al., 2003) that distinguish between small single-digit arithmetic problems (largely solved using fact retrieval) and large double-digit arithmetic problems (largely solved using procedural computation) (Cirino et al., 2015, 2018; Fuchs et al., 2008; Powell et al., 2009). This allowed us to examine whether (and which) phonological processing skills are related to performance on single-digit arithmetic problems that are frequently solved using fact retrieval, but not to double-digit problems that are solved using procedural computation strategies.

Based on the prior literature in typically developing children we expected that phonological processing skills would not necessarily be related to math performance on the Calculation subtest of the Woodcock-Johnson since it evaluates a broad range of math skills. Rather, we expected that phonological processing skills would be related to performance on single-digit (retrieval-based) arithmetic, and not double-digit (procedural computation-based) arithmetic, due to the greater reliance on fact retrieval strategies in small arithmetic problems. Together, we offer a systematic investigation involving several measures of phonological processing and arithmetic in a well-characterized sample of children with impairments in reading and/or math.

2 | METHODS

2.1 | Participants

Sixty-three children were recruited as part of a larger program of research based on a weakness in reading or/and mathematics. To be included in the current study, children needed to have standard scores below 92 (below the 30th percentile) on one of four measures during criteria testing: (1) untimed non-word reading (Word Attack in the Woodcock-Johnson III Tests of Achievement), (2) timed single-word reading (Sight Word Efficiency in the Test of Word Reading Efficiency), (3) untimed calculation (Calculation in the Woodcock-Johnson III Tests of Achievement), or (4) timed single-digit arithmetic (Math Fluency in the Woodcock-Johnson III Tests of Achievement). All children had a Full-Scale IQ above 85 (as measured by the Wechsler Abbreviated Scale of Intelligence). Our selection criteria were deliberately implemented to attain a wide range of abilities, and resulted in the inclusion of some children with mild LD and others with more severe LD. The sample, therefore, represents a heterogeneous group of children of varying ability levels in reading and math, which not only provides greater ecological validity, but is also ideal for correlational analyses. While these cut-offs on standardized tests of reading and math are relatively liberal, the majority of these children (78%) qualified for educational accommodations (e.g., individual education plans or 504 plans). This indicates that most of these children have demonstrated consistent impairments in reading, math, or both. Average measures of these reading and math skills fell in the "low range of normal" or "normal" ranges of the distribution, and one-sample t-tests confirmed that the group's average standard scores for Letter-Word ID, Sight-Word Efficiency, Calculation, and Math Fluency, were all significantly below 100 (p-values all <.019). When considering a 25th percentile cut-off, 54%, 58.7%, 30.2%, and 81% of children were at or below this frequently used threshold on Letter-Word ID, Sight-Word Efficiency,

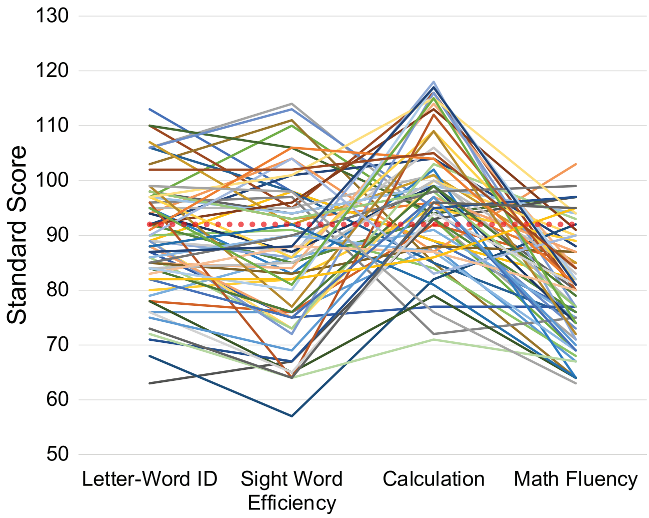

FIGURE 1 Reading and math standard scores for each of the 63 children. The dotted red line represents a standard score of 92

Calculation, and Math Fluency measures, respectively. To illustrate the distribution of the scores for every participant, we plotted their profiles for the reading and math standard scores in Figure 1. The group's average IQ was in the "normal range" (mean standard Full-Scale IQ of 108). Children were between 7.8 and 12.7 years old (M = 10.3, SD = 1.3) with 37 female and 26 male participants. All aspects of the study were approved by Georgetown University's Institutional Review Board and the study conforms to recognized standards under the Declaration of Helsinki. Informed consent was obtained from the legal guardian for all children, and the children gave written assent.

3 | MEASURES

3.1 | Phonological processing

3.1.1 | Phonemic awareness

Phonemic awareness was assessed using the Phoneme Elision subtest of the Comprehensive Test of Phonological Processing 2 (CTOPP-2) (Torgesen et al., 1999). In this test, children are asked to say a real word and then repeat it without a portion of the word (e.g., children are asked to say "bold", then to say "bold" without the sound '/b/'). The test developers report a test-retest reliability of .88 for ages 5–7, and .79 for ages 8–17.

3.1.2 | Phonological memory

We assessed phonological working memory using the Digit Span subtest from the Wechsler Intelligence Scale for Children (WISC-III) (Wechsler, 1991). Children were asked to repeat a list of digits in the same order for digit span forward, and in the reverse order for digit span backwards. Raw scores for forward and backward span were averaged to form one mean digit span score. The test developers report a split-half reliability of .85 for ages 6–15, and a test-retest reliability of .73 for ages 10–11.

3.1.3 | Rapid serial naming

The rate of access to digits and letters was measured through the Rapid Automatized Naming Test (RAN) (Wolf & Denckla, 2005). All four subtests were administered (Objects, Colors, Numbers, Letters), but only Letters and Numbers were used here. Children were asked to say the names of all of the numbers or letters presented on a page sequentially as fast as possible. Completion time formed the raw score on each subtest. Because performance on Letters and Numbers was highly correlated ($r(60) = .75$, $p < .001$, after controlling for age), these scores were averaged (RAN-Alphanumeric) to reduce the number of measures. The test developers report a test-retest reliability of .90 for Letters and Numbers in elementary school students.

3.2 | Arithmetic and mathematics

3.2.1 | Published measures of math ability

We assessed math abilities using two paper-and-pencil subtests of the Woodcock-Johnson III Tests of Achievement (Woodcock et al., 2001). The *Calculation* subtest is an untimed measure that tests a broad range of calculation abilities (e.g., simple arithmetic to geometry and trigonometry). The *Math Fluency* subtest is a timed measure of single-digit arithmetic skills (addition, subtraction, and multiplication) where children answer as many problems as they can in 3 min. The Calculation subtests has split-half reliability estimates of .80–.87 for children between the ages of 7–12, and the Math Fluency subtest has a test-retest reliability of .95 in ages 7–11, as reported by the test developer (Schrank et al., 2001).

3.2.2 | Fact retrieval

To measure children's ability to retrieve arithmetic facts, we administered four subtests from the Grade 3 Math Battery, which uses small numbers (Fuchs et al., 2003, 2005, 2008). This timed paper-and-pencil task consists of two subtests assessing fact retrieval in addition (*Addition Fact Fluency 0–12 and Addition Fact Fluency 0–18*), and two subtests assessing fact retrieval in subtraction (*Subtraction Fact Fluency 0–12 and Subtraction Fact Fluency 0–18*). *Addition Fact Fluency 0–12* had single-digit problems with sums from 0–12 whereas *Addition Fact Fluency 0–18* had single-digit problems with sums from 0–18. *Subtraction Fact Fluency 0–12* had minuends from 0–12 whereas Subtraction Fact Fluency 0–18 had minuends from 0–18. Each subtest consisted of 25 problems on one page. Children had 1 min to write as many answers as possible for each subtest. Scores are the number of correctly answered problems. Scores from all four subtests were averaged to form a robust measure of Fact Retrieval across both operations. Fuchs et al. (2006) report a Cronbach's alpha of .92 for Addition Fact Fluency 0–12 and Subtraction Fact Fluency 0–12 combined.

3.2.3 | Procedural computation

To measure children's computational skills, we administered two additional subtests from the Grade 3 Math Battery, which measured arithmetic with larger numbers (Fuchs et al., 2003, 2008). *Double-Digit Addition* had two-digit by two-digit addition problems with and without regrouping, whereas *Double-Digit Subtraction* had two-digit by two-digit subtraction problems with and without regrouping. Each subtest consisted of 20 problems on one page. Children had 3 min to write as many answers as possible for each subtest. Scores are the number of correctly answered problems. Scores from the two subtests were averaged to form a more robust measure of Procedural Computation across both operations. Fuchs et al. (2006) report a Cronbach's alpha of .93 for Double-Digit Addition and Double-Digit Subtraction combined.

3.3 | Single-word reading

To examine the well-known relationship between phonological processing and reading, we used the *Letter-Word Identification (Letter-Word ID)* subtest from the Woodcock-Johnson III Tests of Achievement (Woodcock et al., 2001) for an untimed measure of single-word reading, and the *Sight Word Efficiency* Test of Word Reading Efficiency 2 (TOWRE-2) (Torgesen et al., 1999) for a timed measure of single real-word reading (i.e., the number of words correctly read aloud in 45 s). Letter-Word ID has split-half reliability estimates of .90-.97 for children between the ages 7–12 (Schrank et al., 2001), and Sight Word Efficiency has a test-retest reliability of .97 for ages 6–9, and .84 for ages 10–18 (Torgesen et al., 1999), as reported by the test developers.

3.4 | Full-scale intelligence (IQ)

IQ was measured using the Wechsler Abbreviated Scale of Intelligence (WASI) (Wechsler, 1999). Verbal IQ subtests consisted of Vocabulary and Similarities while non-verbal subtests consisted of Matrix Reasoning and Block Design. Verbal and non-verbal IQ subtests were averaged to form Full-Scale IQ. The test developers report a test-retest reliability of .91 for verbal IQ and .86 for non-verbal IQ in children between the ages of 6–11 (Wechsler, 1999).

3.5 | Analyses

The retrieval and procedural arithmetic measures discussed above are not standardized. Therefore, to remain consistent across measures, we residualized raw scores for age (regressing the variable on age and saving the unstandardized residuals) for all measures and used these residualized scores for all subsequent analyses.

First, we conducted partial correlations to determine whether phonological processing skills (Phoneme Elision, Digit Span, RAN-Alphanumeric) were correlated with measures of math (controlled for Full-Scale IQ). For this, we used two published tests (Calculation and Math Fluency) and two experimental arithmetic tests (Fact Retrieval and Procedural Computation). Also included in these correlations (for comparison with prior studies) were untimed (Letter-Word ID) and timed (Sight Word Efficiency) single-word reading. Correlations were corrected for multiple comparisons using the Holm-Bonferroni correction (Holm, 1979).

To test our central research question of whether phonological processing skills are related to retrieval-based arithmetic but not procedural-based arithmetic, we conducted two regressions to examine whether measures of phonological processing (Phoneme Elision, Digit Span, RAN-Alphanumeric) predicted unique variance in children's timed retrieval-based arithmetic skills (Arithmetic Fact Retrieval) or procedural-based arithmetic skills (Arithmetic Procedural Computation). To verify whether these phonological processing measures predicted reading ability within this sample, we also conducted another regression with these same three phonological processing measures, this time measuring whether they predict timed real-word reading (for consistency with the regressions using timed math measures). Full-Scale IQ was included in all regression models, and all predictors were entered simultaneously (i.e., using the Enter Method). SPSS 27 was used to analyze all data.

4 | RESULTS

Summary statistics of published measures are shown in Table 1. Performance on the Fact Retrieval and Procedural Computation subtests are shown in Table 2.

4.1 | Correlation analyses

We did not find significant correlations between the phonological processing measures (Phoneme Elision, Digit Span, or RAN-Alphanumeric) and the published measures of math (Calculation or Math Fluency) (see Table 3). We also did not find any significant correlations between phonological processing skills (Phoneme Elision, Digit Span, or RAN-Alphanumeric) and the experimental measures of math (Fact Retrieval or Procedural Computation). While Phoneme Elision was initially significantly correlated with Fact Retrieval ($r(60) = .31$, $p_{\text{uncorrected}} = .015$) as predicted, this correlation did not survive the Holm-Bonferroni correction for multiple comparisons.

As expected, we found positive correlations between phonemic awareness (Phoneme Elision) and measures of untimed (Letter-Word ID) and timed (Sight Word Efficiency) single real-word ability. However, phonological memory (Digit Span) did not correlate with either measure of reading. There was a significant correlation between rapid

TABLE 1 Mean raw and mean standard scores of tests of phonological processing, mathematics, and reading. Standard deviations are shown in parentheses

Construct	Measure	Raw Scores	Standard scores	Standard score range
Phonemic Awareness	Phoneme Elision	12.16 (5.0)	92.22 (16.1)	55–130
Phonological Memory	Digit Span	11.62 (2.2)	92.41 (12.0)	67–120
Rapid Serial Naming	RAN Numbers	36.67 (12.5)	88.30 (11.9)	55–110
	RAN Letters	35.67 (9.7)	87.41 (9.9)	73–108
	RAN Alphanumeric (numbers and letters)	36.17 (10.5)	–	–
Untimed Math	Calculation	16.08 (5.2)	96.44 (11.7)	71–118
Timed Math	Math Fluency	39.14 (13.4)	80.54 (10.1)	63–103
Untimed Word Reading	Letter-Word ID	44.11 (9.9)	89.92 (10.8)	63–113
Timed Word Reading	Sight Word Efficiency	49.14 (18.9)	86.65 (13.8)	57–114
Intelligence	Full-Scale IQ	28.15 (6.1)	107.75 (11.9)	85–136

TABLE 2 Mean number of correct items on the experimental arithmetic measures: four fact retrieval subtests and two procedural computation subtests, as well as their overall average scores. Standard deviations are shown in parentheses

		Measure	Raw scores	Average raw scores	Average raw score range
Fact Retrieval	Addition	Addition 0–12 (/25)	14.35 (5.9)	9.94 (4.4)	0.75–20.75
		Addition 0–18 (/25)	11.10 (4.6)		
	Subtraction	Subtraction 0–12 (/25)	7.46 (4.5)		
		Subtraction 0–18 (/25)	6.86 (4.7)		
Procedural Computation	Addition	Double-Digit Addition (/20)	12.37 (6.0)	9.50 (5.0)	0–20.00
	Subtraction	Double-Digit Subtraction (/20)	6.63 (4.9)		

TABLE 3 Partial correlations (accounting for full-scale IQ). All variables are residualized for age

		1	2	3	4	5	6	7	8	9
1	Phoneme Elision	–								
2	Digit Span	.109	–							
3	RAN-Alphanumeric	–.274	–.302	–						
4	Calculation	.238	.044	.150	–					
5	Math Fluency	.157	–.002	–.172	.333	–				
6	Fact Retrieval	.308	.172	–.156	.269	.715**	–			
7	Procedural Computation	.243	.174	–.069	.434**	.707**	.747**	–		
8	Letter-Word ID	.608**	.216	–.282	.124	-.072	.150	.093	–	
9	Sight Word Efficiency	.439**	.218	–.568**	–.004	.068	.176	.057	.774*	–

Note: Correlations Holm-Bonferroni corrected: ** corrected *p*-value < .001 and * corrected *p*-value < .05.

serial naming of letters and numbers (RAN-Alphanumeric) and timed single-word reading (Sight Word Efficiency). This correlation was negative because RAN raw scores are a measure of time to completion (i.e., less time to complete the test indicates better performance).

We found no significant correlations between measures of untimed (Letter-Word ID) or timed (Sight Word Efficiency) reading ability and untimed (Calculation) or timed (Math Fluency) math. Nor did we find a correlation between these measures of reading and Fact Retrieval or Procedural Computation (all uncorrected *p*-values > .05).

4.2 | Regression analyses

First, we conducted two regression analyses to understand whether phonological processing measures predicted Fact Retrieval but not Procedural Computation skills. The results of these two regressions are shown in Table 4. The first regression with Fact Retrieval as the dependent variable was significant (F (4,58) = 3.08, p = .02), and the model accounted for 18% of the variance in children's Fact Retrieval skills (R^2 = .175). The only significant predictor in the regression was

TABLE 4 Regression analyses predicting fact retrieval and procedural computation. Unstandardized (B) and standardized beta coefficients are reported (*β*)

	Fact retrieval				Procedural computation			
Predictor	*B*	*β*	*t*	*p*-value	*B*	*β*	*t*	*p*-value
Phoneme Elision	.20	.29	2.20	.03	.19	.24	1.81	.08
Digit Span	.20	.13	1.00	.32	.28	.16	1.23	.22
RAN-Alphanumeric	–.01	–.04	–.30	.77	.02	.04	.33	.74
Full-Scale IQ	.10	.15	1.15	.25	.11	.15	1.10	.28

TABLE 5 Regression analyses predicting timed single real-word (Sight Word Efficiency). Unstandardized (B) and Standardized beta coefficients are reported (*β*)

	Sight word efficiency			
Predictor	*B*	*β*	*t*	*p*-value
Phoneme Elision	.98	.32	2.91	.005
Digit Span	.300	.04	.40	.69
RAN-Alphanumeric	–.75	–.48	–4.32	<.001
Full-Scale IQ	–.16	–.05	–.47	.64

phonemic awareness (Phoneme Elision; $B = .20$, $p = .03$), as none of the other phonological processing skills (Digit Span, RAN-Alphanumeric) or Full-Scale IQ predicted unique variance in Fact Retrieval skills. The second regression with Procedural Computation as the dependent variable was not significant (F (4,58) = 2.40, $p = .06$), indicating there were no significant predictors of Procedural Computation (the beta values, *t*-scores, and p-values are shown in Table 4 for completeness).

Finally, we conducted a regression analysis to examine whether phonological processing measures predicted single real-word reading ability, as has been shown in prior studies. This regression model was significant (F (4,58) = 10.14, $p < .001$), and the model accounted for 41% of children's reading skills ($R^2 = .412$) (see Table 5). Phonemic awareness (Phoneme Elision; $B = .98$, $p = .005$) and rapid serial naming (RAN-Alphanumeric; $B = -.75$, $p < .001$) were both significant unique predictors of timed real-word reading skills, however, none of the other measures (Digit Span, Full-Scale IQ) were significant.

5 | DISCUSSION

The phonological processing skills that are correlated with reading abilities (Bus & Van Ijzendoorn, 1999; Koponen et al., 2017; Swanson et al., 2003; Wagner & Torgesen, 1987) have also been shown to be related to math abilities (Hecht et al., 2001), suggesting that phonological processing may support both academic skills. However, the relationship between phonological processing and arithmetic seems to depend on whether the specific arithmetic task is solved through fact retrieval versus procedural computation (De Smedt et al., 2010). Previous research has often not distinguished between these strategies because they are impossible to disentangle with published measures of calculation that encompass a range of skills. It is also unknown whether the relationship between phonological processing and arithmetic depends on the specific phonological processing task. Importantly, the small literature examining relationships between phonological processing and retrieval-based arithmetic has been limited to typically developing children, and no studies to our knowledge have examined these associations in children with LDs. Understanding the relationships between phonological processing and arithmetic is particularly relevant for children with learning disabilities because it may help identify the etiology of their weakness in arithmetic, explain why reading and math disabilities frequently co-occur, and point to possible interventions.

To study whether phonological processing is related to specific aspects of arithmetic in children with LD, we distinguished between arithmetic problems solved with fact retrieval versus those solved with procedural computation strategies. We also examined different phonological processing measures, including phonemic awareness (sound elision) as well as phonological memory (digit span), and rapid serial naming (rapid automatized naming), all of which may potentially be important for arithmetic. Our main finding was that phonemic awareness predicted retrieval-based arithmetic (single-digit arithmetic) but not procedural-based arithmetic (double-digit arithmetic). Specifically, regression analyses with all three phonological measures entered simultaneously revealed a relationship between phonemic awareness (sound elision) and retrieval-based arithmetic, whereas phonological memory (digit span) and rapid serial naming (RAN-Alphanumeric) made no significant contributions. This suggests there is a connection between phonological processing and math, but the relationship is specific to phonemic awareness and retrieval-based arithmetic. However, the strength of this relationship was not as strong as that observed between phonemic awareness and single-word reading. Below, we expand on these findings and discuss their implications for children with learning disabilities.

5.1 | Phonemic awareness is associated with fact retrieval but not procedural computation in learning disabilities

The main objective of this study was to examine whether phonological processing skills (phonemic awareness, phonological memory, and rapid serial naming) predicted fact retrieval but not procedural computation skills. Partial correlations revealed no relationships between phonological processing measures and math abilities at corrected levels. While the correlation between phonemic awareness (sound elision) and fact retrieval did not survive correction for multiple comparisons, the regression analysis (which included all three phonological processing measures and IQ) showed that phonemic awareness significantly predicted unique variance in children's fact retrieval skills but not procedural computation skills. These findings in children with LD mirror research in typically developing children. In a group of fourth and fifth grade children, De Smedt et al. (2010) found relationships between phonemic awareness and small arithmetic problems that are likely solved via fact retrieval, but found no such relationship with large problems that are more likely solved with procedural computation strategies.

While phonemic awareness predicted fact retrieval in the present study, phonological memory did not. Similarly, De Smedt et al. (2010) did not find any correlations between phonological memory and arithmetic, nor did Fuchs et al. (2006) find an association between phonological memory and retrieval or calculation-based arithmetic. A meta-analysis has found evidence for a relationship between phonological memory and arithmetic (Peng et al., 2016), but others have noted that the evidence may be mixed (Vanbinst & De Smedt, 2016).

We also did not observe associations between rapid serial naming and math performance in any of the correlation or regression analyses. This contrasts with prior literature that has documented relationships between rapid serial naming and arithmetic, especially between rapid serial naming and timed single-digit arithmetic (e.g., Koponen et al., 2017; Slot et al., 2016).

Together, these results indicate that phonemic awareness skills are uniquely related to retrieval-based arithmetic in children with LDs. Strong phonemic representations may be important for access to arithmetic facts that are stored in long-term memory as phonological codes (Dehaene et al., 2003). However, our findings also suggest that both phonological memory and rapid serial naming are not strong predictors of retrieval-based arithmetic in children with LD.

To date, Hecht et al. (2001) has provided the largest and most comprehensive study to date on the relationship between phonological processing skills and math abilities in typically developing children. They examined how phonemic awareness, phonological memory, and rapid serial naming related to concurrent math performance, as well as longitudinal gains in math from Grades 2–5. Math performance was measured using an experimental computerized measure of timed simple arithmetic in Grades 4 and 5 (similar to Fact Retrieval in the present study) and a standardized untimed measure of calculation skills in Grades 2–5 (Calculation subtest of Woodcock-Johnson). Unlike the present study, the authors found significant concurrent relationships between all phonological measures and both math measures in these typically developing children. When examining longitudinal gains, Hecht and colleagues found no relationships between phonological measures and gains in timed simple arithmetic, but all three phonological processing measures predicted gains in children's computation skills. The findings of the present study are at odds with these observations, since we would have expected a relationship with gains in timed simple arithmetic but not with gains in calculation skills. However, Hecht et al. (2001) only measured timed simple arithmetic in Grades 4 and 5, and the authors suggest that children had fairly stable performance on simple arithmetic problems during this time. Therefore, there may have been little growth or variability in simple timed arithmetic skills over this period, potentially resulting in a lack of a relationship between phonological skills and gains in timed simple arithmetic. Future longitudinal work will be important in determining how phonological skills predict timed simple arithmetic problems (fact retrieval) and calculation skills (procedural computation) over development. The use of interventions would be especially helpful, and could test specifically whether improving children's phonological awareness skills improves children's retrieval, but not procedural, arithmetic skills. Another potential consideration is that while our sample size was similar to, or larger than some prior studies examining relationships between phonological processing and arithmetic (e.g., De Smedt & Boets, 2010; De Smedt et al., 2010) it is smaller than that in Hecht et al. (2001). Therefore, it is possible we did not observe some of the same relationships due to our smaller sample size and future research with larger samples will help address this question.

While prior research has not examined the relationship between phonological processing skills and arithmetic in children with LDs, a study in adults with dyslexia found associations between phonemic awareness and the use of fact retrieval strategies (De Smedt & Boets, 2010). Specifically, when asking participants to describe the mental steps they took to solve each arithmetic problem, the authors found that phonemic awareness correlated with the reported frequency of fact-retrieval use during multiplication problems (more likely to be solved with fact retrieval), but not during subtraction problems (more likely to be solved using procedural computation) in both adults with and without dyslexia. Further, neither phonological memory nor rapid serial naming predicted retrieval use on either multiplication or subtraction. Together, the findings from De Smedt and Boets (2010) and the present study both show that phonemic awareness is specifically important for retrieval-based arithmetic in learning disabilities.

In contrast with De Smedt and Boets (2010), but similar to De Smedt et al. (2010), we did not assess strategy use through self-reports (as this was not part of the Math Battery Test). So, while we cannot be certain that every Fact Retrieval problem was actually retrieved (that is, some children may have used calculation strategies for larger single-digit problems instead), our main focus was on the relative differences between Fact Retrieval and Procedural Computation. This is consistent with how this Math Battery test was originally designed and implemented (Fuchs et al., 2006, 2008). This approach is also consistent with a large body of literature showing that problem size (e.g., single-digit versus double-digit) affects strategy use, where small arithmetic problems tend to be solved using fact retrieval more than large arithmetic problems (Campbell & Xue, 2001; Caviola et al., 2018; Lemaire & Callies, 2009). As such, there is general agreement that single-digit problems are more likely to be solved with retrieval strategies than double-digit problems, even though there will be some individual differences in strategy use across both problem types. There was, however, evidence that Fact Retrieval and Procedural Computation used different strategies based on the different relationships they had with the subtests of the Woodcock-Johnson. Specifically, Fact Retrieval correlated with Math Fluency (more likely to rely on retrieval strategies as it involves single-digit addition, subtraction, and multiplication), but not Calculation (more likely to rely on calculation-based strategies as it involves more complex arithmetic as well as geometry and trigonometry). On the other hand, the Procedural Computation subtest correlated with both Math Fluency and Calculation subtests (see Table 3). Together, these suggest that Fact Retrieval was more likely to have involved retrieval-based strategies than calculation strategies.

Finally, our results also expand our knowledge of the relationships between phonological processing and arithmetic in children with other types of LD. While prior research has shown that children with dyscalculia have no impairments in phonological processing compared to typically developing controls (Landerl et al., 2009; Szucs et al., 2013; Willburger et al., 2008), these studies did not examine whether individual differences in children's phonological processing skills were related to arithmetic. Our results suggest that phonemic awareness (but not other phonological processing skills) is important for retrieval-based arithmetic in children with a range of learning difficulties.

5.2 | Relationships between phonological awareness measures, and between phonological awareness and reading

In typically developing children, all three tests of phonological processing have been found to correlate with each other, and each skill makes an independent contribution to predicting reading outcome (Wagner & Torgesen, 1987). A surprising result was that the correlations amongst the phonological processing measures in our LD sample were not significant (after correcting for multiple comparisons), suggesting that the strong associations reported for typical children do not extend to children with LD. This may be due to children with LDs having more heterogeneity in their performance profiles, with some children exhibiting weaknesses across several phonological processing measures, but others having a more mixed profile across phonological processing measures.

Consistent with a large body of prior research showing relationships between phonological processing skills and reading (Ehri et al., 2001; Norton & Wolf, 2012; Peng et al., 2018) we found phonemic awareness and rapid serial naming were correlated with, and predictive of, reading ability in this sample of children with LDs. However, phonological memory did not emerge as a significant predictor of reading. Interestingly, we found no relationship between phonological memory and fact retrieval or procedural computation either, demonstrating that phonological working memory predicted neither reading nor arithmetic in our study of children with LD.

Notably, our results showed that phonological processing measures accounted for 41% of the variance in children's reading ability (with phonemic awareness and rapid serial naming making significant unique contributions), while a similar model accounted for 18% of the variance in children's fact retrieval skills (with only phoneme elision making a significant unique contribution). While phonological processing skills are more strongly related to reading than fact retrieval, phonologically-based interventions that are known to improve children's reading skills (Ehri et al., 2001) might have some effects on children's retrieval-based arithmetic skills, even if these gains in arithmetic are not likely to be on the same scale as those for reading. As noted above, the effect of phonological interventions on retrieval-based arithmetic will need to be tested in future research.

The relationship between phonemic awareness and both fact retrieval and reading also has implications for the common co-occurrence of dyslexia and dyscalculia. Specifically, it suggests that comorbidity might be explained by a shared role of phonemic awareness in reading skills and arithmetic skills that rely on fact retrieval. Ashkenazi et al. (2013) describe a verbally-mediated model as one possible account for the comorbidity, where children with poor phonemic awareness skills could subsequently develop difficulties in both reading and math skills. However, weak phonemic awareness as the underlying cause of comorbidity between these LDs is not supported by a large behavioral study, which revealed that poor performance in reading and math is associated with shared deficits in working memory, processing speed, and verbal comprehension (Willcutt et al., 2013). As noted in the Introduction, the specific choice of math measure may play a significant role in these findings. Willcutt and colleagues used the math subtests on the Peabody Individual Achievement Test and the Wide Range Achievement Test to determine math performance, which suggests that retrieval-based arithmetic was not captured. Similar studies with measures that separate fact retrieval from procedural computation in arithmetic will need to be conducted in the future to shed light on whether phonemic awareness explains the co-occurrence of poor reading and poor fact retrieval-based arithmetic.

5.3 | Relationship between reading and mathematics in children with learning disabilities

Though reading and mathematics are seemingly distinct abilities, they are frequently found to be positively correlated with one another (Bull et al., 2008; Duncan et al., 2007; Hecht et al., 2001; Koponen et al., 2007). Longitudinal associations have also been found between the two, where early reading skills predict later math skills (Erbeli et al., 2021; Fuchs et al., 2005; Koponen et al., 2007; Purpura et al., 2011; Rinne et al., 2020; Simmons et al., 2008; Vukovic & Lesaux, 2013) and early math skills predict later reading skills (Duncan et al., 2007; Purpura et al., 2017). There is evidence to suggest that reading and mathematics share some underlying processes, above and beyond intelligence, that could explain why these skills are correlated concurrently and longitudinally. For instance, broad language skills not only play a role in solving mathematical word or story problems (Fuchs et al., 2006; Wang et al., 2016), but also in arithmetic problems presented with Arabic digits (e.g., numbers such as 1 or 4) (LeFevre et al., 2010; Vukovic & Lesaux, 2013). It has been proposed that these relationships emerge because children translate arithmetic problems from Arabic digits into a verbal or speech-based code in order to solve the arithmetic problems (Dehaene, 1992). The relationships between reading and mathematics may also be related to shared associations with phonological processing. However, we found no associations between standard measures of reading and math ability, or experimental measures of math. These results were unexpected given the prior literature in typically developing children (Hecht et al., 2001; Singer & Strasser, 2017), including studies that have used similar measures of reading and math ability (e.g., Fuchs et al., 2006; Hecht et al., 2001). These findings again suggest that relationships observed amongst skills in typically developing populations may not extend to children with LDs. Consistent with this, Donker et al. (2016) examined the relationship between math and reading skills in children with reading and/or math disability (together with some typically developing children) and observed that math skills (simple timed arithmetic and math word problems) were not correlated with reading skills (timed single-word reading and timed single non-word reading). Another study examining the longitudinal relationships between reading and math skills in academically at-risk children illustrated that low reading performance dampened math growth, while average and high reading performance was associated with longitudinal gains in math (Erbeli et al., 2021). These findings indicate that relationships between reading and math may depend on the level of ability, and that reading and math may have strong relationships in typically developing children but not children with learning disabilities.

6 | CONCLUSIONS

This study is the first to examine the relationship between phonological processing and math skills in children with learning disabilities to understand whether phonological processing skills are differentially related to retrieval- and calculation-based arithmetic. We found that phonemic awareness, but not phonological memory or rapid serial naming, was related to arithmetic fact retrieval skills. However, no phonological processing skills were related to procedural computation skills. These results demonstrate that phonemic awareness is a predictor of arithmetic fact retrieval, a relationship that is likely obscured in studies that utilize standard measures of math which capture a range of different skills. Our results raise the possibility that phonemic awareness may be one contributing factor to the co-occurrence of dyslexia and dyscalculia, and that promoting phonemic awareness in children with LDs may not only be beneficial for reading, but also to aspects of arithmetic that rely on retrieval.

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CONFLICT OF INTEREST

The authors declare that they have no competing financial interests or commercial considerations and that this material has not been published (nor is it under consideration for publication) elsewhere.

ETHICS APPROVAL STATEMENT

All experimental procedures were approved by the Institutional Review Board at Georgetown University. Informed consent was obtained from the legal guardian for all children, and the children provided verbal assent.

DATA AVAILABILITY STATEMENT

Data used in this study are available from the corresponding author upon reasonable request.

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