Contents lists available at ScienceDirect



Journal of International Money and Finance

journal homepage: www.elsevier.com/locate/jimf



Nominal exchange rates and net foreign assets' dynamics: The stabilization role of valuation effects $\stackrel{\text{\tiny{$\Xi$}}}{\sim}$

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ARTICLE INFO

JEL classification: F31 F32 F36 F41

Keywords: Nominal exchange rate determination Valuation effects Endogenous portfolio choice Net foreign assets' dynamics Incomplete markets Overlapping generations economies

ABSTRACT

This paper proposes a parsimonious OLG model with output shocks to shed light on the impact of the nominal exchange rate on the dynamics of net foreign assets through valuation effects. We show that an increase in the share of world GDP leads to a trade surplus and negative valuation effects through an appreciation of the nominal exchange rate. The lack of perfect arbitrage in the model implies that the valuation channel is a key component of the process of external adjustment, consistently with the empirical literature. Finally, we provide empirical evidence in support of the role of the share of world GDP in generating trade balance and exchange rate/valuation effects dynamics.

1. Introduction

One of the most relevant developments that characterize the global economy of the recent decades is the rising importance of the so called "valuation channel" in accounting for the dynamics of net foreign assets of many countries.¹ Since the early 1990s, cross-border holdings of assets and liabilities have substantially increased and the traditional method of computing the net foreign assets position of a country, which relied on the cumulation of current account balances over time, has proved to be inaccurate.² As the balance of payments does not record changes in the value of foreign assets and liabilities which can arise due to fluctuations of nominal exchange rates and asset prices, it does not reflect the fact that the valuation channel is becoming more important.

For instance, Fig. 1 shows the discrepancy between the cumulated current account balances and the net foreign assets position of the United States. According to the former measure, the net foreign assets position of the United States amounted to almost -57% of GDP in 2015. However, direct estimates of net foreign assets and liabilities suggest that the net external position was much lower

https://doi.org/10.1016/j.jimonfin.2024.103018

Available online 24 January 2024

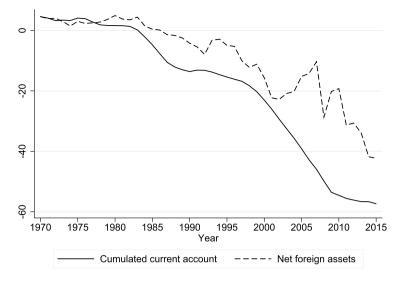
^{*} This paper is a development of the third chapter of my PhD thesis. I thank my supervisor Subir Chattopadhyay for his advice, Michael Reiter for his very useful discussion and Mauro Bambi, Luis Cat ao, Andrea Ferrero, Mark Guzman, Dominik Menno, Roberto Pancrazi, Herakles Polemarchakis, Katrin Rabitsch, Neil Rankin, Steve Spear, Gabriel Talmain, seminar participants at Bologna, Durham, Reading, Warwick, the 2016 EEA-ESEM conference, the 24th CEPR European Summer Symposium in International Macroeconomics (ESSIM), the 2016 Royal Economic Society conference, the CEPR conference "Macro-Financial Linkages and Current Account Imbalances" and the 2014 North American Summer Meeting of the Econometric Society for their comments and insights.

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¹ See Gourinchas and Rey (2015) for a recent survey of the literature.

² Lane and Milesi-Ferretti (2001, 2007) constructed estimates of foreign assets and liabilities for 145 countries for the period 1970-2011 and were among the first to notice the mismatch between the stock measures (at current prices) and the balance of payments data.

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Source. Lane and Milesi-Ferretti's database (2018).

Fig. 1. Net foreign assets' position and cumulated current accounts of the United States as a percentage of GDP, 1970-2015.

and equal to around -42% of GDP. This implies that the US have experienced a substantial wealth transfer from the rest of the world as the value of their foreign assets has risen relatively to the value of their foreign liabilities. The significance of the valuation channel is not specific to the US. It is interesting to observe that e.g. emerging countries in East Asia have faced exactly the opposite situation: while their net external positions have considerably improved over the past decades because of current account surpluses, they have experienced negative valuation effects (Fig. 2). Gourinchas and Rey (2015) make similar observations for other emerging countries. Therefore, valuation effects seem to have a stabilizing effect on countries' net foreign assets positions. In fact, Lane and Milesi-Ferretti (2002) found that the correlation between the trade balance and valuation effects is negative for a cross-section of countries.

It is well known that valuation effects are driven by two main components: fluctuations in asset prices and movements in the nominal exchange rates. The theoretical literature has mainly focused on asset price-driven valuation effects,³ but both channels are known to be important. For instance, Lane and Shambaugh (2010) and Benetrix et al. (2015) built a large database to document countries' currency exposures and found that exchange-rate driven valuation effects account for a significant fraction of the overall valuation effects for a large sample of countries. Moreover, Gourinchas and Rey (2007) have shown that a significant part of the US cyclical external imbalances are eliminated via predictable movements in nominal exchange rates. Importantly, Gourinchas and Rey (2015) have stressed that designing models able to generate meaningful expected exchange rate changes has proved to be a challenge.

This paper takes up this task and proposes a parsimonious two-country OLG model of nominal exchange rate determination and endogenous portfolio choice, where output shocks are the driving force behind the dynamics of exchange rate-driven valuation effects.⁴ One of the main contributions of the paper is that we are able to rationalize in a theoretical model the empirical finding of Gourinchas and Rey (2007) regarding the significance of the valuation channel in the process of external adjustment.⁵

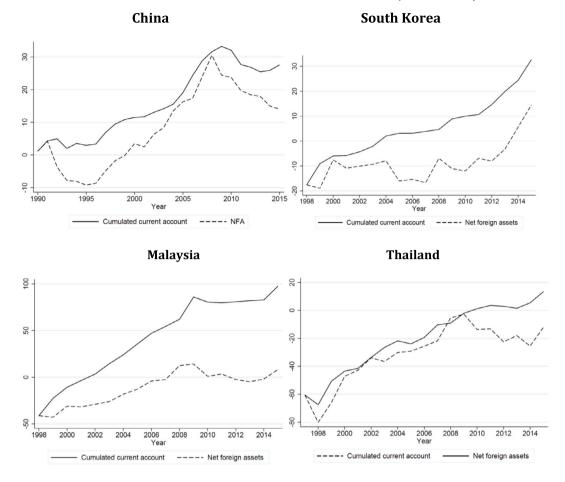
Our two-country model has two key ingredients: incomplete markets and "infrequent portfolio decisions" as in Bacchetta and Van Wincoop (2010). It is well known that complete markets' models are not suitable to explain the dynamics of foreign assets and liabilities (gross positions) as portfolios are constant across states of nature in equilibrium.⁶ Hence, market incompleteness ensures that there is non-trivial portfolio dynamics. We adopt an overlapping generations' structure, since it allows to introduce market incompleteness in a relatively simple and tractable way. Agents receive an endowment of a country-specific good when born, and this depends on the state of nature realized. As agents cannot insure against the realization of the endowment that they receive when born, markets are incomplete. For tractability reasons, we assume that agents have no endowment in the second period of their life. The way in which agents can transfer wealth across periods is to buy a portfolio of currencies. We will refer to a country's holdings of the foreign currency as "foreign assets" while the foreign country's holdings of the domestic currency are the "foreign

³ See e.g. Pavlova and Rigobon (2007, 2010, 2015), Tille and Van Wincoop (2010), Devereux and Sutherland (2010), Nguyen (2011) and Ghironi et al. (2015).

⁴ Tille (2008) analyzes the wealth effects of exchange rate fluctuations but he assumes that portfolios are exogenous. Benigno (2009) examines a model where each country can borrow (lend) in a risk-free nominal bond denominated in its (the other country) currency. However, the asset positions and the nominal exchange rate are indeterminate at the steady state.

⁵ In existing theoretical frameworks focusing on asset price-driven valuation effects, e.g. a country's negative net foreign assets position can only be explained by the ability to run trade surpluses in the future (e.g. the trade channel). E.g. Tille and Van Wincoop (2010), Devereux and Sutherland (2010) and Ghironi et al. (2015).

⁶ See Lucas (1982) for an open economy version of the Lucas asset-pricing model and Judd et al. (2003) for a proof of the same result in a more general version of the model.



Source. Lane and Milesi-Ferretti's database (2018).

Fig. 2. Net foreign assets' position and cumulated current accounts of selected East Asian countries as a percentage of GDP.

liabilities". Because the asset structure is simple, it is very tractable. Quite obviously, the model cannot fully capture the balance sheet of emerging and developing countries which issue a large proportion of their foreign liabilities in foreign currency (e.g. Eichengreen and Hausmann, 1999; Hausmann and Panizza, 2003 on the "original sin").⁷ However, Lane and Shambaugh (2010) and Benetrix et al. (2015) have documented that emerging and developing countries have increasingly a positive *net* foreign currency exposure: although they might still issue a large proportion of liabilities in foreign currencies, they are long in foreign currency due to a positive net foreign assets' position. Therefore, our simple model can capture the dynamics of net foreign assets for East Asian countries such as China, where improvements in the net foreign assets' position have been accompanied by stabilizing valuation effects through a depreciation of the dollar.

The second ingredient relates to the way in which currencies and goods are traded in the economy. When young, agents spend their income (which is the value of the domestic endowment) to buy the domestic and the foreign good, as well as a portfolio of currencies. In the following period, the timing works as follows. Firstly, agents learn about the state of the economy. Secondly, they spend the money balances acquired in the previous period to buy the domestic and the foreign good in a cash-in-advance fashion, i.e. they spend the domestic (foreign) currency to buy the domestic (foreign) good.⁸

Since the old spend the money holdings acquired when young, we are implicitly assuming that they cannot readjust their portfolio after the shock is realized. This implies that only a fraction of agents in the economy (the young) participate in foreign exchange rate markets and make a decision about portfolios in each period. This is similar to Bacchetta and Van Wincoop (2010), where agents

⁷ Despite the extent to which countries are not able to issue liabilities in domestic currency has decreased over time, 49% of foreign liabilities for low income countries were still issued in foreign currency in 2012 (author's calculations based on Benetrix et al. (2015)'s database).

⁸ See e.g. Lucas (1982), Svensson (1985a) and Svensson (1985b). Alvarez et al. (2009) is a more recent treatment of cash-in-advance models. As consumers must decide about their cash holdings before learning about the state of the economy, the timing is similar to Svensson (1985a) and Svensson (1985b), where there is a precautionary demand for money.

who live T periods make a decision in the first period and then never readjust during their lifetime.⁹ This friction can be motivated by empirical evidence showing that investors' behavior is characterized by considerable inertia, in the sense that portfolios are rarely readjusted following changes in market conditions.¹⁰ In a theoretical model, Kim et al. (2016) show that since active management of portfolios is a costly activity, inertia is indeed consistent with optimal behavior and rationality.¹¹ Our two-period lived agents make decisions once, in the first period of their life, while they are characterized by inertia during old age. This is in line with Kim et al. (2016), who argue that active management in old age is relatively more costly because of higher mortality risk and falling efficiency in decision making, thereby reducing old people's asset market participation.¹²

This feature of the model implies that there is no perfect arbitrage between the two currencies. As a consequence, excess returns on the exchange rate can potentially be different from zero in our framework leading to expected valuation effects. In the existing literature, meaningful excess returns cannot be generated precisely because of the perfect degree of arbitrage across assets.¹³ As a consequence, the adjustment of the net foreign assets' position entirely operates through the trade balance channel.¹⁴ In fact, if we allowed agents to adjust their portfolio in their second period of life (or alternatively, to buy any good with the two currencies) then our model would be similar to Manuelli and Peck (1990) where there is perfect arbitrage across currencies. The expected value of the exchange rate (weighted by marginal utility) would be equal to the current exchange rate and the nominal exchange rate would follow arbitrary martingales. This would exclude the possibility of having excess returns in the model.¹⁵

The focus of the analysis will be the stochastic steady state of the model, which is defined as a time-invariant distribution (across state of nature) of nominal prices, exchange rates, consumption and portfolio allocations.¹⁶ Under the assumption that the intertemporal elasticity of substitution is the inverse of the intratemporal elasticity, we show that the model is partially analytically tractable as the demand functions have closed-form solutions. We then search numerically the prices which clear our equilibrium system of non-linear equations.¹⁷

Output shocks is the key mechanism driving the dynamics of net foreign assets' positions and its various components, in line with the theoretical literature on (asset-price driven) valuation effects e.g. Tille and Van Wincoop (2010), Devereux and Sutherland (2010) and Ghironi et al. (2015). There is a vast literature emphasizing that there are other important factors behind the emergence of global imbalances and the dynamics of net foreign assets,¹⁸ but this strand of literature is typically silent on the valuation channel.

To start with, we provide two analytical results on trade balance and exchange rate dynamics. Firstly, we show that a country experiencing an increase in the share of world GDP runs a trade surplus. In order to interpret this result, we need to consider that, in an overlapping generations economy, the trade balance is driven by the behavior of the current as well as the previous generation. In our economy, the young are net savers while the old dissave, as they only consume. Hence, at aggregate level, whether the country is borrowing or lending from abroad depends on how the savings of the young and the dissavings of the old compare and evolve over time. When a country's share of world GDP increases, then the current generation's share of aggregate (world) income is higher with respect to the past. This implies that the young saves more than in the previous period while the consumption of the old is relatively low (since the previous generation was poorer and saved less). This generates a trade surplus.¹⁹ Since the young's savings increase, they accumulate more domestic as well as foreign assets than in the past. This causes an improvement of the net foreign asset position of the country.

Secondly, we prove that, in equilibrium, exchange rate dynamics is linked to expected output in the two countries. In particular, if output in the domestic economy is expected to grow, the price of the domestic good is expected to fall. This implies that the domestic currency will have a higher purchasing power. Under the assumption that the domestic and the foreign goods are substitutes, the demand for the domestic currency increases, leading to a currency appreciation.

Next, we gauge the direction and the importance of valuation effects in net foreign assets' dynamics through a range of numerical examples. As agents only live for two periods, the model does not lend itself to a fully-fledged quantitative analysis. The process of financial globalization, which has been accompanied by large wealth transfers across countries through valuation effects, is a relatively recent phenomenon dating back only to the early 1990s (e.g. Lane and Milesi-Ferretti, 2001, 2007). Since one period lasts for roughly thirty years, it would be difficult to calibrate the model to match specific moments of the data with only two observations. Using overlapping observations or fitting high-frequency models is also not ideal (see e.g. Constantinides et al., 2002).

⁹ In a two-period OLG model, Cooper and Kempf (2003) adopt the same friction to investigate the conditions under which a monetary union emerges. They also show how local currency cash-in-advance constraints can arise endogenously.

¹⁰ E.g. see Agnew et al. (2003) and Bilias et al. (2010).

¹¹ See also Bacchetta and Van Wincoop (2010) on this issue.

¹² Fagareng et al. (2017) show indeed that households' participation in the stock market falls after retirement.

¹³ See Tille and Van Wincoop (2010) and Gourinchas and Rey (2015) for a discussion on this point.

¹⁴ For instance, see Pavlova and Rigobon (2007, 2010, 2015), Tille and Van Wincoop (2010), Devereux and Sutherland (2010), Nguyen (2011) and Ghironi et al. (2015).

¹⁵ See Section 2 in the Supplementary Material for an illustration of this point.

¹⁶ Rabitsch et al. (2015) and Coeurdacier and Rey (2012) point out that one of the advantages of solving the stochastic steady state of models with endogenous portfolio choice is a higher degree of accuracy of the results, both qualitatively and quantitatively.

¹⁷ Although Spear (1985) demonstrated that stationary equilibria do not generically exist in stochastic OLG models with multiple consumption goods, we show that this issue does not arise here due to the friction in currency markets.

¹⁸ See e.g. the survey by Gourinchas and Rey (2015). While most papers have focused on differences in financial markets' development or demographic factors, Zucman (2013) has emphasized that tax evasion can explain the puzzling quantitative importance of statistical discrepancies/measurement errors.

¹⁹ Although the mechanism is slightly different, this is akin to the effect of a transitory (level) output shock in an infinite-horizon economy (see e.g. Devereux and Sutherland, 2010; Ghironi et al., 2015; Amdur and Kiziler, 2014). On the other hand, GDP growth is usually associated with trade deficits instead of trade surpluses as individuals consume more in view of expecting a higher income in the future (Engel and Rogers, 2006; Amdur and Kiziler, 2014).

A proper calibration would require extending our two-period OLG model to a multi-period setting, but this is not straightforward due to the presence of multiple goods. The generic non-existence of stationary equilibria in stochastic OLG models with multiple goods is well known (Spear, 1985) and research is still ongoing on how to find suitable restrictions on the model or equilibrium set to allow researchers to develop algorithms suitable for quantitative analysis.²⁰ The purpose of the numerical exercises is rather to show that, in our simple model, the valuation channel is a key component of the process of external adjustment, consistently with the empirical literature. As such, a key contribution of the paper is that we highlight the model ingredients leading to this result.

The approach that we will take is to work through a simple example, starting from a benchmark parametrization, and then conduct an extensive sensitivity analysis to illustrate the robustness of the mechanism. In particular, we assume that there are only two states of nature, where output can take either a high or a low value. To show the mechanism more clearly, we analyze a situation where the shock is asymmetric, i.e. whenever a country experiences a "boom" the other one experiences a "recession" or viceversa.

We find that an increase in the share of world GDP is associated with an appreciation of the domestic currency for a wide range of parameter values. Therefore, exchange rate-driven valuation effects dampen the improvement in the country's net external position. The result that valuation effects are stabilizing is obtained under the mild condition that there is some degree of output persistence in the economy. In other words, the autocorrelation coefficient for the output process must be positive. When the goods are substitutes, the country that experiences an increase in the share of world GDP is the one whose output is relatively high. Let us think about this as the domestic economy. If there is persistence in the economy, the young expect that the world economy will remain in the current state of nature with higher probability. As the domestic good is cheaper than in the past due to the country's higher output growth, the demand for the domestic currency increases because it is expected that it will have a higher purchasing power. This leads to an appreciation in equilibrium. As a result, the value of the domestic currency held by foreign residents (foreign liabilities) increases relatively to the value of the foreign currency held domestically (foreign assets). Therefore, the surplus country experiences negative valuation effects. When goods are complements, the result still holds.²¹ The stabilizing nature of valuation effects is in line with the empirical findings of Lane and Milesi-Ferretti (2002) and Devereux and Sutherland (2010).

We then show that the model can generate significantly large expected valuation effects. For instance, let us consider the country who has a relatively low share of world GDP in the current period. Our finding is that the country currently runs a trade deficit and has a negative net foreign assets position. However, the country's share of world GDP is expected to increase, since there is a positive probability that the state of nature tomorrow will be different (characterized by high output). The model then suggests that we should expect an improvement of the net foreign assets position through a trade surplus as well as negative valuation effects through an appreciation of the domestic currency. For our benchmark parametrization, the valuation channel accounts for almost half of the net foreign assets position (46%). Our robustness analysis shows that the significance of the valuation channel decreases the lower is the persistence of the output process. However, even in the extreme case the output process is i.i.d. (zero autocorrelation), the valuation channel still explains 8% of the net foreign assets position. Hence, our results are consistent with the empirical literature in pointing out that the valuation channel is a critical component of the process of external adjustment (Gourinchas and Rey, 2007). The imperfect substitutability between the domestic and the foreign currency, which is induced by the friction in currency markets, is what drives the presence of non-zero excess returns. Our theory also suggests a plausible mechanism driving the adjustment of countries' net foreign assets positions through both the trade balance and the valuation channel: expected growth differentials across countries.

Finally, we empirically validate the importance of the share of world GDP in generating trade balance and valuation effects' dynamics. Our results suggest two testable hypotheses. Firstly, there should be a positive correlation between changes in the share of world GDP and the current account. Secondly, increases in the share of world GDP should be associated to domestic currency appreciations and negative valuation effects. We run some panel regressions with fixed effects for a large cross-section of countries and obtain a number of results. Firstly, there exists a positive, statistically significant relationship between the current account balance and changes in the share of world GDP. Secondly, a country whose share of world GDP has increased with respect to the previous period is indeed more likely to experience a currency appreciation. Finally, we calculate exchange-rate driven valuation effects using Benetrix et al. (2015)'s database and investigate whether positive changes in shares of world GDP are associated with negative valuation effects. As one might expect, the sign of the coefficient depends on the net foreign currency exposure. In the model, countries are always long in foreign currency, as money is held in positive net supply. In the data, whenever countries are long in dollars in the previous period, a positive change in the share of world GDP generates negative valuation effects as predicted by our theory. However, when countries are short in dollars, a currency appreciation induced by an increase in output means that the value of foreign liabilities drop, inducing positive valuation effects. This situation applies to those countries who are net borrowers and affected by the "original sin", i.e. countries that are unable to borrow abroad and domestically in their own currency (Eichengreen and Hausmann, 1999; Hausmann and Panizza, 2003). Although the model does not capture this case, our empirical analysis shows that changes in the share of world GDP generates valuation effects even for this set of countries, only with the opposite sign.

In Section 2, we present the model and analyze agents' portfolio problem taking prices as given. After discussing the existence of a stationary equilibrium, in Section 3 we provide model-based definitions of net foreign assets and valuation effects and provide our main analytical results on trade and nominal exchange rate dynamics. In Section 4, we parametrize the model to show that it can rationalize the stabilizing nature of valuation effects as well as make sense of the empirical importance of the valuation channel.

 $^{^{\}rm 20}~$ See Kim and Spear (2021) for some progress and a discussion on this issue.

 $^{^{21}}$ In this case, the country that experiences an increase in the share of world GDP is instead the country whose output is relatively low, since the improvement in the terms of trade dominates the fall in output. In this case, a higher expected price domestically still leads to a currency appreciation - as the demand for the foreign currency falls - which yield negative valuation effects.

In Section 5, we provide suggestive empirical evidence to show that the share of world GDP has indeed a role in generating trade balance, exchange rate dynamics and valuation effects. Section 6 concludes the paper. The proofs are in the Supplementary Material, along with the derivations of the model.

2. The model

We consider the following two-country overlapping generations economy. In each period, an agent with a two-period lifetime is born in each country. Therefore, two young and two old populate the world economy at each date.

The young are born with an endowment of the country-specific good ℓ , which is also the total output of the country. Output is denoted as $y^{\ell}(s)$ as it depends on the state of nature realized, where $s = \{1, ..., S\}$. We will use the superscript ℓ to indicate goods and currencies, while we will refer to agents with the subscript h. We assume that output follows a Markov chain, where transition probabilities are time-invariant and $\rho(ss')$ indicates the probability of transiting from state s to s'. Agents gain utility from the consumption of both goods, although they are only endowed with the country-specific good, as in Lucas (1982).

At time 0, the two governments issue fiat money and distribute it to the initial old. M^{ℓ} is the stock of money issued in country ℓ . As the old have no endowment, money is valued in equilibrium as agents would not be able to consume in their second period of life otherwise. For simplicity, we assume that monetary authorities are inactive after the first period.

Our objective is to characterize the stationary equilibrium of the model. This implies that agents born in the same state of nature but at different dates have the same consumption allocation.

Agent *h* born in state *s* has the following utility function:

$$U_{h}(s) = \frac{c_{1h}(s)^{1-\gamma}}{1-\gamma} + \beta \sum_{s'} \rho(ss') \frac{c_{2h}(ss')^{1-\gamma}}{1-\gamma}$$
(1)

where γ is the coefficient of relative risk aversion. $c_{1h}(s)$ is agent *h*'s consumption in the first period of life while $c_{2h}(ss')$ denotes the consumption of the old. $c_{1h}(s)$ and $c_{2h}(ss')$ are the constant elasticity of substitution (CES) aggregators:

$$c_{1h}(s) \equiv \left[a_h^{1\frac{1}{\sigma}}c_{1h}^{1}(s)^{\frac{\sigma-1}{\sigma}} + a_h^{2\frac{1}{\sigma}}c_{1h}^{2}(s)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$
(2)

$$c_{2h}(ss') \equiv \left[a_h^{1\frac{1}{\sigma}}c_{2h}^{1}(ss')^{\frac{\sigma-1}{\sigma}} + a_h^{2\frac{1}{\sigma}}c_{2h}^{2}(ss')^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$
(3)

where $a_h^1 + a_h^2 = 1$, $\sigma > 0$ and $\sigma \neq 1$.

Taking as given the goods' prices and the nominal exchange rate, agent h born in state s then chooses the consumption vectors and the portfolio of currencies that maximize (1) subject to (2), (3) and the following constraints:

$$\bar{m}_{h}^{1}(s) + e(s)\bar{m}_{h}^{2}(s) = w_{h}(s)$$
(4)

$$m_h^1(s) + p^1(s)c_{1h}^1(s) = \bar{m}_h^1(s)$$
 (5)

$$m_h^2(s) + p^2(s)c_{1h}^2(s) = \bar{m}_h^2(s)$$
(6)

$$p^{1}(s')c_{2h}^{1}(ss') = m_{h}^{1}(s) \ \forall \ s'$$
(7)

$$p^{2}(s')c_{2h}^{2}(ss') = m_{h}^{2}(s) \ \forall \ s'$$
(8)

Agent *h* is born with an initial wealth $w_h(s)$, which is equal to the value of domestic output expressed in units of currency 1 (the numéraire): $w_1(s) \equiv p^1(s)y^1(s)$ and $w_2(s) \equiv p^2(s)e(s)y^2(s)$. $p^1(s)$ ($p^2(s)$) is the price of good 1 (2) expressed in units of currency 1 (2).²² e(s) is the price of currency 2 in units of currency 1 or the nominal exchange rate. Therefore, we say that if e(s) rises then currency 2 (1) appreciates (depreciates). With his wealth, the agent buys the two currencies for the purpose of financing the current consumption of the domestic and foreign good as well as for saving purposes. $\bar{m}_h(s)$ is the portfolio of currencies held at the beginning of the period. The second and the third constraints are the cash-in-advance constraints faced by the young, where $m_h(s)$ is the end-of-period portfolio.

For the old agents, the timing is structured as follows. Firstly, the state of nature realizes. Secondly, the previously accumulated currencies are spent in the respective goods' markets. This structure implies that the old are not allowed to readjust their portfolio. As in Bacchetta and Van Wincoop (2010), agents choose a portfolio only in the first period of their lives. This feature captures in a simple way the fact that actively managing portfolios is a costly activity and hence investors are characterized by considerable inertia.²³ Although only a fraction of agents in the economy (the young) make a portfolio decision each period, the portfolio decision

²² Notice that we are implicitly assuming that the law of one price holds. This is not a restrictive assumption, considering that this is a flexible price, long-run model. For instance, Campa and Goldberg (2005) have shown that there is full exchange rate pass-through in the long-run for many types of imported goods.

²³ Bacchetta and Van Wincoop (2010) and Kim et al. (2016) show that inertia is indeed consistent with optimal behavior. Kim et al. (2016) also show that inertia is stronger in old age because of falling efficiency in decision making and higher mortality risk. See Bilias et al. (2010), Agnew et al. (2003) and Fagareng et al. for empirical evidence surrounding investors' inertia.

of the young at a particular point in time depends on the current realization of the shock. As a result, at country level, portfolios are time-varying.

We can consolidate the constraints of the young (4), (5) and (6) into a single budget constraint and rewrite the problem as follows:

$$p^{1}(s)c_{1h}^{1}(s) + p^{2}(s)e(s)c_{1h}^{2}(s) - w_{h}(s) = -m_{h}^{1}(s) - e(s)m_{h}^{2}(s)$$
(9)

$$p^{1}(s')c_{2h}^{1}(ss') = m_{h}^{1}(s) \quad \forall \ s'$$
(10)

$$p^{2}(s')c_{2h}^{2}(ss') = m_{h}^{2}(s) \ \forall \ s'$$
(11)

For analytical convenience, we will assume that the intertemporal elasticity of substitution, which is the inverse of the coefficient of relative risk aversion γ , is equal to the elasticity of substitution between the traded goods σ^{24} :

Assumption 1. $\frac{1}{\gamma} = \sigma$.

In Section 1 of the Supplementary Material, we compute the first-order conditions of the agents' maximization problem. Combining equations (35), (36) and (37) in the Supplementary Material, we can obtain the following expression for the nominal exchange rate:

$$e(s) = \frac{a_h^2 \sum_{s'} \rho(ss') \frac{c_{2h}^2(ss')^{-\frac{1}{\sigma}}}{p^2(s')}}{a_h^1 \sum_{s'} \rho(ss') \frac{c_{2h}^1(ss')^{-\frac{1}{\sigma}}}{p^1(s')}}$$
(12)

As well as the relative preference for the two goods, the nominal exchange rate depends on the expected purchasing power of the two currencies, weighted by agent *h*'s marginal utilities. For instance, if the price of good 2 is expected to go down (up), this means that currency 2 will have a higher (lower) purchasing power: hence, currency 2 appreciates (depreciates). Notice that the current value of the nominal exchange rate is not tied to its expected value, therefore there is no perfect arbitrage between the two currencies. This paves the way for excess returns on net foreign assets positions to arise in the model.²⁵

In the Supplementary Material (Section 1), we also show how to find closed-form solutions for the demand for the two currencies:

$$m_{h}^{l}(s) = \frac{a_{h}^{1} \beta^{\sigma} \left[\sum_{s'} \rho(ss') p^{1}(s')^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}{A_{h}(s)} w_{h}(s)$$
(13)

$$m_{h}^{2}(s) = \frac{a_{h}^{2}\beta^{\sigma}e(s)^{1-\sigma} \left[\sum_{s'}\rho(ss')p^{2}(s')^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}{A_{h}(s)} \frac{w_{h}(s)}{e(s)}$$
(14)

where

A

$$\begin{split} A_{h}(s) &\equiv a_{h}^{1} \rho^{1}(s)^{1-\sigma} + a_{h}^{2} [p^{2}(s)e(s)]^{1-\sigma} + a_{h}^{1} \beta^{\sigma} \left[\sum_{s'} \rho(ss') p^{1}(s')^{\frac{1-\sigma}{\sigma}} \right]^{\sigma} + \\ &+ a_{h}^{2} \beta^{\sigma} e(s)^{1-\sigma} \left[\sum_{s'} \rho(ss') p^{2}(s')^{\frac{1-\sigma}{\sigma}} \right]^{\sigma} \end{split}$$

Agent *h*'s demand functions for the goods can be derived using (13), (14) and the budget constraints²⁶:

$$c_{1h}^{1}(s) = \frac{a_{h}^{1} p^{1}(s)^{-\sigma}}{A_{h}(s)} w_{h}(s) \qquad \forall \, \ell$$
(15)

$$c_{1h}^{2}(s) = \frac{a_{h}^{2}[p^{2}(s)e(s)]^{-\sigma}}{A_{h}(s)} w_{h}(s) \qquad \forall \ \ell$$
(16)

$$c_{2h}^{1}(ss') = \frac{a_{h}^{1}\beta^{\sigma} \left[\sum_{s'} \rho(ss')p^{1}(s')^{\frac{1-\sigma}{\sigma}}\right]^{-}}{A_{h}(s)} \frac{w_{h}(s)}{p^{1}(s')} \quad \forall s'$$
(17)

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²⁴ This parameter restriction allows us to derive the demand functions for the two currencies analytically, which is very helpful to gain intuition of the main mechanisms. In the sensitivity analysis, we will show that the qualitative and quantitative results obtained under this restriction are robust for commonly assumed values of γ and σ .

²⁵ See Gourinchas and Rey (2015) and Tille and Van Wincoop (2010) on the importance to relax perfect arbitrage to generate expected valuation effects. In Section 2 of the Supplementary Material, we show indeed that if we allowed old agents to manage their portfolio, then we would be back to a framework with perfect arbitrage where the current value of the nominal exchange rate is linked to its expected value.

²⁶ The steps to calculate the demand functions when young are provided in the Supplementary Material.

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$$c_{2h}^{2}(ss') = \frac{a_{h}^{2}\beta^{\sigma}e(s)^{-\sigma} \left[\sum_{s'}\rho(ss')p^{2}(s')^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}{A_{h}(s)} \frac{w_{h}(s)}{p^{2}(s')} \forall s'$$
(18)

As preferences are homothetic, the demand for each good is a linear function of wealth where the multiplicative term is a complicated non-linear function of current and future prices as well as the nominal exchange rate.

Let us now look at the portfolio decision of an agent in more detail. We combine the demand for the two currencies (13) and (14) to get:

$$\frac{m_h^1(s)}{m_h^2(s)} = \underbrace{e(s)^{\sigma}}_{\text{nominal exchange rate}} \cdot \underbrace{\frac{a_h^1}{a_h^2}}_{\text{goods' weights}} \cdot \frac{\left[\sum_{s'} \rho(ss')p^1(s')^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}{\left[\sum_{s'} \rho(ss')p^2(s')^{\frac{1-\sigma}{\sigma}}\right]^{\sigma}}$$
(19)

The portfolio decision of each agent depends on three sets of variables: the nominal exchange rate, the weights of the two goods in the utility function, the expected future prices of the two goods.

Firstly, as currency 2 becomes more expensive (e(s) increases) the relative demand for currency 1 increases. Secondly, the relative demand for currency 1 rises with the weight of good 1 in agent h's utility function. The equation also shows that the impact of future prices on portfolio choices depends on the degree of substitutability between the two goods, which in turn determines the degree of substitutability between the two currencies. In general, if it is expected that a good will be relatively cheaper, then the associated currency will have a higher purchasing power. If $\sigma > 1$, the demand for such currency would increase as the substitution effect is sufficiently high.

Obviously, these arguments about the role of the nominal exchange rate on the portfolio choice of the agents are of a partial equilibrium nature, as agents take prices and the nominal exchange rate as given. Below, we will show the importance of general equilibrium analysis as the nominal exchange rate moves in equilibrium to offset the expected price differentials across countries.

3. Stationary equilibrium: analytical results

Definition 1. A stationary equilibrium is a system of prices and nominal exchange rates $(p, e) \in \mathbb{R}_{++}^{2S} \times \mathbb{R}_{++}^{S}$, consumption allocations and portfolios $(c_{1h}(s), c_{2h}(ss'), m_h(s)) \in \mathbb{R}_{++}^2 \times \mathbb{R}_{++}^{2S} \times \mathbb{R}_{++}^2$ for every h = 1, ..., H and s = 1, ..., S such that:

- (i) agent h maximizes his utility function (1) subject to the budget constraints (9), (10) and (11) in every s;
- (ii) $c_1^{\ell}(s) + c_2^{\ell}(s's) = y^{\ell}(s) \quad \forall s, s' \text{ and } \forall \ell$ (iii) $\sum_h m_h^{\ell}(s) = M^{\ell} \quad \forall s, \ell$

where $c_1^{\ell}(s) \equiv \sum_h c_{1h}^{\ell}(s)$ and $c_2^{\ell}(s's) \equiv \sum_h c_{2h}^{\ell}(s's)$.

In the previous section, we have shown how to compute analytically the demand functions for the goods and the currencies under Assumption 1. The next step is to plug the demand functions into the market clearing conditions to derive our set of equilibrium equations. The number of endogenous variables that we need to compute reduces to 3S, i.e. the two nominal prices and the exchange rate in each state of nature. The number of equations is instead $2S + 2S^2$. 2S refer to the two money markets while $2S^2$ are the two goods' markets, which have to clear for any pair of s and s' as the consumption of the old depends on the state when born s' as well as the current state s. We now demonstrate that many equations that are apparently in excess are actually redundant, so that solving the model reduces to handling a non-linear system of 3S equations and unknowns. Showing this carefully is quite important, since Spear (1985) proved the generic non-existence of a stationary equilibrium in OLG models with multiple goods and one currency. Essentially, the friction in currency markets is what makes the existence of a stationary equilibrium possible.

Firstly, we can reduce the number of equations in the goods' markets by applying Walras Law. Suppose that the money markets clear. If we sum across agents the budget constraints of the young and the old and combine them, we get the following equation:

$$p^{1}(s)[c_{1}^{1}(s) + c_{2}^{1}(s's) - y^{1}(s)] + p^{2}(s)e(s)[c_{1}^{2}(s) + c_{2}^{2}(s's) - y^{2}(s)] = 0 \qquad \forall s', s' \in \mathbb{R}$$

where s is the current state and s' is the previous state. For every pair of (s', s), if the market for good 1 clears then the market for good 2 clears automatically. Therefore, S^2 equations can be made redundant. That would be the same as in Spear (1985).

Given Walras Law, suppose that the independent equations in the goods' markets are those for good 1. The next step is to sum across agents the budget constraints of the old for good 1 in state s so that we obtain the following equation:

 $p^{1}(s)c_{2}^{1}(s's) = M^{1}$

It is easy to see that the aggregate consumption of the old does not depend on the previous state (the state realized when born) as aggregate real money balances only depend on the current state:

$$c_2^1(s's) = \frac{M^1}{p^1(s)} \qquad \Rightarrow \qquad c_2^1(s's) = c_2^1(s)$$

Given that the aggregate consumption of the old for good 1 does not depend on the past, it is enough that *S* equations in the market for good 1 clears (instead of S^2). For instance, assume that the independent equations are those for which s' = s:

$$c_1^1(s) + c_2^1(ss) = y^1(s)$$

Therefore, the solution method involves to find 2S nominal prices and S exchange rates which solve the following system of 3S equations:

$$\sum_{h} c_{1h}^{1}(s) + \sum_{h} c_{2h}^{1}(ss) = y^{1}(s)$$
$$\sum_{h} m_{h}^{1}(s) = M^{1}$$
$$\sum_{h} m_{h}^{2}(s) = M^{2}$$

On the contrary, in an OLG model with one currency as in Spear (1985), the aggregated budget constraint of the old in a two-good setting would be:

$$p^{1}(s)c_{2}^{1}(s's) + p^{2}(s)c_{2}^{2}(s's) = M$$

where p(s) are the prices of the goods expressed in currency units. Since one currency buys two goods, the aggregate consumption of the old for each good will depend on the past. We cannot get rid of further equations for good 1 as we did above using the cash-in-advance constraints. Since the number of prices is 2*S* but the number of equations is $S^2 + S$, there are too many equations with respect to the number of unknowns in Spear (1985). In this paper, the presence of the cash-in-advance constraints where each currency can buy a country-specific good along with the friction in currency markets is critical to ensure that a stationary equilibrium exists.

Next, we will introduce a few key definitions, which will allow us to make some useful remarks, and provide two analytical results on trade balance and exchange rate dynamics.

3.1. Portfolio rebalancing and trade imbalances: a unified view

To start with, let us define the balance of trade of country 1 in state s, where s' is the state of nature in the previous period²⁷:

$$TB_{1}(s's) \equiv \underbrace{p^{1}(s)[y^{1}(s) - c_{11}^{1}(s) - c_{21}^{1}(s's)]}_{\text{exports}} - \underbrace{p^{2}(s)e(s)[c_{11}^{2}(s) + c_{21}^{2}(s's)]}_{\text{imports}}$$

Notice that the sign of the balance of trade depends on the choices that the young make in the current period, but also on the choices made by the generation born in the previous period. Substituting the budget constraints into the trade balance equation, it should be immediate that the above definition can be rewritten as:

$$TB_{1}(s's) = m_{1}^{1}(s) - m_{1}^{1}(s') + e(s)[m_{1}^{2}(s) - m_{1}^{2}(s')]$$
(20)

This leads us to the following remarks:

Remark 1. The balance of trade is zero if: (i) portfolios are constant across states of nature; (ii) this period's realized state is the same as last period's.

Equation (20) shows that there is a close relationship between currency markets and the goods' markets. If, for some reason, there is no portfolio rebalancing in equilibrium, then the balance of trade is always balanced. In this model, this behavior occurs e.g. when utility functions are logarithmic.²⁸ The demand functions become extremely simple as they do not depend on future prices, and the model is fully tractable. Constant portfolios is a prediction at odds with the reality of international financial markets. When the elasticity of substitution is not equal to one, we will show that agents born in different states of nature have different demands for the goods which is then reflected in different demand for the two currencies.²⁹

The second part of the remark is related to the result of Polemarchakis and Salto (2002) for deterministic OLG economies. In a monetary union with no uncertainty, they showed that the balance of trade is always in equilibrium at the monetary steady state. In this paper, the monetary steady state is stochastic and trade imbalances are possible whenever $s \neq s'$.

²⁷ Obviously, by Walras Law we have that $TB_2(s's) = -TB_1(s's)$.

²⁸ Analytical derivations of the log case are available under requests.

 $^{^{29}}$ Our finding for the log case is related to Cass and Pavlova (2004), who showed that the matrix of portfolio returns is degenerate in a two-period economy with N Lucas trees. A similar result holds in the infinite-horizon setting of Pavlova and Rigobon (2007, 2010), who then introduce demand shocks to generate time-varying portfolios.

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We now introduce a further restriction in the model. Suppose that countries give the same weight to the domestic and the foreign good (no home bias):

Assumption 2. $a_h^1 = a_h^2 = 0.5$.

This assumption allows us to derive two analytical results, which are very helpful to develop some key intuitions about the model. The following Proposition establishes that there is a strong relationship between the distribution of world GDP across countries, portfolio holdings and trade imbalances.

Proposition 1. Under Assumptions 1 and 2: (i) a country's portfolio holdings at the end of a period are positively and linearly related to its current share of world GDP.

(ii) If a country has a higher (lower) share of world GDP with respect to the past, it runs a trade surplus (deficit).

The end-of-the-period wealth of the young is equal to their total money holdings, as the two currencies are the means by which they save and therefore finance future consumption. Proposition 1 suggests that the distribution of world wealth at the end of a period is strongly related to the distribution of world GDP.³⁰ If the distribution of world GDP changes across states of nature, then portfolio rebalancing occurs over time and the distribution of wealth will change too.

Proposition 1 sheds light on the dynamics of the trade balance. In our economy, the sign of the trade balance is determined by the savings of the young and the dissavings of the old. If a country's share of world GDP rises, then the current generation has relatively more income than the previous one. This means that the young save more than in the past. At the same time, the previous generation's consumption is relatively low: since it was poorer, it saved less. Hence, the current generation accumulates more assets than in the past. The decrease in foreign liabilities (due to the increase in domestic assets) and the increase in foreign assets cause an improvement in the country's net foreign assets position. As a consequence, the country runs a current account surplus.³¹ In Section 4, we will show numerically that the result highlighted by Proposition 1 still holds in the presence of home bias, while we will explore the significance of the mechanism empirically in Section 5.

Next, we decompose the trade balance equation to highlight valuation effects and net foreign assets' dynamics.

3.2. Net foreign assets dynamics and exchange rate-driven valuation effects

In this section, we explore the relationship between net foreign assets, the balance of trade and valuation effects. Consider the balance of trade of country 1 in state s's, where s' is the past state and s is the current state, as defined in the previous section (equation (20)). Using the fact that $m_1^1(s) + m_2^1(s) = M^1$ for every s, we can rewrite it as follows:

$$TB_{1}(s's) = \underbrace{m_{2}^{1}(s')}_{FL_{1}(s')} - \underbrace{m_{2}^{1}(s)}_{FL_{1}(s)} + \underbrace{e(s)m_{1}^{2}(s)}_{FA_{1}(s)} - \underbrace{e(s)m_{1}^{2}(s')}_{\text{current value } FA_{1}(s')}$$
(21)

where FA(s) and FL(s) stand respectively for "foreign assets" and "foreign liabilities", which, in this context, are a country's holdings of foreign currency and the foreign country's holdings of the domestic currency. Next, define net foreign assets as $NFA(s) \equiv FA(s) - FL(s)$ and rewrite the above as follows:

$$NFA_1(s) = \text{current value } NFA_1(s') + TB_1(s's)$$
⁽²²⁾

Equation (22) states that the end-of-period net foreign assets in country 1 is equal to the current value of the net foreign assets accumulated in the previous period plus the trade balance.

The next step is to rewrite equation (21) to highlight the valuation effects in this model. We can do that by summing and subtracting the foreign assets of country 1 in the previous state $(e(s')m_1^2(s'))$ in the right hand side and using the definition of net foreign assets:

$$TB_{1}(s's) = NFA_{1}(s) - NFA_{1}(s') + [e(s') - e(s)]m_{1}^{2}(s')$$
(23)

This equation can be rewritten as:

$$\Delta NFA_{1}(s's) = TB_{1}(s's) + r(s's)FA_{1}(s') = TB_{1}(s's) + VAL_{1}(s's)$$
(24)

where

³⁰ Because nominal interest rates are zero, then domestic GDP is equal to domestic income. Therefore, the distribution of world GDP is also equal to the distribution of world income.

³¹ One might notice that agents do not have very sophisticated portfolio strategies, as they hold the same share of both money stocks (see the proof in Section 1.3 of the Supplementary Material). The reason is that we have assumed away "home bias" in the preferences: agents hold the two currencies in the same share as they like the two goods equally. We relax this assumption in the numerical exercises below.

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$$r(s's) \equiv R(s's) - 1 \equiv \frac{e(s)}{e(s')} - 1$$

$$VAL_1(s's) \equiv r(s's)FA_1(s')$$
(25)
(26)

The change in the net foreign assets position of country 1 will be determined by the behavior of the balance of trade and the valuation effects, where r(s's) is the return on the foreign assets accumulated in the previous period.³² In the model, valuation effects are entirely determined by exchange rate movements. If the foreign currency has appreciated with respect to the past (i.e. e(s) > e(s')), then the return on the foreign assets accumulated in the previous period is positive and therefore we say that the country experiences positive valuation effects.³³ Conversely, a country experiences negative valuation effects if the foreign currency has depreciated.

Gourinchas and Rey (2015) recently stressed that one of the challenges for open economy models with endogenous portfolio choice is to be able to come up with a model that generates substantial *expected* valuation effects as opposed to *unexpected* valuation effects. In fact, Gourinchas and Rey (2007) have shown empirically that the valuation channel is a critical component of the process of external adjustment of the US, and the exchange rate component is a fundamental force behind that. To be able explore this issue later on, we decompose $VAL_1(s's)$ in equation (24), which are effectively the valuation effects that country 1 would actually experience in the transition from state s' to state s, into an expected and an unexpected component. Firstly, we can write the valuation effects expected in state s' as follows:

$$E_{s's}VAL_{1}(s's) \equiv FA_{1}(s')E_{s's}r(s's)$$
⁽²⁷⁾

where $E_{s's}$ is the expectation operator which is conditional on the world economy being in state s'. Equation (27) indicates that expected valuation effects are not zero in the model as long as the expected return on the foreign assets' position $(E_{s's}r(s's))$ is different from zero.

Hence, the unexpected component is simply the difference between the actual (realized) and the expected valuation effects:

$$\underbrace{UNVAL_{1}(s's)}_{\text{unexpected val. effects}} = \underbrace{VAL_{1}(s's)}_{\text{actual val. effects}} - \underbrace{E_{s's}VAL_{1}(s's)}_{\text{expected val. effects}} = FA_{1}(s')[r(s's) - E_{s's}r(s's)]$$
(28)

We can also derive a forward-looking expression for the net foreign assets position of country 1 in state s' to highlight the role of expected valuation effects in the dynamics of the net foreign assets' position of the country. The change in the net foreign assets position between s and s' must satisfy the following equation:

$$NFA_1(s) - NFA_1(s') = TB_1(s's) + VAL_1(s's) \qquad \forall s$$

Multiplying each equation by the probability that state s realizes next period given that the current state is s' and summing across the S equations, we get:

$$NFA_{1}(s') = E_{s's}NFA_{1}(s) - E_{s's}TB_{1}(s's) - E_{s's}VAL_{1}(s's)$$
⁽²⁹⁾

Equation (29) shows that the net foreign assets position of country 1 in state s' can be written as a function of the expected net foreign assets position, the trade balance and valuation effects.³⁴

Proposition 1 established that the share of world GDP is the driving force behind trade balance dynamics. We will now explore how output shocks influence the dynamics of the nominal exchange rate in equilibrium, which is key to generate valuation effects.

Once again, we assume the absence of home bias for analytical purposes. In Section 4, we will show that the result holds numerically in the presence of home bias.

Proposition 2. Under Assumptions 1 and 2, if output is expected to grow, the domestic currency appreciates when $\sigma > 1$ and depreciates when $\sigma < 1$.

In the Supplementary Material (Section 1.3), we show that the nominal exchange rate in equilibrium satisfies the following equation:

$$e(s) = \frac{M^{1}}{M^{2}} \frac{\sum_{s'} \rho(ss') \left(\frac{1}{f^{2}(s')y^{2}(s')}\right)^{\frac{1-\sigma}{\sigma}}}{\sum_{s'} \rho(ss') \left(\frac{1}{f^{1}(s')y^{1}(s')}\right)^{\frac{1-\sigma}{\sigma}}} \qquad s = 1, ..., S$$
(30)

³² Notice that there is no net income from abroad and therefore the trade balance position is equivalent to the current account position.

³³ As the price of the foreign asset is defined in units of the domestic asset, i.e. the exchange rate, the above rate of return has to be interpreted as the return of foreign assets relatively to the return on foreign liabilities.

³⁴ In Gourinchas and Rey (2007), the first term in the right-hand side does not appear as their analytical expression is based on the assumption that the no-Ponzi game condition holds. While this is always the case when agents are infinitely-lived, we do not have to impose such restriction on the economy in an overlapping generations setting. We will see that this will have important consequences for the dynamics of net foreign assets.

Table 1 Parameter values. Baseline model	
Output in country 1 in state 1	$y^1(1) = y_H$
Output in country 1 in state 2	$y^1(2) = y_L$
Output in country 2 in state 1	$y^1(1) = y_L$
Output in country 2 in state 2	$y^1(2) = y_H$
$y_L = .99$	$y_{H} = 1.01$
Money supply	$M_1=M_2=1$
Elasticity of substitution	$\sigma = \frac{1}{\gamma} = 4$
Share of home goods	$a_1^1 = a_2^2 = 0.72$
Discount factor	$\beta_1=\beta_2=1$
Output persistence	$\rho(ss) = 0.9$

where $f^{\ell}(s')$ is defined as the share of good ℓ consumed by the old in state s'.

The reason behind the result is then the following. If output is expected to grow in the future, then this means that the price of the domestic good is expected to fall. As a result, the domestic currency has a high purchasing power. If the goods are substitutes ($\sigma > 1$), then agents would increase their demand for the domestic good (currency) leading to a domestic currency appreciation. If the goods are complements ($\sigma < 1$), there would be a relative increase in demand for the foreign currency leading to a domestic currency depreciation. In the next Section, we will explore in more detail how changes in the share of world GDP, which occur via shocks to output, lead to significant valuation effects in the model.

4. Valuation effects and external adjustment: numerical results

The aim of this section is to show that our two-country model can rationalize two main features of the data. Firstly, the fact that valuation effects tends to stabilize current account positions. In other words, countries that run current account surpluses (deficits) tend to experience negative (positive) valuation effects (e.g. Lane and Milesi-Ferretti, 2002). Secondly, and most importantly, the valuation channel in the process of external adjustment is quantitatively relevant (Gourinchas and Rey, 2007). While the first fact has been captured by other theoretical papers focusing on asset price-driven valuation effects (e.g. Pavlova and Rigobon, 2010; Devereux and Sutherland, 2010), existing models have not been able to reproduce the second fact so far (see e.g. Gourinchas and Rey, 2015).

As explained in the introduction, it would be difficult to match moments of the data in the context of our simple two-period model. The purpose of the numerical exercises in this section is rather to show that exchange rate-driven valuation effects can significantly contribute to the dynamics of the net external position of a country in a model where the lack of perfect arbitrage across assets gives rise to non-zero excess returns. By conducting a range of sensitivity analysis, we can get a sense of the conditions under which (expected) valuation effects are relatively small or large.

To illustrate the mechanisms as cleanly as possible, we consider an economy with two states of nature. The output of both countries (before the shock) is normalized to 1, and both are subject to a 1% shock. However, this is asymmetric: whenever the output of one country is high the other one is low and viceversa. The parameter values that we choose for the numerical exercise are reported in Table 1.

We normalize both money supplies to 1 since the level of the money supply does not affect the real allocation.³⁵ Since this is a two-period OLG model, our value for the elasticity of substitution between traded goods is more in line with the empirical work based on low-frequency data, which found values in the range between 4 and 15 (see Ruhl, 2008). In Section 4.1, we will show that our main mechanism is robust to different parameter values for the elasticity of substitution.

As it is common in two-period OLG models (see e.g. Gottardi and Kubler, 2011), the discount factor is set equal to 1 and identical across countries (we consider a more standard value in the sensitivity analysis). We assume that the probability that output tomorrow is the same as today is 0.9, which implies that output is somewhat persistent. In other words, if we defined ϕ as the autocorrelation coefficient of the output process, $\phi = 0.8$ given the formula $\phi = 2\rho(ss) - 1$ (see Hamilton, 1994). Although assuming that the output is highly persistent is standard in the real business cycle literature, this might be questionable in a model such as this one. Assessing the persistence of the process for a two-period model is very difficult given the small number of observations that would be available (see e.g. Sánchez-Marcos and Sánchez-Martín, 2006). Hence, we will conduct various robustness exercises on this parameter. Finally, we set the share of home goods in consumption equal to 0.72 as in Corsetti et al. (2008).³⁶

In our sensitivity analysis, we will also study a more general version where the elasticity of intratemporal substitution (between traded goods) is allowed to differ from the elasticity of intertemporal substitution: while the model loses analytical tractability, the qualitative as well as the numerical results are not significantly affected.

 $^{^{35}}$ It can be checked that e.g. changing the money supply of country 1 by a fraction λ will increase p_1 and e by exactly the same fraction with no effects on consumption allocations and the various components of the balance of payments once normalized as percentages of GDP.

³⁶ Our results on the importance of the valuation channel are robust to different degrees of openness of the two economies. Results on this further robustness check are available under request.

Table 2Equilibrium prices. Baselinemodel.

	state 1	state 2	
p^1	1.9848	2.0155	
p^2	2.0155	1.9848	
е	0.9897	1.0104	
ε	1.0050	0.9950	
Notes. ε is the terms of trade			

of country 2 $\varepsilon(s) \equiv \frac{p^2(s)e(s)}{n^1(s)}$.

 Table 3

 The share of "world GDP" and currency holdings. Baseline model.

		state 1	state 2
Country 1	Share of "world GDP"	$\frac{w_1(1)}{\sum_h w_h(1)} = 0.5037$	$\frac{w_1(2)}{\sum_h w_h(2)} = 0.4963$
	Share of domestic currency held	$\frac{m_1^{1}(1)}{M^1} = 0.7221$	$\frac{m_1^{1}(2)}{M^1} = 0.7179$
	Share of foreign currency held	$\frac{m_1^2(1)}{M^2} = 0.2821$	$\frac{m_1^2(2)}{M^2} = 0.2779$

Table 4

The net external positions. Baseline model.

	Trade balance % GDP	Valuation effects % GDP	Change in NFA % GDP
Country 1	$tb_1(12) = -0.4229$	$val_1(12) = 0.2934$	$\Delta n f a_1(12) = -0.1295$
	$tb_1(21) = 0.4166$	$val_1(21) = -0.2877$	$\Delta n f a_1(21) = 0.1289$
Country 2	$tb_2(12) = 0.4166$	$val_2(12) = -0.2890$	$\Delta n f a_2(12) = 0.1276$
	$tb_2(21) = -0.4229$	$val_2(21) = 0.2920$	$\Delta n f a_2(21) = -0.1309$

Firstly, let us consider the impact of output shocks on equilibrium prices (Table 2). In state 1, country 1's output is relatively high. Because of the supply effect, the price of the domestic good (which is exported) is low relatively to the price of the foreign good (imports). As a result, the terms of trade of the country deteriorates (ϵ increases). However, currency 1 appreciates (e falls). In fact, since good 1 is relatively cheaper, currency 1 has a higher purchasing power and money holdings of currency 1 yield more in real terms. Because the current generation expects that the current state realizes tomorrow with a high probability, then the demand for currency 1 increases leading to a currency appreciation.

In Table 3, we report the share of world GDP and the money holdings of the two countries in the two states of nature. In state 1, country 1's share of world GDP increases as its output is relatively high. As a result, the country accumulates more domestic and foreign assets as compared to state 2.3^{37}

Using equations (20) and (24), we can compute the balance of trade, the change in the net foreign assets position and valuation effects for country 1. The key identity that pins down the various components of the net external position of country 2 can be derived following the same steps as for country 1. We report our variables as percentages of the respective country's GDP in Table 4.

As we emphasized above, the sign of the balance of trade depends on the current state as well as the previous state. If the current state happens to be identical to the previous state, no trade imbalances arise. Therefore, the "interesting" situations are those where the state of nature has changed over time. Let us then consider a scenario where the economy transits from state 1 to state 2. As country 1 experiences a fall in output, they run a trade deficit of -0.4229% of domestic GDP. The asset side explanation is that, as the economy becomes relatively poorer, they accumulate less domestic and foreign assets. If country 1 holds less of its domestic currency, then foreign holdings increase which leads to an increase in the foreign liabilities. Along with a fall in foreign asset holdings, the net foreign assets position deteriorates. The goods' markets side explanation is that country 1 imports more and export less than before, since the foreign good has become relatively cheaper.³⁸

The trade deficit is partially offset by substantial positive valuation effects. As currency 2 appreciates, the value of country 1's foreign assets increases relatively to the value of foreign liabilities. Therefore, the country experiences a positive wealth effect that

³⁷ If there was no home bias, the share of the assets held by each country would simply be equal to its share of world GDP according to Proposition 1. Since we have now allowed for the possibility of home bias, this equivalence does not hold precisely. The consequence of home bias in consumption is that each country is holding more domestic than foreign assets in any given period.

³⁸ Plugging the budget constraints into (20), we can write the other definition of the balance of trade as the difference between exports and imports: $tb_1(s's) \equiv p^1(s)[c_{1_2}^1(s) + c_{2_2}^1(s's)] - p^2(s)e(s)[c_{1_2}^1(s) + c_{2_2}^2(s's)]$.

 Table 5

 A decomposition of the net external positions of country 1.

1			1	5	
	$\frac{E_{1s'}NFA_{1}(s')}{NFA_{1}(s)}\%$	-	$\frac{E_{1s'}TB_{1}(1s')}{NFA_{1}(s)}\%$	-	$\frac{E_{1s'}VAL_1(1s')}{NFA_1(s)}\%$
state 1	79.90	-	-65.64	-	45.54
state 2	80.10	-	-64.29	-	44.40
-					

Notes. $\frac{NFA(1)}{GDP(1)}\% = 0.0641$ and $\frac{NFA(2)}{GDP(2)}\% = -0.0651$.

mitigates the negative impact of the trade deficit on the external position. On the other hand, country 2 experiences a negative wealth effect as the country runs a trade surplus.

Therefore, our model shows that exchange rate-driven valuation effects stabilize countries' net external positions. This is consistent with the empirical literature, which has found that the correlation between the trade balance and valuation effects is negative (see e.g. Lane and Milesi-Ferretti, 2002). Papers which have modeled asset price-driven valuation effects had indeed found that valuation effects are stabilizing (e.g. Pavlova and Rigobon, 2010; Devereux and Sutherland, 2010). This paper shows that exchange rate-driven valuation effects are also stabilizing. Moreover, we suggest that one possible mechanism that drives them is differences in output growth rates across countries. In particular, our model suggests that while a country experiencing an increase (decrease) in the share of world GDP will run a trade surplus (deficit), this will be mitigated by an exchange rate appreciation (depreciation) leading to negative (positive) valuation effects.

The most relevant contribution of this paper is that the model can generate meaningful *expected* valuation effects, which substantially contribute to the dynamics of net foreign assets.

Following the analysis in Section 3.2, e.g. let us derive a forward-looking expression for the net foreign assets position of country 1 in state 1.

$$NFA_{1}(1) = E_{1s'}NFA_{1}(s') - E_{1s'}TB_{1}(1s') - E_{1s'}VAL_{1}(1s')$$
(31)

where $E_{1s'}$ is the expectation operator which is conditional on the fact that the world economy is in state 1.

Using our definitions (24), (25) and (27), expected valuation effects can be calculated as follows:

$$E_{1s'}VAL_1(1s') \equiv FA_1(1)E_{1s'}r(1s') =$$

= $FA_1(1)[\rho(11)r(11) + \rho(12)r(12)] = \rho(12)VAL_1(12)$ (32)

since r(11) = 0. Because Table 2 has already established that the nominal exchange rate fluctuates across the two states of nature, $r(12) \neq 0$ implies that expected valuation effects are different from zero.³⁹

Table 5 reports the percentage of the net foreign assets position explained by each component in (31).

The table shows that all three components are important in driving the adjustment of the net external position. In state 1, output remains the same with probability $\rho(11)$ while falls with probability $\rho(12)$. As a result, output is expected to fall in country 1. The model then predicts that we should expect a process of adjustment of the net external position of country 1 driven by a deterioration of the net foreign assets' position due to a trade deficit as well as positive valuation effects.

As $NFA_1(1) > 0$, the net external position remains (changes to) positive (negative) with higher (lower) probability as output is persistent. A great percentage of NFA is explained by the expected NFA position, which is of the same sign as the actual position. However, agents expect that the NFA will be lower than the current NFA as they expect a trade deficit. If the same state realizes next period, then the trade balance will be zero (equation (20)) and agents would regard this as the most likely outcome. However, there is a positive probability that the distribution of wealth would shift in favor of country 2 leading to a trade deficit, hence agents would expect a trade deficit overall. As the expected trade balance component has a negative sign, then the expected stock and flow components of net foreign assets would more than explain the current NFA position. But finally, and most importantly, agents would expect an exchange rate depreciation due to the higher growth in the foreign country, hence the positive expected valuation effects.

In state 2, country 1 has a negative net foreign assets position. Hence, the denominator in the three variables in Table 5 is negative. This implies that the country would then expect a negative position in the future (although lower), a current account surplus and negative valuation effects. This might seem in contradiction with Gourinchas and Rey (2007), who find that future trade surpluses for the US (who have a negative NFA) should be accompanied by positive valuation effects instead. As we explained in section 3.2, our equation is more general than Gourinchas and Rey (2007) since they assume that the no-Ponzi game condition holds, which implies that $E_{ss'}NFA(s') = 0$. Hence, we cannot expect to fully relate our results with their empirical analysis. It is easy to see that the restriction that $E_{ss'}NFA(s') = 0$ automatically implies that $E_{ss'}VAL(ss') > 0$ when $E_{ss'}TB(ss') > 0$, but that is not necessarily the case when $E_{ss'}NFA(s') \neq 0$. Yet, our framework supports their empirical finding that expected valuation effects are a fundamental component of the process of external adjustment. Also, we suggest a possible mechanism based on expected growth differentials. The literature on valuation effects to date has been able to generate large unexpected valuation effects (see e.g. Nguyen, 2011), but

³⁹ If we did not have the friction in currency markets, the exchange rate would be constant across states of nature precluding the possibility of generating expected valuation effects (see Supplementary Material).

Table 6Varying the elasticity of substitution.

	Trade balance % GDP	Val. effects % GDP	ΔNFA % GDP
$\sigma = 0.5$	1.2594	-1.2299	0.0295
$\sigma = 1$	0	0	0
$\sigma = 2$	-0.3021	0.2511	-0.0510
$\sigma = 4$	-0.4229	0.2934	-0.1295
$\sigma = 8$	-0.4836	0.2521	-0.2315
$\sigma = 16$	-0.5186	0.1805	-0.3380

Table 7

Varying output persistence.

	Trade balance % GDP	Val. effects % GDP	$\Delta NFA \%$ GDP
$\rho(ss) = 0.4$	-0.3596	-0.0027	-0.3623
$\rho(ss) = 0.5$	-0.3662	0.0281	-0.3380
$\rho(ss) = 0.6$	-0.3745	0.0674	-0.3071
$\rho(ss) = 0.7$	-0.3856	0.1190	-0.2665
$\rho(ss) = 0.8$	-0.4007	0.1900	-0.2108
$\rho(ss) = 0.9$	-0.4229	0.2934	-0.1295

Table 8

A decomposition of the net external position of country 1 in state 1 for different values of $\rho(ss)$.

	$\frac{E_{1s'}NFA_{1}(s')}{NFA_{1}(1)}\%$	_	$\frac{E_{1s'}TB_1(1s')}{NFA_1(1)}\%$	-	$\frac{E_{1s'}VAL_1(1s')}{NFA_1(1)}\%$
$\rho(ss) = 0.5$	-0.45	-	-108.38	-	8.33
$\rho(ss) = 0.6$	19.90	_	-97.68	_	17.58
$\rho(ss) = 0.7$	39.87	_	-86.98	_	26.86
$\rho(ss) = 0.8$	59.87	_	-76.30	_	36.17
$\rho(ss) = 0.9$	79.90	-	-65.64	-	45.54

the dynamics of net foreign assets is largely driven by the trade balance channel in Tille and Van Wincoop (2010), Devereux and Sutherland (2010) and Ghironi et al. (2015).

4.1. Sensitivity analysis

The aim of this section is to check the robustness of the baseline model to alternative specifications of σ and $\rho(ss)$. Finally, we allow for the intertemporal elasticity of substitution to differ from the intratemporal elasticity of substitution. While the model loses analytical tractability, our results are not affected by the introduction of a more general specification. All the results in the Tables below (except for Table 8) are reported for country 1 in the transition from state 1 to state 2.

4.1.1. The elasticity of substitution

In Table 6, we show that valuation effects always act as a stabilizer of the net external positions of the two countries, independently from whether σ is bigger or lower than 1. As we explained in the previous section, the most realistic values for the elasticity is between 4 and 15 considering that we deal with low-frequency data (see Ruhl (2008)) so we set 4 as our benchmark value. It can be observed that as the elasticity of substitution increases, valuation effects become less important relatively to the trade imbalances. As a consequence, the change in the net foreign asset positions of the two countries increases with the elasticity. However, valuation effects still account for a significant proportion of the dynamics of net foreign assets.

When $\sigma = 1$ (log case), there is "no action" in the countries' net external position as we explained in Section 3.1. It is interesting to observe what happens when $\sigma < 1$, which corresponds to a situation where goods are complements. A decrease in output for country 1 leads to an increase in the share of world GDP (the opposite of "immiserizing growth").⁴⁰ This leads to a current account surplus and negative valuation effects through a currency appreciation. Valuation effects still stabilize the countries' net external position.

⁴⁰ See e.g. Cole and Obstfeld (1991).

Table 9The net external position of country 1. A more general model.

Trade balance % GDP	Valuation effects % GDP	Change in NFA % GDP
$tb_1(12) = -0.3948$	$val_1(12) = 0.2155$	$\Delta n f a_1(12) = -0.1793$
$tb_1(21) = 0.3887$	$val_1(21) = -0.2103$	$\Delta n f a_1(21) = 0.1785$

4.1.2. Output persistence

Table 7 shows that, for valuation effects to stabilize the net external positions of a country, the probability that next period's state is the same as the current period's must be sufficiently high. Valuation effects are stabilizing as long as $\rho(ss) \ge 0.5$ but move in the same direction as the country's trade balance when $\rho(ss) = 0.4$.⁴¹ Therefore, there is some cut-off value of $\rho(ss)$ between 0.4 and 0.5 below which valuation effects are no longer stabilizing.⁴²

The condition under which valuation effects are stabilizing is not stringent at all, as it only requires that output is somewhat persistent in the two countries. In fact, the requirement that $\rho(ss) \ge 0.5$ simply means that the autocorrelation coefficient should be greater or equal to zero given that $\phi = 2\rho(ss) - 1$. (Hamilton, 1994).

Table 7 indicates that valuation effects contribute more to the change in net foreign assets the higher is the persistence of output. The reason is that the expectations of individuals born in different states of nature vary more widely, leading to higher nominal exchange rate volatility.

In Table 8, we explore how the various components of adjustment in the dynamics of the net external position of the US vary with $\rho(ss)$. As $\rho(ss)$ increases, the valuation channel becomes increasingly important in explaining the process of adjustment of net foreign assets' position. Since the realized valuation effects are quantitatively larger (Table 7), they can explain a significant percentage of the net external position. The relative importance of the valuation channel as opposed to the trade balance channel then crucially depends on the nature of the stochastic process of output: when there is high persistence in the economy, the valuation component becomes a crucial adjustment mechanism. However, it is important to stress that even at the opposite at of the spectrum, the valuation channel is still significant. When there is no persistence in the economy, which corresponds to the i.i.d. case (zero autocorrelation), valuation effects would still explain more than 8% of the net external position of the country.

4.1.3. Separating the intertemporal and the intratemporal elasticities of substitution

In this section, we relax Assumption 1 to take into account that the intertemporal elasticity of substitution is typically much lower than the elasticity of substitution between traded goods. Assuming different values for the two elasticities leads to a significant loss in the tractability of the model. Demand functions cannot be computed analytically and finding a solution involves the numerical computation of a large system of nonlinear equations, which includes the first-order conditions of the agents, the budget constraints as well as the market clearing equations. Even in our simple two-states example, the equilibrium system is quite large as it consists of 22 equations and unknowns.⁴³ We show in detail how to solve this generalized version of the model in Section 3 of the Supplementary Material.

For our sensitivity analysis, we set the coefficient of risk aversion γ to 2 as Corsetti et al. (2008). Since this utility function is more realistic, we consider a value for the discount factor which is standard in the real business cycle literature. Since we can assume that a period lasts for 30 years, then a quarterly discount rate of 0.99 implies that $\beta = 0.99^{30\times4} \approx 0.3$ (see also De La Croix and Doepke, 2003; Gottardi and Kubler, 2011). The other parameters are the same as in the baseline model (Table 1).

As Table 9 shows, the numerical results of this more general version of the model is not very different from the baseline.

5. The role of the share of world GDP: suggestive empirical evidence

The analytical and the numerical results obtained in Sections 3 and 4 highlighted how changes in the share of world GDP generate dynamics in the trade balance and in the nominal exchange rate, generating significant valuation effects. The aim of this section is to provide some empirical evidence in support of the key mechanisms of the model.

Firstly, we test the hypothesis suggested by Proposition 1, i.e. that the trade balance should be positively correlated with changes in the share of world GDP. For this purpose, we estimate the following fixed-effects model:

current account balance_{*i*,*t*} = $\alpha_i + \beta_1 \cdot \Delta$ share of the world GDP_{*i*,*t*} + $\beta_2 \cdot$ country size_{*i*,*t*} + $\varepsilon_{i,t}$

where

 Δ share of the world GDP_{*i*,*t*} = share of the world GDP_{*i*,*t*} - share of the world GDP_{*i*,*t*-1}

⁴¹ In this case, state 1 is more likely to occur when state 2 is realized hence agents would expect more inflation in country 2. This would lead to more demand for currency 1, hence an appreciation and negative valuation effects.

⁴² When there is no home bias $(a_h^1 = a_h^2)$, it is easy to see that this cut-off value is exactly 0.5. When $\rho(ss) = 0.5$, agents born in different states attach the same probabilities to future states, which implies that the exchange rate is constant in a stationary equilibrium hence valuation effects are zero (see equations (30) and (24)). When there is home bias, this cut-off value is smaller therefore valuation effects are stabilizing for a larger set of values of $\rho(ss)$.

⁴³ A higher number of countries or states of nature would substantially increase the dimension of the system.

Table 10	
Panel regression of the current account balance, 1981-2011.	

	(1)	(2)	(3) Ex. developing	(4) Ex. China and US
Δ share of world GDP	101.0027***	57.3803***	58.937***	10.0902***
	(6.6998)	(6.9661)	(11.3750)	(4.6484)
Country size		22.8006***	22.8603***	-20.1991***
		(1.30894)	(2.1037)	(0.9534)
Constant	0.16445	-13.5959***	-32.7898***	10.3064***
	(0.3322)	(0.85319)	(3.1572)	(0.4612)
R^2	0.0956	0.1045	0.1185	0.0752
No. countries	183	183	69	181
No. observations	5,038	5,038	1,951	4,992

Source. IMF World Economic Outlook database. The data are annual. The main independent variable is calculated using "Gross domestic product based on purchasing-power-parity (PPP) share of world total". *Notes.* The standard errors are reported in parentheses. *** denotes significance at the 1% level, ** at the 5% level and * at the 10% level. All regressions are estimated with fixed effects.

As a proxy for country size, we use the current share of the world GDP. We report our results in Table 10. We find that β_1 is positive and statistically significant for a large cross-section of countries. In columns (3) and (4), we perform a couple of robustness checks. Firstly, we remove developing countries from the sample. The reason is that there is a large cluster of observations for which the change in the share of world GDP is zero or close to zero, and this is typically the case for many developing countries. It can be observed that the regression results are not significantly affected. Secondly, we remove instead the United States and China. Since these two countries run the largest trade imbalances in the sample, and their shares of world GDP have seen the most dramatic changes, they alone might drive the result. While the estimated coefficient is significantly smaller, the qualitative prediction as well as the statistical significance are not challenged.⁴⁴

Our results suggest a second testable hypothesis related to exchange rate dynamics: an increase in a country's share of world GDP should be associated with the domestic currency's appreciation.

Recall that Proposition 2 showed that an increase in output leads to a domestic currency appreciation when $\sigma > 1$ and depreciation when $\sigma < 1.^{45}$ However, independently from the value of σ , an increase in the share of world GDP always triggers an exchange rate appreciation. When output goes up, the price of the domestic good falls. When $\sigma > 1$, the terms of trade depreciates but less than the relative increase in output. Therefore, an increase in output is associated with an increase in the share of world GDP. Indeed, Table 3 shows that country 1, who has a relatively high output in state 1, has a higher share of world GDP. When $\sigma < 1$, the terms of trade effect dominates and e.g. an increase in output, which is associated with an appreciation when $\sigma < 1$. In fact, Table 6 shows that, when $\sigma = 0.5$, a fall in output for country 1 in the transition from state 1 to state 2 results in a current account surplus (which is associated with an increase in the share of world GDP) and negative valuation effects, i.e. an appreciation of the nominal exchange rate.

In order to test this second hypothesis, we restrict the sample to the countries which have either a floating or a free floating exchange rate regime according to the IMF (2018). Since the majority of countries in the world do not fall into this category, the sample size drops considerably.⁴⁶ We then estimate the following fixed-effects model:

$\%\Delta e_{i,t}^{US} = \alpha_i + \beta_1 \cdot \Delta$ share of the world $\text{GDP}_{i,t} + \beta_2 \cdot \%\Delta$ relative money supply $U^{US} + \epsilon_{i,t}$

where the dependent variable is the percentage change of the nominal exchange rate: $\% \Delta e_{i,l} \equiv \frac{e_{i,l} - e_{i,l-1}}{e_{i,l-1}} \cdot 100$. We take annual (averaged) data for the nominal exchange rate from Lane and Milesi-Ferretti (2018). The exchange rate is defined as the units of the foreign currency equivalent to one USD dollar. If the value of the nominal exchange rate increases, this means that the domestic currency depreciates. If our hypothesis proves to be valid, we should then observe a negative coefficient for β_1 . We also control for changes in the countries' money supplies. This is indeed one of the main determinants of the nominal exchange rate in the model (equation (30)). We take data on broad money from the World Bank and calculate each country's money supply relatively to the US. We then calculate the percentage change in the relative money supply.

We report the results in Table 11. The coefficients related to the share of world GDP are all negative and statistically significant in support of our hypothesis. In the third and fourth columns, we remove the developing countries from the sample. As we argued in the previous set of regressions, the change in the share of world GDP is zero or close to zero for most of these countries. Moreover,

⁴⁴ As a further robustness check, we control for time fixed-effects. Most of the times dummies are insignificant and the significance and the sign of our coefficients are not affected.

 $^{^{45}}$ Let us recall that only in the special case where σ is exactly 1, there is neither trade nor exchange rate dynamics in the model.

⁴⁶ Part of the drop is due to the fact that euro area countries are not included. While changes in the share of world GDP (and trade imbalances) are likely to be driven by domestic factors, fluctuations in the euro/dollar nominal exchange rate are more likely to be driven by the whole economic performance of the euro area as compared to the rest of the world. Moreover, data on money supply is not readily available for euro area countries for the period before they joined the euro.

Table 11

Panel regression of the	percentage change	in the nominal	exchange rate, 1981-2011.

	(1)	(2)	(3) Ex. developing	(4) Ex. developing
Δ share of world GDP	-540.5279***	-235.2918***	-537.9203***	-242.9806***
	(141.6722)	(82.3292)	(162.0402)	(87.7654)
$\%\Delta$ relative money supply		1.0184***		.99501***
		(0.0175)		(0.0188)
Constant	37.4365***	6070	40.8068***	-1.8569
	(7.7949)	(4.1703)	(10.8305)	(5.4213)
R^2	0.0118	0.7489	0.0131	0.7850
No. countries	46	45	30	29
No. observations	1,266	1,207	861	814

Sources. For the share of world GDP, see Table 10. The percentage change in the nominal exchange rate is calculated from Lane and Milesi-Ferretti (2018), where nominal exchange rate data are annual and averaged over the period. Relative money supplies are calculated based on broad money data from the World Development Indicators, World Bank.

Notes. The standard errors are reported in parentheses. *** denotes significance at the 1% level, ** at the 5% level and * at the 10% level. All regressions are estimated with fixed effects.

it is likely that some developing countries have switched exchange rate regime over time. Note that the size of the coefficient is not affected significantly. The inclusion of relative money supply in the regression, while it does not affect the significance of our main variable of interest, increases considerably the explanatory power of the model.⁴⁷ Relative money supply is measured as the money supply of a given country relatively to the US. An increase in relative money supply should then lead to a currency depreciation. Indeed, a positive change in e (as defined in the data) means a domestic currency depreciation. Our estimates suggest that a 1% increase in domestic money supply (relatively to the foreign) roughly leads to 1% depreciation of the nominal exchange rate, consistently with equation (30).

Finally, we test whether exchange-rate driven valuation effects are also driven by the share of world GDP in the data. To calculate valuation effects, we use the methodology of Lane and Shambaugh (2010) on the more up-to-date database of Benetrix et al. (2015). The steps are as follows. Firstly, we calculate the net financial weights:

$$\omega_{ij,t}^F = \omega_{ij,t}^A s_{i,t}^A - \omega_{ij,t}^L s_{i,t}^L$$

where $\omega_{ij,t}^A$ ($\omega_{ij,t}^L$) is the share of currency *j*-denominated assets (liabilities) of country *i*, while $s_{i,t}^A = \frac{A_{i,t}}{A_{i,t} + L_{i,t}}$ and $s_{i,t}^L = \frac{L_{i,t}}{A_{i,t} + L_{i,t}}$ are respectively the share of total assets and total liabilities on total cross-border holdings. $\omega_{ij,t}^F$ then measures the direction of valuation changes for country *i* of a movement in currency *j*: if $\omega_{ij,t}^F > 0$ ($\omega_{ij,t}^F < 0$), this means that country *i* is long (short) in currency *j*. The valuation effects for country *i* from movements in currency *j* is then obtained as follows:

$$VALXR_{ij,t} = \omega_{ij,t-1}^F \cdot \%\Delta e_{ij,t} \cdot IFI_{i,t-1}$$

where $IFI_{i,t-1}$ is the international financial integration of country *i* at time t - 1. The latter variable, together with $\omega_{ij,t-1}^F$, gives a measure of the quantitative exposure of a country to a change of the bilateral exchange rate as measured by $\%\Delta e_{ij,t}$. Since $e_{ij,t}$ is defined as the units of currency *i* which currency *j* can buy, an increase in *e* implies a depreciation of the nominal exchange rate.

Aggregate valuation effects can be calculated by constructing the net financial index $I_{i,i}^F$. In particular, the growth in the net financial index is the sum across currencies of exchange rate changes multiplied by the net financial weight for each currency *j*:

$$\%\Delta I_{i,t}^{F} \equiv \sum_{i} \omega_{ij,t-1}^{F} \cdot \%\Delta e_{ij,t}$$

Therefore, overall valuation effects are:

 $VALXR_{i,t} = \%\Delta I_{i,t}^F \cdot IFI_{i,t-1}$

To start with, we estimate the following equation:

$$VALXR_{i,t} = \alpha_i + \beta \cdot \% \Delta$$
 share of the world $\text{GDP}_{i,t} + \varepsilon_{i,t}$

The results are in Table 12, Column (1). It shows that an increase in the share of world GDP has a positive effect on VALXR. This would seem in contrast with the results of this paper, which shows how an increase in the share of world GDP should be associated

⁴⁷ It is not surprising that this simple set of monetary fundamentals can explain a great deal of nominal exchange rate movements with annual data. See e.g. Cerra and Saxena (2010).

Table 12	
Panel regression of exchange rate-driven valuation effects, 1	990-2012.

	(1) VALXR	(2) VALXR ^{US} (long \$)	(3) VALX R ^{US} (long \$)	(4) VALXR ^{US} (short \$)	(5) <i>VALXR^{US}</i> (short \$)
$\%\Delta$ share of world GDP	0.1604***	-0.1005*	-0.1010*	0.1999***	0.1988***
	(0.0515)	(0.0519)	(0.0519)	(0.0602)	(0.0625)
$\%\Delta$ relative money supply			0.7033		49.9273
			(1.6026)		(62.27154)
Constant	-2.0166***	0.2339	0.1714	-2.9595***	-3.9924***
	(0.2203)	(0.1675)	(0.2200)	(0.2762)	(1.3175)
R^2	0.0013	0.0097	0.0051	0.0024	0.0023
No. countries	100	60	60	93	93
No. observations	2,041	612	612	1,429	1,429

Sources. For the share of world GDP and relative money supply, see respectively Table 10 and 11. Exchange rates for each currency against the euro, the pound, the yen and the Swiss franc, which are used to build *VALXR*, are calculated using end-of-period exchange rates of local currencies against the dollar provided by the International Financial Statistics (IMF). $IFI_{i,i}$ and $\omega_{i,i}^{F}$ are respectively taken and calculated from Benetrix et al. (2015)'s database.

Notes. Long (short) in dollars is the subsample of all observations for which $\omega_{I,J-1}^{FUS} > 0$ (< 0). The standard errors are reported in parentheses. *** denotes significance at the 1% level, ** at the 5% level and * at the 10% level. All regressions are estimated with fixed effects.

with negative valuation effects. However, this regression might not be particularly informative for a number of reasons. Firstly, and most importantly, the impact of a change in the exchange rate depends on the sign of the net financial weights. Suppose that a country has a long position in all foreign currencies. Then, we should expect a currency appreciation and negative valuation effects from an increase in the share of world GDP. However, if a country is short in each currency j ($\omega_{ij,t-1}^F < 0$), then an appreciation of the domestic currency ($\% \Delta e_{ij,t} < 0$) would generate positive valuation effects as the value of the foreign-currency denominated debt would fall. Therefore, the result could be driven by a sample dominated by countries which are short of foreign currencies. Secondly, one of the main drivers of exchange rate changes, and hence exchange-rate driven valuation effects, is monetary policy. As we argued for the previous set of regressions, it is important to control for changes in money supply. Since it would be difficult to construct a meaningful index for this variable, we look at the exposure of each country towards the US. Hence, in Columns (2)-(5), we estimate the following regression model:

$VALXR_{i,t}^{US} = \alpha_i + \beta_1 \cdot \% \Delta$ share of the world $\text{GDP}_{i,t} + \beta_2 \cdot \% \Delta$ relative money supply $U^{US} + \epsilon_{i,t}$

In Columns (2) and (3), we consider only those observations for which $\omega_{i,t-1}^{F US} > 0$. This corresponds to situations where the country is long in US dollar. For instance, suppose that the share of dollars in foreign assets and liabilities is the same: $s_{i,t-1}^{A US} = s_{i,t-1}^{L US}$. Then, a country is long in dollars if $A_{i,t-1} > L_{i,t-1}$. Conversely, suppose that the country has a zero net foreign assets' position in the previous period: $A_{i,t-1} = L_{i,t-1}$. Then, a country is long in dollars if the share of dollars in foreign assets is higher than the share of dollars in foreign liabilities.

In our stylized two-asset model, a country is always long in the foreign currency, as foreign assets are (denominated in) foreign currency and foreign liabilities are (denominated in) domestic currency. Indeed, as our theory would predict, an increase in the share of world GDP would generate negative valuation effects through an appreciation of the domestic currency.⁴⁸

On the other hand, columns (4) and (5) consider observations for which $\omega_{i,t-1}^{F US} < 0$, i.e. when countries are short in US dollars in the previous period. In this case, our main coefficient of interest is instead positive. An appreciation of the nominal exchange rate implies positive valuation effects, as the value of the dollar liabilities fall. Notice that more than two thirds of the observations in our sample fall in this category. This relates to countries that are net borrowers in dollars, i.e. countries affected by the "original sin" (Eichengreen and Hausmann, 1999; Hausmann and Panizza, 2003). Our regression results show how the currency composition of foreign assets and liabilities along with the sign of the net foreign currency exposure matter in determining the sign of valuation effects.

Although our theoretical model does not capture situations where countries live with the "original sin", our results suggest that the mechanism is empirical valid independently from whether a country is long or short in dollars. Changes in the share of world GDP, which are driven by output shocks, are an important determinant of exchange-rate driven valuation effects (Table 12).

6. Conclusions

This paper presents a parsimonious two-country model where exchange rate-driven valuation effects substantially contribute to the dynamics of the net foreign assets' position of a country. If output growth rates differ across countries, the distribution of world

⁴⁸ In fact, notice that if $\omega_{i,t-1}^{F US} > 0$, negative valuation effects occurs when $\% \Delta e_{i,t} < 0$.

GDP varies over time and trade imbalances among the countries arise. At the same time, exchange rate-driven valuation effects stabilize the countries' external positions. In particular, we show that a country runs a trade surplus if its relative position in the world economy improves, as measured by the share of world GDP. As the current generation has a higher income than the previous one, it holds a higher share of the available assets in the world economy and therefore the country's net foreign assets increase. As long as there is some degree of persistence in the economy, an increase in the share of world GDP is also associated with an appreciation of the domestic currency, leading to substantial (negative) valuation effects. We provide evidence to show that the mechanism which generates trade imbalances and exchange rate dynamics in the model is empirically relevant.

Our friction in currency markets implies that there is lack of perfect arbitrage between the two currencies, which are used as stores of value. As a result, this gives rise to non-zero expected valuation effects. The novelty of this paper is that our model is able to rationalize the empirical finding that the valuation channel is an important part of the process of external adjustment of a country, along with the trade balance channel (Gourinchas and Rey, 2007).

Our framework, although admittedly very stylized, explains some relevant trends in international financial markets. Because of its tractability, it shows transparently what are the minimum ingredients required to make the valuation channel matter.

One of the main challenges ahead is to extend the model to a multi-period setting to make it more suitable for quantitative analysis, in view of being able to match certain moments of the data. Secondly, it would be very valuable to have a model with a richer set of assets, in order to simultaneously explain exchange rate and asset price-driven valuation effects. From a theoretical perspective, it would be useful to understand whether or under which conditions these two sources of valuation effects co-move positively or negatively. Finally, another issue is the role of monetary policy in a world where exchange rate fluctuations affect the net external positions of many countries. Do central banks amplify valuation effects or is exchange rate volatility harmless instead? These are all very open questions and beyond the scope of this paper, so we leave them for future research. We hope that this paper can give some directions towards answering these important research questions.

Data availability

Data will be made available on request.

Acknowledgements

This paper is a development of the third chapter of my PhD thesis. I thank my supervisor Subir Chattopadhyay for his advice, Michael Reiter for his very useful discussion and Mauro Bambi, Luis Catao, Andrea Ferrero, Mark Guzman, Dominik Menno, Roberto Pancrazi, Herakles Polemarchakis, Katrin Rabitsch, Neil Rankin, Steve Spear, Gabriel Talmain, seminar participants at Bologna, Durham, Reading, Warwick, the 2016 EEA-ESEM conference, the 24th CEPR European Summer Symposium in International Macroeconomics (ESSIM), the 2016 Royal Economic Society conference, the CEPR conference "Macro-Financial Linkages and Current Account Imbalances" and the 2014 North American Summer Meeting of the Econometric Society for their comments and insights.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.jimonfin.2024.103018.

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