



Letter

Primordial black holes are true vacuum nurseries

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ABSTRACT

The Hawking evaporation of primordial black holes (PBH) reheats the Universe locally, forming hot spots that survive throughout their lifetime. We propose to use the temperature profile of such hot spots to calculate the decay rate of metastable vacua in cosmology, avoiding inconsistencies inherent to the Hartle-Hawking or Unruh vacuum. We apply our formalism to the case of the electroweak vacuum stability and find that a PBH energy fraction $\beta > 7 \times 10^{-80} (M/g)^{3/2}$ is ruled out for black holes with masses $0.8 \text{ g} < M < 10^{15} \text{ g}$.

1. Introduction

Light primordial black holes (PBHs) that evaporated long before the onset of Big Bang Nucleosynthesis (BBN) constitute unique relics of primordial cosmic history, but confirming their existence with observations is challenging, if not impossible, because of their generally small abundance.

Hawking's famous result states that light black holes (BHs) can be extremely hot, $T_H = (8\pi GM)^{-1}$, which would have substantial impact on the local environment. By using the so-called Hartle-Hawking vacuum — in which PBHs are in thermal equilibrium with their surrounding plasma [33] — the authors of Refs. [30,11–13] proposed that PBHs could trigger first-order phase transitions (FOPT) in the early Universe and endanger the stability of the electroweak vacuum, as PBHs have the ability to release part of their mass energy to fully support the formation of true-vacuum bubbles around them. If true, such a claim would severely constrain the formation of PBHs with masses smaller than $O(10) \text{ g}$ in the early Universe or impress the importance of stabilizing the Higg's vacuum via new physics [36].

Since then, this claim has been subject to controversy: PBHs rarely equilibrate with their surrounding and several authors pointed out that the Unruh vacuum [61] — in which PBHs evaporate in an empty Universe — should be used instead, largely mitigating the aforementioned result [42,34,58–60,10]. Moreover, thermal corrections were argued

to increase the energy of true vacuum bubbles for PBHs lighter than $O(10^3) \text{ g}$, effectively rescuing the electroweak-vacuum stability [60].

In reality, PBHs neither live in the vacuum nor are in thermal equilibrium with their surrounding. They deposit energy locally into the thermal plasma via Hawking radiation, forming hot spots around them [24,35]. In this *letter*, we provide a new avenue to calculate false-vacuum decay (FVD) rates around PBHs, building on the results derived in [30,11–13,23], to show that the presence of such hot spots does indeed seed the formation of true electroweak vacuum bubbles, circumventing the aforementioned criticisms.

2. Bubble action at $T < T_H$

Let us consider a homogeneous scalar field configuration ϕ living in a metastable minimum of its potential $V(\phi)$. The decay rate of this configuration is determined in curved spacetime by a saddle point “bounce” solution of the Euclidean action. Around a Schwarzschild BH living in a background plasma with arbitrary constant temperature T , one can write the Euclidean action of a time-independent scalar field bubble configuration as in [13,23]

$$I_b[T] = \beta \int dx^3 \sqrt{-g} \left(-\frac{R}{16\pi G} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + V(\phi) \right), \quad (1)$$

in which g is the determinant of the metric — assumed to be time-independent as well — and $\beta = 1/T$ denotes the Euclidean period-

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icity. At first sight, because β appears in this expression as an overall factor, it is tempting to conclude that a low plasma temperature would lead to a large bubble action, hence suppressing the FVD rate $\Gamma_{\text{FVD}} \propto \exp(-I_b[T])$ exponentially. However, one should note that there are two pieces missing in Eq. (1) [30]. First, the gravitational background action should be subtracted to extract the actual energy of the bubble. Second, if the temperature entering the Euclidean action's periodicity differs from the BH's Hawking temperature, the system features a conical deficit at the horizon that needs to be accounted for in the calculation. Ref. [30] calculated both contributions in the thin-wall approximation. The contribution from the conical deficit was evaluated using a mathematical regularization procedure: using a temperature profile interpolating over a scale ϵ between a constant temperature $T(r) = T$ for $r - r_H \gg \epsilon$, and $T(r_H) = T_H$ at the horizon, one can show that the Ricci contribution to the action is of order

$$I_b[T] \supset -\frac{\beta_H - \beta}{\beta} \frac{\mathcal{A}}{4G} + \mathcal{O}(\epsilon), \quad (2)$$

in the $\epsilon \rightarrow 0$ limit. In this equation \mathcal{A} stands for the BH's area and $\beta_H = 1/T_H$.

After summing up all the contributions, the authors of Ref. [30] proved, in the $\epsilon \rightarrow 0$ limit, that the β -dependency of the total Euclidean action exactly cancels, leading to the seminal result

$$I_b[T] = \frac{\mathcal{A}_+}{4G} - \frac{\mathcal{A}_-}{4G} = I_b[T_H], \quad (3)$$

where \mathcal{A}_+ (\mathcal{A}_-) denotes the area of the BH horizon before (after) bubble formation. This property was then generalized beyond the thin-wall approximation, and in the presence of matter [12]. Despite its remarkable generality, this result was only used in the Hartle-Hawking vacuum where $T = T_H$ to derive constraints on PBHs using the electroweak vacuum stability. In what follows, we will show that in a realistic cosmological set-up, PBHs live in a plasma with a temperature lower than their Hawking temperature, but also that the mathematical smoothing used to derive Eq. (2) actually corresponds to a physical situation.

3. Thermal profile around PBHs

Throughout cosmic history, PBHs are surrounded by two main sources of energy: (i) the Hawking radiation, made of particles with energy $E \sim T_H$, and (ii) the ambient plasma, populated with particles of energy $E \sim T$. Studies considering the Hartle-Hawking vacuum assume that $T = T_H$ and both energy sources are in perfect equilibrium. Instead, studies working with the Unruh vacuum only consider the Hawking radiation, effectively working in the limit where $T \rightarrow 0$.

As a matter of fact, both pictures are incomplete. When PBHs form, the Universe's temperature may be much larger than their Hawking temperature. However, the end of PBH evaporation typically takes place when the Universe is much colder than the initial PBH Hawking temperature. Nevertheless, a big piece is missing in this discussion: in the intermediate period, the quanta that form Hawking radiation unavoidably interact with the surrounding plasma, heating up the surrounding plasma locally. This energy deposition leads to an inhomogeneous temperature profile forming around PBHs that persists long after they have completely evaporated, and can differ by orders of magnitude from the average temperature in the Universe.

This temperature profile evolution was explored in Ref. [35,24]. Assuming a universal coupling constant α to encode particle interactions between Hawking radiation and the plasma, the authors of Ref. [35] showed that for PBHs with masses

$$M \gtrsim M_\star, \quad \text{where } M_\star \equiv 0.8 \text{ g} \left(\frac{\alpha}{0.1} \right)^{-\frac{11}{3}}, \quad (4)$$

the formation of a hot spot is quicker than the BH evaporation. PBHs are thus rapidly surrounded by an initial hot spot featuring an homogeneous temperature

$$T_{\text{plateau}} \approx 2 \times 10^{-4} T_H \left(\frac{\alpha}{0.1} \right)^{\frac{8}{3}} \left(\frac{g_\star(T_H)}{g_\star(T_{\text{plateau}})} \right)^{\frac{2}{3}}, \quad (5)$$

over a distance

$$r_{\text{plateau}} \approx 7 \times 10^8 r_H \left(\frac{\alpha}{0.1} \right)^{-6} \left(\frac{g_\star(T_H)}{g_\star(T_{\text{plateau}})} \right)^{-1}, \quad (6)$$

where $g_\star(T)$ denotes the number of relativistic degrees of freedom at temperature T . Before the evaporation time $t_{\text{ev}} \equiv \Gamma_{\text{ev}}^{-1}$, where $\Gamma_{\text{ev}} \equiv |\dot{M}/M| \propto M^{-3}$ can be calculated from e.g. Refs. [46,45], this temperature is constant. On larger distances, diffusion dominates over energy deposition and the temperature decreases down to the Universe's temperature at infinity.

Later on, the PBH mass starts decreasing, its Hawking temperature increases, and so does the plateau temperature, while its radius decreases. After $M = M_\star$, the plateau temperature then saturates at [35]

$$T_{\text{max}} \approx 2 \times 10^9 \text{ GeV} \left(\frac{\alpha}{0.1} \right)^{\frac{19}{3}} \left(\frac{g_\star(T_{\text{max}})}{106.75} \right)^{-\frac{4}{3}} \left(\frac{g_\star(T_H)}{106.75} \right)^{\frac{5}{6}}, \quad (7)$$

over a radius $r_{\text{max}} = r_{\text{plateau}} \Big|_{T_H = T_{\text{max}}}$. At this point, the BH is not able to provide enough energy to reheat the hot spot further.

To obtain these results, Ref. [35] assumed that Hawking radiation particles deposited their energy at an averaged time $t_{\text{dep.}} \equiv \Gamma(T)^{-1}$, where $\Gamma(T) \sim \alpha^2 T \sqrt{\frac{T}{T_H}}$. We argue that this approximation cannot remain valid near the horizon. Indeed, even if the temperature within the hot spot was perfectly homogeneous, the energy deposition probability of Hawking radiation, $dP \sim \Gamma(T) e^{-(r-r_H)\Gamma(T)} dr$, is a decreasing function of the radius. In addition, the Hawking radiation energy density also decreases like r^{-2} . Energy deposition is thus more efficient close to the horizon, leading to a local increase of the temperature. This increase also corresponds to an enhancement of $\Gamma(T)$. Therefore, the plasma temperature is likely to increase at the horizon, interpolating between the Hawking temperature and the hot spot temperature calculated on larger distances in Eq. (5) and (7). In Fig. 1, we sketch the qualitative temperature profile that must be present around PBHs at $t \ll \Gamma_{\text{ev}}^{-1}$, when the hot spot core temperature is T_{plateau} (upper panel), and at $t \lesssim \Gamma_{\text{ev}}^{-1}$, when the plateau temperature saturates at T_{max} (lower panel). Note that by enforcing the smooth behavior of the plasma $T \rightarrow T_H$ as $t \rightarrow r_H$, we have introduced a physical realization of the regularization procedure used in Ref. [30] to evaluate the conical deficit contribution to the action in Eq. (2). In full generality, one would need to solve the heat equation around PBHs to obtain the exact temperature profile and to evaluate the smoothing size ϵ that is sent to zero to obtain Eq. (3). For simplicity, we assume here that the typical size of this smoothing is negligible as compared to the size of a true-vacuum bubble and leave a precise study of the profile geometry contribution for future work.

4. Bounce solution and nucleation rate

We consider a homogeneous scalar-field configuration ϕ_+ living in a metastable minimum of its potential where $V(\phi_+) = 0$ before tunneling to the true vacuum, located at ϕ_- , for which $V(\phi_-) < 0$. Following Ref. [23], we parametrize the metric as

$$ds^2 = \left(1 - \frac{2G\mu(r)}{r} \right) e^{2\delta(r)} d\tau^2 + \frac{dr^2}{1 - \frac{2G\mu(r)}{r}} + r^2 d\Omega_2^2, \quad (8)$$

with $\mu(r)$ the local mass parameter, and search for the classical solution $\phi(r)$ satisfying the boundary conditions¹

$$\lim_{r \rightarrow \infty} \phi(r) \rightarrow \phi_+, \quad M_+ \equiv \lim_{r \rightarrow \infty} \mu(r). \quad (9)$$

¹ See Ref. [23] for more details.

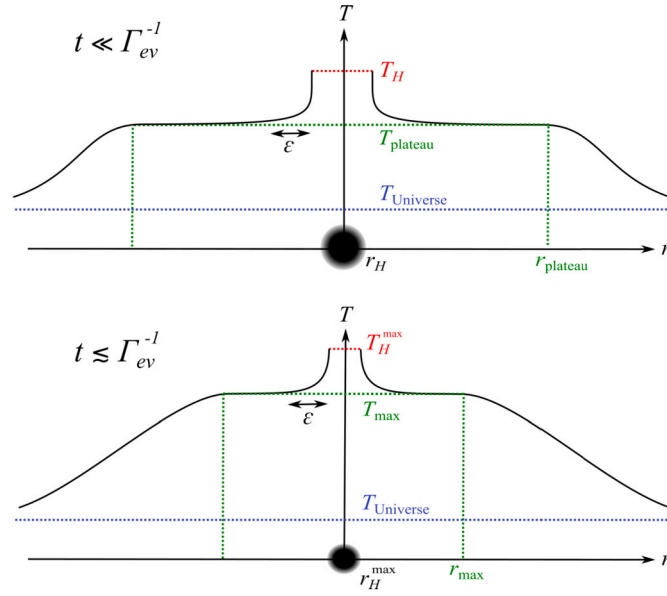


Fig. 1. Sketch of the temperature profile around PBHs throughout the evaporation process.

Assuming that the energy of the bubble is entirely provided by the local mass variation inside the bubble, the BH mass at infinity, M_+ , remains unchanged during the phase transition. In the near horizon limit, due to the presence of a negative cosmological constant $\Lambda_- \equiv V(\phi_-)$, the mass parameter $\mu_- \equiv \lim_{r \rightarrow r_H} \mu(r)$ slightly differs from the physical (ADM) BH mass inside the bubble, M_- , such that $r_H = 2G\mu_- = 2GM_- + \Lambda_- r_H^3/3$. With such a solution in hand, one can calculate the bounce action (1) numerically. Assuming a local thermal equilibrium around PBHs with a temperature profile $T(r)$ such that $\lim_{r \rightarrow r_H} T(r) = T_H$, the metric does not feature any conical singularity, but the contribution of the effective conical deficit at infinity is entirely contained in its contribution to the Ricci scalar. For simplicity, we will assume that the interpolation distance ε is small enough that the regularization procedure described in [30,12] is sufficient, enabling us to use Eq. (3).

Despite the powerful generality of Eq. (3), the FVD rate calculation was always restricted to the Hartle-Hawking case [11,13,12,30,23]. Using dimensional analysis to obtain the prefactor, this rate was obtained by writing Burda et al. [11,13,12]

$$\Gamma_{\text{FVD}}^{\text{HH}} \equiv (GM_+)^{-1} \left(\frac{I_b[T_H]}{2\pi} \right)^{1/2} \exp(-I_b[T_H]). \quad (10)$$

Because $(GM_+)^{-1} \sim T_H$, one can interpret this result as a thermal FVD rate evaluated at $T = T_H$ [43]. By analogy, we propose to generalize this result to the case of a hot spot with temperature $T < T_H$ using instead

$$\Gamma_{\text{FVD}}(T) \approx T \left(\frac{I_b[T_H]}{2\pi} \right)^{1/2} \exp(-I_b[T_H]), \quad (11)$$

where we used Eq. (3) such that $I_b[T_H] = I_b[T]$ in this expression. Consequently, the suppression of the rate in the case where $T < T_H$ is only linear, as the hot spot temperature only appears in the overall prefactor.

Before we use this result for phenomenological purposes, let us discuss what value of T is reasonable to use in Eq. (11): before they evaporate, PBHs with $M \gtrsim 0.8$ g are surrounded by a plasma with temperature $T_{\text{plateau}} \approx 2 \times 10^{-4} T_H$. This temperature remains constant throughout most of the PBH lifetime, enabling the use of the Euclidean formalism. Therefore, one can safely use T_{plateau} to calculate the nucleation rate. However, this choice is extremely conservative. Indeed, PBHs heavier than M_* are expected to evaporate and reach a point where $M = M_*$ and the hot spot temperature reaches T_{max} . Then, $T_{\text{max}} \geq T_{\text{plateau}}$ and $I_b[T_H]|_{M=M_*} \ll I_b[T_H]|_{M>M_*}$, leading to a much larger FVD rate. A legitimate concern is whether the Euclidean formalism is reliable given

that the PBH and its environment is dynamical. To ensure this, we restrict ourselves to cases where the characteristic timescales are much larger than the FVD, i.e. $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}} \gg 1$.

5. The electroweak vacuum

Let us now apply our results to the case of the electroweak vacuum. To calculate Γ_{FVD} , we parameterize the Higgs potential,

$$V(\phi) = \left[\lambda_* + b \left(\ln \frac{\phi}{M_p} \right)^2 + c \left(\ln \frac{\phi}{M_p} \right)^4 \right] \phi^4/4, \quad (12)$$

where we determined the parameters $(\lambda_*, b, c) = (-3.2 \times 10^{-3}, -1.497 \times 10^{-6}, 5.42 \times 10^{-8})$ using updated top-quark mass measurements from CMS [22] with the package PyR@te 3 [54,57] to obtain the RG-improved Higgs potential at three loops. To this zero-temperature potential, we added one-loop thermal corrections, $\Delta V(\phi, T) = \kappa^2 T_{\text{plasma}}^2 \phi^2$, where $\kappa = 0.35$ following [60]. Given these parameters, the electroweak vacuum appears to be metastable, although its lifetime is much longer than the age of the Universe in the absence of PBHs [26,1,28,14]. In Ref. [23], the effect of PBHs on electroweak FVD was used to derive constraints on PBH production in the early universe. Here, we propose to use Eq. (11) to obtain more realistic results.

PBH bounds and discussion. — In Fig. 2, we depict the evolution of $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}}$ with M . As one can see, $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}} \gtrsim 1$ for $M_* < M \lesssim 2$ g. However, Γ_{FVD} is exponentially suppressed for larger masses. Remarkably, this ratio reaches $5 \times 10^5 \gg 1$ at $M = M_*$. PBHs with large initial masses — for which $\Gamma_{\text{FVD}}(T_{\text{plateau}})/\Gamma_{\text{ev}} \ll 1$ — thus seed a much larger FVD rate once $M = M_*$, and the FVD is a much faster process than the PBH evaporation at that point. For PBHs with initial masses $M < M_*$, the hot spot does not have enough time to form in the first place [35]. However, PBHs with initial masses larger than M_* , will eventually enter this region when they evaporate. Then, such PBHs do not have enough energy to reheat the hot spot, whose temperature remains constant at $T = T_{\text{max}}$ throughout the end of the evaporation. In this region, we thus fixed $T = T_{\text{max}}$ in the calculation.

In [60], the effects of thermal corrections were claimed to become sizeable for $M \lesssim 10^3$ g. However, such effects only become relevant for $M \lesssim 0.1$ g $< M_*$ in our case and do not affect any of the conclusions drawn in this letter. Indeed, because we consider the temperature $T_{\text{plateau}} \approx 10^{-4} T_H$, the effect of thermal corrections is shifted to lower

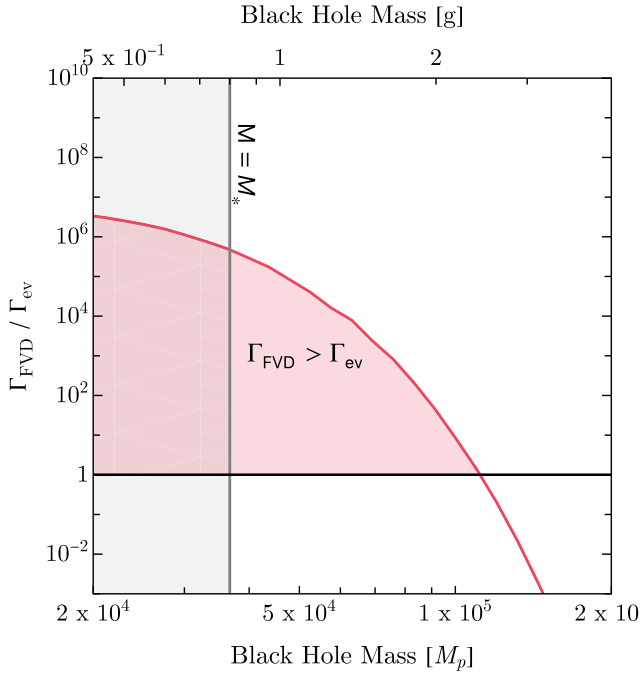


Fig. 2. Evolution of $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}}$ with the PBH mass, using the initial plateau temperature T_{plateau} (red curve). In the gray-shaded area, $M < M_*$, and in the red-shaded area, the ratio is larger than unity.

masses by a factor $O(10^{-4})$, in agreement with our results. Note also that the bubble profiles we obtain always have radii much smaller than r_{plateau} and r_{max} , validating the assumption of homogeneous plateau temperature that we have used throughout this work. We also checked that by subtracting to the action of Eq. (1) the background action and adding manually the conical deficit we recover exactly the result of Eq. (3).

Assuming that the FVD rate calculation is valid over a time Δt , one can calculate the FVD probability around a single PBH as $P_{\text{FVD}} \equiv 1 - e^{-\Gamma_{\text{FVD}}\Delta t}$. Since the hot spot temperature remains constant at T_{plateau} during most of the PBH lifetime, one can use $\Delta t \sim \Gamma_{\text{ev}}^{-1}$ in that case. In that case, $P_{\text{FVD}}(M) = 1 - e^{-\Gamma_{\text{FVD}}(T_{\text{plateau}})/\Gamma_{\text{ev}}}$. Depending on the ratio $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}}$, this probability can vary over orders of magnitude. Once the BH starts evaporating and eventually reaches $M = M_*$, then $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}} \approx 5 \times 10^5$. This ensures that $P_{\text{FVD}} \approx 1$ as long as one considers time scales $\Delta t \lesssim 10^{-6} \times \Gamma_{\text{ev}}^{-1}$. Over such a short time, the PBH mass variation is negligible, and the Euclidean formalism still holds.

Given these decay probabilities, we may now look at the ensuing bounds on the share of the earlier universe energy which collapsed into PBHs $\beta_{\text{PBH}} = \rho_{\text{PBH}}/\rho_{\text{tot}}$. At the earliest, PBHs formed when an overdensity of size comparable to that of a Hubble patch collapsed gravitationally. Assuming the Universe to be radiation-dominated then, $\rho_{\text{tot}} = \rho_{\text{rad}}$, and one can calculate the Universe's temperature at formation to be

$$T_f = \left(\frac{\gamma}{4\pi} \sqrt{\frac{45}{\pi g_*(T_f)} \frac{M_p^3}{M}} \right)^{1/2}, \quad (13)$$

and obtain the value of the density fraction at formation

$$\beta_{\text{PBH}} = \frac{4}{3} \frac{M N_{\text{PBH}} H_0^3}{s_0 T_f} \approx 2 \times 10^{-80} N_{\text{PBH}} \left(\frac{M}{M_*} \right)^{3/2}, \quad (14)$$

where $\gamma = (1/\sqrt{3})^3$ characterizes the gravitational collapse [16,17], N_{PBH} is the total number of PBHs that, if stable, would be contained in a Hubble patch of size H_0^{-1} , $s_0 \approx 2 \times 10^{-38} \text{ GeV}^3$ is the entropy density today, and $H_0 \approx 70 \text{ km}^{-1} \text{ s}^{-1} \text{ Mpc}^{-1}$ is the Hubble constant.

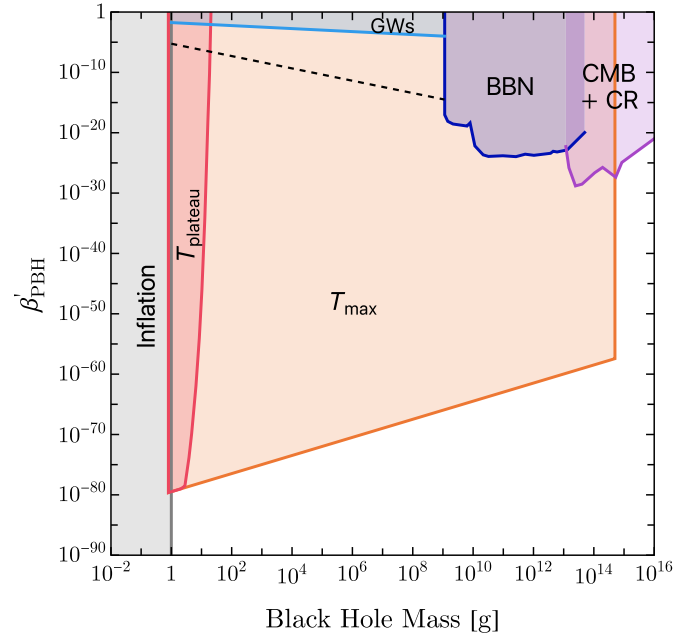


Fig. 3. Constraints on the PBH energy fraction at formation, $\beta'_{\text{PBH}} \equiv \gamma^{1/2} (g_*(T_f)/106.75)^{-1/4} \beta_{\text{PBH}}$. The orange(red)-shaded region corresponds to constraints derived with $T = T_{\text{max}} (T_{\text{plateau}})$.

Demanding that the electroweak vacuum has never decayed in our Hubble patch at 95% confidence level is equivalent to requesting that $N_{\text{PBH}} P_d < 2.7$, or, when $P_d \approx 1$, that

$$\beta_{\text{PBH}} \lesssim 5 \times 10^{-80} \left(\frac{M}{M_*} \right)^{3/2}. \quad (15)$$

In Fig. 3, we depict the corresponding constraints when using T_{plateau} and T_{max} for the hot spot temperature. Note that for $M_* < M < 88 \text{ g}$ (4 g), the temperature $T_{\text{plateau}} (T_{\text{max}})$ is smaller than the plasma temperature at evaporation

$$T_{\text{ev}} \equiv \left(\frac{90}{8\pi^3 g_*(t_{\text{ev}})} \right)^{1/4} \sqrt{\Gamma_{\text{ev}} M_p}. \quad (16)$$

In this case, we slightly modified the result of Eq. (11), by substituting the prefactor T by $\max\{T, T_{\text{ev}}\}$. In the regime where such constraints are relevant, we checked that $\Gamma_{\text{FVD}}/\Gamma_{\text{ev}} \gg 1$, guaranteeing the validity of the Euclidean formalism, and that thermal corrections arising at T_{ev} rather than T_{plateau} or T_{max} still remain subdominant. For comparison, we indicate other constraints from inflation, Big Bang Nucleosynthesis, CMB distortion and γ -rays [15].

Our conclusions are twofold: Should the electroweak vacuum remain metastable given future experimental measurements, such limits exclude any scenario predicting a large abundance of evaporating PBHs in cosmology [39,5,9,51,29,7,40,31,37,18,41,19,48,47,3,21,38,20,53,8,25,50,6,4,2,49,44,27,52,56,32,55] and exclude entirely the possibility that PBHs dominate the Universe before evaporating (above the dotted line in Fig. 3). Alternatively, should such PBH scenario be confirmed by cosmological data, our results may indicate that new physics is required to stabilize the electroweak vacuum throughout cosmic history. Finally, we emphasize that the findings presented in this *letter* can be applied to any FOPT taking place in cosmology.

Declaration of competing interest

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: Louis Hamaide reports financial support was provided by King's College London. Lucien Heurtier reports financial support was provided by

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Data availability

Data will be made available on request.

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