RAND Journal of Economics
Vol. 00, No. 0, August 2024
pp. 1–21

Influencing search

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We show that in search markets an influencer who recommends a product to her followers improves consumer surplus and total welfare despite the firm paying for her recommendation. As consumers learn their value for the product upon search, they will not buy at the recommended firm if they learn their value is low. The threat of search incentivizes firms to offer the influencer a financial contract involving a commission and incentivizes the influencer to be honest in her recommendation. Provided the influencer's search cost is not too high, she also has an incentive to acquire information and give informative recommendations. These informative equilibria are more difficult to sustain if influencers compete with each other.

1. Introduction

■ Influencers on social media, such as Instagram, Twitter, and Facebook, provide recommendations to their followers suggesting which products to buy. Influencers typically focus on one product market, such as cosmetics and personal care products, travel, fashion and lifestyle, or computer games. Although it is difficult to get objective data on the size of the industry, it is clear that influencer marketing is booming. In a 2018 article,[1] the New York Times estimated the industry to reach a turnover of 10 billion USD in 2020,[2] with the most successful influencers individually earning up to 1 million USD per post.[3] The market for influencers is so large that there are even intermediary firms specializing in advising firms which influencers to get involved with.[4]

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We thank Gary Biglaiser, two anonymous reviewers, Heski Bar-Isaac, Ben Casner, Daniel Garcia, Paul Heidhues, José Luis Moraga-González, Daniel Savelle, Anton Sobolev, Tat-How Teh, Jidong Zhou, and seminar participants at the University of Vienna (2020), the Consumer Search Digital Seminar Series (Virtual, 2020), the IIOC (Virtual, 2021), EARIE (Virtual, 2021) Econometric Society European Meetings (Virtual, 2021), and SAET (Canberra, 2022). The authors acknowledge financial support from the Austrian Science Fund FWF under project number I 3487.

[1] See, https://www.nytimes.com/2018/07/15/technology/online-stars-brands.html

[2] The estimate seems to be based on https://mediakix.com/blog/influencer-marketing-industry-ad-spend-chart/#gs.HbV2Xinowhere a range between 5 and 10 billion USD is given.

[3] See, for example, https://www.webfx.com/influencer-marketing-pricing.html

[4] See, for example, https://influencermarketinghub.com/instagram-influencer-marketing-agencies/

This development raises important policy questions, especially related to the possibility of influencers not providing informative recommendations. Countries like Germany, the United Kingdom, and the United States have initiated disclosure rules stating that social media posts must clearly mention if a post was paid for.[5] To evaluate the necessity of these policy rules, it is necessary to better understand why consumers often follow recommendations even if they know that the influencer gets (handsomely) paid for the posting. Are these recommendations informative or are influencers just recommending what the highest paying firm wants them to post? To address the policy questions, it is also important to understand whether influencers have an incentive to make an effort to be informed. In short, we are interested in how influencers affect market outcomes and whether regulators should worry about the impact of influencers on market outcomes and the possibly false information they provide?

This article addresses these questions by focusing on how macro influencers affect consumer search. Macro influencers, that is, individuals with hundreds of thousands of followers, typically are people who are thought of as being able to know what will be trending or what people typically care about when purchasing a product. People take notice of the opinion of influencers as their preferences are correlated with what an influencer likes. That is why an individual follows an influencer; whether the influencer consumes the product herself is irrelevant. It may well be that when inspecting the good a consumer disagrees with the recommendation of an influencer and do not buy as individuals have their own preferences and act according to these. A post by an influencer may, however, affect the order in which people search. We will show that this subtle effect on the search order has important implications for how firms price their product and where consumers buy.

The type of markets we have in mind include travel, beauty, and technology. In each of these markets, there are top influencers with millions of followers.[6] Most of these influencers focus on a subsegment such as luxury accommodations, adventurous destinations, or family destinations for travel and firms are contacting (and pay) influencers to be recommended. If successful and being recommended, this is important news for a firm and it is natural to assume that they will reconsider their pricing strategy if they are recommended.

We build on the seminal work by Wolinsky (1986), and model a market of monopolistic competition where consumers engage in sequential search to discover the products firms offer and the prices they charge. Firms do not know how their product matches the preferences of individual consumers. In our baseline model, the only change we make to that framework is that an influencer is sampling some of the firms' products and recommends one of them. Firms and followers observe the influencer's post. Depending on whether or not they are recommended, firms decide on their pricing; consumers decide where to start searching and how to continue the search process if they are not satisfied with the outcome of their first search.

To understand a key part of our mechanism we first focus on a baseline model where influencers sample a few firms' products and honestly recommend the one they like best. Our first result shows that consumers follow the recommendation and start searching at the recommended firm, despite this firm setting higher prices than its competitors (and consumers expecting them to do so). If upon visiting the firm, the consumers find out they do not have a high match value, they continue to search among the non-recommended firms. Being recommended is good news for a firm as consumers rationally believe the chances this firm produces a product they like are higher, boosting the firm's demand. In response, the firm sets higher prices, but overall welfare

[5] Ershov and Mitchell (forthcoming) study the effects of these regulations on the number of paid posts and their content.

[6] Top influencers in travel are mentioned on, for example, https://www.amraandelma.com/100-top-travel-influencers/, for beauty products see https://smallbiztrends.com/2021/09/top-beauty-influencers.html, and for technology products, see https://www.agilitypr.com/resources/top-influencers/top-10-u-s-social-media-influencers-in-technology/. These macro influencers give recommendations in their specific area and should be distinguished from celebrity influencers who often do not seem to focus on a specific area.

and consumer surplus is higher as consumers more easily find the firm that in expectation is delivering higher value. Thus, influencers help to reduce the total search cost.

We next suppose that (i) firms offer financial contracts and compete for the influencer recommending them and (ii) an influencer has to decide whether or not to acquire information about firms and give consumers a more or less informative recommendation. Thus, in the extended model an influencer chooses a strategy that maximizes the money they make from giving the recommendation minus the monetary value of their search cost. In particular, the influencer follows a threshold strategy recommending the first firm they encounter that has a match value larger than a certain threshold value as such expect to generate so much revenue from such a recommendation that they do not have an incentive to acquire further information about other firms they have not yet visited. We show that provided that the influencers' search cost is not too high, influencers have an incentive to acquire information and give informative recommendations in the sense that they recommend the first firm they inspect that has a match value above a threshold. Consumer surplus and overall welfare is higher even if the recommended firm charges higher prices. Thus, even if there is a potential mismatch in the incentives of influencers and consumers, the influencer effectively recommends the product that is good enough for them (and that is the best indicator for them whether the consumers likes the product). There are two parts to the underlying mechanism. First, firms have an incentive to offer financial contracts that include a positive commission for every consumer that buys: a commission leverages the influencer's information as she only accepts the contract if she expects followers to actually purchase the product with high enough probability.[7] Second, with an informative recommendation it is more likely that a follower will stop searching after having visited the recommended firm at the first visit. As a more informative recommendation generates higher expected sales, influencers are willing to make an effort to get informed.

Importantly, the incentives of influencers and consumers may not be aligned as far as information acquisition is concerned. There are two reasons for this. First, for given prices, consumers would always want the influencer to expend more effort in acquiring information because consumers do not internalize the influencer's search cost. Second, a more informative recommendation leads the recommended firm to charge a higher price and if this effect is very strong (in case the influencer's opinion of a product is strongly correlated with consumers' match values) consumers may actually prefer less informative recommendations. Hence, influencers may acquire too much or too little information from the vantage point of consumers. This potential misalignment is, however, not resolved by the policies on disclosure rules that are implemented around the world requiring that recommendations should clearly mention if a post was paid for.

We also briefly consider competition between influencers in the sense that consumers follow different influencers. Even though a comparison to a monopoly influencer is complicated due to the fact that the threshold values competing influencers choose are different, two lessons seem to appear: (i) as long as informative equilibria exists, consumers benefit from following multiple influencers and (ii) the threshold value at which the influencer stops searching is smaller when multiple influencers compete, so that there are intermediate values of the influencer's search cost for which an informative equilibrium exists when there is one influencer but fails to exist under competition.

Even if our article and its results are cast in terms of social media influencers, the results are equally relevant to advisors that give advice at a more "individual" level, that is, if an advisor gives different advice to different individuals. What is important for our results to apply is that the advisor does not know the precise ranking of different products of an individual and that the individual can inspect the product herself to learn more whether the recommended product suits her. Although these conditions do not apply to pure credence goods, we believe that advice in some financial and health markets fits the two mentioned features. An advisor may know

[7] The blog https://later.com/blog/affiliate-marketing-for-influencers/shows how commissions work for affiliated marketing by influencers and suggests this is a common way influencers are paid.

certain characteristics (age, gender, income, and family situation) of an individual she advises and base the advice on these observed characteristics, but there typically remains enough room for individual preferences to differ from those of the average individual with the same characteristics. Moreover, through information that is available via other sources (e.g., online information about the effects of medicines or financial products) individuals may find out more information about whether the product that is recommended suits her.

The article is related to several strands of literature. A recent article by Teh and Wright (2022) studies similar issues as ours, but arrives at very different conclusions. They assume that the advisor knows exactly the match values of a consumer with each and every firm. The only thing the advisor does not know is an individual shift parameter, which impacts whether or not a consumer wants to buy any product. As, like in our article, the advisor has an incentive to recommend honestly, this assumption of knowing exact and personal match values of all firms implies that in any symmetric equilibrium consumers never search beyond the first firm on their recommendation list, giving the firm that is recommended monopoly power and making consumers worse off relative to a market without influencers. In contrast, in our model influencers do not know the match value of her followers with every firm. Instead, they sample only a finite number of firms. This implies that some consumers search on after visiting the recommended firm, reducing the market power of recommended firms. In addition, the many firms that are not recommended charge their "normal" prices imposing a competitive constraint on the recommended firms.

The article is also related to many recent articles on consumer search, especially in three different directions: (i) search order and pricing, (ii) quality and service provision, and (iii) observational learning in search markets. The typical result in the search literature on how search order affects prices is that the firm that is searched first charges lower prices than their competitors (see, e.g. Armstrong, Vickers, and Zhou, 2009; Armstrong, 2017; Haan and Moraga-González, 2011.[8] This is in contrast to one of the main results in our article, where the recommended firm is searched first, but sets higher prices than competitors. The main reason for these different results is that in the previous literature, the search order is unrelated to a firm's expected quality. In our framework, the reason for searching the recommended firm first is that it is expected to have higher quality and for the average consumer these expectations are realized. Knowing this, the recommended firm has an incentive to charge higher prices. This observation also links the article to the search literature on quality and service provision (see, e.g. Shin, 2007; Janssen and Ke, 2020 and (Moraga-González and Sun, ming)). An important result in that literature is that despite the cost of doing so firms may provide service to consumers, even though other firms may free ride on this service provision. As in our article, despite the higher prices at a service providing firm, the expectation of service provision may affect the search order. In these articles, higher prices are, however, completely driven by the cost difference of service provision and the mechanism we focus on in the current article where recommendations affect consumers perceived valuations is absent. Garcia and Shelegia (2018) study how observational learning affects search and pricing in markets where consumers' valuations are correlated and consumers observe the purchase of a single predecessor. As in our article, search is affected by observations consumers make (or, in our setting by recommendations they get) about other individuals that have somewhat similar preferences. Our article studies, however, the effect of how one individual's (the influencer) action affects search and pricing. In such an environment, observational learning and the implications it has on pricing are not relevant.

It is well known that sequential search with informative intermediation leads to interdependence in the search process. In particular, if the intermediary has information about all firms and

[8] Important exceptions are Arbatskaya (2007) and Casner (2020) who for different reasons (either search cost heterogeneity or vertical differentiation) observe that a firm that is searched first may charge higher prices. Note that Section 3 in (Armstrong, Vickers, and Zhou, 2009) also discusses quality difference as a reason why the first firm that is searched may have higher prices.

recommends a certain firm to consumers, then observing the match value at the recommended firm implies information about the match values at other firms. This learning renders the classic Weitzman (1979) rule unapplicable and optimal search may not be characterized by a reservation price. Our solution to this problem is to have the influencer only have information on a finite number of products. As consumers do not know which other firms the influencer has sampled, they cannot make an inference about the match value of non-recommended firms if there are infinitely many to be explored. Other solutions to avoid complications due to learning either assume that the intermediary has perfect knowledge on product rankings (Teh and Wright, 2022), that the products are homogeneous (Armstrong and Zhou, 2011) or that intermediation truncates the pool of firms (De Corniere, 2016).

The role of influencers is also studied from a network perspective (see, e.g., Fainmesser and Galeotti, 2016; Galeotti, 2010; Chen, Zenou, and Zhou,, 2018; Fainmesser and Galeotti, 2020) modeling influencers as consumers that have many more network connections than others (see, e.g., Campbell, 2013). Connections matter as consumption is characterized by network effects. Knowing the network structure, firms can increase demand by targeting these influencers and offering them better deals. This literature takes it that everyone is an influencer, some more so than others. This may well capture some aspects of what are known as micro- or nano-influencers, that is, individuals that have a few thousands of followers or even less. Many of the more macro influencers, that is, individuals with millions of followers, seem to play a different role, however, as there is a clear asymmetry between them influencing others (and knowing that) and they themselves not being influenced by the decisions that their followers take.

On the role of social influencers, there are also two recent articles that focus on different aspects. Mitchell (2021) studies the dynamic interaction between an influencer and her follower from a principal-agent perspective. The main issue the article focuses on is the trade-off between giving good advice and revenue. Fainmesser and Galeotti (2021) focus on a similar trade-off but in a market setting with many micro influencers.

There is also a literature on credence goods and the way commissions and kickbacks that are paid by firms may shape the advice that is given by intermediaries; see, for example, Inderst and Ottaviani (2012b) and Inderst and Ottaviani (2012a). Our contribution to that literature is that in search markets where consumers can inspect the product themselves by paying a search cost, there is no room for bias and the advisor gives honest and informed advice even if she is only interested in monetary payoffs.

Finally, the article also relates to the literatures on delegated search (see, e.g., Lewis (2012) and Ulbricht (2016)), referral marketing (see, e.g., Schmitt, Skiera, and Van den Bulte, 2011; Mayzlin, Dover, and Chevalier, 2014; Pei and Mayzlin, 2019) and on information intermediaries (see, e.g., Biglaiser (1993) and Lizzeri (1999)). The delegated search literature typically considers a labour market context where a head hunter may be employed by a firm to assist hiring a future employee/manager and focuses on the principal-agent issues that are involved in such hiring processes. Referral marketing studies how a firm can significantly increase referrals from word-of-mouth communication. That literature is mostly empirical, however, and does not address the mechanism by which recommendations by influencers affect search and the associated welfare effects on markets. Finally, the literature on information intermediaries mostly focuses on vertically differentiated products.

The rest of the article is organized as follows. Sections 2 and 3 deal with the baseline model and the main results related to pricing and welfare. Section 4 extends the model to address the issue of paid recommendations and the influencer's incentive to learn her match value for different firms, whereas Section 5 focuses on competition between influencers. Section 6 concludes with a discussion how our analysis may be relevant to advice at a more individual level. All withheld proofs can be found in Appendix A.

2. Baseline model

■ We start this section by introducing the formal model and follow up with a discussion of the main ingredients and their interpretation. The market is comprised of a unit mass of firms,[9] a mass of L consumers,[10] and an influencer. We denote by $v_i \in [\underline{v}, \bar{v}]$ a representative consumer's value for firm i and by $\hat{v}_i \in [\underline{v}, \bar{v}]$ the influencer's value for firm i. Match values are i.i.d. across products. These share a log-concave joint density and are strictly affiliated: $g(v_i, \hat{v}_i)g(v_i', \hat{v}_i') > g(v_i', \hat{v}_i)g(v_i, \hat{v}_i')$ for all $v_i < v_i'$ and $\hat{v}_i < \hat{v}_i'$.[11] The marginal distribution is denoted $G(v_i)$ with density $g(v_i)$. Throughout the article, we illustrate our results using the joint density function $g(v_i, \hat{v}_i) = \alpha(2v_i - 1)(2\hat{v}_i - 1) + 1$, where the parameter α, lying in the unit interval, measures the degree of affiliation. This family of joint density functions has the nice property that for any $\alpha \in [0, 1]$ the marginal distributions of v_i and $\hat{v}_i$ are uniformly distributed on the interval [0,1]; for $\alpha = 0$, the consumers' and the influencer's values are independent of each other and the degree of affiliation is increasing in α.

The influencer examines $k \geq 2$ of the products in the market and recommends one of them. In the baseline model, we assume that the influencer honestly recommends the firm in the sample that yields the highest match value. We introduce a contracting stage in Section 4 where we show why the influencer would do so even if he recommends the firm that pays him most. In Section 4 we endogenize the influencer's decision regarding how many products to examine.

Firms learn whether or not they will be recommended and depending on the influencer's decision they set their prices p_R and p_i, for the recommended and non-recommended firms, respectively, to maximize expected profits. For notational simplicity, we normalize firms' cost to be equal to 0. The influencer then issues the recommendation to consumers, indicating the firm they recommend along with the price charged by this firm.

Consumers are initially uninformed of their match values with firms and can only learn them through costly sequential search. Each search comes at a search cost $s > 0$. To allow the possibility of an active market, we assume throughout that the search cost s is smaller than $\int_{\underline{v}}^{\bar{v}}(1 - G(v))dv$. Consumers have perfect recall when searching. The timing of the interaction in the baseline model is as follows. First, Nature determines the values of all agents. The influencer randomly observes her values for k firms and chooses which of these k firms to recommend. Consumers are unaware of their values until the moment they visit a firm. Second, firms observe the influencer's recommendation and set their prices. Third, consumers observe the influencer's recommendation and the related price and commence their search.

Throughout the article we focus on symmetric Perfect Bayesian Equilibria where firms choose their strategies to maximize expected profits given their information and consumers choose an optimal sequential search strategy. The Prékopa–Leindler inequality ensures the existence of equilibrium (see Lemma A1).

We will now discuss some of the features of the baseline model. First, most markets have multiple macro influencers. This is not an issue for our model, however. All results continue to hold if there is a finite number of these influencers with heterogeneous tastes where each of them is followed by a different segment of consumers who share similar preferences. In particular, consumers continue to benefit from the presence of influencers as they help them to better matched products despite raising market prices.

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[9] The continuum of firms is a common assumption in the search literature and is used not to have to worry about returning consumers (see, e.g., Anderson and Renault, 1999). Our result that the recommended firm charges a higher price may not hold if the number of firms is relatively small (see, e.g., Armstrong, Vickers, and Zhou, 2009).

[10] In the formal analysis, L is just a scaling factor and in the proofs it is normalized to 1. In the numerical analysis presented in the graphs in Sections 4 and 5, L plays a role as explained in footnote 17.

[11] The proofs use the fact that for each match value of the influencer there is a full support posterior over the consumer's match value, which is guaranteed by our assumption. Our results continue to hold for perfect correlation, but that case requires different proofs.

Second, one way to conceive of the correlation between consumer and influencer match values is that a consumer's value for firm i is composed of a common factor ω_i that is shared among all consumers (that follow the influencer) and an idiosyncratic factor η_{ij} that differs between consumers so that the value of consumer j for firm i is $v_{ij} = \omega_i + \eta_{ij}$. The influencer's valuation can be conceived in a similar way so that the correlation between the influencer's and consumers' valuations arises in a natural way. We will use this formulation in the concluding section when discussing the role of individual advice.

Third, we assume that consumers have the same search cost whether or not they follow the recommendation. One may argue, however, that social media influencers typically make it easy for followers to follow their recommendation, for example, by inserting a link to the firm's website in their post. As we show that consumers follow the recommendation even if the search cost of doing so is not smaller than the cost of searching other firms, our results continue to hold if the cost of following the recommendation is smaller.

Fourth, we recognize that recommendations may come in different forms and that our model pertains to influencers recommending a particular product, whereas in other cases influencers place more general ads recommending a lifestyle or a general destination for travel. Most recommendations on a platform like Instagram seem to be, however, of the form we model here.[12]

Fifth, in many instances influencers may not mention the price of the product along with her recommendation. The main technical advantage of influencers in our model "advertising" the recommended firm's price is that it commits the firm to charging that price. Our way of modeling has the advantage that it gets rid of uninteresting equilibria where consumers do not visit a recommended firm as they expect it charges a very high price and the recommended firm charges such a price as anyway no consumer is going to visit. Without pre-commitment of prices, our analysis continues to hold, however, and in particular it captures all equilibria where consumers first search the recommended firm. An alternative way to achieve this would be to assume that following a recommendation comes at no cost (or a cost close to 0) so that following the recommendation is a "no regret" option.

Sixth, in our model firms set prices after knowing whether or not they are recommended. This seems to be natural in situations where firms can easily adapt their prices to new information. Changing the timing of the model so that firms have already chosen prices before a firm is recommended and allow all firms to adapt their prices given the information who is recommended, does not change the results as the non-recommended firms would not want to change their price choices.

Seventh, the fact that we assume k to be known to firms and consumers is of no importance to the results. The only thing that matters is that firms and consumers believe that $k \geq 2$ so that the recommendation is somewhat informative. In Section 4 we explicitly analyze the incentives of the influencer to acquire information.

Finally, to compare the role an influencer plays in our article to the role of an expert advisor in a credence goods market, in the spirit of Inderst and Ottaviani (2012b, 2012a), one could extending our analysis and imagine that a consumer j's match value with firm i is composed of three independent components: $v_{ij} = \omega_i + \gamma_i + \epsilon_{ij}$, where ω_i represents a common component that can be identified by the consumer upon inspection, γ_i represents the "credence good" component that the consumer cannot observe due to a lack of expertise and ϵ_{ij} captures the purely private component, only observable to the consumer. Whether or not our results in this article apply to this extended setting depends on whether the expert can clearly identify what the consumer will and will not observe when inspecting the good. If the expert can clearly separate ω_i from γ_i, then as one might expect from Inderst and Ottaviani (2012b, 2012a), experts without an intrinsic concern for getting the recommendation right do not take the credence good component into account

[12] See, for example, https://www.instagram.com/p/BYYr9JMgNMt/for cameras, https://www.instagram.com/p/CFxQz4sFSo-/for gaming controllers, https://www.instagram.com/p/CJ4JJQylI_W/for hair products, https://www.instagram.com/p/B7mr6DAp4XU/for tire cleaners, or https://www.instagram.com/p/CLE7HNGBST5/for protein powder.

when making a recommendation. She may then provide wrong advice and a policy that forces experts to reveal their commissions may help to correct for this bias. On the other hand, if experts cannot clearly identify what the consumers may be able to find out when inspecting the good, for example, they only observe $\omega_i + \gamma_i$, and not the individual components, then our results apply and experts recommend honestly which product is best for the consumer even if they only care about their own revenue from the recommendation.

3. Honest recommendations, search, and pricing

■ We now show that consumers will follow the recommendation despite the fact that the recommended firm charges a higher price than the other firms. Moreover, consumers are on average better off and total welfare is also increasing because of the presence of the influencer. To do so, we first characterize the optimal search strategy of consumers.

The optimal search strategy for consumers is to follow Pandora's rule (Weitzman, 1979). Firm i's *reservation price* r_i is the highest price at which a consumer is willing to first inspect the firm rather than take an outside option of zero outright. Pandora's rule dictates that at each decision node, a consumer takes the best option among previously inspected firms if that has a higher net value than the net value $r_i - p_i$ of all uninspected firms; otherwise he should continue searching the firm offering the highest uninspected net value. Standard considerations imply that the reservation prices for the recommended and non-recommended firms, denoted by r_R and r, are implicitly defined by

$$\int_{r_R}^{\bar{v}} (1 - G(v_i|R))\mathrm{d}v_i = s = \int_{r}^{\bar{v}} (1 - G(v_i))\mathrm{d}v_i,$$

where $G(v_i|R)$ denotes a consumer's posterior over his match value with the recommended firm, with posterior density

$$g(v_i|R) = \frac{\Pr(R|v_i)g(v_i)}{\Pr(R)}. \tag{1}$$

For intuition, we can explicitly write out the posterior density for the running example where match values are uniformly distributed, $g(v_i|R) = k\int_0^1 g(v_i, \hat{v}_i)G(\hat{v}_i)^{k-1}\mathrm{d}\hat{v}_i = \alpha(2v_i - 1)(2\frac{k}{k+1} - 1) + 1$.

Thus, the search order depends in part on the relationship between r_R and r. As there are a large number of firms, the match values across firms are independent following the recommendation. Letting K denote the set of sampled firms and $\hat{v}_{-i} \equiv \max\{\hat{v}_j\}_{j\in K\setminus\{i\}}$, the chance that $i \in K$ is recommended when v_i is the consumer's match value is $\Pr(R|v_i) = E[1 - G(\hat{v}_{-i}|v_i)]$ where the expectation is taken over $\hat{v}_{-i}$. Because match values are strictly affiliated, $1 - G(\hat{v}_{-i}|v_i)$ is strictly increasing in v_i (Milgrom, 1981). Consequently, the ratio $g(v_i|R)/g(v_i)$ is also strictly increasing, implying the posterior $G(v_i|R)$ has likelihood ratio dominance—and thus, hazard rate and first-order stochastic dominance (Shaked and Shanthikumar, 2007)—over the prior $G(v_i)$. So, we have the following Lemma:

Lemma 1. For any search cost s the recommended firm has a larger reservation price than the other firms, *that is*, $r_R > r$.

Because the influencer reveals the recommended firm's price p_R, charging too high a price will dissuade consumers from following the recommendation. Denoting the price consumers conjecture non-recommended firms will charge by p, consumers first search the recommended firm if $p_R \leq r_R - r + p$, where the RHS is larger than p.[13] As the recommended firm faces expected

[13] For notational clarity, this section assumes $s \leq s_W$ where s_W is the unique search cost below which the market without an influencer delivers positive consumer surplus. The results continue to hold for all values of s between s_W and the upper bound imposed in Section 2.

profit $p_R(1 - G(r - p + p_R|R)) > 0$ when $p_R \leq r_R - r + p$ and zero profit otherwise, it is clear that $p_R \leq r_R - r + p$ and that consumers first search the recommended firm, even though p_R may be larger than p.[14]

Concentrating now on the optimal recommended price, it easily follows that there are two candidates: either the recommended firm charges the interior optimal price $p'_R < r_R - r + p$, which standard considerations (Wolinsky, 1986; Anderson and Renault, 1999), reveal to be equal to

$$p'_R = \frac{1 - G(r - p + p'_R|R)}{g(r - p + p'_R|R)}, \tag{2}$$

or he charges the upper bound $p''_R = r_R - r + p$ that still draws customers. Thus, the recommended firm charges $p_R \equiv \min\{p'_R, p''_R\}$.

After visiting a non-recommended firm a consumer never strictly prefers to subsequently visit the recommended firm nor to make a purchase at a previously inspected firm. Thus, upon being visited, offering value v_i a firm i who charges p_i makes a sale if and only if $v_i - p_i \geq r - p$. From standard calculations (Wolinsky, 1986; Anderson and Renault, 1999), the equilibrium price for non-recommended firms equals

$$p = \frac{1 - G(r)}{g(r)}. \tag{3}$$

It is not difficult to see that in equilibrium, the recommended price is strictly larger than the non-recommended price. If the boundary solution is relevant, it immediately follows from the lemma that $p''_R = r_R - r + p > p$. But this also holds if the recommended price is equal to p'_R. The main reason is that the recommended firm faces a higher demand and because demand is "behaving normally" it reacts by setting higher prices. More technically, a consequence of the hazard rate dominance is that

$$\frac{1 - G(r|R)}{g(r|R)} > \frac{1 - G(r)}{g(r)},$$

which is exactly saying that the marginal profit of the recommended firm in the interior solution evaluated at $\tilde{p}_R = p$ is positive. Thus, we have proved the following proposition.

Proposition 1. In equilibrium, consumers commence their search at the recommended firm, although this firm charges a higher price than the firms that are not recommended, *that is*, $p_R > p$. In addition, the presence of the influencer increases total welfare and industry profits.

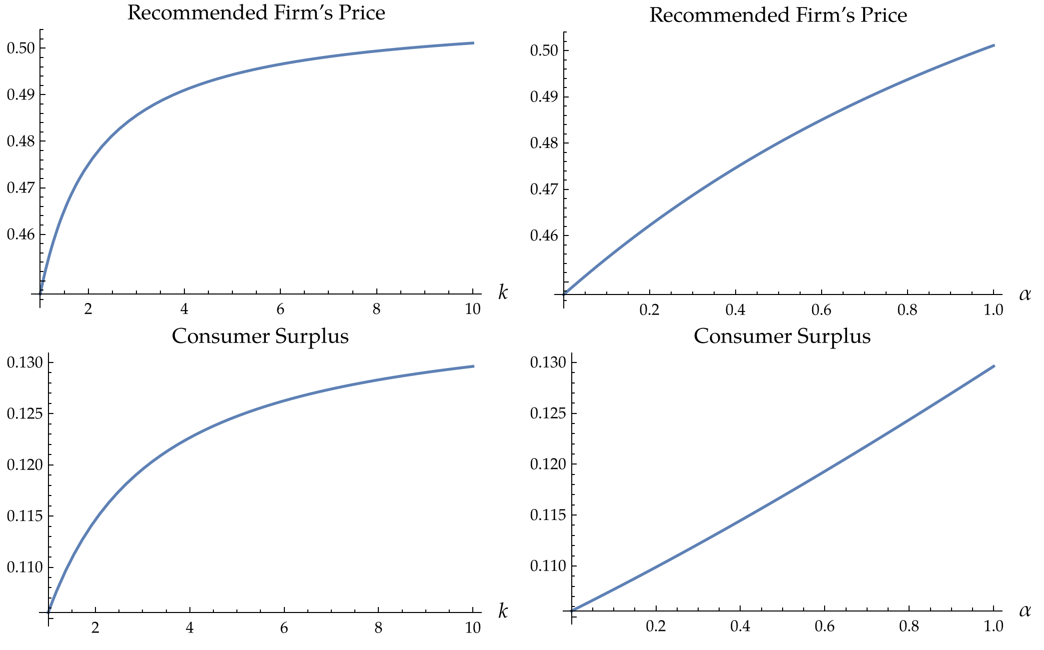
Total welfare increases as consumers are (at least) weakly better off and the influencer and industry profits are also higher as the non-recommended firms are equally well off, whereas the recommended firm is strictly better off. For consumers to *strictly* benefit from the presence of the influencer, the firm must charge the interior optimum with $r_R - p_R > r - p$, so that the price increase does not dominate the increased anticipated match value. Intuitively, for this to be the case the expected demand curve facing the recommended firm must not be too inelastic relative to the demand facing other firms.

Figure 1 depicts how the price p_R of the recommended firm and consumer surplus depend on the number k of firms the influencer samples and on the degree affiliation α. Both α and k can be seen as measures of how informative the recommendation is for consumers, either through the direct affiliation between values or because of the larger sample size. One can clearly see that the more informative the influencer's recommendation the higher the price the recommended firm will charge as he clearly wants to reap the benefits of the recommendation. If $\alpha = 0$, we are in

[14] If the influencer does not announce price and the recommended firm is not committed, then alternative equilibria may exist where the influencer is ineffective as consumers do not follow the recommendation. Our assumption that the recommended firm commits to its price only affects the analysis by avoiding this uninteresting case.

FIGURE 1

The different figures plot the price charged by the recommended firm and consumer surplus when varying the number k of firms sampled by the influencer (for $\alpha = 1$) and the degree of affiliation α between consumers and the influencer (for $k = 10$). In all figures $s = 0.1$.



the Wolinsky equilibrium where the recommended firm and the non-recommended firms charge the same price. As the price that is charged by the non-recommended firms is independent of α, the price increase can also be re-interpreted as the price differential between the recommended and non-recommended firms. Despite the higher recommended price, consumers still want to follow the recommendation and are still better off because of the way their first search is directed to the firm that is more likely to deliver a good match. One can also see that the effects are quantitatively substantial: comparing relatively uninformative outcomes with informative equilibrium outcomes shows that the recommended price can be in the order of 10% higher, although consumer surplus may increase even by 25%. The reason is that even if the price to be paid is higher, consumers are likely to get a much better match value on their first search.

4. Incentives and informative recommendations

■ The welfare gains in the previous section rely on the influencer providing an honest and informative recommendation. In reality, influencers often receive financial compensation from the firm they recommend. Especially in relation to macro influencers with millions of followers, firms often compete with one another to get an explicit endorsement of an influencer. In addition, we assumed influencers always inspects a fixed number of firms before making a recommendation, but they actually may not make an effort to give an *informative* recommendation (even if they are paid).

In this section, we inquire into the incentives of the influencer to acquire information and to provide not only an honest, but also an informative recommendation. To do so, we think of the influencer as an agent who also has a search cost, which we denote by c to distinguish it from the consumer search cost s, and sequentially inspects the products different firms offer. Firms offer a contract to the influencer and the form of the contract may determine whether the influencer inspects a firm in more detail. Although influencers may also care for their reputation or have an intrinsic motivation for being honest, we abstract from these considerations in order to understand the role of firms paying for recommendations and consider that influencers are only purely financially motivated. Consumers and firms only observe the recommendation the influencer provides, but not how many products she has inspected, *that is*, the actual search process is privately observed by the influencer only.

The timing of the interaction is as follows. After Nature determines the values of all agents, all firms simultaneously offer a contract to the influencer. A contract comprises a non-negative lump sum payment and a commission rate on sales revenue generated by the followers of the influencer.[15] Observing the contracts, the influencer optimally decides in a sequential fashion in which order to inspect firms, if at all. The influencer is able to predict what her expected revenue will be from recommending a firm. The influencer accepts the first contract that generates sufficient expected revenue such that it is not optimal to continue to inspect other firms. Firms then set prices, knowing whether or not they are recommended, the influencer issues the recommendation, and consumers commence their search. Consumers know influencers recommend what generates most revenue for them, but do not know the details of the contract between the influencer and the recommended firm.

It is clear that equilibria exist where the influencer does not acquire information. As the influencer cannot credibly convince firms and consumers that they do provide information, there always exist equilibria where firms and consumers believe that influencers do not make an effort and that influencers randomly choose a firm to recommend. Given these beliefs, the influencer does not have an incentive to acquire information. There are two types of uninformative equilibria. One uninformative equilibrium is the Wolinsky equilibrium where consumers randomly search a firm whether or not they are recommended and firms do not pay for a recommendation. Another uninformative equilibrium is where consumers first search the recommended firm (even if they know the recommendation is uninformative) and firms do pay for the recommendation (as they expect consumers still to follow it). In this latter equilibrium, the recommended firm charges a price equal to the Wolinsky price. This equilibrium outcome is similar in spirit to the one in Lizzeri (1999) in that the influencer gets rewarded for not providing any service.

The main question we address in this section is whether there exist equilibria where the influencer does make an effort and acquire information. Whether or not a recommendation is informative depends on the search strategy followed by the influencer. Typically, for a given value of c a search strategy is a cutoff strategy that stipulates continuing to search if, and only if, the highest match value that is observed up to a particular moment during the search process is smaller than a certain cutoff value, denoted by $v^*(c)$. We will say that the recommendation is informative if the influencer follows a search strategy that is such that $v^*(c) > \underline{v}$. Another question is how these informative equilibria, if they exist, compare in welfare terms to the non-informative equilibria.

The next proposition argues that informative equilibria exist if the influencer's search cost is not too large, whereas when they exist they are Pareto superior to uninformative equilibria.

Proposition 2. For influencer search costs c below some threshold value $c_0 > 0$ (that depends on other parameter values), there exists an informative equilibrium where the influencer recommends the first product that has a match value that is larger than $v^*(c)$, consumers follow the recommendation and sellers offer a positive commission rate. Relative to an uninformative equilibrium, consumers and influencers are better off in an informative equilibrium, whereas firms are indifferent.

The main reason why the influencer does not have an incentive to secretly deviate and not make a search effort is that firms offer contracts with positive commissions. Commissions imply

[15] https://later.com/blog/affiliate-marketing-for-influencers/as—unlike in other settings—it is not more difficult to enforce than a commission on revenues as the influencer and firm agree on the price that is charged as it is part of the influencer's announcement.

that an influencer's payment is increasing in the number of sales they expect to generate. As their own match value is the best indicator of expected sales, influencers have an incentive to recommend a firm with a high match value and if their search cost is not too high to make an effort to search for a firm whose match value exceeds the threshold value. As the cost of getting informed is smaller than the marginal increase in revenues, it individually is optimal for the influencer to acquire information even if consumers and firms do not observe the search effort. Competition between firms implies that they want to offer contracts with positive commissions. If competitors would offer contracts without positive commissions it is optimal to deviate as it is then possible to give the influencer a higher expected revenue only if their match value is high enough, which is an indication to the firm it can expect to generate more sales and more profits.

Thus, influencers honestly recommend the product they like best (of all the products they have inspected) and they believe their followers like best. In this way, the assumption of the previous section, namely that influencers recommend honestly, is shown to be the equilibrium outcome of a wider game where firms compete to get recommended by paying for a recommendation and the influencer has to make an effort to inspect firms.

As it continues to be true that the recommended firm will charge $p_R \leq r_R - r + p$, the welfare results of the previous section remain valid. Comparing markets with and without financial contracting, it is obvious that the influencer benefits, the recommended firm's profit decreases and the non-recommended firms are unaffected.

The next two results illustrate the importance of consumer search cost and the influencers' information about their followers' values for the above result. First, for a given value of c and imperfect signals, a necessary condition for Proposition 2 to hold is that the search cost is not too small. If the search cost is small, then consumers are likely to continue their search beyond the first (recommended) firm even if their match value at that firm is reasonably high and if that is so, the value to firms of being recommended decreases. If s approaches 0, this value is so small that firms are not willing to offer enough incentives for influencers to make an effort to get information. Thus, the threshold value c_0 depends on s and converges to 0 when s approaches 0. On the other hand, an informative equilibrium exists for larger values of s than an uninformative equilibrium with trade. As the first search is costly, an uninformative equilibrium where consumers search exists only if s is smaller than the (Wolinsky) threshold value s_W. However, at that and higher search cost levels consumers are still willing to search if they expect to get a better match value than just a random draw.

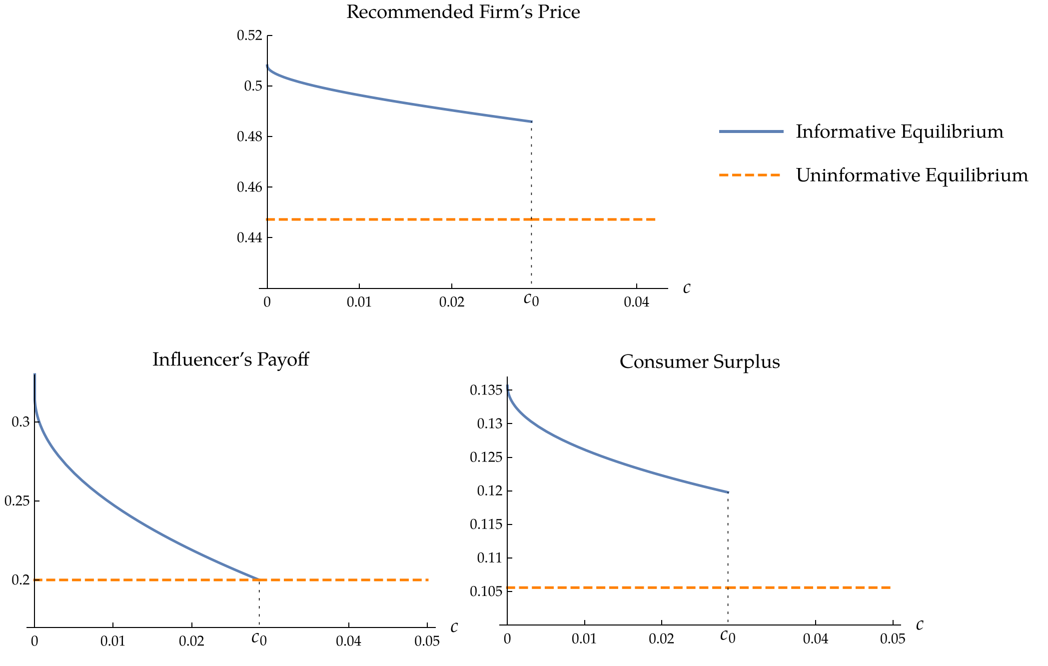
Proposition 3. For any $c > 0$, there exists an $s_0(c) > 0$ such that, if consumer's search cost is smaller than $s_0(c)$, then only uninformative equilibria exist. $\lim_{c \to 0} s_0(c) = 0$. An informative equilibrium exists when $s > s_W$ if c is sufficiently small.

Next, we discuss the effect of the informativeness of the influencer's signal. The easiest way to discuss this issue is to introduce a parameter $\lambda \in [0, 1]$ governing the extent to which match values are affiliated. Specifically, assume $G(v, \hat{v}, \lambda)$ to be (weak*) continuous in λ on [0,1), with match values being independent when $\lambda = 0$, strictly affiliated when $0 < \lambda < 1$, and perfectly correlated when $\lambda = 1$. Clearly, if influencers know very little of their followers' preferences in the sense that λ is close to 0, then there is little information contained in the recommendation and therefore a firm is not willing to pay much for being recommended. Thus, for a given c firms are not willing to sign contracts that give influencers an incentive to make an effort if λ is small enough. On the other hand, if influencers' preferences are (almost) perfectly correlated with the consumers' preferences and c is small enough, an informative equilibrium does exist.

Proposition 4. For every cost c, there is a neighborhood of $\lambda = 0$ within which only an uninformative equilibrium exists. Even if $\lambda = 1$ there is a positive threshold $c_0 > 0$.

FIGURE 2

The different figures plot the price charged by the recommended firm, the influencer's expected payoff, and consumer surplus as a function of the influencer's search cost c and the consumers' search cost set at 0.1 for both the informative and the uninformative equilibrium.



So far, we have established that the influencer may well have an incentive to acquire information and that consumers generally will benefit from this information. The incentives of influencers to acquire information will, however, generally not be aligned with consumers' interests, for two reasons. First, and more obvious, *ceteris paribus* consumers want an influencer to acquire a lot of information so that it can give a very informative recommendation. In this way it is most likely that they can stop searching after the first firm they visit and economize on their search cost. Second, firms that are recommended typically charge higher prices as being recommended is also a signal of consumers probably liking the product. This effect works against consumers' interests. The total effect of these two forces is ambiguous. We finish this section by providing two examples that show this ambiguity and that also illustrate the above propositions.

Example 1. The influencer does not search enough. The first example is based on a numerical analysis and shows a case where the influencer does not search enough. Figure 2 displays the recommended firm's price, the influencer's expected payoff, and the consumer surplus in the case where the joint density function of match values is given by $g(v_i, \hat{v}_i) = (2v_i - 1)(2\hat{v}_i - 1) + 1$ and the consumer's search cost is set at 0.1.[16] As all firms compete to be recommended, their profit equals 0. If the influencer's search cost is too high, the influencer will not make a search effort and recommends a random firm. Thus, in all the plots, the horizontal dashed line represents the outcome in the uninformative equilibrium (which exists for all values of c) where consumers still follow the recommendation. The figure clearly shows that consumers and influencer are much better off in an informative equilibrium, and that the effect is stronger the lower the influencer's

[16] The figure may give the impression that an informative equilibrium only exists if c is (much) smaller than s. Note, however, that the threshold value c_0 can be arbitrarily increased by increasing the mass L of consumers, whereas the consumers' threshold value remains unaffected.

search cost (or the higher her cutoff value v^*), despite the recommended firm setting higher prices. The figure also shows that the difference between informative and uninformative equilibria can be quite significant with effects for consumer surplus and the price of the recommended firm in the same order as we have seen in Section 3, whereas the effect on the influencer's payoff being potentially even stronger and in the order of 50%. The figure also shows the sense in which the influencer does not search enough from the consumers' perspective: consumer surplus is decreasing in the search cost (and effort) of the influencer so that consumers clearly prefer the influencer having lower search cost.

There is one feature of the figure that requires more explanation, which is the wedge between the price of the recommended firm and consumer surplus between the two equilibria at the cutoff value c_0. To understand this wedge, it is important to realize that at c_0 the informative equilibrium still has an interior solution $v^* > \underline{v}$ so that at c_0 the influencer's threshold match value does not continuously transition to $\underline{v}$.

Example 2. The influencer searches too much. Our second example constructs an informative equilibrium for the case where match values are uniformly distributed over [0,1], the influencer gets a perfect signal of the followers' match value ($\lambda = 1$), and the consumer search cost is so large that an uninformative equilibrium does not exist ($s > s_W = 1/8$). This example also illustrates parts of Propositions 3 and 4 demonstrating why an informative equilibrium can exist when the consumer search cost is large and the influencer's signal is very informative. Suppose firms and consumers expect the influencer's threshold value is equal to $1/2 < v^*(c) < 3/4$. It is clear that the recommended firm will then set a price p_R equal to $v^*(c)$, consumers immediately buy if they find out that their match value is indeed larger than or equal to $v^*(c)$ and their expected surplus is $(1 - v^*(c))/2 - s$, which is positive if $s < (1 - v^*(c))/2$. Thus, consumers are best off the lower the threshold value $v^*(c)$. For any value of c that is not too large, any threshold value $1/2 < v^*(c) < 3/4$ can be sustained in equilibrium, however. Given the firms' expectations and their pricing behavior, the influencer does not want to deviate downward as consumers will not want to buy if they find out their match value is below $v^*(c)$, the price charged by the recommended firm. The influencer also does not want to deviate upward as this will only increase the expected search cost and the consumers will anyway buy. Thus, in almost any equilibrium the consumer would be better off if the influencer chooses a lower threshold and the recommendation is less informative.[17]

5. Competing influencers

■ So far, we have assumed that each consumer follows at most one influencer and that there are a finite number of macro influencers. This allowed us to focus on influencers in isolation. In this section, we investigate the extent to which our analysis is robust to consumers following multiple influencers. To do so, we extend the model of the previous section and allow each consumer to get a recommendation from two influencers.

Formally, we assume that influencer j's match value with firm i, $\hat{v}_{ji}$, and the consumer's match value with that firm, v_i, follow a joint distribution $G(v_i, \hat{v}_{ji})$ which is the same for both influencers and either features strict affiliation or perfect correlation between match values. The timing of the game remains as in Section 4. All firms offer a contract to each influencer and each influencer inspects whichever firms they wish in a sequential manner.[18] Influencers then select the firm whose contract they wish to accept and then simultaneously issue their recommendations to

[17] The existence of a continuum of equilibria is an artifact of the assumption that the signal is perfectly correlated with the followers' match values. If the signal is slightly less informative and c is small, the equilibrium would be such that the influencer chooses a much higher threshold than the one consumers would prefer.

[18] As the market includes a continuum of firms, the event in which both influencers end up recommending the same firm occurs with probability zero. As a result, we do not need to place any additional assumptions on the joint distribution of the influencers' match values $(\hat{v}_{1i}, \hat{v}_{2i})$.

the followers. Observing both recommendations, consumers decide in which order to search the firms. We make one departure from the main model by supposing that influencers do not disclose the price charged by the recommended firm.[19] We focus on symmetric equilibria.

We present two sets of results. First, in a numerical analysis we show that with imperfect signals consumers tend to be better off with competing influencers as long as informative equilibria exist. The numerical analysis also indicates, however, that the range of c values for which an informative equilibrium exists with two influencers is smaller than with one influencer. Second, for the case that the influencers' signals are perfectly correlated with their followers' match value we confirm that with two influencers informative equilibria exist for a smaller set of c values. As informative equilibria provide consumers with higher surplus, this result points to a negative aspect of competing influencers.

For the numerical analysis we construct an equilibrium that is analogous to the one constructed in the proof of Proposition 2. In the equilibrium, influencers adopt symmetric strategies of sequentially sampling firms, identified by a threshold value, and recommending the first sampled firm that has a value exceeding the threshold. Firms choose two prices, depending on whether or not they are recommended, while consumers sequentially choose which firms to sample. As consumers do not observe firms' prices, they have to treat both recommended firms symmetrically, but after having visited one recommended firm, they find it optimal to search the second recommended firm before they search non-recommended firms. An important difference with the single influencer case is that consumers now only follow a given influencer's recommendation in their initial search half the time.

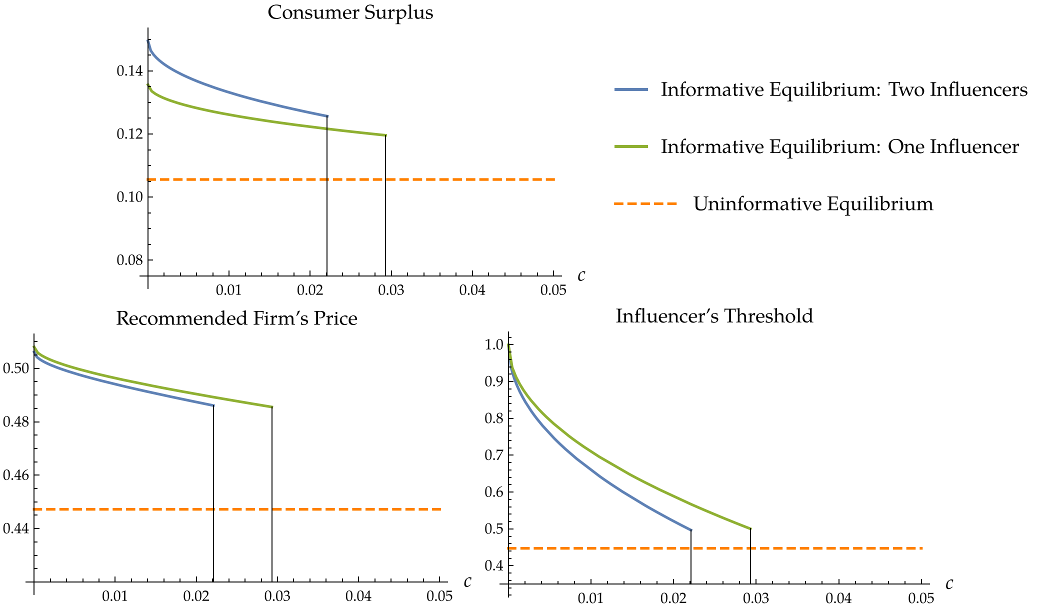
The numerical example presented in Figure 3 uses the same configuration as Example 1 in the previous section and compares market outcomes with one and two influencers for a mass 1 of consumers. Figure 3 clearly shows that if informative equilibria exist in both situations, then consumers are better off with two influencers, whereas informative equilibria may not exist with two influencers even if they do exist with one influencer. At an intuitive level, this is what one would expect and below we first describe the main mechanism why this is the case. The next paragraph describes that there are additional factors at play that complicate a formal, general analysis comparing markets with one and two influencers. With two influencers, the two recommended firms compete for those consumers who after visiting one of the recommended firms first and observing an intermediate match value continue searching the second recommended firm and then choose from which of these two firms to buy. Consumers clearly benefit from this competition effect as it lowers the prices recommended firms set. Influencers, on the other hand, make less profit when their recommended firms compete as recommended firms sell to a smaller fraction of consumers and at lower prices. To find a firm that surpasses a certain threshold value v^*, influencers need to make the same search effort, however, as when they are the only influencer around and therefore will find it optimal *not* to give an informed recommendation for lower values of c.

What complicates a formal analysis, and what the above intuition does not take into account, is that the influencers' threshold value v^* under duopoly will generally be different from the monopoly value and the effect this has on the prices set by the recommended firms. Figure 3 points to the fact that the threshold value v^* and the recommended firms' price move in the same direction: the higher the threshold value v^*, the more valuable the recommendation is to firms. Thus, at least part of the higher value of the recommendation to consumers is offset by the higher prices charged by firms. In Example 2 of the previous section, we have seen that this effect can be so strong that consumers would prefer a less informative recommendation. Whether or not this is true depends, among other things, on the strength of the influencers' signal. In addition, the effect of competition between influencers on the threshold value v^* is also ambiguous. Figure 3

[19] This departure from the model with one influencer does not affect the comparison in results, however, as the equilibria we characterized in the previous section remain equilibria if the price of the recommended firm is not observed (see the fifth comment on the model in Section 2).

FIGURE 3

The different figures plot the price charged by the recommended firm, the influencer's threshold, and consumer surplus as a function of the influencer's search cost c and the consumers' search cost set at 0.1 for the informative equilibrium with two influencers, the informative equilibrium with a single influencer, and the uninformative equilibrium.



illustrates that two influencers may have lower threshold values than one influencer, but this may not be generally true.

For the special case where the influencers' signals are perfectly informative of the match values of their followers our last result formally proves one of the results the numerical example points at, namely that with competing influencers, informative equilibria do not exist for some values of their search cost even if these equilibria do exist in case there is only one influencer.

Proposition 5. If the influencer's match values are perfectly correlated with those of her followers, then the range of influencer search costs for which an informative equilibrium exists is strictly smaller with two influencers than with one.

As we illustrated before in Example 2, the equilibrium construction simplifies when match values are perfectly correlated. In that case the influencer's optimal choice in an informative equilibrium is to sequentially inspect firms and recommend the first whose value net the price she expects the firm to charge exceeds the consumer's value from continuing to search. Any other strategy for the influencer either involves recommending firms with such a low value so that consumers do not buy (thus generating no sales commission) or being unnecessarily restrictive (and generating too high inspection cost) as consumers will even buy at lower values. The additional tractability this generates allows us to identify search costs for which an individual influencer would provide an informative recommendation, but introducing a competitor reduces the influencer's incentive to acquire information, thereby harming consumers.

6. Discussion and conclusion

■ In this article we have argued that social influencers play a beneficial role in directing consumers' search efforts toward products they are likely to like best. This conclusion holds even if influencers are paid by firms for their recommendation and influencers do not have an intrinsic motivation for providing honest recommendations. In addition, influencers have an incentive to provide an informative message even if it is costly for them to acquire information. What is important for our results to hold true is that consumers have their own independent preferences for the good and that they can walk away from the recommendation if after having discovered their own match value is relatively low, they (rationally) believe they have better options. Thus, our results apply to search markets where consumers can learn their match values by paying a search cost, but not to pure credence goods markets where consumers do not have an option to verify whether the product is a good match for them.

The model we propose in this article may also be of relevance for studying the role of platforms in guiding consumer search (see, e.g., De Corniere (2016)), and for position auctions in particular (see, e.g., Athey and Ellison (2011) and Chen and He (2011) for early contributions). The focus of most of this literature is on how the platform prices or auctions a sponsored position, without focusing on the information platforms have about consumers' preferences. Due to the storage of large data sets, many platforms have information about consumers (like the influencer in our article) that is relevant to interpret and evaluate a search query a consumer uses. Acquiring the sponsored position (as in our article) is good news to a firm as it reveals that the platform has information that their product may be liked by the consumer. This aspect of our article is also relevant for the platform literature. To apply our modeling to the platform literature, one has to extend the notion of recommendation to allow for a ranking of multiple slots and allow also the non-sponsored items to be ranked.

Appendix A

Before we prove the propositions, we state and prove a technical lemma that turns out to be useful in the proofs.

Lemma A1. If $g(v_i, \hat{v}_i)$ is log-concave, then $g(v_i|R)$ and $g(v_i|\hat{v}_i \geq v^*)$ are log-concave in v_i.

Proof. Upon sampling k firms and recommending one with the highest match value, the distribution of the influencer's match value with the recommended firm becomes $G(\hat{v}_i)^k$ with density $kg(\hat{v}_i)G(\hat{v}_i)^{k-1}$.

$$g(v_i|R) = k\int g(v_i, \hat{v}_i)G(\hat{v}_i)^{k-1}\mathrm{d}\hat{v}_i \tag{A1}$$

is thus log-concave in v_i as the well-known Prékopa–Leindler inequality establishes that the product and marginals of log-concave functions are log-concave. For this same reason

$$g(v_i|\hat{v}_i \geq v^*) = (1 - G(v^*))^{-1}\int_{\hat{v}^*}^{\bar{v}_i} g(v_i, \hat{v}_i)\mathrm{d}\hat{v}_i \tag{A2}$$

is likewise log-concave in v_i. □

Proof of Proposition 2. For the proof, we shall construct an equilibrium in which all firms offer identical contracts to the influencer who then sequentially inspects products, stopping her search when her match value with a given firm exceeds some threshold. For simplicity, assume $s \leq s_W$ so that consumers prefer to inspected the non-recommended firms to exiting the market. The proof for the case with $s > s_W$ proceeds identically by replacing $r - p$ with zero throughout. Define $p_{\tilde{v}^*}$ to be the optimal price and $\pi(\tilde{v}^*)$ the ensuing expected profit when the influencer's match value is known by both the firm and consumers to exceed the cutoff, that is, $\hat{v}_i \geq \tilde{v}^*$, and consumers break indifference by following the recommendation. Define $\underline{\pi}(\tilde{v}^*) \equiv p_{\tilde{v}^*}(1 - G(r - p + p_{\tilde{v}^*}|\tilde{v}^*))$ to be the influencer's expected payoff when her value is precisely at the cutoff.

An equilibrium where the influencer sequentially inspects firms and accepts a contract if her match value with the firm exceeds $\tilde{v}^*$ yields the influencer an expected payoff of

$$a + \theta\pi(\tilde{v}^*) - \frac{c}{P(\hat{v}_i \geq \tilde{v}^*)}, \tag{A3}$$

where a is some fixed fee. As an equilibrium contract cannot deliver negative expected profit to the firm and firms compete a la Bertrand to be recommended, the contract and cutoff that maximize the influencer's expected payoff sets $a = (1 - \theta)\pi(v^*)$ with cutoff $v^* = v^*(c) \in \arg\max_{\tilde{v}^*} \pi(\tilde{v}^*) - \frac{c}{P(\hat{v}_i \geq \tilde{v}^*)}$. The first-order condition for an interior optimum satisfies

$$\pi(v^*) - \underline{\pi}(v^*) - \frac{c}{P(\hat{v}_i \geq v^*)} = 0. \tag{A4}$$

As the boundaries exhibit $\pi(\underline{v}) - \underline{\pi}(\underline{v}) > 0$ and $\lim_{\tilde{v}^* \to \bar{v}} \pi(\tilde{v}^*) - \underline{\pi}(\tilde{v}^*) = 0$, there is an interior solution if the search cost is small enough, that is, $c < \pi(\underline{v}) - \underline{\pi}(\underline{v})$. Define $u = u(c) \equiv \pi(v^*) - \frac{c}{P(\hat{v}_i \geq v^*)}$ as the influencer's expected payoff.

Let us show that there is an equilibrium in which firms offer a contract so that the influencer indeed wishes to follow this cutoff v^*. In particular, we need to find a contract $x^* = (a^*, \theta^*)$ satisfying $a^* + \theta^* \underline{\pi}(v^*) = u$.

For each contract $x = (a, \theta)$ define v_x^* to be the cutoff the influencer would use if she were to inspect a firm offering this contract when her continuation value is u. Formally, v_x^* either equates $a + \theta \underline{\pi}(v_x^*) = u$ if there is an interior solution, or $v_x^* = \underline{v}$ if the left exceeds the right for all values, or $v_x^* = \bar{v}$ if the right exceeds the left for all values. For the influencer to be willing to follow the cutoff v^* when $a = (1-\theta)\pi(v^*)$, there must exist a $\theta \in (0, 1]$ equating $(1-\theta)\pi(v^*) + \theta \underline{\pi}(v^*) = u$. From the first-order conditions, the contract $x^* = (0, 1)$ achieves the optimal cutoff. We shall show that when the influencer's search cost is sufficiently small there is an equilibrium in which all firms offer x^*.

Notice that when firms offer x^* a decrease in the search cost strictly increases the payoff to searching as $c' < c$ implies $\max_{\tilde{v}^*}(\pi(\tilde{v}^*) - \frac{c'}{P(\hat{v}_i \geq \tilde{v}^*)}) \geq \pi(v^*(c)) - \frac{c'}{P(\hat{v}_i \geq v^*(c))} > \pi(v^*(c)) - \frac{c}{P(\hat{v}_i \geq v^*(c))}$. As $\pi(\underline{v})$ is the influencer's expected payoff in an uninformative equilibrium where firms and consumers optimally react to the uninformative recommendation, accepting a contract x^* without inspection yields a payoff less than or equal to $\pi(\underline{v})$. As a small enough search cost c' guarantees $\pi(v^*(c)) - \frac{c'}{P(\hat{v}_i \geq v^*(c))} > \pi(\underline{v})$ and the payoff to searching is continuous, there exists a cost c_0 satisfying $\pi(v^*(c_0)) - \frac{c_0}{P(\hat{v}_i \geq v^*(c_0))} = \pi(\underline{v})$. For all $c < c_0$, the influencer's best response to all firms offering x^* is to search. For the remainder of the proof, fix $c < c_0$.

Suppose the influencer plays a best response to all firms charging $p_{v_x^*}$ when their contract x is accepted and consumers follow the recommendation. Consider the best response to a contract profile of the form (x_i, x_{-i}^*) wherein firm i offers $x_i = (a, \theta)$ and the remaining firms offer x^*. Let us show that any x_i inducing the influencer to either immediately accept i's contract or to strictly prefer to start her search with i, the firm must obtain a negative expected profit. Accepting i's contract without inspection yields the influencer the expected payoff $a + \theta p_{v_x^*}(1 - G(r - p + p_{v_x^*}))$. If accepting x_i without inspection is strictly preferred to searching, then

$$a + \theta\pi(\underline{v}) \geq a + \theta p_{v_x^*}(1 - G(r - p + p_{v_x^*})) > u > \pi(\underline{v}).$$

The first inequality is due to $\pi(\underline{v})$ being the maximal profit for an uninformative recommendation, the second inequality captures the influencer's strict preference to immediately accept x_i, and the final equality holds when $c < c_0$. Hence, subtracting $a + \theta\pi(\underline{v})$ from the above provides $0 > (1-\theta)\pi(\underline{v}) - a$, that is, firm i's expected profit is negative.

If instead the influencer strictly prefers to start her search with i then

$$(a + \theta\pi(v_{x_i}))(1 - G(v_{x_i})) - c + G(v_{x_i})u > u.$$

But from this and the definition of u it follows that

$$a + \theta\pi(v_{x_i}) - \frac{c}{1 - G(v_{x_i})} > u \equiv \max\left\{\tilde{a} + \tilde{\theta}\pi(\tilde{v}^*) - \frac{c}{1 - G(\tilde{v}^*)} : (1-\tilde{\theta})\pi(\tilde{v}^*) - \tilde{a} \geq 0\right\}.$$

Therefore $(1-\theta)\pi(v_{x_i}) - a < 0$ and so firm i has a negative expected payoff. Therefore, when the influencer best responds to the pricing strategy $p_{v_x^*}$ and all other firms offer x^*, firm i's best response is to also offer x^*.

Thus, we can construct an equilibrium where firms believe the influencer to have inspected and followed the cutoff strategy v_x^* at a firm's information set following the acceptance of contract x, so that it is optimal for them to charge $p_{v_x^*}$. Moreover, we can specify consumers to believe that the influencer follows cutoff v^* so that following the recommendation is a best response. Notice that these beliefs are consistent with play on the equilibrium path.

In terms of welfare, consumers are strictly better off in an informative equilibrium relative to an uninformative equilibrium whenever they strictly prefer to follow the recommendation and are indifferent otherwise. The recommended firm is indifferent as it receives zero profit in both equilibria. The influencer is strictly better off in an informative equilibrium than an uninformative equilibrium as $\pi(v^*(c)) - \frac{c}{P(\hat{v}_i \geq v^*(c))} > \pi(\underline{v})$ holds when $c \leq c_0$. □

Proof of Proposition 3. In an informative equilibrium, the threshold cannot exceed a value v_0^* satisfying $\bar{v}(1 - G(v_0^*)) - c = 0$, otherwise, the influencer would obtain a negative expected payoff. Using the fact that the recommended firm's price lies between the Wolinsky price and the highest match value, we can bound the influencer's expected payoff for a given threshold $\underline{v} < v^* \leq v_0^*$ by

$$\begin{aligned} & p_R(1 - G(r - p + p_R | \hat{v} \geq v^*)) - \frac{c}{1 - G(v^*)} \\ & \leq \bar{v}(1 - G(r|\hat{v}) \geq v^*) - \frac{c}{1 - G(v^*)} \\ & \leq \bar{v}(1 - G(r|\hat{v} \geq v_0^*)) - \frac{c}{1 - G(v^*)}. \end{aligned}$$

As s vanishes, $r \to \bar{v}$, and the above expression becomes negative. Hence, we can conclude that there exists $s_0(c) > 0$ such that, if $s < s_0(c)$ and informative equilibrium does not exist. Moreover, an uninformative equilibrium always exists. That $\lim_{c \to 0^+} s_0(c) = 0$ follows from Proposition 2.

Finally, for $s > s_W$ consumers never search the non-recommended firms. Let r_R denote the reservation price for the recommended firm if the influencer uses a threshold $v^* \in (\underline{v}, \bar{v})$ and $\tilde{p}_R$ an arbitrary positive price less than r_R. Then for any c satisfying $\tilde{p}_R(1 - G(\tilde{p}_R | \hat{v} \geq v^*)) - \frac{c}{1-G(v^*)} \geq 0$, the construction in Proposition 2 provides that there exists an informative equilibrium. □

Proof of Proposition 4. In any informative equilibrium, the difference between the influencer's payoff and the payoff from deviating to providing an uninformative recommendation is bounded above by

$$\max_{\tilde{p}_R, v^* \in [\underline{v}, \bar{v}]} \left(\tilde{p}_R(1 - G(r - p + \tilde{p}_R | \hat{v} \geq v^*, \lambda)) - \frac{c}{1 - G(v^*)} - \tilde{p}_R(1 - G(r - p + \tilde{p}_R)) \right). \tag{A5}$$

Applying the Berge Maximum Theorem, we find that the above expression is continuous in λ and, by inspection, is negative at $\lambda = 0$. Hence, for λ in a neighborhood of 0, there cannot be an informative equilibrium.

Finally, we extend the conclusion of Proposition 2 to the setting with perfectly correlated match values, arguing that for every fixed search cost for consumers s, there exists an informative equilibrium equilibrium when the influencer's search cost lies in $(0, c_1]$ for some $c_1 > 0$. Following the same construction in Proposition 2, if an informative equilibrium exists at a search cost c then there is an informative equilibrium in which all firms offer zero fixed payment and a full commission and the influencer uses her profit maximizing search threshold. To simplify notation, assume that $s < s_W$ so that $r - p \geq 0$. If $s \geq s_W$, replace $r - p$ with zero (the consumer will never inspect non-recommended firms), and the proof proceeds identically.

An informative equilibrium is characterized by threshold v^* and a price p_R such that the influencer sequentially inspects firms, stops her search and recommends if and only her match value with the firm exceeds v^*, and the recommended firm charges p_R. Given the influencer's threshold, the recommended firm's objective is to maximize[20]

$$\tilde{p}_R \left(1 - \frac{G(r - p + \tilde{p}_R) - G(v^*)}{1 - G(v^*)} \cdot \mathbf{1}_{\{r - p \geq v^* - \tilde{p}_R\}}(\tilde{p}_R) \right) \cdot \mathbf{1}_{\{r_R - \tilde{p}_R \geq r - p, 0\}}(\tilde{p}_R). \tag{A6}$$

By inspection, we find that whenever $v^* < r$, the recommended firm's optimal price coincides with those of the non-recommended firms. Such a threshold cannot support an equilibrium because recommending a firm with which the influencer's match value is v^* delivers the influencers the zero expected profit (she knows her followers will not purchase the good) whereas she obtains a positive expected profit from deviating and recommending a random firm that has not previously been inspected. Thus any equilibrium must involve a threshold $v^* \geq r$. For any such threshold, the recommended firm's optimal price lies on the boundary ensuring that the influencer's followers always purchase the good: $v^* - p_R = r - p$. At this price, consumers are willing to follow the recommendation as $r_R - p_R > v^* - p_R = r - p \geq 0$.

The influencer's expected profit in an informative equilibrium is $p_R - \frac{c}{1-G(v^*)} = v^* - (r - p) - \frac{c}{1-G(v^*)}$. Given the price, the influencer cannot increase her expected profit from unilaterally deviating to a different threshold. Thus, an informative equilibrium exists if and only if the influencer does not wish to deviate to offering an uniformed recommendation, which would yield expected profit $p_R(1 - G(r - p + p_R)) = (v^* - (r - p))(1 - G(v^*))$. Defining the function

$$\begin{aligned} f(v^*, c) &\equiv v^* - (r - p) - \frac{c}{1 - G(v^*)} - (v^* - (r - p))(1 - G(v^*)) \\ &= (v^* - (r - p))G(v^*) - \frac{c}{1 - G(v^*)}, \end{aligned}$$

it is clear that the above expression is positive for c small enough proving the claim. □

Proof of Proposition 5. Let f be as defined in the proof of Proposition 4. The highest influencer search cost for which an informative equilibrium can exist, denoted by c_1, satisfies

$$\max_{v^* \in [r, \bar{v}]} f(v^*, c_1) = 0.$$

We now turn to the case with two influencers and show that there does not exist a symmetric informative equilibrium when the influencers' search cost is c_1. In an informative equilibrium, it remains true that the influencers' threshold v_D^* must exceed r and that the price charged by the recommended firms p_R must satisfy $v_D^* - p_R \geq r - p$ and thus $v_D^* - (r - p) \geq p_R$. Let β be the influencer's expected payment in an informative equilibrium. In equilibrium, recommended firms cannot generate an expected loss, implying $\beta \leq \frac{1}{2} p_R$. Deviating to an uninformative equilibrium provides

[20] For sets $S \subset X$, the *indicator function* $\mathbf{1}_S : X \to \{0, 1\}$ takes the value $\mathbf{1}_S(x) = 1$ when $x \in S$ and $\mathbf{1}_S(x) = 0$ when $x \in S^c$.

the influencer an expected payoff of $\beta(1 - G(v_D^*))$. Define the function

$$h(\beta, v_D^*, c) \equiv \beta - \frac{c}{1 - G(v_D^*)} - \beta(1 - G(v_D^*)) \tag{A7}$$

$$= \beta G(v_D^*) - \frac{c}{1 - G(v_D^*)}. \tag{A8}$$

Using the fact that $\beta \leq \frac{1}{2} p_R$ and $p_R \leq v_D^* - (r - p)$, we find that

$$h(\beta, v_D^*, c_1) \leq h\left(\frac{1}{2} v_D^*, v_D^*, c_1\right) < f(v_D^*, c_1) \leq \max_{v^* \in [r, \bar{v}]} f(v^*, c_1) = 0 \tag{A9}$$

and thus there cannot be an informative equilibrium at $c = c_1$ when there are two influencers.

Now suppose that c is a search cost for which an informative equilibrium exists with two influencers. Then because

$$h\left(\beta, v_D^*, c\right) \leq h\left(\frac{1}{2} v_D^*, v_D^*, c\right) < f(v_D^*, c) \leq \max_{v^* \in [r, \bar{v}]} f(v^*, c) \tag{A10}$$

it follows that an informative equilibrium exists with a single influencer. □

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