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MODELLING POST-FAILURE BEHAVIOUR OF CHALK CLIFFS WITH THE MATERIAL POINT METHOD

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Abstract. This work evaluates the use of the Material Point Method (MPM) with continuum damage-plasticity to model fracture for the use of a combined pre- and post-failure simulation. MPM is used to allow for large deformations and geometry changes without mesh distortion and damage diffusion. An integral non-local continuum damage model is used to model brittle fracture, which avoids the mesh-dependency issues exhibited by local models. The modelling approach is demonstrated on chalk cliff collapse problems, where the final state of the rock formation after the failure is of importance and critically linked to further failure processes.

Key words: *chalk cliff collapse; post failure, large deformation, brittle fracture, damage-plastic*

1 Introduction

Cliff collapse represents a significant danger to communities that are situated or operate near cliff fronts [1], causing damage when the debris falls down onto the beach-front below, but also posing a hazard to structures and people situated at the top of the cliff. Coastal chalk cliffs ranging from 10 m to 100 m tall are a prominent feature around South-Eastern United Kingdom, Northern France, and Germany [2]. These cliffs experience very high rates of erosion, making them a geo-hazard that must be evaluated carefully. The chalk cliffs tends to undergo brittle failure when under moderate shear and tensile loads of ≈ 1 MPa [3]. Collapse causes large piles of debris to form around the base of the cliffs, which provides stability and helps protect the cliff from further collapse [2]. It is therefore important to not only understand the initiation of failure, but also the mass transport and post-failure behaviour of the collapse to accurately model recurrent failures. Modelling both the fracture and mass transport of the collapse is not trivial with the Finite Element Method (FEM), due to the large deformation run-out causing highly distorted meshes that introduce numerical issues. Pre-failure FEM-based models of chalk cliffs have been presented in [3] and [4], evaluating the effects of geometry and shear strength in a linear elastic or elasto-plastic setting. This paper uses the Material Point Method (MPM) [5] to allow for large deformation and geometry changes, while using damage and plasticity to model failure and collapse.

2 Material Point Method

The MPM [5] combines an Eulerian computational mesh with Lagrangian material points (MPs) that are allowed to move through the mesh. Multiple discrete bodies in a system are approximated as groups of MPs, and as such geometry changes are represented by the motion of MPs. As the mesh is reset at the beginning of every step, it may never become degenerate. A key advantage of the MPM is that history dependant variables are stored and used on the MPs, so fields like damage cannot suffer any numerical diffusion.

The MPM uses finite element machinery, with interpolation (or shape) functions, S_{vp}, linking the vertices of the mesh, v, with the MPs, p. The momentum balance equation is solved at the nodes of the background mesh, with the strong form:

$$\rho \frac{Dv}{Dt} = \nabla \cdot \sigma + f_b, \tag{1}$$

where ρ is the density of the body with a velocity, v, which is subject to body forces, f_b that generate a Cauchy stress field, σ. Applying Galerkin's method, discretising and approximating the volume integrals over the body Ω as summations over MPs, each representing a volume, V_p, and associated mass, m_p, we arrive at an equation that may be solved explicitly at the nodes

$$M_v a_v = \sum_{p}^{N_{mp}} \nabla S_{vp} \sigma_p V_p + \sum_{p}^{N_{mp}} S_{vp} m_p g, \tag{2}$$

where M_v is the nodal mass matrix (in this case lumped and diagonal). Once accelerations a_v, and subsequently velocities are found at the nodes, the velocity of the MPs are found by interpolating from nodes to MPs. The MPs are then advected in space, and the original computational mesh discarded and reset (or redefined).

3 Continuum damage

The use of continuum damage combined with plasticity allows for modelling progressive failure of material under combined tension and shear loading [6]. In linear elastic damage, based on the strain equivalence hypothesis that the undamaged strain is equal to the damaged strain $\overline{\varepsilon} = \varepsilon$, the undamaged stress $\overline{\sigma}$ can be related to the actual damaged stress state σ via

$$\sigma = (1-d)\overline{\sigma} = (1-d)[D^e]\varepsilon = (1-d)[D^e]\overline{\varepsilon}. \tag{3}$$

Here a uniform single scalar isotropic degradation function is shown, where $0 \leq d \leq 1$ is the scalar damage (0 represents an undamaged material), and $[D^e]$ is the elastic constitutive matrix. As in [6], this is weakly coupled with a plasticity model based on a Mohr-Coloumb yield surface where plasticity acts in the undamaged stress space, and damage effectively acts as a softening law. By separately degrading the volumetric and deviatoric components it is possible for a tensile-compressive split in degradation, such as

$$\sigma = (1 - g_v(d, tr(\overline{\sigma})))tr(\overline{\sigma}) + (1 - g_d(d))(\overline{\sigma} - \frac{1}{3}tr(\overline{\sigma})) \tag{4}$$

The aim of the model is to have some residual shear and compressive strength, governed by the limits of the functions g_v and g_d as $d \to 1$. Stress is updated with a local elastic-predictor plastic-corrector, and Eq. (4) is then used as a non-local damage-corrector to map the undamaged plastic stress into damaged space. The current damage level is set by a viscous-regularised maximum stress history κ and a softening parameter η, via an exponential softening function. A characteristic time τ enforces a maximum damage increment rate. A damage criteria is required to define the driving stress Y which updates κ, here a form of Drucker-Prager criterion is used

$$Y = \frac{3}{3 + \tan\theta} \left(\sqrt{3J_2} + \frac{I_1}{3} \tan\theta \right) \tag{5}$$

where I_1 and J_2 are the first and second invariant of the undamaged Cauchy stress and its deviator, and θ is the damage frictional angle. The driving stress is regularised with a non-local integral scheme [6], to avoid mesh dependency in the strain softening.

4 Numerical results: Joss Bay case study

An example of chalk cliff collapse measured in [7] and analysed in [4] with FEM and a Mohr-Coloumb model is analysed using the proposed MPM framework. The cliff is modelled as a homogeneous chalk material, of density $1700\,\mathrm{kg\,m^{-3}}$, with initial Young's modulus $E = 1\,\mathrm{GPa}$ and Poisson's ratio of $\nu = 0.24$ [4]. A 2D plane strain section of height $H = 15.5\,\mathrm{m}$, with a length of $2H$ is considered. The front of the cliff has three main features: a sloped lower section with an angle of around 78 degrees, a wave cut notch $L_n = 0.5\,\mathrm{m}$ at the foot with an angle of 45 degrees, and an initial tension crack $2.2\,\mathrm{m}$ back from the cliff front [7].

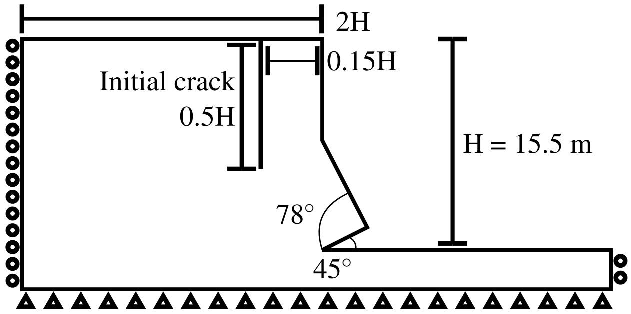


Figure 1: Numerical problem setup

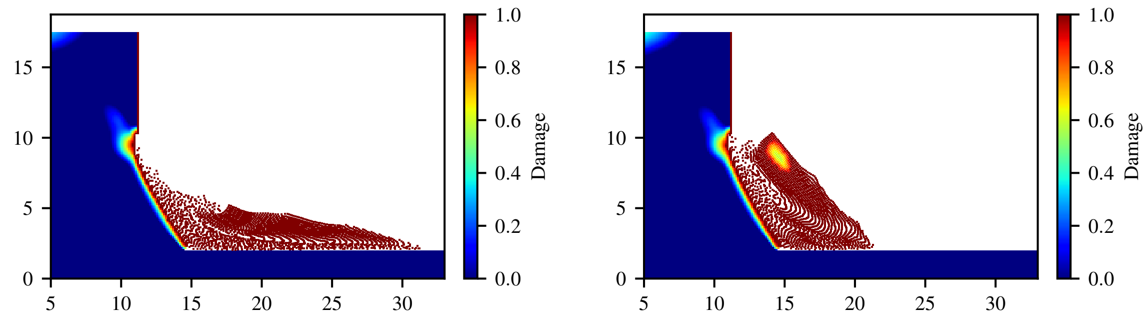
Strength parameters of the plastic model are taken as the highest possible bound of cohesion $c = 1000\,\mathrm{MPa}$, and friction angle $\phi = 50°$, with zero dilatancy. It is assumed that under compression the residual bulk modulus of the material is reduced to 1 %, the tensile bulk modulus vanishes to 1×10^{-7}%, and the shear modulus of the material is varied from $1-0.5$ %.

The damage criteria parameters are: a frictional angle $\theta = 60°$ and tensile initiation stress $\sigma_f = 20\,\mathrm{kPa}$ inferred from [4], a ductility of $\eta = 5$, giving a very low fracture energy $\simeq 10\,\mathrm{J\,m^{-2}}$, and the viscous characteristic time is taken as $\tau = 1$ s. Experimentally finding a length scale is possible, however here it is numerically taken as $l_c = 0.18$ m - a patch size roughly 4 times the mesh resolution.

As seen in Fig. 2, the shape of the debris pile is highly sensitive to the residual strength of the chalk. In Fig. 2a the very small residual strength causes a highly mobilised flow of chalk forming a debris pile similar in angle $\theta \simeq 15°$ to larger collapses in [2]. In Fig. 2b the larger residual shear strength causes a steeper pile - more closely matching the measured 44° debris angle in [7], with a qualitatively more intact debris texture.

5 Observations

The MPM shows promise for modelling cliff collapse in pre and post failure behaviour, and that it may be used for forward modelling of other brittle cliff collapse such as in marine ice cliffs. It was found the use of plasticity and isotropic damage allows for the modelling of shear failure under gravity driven



(a) Residual shear strength 0.5 %
$\phi_{residual} = 30^\circ$

(b) Residual shear strength 1 %
$\phi_{initial} = \phi_{residual} = 50^\circ$

Figure 2: Numerical results

loads, with the damage model allowing for non-local softening behaviour and plasticity de-activating the damage at large inelastic strains. Numerically the shape of the chalk debris post failure is highly sensitive to the residual shear strength, further work should find a meaningful way of calibrating this parameter.

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