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A non-hysteretic simplification to the Glasgow Coupled Model (GCM)

Takudzwa Mutsvairo[1*], Marti Lloret-Cabot[1], Paul Hughes[1], Sravan Muguda-Viswanath[1], Ana Sofia Dias[1]

[1] Department of Engineering, Durham University, Durham, DH1 3LE, UK
takudzwa.m.mutsvairo@durham.ac.uk; marti.lloret-cabot@durham.ac.uk;
paul.hughes2@durham.ac.uk

Abstract

The Glasgow Coupled Model (GCM) is an established elasto-plastic constitutive model developed to capture the coupled hydro-mechanical response of unsaturated soils. This study highlights the unique capabilities of the GCM in modelling the water retention and mechanical behaviour of unsaturated soils. Namely, its ability to capture the scanning, main wetting and main drying water retention responses, the mechanical response (including volume change and shear strength), and the coupling between the water retention and mechanical behaviour. However, due to the multiple yielding scenarios that a given stress path can trigger, numerical integration of the model is challenging and adjustments towards a more simplified approach are desirable if the model is to be used in geotechnical practice. For this purpose, the capabilities of a simplified non-hysteretic version of the GCM, designed to streamline its formulation, are investigated in this paper. The non-hysteretic model requires less parameters and simplifies the identification of the elasto-plastic mechanism activated by a stress path, at the expense of limiting some of the original modelling abilities.

Key words: *Constitutive modelling, Unsaturated soils, Water retention behaviour, Mechanical behaviour, Hysteresis, Hydromechanical-Coupling.*

1 Introduction

Soils used in the construction of civil engineering infrastructure such as road and railway embankments are compacted, thus under unsaturated conditions, where voids between solid particles are filled with both liquid (water) and gas (air). The mechanical behaviour of unsaturated soils is greatly influenced by the amount of water present within the voids, and this is described by the Soil-Water Retention (SWR) response that relates water content (w) or degree of saturation (S_r) to matric suction (s) defined as the difference between pore air pressure (u_a) and pore water pressure (u_w). An important feature of unsaturated soils is that their soil-water retention curve exhibits hysteresis, where the evolution of S_r against s measured during drying differs from that measured during wetting. In order to capture this behaviour, the Glasgow Coupled Model (GCM) assumes that the hysteresis can be represented as an elasto-plastic process (as illustrated in Figure 1) [1].

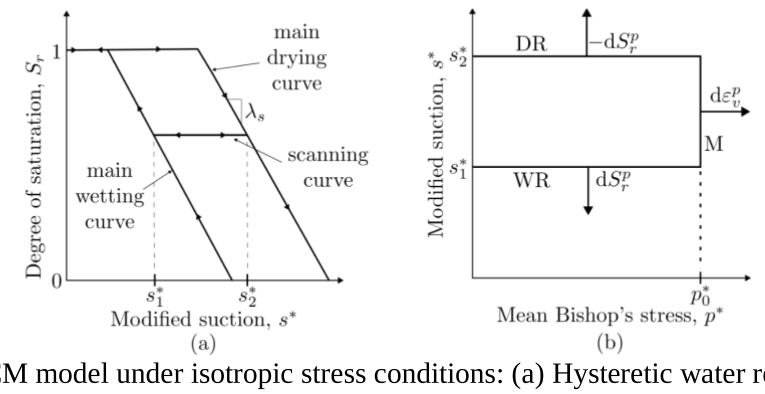

Figure 1: The GCM model under isotropic stress conditions: (a) Hysteretic water retention behaviour; (b) Mechanical (M), Drying Retention (DR), and Wetting Retention (WR) yield curves (after [1])

The GCM is able to represent water retention hysteresis through the use of two yield curves: one to represent plastic (or irreversible) decrements of degree of saturation ($-dS_r^p$) during drainage of pore water (Drying Retention, DR yield curve); and one to represent plastic increments of degree of saturation (dS_r^p) during filling of voids with water (Wetting Retention, WR yield curve) (Figure 1). One additional mechanical yield curve (M) is used to represent irreversible increases of plastic strain ($d\varepsilon_v^p$). The GCM is also able to represent the couplings between water retention and mechanical behaviour through a number of coupled movements between the three yield curves [1].

2 The GCM

The original formulation of the GCM was developed for isotropic stress states [1], then later extended to general stress states [2]. The GCM constitutive stress variables for isotropic stress states include the mean Bishop's stress (p^*) and modified suction (s^*) defined as follows:

$$p^* = p - S_r u_w - (1 - S_r)u_a = \bar{p} + S_r s \tag{1}$$

$$s^* = n(u_a - u_w) = ns \tag{2}$$

Where p is the mean total stress, $\bar{p}$ is the mean net stress, and n is the porosity. Note that when the soil becomes saturated ($S_r = 1$), Equation 1 for p^* becomes $p' = p - u_w$, where p' is the saturated mean effective stress for saturated soils. A detailed description of the elasto-plastic framework, including the governing equations, can be seen in previous works [2] and only a very brief description of the most relevant features for this work is included next. Elastic increments of volumetric strains ($d\varepsilon_v^e$) are related to the gradient of the elastic swelling line (κ) in v: lnp^* plane, where v is the specific volume. As discussed in [3], elastic variations of degree of saturation (dS_r^e) are assumed to be zero in the GCM (Figure 1a) to avoid inconsistent representations of the transitions between saturated and unsaturated conditions. The hardening parameters of the M, DR and WR yielding curves are p_0^*, s_1^*, and s_2^*, respectively (Figure 2).Yielding only on the M curve under isotropic stress conditions, produces plastic increments of volumetric strain ($d\varepsilon_v^p$) which are related to the gradient of the normal compression line (λ) in v: lnp^*plane. Yielding on the DR or WR yielding curve alone would produce plastic decrements or increments of S_r and these are related to the gradient of main drying and wetting retention curves (λ_s) in the S_r: lns^* plane (Figure 1a).

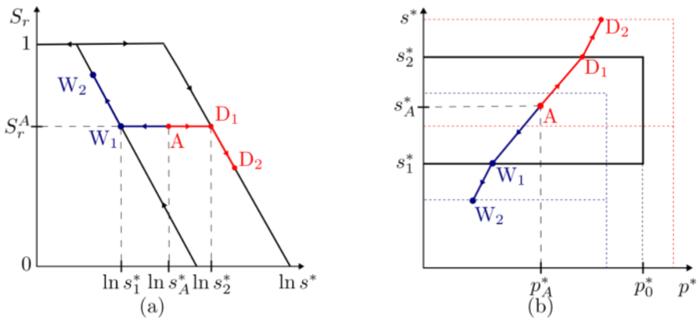

Figure *2*: The GCM model for isotropic stress conditions: (a) Typical wetting and drying water retention behaviour; (b) Typical wetting and drying paths involving water retention yielding.

To demonstrate how hysteresis is represented within the GCM, a soil under unsaturated conditions ($S_r <$ 1) is considered, which state is on a scanning curve in the water retention plane S_r: lns^*, and located within the elastic domain in the s^*: p^* plane as shown by point A in Figure 2. During drying (without any mechanical yielding), plastic decrements of degree of saturation ($-dS_r^p$) start to occur after reaching the main drying curve at point D₁ (indicated by the drying retention hardening parameter s_2^*). Yielding on DR causes a coupled upward and outward movement of the WR and M respectively. This coupling mechanism represents the stabilising effect that the increasing number of meniscus water bridges during drying has on the soil skeleton (as $-dS_r^p$ hinders slippage at the interparticle contacts causing an increase in the mechanical hardening parameter p_0^*). An equivalent behaviour is observed for the typical wetting path A-W_1 represented in Figure 2. Upon reaching the main wetting curve at point W_1, plastic increments of degree of saturation (dS_r^p) occur causing a downward and inward movement of the DR and M respectively. Due to the yielding on WR during wetting, the additional stabilising force offered by the number of meniscus water bridges at the interparticle contacts decreases, resulting in a lower value of the mechanical hardening parameter. During mechanical yielding, slippage occurs at the interparticle and interpacket contacts, causing a reduction in the size of the voids within the soil. This leads to a shift of the SWR response to higher values of modified suction [1]. These couplings between the water retention and mechanical behaviour are intrinsic within the GCM formulation and are controlled by the model parameters k_1 and k_2. Coupled movements of M during yielding on the DR and WR yield curves are given by coupling parameter k_1, whereas coupled movements of DR and WR during yielding on M are controlled by the second coupling parameter k_2. The number of parameters, yield curves and coupled movements result in a challenging mathematical framework for calibration and computational purposes [3,4]. The main computational challenge is associated with the determination of which is the correct model response activated by the stress path and adjustments towards a more simplified approach are desirable if the model is to be used more broadly, including in geotechnical practice. The study of such simplification by means of a non-hysteretic version of the GCM is the main goal of this paper.

3 A non-hysteretic form of the GCM

To adapt this non-hysteretic simplification, the two yield curves of the GCM associated with the water retention response (DR and WR) are merged to a single yield curve (Figure 3), streamlining the representation of yielding occurrences while simplifying the characterisation of the initial stress state. Any stress path in the water retention plane (S_r: $\ln s^*$), whether wetting or drying, would involve water retention yielding and the corresponding evolution of $S_r$would follow a single main water retention curve as illustrated in Figure 3a. For instance, the equivalent of the generic initial stress state A included in Figure 2, would correspond to A=D_1=W_1 when hysteresis is not considered (Figure 3), meaning that the initial value of s^* at A defines the initial position of WR and DR yield curves (as $s_A^* = s_1^* = s_2^*$).

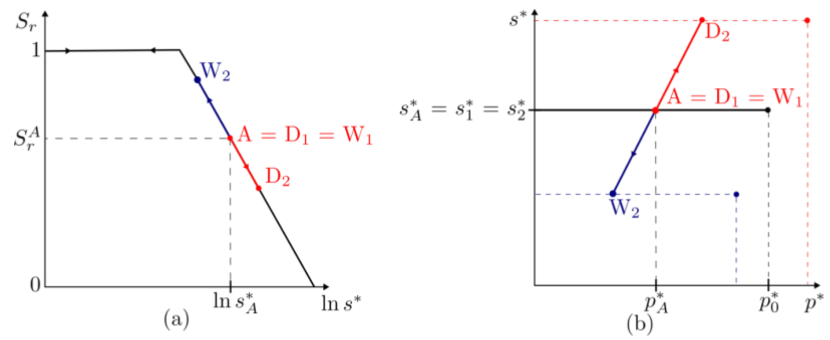

Figure 3: The non-hysteretic GCM version for isotropic stress conditions: (a) Typical wetting/drying retention behaviour in the S_r: $\ln s^*$plane; (b) Typical DR and WR yielding stress paths in the s^*:p^*plane.

Six different material responses are possible within the GCM, including elastic behaviour, water retention yielding (DR and WR only), mechanical yielding (M only), and simultaneous yielding (M and WR) and (M and DR) as illustrated in Figure 4a. The coupling parameter k_1 marks the trajectory of the top and bottom corners in the $\ln s^*$: $\ln p^*$ plane (i.e., intersection between M and DR or between M and WR, respectively) during water retention yielding. Similarly, k_2 marks the trace of top and bottom corner movements during mechanical yielding (Figure 4a).

If hysteresis is not considered, the possible model responses simplify because of the highly unlikely occurrence of the elastic response and of yielding on only the M (Figure 4b). Elasto-plastic variations of degree of saturation and plastic volumetric strains are still represented. For instance, any increases of modified suction during drying would cause yielding on DR, producing irreversible decreases of S_r. Irreversible shrinkage during drying (i.e., simultaneous yielding between M and DR yield curves) is also still possible with the non-hysteretic version of the GCM. However, this model response would be highly dependent on the stress path and how it compares with the gradient of k_1 in $\ln s^*$: $\ln p^*$ plane (see Figure 4b).

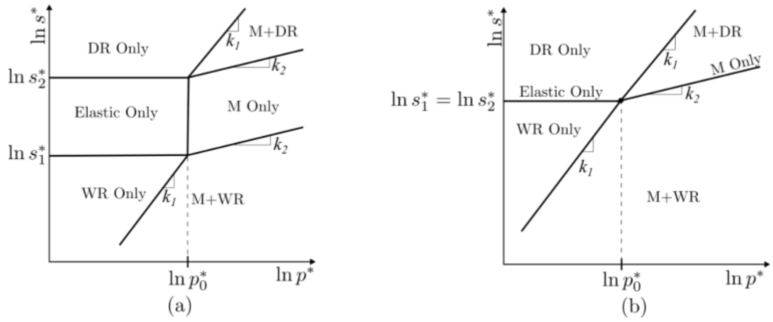

Figure *4*: Elastic and Elasto-plastic mechanisms in $\ln s^*: \ln p^*$plane: (a) GCM; and (b) non-hysteretic version of the GCM.

3.1 Discussion

Within the framework of the GCM, the separation of the DR and WR yield curves in the $s^*: p^*$plane to account for hysteresis allows for a wide range of modified suction values, at which scanning behaviour can be represented. This is particularly useful when modelling soils with a marked hysteretic behaviour and which are not possible to represent with the simplified version of the GCM presented here. More interesting, is the range of possible values of modified suction that a drying stress path (i.e., increases of suction) can experience when mechanical yielding begins to occur (see Figure 2b). A consequence of the range of modified suction values is that many different drying paths can activate yielding on M and cause plastic compression on drying (note that during a drying path, $p^*$and s^* would both increase as a consequence of the suction increase during drying as seen in Equations 1 and 2). Yielding on only the M during drying is highly unlikely to be predicted by the non-hysteretic version presented here, as the M yield curve is reduced to a single point and it will only be predicted when the direction of the drying stress path coincides exactly with the gradient of k_2 in the $\ln s^*: \ln p^*$plane as indicated in Figure 2b. However, it is still possible to represent plastic shrinkage on drying by the non-hysteretic GCM if the stress path activates simultaneous yielding between M and DR curve (see Figure 2b). Future research is needed in this direction to further investigate the likelihood of representing irreversible shrinkage on drying and to investigate how well the non-hysteretic GCM responses compare against experimental results.

4 Conclusions

This study introduces a non-hysteretic simplification to the GCM, where the two water retention yield curves (DR and WR) are merged into a single yield curve. This simplification reduces the number of parameters required and removes the occurrence of elastic changes in degree of saturation. It also makes the determination of the model mechanism active a much simpler process as less intersections of the stress path with a particular yield curve are needed (given that the initial stress state is already on the WR and DR yield curves). However, this simplification leads to some modelling limitations especially in the representation of plastic volumetric strains during drying paths which are only possible for simultaneous yielding on M and DR.

References

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