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II—Sara L. Uckelman

What Logical Consequence Could, Could Not, Should, and Should Not Be

In 'Logical Consequence (Slight Return)', Gillian Russell asks 'What is logical consequence?', a question which has vexed logicians since at least the twelfth century, when people first began to wonder what it meant for one sentence (or proposition) to follow from another sentence (or proposition, or set of sentences, or set of propositions), or whether it was possible to put down rules determining *when* the relation of 'follows from' (or 'is antecedent to') holds. Her aim is threefold: (1) to explain what an answer to the question 'What is logical consequence?' would need to be able to do in order to be a satisfying answer; (2) to identify previous answers to the question; and (3) to demonstrate why these previous answers are inadequate to do what the answer needs to be able to do, and to offer a new answer. In the present paper, I respond to these aims in two ways. The first is to say something about where Russell's central question even comes from, because this is not a topic that is often discussed by twentieth- and twenty-first-century logicians, and even historians of logic tend to not have had much to say about when—and why—this question even comes about in the first place. The second is to evaluate the accounts proposed and discussed by Russell, including her new proposal. In the end, I will argue that she has reached the right account of the nature of logical consequence, but not necessarily for the right reasons.

I

Setting the Scene. This paper takes as its springboard Gillian Russell's 'Logical Consequence (Slight Return)' (Russell 2024). It is both a commentary on the details of the paper and a response to the broader programme that it's concerned with.

The central question of Russell's paper is 'What is logical consequence?', a question which has vexed logicians since at least the twelfth century, when people first began to wonder what it meant for one sentence (or proposition) to follow from another sentence (or

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proposition, or set of sentences, or set of propositions), or whether it was possible to put down rules determining *when* the relation of 'follows from' (or 'is antecedent to') holds.

Already from this brief summary of Russell's central question—so easily stated, its simplicity hiding a wealth of complications—we have some insight into what that complexity is. Is logical consequence a relation? If it is a relation, what are its relata, and what is its arity? Does it relate sentences—specific tokens which are written, spoken or thought, and hence come into and go out of existence—or does it relate something more abstract, such as propositions? Or possibly facts, or states of affairs? If it is a relation, what grounds it? Is it a relation based on truth? On modality? On something else? How does logical consequence relate to other key logical concepts such as validity, entailment, tautology, necessity, logical truth, and so forth?

We can't answer all of these questions in the scope of the present paper, so we won't attempt to. Instead, I take my cue from Russell's project as set out in her paper, which consists of three parts:

- To explain what an answer to the question 'What is logical consequence?' would need to be able to do in order to be a satisfying answer.
- To identify previous answers to the question.
- To demonstrate why these previous answers are inadequate to do what the answer needs to be able to do, and to offer an alternative, new answer, which she describes as 'a hybrid of the two standard views in the philosophy of logic' (Russell 2024, p. 233), the semantic and the metaphysical.

There are two main things I hope to accomplish in the present paper. The first is to say something about where this question even comes from, because this is not a topic that is often discussed by twentieth- and twenty-first-century logicians, and even historians of logic tend to not have had much to say about when—and why—this question even comes about in the first place. I think that discussing this will allow us to have a broader perspective on Russell's project and what she is trying to accomplish, and provide some further support for the way in which she attempts to carry out her project. This will be the subject of §II, which is primarily historical in nature, and §III, which discusses one method of determining when we *don't* have logical consequence.

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Then I turn to the specific accounts proposed and discussed by Russell; first, in §IV, I look at previous answers to the question 'What is logical consequence?' and how she rejects them. This will allow us to see her motivation for her positive account, as well as raise some questions about her characterizations of the alternative accounts, so that in §VI we can evaluate her proposal. In the end, I will argue that she has reached the right account of the nature of logical consequence, but not necessarily for the right reasons.

II

What Is Logical Consequence? It's hard to talk about the concept of logical consequence without making reference to two recent (from my perspective) important works which bear this phrase or variants of it as their names: Tarski's 1936 article 'On the Concept of Logical Consequence' (in its English translation) and Etchemendy's 1990 book *The Concept of Logical Consequence*. Tarski, Corcoran (1956) paper is taken to be 'the only serious attempt to state, in its most general form, the analysis underlying the standard [model-theoretic] definitions, and to put forward a detailed philosophical justification for that analysis' (Etchemendy [1990] 1999, p. 7).

It is therefore of no surprise that his definition has reached a state when it is 'no longer seen as the result of conceptual analysis—when the need for analysis is forgotten, and the definition is treated as common knowledge' (Etchemendy [1990] 1999, p. 1). Once we have this model-theoretic account of consequence, it does the job so well that we sometimes cease to even remember that it was an answer to a question that needed to be asked. This is not to say that Tarski's definition sprang fully formed from his head like Athena from Zeus: as Etchemendy points out, Tarski's definition builds on work of earlier logicians such as 'Padoa (1901), Bernays (1922), Hilbert and Ackermann (1928), and Gödel (1929)', and shows many similarities to work by Bolzano (1837), though Tarski was not aware of Bolzano's work (Etchemendy [1990] 1999, p. 7). Nevertheless, Tarski's work is often treated as the first real solution to the problem, a problem that 'confronted early, formal logicians' whose work was 'driven by an interest in the intuitive notions of logical truth and logical consequence, but the only precise access to these notions was through specific, proof-theoretic characterizations' (Etchemendy

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[1990] 1999, p. 5)—specific proof systems that didn't necessarily capture the intuitive notions.

Some people may laugh at my calling 1936 'recent', given both that this is almost 100 years ago and the seismic shifts in the development of logic as a discipline between the late nineteenth century and the early twenty-first century—in a sense, Tarski's developments are both conceptually and temporally remote from the current working practices of modern logic.[1] But from the perspective I wish to adopt, even the work of Bolzano is recent. For the nineteenth and twentieth centuries were not the first time that concerns about how to capture the notion of logical consequence arose. What I will argue in this section is that the radical new developments in logic in the twelfth and thirteenth centuries were significantly driven by the same problem: what is logical consequence? Or, to put it in more medieval terms, what does it mean for one sentence to follow from another, and how do we know when this happens? I will argue that despite the fact that it wasn't until the early fourteenth century that we first begin to see explicit treatises 'On Consequences' (*De consequentia*) (Klima 2016, pp. 317–18), the motivations for the ideas captured in these treatises go back a further century and a half—and these motivations themselves have their roots in Aristotle.

Aristotle himself does not give a *general* account of anything that we would call logical consequence (whatever that turns out to be); he is interested (in the *Prior Analytics*) in a particular, narrow view of logical relationships between sentences, namely, those that are expressed in the familiar syllogistic moods and the relations of opposition that hold between pairs of sentences—contrary, contradictory, subcontrary, and subaltern. He is also concerned (in the *Topics* and the *Categories*) with categorizing certain types of methods of generating middle terms for syllogisms. Finally, in the *Sophistical Refutations*, we also see an interest in when reasoning goes wrong; but there is no attempt to provide a synthesis of all these different aspects of reasoning and inference into a single, general theory.

The next major advances in logical theorizing arose with the Stoics, who expanded the arsenal of valid argument forms by incorporating propositional connectives in a first move towards propositional

[1] This is not to say that Tarski's definition is either archaic or anachronistic; simply that the motivations which led to its development are no longer significant drivers in contemporary logical practice, in part *because* Tarski's account of consequence works so well!

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logic (see Mates 1961). While we do not have access to the original Stoic source material in the way that we do the Aristotelian corpus, from later commentators we have a clear picture of the Stoics being preoccupied with the correct interpretation of conditional sentences, with at least four different accounts being put forward (Hájek 2009, p. 206). One of these accounts is obscure and 'seemingly of little historical interest' (2009, p. 206), but the other three are of not only historical but also properly logical interest. The Philonian account interprets conditionals as material, with a conditional sentence being true in all cases except when the antecedent is true and the consequent false. The Diodorean account strengthens the conditional from material implication to strict implication. Finally, the Chrysippean account, in which 'a conditional is sound whenever the contradictory of its consequent conflicts with its antecedent' (Hájek 2009, p. 212, quoting Sextus Empiricus), introduces a notion of connection or relevance between the antecedent and the consequent which interestingly mirrors some twentieth- and twenty-first-century developments in connexive and relevant logic. Nevertheless, even given this preoccupation with conditionals and implication, the Stoics did not address the general question of how to identify when one sentence (logically) follows from or is a consequence of another, and hence also did not consider the question of the relationship between implication and consequence.

It wasn't until the middle of the twelfth century, with the increased dissemination of both old and new Latin translations of Aristotle's corpus, especially the logical works, that we first see logicians becoming concerned with the generalized question of how to determine what follows from what. Prior to the twelfth century, the only logical works of Aristotle available in Latin were Boethius's translations of the *Categories* and *On Interpretation*. In the 1120s, his translations of the *Prior Analytics*, *Topics*, and *Sophistical Refutations*—all of which, in some fashion or another, implicitly consider the question of the limits of logical consequence—began to circulate again, and the *Organon* was made fully available to the Latin west with James of Venice's translation of the *Posterior Analytics* around 1150 (Dod 1982, p. 46). These texts were copied, circulated, and quickly adopted into the logical curricula of the cathedral and convent schools that were themselves the ancestors of the first universities, founded in the early thirteenth century. Thus in the second half of the twelfth century, between an increase in the availability (and novelty!) of logical

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texts and in the concentration of scholars devoted to studying and teaching these texts, we have the perfect storm necessary to begin to ask these fundamental questions about the nature of logic.

It is in this context that we see the first genuinely innovative developments in post-Aristotelian western logic—grounded in the building blocks of Aristotle's categories, topics and syllogisms, but constructing edifices that Aristotle himself could never have imagined. One such edifice is the theory of *disputationes de obligationibus*, obligational disputations. Though authors of treatises on *obligationes* often point to Book 8 of the *Topics* as their conceptual starting point (de Rijk 1974, p. 94), what they actually do in these treatises goes far beyond the limits of Aristotle's *Topics*.

Here is not the place to rehearse all the intricate details of these disputations (for an overview of the genre and its variations, see Uckelman 2012), but suffice it to say that they are two-player 'games' between an Opponent and a Respondent, where the Opponent puts forward a series of statements and the Respondent is obliged to concede, deny or remain agnostic about each one of them, depending on the logical relationship each proposed sentence has with what has been conceded and denied already in the disputation. In the most basic version, sentences which follow from what has gone before must be conceded, while those which are inconsistent with what has already been conceded along with the negations of what has been denied must themselves be denied. Anything which neither follows from nor is inconsistent with the disputation so far should generally be conceded if it is true, denied if it is false, and if the Respondent doesn't know (for instance, in the case of a contingent sentence such as 'The king is sitting'), then they should remain agnostic.

Obligational disputations have puzzled recent commentators ever since serious attention first began to be devoted to them (Boehner 1952; de Rijk 1974; Spade 1982, 1993, 2000; King 1991; Yrjönsuuri 1994), and various accounts of their purpose and function have been put forward (see Dutilh Novaes and Uckelman 2016 for a recent overview). The confusion arises in part because very few texts say anything explicit about what the goal or purpose of these 'games' is, and we must look elsewhere—to treatises on sophisms and insolubles, and, later, to treatises on consequences, as well as to non-logical philosophical treatises (p. 4), to see the techniques and vocabulary of obligational disputations being deployed.

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It is my contention here that one of the—implicit, and hence hardly recognized—functions of these logical games/puzzles/dialogues/disputations/whatever you want to call them (a function whose identification and articulation subsumes some of the other functions that have been put forward in the secondary literature) is as a means of beginning to work out a generalized notion of logical consequence, by working through concrete examples of propositions following or not following from an initial proposition. In fact, one exception to the general rule that medieval logicians writing on *obligationes* 'don't ever tell us what they're doing' (Spade 2000, p. 1) explicitly supports this contention. In this early anonymous treatise, edited by de Rijk under the title *Tractatus Emmeranus de Positio Impossibilis*, and dated to 'not later than about the 1220s' (de Rijk 1974, p. 102),[2] there is a discussion of a non-standard type of *obligatio*, where the game starts not from a false contingent proposition, as is typical, but rather from an impossible proposition. The author justifies this with an appeal to two Aristotelian principles, the first of which is that

> Just as we say that something possible must be conceded in order to see what follows from it, similarly we have it from Aristotle that something impossible must be conceded in order to see what happens then. (Anonymous 2001, p. 217)[3]

Here we have a clear and succinct statement of the purpose of these disputations: to see what follows from what.

This understanding of the function and purpose of *obligationes*, at least in their early developments and motivations, helps us to dispel some of the confusions contemporary modern commentators have of their purpose. Spade points out that some of the texts explicitly say that the purpose of *obligationes* is to test Respondent—but he then asks, test them *in what*? (Spade 2000, p. 4). He says:

> [I]f we look closely, it seems that all that is really being exercised or tested by an obligational disputation is the respondent's skill at disputing according to the rules of obligational disputations! (Spade 2000, p. 5)

[2] An English translation appears as Anonymous (2001).

[3] *Sicuti enim nos dicimus quod possibile est concedendum ut videatur quid inde sequitur, similiter habemus ab Aristotile quod impossibile est concedendum ut videtur quid inde accidat* (de Rijk 1974, p. 117). Though the author (and others) attributes this principle to Aristotle, in truth no such principle has been identified in Aristotle; the most similar statement is found in Boethius's *De Hypotheticis Syllogismis*, attributed to Eudemus (Martin 2001, p. 64).

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Spade thinks such a response is tautological, but if the rules of obligational disputations are intended to help us articulate and identify logical consequence, then it is unsurprising that what is being tested are these very rules. This is clearer when we look at what happens when things go wrong in a disputation, that is, when the Respondent has 'responded badly', to use the technical term. This happens when the Respondent has either conceded and denied the same statement at different steps in the disputation or has conceded some statement at some step and conceded its negation at another—that is, they have ended up in an inconsistent state. This can happen either because the Respondent has simply made a mistake—conceding when they should have denied or denying when they should have conceded—or because Respondent hasn't made a mistake but the initial statement contains a hidden inconsistency which can only be made explicit by drawing out its logical consequences.

This account of the function of *obligationes* requires us to be able, in advance of having a *theory* or *account* of logical consequence, to identify and recognize at least some—if not the majority of—individual instances of logical consequence and logical repugnance (that is, inconsistency). One might think that this is setting the cart before the horse, but in fact it is no more different than for any other case of scientific theorizing which begins with the phenomena and builds continual hypotheses which explain and account for this phenomena.

Seen in this light, *obligationes* shift from being a strange, idiosyncratic oddity in thirteenth- and fourteenth-century logic to a central step in the shift from the Aristotelian-Stoic approach to logic, which looked at isolated types of logical consequence and theorized about them, to an approach that looks for a generalized and generalizable account. Without this conceptual shift, it would not have been possible for treatises specifically devoted to consequences to have been written in the fourteenth century.

III

What Logical Consequence Can't Be. Let us grant, then, that one of the functions of *obligationes* was to provide a mechanism for exploring, empirically and concretely, the boundaries of logical consequence. In this section, we will look at another such mechanism, introduced by Russell in her recent book (Russell 2023): barriers to entailment.

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A barrier to entailment is a claim that from premisses of a certain type, conclusions of another type never follow. Examples of such barriers include (pp. 236–7):

- Hume's Law: no *ought* from an *is*.
- Various Aristotelian syllogistic principles: no general conclusions from partial premisses; no negative conclusions from affirmative premisses.
- Modal-temporal barriers: no conclusions about the future from premisses about the past; no necessary conclusions from assertoric premisses.[4]
- Linguistic barriers: no indexical conclusions from non-indexical premisses.

These barriers can all be expressed in the form '*X*/*Y*' barriers, indicating that no argument from a set of premisses all of which are drawn from the class *X* to a conclusion from the class of sentences *Y* is valid—except perhaps under restricted, well-defined circumstances or contexts. A counterexample to an *X*/*Y* barrier, then, is such a valid argument. These barriers to entailment are useful because—if we are still trying to figure out what entailment or logical consequence or 'following from' *is*—they allow us to carve up a large amount of the logical space and say that these are *not* instances of the phenomenon in question.

In this sense, we can see investigating these barriers to entailment as playing the same role that the obligational disputations did—they are an empirical mechanism for identifying where the boundaries of the 'entailment' concept are. We can use these barriers, then, to carve up the linguistic space with respect to logical consequence; while this doesn't necessarily tell us where consequence *does* exist, it does divide up the space into the space where consequence *cannot* exist and the space where it *might* exist.

As Russell has argued elsewhere (Russell 2023), these barriers can be expressed in a general form which allows for counterexamples under certain conditions. This general form is (Russell 2024, p. 237):

$$\text{If } \Gamma \in X, \text{ and } \varphi \in Y, \text{ then } \Gamma \nvDash \varphi \text{ unless } C. \qquad (1)$$

[4] This one, however, is problematic, for any logician who accepts the principle that necessities follow from everything.

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That is, if Γ is a set of sentences in the class *X* of (sets of?) sentences,[5] and *φ* is a member of the class *Y* of (sets of?) sentences, then Γ will not entail *φ* unless condition *C* holds.

This provides a nice characterization of entailment barriers *for the logician*, who likes things captured in neat formalizations. But many examples to putative barriers are drawn from natural language—Russell points out that this is often the case for putative counterexamples to Hume's Law, or the *is*/*ought* barrier. These counterexamples often 'exploit thick normative expressions, embedded sentences and truth attributions, and performative speech acts' (Russell 2024, p. 236), things which often go beyond the expressive power of traditional symbolic logics. (Interestingly, of the four counterexamples to the *is*/*ought* barrier that Russell gives, only two of them have conclusions that are explicitly about 'ought' claims; the other two have conclusions which are *evaluative* in character, but this doesn't mean they are necessarily *normative*.)

There is then a worry that this theorem raises for Russell, which is whether she can say anything about the putative *informal* arguments, those in natural language that are not formalized and are potentially unformalizable, because 'natural language … has expressive capacities that no extant formal language possesses' and some of the words and concepts involved in the putative counterexamples to the entailment barriers are 'not easily formalized' (2024, pp.236–8).

Russell attempts to address this worry by developing an analogy between logical models, such as the formalized representation of the general barrier form above, and scientific models; just as scientific models can provide us with simulations of, for instance, pandemics or hurricanes, logical models provide us with simulations of natural language (Russell 2024, pp. 239–42). One difference, though, is that in the scientific case, we measure things in the real world, and input that data into the model, and perform the simulation—but then we measure things *again*, and input the update data to perform a new simulation, with potentially different consequences. For instance, one can simulate the spread of a disease through a population and compare the results to what has actually occurred in the real world; but where the simulation results come apart from the

[5] The precise nature of *X* and *Y*—are they classes of sentences or classes of sets of sentences?—is not made explicit in Russell (2024), and we might think that the general form of the claim would make more sense if the first conjunct of the antecedent was $\Gamma \subseteq X$ rather than $\Gamma \in X$.

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actual world, we have to revisit either the data we input into the simulation or the model itself. This dynamic update is an important—perhaps even crucial or integral—part of the simulating process in the empirical sciences. If we try to build an analogy between scientific modelling and logical modelling, then we must ask: What is the correlate for logical models? What is our data in the real world, how do we input it, and how do we update our models on the basis of it? Unfortunately, this is one way that in which the analogy breaks down: natural language doesn't work in the way that weather cycles or infection rates do.

Russell herself mentions another point at which the analogy breaks down: the process via which an informal argument is translated into logic and evaluated across sets of models is not done via computation or algorithm as it is in scientific modelling (see Russell 2024, n. 2)—instead, we have to search for a counterexample or give a deductive proof. Neither of these processes has correlates in the hurricane or illness cases. This doesn't mean, though, that the search for counterexamples cannot provide us with illuminating information about the limits of our logical models. Russell discusses such natural language words such as 'bilker' and 'brave', as examples of things not easily formally expressible in logic. Another of her examples is a curious one: 'promise'. While promising is not treated as a potential logical constant in modern logic (Russell 2024, p. 238), the logic of promising *was* systematically discussed from the late fourteenth century onwards (Ashworth 1976a, 1976b). We can conclude from this that, concerning at least some of the examples Russell discusses, where it seems as though natural language is more expressive than any formalization, (i) maybe we just haven't found the right formalization yet, and (ii) one doesn't need very thick formalization (and certainly not symbolization) to treat things in a logical fashion. In a sense, just as formalized entailment barriers give us information about what falls outside the scope of the concept 'entailment' or 'logical consequence', so informal examples of informal entailment barriers can provide us with information about what falls outside the scope of our logical modelling; and just because certain things fall outside the scope of formal logic, it doesn't mean that they aren't still amenable to logical consideration.

This provides the motivation for one of Russell's aims in the paper under discussion, namely, to develop an informal version of

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the formal theorem in the book (that all *X*/*Y* barriers are of the form given in (1)). In order to do so, she says she needs to define a consequence relation with the following two features (Russell 2024, p. 238):

(1) It has to be applicable to natural language sentences without the intermediary of formalization.
(2) It has to be able to hold in virtue of the meanings of expressions like *brave*, *bilker*, and *promised*.

So now we have some notion of what consequence *can't* be, as well as what it *must* be. Let us then take a look at what 'logical consequence' *might* mean, taking into account these two desiderata.

IV

What Logical Consequence Might Be. Russell begins by identifying two things people might think of when they think of 'logical consequence':

(1) The 'modal slogan': an argument is valid when 'it is impossible for the premisses to be true and the conclusion false' (2024, p. 234).
(2) The 'model-theoretic definition': 'a set of premisses entails a conclusion just in case every model of the premisses is a model of the conclusion' (2024, p. 234).

Before we look at Russell's reasons for rejecting both of these accounts, a point about the language being used here. As philosophers, precision in language matters; we know that 'logical consequence' is the central concept under consideration here, but it is precisely because defining it is part of our goal that we must be careful how we speak about it.

The two accounts related by Russell here each introduce their own potential linguistic confusion or conflation. The modal slogan, on the one hand, doesn't mention logical consequence, but rather shifts attention from logical consequence to the validity of an argument. On the other hand, in the second definition, 'logical consequence' is defined in terms of 'entailment', and in fact Russell seems to think that the programme of defining what logical consequence is just *is* the programme of 'capturing the entailment

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relation' (2024, p. 231). Are these—logical consequence, entailment, argument validity—the same thing? If we are still in a position where we don't yet know what logical consequence—or validity or entailment!—is, then we aren't yet in a position to say whether they are the same thing. This rather pedantic point is one which we feel compelled to harp upon, perhaps to the point of annoyance, because it illustrates the difficulty inherent in the overall project: we have so many interrelated notions which we can use with more or less intuitive ease, that this inhibits us, sometimes, in making the distinctions necessary to draw these slippery notions and concepts out into a more precise form. Nevertheless, even though it is not immediately obvious that these various notions will coincide, I will attempt to follow Russell as much as possible in not separating these notions; it is only when definitions or objections rely on these notions being conflated that I will raise this point again.

On to the two definitions themselves. Russell objects to the modal slogan 'because it is false' (Russell 2024, p. 234). This is because, she argues, if it were true, then 'something is a logical truth if and only if it is necessary' (Russell 2024, p. 234). However, it's not clear why this follows (and not just because we don't yet know what 'follows' is yet), for two reasons. First, it's a statement about logical truth, whereas the definition was given of validity; as noted above, we need to know what the link or connection is between the two. One could define something to be logically true if it is the conclusion of a valid argument with no premisses, in which case we *would* get that 'if something is a logical truth, then it is necessary', because the modal account of the validity of an argument with no premisses collapses to 'it is impossible for the conclusion to be false', which—at least on most standard accounts of modality—is equivalent to 'it is necessary for the conclusion to be true'. But this only gets us one direction of the biconditional—why does the modal slogan force us into accepting that every necessity is a *logical* truth? It is not clear that it does. Be that as it may, Russell wants to reject the counterfactual claim altogether, because 'there are necessary truths which are not logical truths, including necessary a posteriori truths' (Russell 2024, p. 234). Once we drop the belief that all necessity is *logical* necessity, then we shouldn't have any trouble accepting the necessity of, for example, mathematical truths without being forced into calling them *logical* truths.

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On the other hand, Russell has 'no plans to challenge the model-theoretic definition' (2024, p. 235), but instead to start her investigation from this basis. Russell says she is going to *assume* that the answer to the question 'What is logical consequence?' is 'truth-preservation over *X*', and that the only question left to answer, then, is 'what [the right value of] *X* is' (2024, p. 235). That is, the question becomes 'What do the models we use in logic *represent* such that truth-preservation over them is a good way to capture logical consequence'? (2024, p. 235).

Etchemendy identifies two options (Russell 2024, p. 235), to which Russell adds a third and fourth:

(1) Metaphysical view: the models represent ways the world could be.
(2) Semantic view: the models represent ways we can interpret formal languages.
(3) Deflationary view:[6] the models represent nothing at all, they are just tools and machinery.
(4) ??? view: the models represent 'a fourth secret thing' (Russell 2024, p. 235).

This is an interesting collection of possibilities, because it is not clear that they are either exhaustive or exclusive. For instance, if the metaphysical view is that models represent 'ways the world could be', one might think that the semantic correlate of this view is that the models represent 'ways *we can describe* how the world could be', which is not necessarily the same thing as ways we can interpret our languages.

Russell considers each of these options, adopting the fourth as her final position; I look at her reasons for doing so in the next section (§V). Ultimately, I will argue that she got to the right conclusion, but for the wrong reasons.

V

What Logical Consequence Shouldn't Be. Each of the options outlined at the end of the previous section represent an answer to the question, 'What do models, as wholes, represent?' What I call the

[6] This is my term, not Russell's.

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deflationary view, that they represent nothing at all, is the null answer. Russell rejects this account because it is explanatorily useless, and I agree that there is nothing to be gained from considering this case further. However, Russell's rejection of the non-deflationary accounts (the metaphysical and semantic views) does deserve some substantive consideration.

The metaphysical view's answer is that what the models represent is 'different ways things might be' (Russell 2024, p. 243), that is, possible worlds. Russell rejects the metaphysical answer because, she argues, it gets identities wrong. It is possible to give a model where 'George Eliot is Mary Ann Evans' is true and one where it is false, which is as we would expect, because it 'is neither a logical truth nor unsatisfiable' (p. 243). Nevertheless, she says the metaphysical account gets this sentence wrong, because 'identities like this are necessary if true, and necessarily false if false' (p. 243). But this is a strange, and unjustified, claim. It appears to be based on her belief that 'names are rigid over possible worlds' (p. 246), but this a position which one need not maintain: it is quite easy to imagine Mary Ann Evans living in a possible world which is not so dominated by misogyny that she must publish under a male pseudonym in order to be taken seriously. While imagination should not be taken as a foolproof guide to possibility, in this case it is hard to see what could go wrong with our imagination such that this imagined scenario does not represent a different way that things might be. Unfortunately, it is required for Russell's argument that a statement like 'George Eliot is Mary Ann Evans' is not contingent, because she concludes from the fact that it is true in some models and false in others that these models *cannot be* possible worlds—a strange conclusion indeed. If one wishes to maintain that equations of proper names like this are not contingent, then at the very least, one should not say that the necessity involved in 'George Eliot is Mary Ann Evans' is *logical* necessity; it is, at most, some sort of metaphysical necessity, and more likely merely linguistic necessity, which—if we take seriously the principle that names are rigid over possible worlds—requires us to hold fixed what our language refers to, in order to properly investigate it. But then this scenario is no longer about metaphysics, but about language.

Which brings us to the next answer that Russell considers, the semantic one, which takes models to be representations of ways of interpreting language—except, however, we can't reinterpret *all* parts of the language, or nothing would come out as a logical truth.

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Instead, 'logical consequence is relative to a set of logical constants' (Russell 2024, p. 245). This means that before we can use the semantic view, we must have some advance idea of what the logical constants are, which is definable before we attempt to define logical consequence (so no notion of consequence can be involved in our picking out of the logical constants). This is not an insurmountable difficulty, but it is certainly a difficulty.

A more problematic difficulty for Russell is that 'the semantic view can't explain why sentences that "say" how many objects there are … are true in some models but false in others' (Russell 2024, p. 246)—because how many objects there are in a world is not a function of the interpretation of the logical constants. That is, there are things in a model we can change which result in the change of the truth value of a sentence. So we can completely fix the interpretation of a language, and yet by varying the models change the truth-values of certain sentences (namely, sentences saying 'There are *n* distinct objects in the domain')! This goes against what Russell wants the semantic account to be able to do, but also gives us an insight into where things are beginning to go wrong: it's because there are two ways things can change that can cause a change in a sentence's truth-value—'change in what it means, and change in the way the world is' (Russell 2024, p. 247)—and these two ways are themselves *both* types of *model update* (see Uckelman 2011). (We will return to this issue in the next section.)

In the end, neither the semantic nor the metaphysical answers are acceptable to Russell—each has their benefits and drawbacks, such that neither trumps the other. For this reason, her preferred solution is a combination solution, in which models 'represent ways the world and language can be combined' (Russell 2024, p. 247). As I said at the beginning of this section, I agree that this is the right solution, but I disagree with her route to it; her arguments for her positive account rely on taking the metaphysical and semantic views in a narrow, blinkered way. It is not so much that the metaphysical and semantic views are *wrong*, but that as Russell has articulated them, they are incomplete, and once we complete them we realize that the result is essentially the same as her combination account.

VI

What Logical Consequence Should Be. Russell's distinction between the metaphysical and the semantic approaches requires there to be

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a separation between language and the world. But what the concern that Russell levels against the semantic view—that one can change the truth-value of a sentence without changing the interpretation of the language—shows is that *one cannot separate language from the world*. There are two parts to this claim:

(1) Our only access to the world is through the language. We cannot get to a representation of the world that doesn't, at some stage, go through language, whether it is English as a metalanguage, or a hybrid English-symbolic language that involves set-theoretic notation, or whether we're fully within a symbolic context.
(2) Language is *part* of the world. Even if we had access to the world through something other than language, if we had another way of describing, using and manipulating it, we can still change a world by changing language, because language doesn't sit apart from what goes into making up a world.

Let us flesh these two claims out.

The first is a relatively innocuous, but nevertheless undiscussed point, which is that we can only differentiate two models if we have the language necessary to do so. Suppose we have two worlds that differ only in their underlying set theory. While we can describe this difference in English as the metalanguage, or using a hybrid English-symbolic metalanguage, unless our object languages differ (either in the language itself or how it is interpreted: both of these are 'parts of the language'), and they are not going to be able to distinguish the two models. Suppose, for example, that we have one model with a single-element domain D and another model with a two-element domain D', and a language that contains a single unary predicate, P, whose interpretation, on both models, is the entire domain. Without equality, there is no way to distinguish these models on the basis of their theories (that is, the sets of sentences that they make true).

The second claim brings together the two issues highlighted in the previous section, about the rigidity of proper names and the fact that changing an underlying domain can change the interpretation of the language. Both of these issues arise because we were attempting to look at the metaphysical aspects and the linguistic aspects of models as if they are two distinct things, rather than two aspects of the same thing—that is, forgetting that an interpretation just *is*

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a part of a model! Names (that is, constants) don't have to be rigid designators, and Russell errs in assuming (following Kripke) that they do. At the level of language, constants are constants in order to distinguish them from variables, *not* from predicates. If you have a varying-domain model, your constants can easily have different interpretations at each world, just as predicates can. There is nothing in how models are defined that forces them to be fixed. There is nothing that prevents us, as omnipotent modellers, from changing whether names are rigid designators from model to model; in some models, they might be, in other models, they might not be. Or, in the terminology of Russell, in some worlds names might be rigid and in some worlds they might not be.

The moral of the story is this: how languages can be interpreted will depend on the underlying metaphysical structure; you cannot have one without the other. If we recognize this, then the combination approach looks less like a genuine alternative to the metaphysical and semantic approaches, rather than these other approaches being, essentially, blinkered views that (deliberately?) ignore a component of what is being varied. Russell's first argument for her combination view—the one which she says sets her account apart from other, similar accounts (Russell 2024, n. 6)—could be obviated by simply recognizing that models *include* their interpretations. So recognizing that 'what a model represents' is a world + interpretation combination is no more than recognizing what the components of a model are, and not systematically ignoring one or the other.

This point can be driven home by noting that if we say, as we would on the metaphysical view, that models represent possible worlds, then it would not make sense to speak of *possible worlds models*, that is, models which are made up of things which we say represent possible worlds, since a model is supposed to represent just one possible world. Yet this is what would be required on Russell's account of the metaphysical view. This highlights a complication of the distinction she's trying to make: her approach makes traditional modal semantics come out *weird*. The only way it makes sense to talk about names being rigid or not is if we have a possible worlds-like structure to our models—but then, if the model is 'the world', what are the 'possible worlds' that we have in the model? How can a possible world contain other possible worlds? Do we have worlds within worlds? If we do, then worries arrive that we end up in a

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multiverse setting, with all the problems concomitant with that (see Uckelman 2020).

If, on the other hand, we allow 'model' to operate at the highest level of generality, then the models incorporate both domains and interpretations, and if we change the interpretation, we've changed the model, and we can speak of the result as being a different possible world, just one where *only* the language has changed. It is artificial to exclude the semantic aspects of worlds from the worlds themselves, and it also seems artificial to restrict our attention to models that can correspond only to individual possible worlds, rather than allowing for modal models, which contain many possible worlds.

To say, then, that 'truth in all models is truth on all combinations' is just to say 'truth in all models is truth in all models', because models *already are* combinations of metaphysical and semantic aspects. The error arises in trying to separate out these two aspects, when they are in fact inextricably linked. We cannot describe the world without language, and we cannot understand language without the world.

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