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SPATIAL VARIABILITY OF SOIL PROPERTIES IN SAUDI ARABIA: ESTIMATION OF CORRELATION LENGTH

Kun Zhang[1]*, Marti Lloret-Cabot[1], Whangcheng Zhang[1], Alaa Kourdey[1]

[1] Department of Engineering, Durham University, Durham, DH1 3LE, UK.
kun.zhang@durham.ac.uk

Abstract. Spatial variability of soil properties plays an important role in geotechnical engineering, influencing the design and performance of geo-structures. This study investigates the characterisation of this spatial variability by examining in-situ soil data obtained from 74 Cone Penetration Tests (CPTs) conducted in a reasonably homogeneous granular fill of a large construction project in Saudi Arabia. The primary focus of this work is to assess, from the available in-situ CPT data, the spatial variability of this sand fills in terms of the vertical correlation length (θ_v). The autocorrelation function (ACF) method is used to estimate θ_v, and the geo-statistical patterns of all estimated θ_vs are analysed through the values of its mean (μ), standard deviation (σ), coefficient of variation (CoV), and fitted probability density function (pdf). An assessment of the accuracy in the estimated θ_vs is then carried out from the statistical analysis of $1,000$ one-dimensional ($1-D$) random fields with spatial statistical information similar to the in-situ fill. The study shows that as much as 25% error can be expected for the specific depth ($D = 10$m) and sampling interval ($dx = 0.01$m) of the 74 CPTs considered.

Key words: *Correlation length; Spatial variability; Random field; CPT; Saudi Arabia*

1 Introduction

Natural soils are heterogeneous and their characteristics exhibit spatial variability [1]. This inherent spatial variability is a significant source of uncertainty in stochastic geotechnical engineering, making its characterisation important for the design of geotechnical structures [2]. The correlation length serves as a key parameter in simulating soil spatial variability as it measures the distance between regions with similar geotechnical properties [2]. Even though the estimation of θ_v from in-situ CPT data has been extensively researched in the literature (e.g., [3]), studies quantifying the spatial variability of Arabian sandy soils are scarce.

The main objective of this research is therefore to statistically characterise the spatial variability of an Arabian granular fill in terms of the θ_v estimated from 74 CPTs available at a site of a very large construction project in Ras Al Khair, Saudi Arabia. An assessment of the accuracy of the estimated θ_vs is then carried out, including an investigation of how varying the number of CPTs (referred to as n in this paper) would affect the accuracy of estimated θ_v. The conventional ACF method (e.g., [3]) is the strategy applied to estimate θ_v from all the in-situ CPT profiles but it is unclear whether the obtained estimates are accurate or not with the limited set of CPT data used. To study this effect, a sufficiently large sample size of CPT data (e.g., $n = 1,000$) is desirable. For this purpose, $1,000$ artificial CPTs containing spatial statistical information similar to that of the in-situ sand fill are generated using the LAS method [4] and a statistical analysis is then carried out to quantify the influence that the number of CPTs (n) considered has in the accuracy of the estimated θ_vs.

2 Estimation of vertical correlation length from CPT data

2.1 In-situ geotechnical information available

A very large shipyard project is currently under construction in Ras Al-Kahir city, located in the Easter coastal part of Saudi Arabia. The huge dimensions of the project (covering an area of about $1,125$ha), the associated costs and the complex geotechnical conditions have allowed for an extensive soil investigation campaign. Seventy-four CPTs are available from an area of the project with a reasonably homoegenous granular soil and these 74 CPTs are used in this work to quantify the soil heterogeneity of the site in terms of vertical correlation length. Each of these CPTs included information on cone tipe resistance (q_c), sleeve fiction (f_s), friction angle (R_f) and pore water pressure (u) but only the tip resistance is considered here. For consistency, only the first $10m$ of measured tip resistances from the ground surface ($D = 10m$) are used in all CPTs. The sampling interval is also the same in all CPTs considered ($dx = 0.01m$).

2.2 Geo-statistical characterisation

The autocorrelation function (ACF) method is one of the most generally employed methods for estimating correlation length of soil properties from in-situ data (e.g. [3, 5, 6]), and is also applied in this study to determine θ_v from the CPT data. The application of the method requires that the data used is stationary, meaning that the mean and standard deviation of the data are constant in the space. As discussed, for instance in [3], it is common in geotechnical engineering to assume stationarity after removing a linear trend to the data and normalising the de-trended data by the standard deviation (calculated from the de-trended data). Applying this process to each individual CPT, leads to a set of de-trended and normalised tip resistances with an approximate zero mean and unit standard deviation (and hence satisfying the stationarity conditions). The stationary data is used to estimate the experimental correlation function $\hat{\rho}(\tau)$ approximated here as:

$$\hat{\rho}(\tau_j) = \frac{1}{k-j} \sum_{i=1}^{k-j+1} X_i X_{i+j}, \quad for \quad j = 0, \ldots k-1 \tag{1}$$

where τ is the lag distance between two points, X_i and $X_{(i+j)}$ are the CPT measurements, k is the total number of measured points and i is the total number of pairs of points.

This experimental correlation function $\hat{\rho}(\tau)$ is then best-fitted to a theoretical correlation model from where θ_v is found. From the different possible alternatives for $\rho(\tau)$, this research assumes a Markovian correlation structure defined by:

$$\rho(\tau) = \exp\left(-\frac{2|\tau|}{\theta}\right) \tag{2}$$

Fig. 1a shows the CPT tip resistance q_c for all in-situ CPTs investigated. In the plot, the thin lines indicate q_c values for each individual CPT profile and the thicker dashed line indicates the linear mean trend. Fig. 1b shows the de-trended and normalised q_c values. Fig. 1c shows the individual experimental correlation function for the vertical direction for each of the 74 CPTs used together with the best fitted theoretical correlation function (corresponding to the value of θ_v estimated from an average experimental correlation function which is indicated in the figure by a thicker dashed line). Fig. 1d illustrates the histogram based on all the estimated vertical correlation lengths as well as the fitted normal distribution showing that the estimated θ_vs are reasonably well-represented by a normal distribution with mean (μ=0.848m), standard deviation (σ=0.502m), coefficient of variation (CoV=0.592).

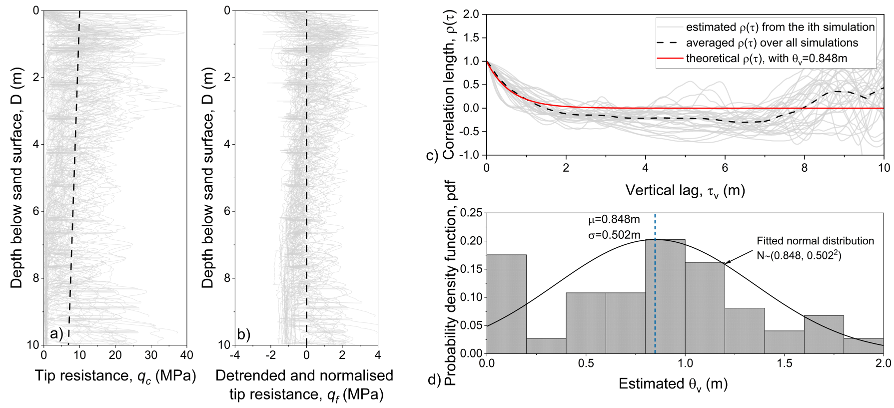

Figure 1: Geostatistical characterisation of in-situ CPT data: (a) tip resistance; (b) de-trended and normalised tip resistance; (c) Experimental and theoretical correlation functions and (d) histograms of estimated θ_vs.

3 Accuracy assessment based on fictitious CPT data

3.1 Artificial CPTs generated using 1-D LAS

The analysis of the above information raises the question of how accurate are our estimations for θ_v and, more interestingly, how an increased number of CPT would improve the accuracy of the obtained mean estimated θ_v. Quantitative evidence to answer this question can be obtained following the ideas discussed in [7] on generating $1-D$ random fields using the Local Average Subdvivision (LAS) method [4] and interpret them as artificial CPTs. In this context, LAS is utilised to generate $1{,}000$ $1-D$ random fields representing the fictitious CPT data with zero mean, unit standard deviation and a vertical correlation length θ_0=0.848m. The depth D and spacial sampling dx of the generated CPTs are as the ones used for the in-situ data (i.e., $D = 10m$ and $dx = 0.01m$).

3.2 Influence of number of CPTs

From the generated $1{,}000$ artificial CPTs it is possible to study the influence of n on the estimated θ_v, noting that D and dx are kept fixed in this analysis, so that the influence of these variables on the accuracy of the estimated θ_v (see e.g., [6]) does not affect the results. In order to analyse this influence, it is first necessary to estimate the θ_v in each artificially generated CPT following the same approach discussed for the in-situ data.

Fig. 2 shows the information from all estimated θ_v in a form that facilitates the discussion. The vertical axis corresponds to the ratio between the mean estimated θ_v over a specific number of CPTs (referred to $\bar{\theta}$ as in the figure), and the value of the θ_v used as input in LAS (indicated as θ_0). The horizontal axis corresponds to the number of CPTs n considered (using a logarithmic scale). The shadowed grey area in Fig. 2 shows the range of possible values of $\bar{\theta}/\theta_0$ for a given value of n. This area is bounded by two curves (indicated as dashed lines in the figure) that correspond to maximum and minimum values that the ratio $\bar{\theta}/\theta_0$ can take for a given value of n, amongst all $1{,}000$ CPTs. The continuous line within this shadow area shows how the ratio $\bar{\theta}/\theta_0$ varies with increasing n clearly illustrating the improvement on the averaged estimated θ_v with increasing n. For the particular case of n = 74, the analysis shows that the expected error in the averaged estimated θ_v can be as much as 25%.

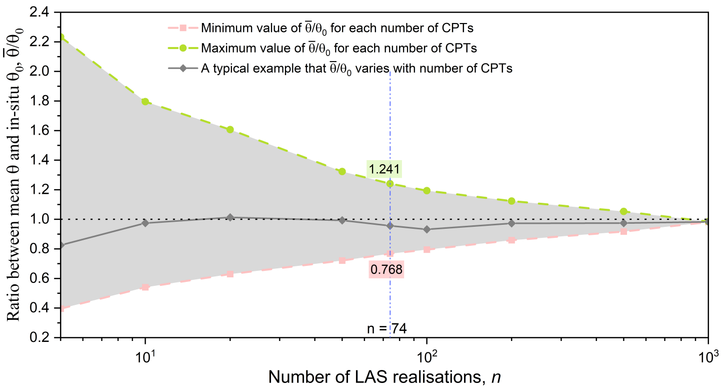

Figure 2: The upper and lower boundaries for the values of $\bar{\theta}/\theta_0$ varying with number of LAS realisations.

4 Conclusions

This work has presented an analysis of the spatial variability in the vertical direction of the tip resistances measured in a sandy deposit of a construction site in Saudi Arabia. Using 74 CPTs with a sampling distance of $dx = 0.01m$ and a total depth $D = 10m$, the estimated vertical correlation length obtained is $\theta_v = 0.848m$. Based on the results from a subsequent statistical analysis using $1,000$ $1-D$ random fields, the expected error in this estimation can be as much as 25%.

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