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SIMULATION OF STRAIN LOCALISATION WITH AN ELASTOPLASTIC MICROPOLAR MATERIAL POINT METHOD

Ted J. O'Hare[1*], Panos A. Gourgiotis[2], William M. Coombs[1] and Charles E. Augarde[1]

[1] Department of Engineering, Durham University, Durham, DH1 3LE, UK.
*ted.o'hare@durham.ac.uk, w.m.coombs@durham.ac.uk, charles.augarde@durham.ac.uk
[2] Mechanics Division, National Technical University of Athens, Zographou, GR-15773, Greece.
pgourgiotis@mail.ntua.gr

Abstract. The thickness of shear bands, which form along slip surfaces during certain modes of geotechnical failure, depends directly on the size of the soil particles. Classical continuum models, however, are invariant to length scale, so the strain localisation zone cannot converge to a finite size when employing numerical techniques such as the finite element method. Instead, the present approach adopts the micropolar (Cosserat) continuum, a weakly non-local higher-order theory which incorporates a characteristic length and allows independent rotations of the material micro-structure as well as transmission of couple stresses. As a result, strain can localise naturally in micropolar continua to form realistic finite-sized shear bands. By extending an elastic finite-strain micropolar implementation of the material point method (a numerical method well-suited to modelling large deformation problems) with an elasto-plastic constitutive model suitable for geomaterials, this novel combined approach will provide a powerful tool to analyse numerically challenging localisation problems in geotechnics.

Key words: *micropolar; Cosserat; material point method; strain localisation*

1 Introduction

When modelling geotechnical failure events with conventional mesh-based techniques underpinned by a classical continuum theory, two key problems arise. The first concerns the magnitude of deformation generally brought about by such events. Attempts to use numerical techniques like the finite element method (FEM) here can lead to severe distortion of the mesh – and, in extreme cases, element inversion – such that the method begins to falter and accuracy is not guaranteed, if a solution can even be produced at all. Moreover the remedial task of subsequently re-meshing the deformed domain only brings further problems, particularly surrounding projection of history variables and the increase in computational cost and algorithmic complexity. Far better suited to modelling such large-deformation problems are the various *particle*-based methods which are not, conversely, hindered by any deviation from an initial geometry. Here we introduce the material point method (MPM) [1], a particle method which utilises a mesh only for the purpose of computations – not for tracking the material – which is reset for each time- or load-step and does not therefore experience significant distortion. Although the material body is discretised into particles, the MPM has a commonality with much of the FE idiom through the way it operates on the mesh, readily allowing for implementation of FE formulations. This is the method adopted for this work.

The second problem is less insidious but far more fundamental and more complicated to remedy. Shear failure in geomaterials usually occurs in concentrated regions called *shear bands* in a process known as *strain localisation*. But because shear bands represent a sharp discontinuity in the displacement field, if the underlying partial differential equation (PDE) used to describe the event has the local displacement as its sole primary field variable – and does not impose an artificial smoothing technique – then it loses one of its conditions for ellipticity (or hyperbolicity in the case of dynamic analysis). The governing system is now ill-posed for the problem at hand and will not produce reliable results; numerical simulations do not converge to a particular failure load or shear band thickness with mesh refinement, and are instead fully mesh-dependent.

Our chosen solution is to supplant the classical approach with micropolar (or Cosserat) theory [2], which supposes that the rotation of each element of the micro-structure is independent of the rotation of the surrounding continuum. The independent *micro-rotations* and their spatial gradient (*curvature*) regularise the ill-posedness of the PDE, smoothing the solution field around the shear band with respect to a length scale which is generally taken to be indicative of the size of the micro-structure (e.g. the diameter of a soil particle). Numerical simulations based on the micropolar continuum can therefore reliably predict shear bands with a thickness depending on the scale of the constituent micro-structure, as observed in real localisation events.

This paper details an approach building on [3–6], whereby the geometrically-exact micropolar theory is extended for elasto-plasticity with a pressure-dependent yield surface, and implemented within the MPM. A cursory overview of the adopted continuum theory and numerical method is given, and further details including numerical examples will be provided during the oral presentation.

2 The micropolar continuum

2.1 Kinematics

With reference to Figure 1a, a micropolar continuum occupies a volume Ω in its current (deformed) configuration. The translation vector u_i emanates from the Cartesian reference position X_i of each point in the undeformed volume Ω_0 to its current position x_i in Ω, and the deformation gradient tensor $F_{i\theta} = \frac{\partial x_i}{\partial X_\theta}$ provides the fundamental link between reference and current coordinates. At every point in the micro-continuum there exists a rigid body, attached to which is a set of axes that are free to rotate independently of deformation occurring at the continuum scale. Each rotated axis w_i in the current configuration is related to its counterpart W_ψ in the reference configuration via $w_i = Q_{i\psi} W_\psi$, where $Q_{i\psi} \in \mathrm{SO}(3)$ is a proper orthogonal tensor termed the *micro-rotation* tensor. The rotation may also be parameterised as a vector φ_k, identified as the axis of rotation with the angle its magnitude. A skew-symmetric tensor $\Phi_{ij} = -e_{ijk}\varphi_k$ (where e_{ijk} is the third-order Levi-Civita, or *permutation*, tensor) is then used to compute the micro-rotation tensor using the (Euler-)Rodrigues formula

$$Q_{i\psi} = \delta_{i\psi} + \frac{\sin|\varphi|}{|\varphi|}\Phi_{i\psi} + \frac{1-\cos|\varphi|}{|\varphi|^2}\Phi_{ij}\Phi_{j\psi}, \tag{1}$$

where $\delta_{i\psi}$ denotes the Kronecker delta and $|\varphi|$ is the magnitude of φ_k. For our purposes, two spatial measures are used to quantify micropolar deformation: a stretch tensor, and a measure of the rotation gradient named the *left curvature tensor* which endows the theory with its non-local property

$$V_{ij} = F_{i\theta} Q_{j\theta} \qquad \text{and} \qquad k_{ij} = -\frac{1}{2} Q_{i\gamma} e_{\gamma\tau\eta} Q_{p\tau} \frac{\partial Q_{p\eta}}{\partial X_\pi} Q_{j\pi}. \tag{2}$$

A multiplicative elasto-plastic split is assumed for both the deformation gradient $F_{i\theta} = F^e_{iA} F^p_{A\theta}$ and the micro-rotation tensor $Q_{i\theta} = Q^e_{iA} Q^p_{A\theta}$, where the superscripts denote the elastic and plastic parts. Hence the elastic stretch is defined $V^e_{ij} = F^e_{iA} Q^e_{jA}$ and the curvature simply decomposes additively:

$$k_{ij} = k^e_{ij} + k^p_{ij}. \tag{3}$$

2.2 Elastic constitutive laws and balance equations

The Cauchy stress σ_{ij} and couple-stress m_{ij} (moment per unit area) are obtained from the elastic deformation measures using a neo-Hookean hyperelastic model [3]

$$J\sigma_{ij} = \frac{\lambda}{2}(J^2 - 1)\delta_{ij} + \mu(V^e_{ik}V^e_{jk} - \delta_{ij}) + \frac{\kappa}{2}(V^e_{ik}V^e_{jk} - V^e_{ik}V^e_{kj}) \tag{4}$$

$$Jm_{ij} = V_{ik}(\alpha k^e_{ll}\delta_{kj} + \beta k^e_{kj} + \gamma k^e_{jk}) \tag{5}$$

where $J = \det(F)$ is the volume ratio between the original and deformed states, and λ (first Lamé constant), μ (second Lamé constant), κ, α, β and γ are constitutive parameters. An internal length scale L is then given by $L = \sqrt{(\beta + \gamma)/2\mu}$. The spatial forms of linear and angular momentum balance in the quasi-static case read

$$\frac{\partial \sigma_{ij}}{\partial x_j} + p_i = 0 \qquad \text{and} \qquad \frac{\partial m_{ij}}{\partial x_j} - e_{ijk}\sigma_{jk} + q_i = 0, \tag{6}$$

where p_i and q_i are the body force and body couple respectively.

2.3 Elasto-plastic constitutive model

This formulation uses a conventional elastic predictor-plastic corrector algorithm to map the stress state at a material point onto the yield surface f, which has the Drucker-Prager form [5]

$$f = \sqrt{3J_2} + \frac{A}{3}\sigma_{kk} - c \tag{7}$$

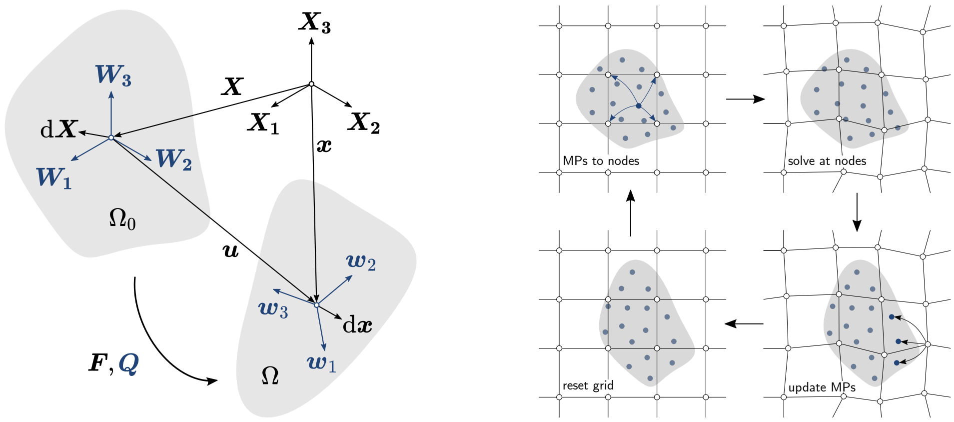
using the modified second invariant J_2 of deviatoric stress $s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}$,

$$J_2 = a_1 s_{ij}s_{ji} + a_2 s_{ij}s_{ij} + \frac{a_3}{L^2} m_{ij}m_{ji} \tag{8}$$

where a_1, a_2 and a_3 are heuristics, and constants A and c which are related to the material's internal friction, dilatancy and cohesion. The yield function is satisfied through the use of plastic flow rules which chart the evolution of the elastic stretch and plastic curvature – see [3] for an implicit implementation.

3 Numerical formulation

To initialise an MPM analysis, the material is discretised into a number of Lagrangian material points which occupy a grid of elements joined together at nodes. All history variables including volume, stress, strain, force and translation are tracked using the MPs. In each step, the requisite quantities are mapped from the MPs to the nodes using grid shape functions in order to perform a standard FE-type computation. Once the nodal solution is obtained, it is then mapped to the MPs and their positions and state variables are updated. At this point, the grid is reset to its initial position ready for the next step. This process



(a) (b)

Figure 1: (a) the total kinematics of a micropolar continuum, with the rotational elements shown in blue; (b) the main steps of an MPM algorithm, reproduced from [6].

is then repeated for as many time- or load-steps the analysis requires. See Figure 1b for a graphical overview of a general MPM algorithm.

Although our general approach to modelling strain localisation in geomaterials has been set out, the format of this contribution limits any further elaboration of the formulation or presentation of examples. Specific details of the implementation of elasto-plastic geometrically-exact micropolar theory within the MPM will instead follow in the oral presentation. It is hoped that this novel numerical tool will offer a more robust and reliable way to analyse challenging localisation problems involving large deformations such as landslides.

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REFERENCES

[1] Sulsky, D., Chen, Z. and Schreyer, H.L. A particle method for history-dependent materials. *Computer Methods in Applied Mechanics and Engineering* (1994) **118**(1-2):179-196.

[2] Cosserat, E. and Cosserat, F. *Théorie des corps déformables*. A. Hermann et fils, Paris (1909).

[3] Bauer, S., Dettmer, W.G., Perić, D. and Schäfer, M. Micropolar hyperelasto-plasticity: constitutive model, consistent linearization, and simulation of 3D scale effects. *International Journal for Numerical Methods in Engineering* (2012) **91**:39-66.

[4] Neuner, M., Rigueiro, R.A. and Linder, C. A unified finite strain gradient-enhanced micropolar continuum approach for modeling quasi-brittle failure of cohesive-frictional materials. *International Journal of Solids and Structures* (2022) **254-255**:111841.

[5] de Borst, R., Sabet, S.A. and Hageman, T. Non-associated Cosserat plasticity. *International Journal of Mechanical Sciences* (2022) **230**:107535.

[6] O'Hare, T.J., Gourgiotis, P.A., Coombs, W.M. and Augarde, C.E. An implicit Material Point Method for micropolar solids undergoing large deformation. *Computer Methods in Applied Mechanics and Engineering* (2024) **419**:1166688.