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IMMERSED TRACTION BOUNDARY CONDITIONS IN PHASE FIELD FRACTURE MODELLING

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Abstract. Phase field approaches are an increasingly popular method for modelling complex fracture problems and have been applied to a number of real-world settings. In some applications, pressure forces, or more generally traction terms, must be considered on the crack faces. However, application of appropriate boundary conditions to represent these tractions is non-trivial, since phase field models do not include a direct description of the fracture surface due to their diffuse nature. This paper summarises one- and two-domain approaches to including immersed traction boundary conditions and states the authors' intention to implement and evaluate these methods in an hp-adaptive discontinuous Galerkin finite element framework.

Key words: *fracture; phase field; traction; boundary conditions; fluid-filled fractures*

1 Introduction

Phase field fracture models have gained popularity in recent years as an approach to the numerical modelling of complex fracture problems. The method evolved from Griffith's energy-based theory for fracture [1] and the subsequent re-framing of the theory as an energy minimisation problem by Francfort and Marigo [2].

Phase field fracture models are an example of a diffused fracture modelling approach. This approach can be contrasted to discrete fracture models, such as the extended finite element method (XFEM) or configurational force approach, where cracks are modelled as discontinuities in displacement, incorporated using enrichment functions in the mesh or introducing discontinuities in the mesh through mesh splitting. In the phase field approach, the fracture surfaces are represented over an approximate volume, meaning no special treatment of the mesh is required. As a result, the phase field approach is also better suited to modelling crack nucleation, and complex fracture patterns such as branching or merging.

The growing popularity of the phase field approach has led to its application to a number of real-world problems, including the modelling of calving events in ice shelves and glaciers [3–5], where large chunks of ice are lost to the ocean as a result of fractures propagating through the full thickness of the ice. These calving events are driven both by gravitational body forces in the ice, as well as fluid pressure forces from water that collects in existing fractures. However, the lack of a discrete fracture surface in phase field fracture models means that the imposition of appropriate boundary conditions, representing the fluid pressure forces, becomes challenging.

Several approaches have been taken in the existing literature, largely in the context of hydraulic fracturing, including the transformation of the fluid pressure from a surface term into a volume term through an attempt to reconstruct the fracture surface as in [6, 7], and poroelasticity approaches where the pressure term acts through the volume, controlled by the phase field value as in [3].

This paper presents these approaches and will state the authors' intention to investigate incorporating them within an hp-adaptive discontinuous Galerkin finite element phase field fracture code [8].

2 Phase field fracture modelling

In 1920 Alan Arnold Griffith published his energy-based theory for brittle fracture from which the field of fracture mechanics developed [1]. For an isothermal, quasi-static case, Griffith's theory states that a fracture will propagate when the release of elastic strain energy due to crack growth, is equal to or greater than the energy required to create new fracture surfaces.

Considering the arbitrary domain $\Omega \subset \mathbb{R}^{\delta}$ where $\delta \in \{1,2,3\}$ with the boundary $\partial\Omega \subset \mathbb{R}^{\delta-1}$ and an internal fracture $\Gamma \subset \mathbb{R}^{\delta-1}$, the work of Francfort and Marigo [2] allows Griffith's theory to be stated in variational form as an energy minimisation problem. The total energy of the system Π can be expressed using the functional

$$\Pi = \int_{\Omega} \Psi(\boldsymbol{\varepsilon})\, \mathrm{d}\Omega + \int_{\Gamma} G_c\, \mathrm{d}\Gamma - \int_{\Omega} \boldsymbol{f} \cdot \boldsymbol{u}\, \mathrm{d}\Omega - \int_{\partial\Omega} \boldsymbol{\tau} \cdot \boldsymbol{u}\, \mathrm{d}\partial\Omega, \tag{1}$$

where Ψ represents the strain energy density, $\boldsymbol{\varepsilon}$ the strain tensor, $\boldsymbol{f}$ the body force, $\boldsymbol{\tau}$ the surface traction and $\boldsymbol{u}$ the displacement field. G_c is known as the critical energy release rate, in other words the amount of energy required to create new fracture surfaces, per unit area of the surface.

The minimisation of Π is however challenging in its current form, due to the difficulty in integrating over an evolving discrete fracture surface Γ. A scalar field, $\phi \in [0,1]$ can therefore be introduced in order to represent the discrete surface over an approximate volume. The scalar variable ϕ is known as the phase field, and represents the transition from intact to fully damaged states, where $\phi = 0$ at an intact material point and $\phi = 1$ at a fully cracked point in space. The introduction of the phase field allows the functional to be regularised such that the problem becomes numerically tractable

$$\Pi_l = \int_{\Omega} [g(\phi) + \kappa]\, \Psi_0(\boldsymbol{\varepsilon})\, \mathrm{d}\Omega + G_c \int_{\Omega} \gamma(\phi, \nabla\phi)\, \mathrm{d}\Omega - \int_{\Omega} \boldsymbol{f} \cdot \boldsymbol{u}\, \mathrm{d}\Omega - \int_{\partial\Omega} \boldsymbol{\tau} \cdot \boldsymbol{u}\, \mathrm{d}\partial\Omega \tag{2}$$

where $\gamma(\phi, \nabla\phi) = \frac{1}{2l}\phi^2 + \frac{l}{2}|\nabla\phi|^2$ and the subscript 0 denotes a quantity for undamaged material. A length scale l is introduced which controls the width of the cracked region approximating the discrete fracture surface. A monotonically decreasing function, $g(\phi) = (1-\phi)^2$ is used to degrade the material stiffness with increasing damage. A small positive constant κ is used to avoid full degradation of the material stiffness, ensuring the problem remains well-posed.

The difference in crack propagation behaviour under tensile and compressive loading can be accounted for through decomposition of the strain tensor into tensile and compressive parts, which in turn allows an equivalent split of the strain energy density into positive, Ψ_0^+, and negative, Ψ_0^-, components as in [9]. A strain history term $\mathcal{H}$ can also be introduced as in [9] to prevent reversibility of damaged regions. $\mathcal{H}$ tracks the maximum tensile strain energy density where $\mathcal{H} = \max_{t \in T}(\Psi_0^+(\boldsymbol{\varepsilon}, t))$ and T represents the current time.

3 Immersed traction boundary conditions

Including a pressure or traction term along crack surfaces in phase field fracture modelling is non-trivial due to the lack of discrete fracture surface in the model. As detailed in [10], the approaches taken in existing literature to apply pressure forces to a crack surface in phase field models can be categorised into one-domain and two-domain approaches.

In a one-domain approach, the fracture is taken to be a part of a single porous domain, simply with higher porosity and permeability at the fracture location. In an example of one such approach [3], which details a phase field model for predicting fracture propagation in ice shelves, pressure forces from meltwater are incorporated in damaged regions through Biot's theory of poroelasticity. The effective stress tensor, $\boldsymbol{\sigma}$, is defined as

$$\boldsymbol{\sigma} = g(\phi)\boldsymbol{\sigma}_0 - [1 - g(\phi)]\, p\alpha \boldsymbol{I}, \tag{3}$$

where p is the fluid pressure, $\boldsymbol{\sigma}_0$ is the undamaged total stress and α is the Biot coefficient. The degradation functions in Equation (3) ensure that both the load carrying capacity is degraded with damage, but also that the pressure forces are constrained to damaged regions, with the pressure effects increasing with increased damage. This modified stress tensor is incorporated when solving for the minimum of the energy functional in Equation (2). The pressure, p, is modified based on the depth of meltwater in the crack, as detailed in [3].

By contrast, two-domain approaches attempt to reconstruct or approximate the fracture surface in some way, as a clearer interface between the solid and fluid constituents, over which a pressure can be applied. An example of this approach is given in [11]. In this approach, the work done by the pressure force on a crack surface is given by

$$\int_{\Gamma} p(\boldsymbol{u}^{+} - \boldsymbol{u}^{-}) \cdot \boldsymbol{n}_{\Gamma}\, \mathrm{d}\Gamma, \tag{4}$$

where $\boldsymbol{u}^{\pm}$ denotes the displacement on the positive and negative crack faces and $\boldsymbol{n}_{\Gamma}$ the normal to the crack face. This expression can then be modified using the gradient of the phase field to approximate the crack volume and added to the regularised total energy functional as an additional work term. The functional then becomes

$$\Pi_l = \int_{\Omega} [g(\phi) + \kappa]\, \Psi_0(\boldsymbol{\varepsilon})\, \mathrm{d}\Omega + G_c \int_{\Omega} \gamma(\phi, \nabla\phi)\, \mathrm{d}\Omega - \int_{\Omega} \boldsymbol{f} \cdot \boldsymbol{u}\, \mathrm{d}\Omega - \int_{\partial\Omega} \boldsymbol{\tau} \cdot \boldsymbol{u}\, \mathrm{d}\partial\Omega + \int_{\Omega} p\boldsymbol{u} \cdot \nabla\phi\, \mathrm{d}\Omega. \tag{5}$$

This approach in [11] has also been extended in [6] and [7] to include porosity. Other two-domain approaches have used level-set functions [12] and phase field contours [13] to reconstruct fracture surfaces that act as an interface over which a pressure can be imposed.

4 Observations

The inclusion of traction boundary conditions, including pressure forces, along fracture surfaces in phase field models, is challenging due to the lack of discrete fracture surface. Existing approaches fall broadly into two categories - one-domain and two-domain methods. While one-domain methods could be considered simpler to implement, particularly for porous materials where poroelasticity and immersed boundary conditions can be handled simultaneously, two-domain methods could on the other hand be interpreted as more physically representative of the mechanical processes involved, and more readily extended to general traction boundary conditions, instead of pressure forces. These different approaches will be implemented and evaluated in a discontinuous Galerkin finite element phase field model [8] and their suitability for wider applications, such as the modelling of calving events in ice shelves, considered.

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