

# Tri-boson and WH production in the $W^+W^+jj$ channel: predictions at full NLO accuracy and beyond

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**ABSTRACT:** In this work, we present the first full NLO predictions for the process  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at the LHC in a typical tri-boson phase space. The NLO corrections reach 50% at the level of the fiducial cross section and have a very different hierarchy with respect to vector-boson-scattering phase spaces. By comparing the cross section of the full off-shell process with the sum of contributing on-shell electroweak-boson production subchannels, we find that the process is dominated by WWW and WH production, while vector-boson-scattering topologies still play a non-negligible role. In addition, NLO QCD predictions matched to parton shower which are supplemented by approximate electroweak corrections are provided. For the fiducial cross section, the electroweak corrections turn out to be small but the QCD corrections reach 47%. For the inclusive cross section, matching to parton shower affects the predictions by 7%. However, for differential distributions corrections due to the parton shower can be much more sizeable, depending on the region of phase space.

**KEYWORDS:** Higher Order Electroweak Calculations, Electroweak Precision Physics, Higher-Order Perturbative Calculations

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Features of the calculations</b>	<b>3</b>
2.1	LO contributions	3
2.2	Full NLO predictions	5
2.3	Analysing the composition of the off-shell calculation	6
2.4	NLO QCD matched to parton shower with virtual EW approximation	7
2.5	Technical aspects and tools	7
2.6	Setup	8
<b>3</b>	<b>Results</b>	<b>10</b>
3.1	LO contributions	10
3.2	On-shell approximations	10
3.3	Full NLO predictions	19
3.4	NLO QCD matched to parton shower with virtual EW approximation	26
<b>4</b>	<b>Conclusion</b>	<b>33</b>

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## 1 Introduction

The physics programme at the Large Hadron Collider (LHC) relies on the comparison of experimental measurements and theoretical predictions. While this task might sound trivial, it is actually extremely complex as typically different objects/concepts are considered on both sides. In particular, the definition of the process measured or computed is at the core of these comparisons.

Experimentally, only the final states retained and the event selection applied to them define the experimental signature and the associated process measured. On the other hand, theoretical predictions require knowledge of all the external states, including the initial ones, as well as the order in perturbation theory to be considered. None of these definitions actually refer to the content of the intermediate virtual particles as these are not physically manifest since only their decay products can be observed experimentally.

Nonetheless, processes are usually claimed to be measured or computed based on their content of intermediate particles, e.g. top-pair production or di-boson production. This becomes meaningful when selecting phase-space regions that enhance the contributions of interest. In addition, on the experimental side, irreducible backgrounds are often removed using theoretical inputs. On the theory side, instead of computing the full off-shell process, several methods can be used in order to single out certain contributions. It should be kept in mind that all these treatments rely on approximations that eventually aim at simplifying the interpretation while blurring the physical meaning of the comparisons.

For our purpose, the final state under consideration is  $\mu^+\nu_\mu e^+\nu_e jj$ , mostly known as the signature of vector-boson scattering (VBS) of same-sign W bosons, which is the golden

channel for VBS measurements at the LHC [1]. In such analyses, the phase-space requirements are rather extreme with large invariant mass and rapidity separation for the two tagging jets in order to single out the electroweak (EW) production from the QCD-induced background. On the other hand, for different phase spaces such as the one used here, which is a simplified version of a tri-boson measurement, the process is actually dominated by Higgs-strahlung (WH) and tri-boson contributions.

On the experimental side, all three processes (VBS, tri-boson, WH) have been measured within various phase spaces and/or final states. The production of a W boson in association with a Higgs boson has long been measured by both ATLAS and CMS [2–4]. Nonetheless, until now and to the best of our knowledge, no experimental measurement has been designed to probe WH production in the  $W^+W^+jj$  channel. For VBS, the same-sign WW channel has provided the first VBS measurements performed at the LHC [5–8]. For what concerns tri-boson measurements, the process has been searched for and was ultimately observed in various channels [9–11]. However, only the latest measurement of the ATLAS collaboration [11] is actually sensitive to the semi-leptonic decay channel under consideration here. Also, it is worth mentioning that in the latter analysis, a tension has been found between the experimental data and the Standard Model (SM) expectations.

On the theory side, the process  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  is known at full next-to-leading-order (NLO) accuracy [12, 13] in a VBS phase space.<sup>1</sup> The EW component has been found to feature particularly large EW corrections [18],<sup>2</sup> while they turned out to be of the expected size for off-shell tri-boson production in the leptonic channels [21, 22]. For these processes, the state of the art is NLO QCD+EW in a fully off-shell calculation [22]. For WH production with on-shell Higgs and leptonically decaying W boson, the inclusive cross section has been computed in the threshold limit at N<sup>3</sup>LO in QCD [23], while it is available fully differentially [24, 25] at NNLO QCD accuracy. The latter calculation has been matched to a parton shower [26]. Furthermore, soft-gluon resummation results are also available for this process [27, 28]. Concerning EW corrections, full NLO results for on-shell Higgs production exist [29] and have been combined with a QCD+QED parton shower [30] in the POWHEG framework [31–33]. To the best of our knowledge, there exist no specific computations in the literature discussing off-shell Higgs-strahlung processes. In ref. [34], triple-boson production through Higgs strahlung was studied in an on-shell approximation with NLO multijet merged predictions. Finally, the QCD corrections to the QCD background are known for some time [35, 36], and an implementation at NLO QCD plus parton shower accuracy [37] is available in POWHEG [31–33].

The process  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  is interesting for several reasons. First and as argued before, the process shares several subprocesses with different physics aspects. Second, WH and tri-boson production have never been computed for this final state at full NLO accuracy. In that respect, it is particularly interesting to reconcile several findings on the size of various corrections for different processes in the literature. In view of recent experimental analyses [11], an improvement and reassessment of the SM predictions is in order.

<sup>1</sup>NLO QCD predictions in an inclusive phase space based on the same calculation have been presented in ref. [14]. Previous calculations [15–17] relied on the VBS approximation and are thus not valid in different phase spaces.

<sup>2</sup>We note that these EW corrections are publicly available [19] in the resonance-aware version of POWHEG [20]. While there is in principle no restriction on the phase space in this implementation, it has only been tested in VBS phase space. Other regions might suffer from inefficiencies.

To that end, we have computed the full NLO corrections to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  in a triboson phase space which turned out to enhance both triboson and WH contributions. We remark that full NLO accuracy for processes involving more than two coupling-constant orders has only been obtained for few cases in the on-shell description [38, 39], in the narrow-width approximation [40] and for full off-shell processes [12, 13, 41–46]. To complement our fixed-order results and to account for multiple QCD emissions, we provide predictions compiled with SHERPA [47, 48] at NLO QCD matched with the parton shower thereby including approximate NLO EW corrections. The latter are treated in the so-called EW virtual approximation, first presented in ref. [49] and applied to a variety of processes in the meantime [50–52], that captures the exact NLO EW virtual corrections and integrated approximate real-emission subtractions but discards hard real-emission configurations. In turn, we here present the current state of the art in fixed-order and parton-shower-evolved predictions for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  production at the LHC.

The article is organised as follows: in section 2, all definitions and details of the calculations are provided. Section 3 is devoted to the discussion of numerical results. These range from leading-order (LO) studies to full NLO predictions, including detailed analyses on the impact of off-shell contributions, to NLO QCD parton-shower matched results. Finally, section 4 contains a summary and concluding remarks.

## 2 Features of the calculations

### 2.1 LO contributions

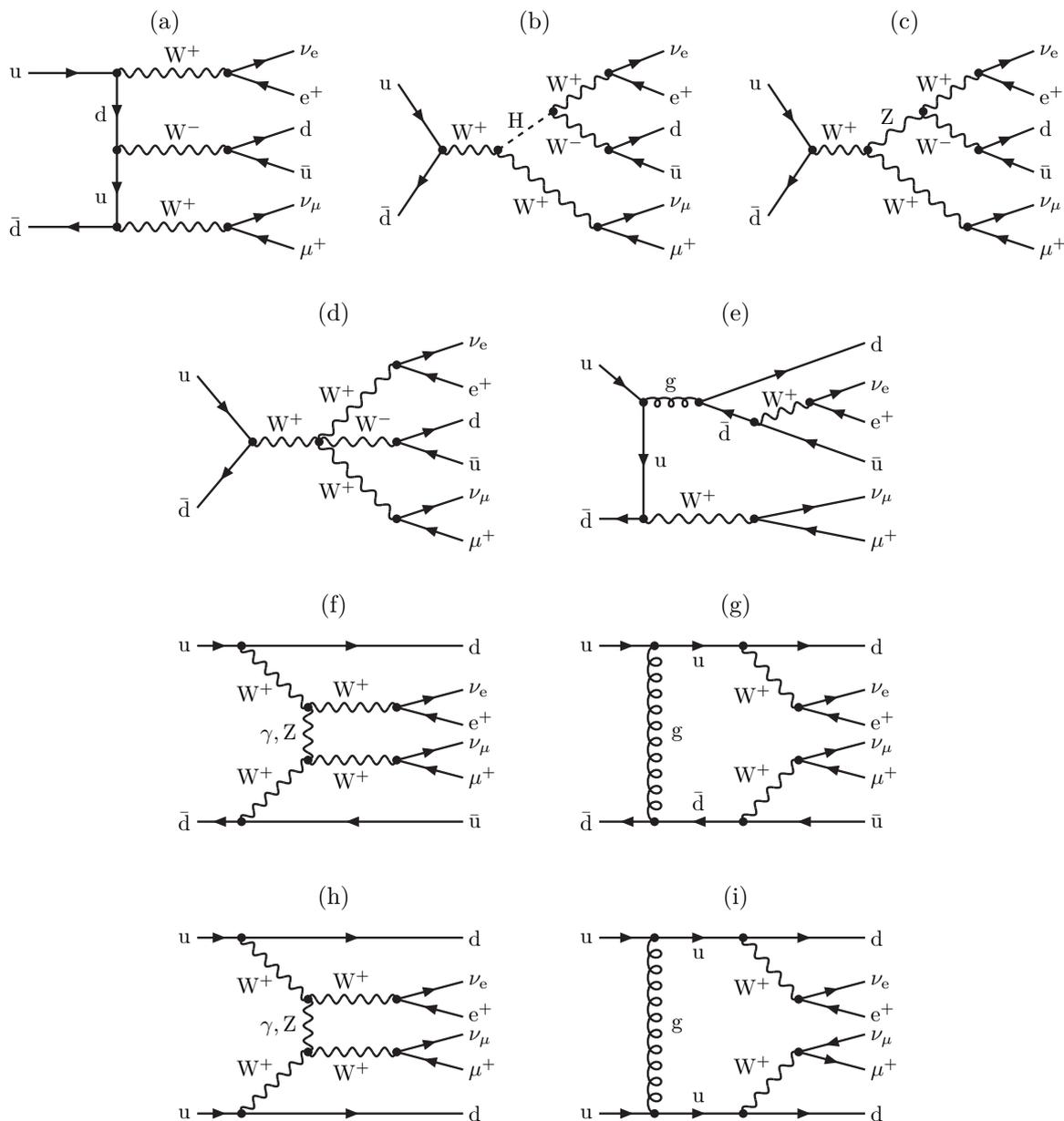
The process under investigation in this work is

$$pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj \tag{2.1}$$

at the LHC. This process is of particular interest as it contains contributions with three resonant W bosons,  $pp \rightarrow W^+(\rightarrow \mu^+ \nu_\mu) W^+(\rightarrow e^+ \nu_e) W^-(\rightarrow jj)$ , as shown in figure 1(a). The process also involves Higgs strahlung, i.e.  $pp \rightarrow W^+(\rightarrow \mu^+ \nu_\mu) H[\rightarrow W^+(\rightarrow e^+ \nu_e) W^-(\rightarrow jj)]$  (as well as  $e^+ \leftrightarrow \mu^+$  and  $\nu_e \leftrightarrow \nu_\mu$ ), and  $W^+Z$  production as subprocesses (for representative Feynman diagrams see figures 1(b) and 1(c)). In the phase space considered in this article, the triple-resonant WWW contribution dominates with roughly 50%, the Higgs-strahlung process contributes about 40%, while the  $W^+Z$  contribution is very small. Besides, many other types of contributions are present in the EW process at order  $\mathcal{O}(\alpha^6)$ , such as diagrams with quartic gauge couplings (figure 1(d)) or VBS ones (figures 1(f) and 1(h)). We note that triple-W production and WH production only appear in partonic channels that contain  $s$ -channel contributions, i.e. only in quark-anti-quark annihilation channels.

In addition, the process (2.1) receives contributions of order  $\mathcal{O}(\alpha_s^2 \alpha^4)$ , which are usually referred to as irreducible QCD background. Examples of corresponding diagrams are depicted in figures 1(e), 1(g), and 1(i). It is worth emphasising that the relative size of the QCD background depends strongly on the considered phase space.

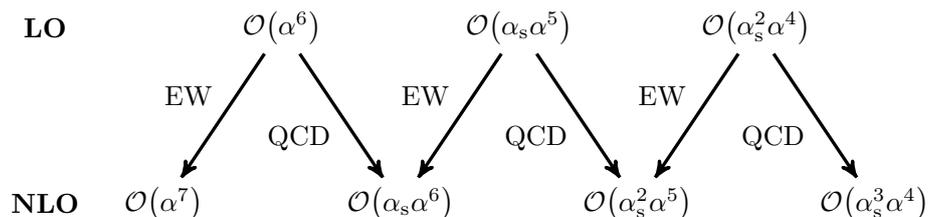
Finally, interference contributions between the EW and QCD amplitudes arise at LO, which are of order  $\mathcal{O}(\alpha_s \alpha^5)$ . Owing to colour algebra, these interferences are non-zero only for those partonic channels that receive contributions from different kinematic channels, either



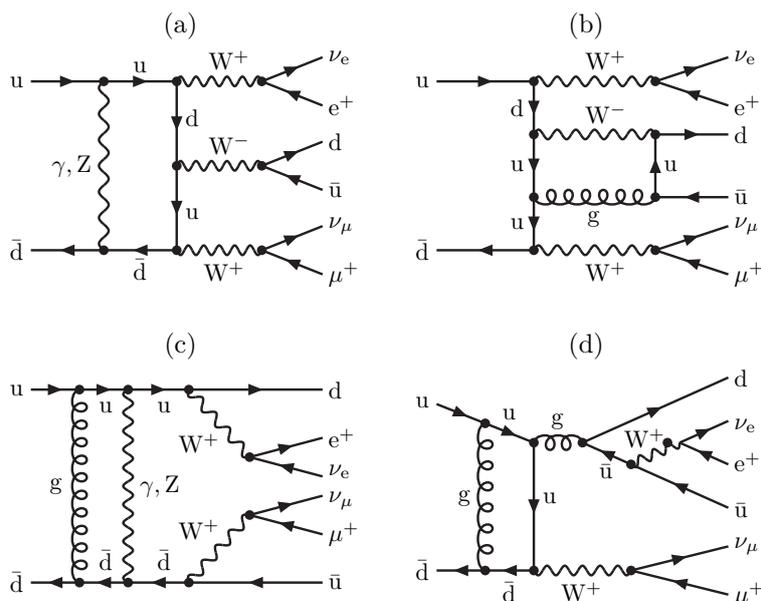
**Figure 1.** Examples of tree-level diagrams contributing to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at  $\mathcal{O}(e^6)$  (a, b, c, d, f, h) and  $\mathcal{O}(g_s^2 e^4)$  (e, g, i).

from  $t$  and  $u$  channels or from  $t$  and  $s$  channels. The complete set of partonic contributions is listed in table 1 of ref. [12] along with their potential interferences at  $\mathcal{O}(\alpha_s \alpha^5)$  and their kinematic channels.

In quark-quark and anti-quark-anti-quark channels, only  $t$ - or  $u$ -channel diagrams like those in figures 1(h) and 1(i) contribute as well as interferences between EW and QCD amplitudes with identical final-state quarks.



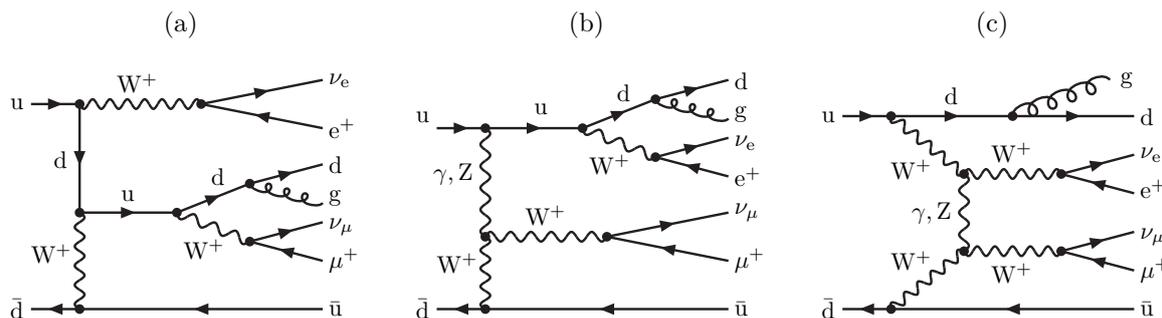
**Figure 2.** Perturbative orders contributing at LO and NLO for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ .



**Figure 3.** Examples of loop diagrams contributing to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at  $\mathcal{O}(e^8)$  (a),  $\mathcal{O}(g_s^2 e^6)$  (b,c), and  $\mathcal{O}(g_s^4 e^4)$  (d).

## 2.2 Full NLO predictions

The three different LO contributions give rise to four terms contributing at NLO accuracy arising from both QCD and EW corrections. A pictorial representation of this is provided in figure 2. Each NLO order is either made of pure QCD or EW corrections or a mixture of the two as for the orders  $\mathcal{O}(\alpha_s \alpha^6)$  and  $\mathcal{O}(\alpha_s^2 \alpha^5)$ . For example, the NLO corrections of  $\mathcal{O}(\alpha_s \alpha^6)$  are QCD corrections to the LO process of  $\mathcal{O}(\alpha^6)$  and EW corrections to the LO process of  $\mathcal{O}(\alpha_s \alpha^5)$  simultaneously. In figure 3 we show examples of loop diagrams of the orders  $\mathcal{O}(e^8)$ ,  $\mathcal{O}(g_s^2 e^6)$ , and  $\mathcal{O}(g_s^4 e^4)$  in figures 3(a), 3(b) as well as 3(c), and 3(d), respectively. The diagram in figure 3(c), for example, can be viewed as an EW correction to the LO diagram in figure 1(g) or as a QCD correction to a similar LO diagram with the gluon replaced by a photon or Z boson. A similar statement holds for the diagram in figure 3(b). Interfering the diagrams in figures 3(c) and 3(b) with those in figures 1(a) and 1(f) yields contributions of order  $\mathcal{O}(\alpha_s \alpha^6)$ , while interfering them with those in figures 1(g) and 1(e) gives contributions of order  $\mathcal{O}(\alpha_s^2 \alpha^5)$ .



**Figure 4.** Examples of real-radiation diagrams contributing to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at  $\mathcal{O}(g_s e^6)$ .

In this work, we compute the full NLO predictions to the process of eq. (2.1), i.e. all four NLO contributions of orders  $\mathcal{O}(\alpha^7)$ ,  $\mathcal{O}(\alpha_s \alpha^6)$ ,  $\mathcal{O}(\alpha_s^2 \alpha^5)$ , and  $\mathcal{O}(\alpha_s^3 \alpha^4)$  are fully taken into account. Our fixed-order off-shell calculation follows closely the one presented in ref. [12] for VBS with the same final state. As explained there in detail, all virtual and real contributions are included and, in particular, all off-shell and non-resonant contributions are taken into account. Some sample diagrams are shown in figures 3 and 4. In addition, we also compute the photon-induced contributions for all relevant orders [ $\mathcal{O}(\alpha^7)$ ,  $\mathcal{O}(\alpha_s \alpha^6)$ , and  $\mathcal{O}(\alpha_s^2 \alpha^5)$ ].

One of the calculations presented in this work thus provides the full NLO and off-shell predictions for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at the LHC in a tri-boson phase space in section 3.3. To consistently treat off-shell contributions, the complex-mass scheme [53–55] is used throughout.

### 2.3 Analysing the composition of the off-shell calculation

In order to investigate the complexity and composition of the complete off-shell calculation, both at LO and NLO, we decompose the EW production mode taking into account all contributing on-shell channels, i.e. WWW production, WH production, WZ production, and  $W^+W^+jj$  production in the VBS approximation. Besides discarding non-resonant contributions, this approach also neglects interferences between different resonant channels. The details of this approximation and its quality to capture the full off-shell results are discussed in section 3.2.

In these on-shell calculations, the W, Z, and/or Higgs bosons are produced on their mass shell, and their decays are calculated using LO  $1 \rightarrow 2$  and  $1 \rightarrow 4$  decay matrix elements, including full LO spin correlations [56]. We adjust the branching ratios to account for the respective LO decay width and the measured total width used in the LO off-shell calculation (see section 2.6 for details).

Furthermore, we also study a decomposition of the fully off-shell calculation in terms of  $s$ - and  $t/u$ -like partonic channels of the underlying di-jet production topologies. Thereby we still treat all internal propagators as fully off shell. While the  $s$  channel comprises off-shell WWW, WH, and WZ topologies as well as their interferences, the  $t/u$  channel consists of the off-shell  $W^+W^+$  VBS process (for details see the discussion in section 3.2.3). This decomposition forms the basis for our parton-shower matched predictions of the EW production mode.

## 2.4 NLO QCD matched to parton shower with virtual EW approximation

Besides the aforementioned fixed-order calculations, we provide parton-shower matched predictions for the  $\mu^+\nu_\mu e^+\nu_e jj$  final state, allowing one to model fully exclusive hadron-level events. For the NLO QCD accurate parton-shower simulations with SHERPA we separate the full process into its QCD and EW production modes, corresponding to the  $\mathcal{O}(\alpha^6)$  and  $\mathcal{O}(\alpha_s^2\alpha^4)$  terms at LO, respectively. The interference contribution of  $\mathcal{O}(\alpha_s\alpha^5)$  is not included.

In addition to the NLO QCD corrections, which are matched to the SHERPA dipole shower [57], using the methods detailed in ref. [58], for the EW production mode we furthermore incorporate NLO EW corrections through the  $\text{EW}_{\text{virt}}$  approximation [49, 59], which, in particular, captures the dominant effects in the high-energy limit [51, 52].

While for the EW production mode we use an incoherent decomposition into pure  $s$ - and  $t/u$ -channel-like contributions, the QCD production process is treated in full generality, including NLO QCD corrections and shower evolution, as well as resolved final-state photon emissions off the charged leptons in the YFS [60] formalism. However, for this channel we do not account for EW corrections, as these cannot unambiguously be assigned to the QCD production mode (cf. section 3.4).

The calculational setup used and described here, based on the SHERPA event generator, is fully realisable in the computing frameworks of the LHC experiments, such that corresponding particle-level simulations can be obtained and utilised in future data analyses.

## 2.5 Technical aspects and tools

For the full NLO computation at fixed order, the combination of codes `MOCANLO+RECOLA` has been used. `MOCANLO` is a general NLO Monte Carlo program which has been shown to be particularly adapted for high-multiplicity processes such as  $pp \rightarrow VV'jj$  at NLO QCD and EW accuracy [12, 18, 43, 61–63]. To render the numerical integration of such complex processes efficient, it uses multi-channel phase-space mappings as introduced in refs. [53, 64, 65]. To ensure the convergence over the full phase space all relevant integration channels have to be included, usually by introducing one integration channel for each kinematically different Feynman diagram. Furthermore, for contributions like  $pp \rightarrow W^+H(\rightarrow W^+W^-)$ , with different mutually exclusive resonance structures, a permutation of the order of the generated resonances has been implemented in `MOCANLO` following the ideas of ref. [66]. Moreover, resonant contributions in the dipole-subtraction terms of the real NLO corrections are taken care of by tailored integration channels [67]. Both types of extra channels turned out to be particularly relevant for contributions involving a potentially resonant Higgs boson and two potentially resonant  $W$  bosons. The infrared (IR) singularities arising from the real QCD or QED radiations are treated by the dipole-subtraction method [68–71]. Note that for the present computation, we did not make use of a recent FKS-scheme [72] implementation [73] in `MoCaNLO`. For all amplitudes (either tree or loop ones), the matrix-element generator `RECOLA` [74, 75] has been used. It employs the `COLLIER` library [76, 77] for the numerical evaluation of the one-loop scalar [78–81] and tensor integrals [82–84].

The NLO QCD computations of the EW production process in the on-shell approximation and  $s$ - and  $t/u$ -channel decomposition, as well as all parton-shower-matched NLO QCD calculations, including approximate NLO EW corrections, have been performed within the

SHERPA framework [47, 48]. Tree-level amplitudes and phase-space integration channels are provided by AMEGIC [85] and for the real-emission subtraction by COMIX [86]. All one-loop contributions are obtained from RECOLA [74, 75] using a general interface to SHERPA [87]. Infrared QCD and QED singularities are treated according to the dipole-subtraction formalism [68–71] with the dedicated implementation in SHERPA [88, 89]. The full QCD NLO matrix elements get matched to the SHERPA dipole shower [57] based on the MC@NLO formalism [90]. NLO EW corrections are included in the virtual approximation (see for example refs. [51, 52] for a detailed description). QED corrections to final-state charged leptons are included via the YFS formalism [91, 92], including the photon-splitting corrections of ref. [93]. When considering the on-shell approximation, the decays of massive particles produced on shell in the matrix-element calculation are treated by SHERPA’s internal decay-handler module, thereby accounting for spin correlations and invoking a Breit-Wigner smearing for the intermediate resonances [56]. To analyse events we make use of the RIVET package [94].

## 2.6 Setup

**Numerical inputs.** The results obtained in the present work are for the LHC running at a centre-of-mass energy of  $\sqrt{s} = 13.6$  TeV. We use the NNPDF31\_nnlo\_as\_0118\_luxqed parton distribution function (PDF) set [95] via the LHAPDF interface [96]. This PDF set employs  $\alpha_s(M_Z^2) = 0.118$  for the strong coupling constant and the method of ref. [97] for the extraction of the photon distribution. The renormalisation and factorisation scales have been set to

$$\mu_R = \mu_F = m_{T,ij} + m_{T,\nu_e e^+} + m_{T,\nu_\mu \mu^+}, \quad (2.2)$$

where  $m_{T,ij} = \sqrt{m_{ij}^2 + p_{T,ij}^2}$  is the transverse mass for the particle pair  $i$  and  $j$  with invariant mass  $m_{ij}$ . This scale is similar to the one used in ref. [22] for the calculation of NLO QCD and EW corrections to triple-W-boson production with leptonic decays at the LHC. In order to study the validity of the on-shell approximation we perform calculations using an alternative fixed-scale definition that is in particular independent of the decay-product kinematics, namely

$$\mu_R = \mu_F = 3M_W. \quad (2.3)$$

In the off-shell computation, the following masses and widths are used:

$$\begin{aligned} m_t &= 173.0 \text{ GeV}, & m_b &= 0 \text{ GeV}, \\ M_Z^{\text{OS}} &= 91.1876 \text{ GeV}, & \Gamma_Z^{\text{OS}} &= 2.4952 \text{ GeV}, \\ M_W^{\text{OS}} &= 80.379 \text{ GeV}, & \Gamma_W^{\text{OS}} &= 2.085 \text{ GeV}, \\ M_H &= 125.0 \text{ GeV}, & \Gamma_H &= 4.07 \times 10^{-3} \text{ GeV}. \end{aligned} \quad (2.4)$$

Note that in the process (2.1) no bottom or top quarks enter in tree-level amplitudes. Since they only appear within loops, their respective widths are set to zero in the calculation. The values of the Higgs-boson mass and width are taken from ref. [98]. The pole masses and widths of the W and Z bosons that are utilised in the numerical calculations are obtained from the measured on-shell (OS) values via [99]

$$M_V = \frac{M_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/M_V^{\text{OS}})^2}}, \quad \Gamma_V = \frac{\Gamma_V^{\text{OS}}}{\sqrt{1 + (\Gamma_V^{\text{OS}}/M_V^{\text{OS}})^2}}, \quad (2.5)$$

with  $V = W, Z$ . The EW coupling is fixed through the  $G_\mu$  scheme [100, 101] upon

$$\alpha = \frac{\sqrt{2}}{\pi} G_\mu M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) \quad \text{and} \quad G_\mu = 1.16638 \times 10^{-5} \text{ GeV}^{-2}. \quad (2.6)$$

For the on-shell calculations in the narrow-width approximation, the widths of the W, Z, and Higgs bosons are set to zero in the matrix-element computations. The produced bosons are subsequently decayed using the algorithm of ref. [56], preserving the LO spin correlations. The branching ratios for each decay are determined by taking the ratio of the LO decay width of the chosen decay channel over the total width in the pole scheme of the decaying boson. The LO decay width of the Higgs boson into 4 fermions is calculated with PROPHECY4F [102], resulting in

$$\Gamma(\text{H} \rightarrow e^+ \nu_e \bar{u} d) + \Gamma(\text{H} \rightarrow e^+ \nu_e \bar{c} s) = 0.06125 \text{ MeV}. \quad (2.7)$$

In addition, we adjust the kinematics of the intermediate resonance according to a Breit-Wigner distribution using its pole mass and width as input to mimic kinematic off-shell effects. This LO treatment of the boson decays is also applied for on-shell NLO calculations.

**Event selection.** The event selection for the present calculation is a simplified version of the kinematic cuts used in the ATLAS measurement of ref. [11]. The recombination of the QCD partons and photons (with  $|y| < 5$ ) is performed in two steps:

1. First, photons and jets are recombined with the anti- $k_T$  algorithm [103] and a radius parameter  $R = 0.4$ .
2. Then, the non-clustered photons are recombined with the charged leptons with radius parameter  $R = 0.1$  using the Cambridge-Aachen algorithm [104].

In general, the experimental signature consists of two identified jets, two same-sign charged leptons (positron and anti-muon in the present case), and missing transverse energy (which is not explicitly required in the present selection). In detail, the two dressed leptons must fulfil the conditions

$$p_{T,\ell^+} > 20 \text{ GeV} \quad \text{and} \quad |y_{\ell^+}| < 2.5, \quad (2.8)$$

where  $y$  is the rapidity and  $p_T$  the transverse momentum. In addition, their invariant mass is constrained to be in the window

$$40 \text{ GeV} < m_{\ell^+ \ell^+} < 400 \text{ GeV}. \quad (2.9)$$

Finally, the identified anti- $k_T$  QCD jets are defined through the two conditions

$$p_{T,j} > 20 \text{ GeV} \quad \text{and} \quad |y_j| < 4.5. \quad (2.10)$$

From the list of all identified jets in an event [fulfilling eq. (2.10)], the two hardest jets (in transverse momentum) are further required to respect the conditions

$$m_{jj} < 160 \text{ GeV} \quad \text{and} \quad |\Delta y_{jj}| < 1.5. \quad (2.11)$$

order	$\mathcal{O}(\alpha^6)$	$\mathcal{O}(\alpha_s\alpha^5)$	$\mathcal{O}(\alpha_s^2\alpha^4)$	sum
$\sigma_{\text{LO}}[\text{fb}]$	0.78549(9)	0.00732(1)	0.25925(3)	1.05206(9)
$\sigma/\sigma_{\text{LO}}^{\text{sum}}[\%]$	74.7	0.7	24.6	100

**Table 1.** Cross sections at LO accuracy for  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  at the LHC for the three contributing orders and their sum using the dynamical scale defined by eq. (2.2). The second line contains the absolute predictions, while the third one provides the relative numbers normalised to the sum of all contributions.

### 3 Results

#### 3.1 LO contributions

In this section, we first discuss results of the off-shell description of  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  at LO accuracy at the LHC. As explained in section 2, at LO the partonic process contains three different contributions of order  $\mathcal{O}(\alpha^6)$ ,  $\mathcal{O}(\alpha_s\alpha^5)$ , and  $\mathcal{O}(\alpha_s^2\alpha^4)$ , respectively. The corresponding fiducial cross section, as well as those of the separate parts, are provided in table 1.

With the event selection defined in section 2.6, the EW contribution is about 75% of the fiducial cross section. As a comparison, in ref. [12] where the same final state is computed in a VBS phase space, the EW part amounts to a bit more than 85%. Typical VBS event selections require large invariant masses and large rapidity differences of the two jets to single out the EW component. Taking advantage of the high-energy behaviour of VBS amplitudes over QCD-mediated ones allows one to obtain samples highly enriched in EW contributions. On the other hand, the tri-boson phase space necessarily requires a low invariant mass for the two jets (of the order of the W-boson mass) where the QCD component is not suppressed [14] resulting in lower purity. The interference contribution is small, below 1%. Note that in the present case, in addition to being simply colour suppressed, there is a further cancellation between partonic channels. While the  $t$ - $u$  interference channels come with a positive contribution, those involving  $s$  channels are negative. Combining all contributions, the full LO prediction reads

$$\sigma_{\text{LO}}^{\text{sum}} = 1.05206(9)_{-3.8\%}^{+4.9\%} \text{ fb.} \tag{3.1}$$

The subscript and superscript indicate the 7-point scale variation, which amounts to taking the envelope of the predictions obtained by scaling the renormalisation and factorisation scales defined in eq. (2.2) by the factors  $(\xi_F, \xi_R) \in \{(1/2, 1/2), (1/2, 1), (1, 1/2), (1, 1), (1, 2), (2, 1), (2, 2)\}$ . This rather small scale dependence at LO is driven by the fact that the dominating EW contribution does not carry a renormalisation-scale dependence.

#### 3.2 On-shell approximations

In this section, we discuss the quality and implications of calculating the production of  $\mu^+\nu_\mu e^+\nu_e jj$  in the on-shell approximation at LO for the EW production mode. Because of the presence of multiple sequential and competing resonances in the full process (see figure 1), we include four separate on-shell production channels at  $\mathcal{O}(\alpha^6)$ :

- a)  $W^+W^+W^-$ , where the  $W^+$  bosons decay into electron/electron neutrino and muon/muon neutrino, while the  $W^-$  boson decays hadronically.
- b)  $W^+H$ , where the  $W^+$  boson decays into electron/electron neutrino or muon/muon neutrino, while the Higgs boson decays into four fermions semi-leptonically, containing a muon/muon neutrino or electron/electron neutrino pair, respectively. The Higgs decay width into 4 fermions is calculated with PROPHECY4F [102].
- c)  $W^+Z$ , where the  $W^+$  decays into electron/electron neutrino or muon/muon neutrino, while the Z boson decays into four fermions semi-leptonically, containing a muon/muon neutrino or electron/electron neutrino pair, respectively. The Z-boson decay is thereby approximated by the production of an on-shell W boson (with subsequent decay) and two fermions. Both cases of an on-shell  $W^-$  and on-shell  $W^+$  boson are added.
- d)  $W^+W^+$  production in the VBS topology. Here all  $s$ -channel Feynman diagrams, i.e. diagrams where the incoming quarks are detached from the outgoing quarks, are discarded, while all  $t$ - and  $u$ -channel diagrams and interferences between them are retained.<sup>3</sup> This ensures that this category has no overlap with categories a–c. The  $W^+$  bosons decay into electron/electron neutrino and muon/muon neutrino.

At  $\mathcal{O}(\alpha_s^2\alpha^4)$  and  $\mathcal{O}(\alpha_s\alpha^5)$ , in the absence of other resonant channels, we include all contributions to the complete  $W^+W^+jj$  final state. As mentioned before, when investigating the quality of the on-shell approximation, a fixed renormalisation and factorisation scale of  $\mu_R = \mu_F = 3M_W$  is used.

### 3.2.1 On-shell approximations at LO

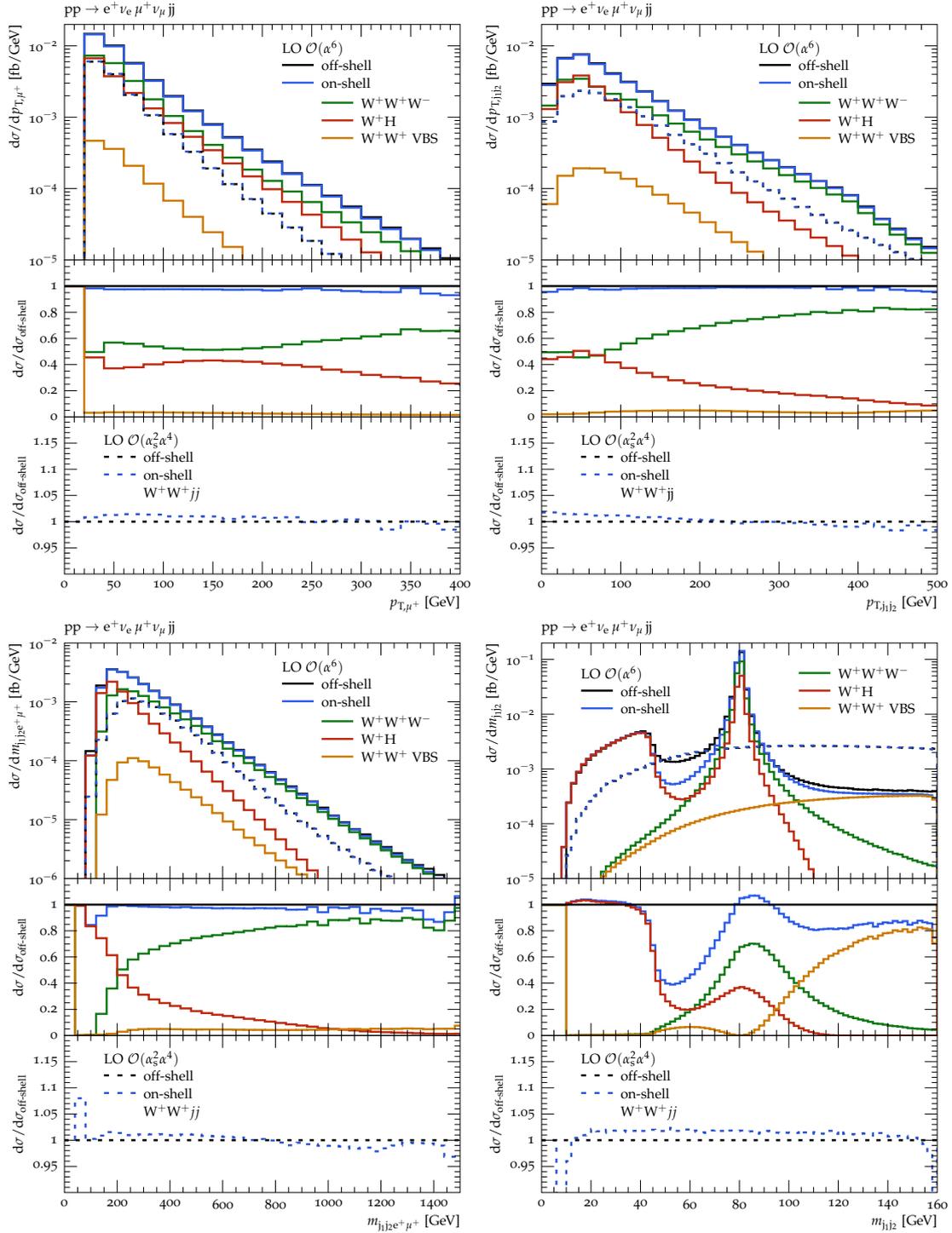
In table 2, the cross sections for all contributing on-shell processes are compared to the off-shell one at LO accuracy. We notice that the on-shell approximation reproduces the full off-shell results for the contributions of order  $\mathcal{O}(\alpha^6)$  within less than 3%, which is in agreement with the expected precision of the on-shell approximation. Most strikingly, tri-boson production makes up only 53% of the full  $\mathcal{O}(\alpha^6)$  off-shell cross section. On the other hand, WH production amounts to 41%. Thus, for this phase space, which is supposedly a tri-boson one, almost half of the cross section actually results from the Higgs-strahlung process. We note that the contribution of VBS production is at a level of few per cent only, and the WZ contribution is negligible.

For the second largest contribution to  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$ , of order  $\mathcal{O}(\alpha_s^2\alpha^4)$ , the on-shell approximation reproduces the full off-shell result very well, yielding a cross section that is larger by only 0.9%. For the interference contributions of order  $\mathcal{O}(\alpha_s\alpha^5)$ , which amount to less than one per cent of the cross section, the difference between on-shell and off-shell calculation is 4%.

Figures 5 and 6 inspect the same aspect at the differential level. The upper panels contain absolute predictions at order  $\mathcal{O}(\alpha^6)$  for the off-shell process, for the contributing on-shell processes (apart from the completely negligible  $W^+Z$  contribution) and their sum.

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<sup>3</sup>This is at variance to the traditional VBS approximation, where also interferences between  $t$ - and  $u$ -channel diagrams are neglected.



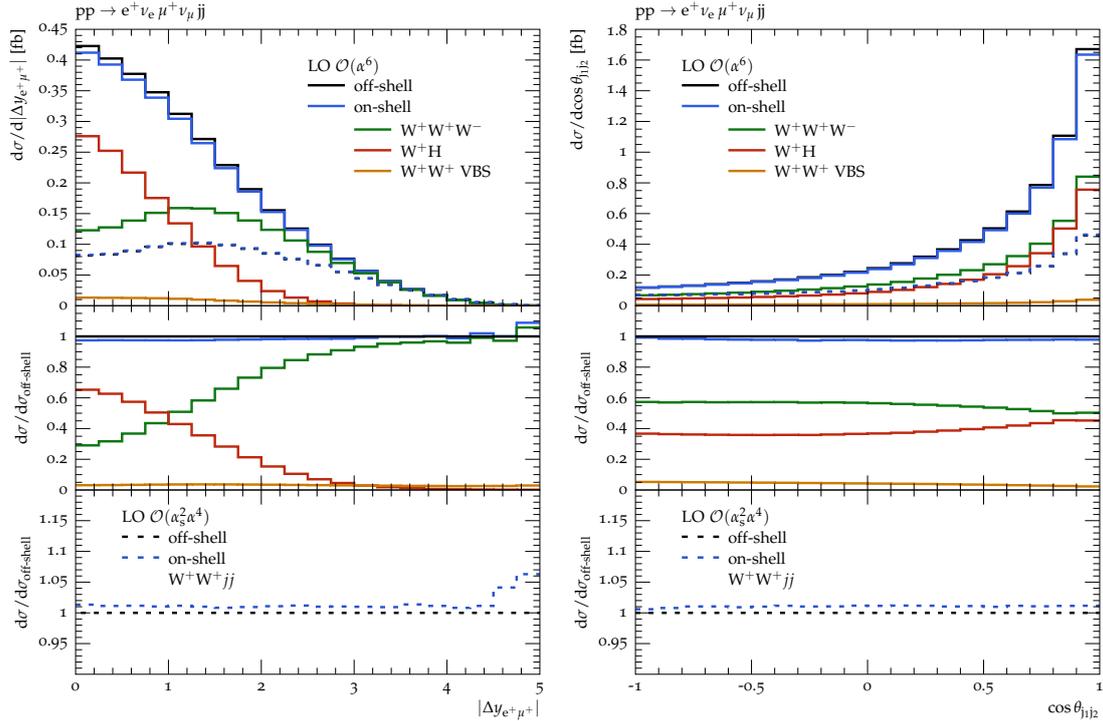
**Figure 5.** Differential distributions for the full off-shell process  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  and its relevant on-shell sub-contributions at LO, using  $\mu_R = \mu_F = 3M_W$ . The observables are: the transverse momentum of the anti-muon (top left), the transverse momentum of the two jets (top right), the invariant mass of the two jets and two charged leptons (bottom left), and the invariant mass of the two jets (bottom right).

$\mathcal{O}(\alpha^6)$	off shell	on shell	on-shell subprocess			
Process	$\mu^+\nu_\mu e^+\nu_e jj$	sum	$W^+W^+W^-$	$W^+H$	$W^+Z$	$W^+W^+$ VBS
$\sigma_{\text{LO}}[\text{fb}]$	0.7917	0.7738	0.4207	0.3265	$5 \cdot 10^{-7}$	0.0266
$\sigma/\sigma_{\text{LO}}^{\text{off shell}}[\%]$	100	97.7	53.1	41.2	$7 \cdot 10^{-5}$	3.3

$\mathcal{O}(\alpha_s^2\alpha^4)$	off shell	on shell	$\mathcal{O}(\alpha_s\alpha^5)$	off shell	on shell
Process	$\mu^+\nu_\mu e^+\nu_e jj$	$W^+W^+jj$	Process	$\mu^+\nu_\mu e^+\nu_e jj$	$W^+W^+jj$
$\sigma_{\text{LO}}[\text{fb}]$	0.2912	0.2938	$\sigma_{\text{LO}}[\text{fb}]$	0.0071	0.0074
$\sigma/\sigma_{\text{LO}}^{\text{off shell}}[\%]$	100	100.9	$\sigma/\sigma_{\text{LO}}^{\text{off shell}}[\%]$	100	104.2

**Table 2.** Cross sections for off-shell production and on-shell approximations for  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  at LO using  $\mu_R = \mu_F = 3M_W$  throughout. The statistical integration errors are at most one in the last digits shown.



**Figure 6.** Differential distributions for the full off-shell process  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  and its relevant on-shell sub-contributions at LO, using  $\mu_R = \mu_F = 3M_W$ . The observables are: the modulus of the rapidity difference between the two charged leptons (left) and the cosine of the angle between the two jets (right).

The middle panels show the ratio of the on-shell contributions to the off-shell cross section at order  $\mathcal{O}(\alpha^6)$  and the lower panels the ratio at order  $\mathcal{O}(\alpha_s^2\alpha^4)$ . We do not investigate the small interference contributions at the differential level.

While the inclusive cross section of the off-shell calculation is reproduced on the level of 2%, differential distributions show deviations of up to 20% in various phase-space regions. The differences are largest in the invariant-mass distribution of the two jets where they reach 60% for  $m_{j_1j_2} \approx 50$  GeV. They are driven by missing off-shell contributions as well as missing interference effects between the various on-shell channels which have different resonance structures. Note also that the on-shell approximation overestimates the cross section for  $m_{j_1j_2}$  below 40 GeV and close to the W-boson mass. In the distributions in  $p_{T,\mu^+}$ ,  $p_{T,j_1j_2}$ , and  $m_{j_1j_2e^+\mu^+}$ , the differences between on-shell approximation and off-shell calculation somewhat grow with the variables values.

The two transverse-momentum distributions (top row in figure 5) display a similar qualitative behaviour with WH production contributing up to 50% of the cross section at low transverse momenta while becoming small towards high  $p_T$  where tri-boson production is dominant. For the distribution in the invariant mass of the two jets and two charged leptons (bottom left in figure 5) WH production dominates for low invariant masses (below the triple-W threshold) but becomes negligible for large invariant masses. The fraction of  $W^+W^+$  VBS is roughly constant except for small invariant masses. As mentioned previously, the distribution in the invariant mass of the jet pair (bottom right in figure 5) is characterised by several resonance structures that interfere strongly. For  $M_H - M_W \lesssim m_{j_1j_2} \lesssim M_W$ , all on-shell production modes are suppressed (with at least one off-shell W or Higgs boson). On the other hand, for  $m_{j_1j_2} \gtrsim 120$  GeV, VBS production is becoming dominant as expected for high di-jet masses. We stress that this picture holds only true at LO while higher-order QCD corrections significantly modify it. In particular, tri-boson contributions are still very large for  $m_{j_1j_2} > 100$  GeV at NLO QCD as shown later (or as observed in refs. [14, 62]).

Turning to angular distributions in figure 6, WH production is dominant (at the level of 60%) at low rapidity difference between the charged leptons. At large rapidity difference tri-boson production is overwhelming, reaching more than 90% at  $|\Delta y_{e^+\mu^+}| \gtrsim 3$ . On the other hand, the distribution in the cosine of the angle between the two jets does not show strong variations in the composition.

The difference between off-shell and on-shell calculations at  $\mathcal{O}(\alpha_s^2\alpha^4)$  remains below 2% except in phase-space regions with very small cross sections.

### 3.2.2 On-shell approximations at NLO QCD

We turn to the discussion of the quality of the on-shell approximations, as defined in the beginning of this section, at NLO QCD. We start by comparing fiducial off-shell and on-shell cross sections in table 3. While the  $K$  factors are almost equal in both calculations, the difference between both predictions rises to 4% at NLO. This can be explained by the fact that we do not include NLO QCD corrections to the decays of the W boson and the Higgs boson. For an inclusive W-boson decay these corrections amount to  $\alpha_s/\pi \approx 3\%$ . The  $K$  factors for the on-shell subprocesses  $W^+W^+W^-$  and  $W^+H$  are in reasonable agreement with literature results [24, 105, 106]. The contribution of  $W^+W^+$  VBS is increased by more than a factor of

$\mathcal{O}(\alpha^6 + \alpha_s \alpha^6)$	off shell	on shell	on-shell subprocess			
Process	$\mu^+ \nu_\mu e^+ \nu_e jj$	sum	$W^+ W^+ W^-$	$W^+ H$	$W^+ Z$	$W^+ W^+$ VBS
$\sigma_{\text{NLO}}[\text{fb}]$	1.123	1.080	0.542	0.451	$1.8 \cdot 10^{-6}$	0.086
$K_{\text{NLO}}$	1.42	1.40	1.29	1.38		3.25
$\sigma/\sigma_{\text{NLO}}^{\text{off shell}}[\%]$	100	96.2	48.3	40.2	$1.6 \cdot 10^{-6}$	7.7

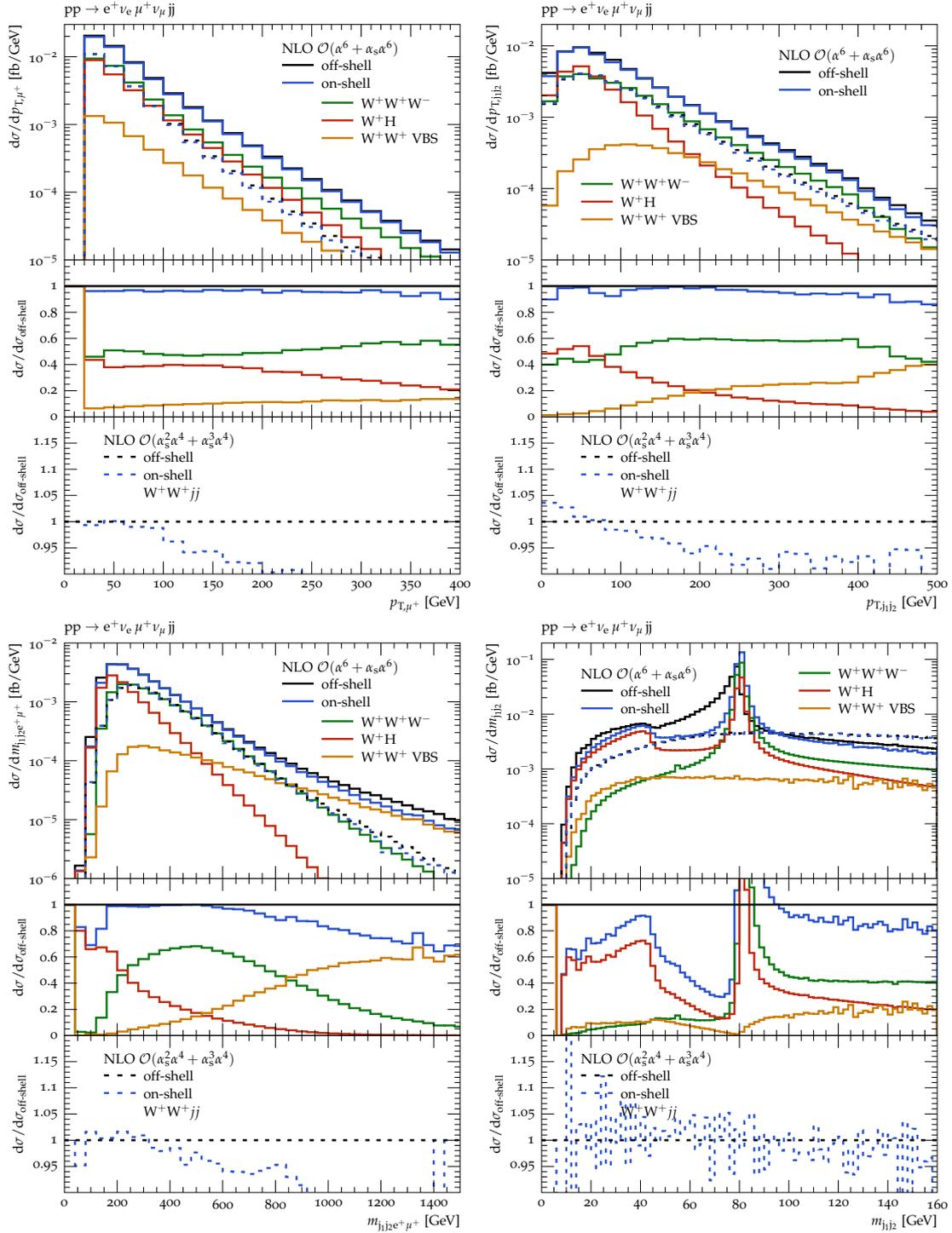
$\mathcal{O}(\alpha_s^2 \alpha^4 + \alpha_s^3 \alpha^4)$	off shell	on shell
Process	$\mu^+ \nu_\mu e^+ \nu_e jj$	$W^+ W^+ jj$
$\sigma_{\text{NLO}}[\text{fb}]$	0.525	0.520
$K_{\text{NLO}}[\text{fb}]$	1.80	1.77
$\sigma/\sigma_{\text{NLO}}^{\text{off shell}}[\%]$	100	99.0

**Table 3.** Cross sections for off-shell production and on-shell approximations for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at NLO using  $\mu_R = \mu_F = 3M_W$  throughout. The statistical integration errors are at most one in the last digits shown. Please note that, contrary to all other NLO cross sections, the off-shell  $\mathcal{O}(\alpha_s \alpha^6)$  process does not only comprise QCD-type corrections to its listed LO process but also EW-type corrections to a different Born process. The NLO  $K$  factor is defined as the ratio of the quoted LO and NLO cross sections.

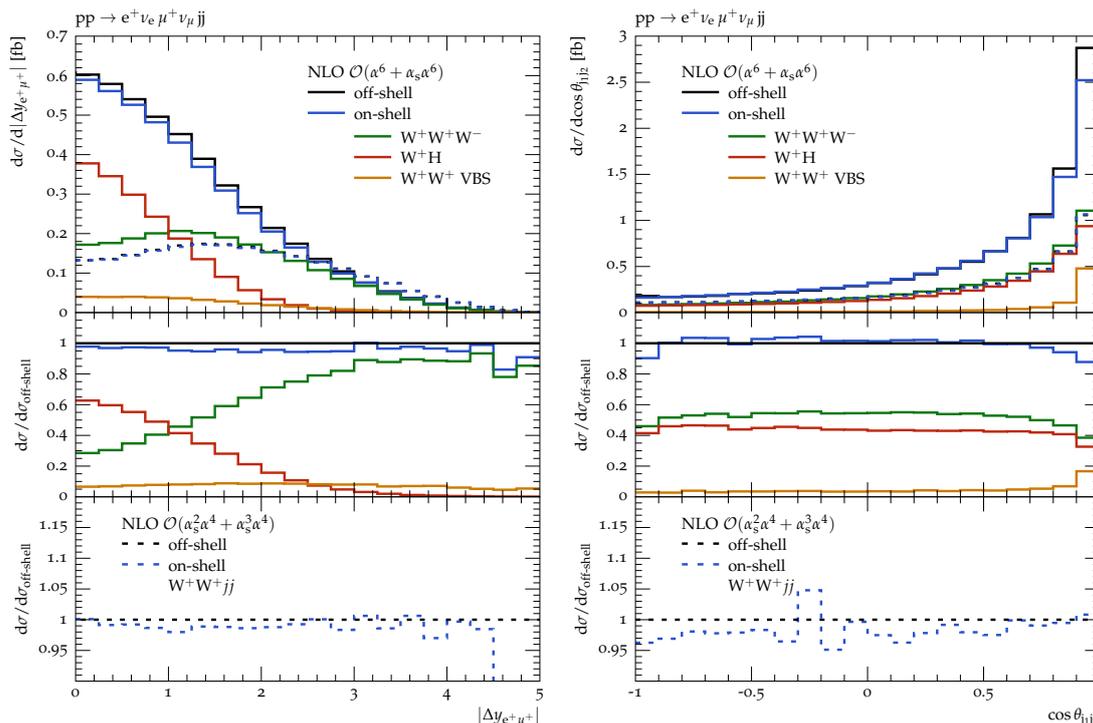
three. This can be explained as follows: at LO the VBS process is suppressed by the cuts (2.11). At NLO an additional radiated gluon can play the role of one leading jet, allowing the two quark jets to have a large pair invariant mass and rapidity separation and thus being in a phase-space region where VBS is enhanced. At orders  $\mathcal{O}(\alpha_s^2 \alpha^4 + \alpha_s^3 \alpha^4)$  the difference between on-shell and off-shell cross sections is one per cent, and the  $K$  factors agree at this level.

In figures 7 and 8 we present an analysis of the on-shell approximation at  $\mathcal{O}(\alpha^6 + \alpha_s \alpha^6)$  and  $\mathcal{O}(\alpha_s^2 \alpha^4 + \alpha_s^3 \alpha^4)$  at the differential level. We consider the same distributions as in section 3.2.1 and use the same layout as in figures 5 and 6.

At  $\mathcal{O}(\alpha^6 + \alpha_s \alpha^6)$  the on-shell approximation is considerably enhanced for values of  $m_{j_1 j_2}$  close to and slightly above  $M_W$ . This results from contributions where one of the hardest jets is a gluon allowing the invariant mass of the  $W^-$ -boson decay jet pair to be close to the resonance while  $m_{j_1 j_2}$  is larger. Obviously, this enhancement mechanism does not apply to  $W^+ W^+$  VBS. On the other hand, the on-shell approximation is strongly suppressed in the range  $M_H - M_W \lesssim m_{j_1 j_2} \lesssim M_W$ . While this region is filled in the off-shell calculation by the usual redistribution of events with real final-state radiation, this contribution is missing in our on-shell calculation that does not include QCD corrections to the bosons' decays. A similar though smaller effect is seen near the peak at  $m_{j_1 j_2} = M_H - M_W$ . In the distributions in  $p_{T, \mu^+}$ ,  $p_{T, j_1 j_2}$ , and  $m_{j_1 j_2 e^+ \mu^+}$ , the differences between on-shell approximation and off-shell calculation grow stronger with the variables than at LO. This behaviour is even more pronounced at  $\mathcal{O}(\alpha_s^2 \alpha^4 + \alpha_s^3 \alpha^4)$ , where the difference exceeds 10% in the tails. We attribute these differences



**Figure 7.** Differential distributions for the full off-shell process  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj (+j)$  and its relevant on-shell sub-contributions at NLO QCD, using  $\mu_R = \mu_F = 3M_W$ . The observables are: the transverse momentum of the anti-muon (top left), the transverse momentum of the two jets (top right), the invariant mass of the two jets and two charged leptons (bottom left), and the invariant mass of the two jets (bottom right).



**Figure 8.** Differential distributions for the full off-shell process  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj (+j)$  and its relevant on-shell sub-contributions at NLO QCD, using  $\mu_R = \mu_F = 3M_W$ . The observables are: the modulus of the rapidity difference between the two charged leptons (left) and the cosine of the angle between the two jets (right).

to contributions of off-shell diagrams that are not present in an on-shell approximation. Similar effects have, for instance, been found in high-energy tails in  $W^+W^-$  di-boson production when comparing the off-shell calculation with the double-pole approximation [107].

Looking at individual on-shell contributions we observe the following: the NLO QCD corrections raise the contribution of WH production with respect to WWW production as compared to the LO ratio. The contributions of  $W^+W^+$  VBS are enhanced for large  $p_{T\mu^+}$ , and  $m_{j_1j_2}$  close to and below  $M_W$  and, in particular for large  $p_{Tj_1j_2}$  and large  $m_{j_1j_2e^+\mu^+}$ . The mechanism of this enhancement is the same as for the NLO corrections to the full process explained in section 3.3. Note that for large  $m_{j_1j_2}$ , the fraction of  $W^+W^+$  VBS is actually reduced.

### 3.2.3 $s$ - and $t$ -channel di-jet production modes

While the decomposition of  $\mu^+ \nu_\mu e^+ \nu_e jj$  production into its on-shell channels is useful to understand the contributions of the underlying topologies, it inherently limits the accuracy of the theoretical predictions in our fiducial region. Hence, if the production of the  $\mu^+ \nu_\mu e^+ \nu_e jj$  signature is to be understood on the per-cent level this picture has to be abandoned. It is still useful, however, to distinguish between  $s$ - and  $t$ - (and/or  $u$ -) channel topologies regarding the production of the di-jet system. In particular, this distinction converts the NLO corrections of  $\mathcal{O}(\alpha_s \alpha^6)$  into pure QCD corrections to the respective  $\mathcal{O}(\alpha^6)$  Born process, which allows us

$\mathcal{O}(\alpha^6 + \alpha_s \alpha^6)$	coherent	incoherent	subchannels	
Process	$ s + t/u ^2$	$ s ^2 +  t/u ^2$	$ s ^2$	$ t/u ^2$
$\sigma_{\text{LO}}[\text{fb}]$	0.7855	0.7841	0.7576	0.0265
$\sigma_{\text{NLO}}[\text{fb}]$	1.091	1.084	0.998	0.086
$K_{\text{NLO}}$	1.39	1.38	1.32	3.3
$\sigma/\sigma_{\text{LO}}^{\text{off shell}}[\%]$	100	99.8	96.5	3.3
$\sigma/\sigma_{\text{NLO}}^{\text{off shell}}[\%]$	100	99.4	91.5	7.9

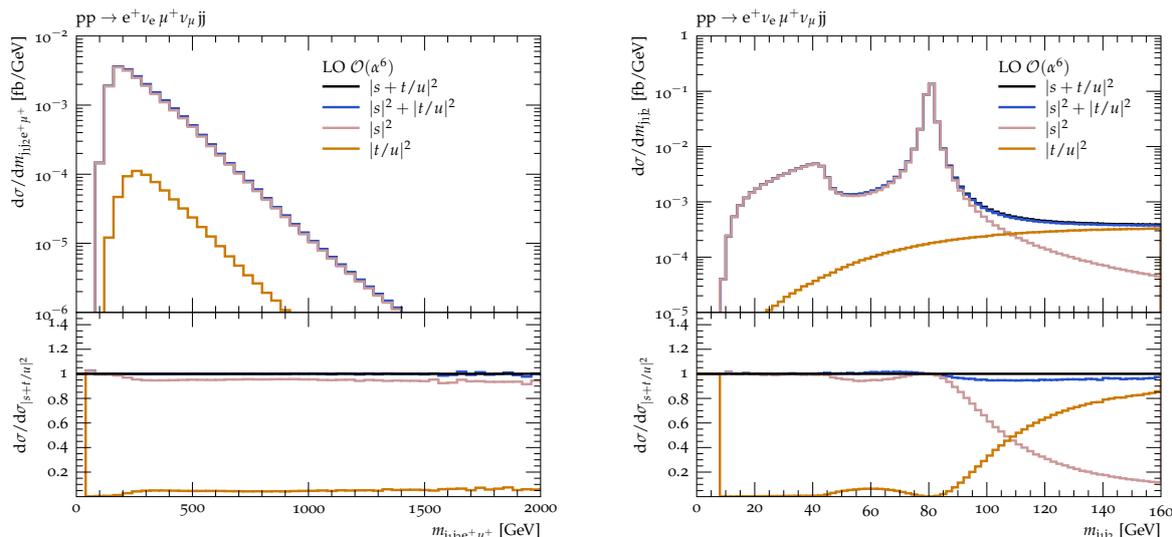
**Table 4.** Cross sections for off-shell  $s$ - and  $t/u$ -channel approximations as well as their coherent and incoherent sum at LO and NLO QCD with  $\mu_R = \mu_F = m_{T,jj} + m_{T,\nu_e e^+} + m_{T,\nu_\mu \mu^+}$ . The statistical integration errors are at most one in the last digits shown. Please note that, contrary to all other NLO cross sections, the off-shell coherent sum of  $s$  and  $t/u$  channels at  $\mathcal{O}(\alpha_s \alpha^6)$  does not only comprise QCD-type corrections to its listed LO process but also EW-type corrections to a different Born process. The NLO  $K$  factor is defined as the ratio of the quoted LO and NLO cross sections.

to employ the parton-shower matching procedure available in SHERPA without modification (see section 3.4).

While most quark-induced partonic processes comprise either  $s$  or  $t/u$  channels, and therefore such a separation is straight-forward, the partonic processes  $u\bar{d}/\bar{d}u \rightarrow \mu^+ \nu_\mu e^+ \nu_e d\bar{u}$  contain both (see table 6). We incoherently decompose these into their  $s$ - and  $t/u$ -channel contributions, thereby substituting the full squared matrix element by the sum of corresponding pure  $s$ - and  $t/u$ -channel squared matrix elements. For example, to reduce the matrix element of the partonic process  $u\bar{d} \rightarrow \mu^+ \nu_\mu e^+ \nu_e d\bar{u}$  to its  $s$ -channel contribution it is calculated by using the  $u\bar{d} \rightarrow \mu^+ \nu_\mu e^+ \nu_e s\bar{c}$  amplitude instead, while using the  $u\bar{s} \rightarrow \mu^+ \nu_\mu e^+ \nu_e d\bar{c}$  amplitude allows to calculate its  $t$ -channel part. Its original PDF is kept irrespective of a possible change of parton flavour in the stand-in matrix element. In this way, the separation is gauge invariant and all topologies are accounted for on a diagrammatic level at LO.<sup>4</sup> Of course, interferences between  $s$  and  $t/u$  channels are omitted in this approach. They, however, turn out to be negligible in most of the considered phase space. In terms of included on-shell equivalents, the  $s$  channel thus contains the WWW, WH, and WZ processes as well as all their respective interferences and off-shell contributions. Conversely, the such defined  $t/u$  channel corresponds to the VBS process defined above.

Results for the fiducial cross sections at  $\mathcal{O}(\alpha^6)$  and including NLO QCD corrections are shown in table 4. The difference between the off-shell calculation (coherent) and the incoherent sum of  $s$ -channel and  $t$ -channel contributions amounts to only 0.2% at LO and 0.6%

<sup>4</sup>Note that such a splitting based on partonic processes is not possible at NLO owing to the appearance of quark-gluon channels that involve diagrams with both  $s$ -channel and  $t$ -channel vector-boson propagators. Hence, for these partonic processes, we rely on a diagram selection only. In the  $s$ -channel contribution we include diagrams featuring no EW  $t/u$ -channel propagator (setting the SHERPA parameter `Max_N_TChannels=0`), whereas for the  $t$ -channel mode we demand diagrams to have at least one such propagator, imposed through the setting `Min_N_TChannels=0`. We employ this separation for calculating both the NLO QCD fixed-order cross sections of this subsection and the parton-shower-matched results of section 3.4.



**Figure 9.** Differential distributions for the full off-shell process  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj (+j)$  and its decomposition into  $s$  and  $t/u$  channels at LO  $\mathcal{O}(\alpha^6)$ , using  $\mu_R = \mu_F = m_{T,jj} + m_{T,\nu_e e^+} + m_{T,\nu_\mu \mu^+}$ . The observables are: the invariant mass of the two jets and two charged leptons (left), and the invariant mass of the two jets (right).

at NLO. The corresponding  $K$  factors are practically identical. Note also that the results for the off-shell  $t/u$ -channel contributions in table 4 are almost identical to those of the on-shell calculation in tables 2 and 3. The differences between the off-shell  $s$ -channel contributions in table 4 and the sum of the corresponding terms in tables 2 and 3 are at the level of 1%.

In figure 9 we depict the contributions of  $s$  and  $t$  channels as well as their coherent and incoherent sum for the distributions in the invariant masses of the charged leptons and the two leading jets as well as the di-jet system at LO  $\mathcal{O}(\alpha^6)$ . The upper panels show the nominal distributions and the lower ones when normalising to the coherent sum, i.e. the full off-shell result. For the distribution in  $m_{j_1 j_2 e^+ \mu^+}$ , as for all other distributions considered in figures 5 and 6, practically no differences between the coherent and incoherent sum are visible. The noticeable difference in the  $m_{j_1 j_2}$  distribution for invariant masses above  $M_W$  can be attributed to interferences between  $s$ -channel and  $t/u$ -channel contributions. Nonetheless, the size of the  $s$ - $t/u$  interference does not exceed 5% locally in this region, indicating the usefulness of this separation.

### 3.3 Full NLO predictions

We turn to NLO corrections to the full process (2.1) which consist of four contributions at orders  $\mathcal{O}(\alpha^7)$ ,  $\mathcal{O}(\alpha_s \alpha^6)$ ,  $\mathcal{O}(\alpha_s^2 \alpha^5)$ , and  $\mathcal{O}(\alpha_s^3 \alpha^4)$ . Their absolute and relative values with respect to the full LO prediction are given in table 5. The largest contributions are the ones of order  $\mathcal{O}(\alpha_s \alpha^6)$  and  $\mathcal{O}(\alpha_s^3 \alpha^4)$  and amount to 29.0% and 21.5%, respectively. Even if the order  $\mathcal{O}(\alpha_s \alpha^6)$  is a mixture of EW and QCD corrections, it is to a good approximation dominated by the QCD ones. Hence the NLO corrections are dominated by the QCD ones. It is worth emphasising that the relative corrections are normalised to the full LO prediction. If normalised to the corresponding Born contribution of order  $\mathcal{O}(\alpha_s^2 \alpha^4)$ , the  $\mathcal{O}(\alpha_s^3 \alpha^4)$  corrections

order	$\mathcal{O}(\alpha^7)$	$\mathcal{O}(\alpha_s\alpha^6)$	$\mathcal{O}(\alpha_s^2\alpha^5)$	$\mathcal{O}(\alpha_s^3\alpha^4)$	sum
$\delta\sigma[\text{fb}]$	-0.035(1)	0.305(1)	-0.0032(3)	0.2260(3)	0.493(2)
$\delta\sigma/\sigma_{\text{LO}}^{\text{sum}}[\%]$	-3.4	29.0	-0.30	21.5	46.9

**Table 5.** Cross sections at NLO accuracy for  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  at the LHC for the four different orders and their sum using the dynamical scale of eq. (2.2). The second line contains the absolute corrections while the third one contains the relative corrections normalised to the full LO prediction.

almost reach 90%. Such large QCD corrections have actually been already observed in the literature for low invariant mass (see figure 10 of ref. [36]). The corrections of order  $\mathcal{O}(\alpha^7)$  are negative and amount to about 3.5%. This is in the same ballpark as the EW corrections for triple-W production with fully leptonic decays [21, 22] or W-pair production [107]. On the other hand, the mixed corrections at order  $\mathcal{O}(\alpha_s^2\alpha^5)$  are at the per mille level. The smallness of this contribution has also been observed in the VBS phase space [12] but is due to an accidental cancellation. For VBS ZZ [43, 62], these corrections are, as expected, negative at the level of few per cent with respect to the Born contribution of order  $\mathcal{O}(\alpha_s^2\alpha^4)$ . In general, for the present case, the hierarchy of the corrections at full NLO accuracy is rather natural and in stark contrast to the one observed in the VBS phase space [12].

The full NLO prediction for the production cross section reads

$$\sigma_{\text{NLO}} = 1.545(2)_{-5.1\%}^{+6.1\%} \text{ fb.} \tag{3.2}$$

The total corrections are about 47%, and the scale uncertainty slightly increases with respect to the LO. It is worth noticing that in the VBS phase space, the scale uncertainty is [+11.66%, -9.44%] at LO and [+1.2%, -2.7%] at NLO [12]. In the latter case, the small NLO scale variation is related to the small size of the QCD corrections.

At first sight, these findings seem to be in contradiction with those of ref. [12]. In particular, the EW corrections here have a moderate size, while they have been shown to be intrinsically large for VBS at the LHC [18]. In the present case, the EW corrections amount to -4.6% when normalised to the LO of order  $\mathcal{O}(\alpha^6)$ . It is worth noticing that the photon-induced corrections contribute +2.6%. Thus, the corrections for the  $qq'$  channels are -7.2%, which is very close to the -7.8% stated in table 3 of ref. [22] for the fully leptonic final state. Also, NLO EW corrections for WH production have been found to be at the level of -7% [29, 108]. As explained previously, tri-boson production, WH production, and VBS share identical final states and differ only in their phase space. In ref. [18], the fiducial cross section at LO is 1.5348(2) fb, and the relative EW corrections -16.0%. In table 6, absolute predictions and relative EW corrections at order  $\mathcal{O}(\alpha^7)$  are presented for each partonic channel in the present setup and in the one of ref. [18]. It is interesting to observe that the relative EW corrections do not vary significantly over the partonic channels. This has already been found in refs. [62] and [63] for VBS ZZ and opposite-sign VBS WW. Thus, large EW corrections are an intrinsic feature of VBS at the LHC, when VBS contributions are dominating. In contrast, in phase spaces where VBS contributions are suppressed while WH or tri-boson ones are enhanced, the EW corrections typically are below 10%.

		tri-boson phase space		VBS phase space	
partonic channel	kin.	$\sigma_{\text{ch}}/\sigma_{\text{all}}[\%]$	$\delta_{\text{NLO EW}}[\%]$	$\sigma_{\text{ch}}/\sigma_{\text{all}}[\%]$	$\delta_{\text{NLO EW}}[\%]$
uu $\rightarrow \mu^+\nu_\mu e^+\nu_e d\bar{d}$	$t, u$	1.2	-6.3	67.4	-17.1
uc/cu $\rightarrow \mu^+\nu_\mu e^+\nu_e d\bar{s}$	$t$	0.44	-5.3	5.8	-13.5
u $\bar{d}$ / $\bar{d}u$ $\rightarrow \mu^+\nu_\mu e^+\nu_e d\bar{u}$	$t, s$	49.0	-7.2	16.7	-13.9
u $\bar{d}$ / $\bar{d}u$ $\rightarrow \mu^+\nu_\mu e^+\nu_e s\bar{c}$	$s$	48.2	-7.1	0.007	-30.1
u $\bar{s}$ / $\bar{s}u$ $\rightarrow \mu^+\nu_\mu e^+\nu_e d\bar{c}$	$t$	0.51	-5.2	8.3	-13.1
$\bar{d}\bar{s}/\bar{s}\bar{d}$ $\rightarrow \mu^+\nu_\mu e^+\nu_e \bar{u}\bar{c}$	$t$	0.27	-3.6	0.7	-12.2
$\bar{d}\bar{d}$ $\rightarrow \mu^+\nu_\mu e^+\nu_e \bar{u}\bar{u}$	$t, u$	0.31	-4.1	1.0	-12.1
pp $\rightarrow \mu^+\nu_\mu e^+\nu_e j\bar{j}$	-	100	-7.1	100	-16.0

**Table 6.** Partonic channels of  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e j\bar{j}$ . For each of them, its kinematic channels are indicated. For the tri-boson (present work) and the VBS phase space [18] the relative contribution of each partonic channel with respect to the hadronic process at order  $\mathcal{O}(\alpha^6)$  and the relative EW  $\mathcal{O}(\alpha^7)$  corrections of this channel are provided. In this table, the partonic channels of different quark generations, related by simultaneously replacing  $u \leftrightarrow c$  and  $d \leftrightarrow s$ , are merged together as they only differ by their PDF.

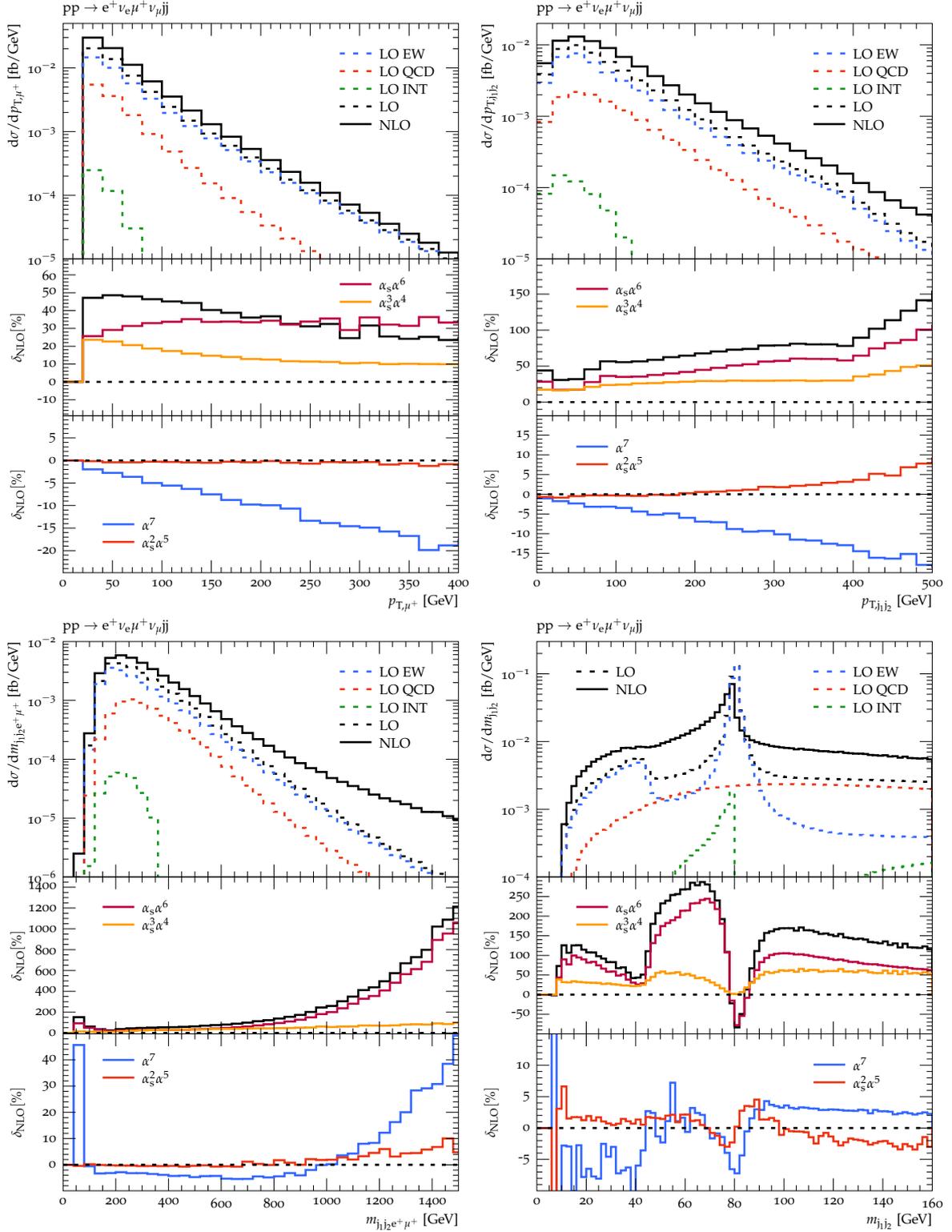
Note that the QCD corrections at order  $\mathcal{O}(\alpha_s\alpha^6)$  are significantly larger than in the VBS phase space (29% here against -3.5% for the VBS selection). Nonetheless, this should not come as a surprise as tenths of per cent is the usual size of QCD corrections at the LHC. Rather, the corrections are exceptionally small for VBS.

Finally, the corrections of order  $\mathcal{O}(\alpha_s^3\alpha^4)$  are much larger in the present setup than in the VBS phase space. This can be understood from figure 10 of ref. [36] where the QCD corrections have been found to become large when approaching the region of low invariant mass of the two jets. In the present setup, the invariant mass of the two jets is around the W-boson mass while in a typical VBS phase space it is required to be above 500 GeV. In ref. [36], the reason for the large corrections for low invariant mass is attributed to gluons splitting from quarks, forming hence a small invariant-mass jet pair, while the other quark jet is not tagged.

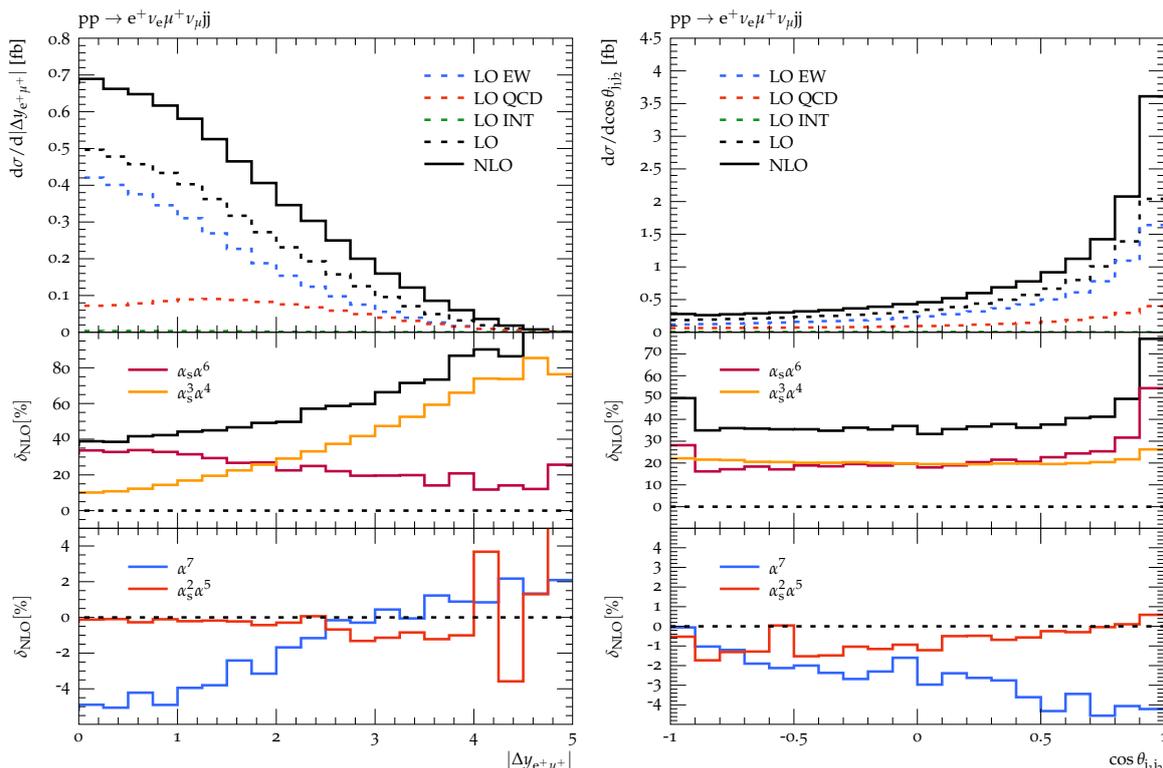
In the following, several differential distributions are discussed. First, in figures 10 and 11, all LO and NLO contributions are shown separately. While the upper panels contain the full NLO predictions and the separate LO contributions, the middle and lower panels show the full NLO corrections relative to the full LO along with the contributions of individual NLO orders.

The distribution in the transverse momentum of the anti-muon (top left in figure 10) rapidly falls off towards high energy. The  $\mathcal{O}(\alpha_s\alpha^6)$  corrections exceed 20% in the lowest bin and increase up to almost 40% at 200 GeV where they reach a plateau. The  $\mathcal{O}(\alpha_s^3\alpha^4)$  corrections are at the same level as the  $\mathcal{O}(\alpha_s\alpha^6)$  ones in the first bin but decrease for higher transverse momenta. The  $\mathcal{O}(\alpha_s^2\alpha^5)$  corrections are essentially zero across the whole phase space as for the fiducial cross section. The  $\mathcal{O}(\alpha^7)$  corrections, on the other hand, display the typical Sudakov behaviour with negative corrections reaching -15% at  $p_{T,\mu^+} \approx 300$  GeV.

For the distribution in the transverse momentum of the two jets (top right in figure 10),



**Figure 10.** Differential distributions at full NLO accuracy for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ . The observables are: the transverse momentum of the anti-muon (top left), the transverse momentum of the two jets (top right), the invariant mass of the two jets and two charged leptons (bottom left), and the invariant mass of the two jets (bottom right).



**Figure 11.** Differential distributions at full NLO accuracy for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ . The observables are: the modulus of the rapidity difference between the two charged leptons (left) and the cosine of the angle between the two jets (right).

the  $\mathcal{O}(\alpha^7)$  corrections display a similar qualitative behaviour. On the other hand, the  $\mathcal{O}(\alpha_s^2 \alpha^5)$  corrections are at the per-mille level up to 250 GeV and thereafter increase slightly to reach 5% close to 450 GeV and almost 15% for  $p_{T,j_1 j_2} = 600$  GeV (not shown). The corrections of order  $\mathcal{O}(\alpha_s^3 \alpha^4)$  increase to almost 30% at 400 GeV, while the  $\mathcal{O}(\alpha_s \alpha^6)$  ones are larger, reaching about 60% at this transverse momentum. At about 70 GeV in the second and third bin, there is a dip in the distribution, which is driven uniquely by the corrections of order  $\mathcal{O}(\alpha_s \alpha^6)$ . This is related to the presence of the W boson which decays hadronically, a configuration which does not exist for the QCD-induced contribution. While at LO the two leading jets dominantly result from the decay of the W boson that originates from a Higgs boson, at NLO one of the leading jets can be a bremsstrahlung jet and  $p_{T,j_1 j_2}$  does not correspond to the transverse momentum of the W boson and tends to be higher. The stronger increase of the relative corrections above 400 GeV is related to a faster drop of the LO EW cross section in this region. The latter results from an interplay of the cut (2.11) on  $m_{jj}$  and the jet recombination parameter  $R = 0.4$ . For  $p_{T,jj} \gtrsim 400$  GeV, jet pairs with  $m_{jj} < 160$  GeV get more and more recombined and the corresponding events are cut away.

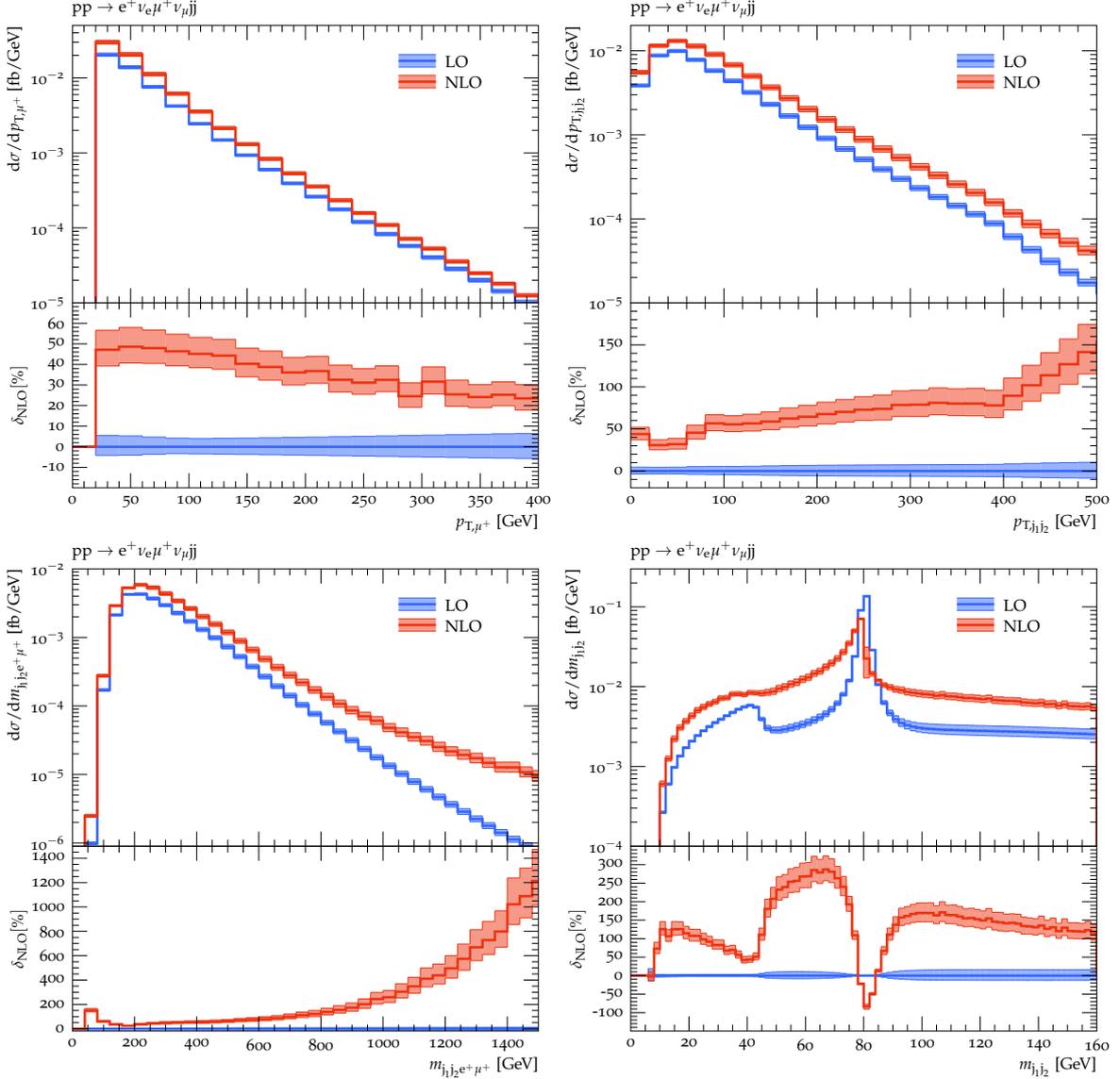
The distribution in the invariant mass of the visible system (two charged leptons and two jets, shown in the lower-left panel of figure 10) is particularly interesting. At high energy, the corrections of orders  $\mathcal{O}(\alpha_s^3 \alpha^4)$  and  $\mathcal{O}(\alpha_s \alpha^6)$  become very large. This is particularly true for the latter one which exceeds 100% at 900 GeV. This dramatic effect is due to contributions of  $t$ -channel topologies, i.e. partonic processes where quark lines

run from the initial to the final state (see figures 4(a) and 4(b)). The hadronically decaying W boson is faked by a quark-gluon pair, while the second quark jet is subleading or cut away. These diagrams are enhanced by a  $t$ -channel W boson similarly to VBS topologies (see figure 4(c)). Obviously, such contributions are neither present in genuine triple-W-production contributions nor at LO in the full process. It is worth noticing that the EW corrections of order  $\mathcal{O}(\alpha^7)$  do not become negatively large under the influence of Sudakov logarithms in the high-energy region of this distribution. The reason for this is twofold: on the one hand, photon-induced contributions become large, reaching +20% at 1 TeV, and compensate the negative Sudakov-like contributions. Note that at this energy, the cross section is strongly reduced, and so even if relatively significant, photon-induced contributions remain small in absolute terms. On the other hand, a large invariant mass does not imply the Sudakov regime, because some invariants may still be rather small [107]. It has been demonstrated that logarithmic terms of the form  $\alpha \log^2(t/s)$  may cause large corrections for processes involving  $t$ -channel propagators [109, 110] that can be of the order of 10% at LHC energies [110].

The distribution in the invariant mass of the two leading jets (bottom right in figure 10) receives large corrections below the W-mass peak. This is characteristic for final-state radiation that carries away energy and therefore shifts events from the peak to below it. In addition, large corrections of order  $\mathcal{O}(\alpha_s \alpha^6)$  [and to a lesser extent  $\mathcal{O}(\alpha_s^3 \alpha^4)$ ] are observed above the peak. This region opens up at NLO, owing to the decaying quarks of the W boson recombining into one jet while the QCD radiation makes up the second hard jet [14, 62], while it is suppressed at LO. The EW corrections of  $\mathcal{O}(\alpha^7)$  have a similar structure as the corrections of order  $\mathcal{O}(\alpha_s \alpha^6)$  albeit at a reduced level but do not rise towards very small jet-jet invariant masses. The even smaller variation of the relative  $\mathcal{O}(\alpha_s^2 \alpha^5)$  corrections follows the one of  $\mathcal{O}(\alpha_s^3 \alpha^4)$  except for the region of high  $m_{j_1 j_2}$ . As mentioned above (see figure 5), the peak in the distribution around 40 GeV is due to the WH contribution [63, 111]. All in all, we can observe that the full LO structure with two sharp structures is strongly distorted by higher-order corrections which smear these shapes.

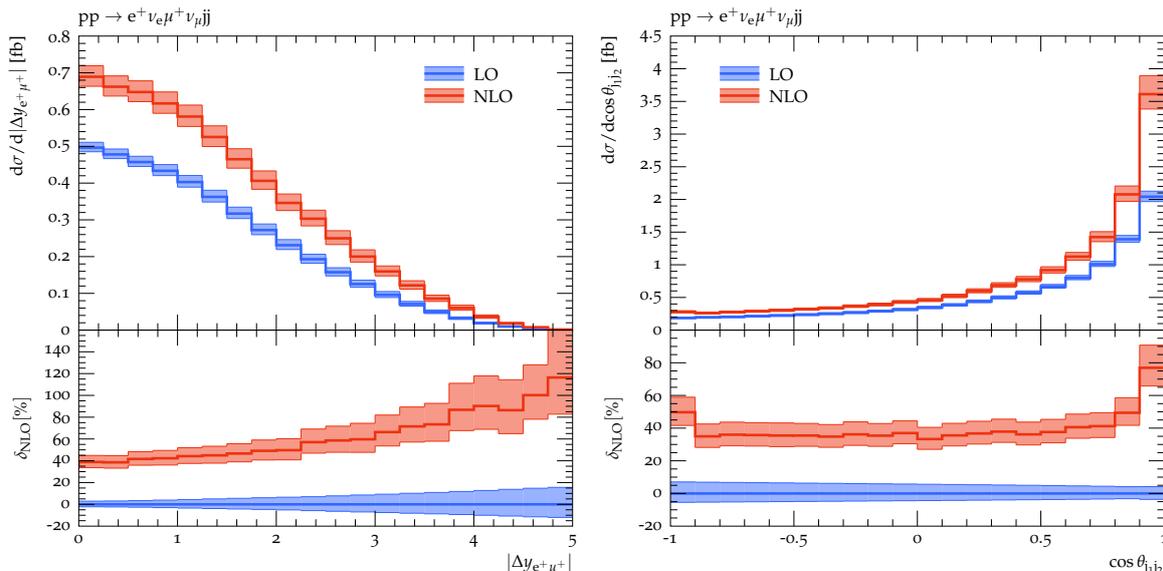
For the distribution in the rapidity difference of the two charged leptons, presented in the left panel of figure 11, the  $\mathcal{O}(\alpha_s^2 \alpha^5)$  corrections remain very small across the whole phase space. The EW corrections of order  $\mathcal{O}(\alpha^7)$  are only negative in the region of low rapidity difference where the bulk of the cross section sits. This is due to the large positive ( $\sim +7\%$ ) photon-induced contribution in the high-rapidity regions. Finally, the corrections of order  $\mathcal{O}(\alpha_s^3 \alpha^4)$  and  $\mathcal{O}(\alpha_s \alpha^6)$  display an opposite behaviour. The former are minimal at low rapidity differences and larger at high differences, while the latter reach their maximum for small and decrease for larger rapidity separations.

Finally, the distribution in the cosine of the angle between the two jets (right in figure 11) is maximal at  $\cos \theta_{jj} \sim 1$ , i.e. when the two jets are close to each other. All the different NLO corrections show only small variations in this distribution with the noticeable exception of those of order  $\mathcal{O}(\alpha_s \alpha^6)$ , which increase towards  $\cos \theta_{jj} \sim 1$  and reach about 55% in the last bin. This results again from real contributions with a gluon jet collinear to a quark jet that fake the hadronically decaying W boson and thus evade the invariant-mass cut (2.11). Note that in this region, the EW corrections are compensated by photon-induced contributions, which reach +4% in the right-most bin.



**Figure 12.** Differential distributions at full NLO accuracy (combined) for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ . The observables are: the transverse momentum of the anti-muon (top left), the transverse momentum of the two jets (top right), the invariant mass of the two jets and two charged leptons (bottom left), and the invariant mass of the two jets (bottom right).

In figures 12 and 13, the same distributions as in figures 10 and 11 are displayed, but for the full NLO and LO predictions including the respective scale uncertainty. While the upper panels show the absolute LO and NLO predictions, the lower panels display these contributions normalised to the LO predictions at the central scale. For the distribution in the transverse momentum of the anti-muon (top left of figure 12), the NLO corrections are around 50% at low values and decrease smoothly under the influence of the  $\mathcal{O}(\alpha_s^3 \alpha^4)$  and  $\mathcal{O}(\alpha^7)$  corrections to reach 20% at 400 GeV. The NLO corrections to the distribution in the transverse momentum of the two jets (top right of figure 12) display an opposite behaviour. At about 50 GeV, there appears the dip originating from the  $\mathcal{O}(\alpha_s \alpha^6)$  corrections



**Figure 13.** Differential distributions at full NLO accuracy (combined) for  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ . The observables are: the modulus of the rapidity difference between the two charged leptons (left) and the cosine of the angle between the two jets (right).

discussed above. For smaller  $p_{T,jj}$ , the total corrections are slightly below 50% while above they grow very large to exceed 80% above 400 GeV. For the distribution in the invariant mass of the visible system (bottom left of figure 12), the corrections are also very large. They are at the level of 80% at 600 GeV, and the NLO cross section is an order of magnitude larger than the LO one at 1.5 TeV. The distribution in the invariant mass of the two jets (bottom right of figure 12) receives positive corrections apart from the bin around the W mass where the corrections are at the level of  $-50\%$ . This feature is largely due to the  $\mathcal{O}(\alpha_s \alpha^6)$  corrections. The corrections to the distribution in the rapidity difference of the two charged leptons (left of figure 13) are significant. They are minimal for small  $|\Delta y_{e^+ \mu^+}|$ , where they reach 40% in the bulk of the cross section. For more extreme phase-space regions with large rapidity differences, the corrections are very large and grow up to almost 100%. Finally, the corrections to the distribution in the cosine of the angle between the two jets (right of figure 13) are rather flat in most of the phase space. They are at the level of 40% almost everywhere apart from the two bins close to  $\cos \theta = 1$ , where most of the cross section is located and where the corrections reach 80% mainly driven by the  $\mathcal{O}(\alpha_s \alpha^6)$  corrections. Note that the NLO corrections are in general much larger than the LO scale variation, i.e. the scale variation does not provide a good measure of the uncertainty of the calculation.

### 3.4 NLO QCD matched to parton shower with virtual EW approximation

In this section, parton-shower-matched predictions for off-shell  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  production, supplemented by EW corrections in the virtual EW approximation, are presented. In particular, they are compared against the full fixed-order results. While in a full computation, EW and QCD contributions cannot be separated unambiguously beyond LO [12], this distinction is nonetheless typically made in experimental analyses or new-physics studies.

To this end, we follow the approach of ref. [45] and separate all contributions (see figure 2) into QCD and EW production processes as follows:

**QCD production.** We define the QCD production mode at LO by the  $\mathcal{O}(\alpha_s^2\alpha^4)$  process. Its QCD corrections include all NLO contributions of  $\mathcal{O}(\alpha_s^3\alpha^4)$  while those of  $\mathcal{O}(\alpha_s^2\alpha^5)$  constitute its EW corrections. The latter can also be understood as QCD corrections to the LO QCD-EW interference of  $\mathcal{O}(\alpha_s\alpha^5)$  and, accordingly, contain diagrams with up to three resonant W bosons interfered with diagrams involving a gluon exchange and at most two resonant W bosons, e.g. the interference of diagrams illustrated in figures 3(b) and 1(e). Our parton-shower-matched MC@NLO calculation includes the full off-shell LO and NLO QCD contributions, but the  $\mathcal{O}(\alpha_s^2\alpha^5)$  EW corrections, owing to this feature, are omitted. However, as shown in section 3.3, the  $\mathcal{O}(\alpha_s^2\alpha^5)$  corrections are rather small. Nonetheless, QED corrections are accounted for through SHERPA’s YFS soft-photon resummation. Note, in the MC@NLO simulation of this QCD production process, in contrast to the fixed-order calculations, all QCD couplings get evaluated at the reconstructed emission scale associated to the corresponding parton, i.e. its relative transverse momentum [112, 113]. This applies to the two powers of  $\alpha_s$  present at Born level, the QCD NLO correction, as well as all subsequent parton-shower splittings.

**EW production.** Likewise, the EW production mode is defined at LO by the  $\mathcal{O}(\alpha^6)$  process, and its QCD and EW corrections at  $\mathcal{O}(\alpha_s\alpha^6)$  and  $\mathcal{O}(\alpha^7)$ , respectively. This time, the QCD corrections can also be interpreted as the EW corrections to the LO QCD-EW interference, containing, for example interferences of diagrams with three resonant  $s$ -channel W bosons (such as illustrated in figure 1(a)) and those missing the hadronic resonant decay (see figure 3(c)). The parton-shower-matched MC@NLO calculation is split into its  $s$ - and  $t/u$ -channel components, as explained in section 3.2.3, but otherwise treated fully off shell. We neglect the  $s$ - $t/u$ -channel interference, which has been found to be small in section 3.2.3. Foremost, this separation implies that QCD radiation off the  $s$ -channel resonant W decay does not interfere with any other source of radiation, allowing for a standard matching prescription as the virtuality of intermediate resonances is preserved. Likewise, it removes all EW-type divergences described above in the  $\mathcal{O}(\alpha_s\alpha^6)$  corrections, reducing the complexity of the combination with the parton shower to a standard QCD MC@NLO matching. In each case, the calculation contains the full respective LO and NLO QCD contributions, while the EW corrections are added in the  $\text{EW}_{\text{virt}}$  scheme. As above, SHERPA’s YFS soft-photon resummation provides QED corrections. As for the QCD production channel, in the matched calculation all coupling factors for QCD emissions get evaluated at their relative transverse momentum. For the EW production mode, this affects the NLO QCD correction and all parton-shower emissions.

In line with the above findings, we neglect the LO QCD-EW interference of  $\mathcal{O}(\alpha_s\alpha^5)$  in the parton-shower-matched predictions.

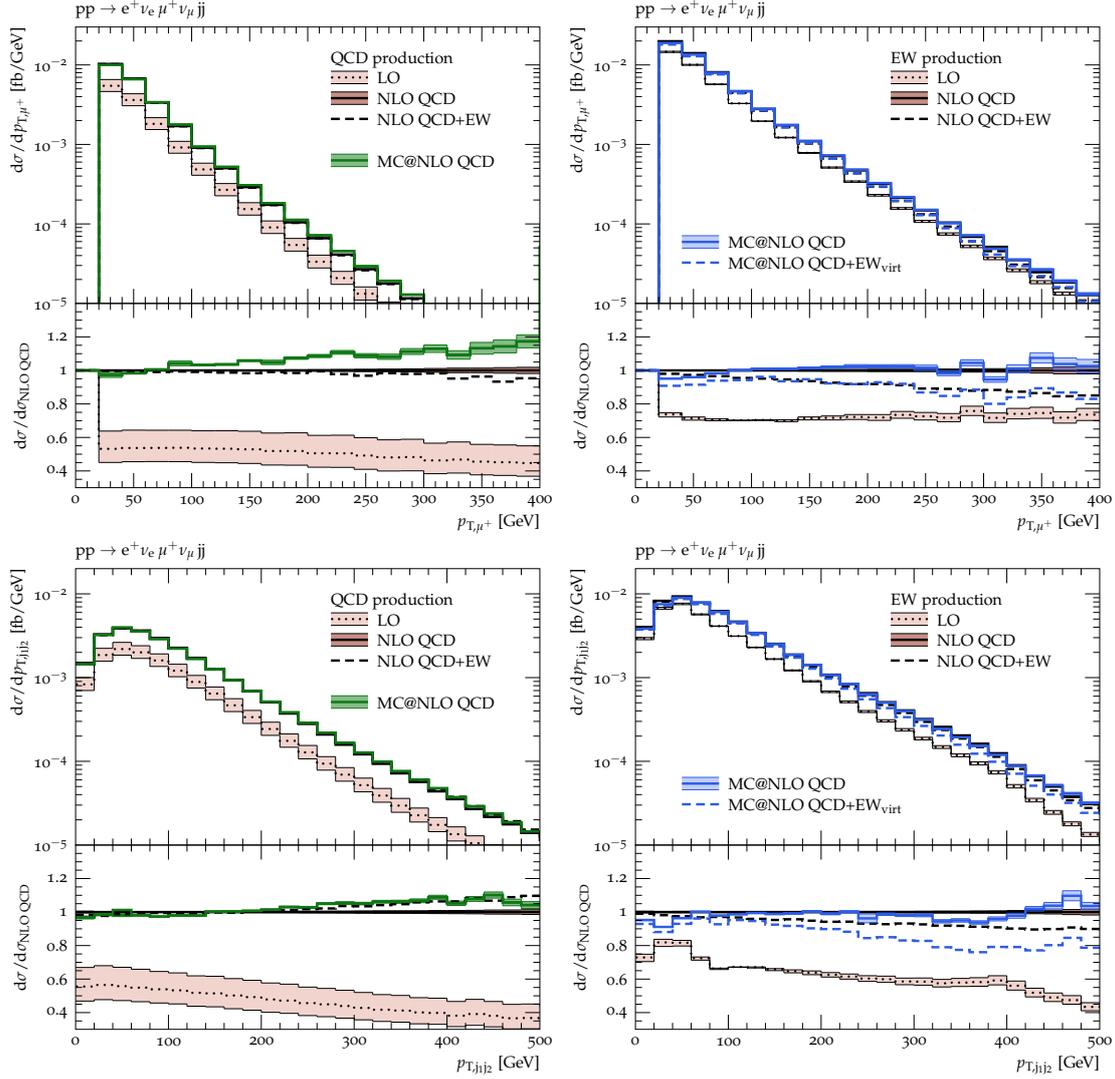
In table 7 we collate fiducial cross sections for the QCD and EW production modes, comparing the results obtained with the shower-matched calculation and the off-shell NLO QCD and full NLO prediction. Considering the separation into QCD and EW production, we here ignore the mixed LO contribution of order  $\mathcal{O}(\alpha_s\alpha^5)$ , which contributes about 0.007 fb

		NLO		MC@NLO		
		QCD	QCD+EW	QCD	QCD+EW <sub>virt</sub>	
QCD prod.	$\sigma[\text{fb}]$	0.485	0.482	0.484	—	
EW prod.	$\sigma[\text{fb}]$	$ s + t/u ^2$	1.091	1.056		
		$ s ^2 +  t/u ^2$	1.084	—	1.017	0.973
		$ s ^2$	0.998		0.882	0.842
		$ t/u ^2$	0.086		0.135	0.131

**Table 7.** Comparison of fiducial cross sections for the QCD and EW production processes at full NLO and from MC@NLO calculations with SHERPA. For the EW production MC@NLO result a break down into the kinematic  $s$ - and  $t/u$ -channel contributions is provided.

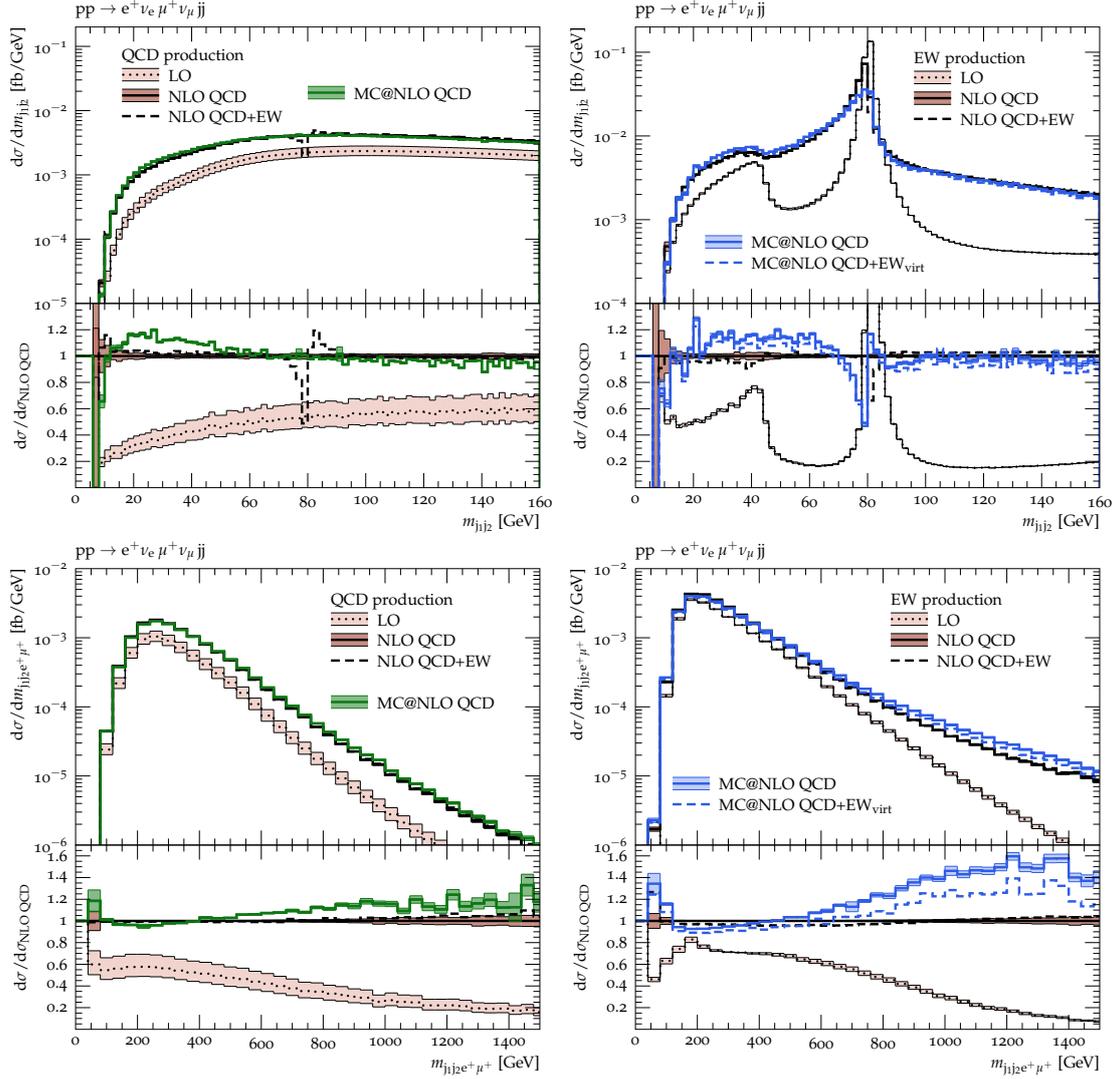
to the total cross section (see table 1). Notably, for the QCD production mode the full NLO QCD calculation and the MC@NLO result agree very well to within 1%. We do not include EW corrections for the shower-matched calculations for this component, which, however, for the full calculation amount to  $-0.6\%$  only.

In the EW production process, at NLO QCD accuracy, the two calculations differ by 7% (1.017 fb versus 1.091 fb), with the MC@NLO simulation predicting a lower cross section. When studying the quality of the incoherent  $s$ - and  $t/u$ -channel approximation in section 3.2.3 for the fixed-order calculation, we instead observed a difference between the coherent and the incoherent result of only 0.6% (see table 4). In consequence, the larger difference observed is attributed to the additional parton-shower corrections beyond NLO accuracy. Similarly, the admixture of the  $s$ - and  $t/u$ -channel contributions is altered in the MC@NLO simulation. While the  $t/u$ -channel component contributes about 8% to the fixed-order NLO result, it makes up 13% of the full MC@NLO result. This is a consequence of the fact that the  $t/u$ -channel process already experienced much larger NLO corrections than the  $s$ -channel process which was driven by the emission of an additional parton as that parton made the presence of a central jet-pair within the required mass window much more likely. The scale choice for the QCD coupling in the MC@NLO calculation and additional shower emissions further increase this probability and hence the cross section of this subprocess by another 52% wrt. the fixed-order NLO QCD result. Conversely, the  $s$ -channel process loses 12% of its events through the explicit modelling of additional radiation with the parton shower. Explicitly resolving the multiple-emission kinematics leads to a more precise modelling of the radiative energy loss of the jet through out-of-cone radiation, i.e. radiation at angles large enough not to be recaptured by the jet recombination procedure, leading to a significant number of jets falling below the jet- $p_T$  threshold and thus reducing the fiducial cross section. Through their respective characteristic di-jet correlations this leads to a larger impact in the  $s$  channel, dominated by WWW and WH topologies, as compared to the  $t$  channel. Taken together, this changes not only the composition of the combined sample, but also reduces its combined cross section by about 6% (1.017 fb versus 1.084 fb).



**Figure 14.** Parton-shower-matched predictions for the QCD and EW production modes contributing to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$ . For the QCD channel (left panels) we compare the MC@NLO results at NLO QCD accuracy obtained from SHERPA with the LO and NLO QCD and QCD+EW predictions. For the EW channel (right panels) we in addition include approximate EW corrections in the MC@NLO calculation, labelled as MC@NLO QCD+EW<sub>virt</sub>. Results are shown for the transverse momentum of the anti-muon (top row) and the transverse momentum of the di-jet system (bottom row).

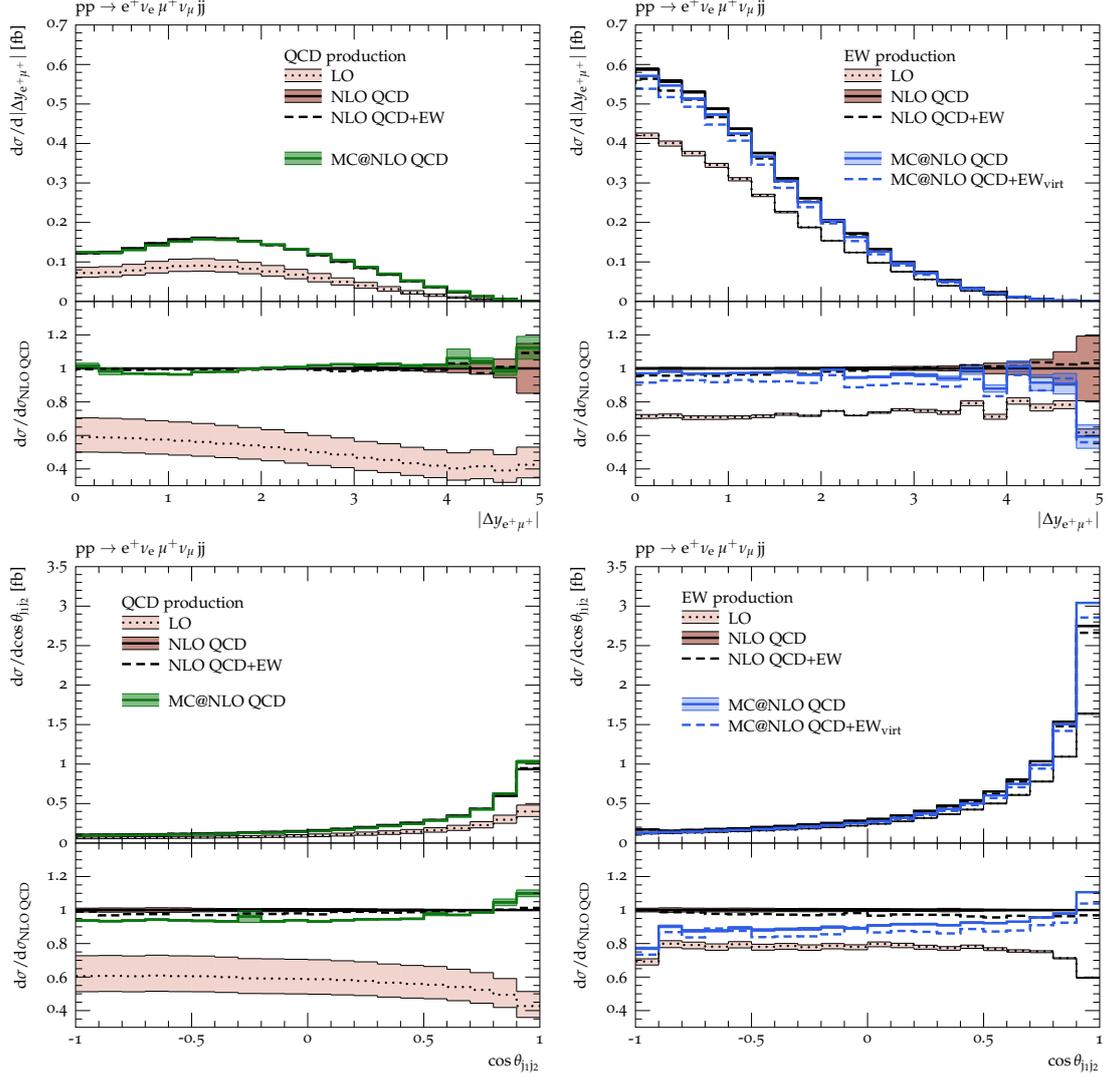
The EW corrections to the EW production process are  $-3.2\%$  at fixed order, of which  $+1.9\%$  are contributed by the photon-induced channels, while they amount to  $-4.3\%$  for the full MC@NLO result, where the photon-induced channels are not accounted for. For the  $s$ -channel MC@NLO we obtain  $-4.5\%$  and for the  $t$ -channel contribution  $-3.0\%$ . In light of the fact that the EW<sub>virt</sub> approximation is designed to recover the NLO EW corrections in the Sudakov limit, this agreement for the inclusive cross section is reasonable and does not spoil the overall accuracy of the predictions. Altogether, the final SHERPA prediction of NLO QCD+EW<sub>virt</sub> accuracy is  $8\%$  lower than the NLO QCD+EW result.



**Figure 15.** Parton-shower-matched predictions for the QCD and EW production modes contributing to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  in comparison to LO and NLO QCD and NLO QCD+EW predictions. Results are shown for the invariant masses of the di-jet system (top row) and the system formed by the two leading jets and the two charged leptons (bottom row).

We turn our discussion to differential distributions of physical observables. To this end, figures 14–16 contrast the fixed-order predictions with MC@NLO simulations for both the QCD and EW production processes in the left and right panels, respectively. At fixed order we show corresponding results at LO (dotted), NLO QCD (solid) and NLO QCD+EW (dashed). For the shower simulations we present results at MC@NLO accuracy, where for the EW production mode we furthermore include EW corrections in the  $EW_{\text{virt}}$  approximation.

We begin the discussion of differential distributions with the transverse momentum of the anti-muon in the top-left and top-right panels of figure 14. Here we observe that the shower-matched calculation predicts an increase in the differential cross section of 10% or more for  $p_{T,\mu^+} \gtrsim 250$  GeV in the QCD production mode. This difference roots in the different



**Figure 16.** Parton-shower-matched predictions for the QCD and EW production modes contributing to  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  in comparison to LO and NLO QCD and NLO QCD+EW predictions. Results are shown for the modulus of the rapidity difference between the charged leptons (top row) and the cosine of the angle between the two leading jets (bottom row).

arguments used for the strong coupling in the fixed-order calculation and the MC@NLO simulation. For the former the unique scale  $\mu_R$  is determined by eq. (2.2), which for large  $p_{T,\mu^+}$  gets large. In the matched calculation, however, strong-coupling factors get evaluated at the respective jet-emission scales, which are significantly smaller than  $m_{T,jj} + m_{T,\nu_e e^+} + m_{T,\nu_\mu \mu^+}$ , resulting in a relative enhancement of such events in the MC@NLO prediction. For the EW channel, on the other hand, both the fixed-order calculation and the shower-matched one agree very well for  $p_{T,\mu^+} \gtrsim 70 \text{ GeV}$  but deviate by about 5% for smaller  $p_{T,\mu^+}$  values, in agreement with the differences observed for the inclusive cross section. Notably, the exact EW corrections are well-reproduced by the  $\text{EW}_{\text{virt}}$  approximation for  $p_{T,\mu^+} \gtrsim 70 \text{ GeV}$ .

In the lower-left and lower-right panels, figure 14 furthermore shows the transverse momentum of the leading-jet pair in the QCD and EW production mode, respectively. The MC@NLO calculation predicts a slightly larger cross section at high- $p_{T,jj}$  in the QCD channel, again related to the above explained differences in the scale-setting prescription, while it is generally well reproduced in the EW production process for  $p_{T,jj} \lesssim 300$  GeV. However, the EW corrections now show a different behaviour. For the QCD production process they are positive and accidentally of a similar size as the QCD parton-shower corrections. For the EW production they are negative, but in fact not well reproduced by the  $EW_{\text{virt}}$  approximation. The exact NLO EW corrections are only half of what their Sudakov-approximation in the  $EW_{\text{virt}}$  approximation suggests, with the difference being made up by photon-induced contributions missing in the  $EW_{\text{virt}}$  ansatz.

Turning to invariant-mass distributions in figure 15, the di-jet invariant mass displays again many aspects of the dynamics of this process. While being generally featureless and rather flat in the QCD production mode, the fixed-order calculation agrees quite well with the shower-matched one. For invariant masses above  $M_W$  we observe a reduction at the level of 5%. In contrast, for smaller values of  $m_{j_1j_2}$  we find a notable increase in cross section reaching up to 15%. In general, out-of-cone final-state radiation will shift the di-jet invariant mass to somewhat lower values, here leading to an accumulation of events where the rather flat distribution starts to drop off. The EW corrections introduce a characteristic structure through the interference of diagrams with  $s$ -channel resonances and diagrams without them. These contributions do not give rise to a Breit-Wigner peak. Instead, a dip-peak structure emerges in the NLO QCD+EW prediction, i.e. we observe a depletion of the cross section right below the  $W$  mass and an increase just above it. While this structure integrates to a very small inclusive contribution (for a vanishing  $W$  width this would form an integrable singularity), its impact on the differential distribution is rather sizeable. In the MC@NLO QCD+ $EW_{\text{virt}}$  algorithm, such mixed QCD-EW correction would be applied prior to matching the QCD emission to the shower, and would thus be smeared throughout the entire phase space. This rather unwanted behaviour leads us to abandon the  $EW_{\text{virt}}$  approximation for the MC@NLO description of the QCD production process.

The EW production process, on the other hand, features the full Breit-Wigner shape at the  $W$  mass already at LO. The NLO QCD corrections are very sizeable and basically overshadow any potential feature from mixed QCD-EW contributions. The separation of the MC@NLO calculation in  $s$ - and  $t/u$ -channel processes removes this feature as well, and the resulting shower-matched result agrees rather well with the fixed-order one. Notable differences, however, induced by multiple-emission effects, can be seen in the region below  $M_W$ , and the further distortion of the shape of the distribution around the  $W$ -mass peak itself. As for the QCD process, multiple shower emissions migrate events from higher to lower invariant masses, resulting in about 15% higher predicted cross section in the MC@NLO calculation below the Breit-Wigner peak, i.e. for  $m_{j_1j_2}$  between 20 and 60 GeV. The EW corrections are moderate in this observable and well reproduced at small invariant masses. For  $m_{j_1j_2} \gtrsim M_W$ , the exact EW corrections are small but positive, while the  $EW_{\text{virt}}$  approximation predicts small negative corrections. Again, the difference originates in the photon-induced contributions only present in the exact calculation. This is nevertheless unproblematic, as

the EW corrections are moderate and this observable is outside the validity of the  $EW_{\text{virt}}$  approximation throughout its range considered here.

The bottom two plots of figure 15 show the invariant-mass distribution of the system formed by the two leading jets and the positron and anti-muon. Similarly to the  $p_T$ -type distributions in figure 14, for the QCD production channel the difference between NLO and MC@NLO predictions is within  $\sim 5\%$  for small invariant masses, but increases to 20% for higher values of  $m_{j_1 j_2 e^+ \mu^+}$ . As before, the EW corrections are miniscule throughout the investigated range. For the EW production process, the agreement between the NLO and MC@NLO predictions is worse, driven entirely by the  $t/u$ -channel contribution. For  $m_{j_1 j_2 e^+ \mu^+} \gtrsim 500$  GeV, the further increase in the cross section through multiple emissions described only in the shower-matched calculation modifies the spectrum at the level of up to 50%. In the exact fixed-order calculation, the net EW corrections are very small, exceeding  $\pm 5\%$  only in the very first bin. This roots in the fact that their large Sudakov logarithms are calculated as a correction to the Born process. Their impact is then countered with moderately sized positive photon-induced contributions, leading to a small overall effect. On the contrary, the MC@NLO matched  $EW_{\text{virt}}$  approach effectively applies the large Sudakov logarithms on showered events that also carry the bulk of large QCD corrections. As a consequence, the effect of the Sudakov corrections is amplified by the additional QCD corrections of the shower. Nonetheless, the  $EW_{\text{virt}}$  approach still misses positive photon-induced corrections of up to 8% in the spectrum.

Finally, figure 16 shows the distributions in the rapidity separation of the two charged leptons and the cosine of the opening angle between the two jets for both production modes. For the leptonic observable, both for the QCD and EW production mode the NLO predictions agree well with the MC@NLO calculations. In particular, for the QCD channel the EW corrections are very small. Also in the EW channel they are very moderate and well reproduced by the  $EW_{\text{virt}}$  approximation, so their overall size is larger by about 6%, as already seen for the total cross sections in table 7.

The situation is somewhat different for the angle between the QCD jets. Already for the QCD channel we observe an enhancement of events with  $\cos \theta_{j_1 j_2} \approx 1$ , i.e. rather collinear jets, at the expense of a slight suppression for larger angles, i.e. small  $\cos \theta_{j_1 j_2}$ . This effect can be traced back to collinear shower emissions off the hard-process partons that ultimately form one of the two leading jets entering the distribution. The impact of multiple shower emissions is even more pronounced for the EW production mode. Here the suppression on the left tail of the distribution reaches up to 10% relative to the NLO QCD prediction, while the bin close to  $\cos \theta_{j_1 j_2} = 1$  gets an enhancement of the same size. On the other hand, the EW corrections both for the fixed order and the MC@NLO calculation are rather constant and uniform throughout almost the entire observable range. Their size being again somewhat larger for the MC@NLO prediction, in line with the results shown in table 7.

## 4 Conclusion

This article presents a detailed study of the process  $pp \rightarrow \mu^+ \nu_\mu e^+ \nu_e jj$  at the LHC. While this final state is usually associated to vector-boson scattering and its same-sign channel, it also involves WWW production which has been analysed in specific experimental measurements.

The tri-boson contributions are, in particular, relevant for the study of anomalous quartic gauge-boson couplings.

In this article, we have first studied the various production mechanisms at LO and NLO accuracy and find that for the considered phase space, targeted to tri-boson measurements, a large fraction (about 40%) of the cross section can actually be attributed to WH production. Obviously, the  $\mu^+\nu_\mu e^+\nu_e jj$  final state is far from trivial as it contains many intricate production mechanisms.

We have further computed the full NLO corrections to the off-shell production. This allows us to get a deep understanding of the structure of higher-order corrections for this final state. In particular, we reconcile a-priori contradicting observations: tri-boson production has relatively small EW corrections, while vector-boson scattering has intrinsically large ones. We confirm that both statements are correct but point out that different phase-space regions enhance production mechanisms that have different dynamics and therefore different EW corrections. The higher-order corrections can reach 50% at the level of the total cross section and show a very different hierarchy with respect to the one in VBS phase spaces. At the level of differential distributions, the corrections exceed 100% in certain phase-space regions owing to real radiation.

Differences between the off-shell and on-shell calculations are at the level of few per cent for fiducial cross sections. In distributions, deviations are found at the level of 10% for large transverse momenta and up to 60% in invariant-mass distributions away from the resonances.

Moreover, we provide NLO-QCD matched predictions for the EW and QCD production mode supplemented with approximate EW corrections for the former, where they are relevant. Inclusion of the parton shower reduces the fiducial cross section by 6–8%. Differential distributions are modified at the level 10–20% in phase-space regions with appreciable cross sections. The  $\text{EW}_{\text{virt}}$  approximation performs reasonably well in capturing the dominant NLO EW corrections for the considered phase space. To further improve the shower-matched simulations, photon-induced contributions would need to be included, as these can yield sizeable effects in the tails of some observable distributions.

All in all, the present study delivers state-of-the-art predictions for  $pp \rightarrow \mu^+\nu_\mu e^+\nu_e jj$  at the LHC. We hope that they will foster comparisons between Standard Model predictions and experimental data in the future. It is worth stressing that all matched predictions have been obtained with the general-purpose Monte Carlo SHERPA and that they can readily be reproduced and used in experimental analyses.

Finally, we would like to point out that the present work illustrates perfectly the richness of the interplay between experimental data and theoretical predictions. In particular, the interpretation of the experimental data in terms of simple production mechanisms turned out to be non-trivial and therefore requires an intricate work between the experimental and theory community.

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