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Optimal Sovereign Debt Relief and Exclusion with Unobservable Physical Capital

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Abstract

We investigate the optimum lending arrangements when there is the possibility of partial default, in addition to full default when physical capital is unobservable. In a model calibrated on Argentina, we find an optimal debt reduction of 39%, and optimal re-entry probability of 0.10. Full default is more likely when total factor productivity is very low, and either debt is low or very high. Partial default is more likely when debt is moderate. Monte Carlo simulations under the optimum lending arrangements indicate the economy spends 47.90% of the time in partial default, translating into an average partial default probability of 9.12%. This is quantitatively close to what emerging economies have experienced, thus suggesting that current arrangements are close to optimal. In fact, if there is a competitive market for borrowers, we would expect risk-neutral lenders to offer schemes giving higher utility to the borrower, and thus the competitive market should converge to the optimal scheme.

Keywords

Optimal Sovereign Default, Debt Relief, Re-Entry Probability, Unobservable Physical Capital

1. Introduction

Sovereign default has been widely discussed in the international financial market since the 1920s, when many European countries faced extensive debt rescheduling.[1] Overtime, numerous countries have experienced sovereign defaults and asked

[1]Typically, debt relief is achieved by a contract between debtor and creditor countries and involves a period of exclusion (penalty) from the international financial market. It can be empirically measured by looking at creditors' haircuts (Cruces & Trebesch, 2013).

for bailouts or debt reliefs.[2] Average debt relief for middle-high income countries in recent decades is approximately 36.1% of external public debt (Reinhart & Trebesch, 2016). Many countries (see for example the case of Argentina, Brazil and Mexico) have made agreements with international bondholders and spent several years in debt restructuring before re-entering the international financial market after a default. We often observe that many borrowers partially default and are offered debt restructuring (Cruces & Trebesch, 2013; Asonuma & Trebesch, 2016).

When it comes to sovereign debt the lender cannot seize collateral assets from defaulters. Hornbeck (Hornbeck, 2004), Gelpern (Gelpern, 2005) and Miller & Thomas (Miller & Thomas, 2007), state that trade intervention and public assets seizures by sovereign creditors are typically not implementable. Indeed, trade intervention against defaulters is illegal under WTO regimes and seizures of land and state-owned enterprises are often precluded by domestic laws and court judgements. Furthermore, the scope of international creditors' litigation against sovereign borrowers is in general very limited (Alfaro, 2015). Therefore, exclusion from international from financial markets may provide a deterrence to default and debt restructuring schemes seem to be the most viable avenue expectedly leading to debtors' exit from debt hangovers and creditors' partial recovery of losses (haircuts).

Reinhart et al. (Reinhart et al., 2003) and Porzecanski (Porzecanski, 2006) provide interesting insights pertaining to sovereign indebtedness. From the 16th to 19th centuries, many developed countries, such as France, Germany and Spain, experienced sovereign default several times. Borrowing by these countries often occurred because of wars and later on due to the need to finance the transition from an agrarian to an industrial economy. In the late 20th century, sovereign defaults were mainly experienced by low-middle-income countries and later also by developed countries.

In 1995, the ratio of external debt to GDP for Africa, Developing Asia, Middle East, Latin America and Emerging Europe was 72%, 33%, 58.5%, 37% and 38% respectively (IMF, 2004). As a result, countries in Africa and Latin America often faced debt hangover, default and exclusion more than other regions. Thus, the 1990 Brady Plan enabled numerous proposals for debt relief from highly indebted poor countries (HIPC). For the period 1820-2012, Tomz and Wright (Tomz & Wright, 2007, 2013) found that the unconditional probability of default was around 1.8, with Meyer et al. (Meyer et al., 2019) recording 313 defaults in the period 1815-2016 with instances of serial defaults (see on this also Reinhart & Rogoff, 2004).

Reinhart and Rogoff (Reinhart & Rogoff, 2009) find evidence for financial crisis and consequent increase in government debt triggering defaults, while the evidence for defaults occurring during recessions is not clear cut (see Tomz &

[2]Porzecanski (Porzecanski, 2006) suggests that sovereign default can be inferred from the debt to government revenue ratio and debt to GDP ratio. Cantor et al. (Cantor et al., 2008) shows that credit ratings for emerging countries is significantly correlated with the debt to sovereign revenue ratio. Sovereign bonds with lower credit rating will face a higher bond premium, requiring higher debt service and being more difficult to roll-over.

Wright, 2007), even if countries typically borrow more during bad times. Some studies (Tomz & Wright, 2007; Yeyati & Panizza, 2011; Reinhart & Rogoff, 2011) find evidence that sovereign defaults have occurred from high public debt and negative output shocks, since the early nineteenth century. Therefore, the overall level of public debt and low output therefore can be indicators of default risk. Nevertheless, when one looks at the external debt-to-output ratio evidence is more mixed. For example, Russia in 1991, Turkey in 1978, Chile in 1972 and The Dominican Republic in 1982 chose to default with relatively low external debt ratios of 12.5%, 21.0%, 31.1% and 31.8% respectively. On the contrary, Guyana in 1982, Jordan in 1989, Costa Rica in 1981 and Egypt in 1984 defaulted with external debt to GNP ratios of 214.3%, 179.5%, 136.9% and 112.0% respectively. Moreover, Reinhart and Rogoff (Reinhart & Rogoff, 2011) also find an insignificant direct relationship between default probability and public debt crises. The authors explain that there had been a change in the safe thresholds of external debt over the past century which makes intertemporal comparisons difficult. For example, presently, many countries can hold higher debt ratios with lower bond risk premiums than in the past.

Perhaps the strongest evidence of a negative correlation between output and the occurrence of sovereign default is in Yeyati and Panizza (Yeyati & Panizza, 2011), who investigated the default experience of 39 countries between 1970 and 2005. Output significantly drops one year before the default and keeps decreasing in and after the default year. Moreover, if the sovereign default occurred during the banking this negative effect is enhanced. On the contrary, analysis based on quarterly data shows that output significantly increases after default with default representing a signal for an imminent recovery. The debtor countries can therefore use default as a strategy to maximise their welfare.

After default, sovereign debtors would be temporarily excluded from the international credit market, typically experiencing low economic performance (cost of default). Some countries spent over 50 years in debt exclusion (e.g. following defaults in Russia in 1918, Greece in 1826 and Honduras in 1873). In modern time, countries in Latin America spend over 40% of the time in debt restructuring whilst Asian countries 10% (Reinhart & Rogoff, 2008). Tomz and Wright (Tomz & Wright, 2013) find that the average duration of exclusion is 9.9 years. The default countries can regain access to the international credit markets after completing debt renegotiation with lenders.

Reinhart and Trebesch (Reinhart & Trebesch, 2016) divide periods of debt relief over the past century into two main eras.[3] The first era includes the period between 1920s and 1930s, in which the US and UK provided war debt relief to

[3]Within those eras, Reinhart and Trebesch (Reinhart & Trebesch, 2016) identify 4 landmark debt-relief programmes: The 1920s debt rescheduling programme, the 1931 Hoover Moratorium, the 1986 Baker Plan and the 1990 Brady plan. The last two programmes were widely acknowledged as the main debt relief schemes for emerging markets over the last few decades. In the 1990s, 16 countries signed on for the Brady initiative plan of debt relief (Reinhart & Trebesch, 2016). 22 HIPCs signed on in 2002 for conditional agreement of debt relief (Edwards, 2003).

several European countries, such as Austria, Belgium, France, Greece, Italy, Germany and Portugal, in the range of 17.6% and 28.6% of GDP. During this time, the resolution was typically a restructuring of the debt by replacing the non-performing debt with new bonds characterised by a reduced interest rate and a longer length of maturity. The second era includes the period between 1978 and 2010, in which sovereign defaulters were middle-high-income emerging markets economies, especially in Latin America and Africa. The average debt relief to these countries was around 36% of external public debt. In this period, resolutions typically were in a softer form with many debt-rescheduling enacted through a temporary freeze of payments or bridging loans.

The amount of debt relief can be captured by the extent of the losses to sovereign lenders (haircuts) According to Cruces and Trebesch (Cruces & Trebesch, 2013), between 1978 and 2010 creditors faced average losses (haircuts) of 37%. They found that the higher the size of the haircuts the lower the probability of re-entry into the sovereign debt market (and the higher is the length of the exclusion period). Their findings fit the default events of several countries as underlined by Porzecanski (Porzecanski, 2005). Pakistan in 1999, Ukraine in 1998 and Uruguay in 2003 had to repay creditors in full and re-entered the market within 1, 3 and 9 months, respectively. Argentina in 2005, Ecuador in 2000 and Russia in 1998 returned to the market within 38, 10 and 18 months with debt reductions of 66.3%, 40% and 37.5%, respectively. The latter facts may suggest that a long period of debt negotiation (exclusion) translates into loss of opportunities for bondholders to recover their investment and ultimately into a high amount of haircuts (debt reduction) (Porzecanski, 2012). This effect is also found in Benjamin and Wright (Benjamin & Wright, 2009) who used a sample of 73 countries between 1989 and 2006 to show a positive relationship between debt restructuring and debt relief. If sovereign borrowers spend longer in debt renegotiation during default, the rate of debt reduction will increase. An average of eight years in debt restructuring, results in an average haircut of 44%.

When reviewing the historical experience on debt default and relief reported above, a main question arises on whether a contract can be designed that includes the optimal size of debt default and optimal re-entry probability (thereby the expected length of the exclusion period). This paper addresses the above question and is related to the existing literature on DSGE models of sovereign default[4]. The seminal DSGE model of endogenous sovereign debt and default was provided by Eaton and Gersovitz (Eaton & Gersovitz, 1981) and further developed by Aguiar and Gopinath (Aguiar & Gopinath, 2006) and Arellano (Arellano, 2008), among others. In these papers borrowers strive to maximize intertemporal utility by using foreign assets (as physical capital is not present). The authors model full default with zero repayment and include

[4]For a comprehensive review and latest developments of DSGE models of sovereign default see Aguiar and Amador (Aguiar & Amador, 2021).

an exogenous probability of re-entry in the international credit market after some periods of debt exclusion. They find that countries will borrow more during bad times and repay during good times (countercyclical policy) and, due to the absence of capital, the only option is to borrow for consumption.

More recent papers, see for example Bai & Zhang (2012), Romero-Barrutieta et al. (2015), Park (2017), Gordon & Guerron-Quintana (2018), include physical capital as an asset available to borrowers in addition to foreign bonds. The role of lenders is implicit in these models as the market can fully observe borrowers' assets and always accept the requested amount of borrowing by adjusting the risk premium, providing borrowers do not default in the next period. Observable capital makes the bond price schedule contingent on physical capital and provides a direct incentive for borrowers to increase capital investment in order to reduce the cost of capital. Park (Park, 2017) highlights that if borrowers hold high physical capital, they are allowed to borrow more from the international credit market and enjoy a lower risk premium. Therefore, in models where capital is present and observable by the lenders, countries typically have an incentive to over-accumulate physical capital. Interestingly, Romero-Barrutieta et al. (Romero-Barrutieta et al., 2015) find that sovereign borrowers will accumulate debt (borrow more) for consumption rather than investment if they can obtain debt relief from lenders. Yaisawang et al. (Yaisawang et al., 2021) developed a model where physical capital is unobservable to the lender. Consequently, the bond price schedule is independent of physical capital. It opens up the possibility that countries borrow for consumption rather than investment, as the loan terms (bond price) remain unchanged. They show, however, that this is not the case. The country would borrow both for consumption and investment.

To our knowledge, previous DSGE models of sovereign default have typically not included modelling of partial default, with the exception of the literature on debt renegotiation, Romero-Barrutieta et al. (Romero-Barrutieta et al., 2015), Adam and Grill (Adam & Grill, 2017) and Arellano et al. (Arellano et al., 2023). The debt renegotiation literature (Bulow & Rogoff, 1989, Yue, 2010; Benjamin & Wright, 2009, Asonuma & Trebesch, 2016; Asonuma & Joo, 2020, 2021) accounts for some debt repayment emerging after default through negotiation between borrowers and lenders. In this way, debt relief is endogenized. Romero-Barrutieta et al. (Romero-Barrutieta et al., 2015) developed a DSGE model calibrated on Uganda data between 1982 and 2006 to investigate the effect of different levels of default probability and debt relief on the macroeconomy in the presence of technology shocks. They show that debt relief leads to more debt, as it incentivises consumption and hampers investment and long-run growth. Long-run debt is twice as large with debt relief than without. Adam and Grill (Adam & Grill, 2017) solve for the borrower's optimal state-contingent path of default decisions. The borrower decides at the beginning of time on the default decision (full or partial) for every future date and for each level of in the future realised productivity shock. Thus, it is assumed that the government can commit to such policies (and is unable to re-optimise). Rather than being excluded for a time interval from the international credit market in case of default, it is assumed that there is a cost of default proportional to the level of debt not repaid. Arellano et al. (Arellano et al., 2023) present a model of partial default where the borrower re-optimises in every time period (time consistent solution). Physical capital is however not present, so the purpose of borrowing is to smooth consumption. There is no exclusion (and thus no associated probability of re-entry), but instead an output cost large enough to avoid default in all states of nature. The main aim of the paper is to find the debt reduction to match the data, rather than its optimal value.

The model of our paper has two main innovations with respect to the above literature: physical capital is unobservable by lenders (as in Yaisawang et al., 2021) and debt reductions and average exclusion periods are optimised, in an economy where partial and full default decisions are Markov perfect. Having capital as unobservable account for the controversies surrounding the measurement of capital stock series. Indeed, capital stock is often not directly measured, but instead computed by national statistics institutes from investment data, subject to local methodology and assumptions (see OECD, 2009). As standardization is not yet widespread, questions may arise on the reliability of capital stock estimates. It is argued that a country, especially if less developed, might incorrectly estimate capital accumulation due to problems with raw data availability, measurement, and recording (see for example, Blavy, 2006; Escribá-Pérez et al., 2023). Thus, one of the merits of our approach is in providing a new framework for scrutinizing sovereign-debt decisions in the absence of information on capital stock. Besides, we take a normative view in that to our knowledge this is the first study that focuses on the optimal debt relief and the probability of re-entry in a DSGE model of sovereign default.

The remaining of the paper is structured as follows. Section 2 presents our model, including options for full and partial defaults and exclusions. The parameters of the model and the state space are presented in Section 3 based on the Argentine economy between 1980 Q1 and 2017 Q4. Section 4 presents the quantitative results (value functions, default probabilities, bond price schedules, the Monte Carlo simulations, and welfare comparisons). Section 5 concludes.

2. The Model of Sovereign Partial Default

This section includes our model. **Figure 1** below describes the timing of default decisions. The borrower enters period t with a physical capital stock k and foreign assets b (with $b < 0$ meaning debt). The borrower chooses between paying $(-b)$ or defaulting (def). If defaulting, the borrower faces a choice of full default ($b = 0$) or partial default (where debt is reduced to a fraction λb, with $0 < \lambda < 1$). In both cases, the borrower is excluded from the international market (autarky) and the only choice variable is k'. In the next period, $t + 1$ the borrower can return to the international market with probability θ_1 if the default was full, and θ_2 if it was partial (with $\theta_1 < \theta_2$). When returning from full default b is equal to 0, when returning from partial default the liability is λb.

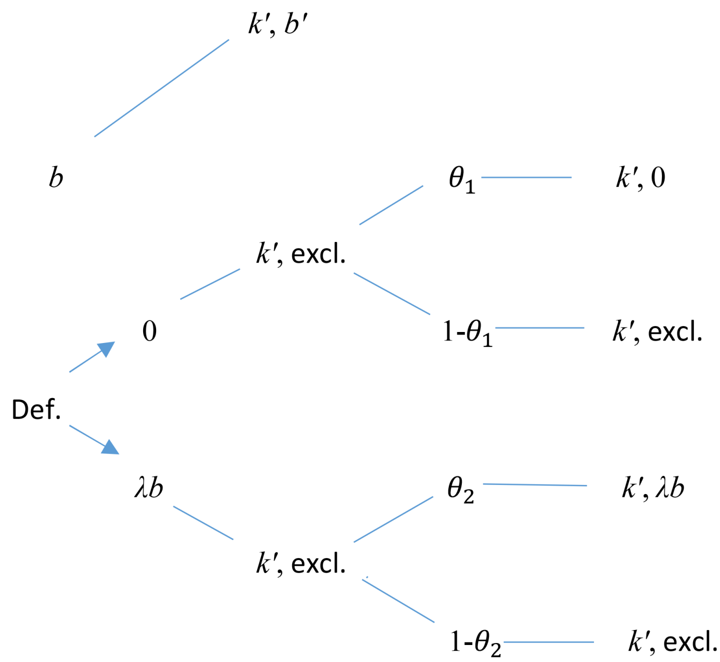


Figure 1. Timing of default decisions.

2.1. Preferences

The representative individual of the borrowing country has the following utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma} \tag{2}$$

where c is consumption, γ is the relative risk aversion parameter and β the discount factor.

2.2. Production Functions

The production function is assumed to be of the Cobb-Douglas specification:

$$y = ak^{\alpha} \tag{3}$$

where y is output, k is physical capital and a is the (stochastic) total factor productivity (TFP).

TFP follows an AR (1) process with the Gaussian distribution:

$$\ln a' = \mu + \rho \ln a + \epsilon \tag{4}$$

where a' denotes the net period's TFP, and $\epsilon \sim N(0, \sigma_{\epsilon}^2)$.

If the country defaults on its debt (partially or fully), it will be excluded from the international credit market for an interval of time. We further assume (as in the previous literature) that during exclusion, TFP is below its normal level. Denoting TFP during default by a^{def}, production is:

$$y^{def} = a^{def} k^{\alpha} \tag{5}$$

Stylised facts indicate that the output of defaulting countries tends to be lower during default periods (Borensztein & Panizza, 2009; Reinhart & Rogoff, 2011; Tomz & Wright, 2007; Zymek, 2012). The boundaries of TFP during repayment and default periods are written as: $a \in (\underline{a}, \overline{a})$ and $a^{def} \in (\underline{a}, \overline{a}^{def})$, respectively, where $\overline{a}^{def} \le \overline{a}$.

2.3. The Resource Constraints of the Sovereign Borrower

2.3.1. Closed Economy

When the country is excluded from international borrowing and lending, the economy is modelled as a closed one, with access to only one asset (physical capital). The resource constraint is then:

$$c = y^{def} + (1-\delta)k - k' - \Phi(k, k') \tag{6}$$

where k' denotes next period's physical capital, and δ the depreciation rate. We also assume that there is a capital adjustment cost $\Phi(k', k)$.

2.3.2. Open Economy

When the country has access to international borrowing and lending, there is a further asset, denoted b, where b takes a negative value if the country borrows. We introduce a bond-price schedule q. So, say that the country wishes to enter a loan contract stipulating payment of b' in the following period, the amount of money available for the country in the present period is $q(-b')$. We allow the bond price schedule to be a function of b' and a, but not on physical capital as it is unobservable in our model.[5] Consequently, the open economy resource constraint is:

$$c = y + b - q(b', a)b' + (1-\delta)k - k' - \Phi(k, k') \tag{7}$$

2.4. The Decision Functions of the Sovereign Borrower

The sovereign borrower has the choice of honouring the debt (repaying) or defaulting $b = 0$. If repaying, access to the international market is granted. If defaulting, the borrower has the choice of full default or partial default. If full default the borrower is excluded from the international market and will be allowed to re-enter (with debt level zero) with probability θ_1. If opting for partial default, re-entry occurs with probability θ_2, but at re-entry, a fraction of the initial debt defaulted upon is paid (with no possibility of borrowing for repayment). $\theta_2 > \theta_1$ implying that the (expected) exclusion period for full default on average will be longer than that of partial default. Thus, a country will trade off the gain from full default against longer exclusion. This type of debt contract expands the choice set for the borrower without lowering the expected return for the lender

[5]We could have introduced a third asset, a domestic fund distinct from physical capital. This would allow the borrower to pay the international debt in case of a negative shock, thus lowering the probability of default and consequently facing a lower cost of borrowing. However, this is equivalent to our current set up, with $-b$ being net borrowing (international debt minus domestic fund).

(as it always will be equal to r in equilibrium). The value function at any date is

$$v_g^0\left(b,k,a\right)=\max_{\{d,r\}}\left\{v_g^d\left(0,k,a\right),v_g^r\left(b,k,a\right)\right\} \tag{8}$$

where $v_g^d\left(0,k,a\right)$ and $v_g^r\left(b,k,a\right)$ are the value functions of default and repayment, respectively.

Denoting partial default debt repayment by b'_{pd}, the value function of default (choosing between full and partial default) is

$$v_g^d\left(b,k,a\right)=\max_{\{k'\}}\left\{u\left(c\right)+\beta\max\left\{v_g^{fd},v_g^{pd}\right\}\right\} \tag{9}$$

subject to

$$v_g^{fd}=\mathbb{E}\left[\theta_1 v_g^0\left(0,k',a'\right)+\left(1-\theta_1\right)v_g^{fd}\left(0,k',a'\right)\right] \tag{10}$$

$$v_g^{pd}=\mathbb{E}\left[\theta_2 v_g^0\left(b'_{pd},k',a'\right)+\left(1-\theta_2\right)v_g^{pd}\left(b'_{pd},k',a'\right)\right] \tag{11}$$

$$c,k'\geq 0 \tag{12}$$

$$-b>-b'_{pd}>0 \tag{13}$$

and Equation (6).

We assume a constant fraction of debt being re-payable in the case of partial default:

$$\lambda=b_{pd}/b \tag{14}$$

Consequently, the debt relief percentage is $1-\lambda$.

If the borrower is honouring the debt, the value function of repayment is:

$$v_g^r\left(b,k,a\right)=\max_{\{b',k'\}}\left\{u\left(c\right)+\beta\mathbb{E}v_g^0\left(b',k',a'\right)\right\} \tag{15}$$

subject to:

$$c,k',q\left(b',a\right)\geq 0 \tag{16}$$

and Equation (7).

If the borrower is re-entering the international market under partial default, it solves the problem in (14), subject to (15), (7) with $b=b_{pd}$, and an extra constraint that $b'\geq 0$. This constraint ensures that the borrower is not borrowing for repayment.

2.5. Capital Adjustment Cost

We assume the capital adjustment cost $\Phi\left(k',k\right)$ is quadratic:

$$\Phi\left(k',k\right)=\frac{\Phi}{2}\left(\frac{k'-\left(1-\delta\right)k}{k}\right)^2 k \tag{17}$$

2.6. Bond price Schedule

International lenders are assumed to be risk-neutral. They will be indifferent between getting $(1+r)q$ for sure and getting the expected repayment from lending to the sovereign borrower. Denoting by Ψ^{fd} and Ψ^{pd}, the probability of full default and partial default, respectively, we have

$$q(1+r)=\Psi^{fd}\mathbb{E}\left[R^{fd}\right]+\Psi^{pd}\mathbb{E}\left[R^{pr}\right]+\left(1-\Psi^{fd}-\Psi^{pd}\right)\mathbb{E}\left[R^{fr}\right] \tag{18}$$

where $\mathbb{E}\left[R^{fd}\right]=0$ and $\mathbb{E}\left[R^{fr}\right]=1$. Hence, this can be re-written as

$$q(1+r)=\Psi^{pr}\mathbb{E}\left[R^{pr}\right]+1-\Psi^{fd}-\Psi^{pd} \tag{19}$$

Moreover, the expected return on partial default is derived from the partial repayment (bailout) and the probability of re-entry as follows. As re-entry happens with probability θ_2, the (reduced) payment of λ takes place in the next period with probability θ_2. This payment is discounted at rate r. With probability 1 − θ_2 we move one further period ahead with a new chance of re-entry (with probability θ_2), this is then discounted further at rate r, and so on. Consequently

$$\mathbb{E}\left[R^{pr}\right]=\frac{1}{1+r}\left\{\theta_2\lambda+\left(\frac{1-\theta_2}{1+r}\right)\theta_2\lambda+\left(\frac{1-\theta_2}{1+r}\right)^2\theta_2\lambda+\cdots+\left(\frac{1-\theta_2}{1+r}\right)^\infty\theta_2\lambda\right\} \tag{20}$$

which becomes[6]

$$\mathbb{E}\left[R^{pr}\right]=\frac{\theta_2\lambda}{r+\theta_2} \tag{21}$$

Then (19) can be re-written as

$$\begin{aligned} q(1+r)&=\Psi^{pd}\frac{\theta_2\lambda}{r+\theta_2}+1-\Psi^{fd}-\Psi^{pd} \\ &=1-\Psi^{fd}-\Psi^{pd}\left(1-\frac{\theta_2\lambda}{r+\theta_2}\right) \end{aligned} \tag{22}$$

2.6. Perceived Equilibrium Default Probabilities and Equilibrium Bond Price Schedule

The bond-price schedule is a function of the full default and partial default probabilities:

$$q(b',a)=\left[1-\Psi^{fd}(b',a)-\Psi^{pd}(b',a)\left(1-\frac{\theta_2\lambda}{r+\theta_2}\right)\right]\Big/(1+r) \tag{23}$$

Since k is not observed by the lender, in finding the (perceived by the lender) default probabilities, we assume that the lender acts agnostically with a best guess that k is at its steady state level k^*. Consequently, we need to find the default probabilities $\Psi^{fd}(b',a)$ and $\Psi^{pd}(b',a)$ when $k=k^*$. The borrower's choice function at k^* is:

$$v_g^0(b,a;k^*)=\max_{\{d,r\}}\left\{v_g^d(0,a;k^*),v_g^r(b,a;k^*)\right\} \tag{24}$$

where $v_g^d(0,a;k^*)$ and $v_g^r(b,a;k^*)$ are derived below.

The value function of default is

$$v_g^d(0,a;k^*)=\max_{\{k'\}}\left\{u(c)+\beta\max\left\{v_g^{fd},v_g^{pd}\right\}\right\} \tag{25}$$

[6]This could of course be derived by realising that $\mathbb{E}\left[R^{pr}\right]=\theta_2\frac{\lambda}{1+r}+(1-\theta_2)\frac{1}{1+r}\mathbb{E}\left[R^{pr}\right]$.

subject to

$$v_g^{fd} = \mathbb{E}\left[\theta_1 v_g^0\left(0,a';k^*\right)+\left(1-\theta_1\right)v_g^{fd}\left(0,a';k^*\right)\right] \tag{26}$$

$$v_g^{fd} = \mathbb{E}\left[\theta_2 v_g^0\left(b'_{pd},a';k^*\right)+\left(1-\theta_2\right)v_g^{pd}\left(b'_{pd},a';k^*\right)\right] \tag{27}$$

$$c = a^{def} k^{*\alpha} - \delta k^* \tag{28}$$

$$-b > -b'_{pd} > 0 \tag{29}$$

The value function of repayment is

$$v_g^r\left(b,a;k^*\right) = \max_{\{b'\}}\left\{u\left(c\right)+\beta\mathbb{E}v_g^0\left(b',a';k^*\right)\right\} \tag{30}$$

subject to:

$$c = ak^{*\alpha} + b - q\left(b',a\right)b' - \delta k^* \tag{31}$$

$$c,k',q\left(b',a\right) \ge 0 \tag{32}$$

where $q\left(b',a\right)$ is given by Equation (23). Again, if the borrower re-enters under partial default, the extra constraint $b' \ge 0$ is imposed. We are then able to compute the default probabilities for an economy where $k = k^*$. These default probabilities then give the bond price schedule.

3. Parameters

We use the parameter values from the existing literature on Argentina reported in **Table 1**. For solving the model numerically, we use a discretised state space for solving the model numerically, where the range of physical capital $\{\underline{k},\overline{k}\}$ and foreign assets $\{\underline{b},\overline{b}\}$ is set to {2, 13} and {−4.5, 0.5}, respectively. The number of grid points for physical capital and foreign assets are 71 and 51, respectively. We also discretise the continuous stochastic process for TFP by using the quadrature method (Tauchen, 1986; Tauchen & Hussey, 1991) to obtain a finite state Markov chain approximation,[7] with 31 grid points.

As can be seen in **Table 1**, the probability of re-entry from full default is set at 0.05 (Yaisawang et al., 2021), whilst the parameters θ_2 (probability of re-entry under partial default) and λ (debt repayment ratio) will vary in our computations. In fact, determining the optimal combination (in terms of welfare) of θ_2 and λ is the focus of this paper.

4. Quantitative Results

This section will provide the quantitative results of the computation and simulations. It includes five main parts. First, we present the value function evaluated at the respective steady state quantities for different combinations of debt relief and re-entry probability. This allows us to find the optimal scheme. Second, we present the default probabilities (as functions of the state variables a, b and k).

[7]This method has been widely used by the previous literature on nonlinear model with a discrete-valued Markov chain (see Aguiar & Gopinath, 2006; Arellano, 2008; Schaltegger & Weder, 2015; Gordon & Guerron-Quintana, 2018, among others).

Table 1. Model specific parameter values.

Parameters		Values	Source
Stochastic structure (TFP)	ρ, σ_ε	0.982, 0.014	Yaisawang et al. (2021)
Discount factor	β	0.95	Arellano (2008)
Risk aversion of the borrower	γ	2	Aguiar & Gopinath (2006)
Risk-free rate	r	0.01	Aguiar & Gopinath (2006)
Capital share	α	0.35	Park (2017)
Capital depreciation	δ	0.05	Romero-Barrutieta et al. (2015)
Capital adjustment cost	Φ	2.4	Yaisawang et al. (2021)
Output default cost	χ	7%	Tomz & Wright (2007)
Probability of re-entry from full default	θ_1	5%	Yaisawang et al. (2021)
Probability of re-entry from from partial default	θ_2	{0, 100%}	Variable in present paper
Partial debt repayment ratio	λ	{0, 100%}	Variable in present paper

Source: Various sources as per table.

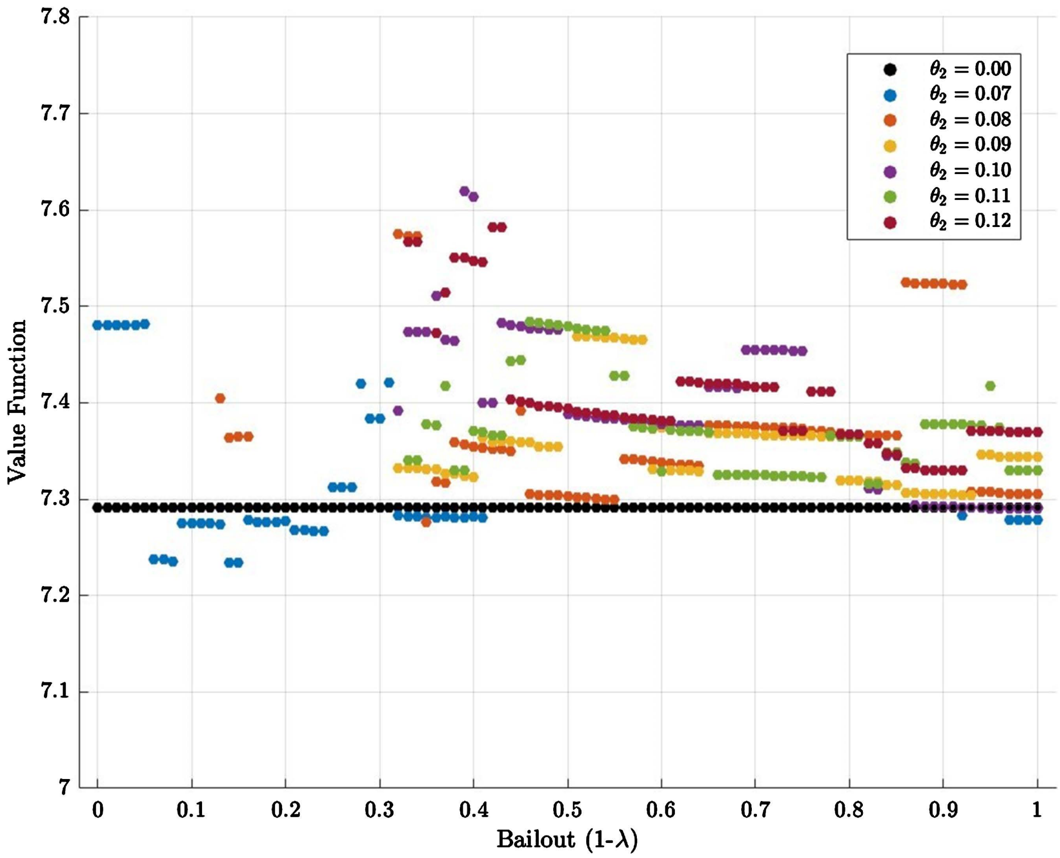
Third, we show the bond price schedules and how they change with debt relief and re-entry probability. Fourth, we report the results from the Monte Carlo simulations. Fifth, we compare the economy under the optimal scheme with the benchmark (only full default). Here we also give a measure of the welfare gain (certainty equivalent steady state consumption).

4.1. Optimal Debt Relief and Probability of Re-Entry

As lenders are risk neutral and receive the expected return of r, the relevant welfare measure is the borrower's maximised expected utility (the value function), taking into account the choices of repayment, full default and partial default, $v_g^0(b,k,a)$, as given by Equation (8). We are seeking the optimal combination of partial default arrangements, complementing the existing choice of full default. In focusing on the value function at the steady-state quantities, we take on a 'long-run' perspective. We could have found different optima computing the value function for b, k, and a at starting values outside the steady state.

Figure 2 shows the scatter plots of the value function at the steady state level of capital (k), foreign assets (b), and TFP (a) against the level of debt relief ($1-\lambda$), for different values of the partial-default re-entry probability (θ_2). Each colour corresponds to a different value of θ_2.[8] For all computations we keep the probability of re-entry from a full default (θ_1) at 0.05 (**Table 1**). As can be seen, the black dots in the plot illustrate the value functions when $\theta_2 = 0$, which means there will be no re-entry under partial default. Consequently, partial default is

[8]For some values of θ_2 and $1-\lambda$ the steady state may be unstable to shocks to total factor productivity. We have excluded those in **Figure 2**.



Source: Authors' computations.

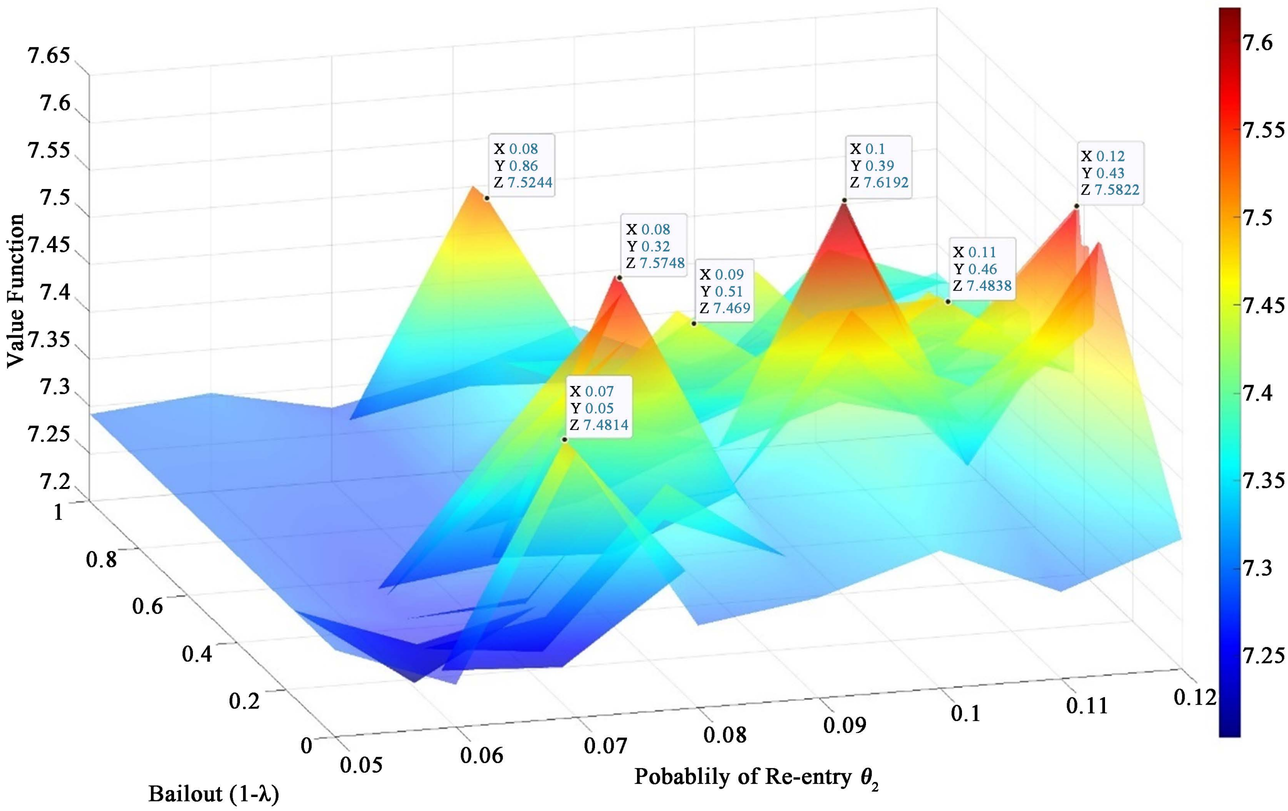
Figure 2. Value functions with partial default at the steady state for the given θ_2 and $1 - \lambda$.

never chosen and the model collapses to one with where only full default or re-payment are possible. The partial re-payment then becomes irrelevant and the value function is constant at 7.29 for all value of debt relief, $1 - \lambda$.[9] Among the different θ_2, the value of 0.1 is of interest. Here (indicated by purple dots), the value function reaches a maximum (7.62) at debt relief of 0.39. We notice that when θ_2 is 0.07, there are instances when the value function is below its value when only full default is possible, especially for low levels of debt relief. Figure 2 also illustrates the maximum value function under the steady-state at 7.59 from the given probability of re-entry (θ_2) and debt relief ($1 - \lambda$) at 0.10 and 0.39, respectively. The value functions from θ_2 at 0.10 are plotted with red dots.[10]

Figure 3 shows the three-dimensional data with a heat map over the value functions by giving debt relief ($1 - \lambda$) from 0 to 1 and probability of re-entry (θ_2) between 0.07 and 0.12. The above three-dimensional heat map plot clearly shows

[9]This value of the value function can equivalently be generated by setting $\theta_2 = 0.05$, and $1 - \lambda = 1$, as it is equivalent to the model with only full default versus repayment.

[10]We should point out that $\theta_2 = 0.10$ and $1 - \lambda = 0.39$ are optimal if the economy starts at its steady state level. Other combinations will be optimal for starting at a different point (or optimal at different point in time). As mentioned earlier we take on a "long-run" perspective, focusing on the value function at the steady state quantities. These steady state quantities will of course vary with θ_2 and $1 - \lambda$, and we present those results in a table in the next sub-section.



Source: Authors' computations.

Figure 3. 3D heat map of value functions for different θ_2 and $1 - \lambda$.

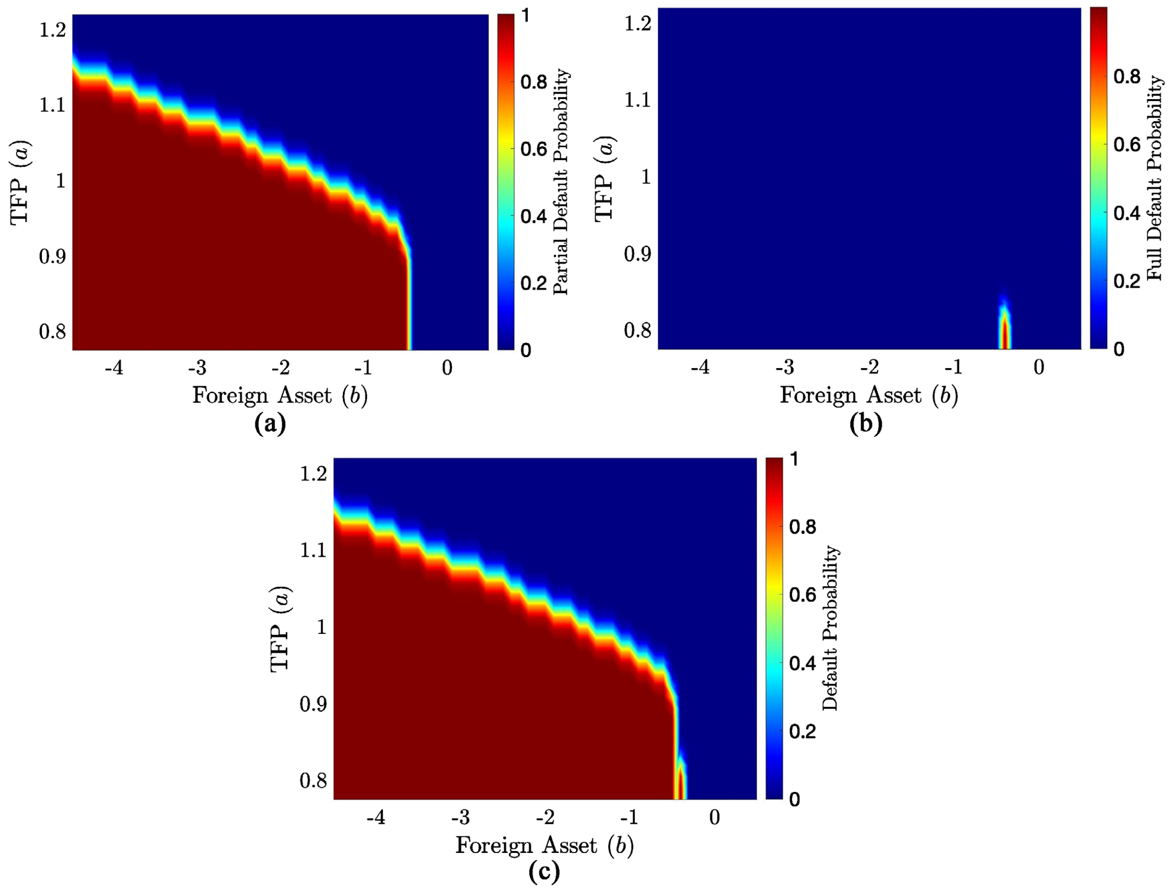
the range of value functions with colour. The blue area in **Figure 3** indicates a low value of utility; most of the blue area is approximately 7.29 as the baseline from a full default model. On the opposite side, the red area indicates the highest values.

The figure clearly shows the peak of value functions for each level of θ_2. For a given θ_2 between 0.07 and 0.12, the optimal debt reliefs $(1 - \lambda)$ are 0.05, 0.32, 0.51, 0.39, 0.46 and 0.43, respectively. The highest peak of value functions in **Figure 3** is 7.62 at debt relief and re-entry probability of 0.39 and 0.10, respectively.

4.2. Partial and Full Default Probabilities

We explore the full and partial default probabilities as functions of a, b, and k, in order to determine which underlying economic variables influence the likelihood of which default (i.e. when a country is more likely to opt for partial default rather than full default).

Figure 4 illustrates the probabilities for partial and full default at the steady-state level of capital as a function of total factor productivity (a) and debt ($-b$), given the probability of re-entry (θ_2) at 0.08 and debt relief $(1 - \lambda)$ at 0.86. We pick this higher level of debt relief to show an area of full default likelihood, which is visually more difficult to see in the full graph later. **Figure 4(a)** shows



Source: Authors' computations.

Figure 4. Default probability with partial and full default at the steady state level of capital (k) and foreign asset (b). (a) Partial default prob.; (b) Full default prob.; (c) Partial and full default prob.

the probability of partial default. As can be seen in the colour bar, the blue shade in the figure indicates low probability of partial default, whilst the red area indicates high. The red area is for low levels of TFP and low levels of b (high debt). The steady state level of TFP is 1, and b is −0.8, thus the probability of default at the steady state is low.

Partial default is more likely if the TFP is below its steady-state level. Partial default is also more likely at larger debt levels. Even if TFP is at its steady state level (of 1), a level of b below −1.4 is likely to trigger partial default. For very large debt levels default is likely even if TFP is above its steady state level. It should be noted that these coincide with perceived default probabilities (perceived by the lender) and thus are part of the bond-price schedule.

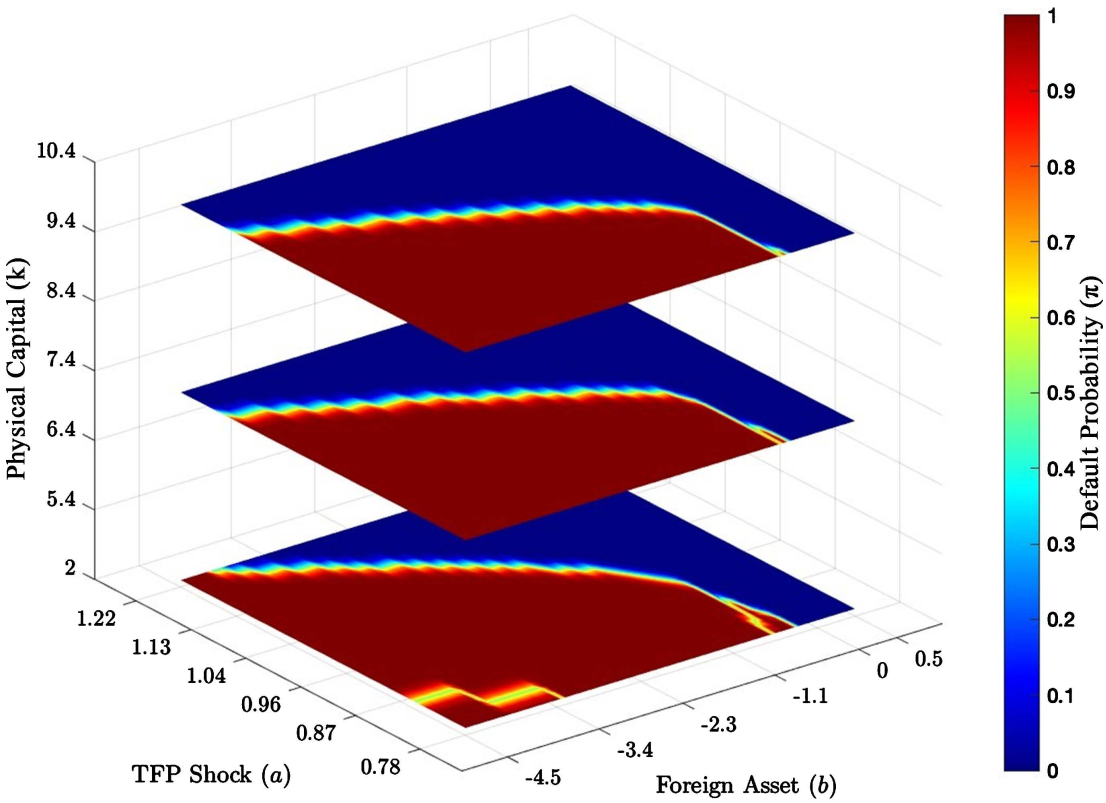
Figure 4(b) illustrates the probability of full default. As can be seen in the figure, the possibility of the full default option is significantly lower than the partial default in Figure 4(a). At the steady-state level of capital (k), the borrower will decide to fully default without any guarantee for future repayment when an extremely severe shock occurs at foreign assets between −0.50 and −0.30. Furthermore, in order to fully illustrate the default probability under the two options, the possibility of partial and full defaults can be combined in Figure 4(c).

In addition, we provide illustrations of partial and full default probabilities at different levels of physical capital. Figure 5 gives the three-dimensional plot of a heat map of default probabilities with respect to physical capital (z-axis), TFP (y-axis) and foreign assets (x-axis).

When the level of physical capital is 7.10 (its steady-state level), the middle layer of the figure coincides with Figure 4(c). In Figure 5, at lower than the steady-state level of physical capital (first layer from the bottom), there are two regions of high probability for full default. One at a very high level of debt ($-b >$ 3.4) and another at a smaller debt level ($-b$ around 0.5). Both are at the low level of TFP ($a < 0.84$). The regions for full default and partial default are larger than for a steady-state level of capital. On the contrary, for a capital stock higher than its steady state level (the top layer in the figure), the region for default is smaller, particularly for full default.

The reason is that an economy with a high level of capital will have sufficient assets in order to sustain consumption in the event of a negative TFP shock. This result is consistent with the previous literature and also with Yaisawang et al. (Yaisawang et al., 2021), where only full default is possible.

A consequence is that the default probability is under- (over-) estimated (by the lender) when capital is higher (lower) than the steady state level (as capital



Source: Authors' computations.

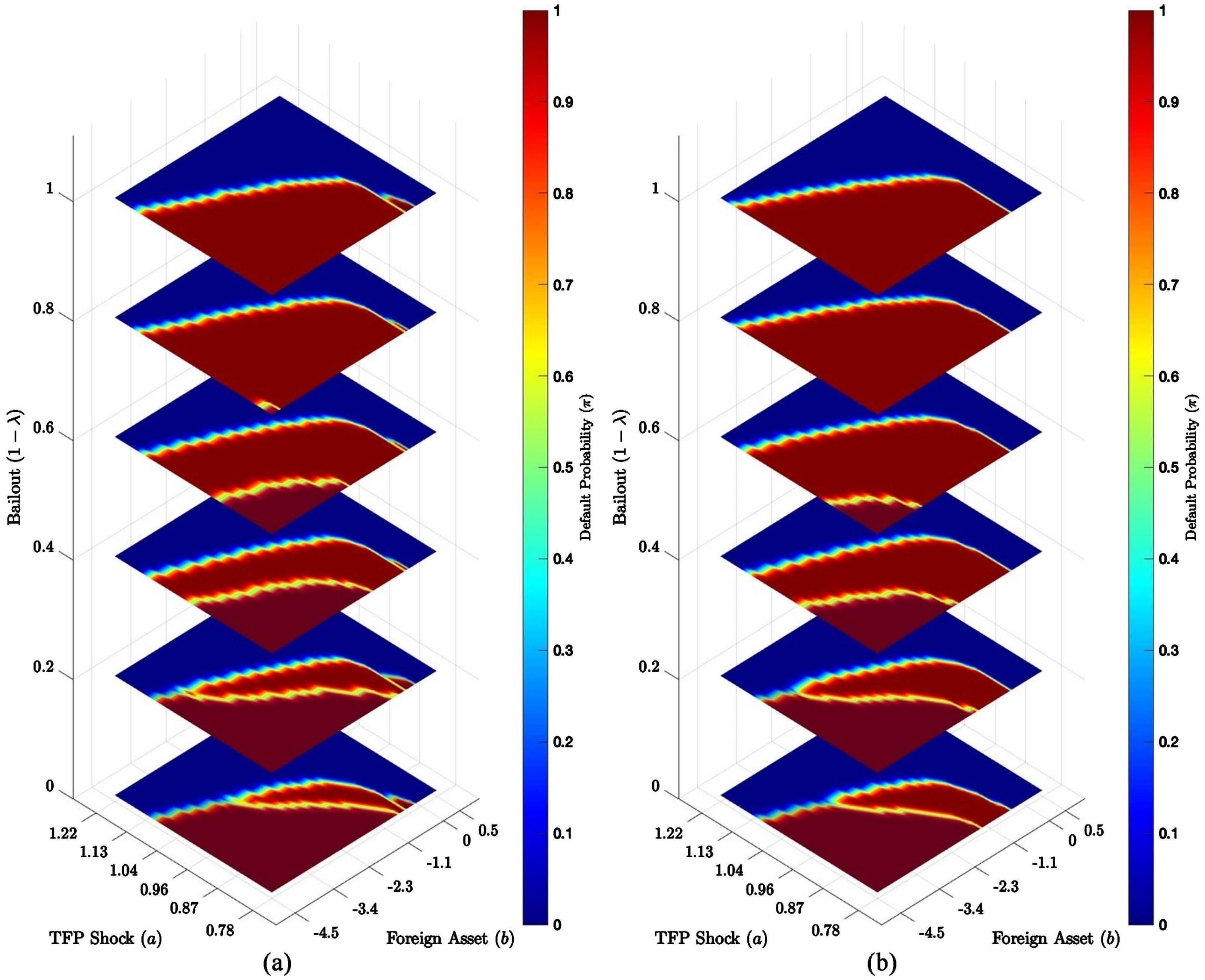
Figure 5. Partial and full default probability at different levels of physical capital (k).

is unobservable). Consequently, the cost of borrowing is higher (lower) than it should be (if it was observable) when physical capital is higher (lower). This limits the incentive to borrow for investment.

We next explore the default probability at different levels of debt relief ($1-\lambda$) and the probability of re-entry (θ_2). When these are high, partial default becomes more attractive.

Figure 6 provides a three-dimensional heat map of the default probability at two different levels of θ_2 with foreign assets (x-axis), TFP shock (y-axis) and the amount of bailout (z-axis). Similar to the previous figures, the red area represents a high probability of default, whilst the blue area indicates low default probability.

Figure 6(a) and Figure 6(b) are for the partial default re-entry probabilities (θ_2) of 0.08 and 0.10, respectively. The results are plotted at their respective



Source: Authors' computations.

Figure 6. Partial and full default probabilities for different bailouts and re-entry probabilities. (a) Default Prob. at $\theta_2 = 0.08$; (b) Default Prob. at $\theta_2 = 0.10$.

steady state level of physical capital (k) and the probability of returning from full default (θ_1) is set at 0.05, as before. We see that when bailout decreases, the area for partial default shrinks, and the dark red area for full default expands. For example, in Figure 6(a) for bailout at 0.80 full default is likely to occur for either very large debt or low debt when TFP is very low. These two areas monotonically increase as bailout is reduced. This is also the case in Figure 6(b). It should also be noticed that the total probability of default is only slightly reduced as bailout is reduced.

Next, comparing Figure 6(a) with Figure 6(b), for every level of bailout the partial default area increases. The reason is that partial default is more attractive when the re-entry probability is higher.

At the optimal debt contract ($\theta_2 = 0.1$, $1 - \lambda = 0.39$), the fourth layer from the top in Figure 6(b), there is a relatively large region where partial default is likely and a relatively smaller region (for low a and large $-b$) where full default is likely (the dark red region). This shows that it is never optimal to construct the debt contract so that only partial or only full default happens.

Finally, the result that the region for partial default is relatively large is consistent with the stylised fact that several countries often choose to receive bailout or some amount of debt reduction (Edwards, 2003; Reinhart & Trebesch, 2016).

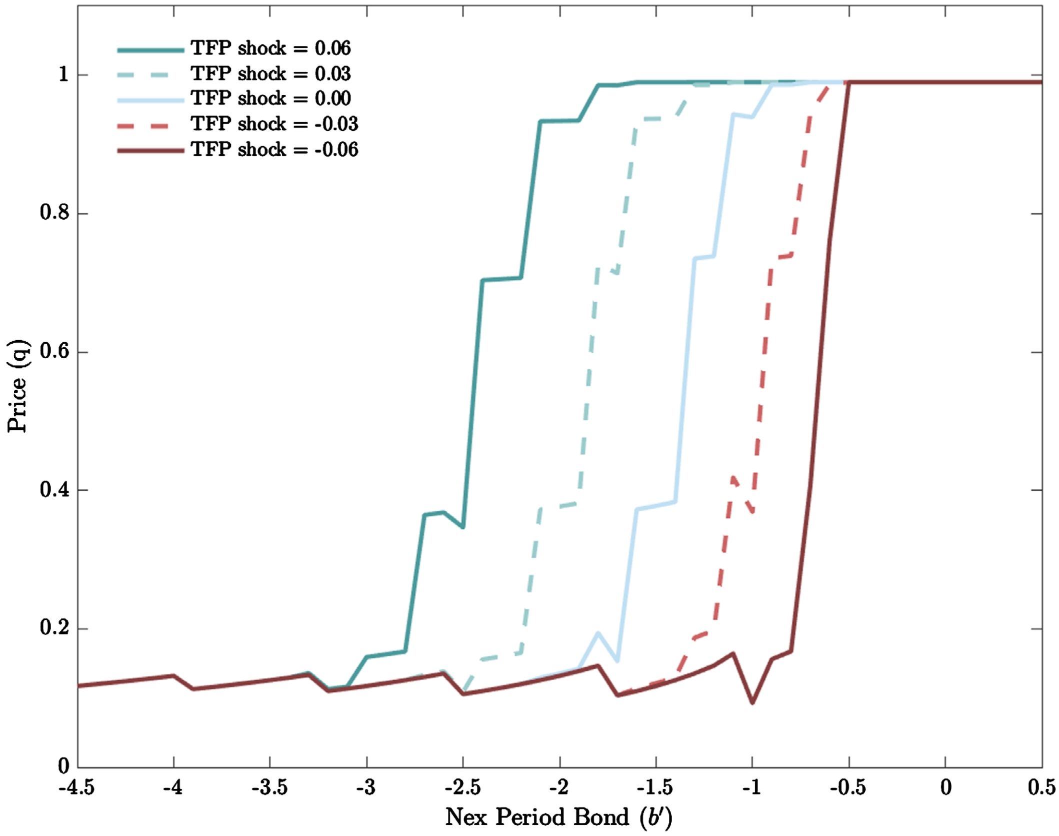
4.3. Bond Price Schedules

In this sub-section, we present computations of the bond-price schedule (equation (22)), where the probabilities of full and partial default are evaluated to the steady state level of capital (as the lender does not observe physical capital). In Figure 7 the current bond price (q) is on the vertical axis and the foreign assets (b') on the horizontal (negative value indicating borrowing). Each coloured graph corresponds to a different level of TFP (a). The bond premium, $1/(1 + r) - q$, will be low when borrowing is less (larger b'), whilst a high volume of debt will lead to a high bond premium.

In addition, Figure 7 indicates a shift in the bond price schedule when there is a TFP shock. A positive TFP shock will shift the bond price schedule to the left, lowering the cost of borrowing at each level of b'. The reason is that default (partial or full) is less likely at higher TFP.

Figure 8 provides four diagrams of bond price schedules with different sets of parameters. As in the previous figure (Figure 7), the figure shows multiple lines of the bond price schedule, for various TFP shocks and the amount of borrowing. The bond price schedule is computed at the respective steady-state levels of both physical capital (k) and foreign assets (b). From the figure, it is clear that the bond premium will be higher if a negative shock occurs or there is a higher amount of borrowing (lower foreign assets).

Figure 8(a) shows a comparison of the bond prices at two different levels of θ_2. The solid lines are for $\theta_2 = 0.08$, whilst the dashed lines are for $\theta_2 = 0.10$. We

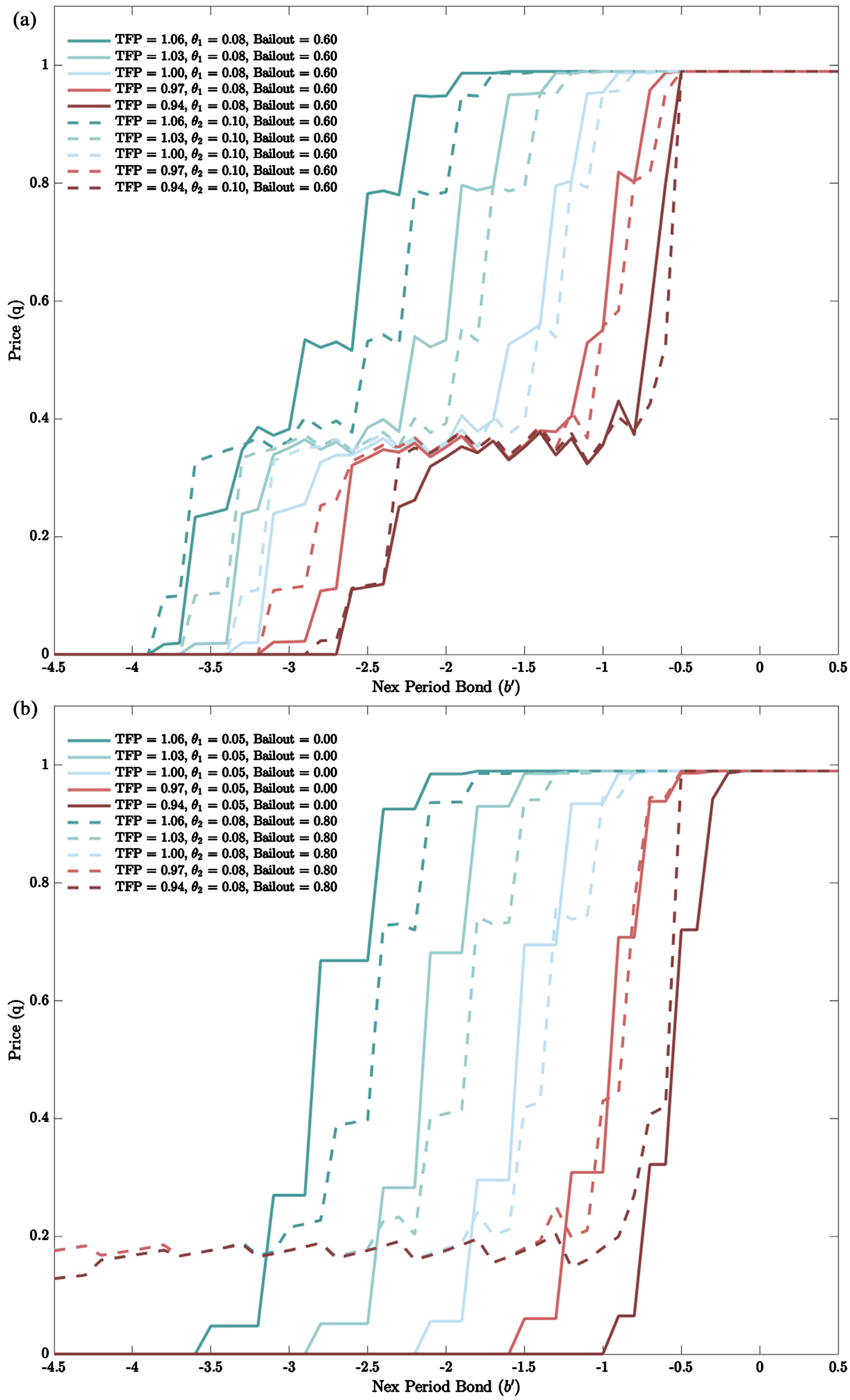


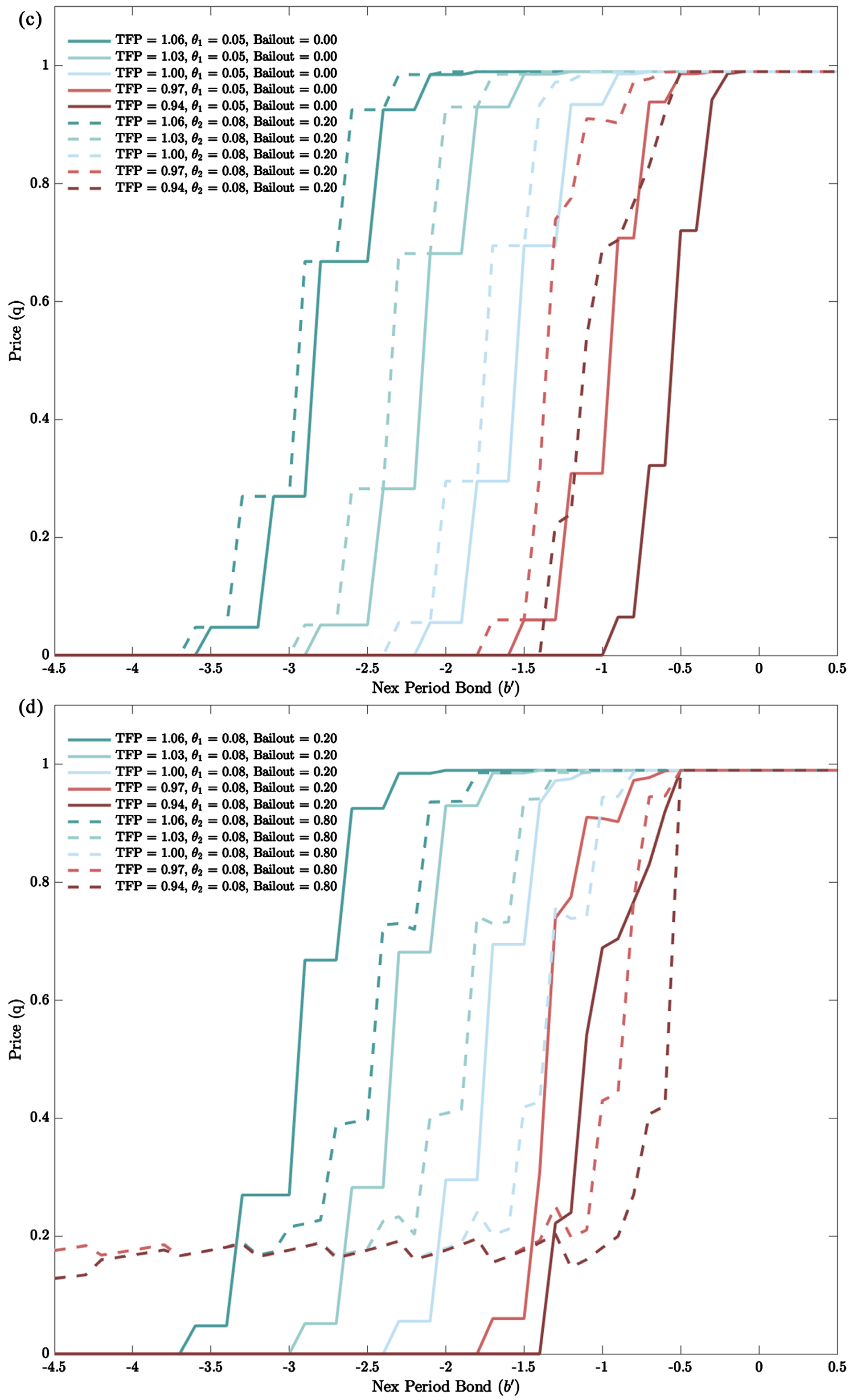
Source: Authors' computations.

Figure 7. Bond price schedule at different levels of TFP shock (ϵ).

see that when q is above 0.40, i.e. the default probability is not too large, the bond premium increases when θ_2 is raised. The reason is as follows. First, a higher θ_2 makes partial default more likely (as we saw in the previous sub-section), as there is a greater chance of returning to the market after partial default. Second, an increase in θ_2 reduces the expected exclusion period, and repayment occurs earlier. Because payments are discounted at r, this effect tends to increase q (if r was zero, this second effect on q would be absent). When the default probability is not too large, the first effect dominates. When the default probability is very large, it changes little with θ_2, consequently, the second effect dominates. This is seen for bond prices below 0.40.

Figure 8(b) shows a comparison of bond price schedules at different levels of debt relief. The solid and solid lines denote the schedules at debt relief of 0.20 and 0.80, respectively, with the probability of re-entry from partial default of 0.08. From this figure, it is clearly shown that a higher rate of debt relief at the same level of θ_2 will cause an increased bond premium in general. A higher bailout makes partial default relatively more attractive. Consequently, the probability of partial default increases and that of full default decreases (as was seen in the previous sub-section). The overall effect, everything else equal, is that the sum of probabilities remains roughly the same. The reason for q falling is as


(a)
TFP = 1.06, θ_1 = 0.08, Bailout = 0.60
TFP = 1.03, θ_1 = 0.08, Bailout = 0.60
TFP = 1.00, θ_1 = 0.08, Bailout = 0.60
TFP = 0.97, θ_1 = 0.08, Bailout = 0.60
TFP = 0.94, θ_1 = 0.08, Bailout = 0.60
TFP = 1.06, θ_2 = 0.10, Bailout = 0.60
TFP = 1.03, θ_2 = 0.10, Bailout = 0.60
TFP = 1.00, θ_2 = 0.10, Bailout = 0.60
TFP = 0.97, θ_2 = 0.10, Bailout = 0.60
TFP = 0.94, θ_2 = 0.10, Bailout = 0.60
Price (q)
Nex Period Bond (b')
(b)
TFP = 1.06, θ_1 = 0.05, Bailout = 0.00
TFP = 1.03, θ_1 = 0.05, Bailout = 0.00
TFP = 1.00, θ_1 = 0.05, Bailout = 0.00
TFP = 0.97, θ_1 = 0.05, Bailout = 0.00
TFP = 0.94, θ_1 = 0.05, Bailout = 0.00
TFP = 1.06, θ_2 = 0.08, Bailout = 0.80
TFP = 1.03, θ_2 = 0.08, Bailout = 0.80
TFP = 1.00, θ_2 = 0.08, Bailout = 0.80
TFP = 0.97, θ_2 = 0.08, Bailout = 0.80
TFP = 0.94, θ_2 = 0.08, Bailout = 0.80
Price (q)
Nex Period Bond (b')



Source: Authors' computations.

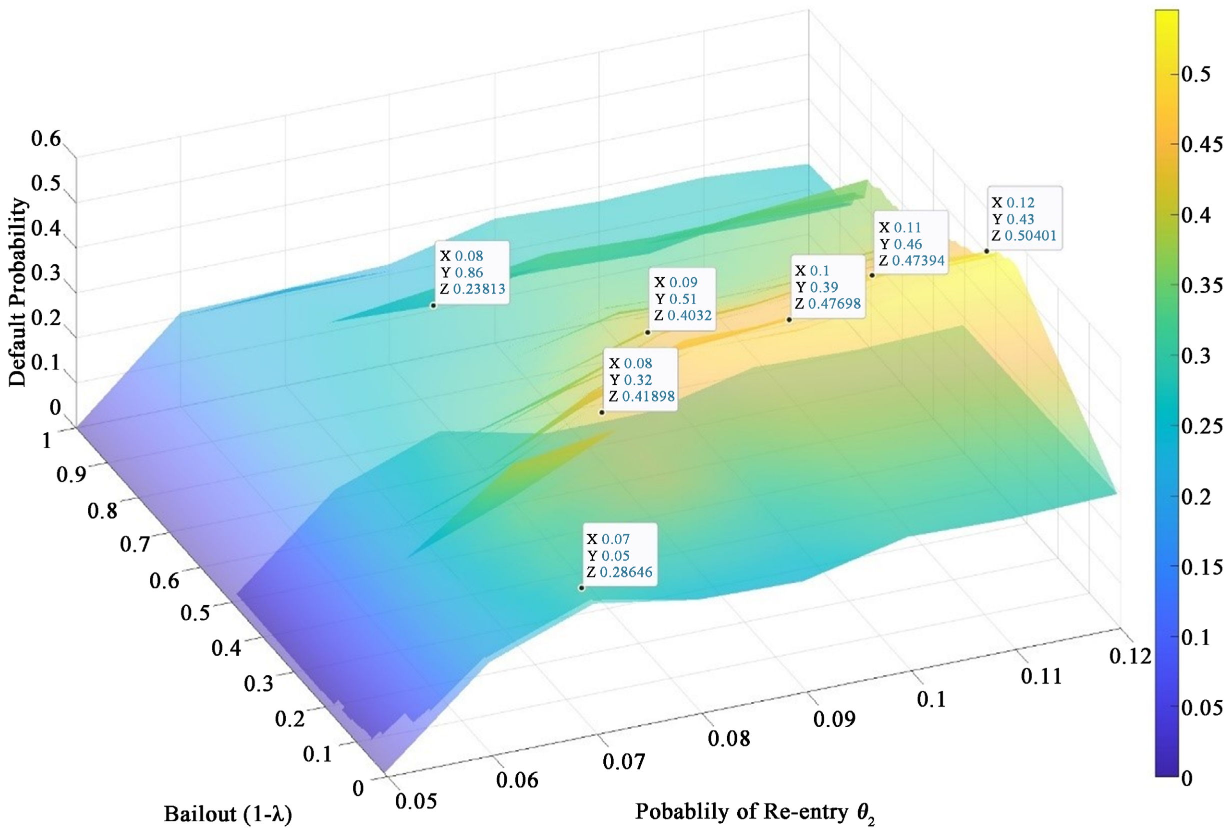
Figure 8. Bond price schedules with partial default and different sets of parameters. (a) Bond price at different levels of θ_2; (b) Bond price at different. levels of bailout; (c) Bond price with bailout at 0.20; (d) Bond price with bailout at 0.80.

follows: First, the bond price adjusts directly because of the bailout (when there is partial default only a fraction λ will be returned to the lender), yielding a lower q. Second, partial default is more attractive to the lender, at least a fraction is paid back. This works to increase q when the partial default probability increases (and the full default probability decreases). The first effect dominates when the default probability is not too large (when q is greater than 0.20).

Figure 8(c) and Figure 8(d) compare the bond price schedules for two economies. One with the partial default option with θ_2 at 0.08 and one where only full default is possible. For low levels of bailout ($1 - \lambda = 0.20$), the bond premium is greater under full default only (q is lower) wile for large levels of bailout ($1 - \lambda = 0.80$), it is the other way around.

4.4. Default Periods: Monte Carlo Simulations

We run Monte Carlo simulations for 100,000 periods for each of the given sets of debt relief and re-entry probability. Figure 9 and Figure 10 show the percentage of time the economy is staying in partial and full default, respectively. These three-dimensional heat maps provide the plot of θ_2 (x-axis) and bailout (y-axis) against the share of time periods in default (exclusion) (z-axis). The yellow area in the figure indicates a high share of the time periods in default, whilst the blue



Source: Authors' computations.

Figure 9. Simulated share of time in partial default.

area indicates a low share.

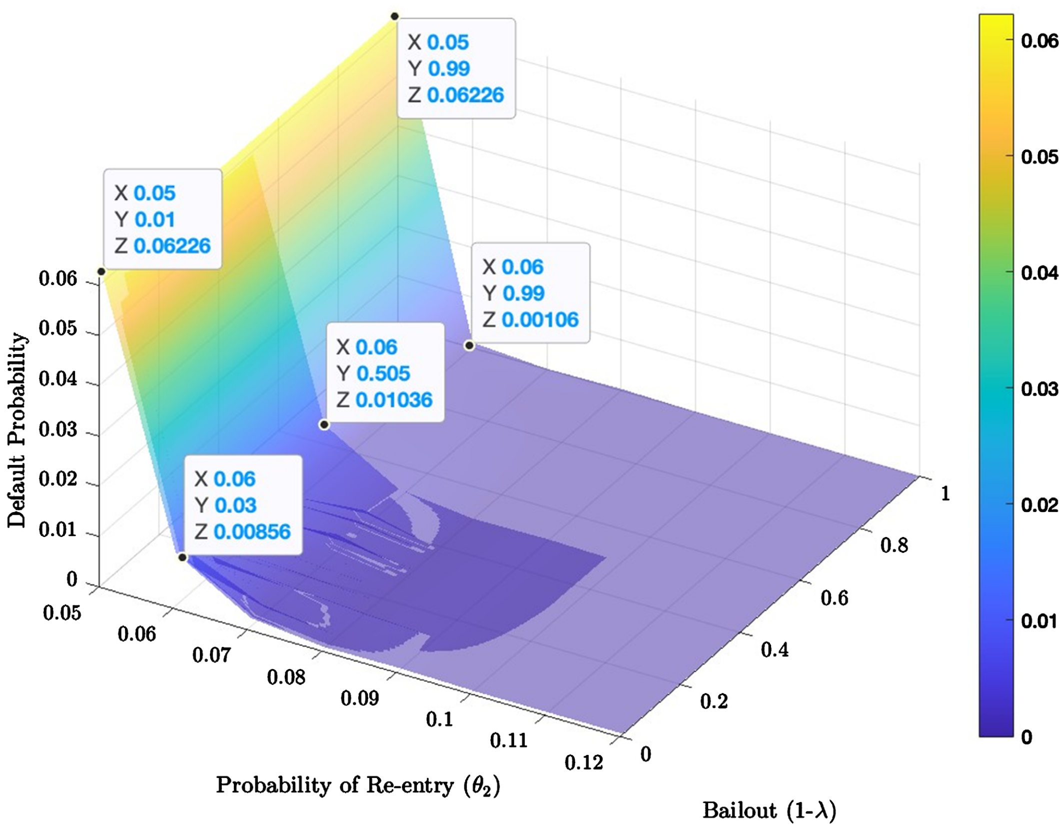
Figure 9 has the maximum percentage of exclusion periods due to partial default (yellow area) at 55%. For lower levels of θ_2 the share of partial default time periods is relatively low, while for θ_2 greater than 0.08 it is relatively high. Under the optimal scheme, the economy stays in partial default 47.7% of the time periods.

Figure 10 shows the share of the time periods in full default. The maximum is 6.23%, whilst the maximum percentage density of full default in Figure 10 is only 6.23%, and decreases rapidly in θ_2. If fact, for θ_2 greater than 0.07, full default is extremely rare.

It should be noticed that if an economy spends 47% of the time in partial default, it does not mean that the default probability is the same. In fact, being excluded from the market 47.70% of the time periods, with a re-entry probability of 0.10, translates into (on average) a probability of default of 9.12%.[11]

4.5. Welfare Comparisons

In this section we seek to quantify the welfare gains moving from a regime where



Source: Authors' computations.

Figure 10. Simulated share of time in full default.

[11]To see this, denote the total number of time periods by T, the fraction of time in default by ω, and the number of times default is triggered by n. For every default, the expected number of time periods being excluded is θ_2. So, $n/\theta_2 = \omega T$. The fraction of time in the market is $1-\omega$, consequently the simulated average partial default probability $\Psi^{pd} = n/[(1-\omega)T] = \theta_2\omega/(1-\omega)$.

only full default is possible in a regime with the option of also partial default. One measure of interest is the percentage change in steady state consumption. However, since different regimes imply different degrees of uncertainty (different default probabilities), it is not capturing the real welfare gain. We therefore also look at the percentage changes in the certainty equivalent steady state level of consumption. The results are presented in the table below (Table 2).

For the optimal scheme (θ_2 = 0.10, λ = 0.39), the steady state consumption level increases by 1.67% while the certainty equivalent consumption change is 2.64%. For other levels of λ and θ_2 there are also gains, in the order of 1.5% - 2%. This suggests there are significant welfare gains by including the partial default option. The table also presents the debt/GDP level and the bond price. The debt/GDP ratio varies between 35% and 55% and is 40% at the optimal scheme. In an economy with only the full default option, it is 46.25% (not presented in the table). In the region 0.08 - 0.12 the bond price is monotonically decreasing in $1-\lambda$. Finally, we also present the fraction of time being in default, as well as the implied average default probability from the simulations. The implied partial default probability (see footnote 11) varies between 2.81% and 12.19%, and is 9.12% at the optimal scheme.

Table 2. Welfare and other quantities.

$1-\lambda$	θ_2	% change in s.s. cons.	% change in cert. equiv. ss. cons.	$-b/y$	Q	% time in default	Implied def. prob. (%)
0.05	0.07	1.01	1.51	−0.55	0.98608	28.65	2.81
0.32	0.08	1.60	2.36	−0.45	0.98846	41.99	5.79
0.39	0.10	1.67	2.64	−0.40	0.98836	47.70	9.12
0.43	0.12	1.17	2.34	−0.35	0.98820	50.40	12.19
0.46	0.11	0.79	1.53	−0.41	0.98792	47.39	9.92
0.51	0.09	0.81	1.48	−0.41	0.98789	40.32	6.08

Source: Authors' computations.

5. Conclusions

We presented a model where a sovereign borrower has the option of partial default in addition to the option of full default. Physical capital is unobservable by the lender. If partial default is chosen, the probability of returning to the international market is greater than the one for full default. This expands the choice set of the borrower. In fact, we found the steady state value function being higher with the option of partial default. As borrowers are risk-neutral and obtain on average the exogenous international risk-free rate, the welfare measure we adopt is the value function of the borrower. The optimal scheme (when the value function is highest) is where the probability of returning is 0.10, with a debt relief of 0.39.

In order to establish which type of default is triggered, we examined the probability of default.

We found that full default is more likely for either small debt levels and very large negative technology shock or for very large levels of debt and large negative technology shocks. Partial default is more likely for moderate debt levels and moderate negative technology shocks. A higher level of physical capital reduces both default probabilities, as the country can better absorb negative shocks. This latter effect is not present in the bond price schedule, as physical capital is unobservable by the lender. We also found that the partial default probability is increasing while the full default probability is decreasing when either the bailout or the return probability in case of partial default increases. In the former case, the overall default probability increases slightly, while in the latter it stays about the same. In investigating the bond price schedule, we found (apart from the premium being decreasing in total factor productivity and increasing in borrowing) that an increase in either debt relief or the partial default return probability increases the bond premium, if the default probability is not too large. When the default probability is very large, the relationship is reversed.

We simulated the model 100,000 times for each combination of partial default, return probability and the associated optimal bailout. We found that for the optimal arrangement, the economy spends 47.70% of the time in partial default (and 0 in full). This translates into an average partial default probability of 9.12%.

We finally compared the differences in economic outcomes between economies with both default options and one where there is only the full default option. For the optimal scheme, the steady state debt to GDP ratio is 40% compared to the benchmark (only full default) of 46.25%. The welfare gain of having both options with optimal bailout and return probability corresponds to a 2.64% increase in certainty equivalent steady state consumption. This suggests there are significant welfare gains from a partial default scheme.

Our optimal re-entry probability of 0.10 implies that on average the country would be excluded for 10 periods, which translates into 30 months (as we use quarterly data). Cruces and Trebesch (Cruces & Trebesch, 2013) report that the Argentina returned to the market within 38 months of the default in 2005, but this was with a debt reduction of 66.3%. Other literature has found longer average exclusion periods: Tomz and Wright (Tomz & Wright, 2013) 9.9 years and Benjamin and Wright (Benjamin & Wright, 2009) 8 years. Our optimal debt reduction of 0.39 is very close to actual default experiences reported in the literature. Cruces and Trebesch (Cruces & Trebesch, 2013) report that between 1978 and 2010 creditors faced average losses (haircuts) of 37%. Reinhart and Trebesch (Reinhart & Trebesch, 2016) found the average debt relief for the middle-high income emerging markets economies of 36.1%. Our simulations yielded a 47.70% of time being in default (exclusion). This is not too far from actual default experiences reported in the literature. Reinhart and Rogoff (Reinhart &

Rogoff, 2008) report that in modern time, countries in Latin America spend over 40% of the time in debt restructuring. Our model under the optimal scheme yielded an equilibrium steady state debt to GDP ratio of 40%. This is not too far off the IMF (IMF, 2004) estimates for Latin America and Emerging Europe of 37% and 38%, respectively.

We conclude that our proposed optimum, with the resulting equilibrium is close to what countries are actually experiencing. Thus, current international arrangements are (at least close to) optimal. Indeed, if there is a competitive market for borrowers, risk neutral lenders would offer schemes giving higher utility to the borrower, and thus the competitive market should converge to the optimal scheme.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Appendix: Approximation of Partial Debt Repayment

In the model of partial default, when the sovereign borrower chooses to negotiate with an international lender for the partial repayment, the borrower is obligated to partially repay the debt as the percentage of the initial amount. The function of partial repayment or debt with bailout is defined as $b_{pd} = \lambda b$, where b is the amount of initial debt, b_{pd} is the amount of debt after re-entry from partial default and λ is a fraction of debt repayment ($0 \le \lambda \le 1$). Hence, if the partial default is decided, the lender has to sacrifice a percentage of lending amount at $1 - \lambda$. However, with the discrete model, the amount of partial repayment has to be approximated in order to locate equivalently in the given grid. The amount of partial repayment (b_{pd}) in the grid form is approximately located to the nearest grid within ±3% of the required λ. In addition, the minimum amount of initial debt ($\underline{b}$) for partial default is set at −0.50. There will be only an option of full default, if the initial debt is smaller than the minimum amount ($-0.50 < b$). The discrete grid of the initial debt (b) and the partial debt approximation (b_{pd}) is illustrated as below:

$$\begin{array}{ccccc} b & & b_{pd}\left(1-\lambda\right)\approx & & 1-\lambda \\ \begin{bmatrix} -4.5 \\ -4.4 \\ -4.3 \\ -4.2 \\ \vdots \\ -0.50 \\ \vdots \\ -0.10 \\ 0 \\ 0.10 \\ \vdots \\ 0.50 \end{bmatrix} & \Rightarrow & \begin{bmatrix} -2.2 \\ -2.2 \\ -2.1 \\ -2.1 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0.10 \\ \vdots \\ 0.50 \end{bmatrix} & \Rightarrow & \begin{bmatrix} 0.4889 \\ 5.0000 \\ 0.5116 \\ 0.5000 \\ \vdots \\ 1 \\ \vdots \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} \end{array}$$

As can be seen in the above vectors, it can illustrate the value of each element in the grid of the initial debt (b), debt after re-entry from partial default (b_{pd}) and amount of bailout or debt reduction ($1 - \lambda$). The value of b_{pd} is required to locate within the finite set of possible debt levels within the interval $[\underline{b}, \overline{b}]$, i.e. [−4.5; 0.50] with the distance for each element at 0.10. Therefore, with the given debt relief, the amount of debt repayment with debt relief will be approximated to the nearest grid as can be seen in the vector of b_{pd}. Moreover, the percentage of debt reduction to the initial debt ($1 - \lambda$) is then computed as follows:

$$1-\lambda = 1 - b_{pd}/b$$

when the foreign asset (b) is positive, there is no such thing as debt reduction so $1 - \lambda = 1$.