Corporate Investment Decisions with Switch Flexibility, Constraints, and Path-dependency

Abstract

We model sequential, corporate investment decisions with time-to-build delays, operating scale mode switching, operating constraints, and path dependencies. We also account for stochastic salvage (abandonment) values that are utilization (path) dependent. Our results highlight a key link between economic depreciation, stochastic salvage values and operational flexibility with asymmetric switching costs. We further identify conditions uncovering a non-conventional impact of resulting path-dependencies on the investment-uncertainty relationship: higher uncertainty and lower asset return shortfall ("dividend yield") may expedite, rather than delay, corporate investment. High switching costs, operating constraints, and economic depreciation may reduce or eliminate these non-conventional effects.

Keywords: Real options, multistage decisions, operating and production flexibility, stochastic salvage, switching costs.

JEL Classification: D81, G31

1. Introduction

The impact of real options and associated operational flexibility on corporate investment and production decisions occupies a central role in corporate finance decisions (e.g., Trigeorgis and Tsekrekos, 2018; Bengtsson 2001, Seshadri and Subramanyam, 2005, and Zhao and Huchzermeier, 2015, provide reviews at the interface between finance and operations management decisions). The current paper focuses on the effects of switching-mode operational flexibility on firm capital investment and production decisions. We analyze the operating flexibility to switch among several production or operating scale modes when the cost of switching is partially reversible and stochastic. Economic depreciation driven by asset utilization under operating or resource constraints leads to path-dependencies. The paper contributes to the real options literature concerning the impact of operating flexibility on corporate investment and operating policy. It highlights a link between economic depreciation and operational flexibility with important path dependencies that affect the investment-uncertainty relationship in non-conventional ways: higher uncertainty and lower asset ("dividend") payout may expedite -- rather than delay -- investment in a sequential, multi-stage decision setting. Specifically, we analyze sequential (dis)investment decisions involving technology or production capacity choices and path-dependent stochastic switching and abandonment costs. We consider time-to-build delays, operating or resource constraints, and interactions among investment timing and operating decisions in a real options framework (see Dixit and Pindyck, 1993; Trigeorgis, 1996).

Brennan and Schwartz (1985) analyze the value of natural resource investments, showing that the classic Net Present Value (NPV) rule fails under uncertainty and irreversibility. In many sequential investment decision contexts, complete or partial irreversibility of capital is often assumed (e.g., Dixit and Pindyck, 1993). Dixit (1989a, 1989b, 1989c) discusses *hysteresis* characterizing a zone of inaction when decisions are path-dependent and capital is partly reversible. He also discusses optimal capital (dis)investment decisions with limited capital mobility involving costly entry and exit decisions.Kulatilaka (1988), Kulatilaka and Marcus (1988), Triantis and Hodder (1990), and Trigeorgis (1993) analyse various situations involving sequential investment decisions. Mauer and Triantis (1994) provide an extension in a corporate finance context. Related issues addressed in the literature involve capacity choice (Pindyck, 1988), path-dependent problems with time-to-build delays and learning-by-doing (Majd and Pindyck, 1987, 1989), dynamic R&D policies (Childs and Triantis, 1999), sequential investment with time-to-build delays (Bar-Ilan and Strange, 1998), dynamic choice among manufacturing locations under exchange rate fluctuations (Kogut and Kulatilaka, 1994), holding intermediate inventories (Cortazar and Schwartz, 1993), and the choice among mutually exclusive projects of different scale (Dixit, 1993; Dangle, 1999). Some authors investigate interactions between time-to-build delays and capacity choice (Bar-Ilan et al., 2002), interactions of investment and operational flexibility with multiple rare events (Martzoukos and Trigeorgis, 2002), partially-reversible investments (Hartman and Hendrickson, 2002; Kandel and Pearson, 2002), flexible supply contracts (Kamrad and Siddique, 2004), disinvestment flexibility and its effects on NPV (Keswani and Shackleton, 2006), modular decisions (Gamba and Fusari, 2009; Xu et al., 2012) and asset replacement under varying depreciation schedules (Adkins and Paxson, 2013). Chung and Johnson (2011) review and extend complex sequential options with irreversibility involving analytic solutions, extending Geske (1979). Khan and Thomas (2008) further examine lumpy investments involving productivity shocks, offering insights on the influence of adjustment costs on corporate investment. Bloom (2009) examines changes in economic output and investment activity following uncertainty shocks. Huchzermeier and Loch (2001) analyse decisions in an R&D context, Harrison and Sunar (2015) examine learning aspects, Koussis et al. (2013) consider R&D problems with path-dependency, and Luo et al. (2019) use regime switching to examine partial information about the project return. Hult et al. (2010) analyze supply chain decisions, Wang et al. (2012) examine the switching behaviour of managers, Yayla-Kullu et al. (2013) discuss the impact of operating constraints on production decisions, de Treville et al. (2014) consider the role of lead-time reduction under evolutionary demand, and Li et al. (2015) discuss the impact of construction lead times. Kouvelis and Tian (2014) examine flexible capacity investment, Chaturvedi and Martinez-de-Albeniz (2016) discuss interactions between demand uncertainty and capacity choice, and Bensoussan and Chevalier-Roignant (2019) examine optimal product-mix and sequential capacity expansion. Maier at al. (2020) assume mean-reverting cash flows and study a problem with development, operation, mothballing, and state-dependent abandonment.

We focus on costly reversibility and path-dependencies when capital depreciates stochastically according to economic use. In our setting, economic depreciation results in path-dependent (utilization-dependent) stochastic switching costs and salvage values. Such problems have no analytic solutions. We therefore use a general multi-stage numerical approach to study optimal scale, expansion and contraction policies among several alternative operating modes, considering time-to-build delays and operating constraints under limited resources. Each of the above interdependent issues is of interest in itself. Ramey and Shapiro (2001) study aerospace plant closings and the process of shifting physical capital to a new use. They observe that age-related accounting depreciation does not accurately reflect the market value of used or displaced capital. This motivates the need to employ utilization-based economic depreciation, rather than accounting-driven (historical) depreciation practices.

The main message of our paper is that in every complex model of investment, production or disinvestment decisions, there is a trade-off between reversibility (flexibility, optionality) and irreversibility. When irreversibility prevails, we have the traditional results in (American) option pricing where higher volatility and/or lower dividend yield delay exercise decision; when reversibility prevails, higher volatility and/or lower dividend yield bring decisions earlier. These factors should be accounted for in the design stage of a new investment so that flexibility (and its relevant cost) is added in the system, in order to add value and enable optimal investment decisions.

The rest of the paper is organized as follows: Section 2 presents our base model of sequential capital investment with partial reversibility that involves interactions among uncertainty, flexibility and stochastic switching costs under constraints. Section 3 provides two applications of the model, learning-by-doing and tanker fleet choice, with a comparative-static discussion. Section 4 extends the base model to incorporate utilization-dependent economic depreciation, involving stochastic switching costs and abandonment values with path-dependencies. The last section concludes.

2. A Model of Switching Decisions with Partial Reversibility

In this section, we model a general investment problem in a multistage (sequential) decision setting, involving several alternative operational modes with flexibility to switch (among them) at a cost. Option value derives from potential investing to capture cash flows that follow a geometric Brownian motion process. This generic model embeds the essence of several common real options from the literature as special cases: waiting to invest (McDonald and Siegel, 1986), choosing the best among several investment alternatives (strategies, technologies or operating scales) as in Stulz (1982) but implemented in a one-dimensional setting due to efficiency considerations; the flexibility to switch back and forth among the various alternatives (with varying degrees of capital reversibility) as in Dixit (1989a) and Kandel and Pearson (2002) within a finite horizon; sequential compound options as in Geske (1979) and Chung and Johnson (2011); investment with time-to-build delays as in Bar-Ilan and Strange (1998) or with time-to-build delays and capacity choice as in Bar-Ilan et al. (2002); modular investment decisions as in Gamba and Fusari (2009) and Xu et al. (2012). Our approach also captures features addressed in Maier (2020) who study a problem with development, operation, mothballing, and state-dependent abandonment. Our distinguishing feature and contribution is that we focus on the interactions among investment timing and operating scale choices (modes of operation) with the flexibility to costly switch back and forth between these scale choices, while also incorporating time-to-build delays, operating constraints, mothballing, and utilization-dependent (both state- and path-dependent) abandonment and switching costs.

Asymmetric switching costs and partial capital reversibility lead to path-dependencies. Switching costs and salvage (abandonment) values are also path-dependent due to economic depreciation being dependent on capital utilization. Switching decisions take place at finite decision points. Delays in project construction (time-to-build) and operating constraints enhance these path-dependencies. Thus, various aspects of the problem require keeping track of the complete path of these interdependent decision choices.

Valuation of such investment problems involving interdependent switching and other real options depends on stochastic variables typically assumed to follow Ito processes (Black and Scholes, 1973, Merton 1973a, b; McDonald and Siegel, 1986). A review of investment under uncertainty is given in Dixit and Pindyck (1994) and Trigeorgis (1996).[1] We here assume a single stochastic underlying asset S, that represents the discounted present value of expected cash flows per operating period (per stage in our multi-stage setting). Stochastic variable S follows (under risk-neutrality) a geometric Brownian motion (GBM) of the form

$$\frac{dS}{S} = (r - \delta)dt + \sigma dz \tag{1}$$

where r is the riskless interest rate and σ^2 is the variance per unit time. The difference between the investors' required return on the asset (or similar traded assets correlated with S) and its actual growth rate, or return shortfall, is denoted by δ. This represents an opportunity cost of deferring investment in the cash-producing project (see McDonald and Siegel, 1984). This return shortfall is analogous to a dividend yield on a financial asset like a stock, as it benefits the asset holder but

[1] Standard assumptions include continuous-time capital asset pricing (Merton, 1973b, Breeden, 1979), absence of market imperfections (taxes etc.), market completeness (spanning), and an all-equity firm facing proprietary investments.

not the holder of a call option on the asset. Brennan and Schwartz (1985) and Brennan (1991) use a convenience-yield variant of the opportunity cost δ in the context of commodity investments.

Our switching-option model is cast in a multi-stage setting that allows several alternative operating actions like expansion, contraction, and abandonment at an optimal time. Switching from production or operating mode *i* to mode *j* involves incurring a switching cost $I^{i\to j}$ in exchange for receiving value V^j. A superset of admissible action paths (*M)* specifies what decisions are admissible, given any sequence of decisions. At each point in time, M_t^- represents a subset of *M* that includes the history of investments up to time *t*. Given M_t^- and the switching mode decision taken at *t*, m_t, subset M_t^+ denotes the remaining (future) admissible decisions. V^{m_t} specifies the payoff under operating mode m_t.

The firm's objective is to determine the optimal investment value V^* over the set M_t^+ of future admissible investment choices. This includes staying in the same mode *i* or switching to any other feasible alternative mode *j* (including abandonment for salvage value A^i):

$$V^*(S_t, t|M, M_t^+, M_t^-) = \max_{M_t^+}\{V^{m_t}\} \tag{2}$$

Future action subset M_t^+ includes: a) switching from operating mode *i* to any other future mode *j*, b) staying in the same mode *i*, c) becoming idle (not producing temporarily) or d) abandoning capital for the stochastic salvage value A^i. The above is described with the following set of equations:

$$\begin{gathered}
V^{j_1}(S_t, t|M, M_t^+, M_t^-) - I^{i\to j_1}(S_t, t|M, M_t^+, M_t^-) \\
V^{j_2}(S_t, t|M, M_t^+, M_t^-) - I^{i\to j_2}(S_t, t|M, M_t^+, M_t^-) \\
\dots \\
V^{i}(S_t, t|M, M_t^+, M_t^-) - I^{i\to i}(S_t, t|M, M_t^+, M_t^-) \\
V^i_{idle}\left(S_t, t\,|\,M, M_t^+, M_t^-\right) - I^{i\to i_{idle}}\left(S_t, t\,|\,M, M_t^+, M_t^-\right) \\
A^i(S_t, t|M, M_{t-\Delta t}^-, m_{t-\Delta t} = i, S_{t-\Delta t}, S_{t-2\Delta t}, \dots).
\end{gathered} \tag{3}$$

If a state of operating inaction is reached (i.e., a state of idleness, temporary shutdown or mothballing), the cash flows in that mode are determined by the preceding operating mode *i*. The overall process begins at an initial mode "wait to invest" (*W*), which can be maintained (option to wait to invest). The investment opportunity value at each operating mode *j* is a function of the cash flows obtained in that mode plus the discounted expected value of the claim at the next date:

$$\begin{aligned}
&V^j(S_t, t|M, M_t^+, M_t^-) \\
&= R_t^j(S_t, t) - X_t^j(S_t, t) + e^{-r\Delta t}E_t[V^*(S_{t+\Delta t}, t + \Delta t|S_t, M, M_{t+\Delta t}^+, M_{t+\Delta t}^-)]
\end{aligned} \tag{4}$$

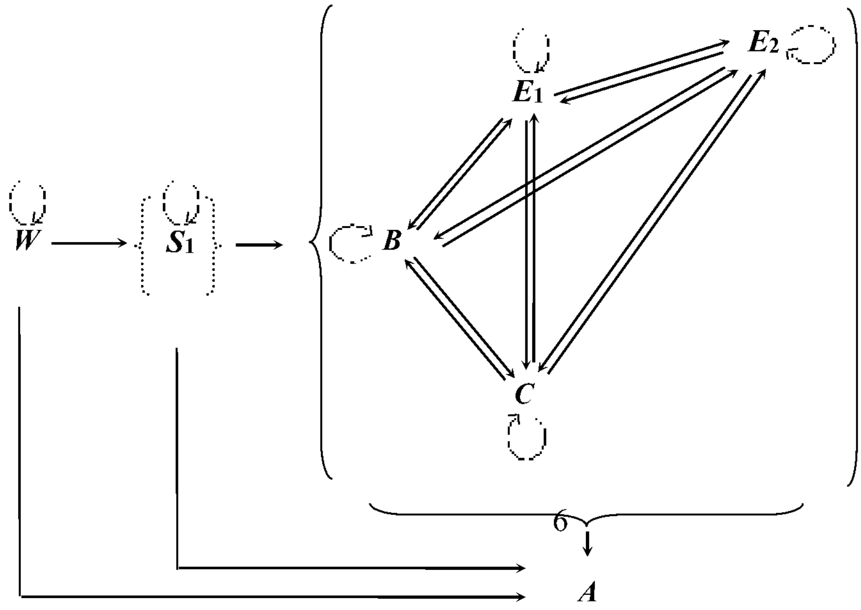
Operating revenues *R* minus operating costs *X* determine the value of net cash flows until the next decision revision. *R* is a function of state-variable *S*, thus allowing different technologies or operating scales to depend on *S*. At the end of the operating project's life (T), the last term (involving discounted expected value) vanishes. At the boundary (critical threshold of project value S) separating the regions where two alternative decisions, i and j, are optimal, the value matching condition gives $V^i = V^j - I^{i \to j}$.

In many multi-stage capital investment problems, operations cannot start until an initial (e.g., construction or infrastructure) stage S_1 is completed, and this initial stage takes time-to-build (e.g., Majd and Pindyck, 1987). In such cases, the attainable set of decisions M may differ before vs. after completion of the initial stage S_1. Prior to the completion of the project construction, the set of available decision choices M includes only: waiting to invest W, completing construction stage S_1, and potentially abandoning for a salvage or resale value (out of waiting mode W) A^W. Following completion of the initial (construction) stage, M includes all the available investment modes described previously. The available switching decisions and admissible choices are summarized in Figure 1. The base configuration involves four alternative operating modes or technologies (C, B, E_1, and E_2), differing in the degree of installed capacity. These are: Contracted (small) scale C, Base-scale B, Expanded scale E_1, and Very Expanded (large) scale E_2.

Operating constraints are used to account for the exhaustibility of corporate resources. The project's economic life is a function of the active operation of the project and utilization of installed capital. If the firm enters an idle (mothballing) phase after initial capital investment, exploitable resources are not depleted during the period that operations are off. The extent to which an operation stays in an active mode is part of the admissible action set M. In the optimisation process, M_t^- and m_t keep track of past (interacting and path-dependent) operations, while M_t^+ includes the set of forward actions still admissible. Stochastic switching costs, I, and abandonment (salvage) values, A, are also utilization-dependent (path-dependent). M_t^- and m_t (similar to operating or resource constraints) keep track of past operations. M_t^+ specifies future admissible actions, switching costs, I, and abandonment values, A, as a function of the actual use of installed capital.

Figure 1

Configuration of corporate investment decisions with switching flexibility among four alternative operating modes



Note: The process starts from the wait-to-invest mode (W). This can be followed by a staged investment (S_1). Subsequently, management can choose among alternative production scale modes: Contracted (small scale) C, Base-scale B, Expanded scale E_1, and very expanded scale E_2. Once in a mode, one can stay in this mode by keeping operations active or idle (mothballing). In mode W, one can stay for as long as it is optimal in order to make the first investment decision. The feasible decision set additionally includes the option to abandon (A) from any mode (abandonment is an absorbing state).

A switching-cost matrix is used to specify all switching costs in a logically (economically) consistent manner. For example, $I^{1\to 2}$, $I^{2\to 3}$ and $I^{1\to 3}$ specify the costs of switching from the first mode to the second, from the second to the third, and from the first mode directly to the third (in ascending order of scale or productive capacity). For coherent economic meaning, it is useful to compare $I^{1\to 3}$ with $I^{1\to 2} + I^{2\to 3}$. For example, when $I^{1\to 3} > I^{1\to 2} + I^{2\to 3}$, cost efficiencies can be achieved in sequential investing due to learning-by-doing. When $I^{1\to 3} < I^{1\to 2} + I^{2\to 3}$, scale efficiencies may be attained. Thus, a meaningful economic definition of switching costs across admissible paths is required, including mothballing states (later denoted by N, see fig. 3a and 3b).

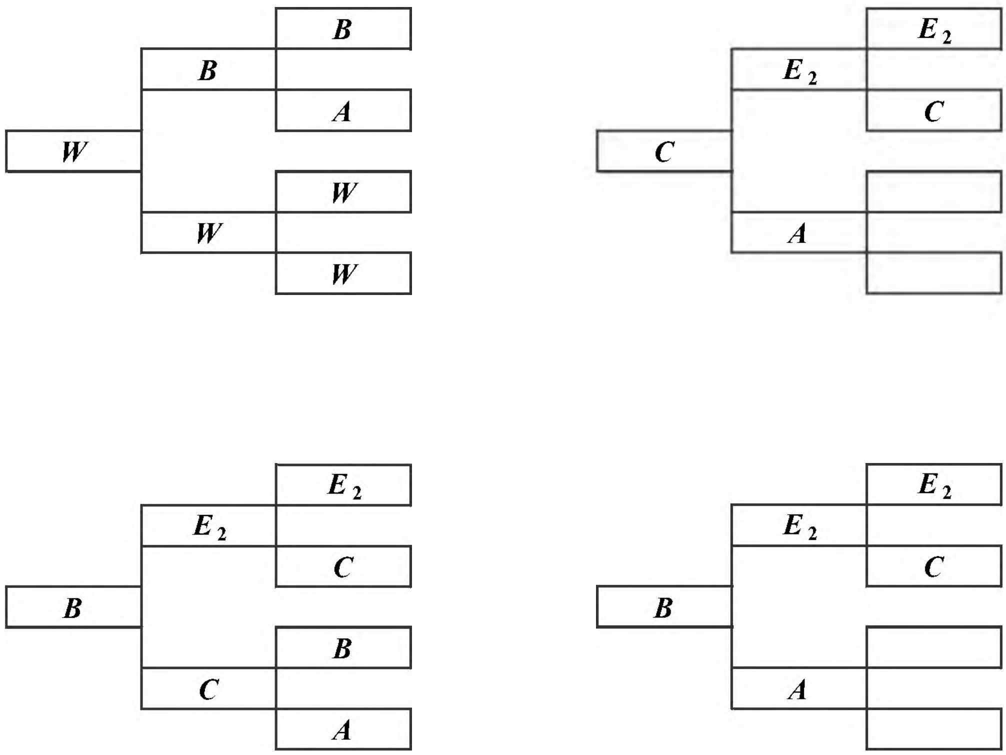
Numerical solutions for real option problems involving path-dependency based on partial differential equations (PDE) have been employed in Mauer and Triantis (1994) and Majd and Pindyck (1987 and 1989). Financial (Asian) options have been solved numerically since Ingersoll (1987) and Alziary et al. (1997). A simulation-based regression method has been recommended in Longstaff and Schwartz (2001) and a variant has been implemented in Maier et al. (2020). These authors show that each decision point involving path-dependency adds considerably to the dimensionality of the numerical solution. The complexity of the problem we consider here and the need for accommodating a considerable number of intermediate decision points makes the use of such methods rather impractical. We therefore follow the recommendation of exhaustive search in Trigeorgis (1996) (see also real R&D options applications in Koussis et al., 2013). We introduce a discretized lattice-based numerical scheme approximating the continuous state-space that allows decisions to be made at limited discrete points in time. We solve this discrete-time multi-stage optimization problem through a forward-backward looking algorithm of exhaustive search, which accounts for the path-dependencies, optimal timing and early exercise features of the problem. Path-dependency is accounted for at the specified decision points. In-between these decision points, a dynamic programming approach is implemented with a binomial tree lattice that evolves with an arbitrary number of time steps (see Cox and Rubinstein, 1985), in order to improve numerical accuracy. We thus need to consider only the lattice points at the discrete decision times. The relevant option values for every feasible path of past decisions are calculated at each such decision point, via exhaustive search. This effectively creates auxiliary variables that keep track of all decisions and paths. Although this increases memory requirements, the computational burden remains comfortably within the reach of a personal computer. The algorithm used for the forward-backward exhaustive search is described below.

In the forward run, at each decision time (starting at time 0 and proceeding toward the option maturity T), for each lattice point (spanning from low to high values of the state-variable *S*), each admissible decision is considered. By the time we reach option maturity T, the payoffs of all decision combinations have been considered (including operating revenues, fixed and operating costs). Auxiliary variables created keep track of the path of past decisions for each feasible path. At each decision time for each lattice node (state), many paths of admissible past decisions are considered and saved. An exponential increase in the number of these paths gives rise to the computational intensity of the exhaustive search solution.

When proceeding backwards, option values are calculated (starting from option maturity T and stepping earlier towards time 0) at each lattice node for each past decision path. Given the previous decision path, an optimal decision is determined (providing the optimal option value going forward). Option values are specified as functions of the received cash flows net of incurred switching costs (given a decision and the previous path) plus the expected continuation (option) value estimated as a (probability) weighted average of the optimal forward values for lattice points at the next decision times. At time 0, the optimal value and the optimal investment decision are determined. A similar grid search, above and below the starting lattice point, provides the critical thresholds for alternative operating decisions.

Figure 2

Illustration of optimal strategies with four flexible operating modes



Note: The figure shows four simplified (two-stage) illustrations of possible investment decisions with hysteresis for different paths of the state-variable S on a binomial lattice. At the end of the lattice, the underlying takes only three values (the lattice reconnects), but for the middle value there can be different investment decisions, due to path-dependency induced by switching costs and partial reversibility. The process begins initially from the wait-to-invest mode (W). Then management can choose amongst differing scale alternative operating modes $\{C, B, E_1, E_2\}$. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. The decision set also includes the option to abandon (A) from any mode.

First, a simple example -- without time-to-build delays or an idle mode (mothballing) -- is illustrated with the optimal decisions. To keep things simple, only two decision stages (before maturity T) are considered (for now). The number of decision stages is increased in the next section. Figure 2 illustrates the path-dependent optimal decision sequence in this simplified two-stage case. The lattice would normally recombine with respect to the asset value S but the evolution (path) of S drives the optimal investment and disinvestment (or abandonment) decisions; due to the path-dependency of these investment decisions, the lattice in effect *is not* recombining. The optimal investment decisions (and option values) thus differ depending on the realized path (hysteresis) of S and on the history of past decisions. These are sensitive to the level of the underlying asset value S and to the value of some key parameters, including uncertainty.

Let us provide a review of the solution methodology. We first create a figure like fig. 1 that defines available actions for each problem; then we create on a lattice going forward all possible paths of the stochastic state variable and all possible decisions (keeping this path-dependency of actions into account) which is a very memory-intensive process; and then we solve backwards by identifying at each point in time and each value of the state variable the optimal decisions given prior actions on that path. This exhaustive search approach provides optimal decisions and value at each point in time and for each value of the state variable given the previous path of the state variable and previous actions; finally, we get the optimal decision and value at time zero. Our fig. 2 provides a simplified example with a few available actions only (wait, operate in one of four possible operating modes, and possibly abandon after operation has started). In that figure, at time zero and depending on the value of the state variable, optimal actions like wait (for low values of the state variable) or invest either in base-scale or in contracted (small) scale (for higher values of the state variable) are activated. Expanded scale operations are not optimal to start with, unless the state variable reaches an unrealistic and very high level (and thus it is not shown at time zero in that simplified example).

In the next section, this framework is extended to accommodate five decision points in the context of two specific applications, studying the impact of stochastic switching costs driven by learning-by-doing and/or utilization-dependent economic depreciation; as well as operating or resource constraints and lead construction times.

3. Applications: Learning-by-Doing and Market Niche in Shipping

The first application involves implementation of the generic problem of Figure 1 in the context of *learning-by-doing* as a result of economic and/or technological efficiencies associated with sequential (phased) project development. Accumulated experience may enhance revenues or (as in our case) reduce construction costs (see, for example, the case discussed in the two-period problem in Martzoukos and Zacharias, 2013, and references therein). In our context the accumulated capital outlay of installing the highest capacity (E_2) is lower when built in a sequential fashion than when it is built all at once. The added value in such sequential investment due to learning-by-doing represents a situation with a high degree of embedded optionality. By also incorporating operating constraints due to limited depletable resources, the degree of unconstrained optionality affects switching option values and sequential investment decisions. The cost parameters used are representative of this application context.

The second application (also a variant of the general configuration of Figure 1) is cast in the shipping context: it involves optimal operation of a tanker with a mothballing option, as described in Dixit and Pindyck (1994, pp. 237-242), but extended to four tanker technologies. A tanker's flexibility to switch back and forth is reduced due to considerable switching costs associated with lower abandonment (resale) values. The overall setup is useful for analyzing not only the decisions of an individual tanker operator who has the option to switch among freight technologies, but also those of a shipping manufacturer who considers *offering* a new tanker technology (taking into account other competing technologies). By looking at the problem from the buyer´s (tanker operator's) perspective, the manufacturer can better assess the extent to which the new technology will be adopted by tanker operators, given the prevailing market conditions and demand uncertainty. The tanker manufacturer can also consider the extent and circumstances under which the new tanker technology might become attractive in the future. The manufacturer would be interested to know if a market niche can be captured and whether (and indeed when) to invest in the development of a new tanker technology.

Figure 3a

Case 1: Capital costs and net operating revenues for learning-by-doing case involving four alternative operating modes (with and w/o Time-to-Build)

from \ *to*	CAPITAL COST											
	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
	Initial Capital Cost											
W	-	10	-	12	20	65	170	-	-	-	-	0.0
S_1	-	-	2	2	10	55	160	-	-	-	-	-2.5
N_{S1}	-	-	-	5	13	58	163	-	-	-	-	-2.5
	Switching Capital Cost											
C	-	-	-	-	5	35	85	2	-	-	-	-3.0
B	-	-	-	-5	-	30	75	-	2	-	-	-5.0
E_1	-	-	-	-35	-30	-	40	-	-	2	-	-16.25
E_2	-	-	-	-85	-75	-40	-	-	-	-	2	-42.5
N_C	-	-	-	3	8	38	88	-	-	-	-	-3.0
N_B	-	-	-	-2	3	33	78	-	-	-	-	-5.0
N_{E1}	-	-	-	-32	-27	3	43	-	-	-	-	-16.3
N_{E2}	-	-	-	-82	-72	-37	3	-	-	-	-	-42.5

	NET OPERATING REVENUES											
	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
Benchmark - B	*Expansion factors for the Operating Revenues (S)*											
	-	-	-	**-50.0%**	**0.0%**	**50.0%**	**100.0%**	-	-	-	-	-
	0.000	-	-	0.607	1.000	1.649	2.718	-	-	-	-	-
	Expansion factors for the Operating Costs (X)											
	-	-	-	**-50.0%**	**0.0%**	**50.0%**	**100.0%**	-	-	-	-	-
	0.000	-	-	0.607	1.000	1.649	2.718	0.050	0.050	0.050	0.050	-

Figure 3b

Case 2: Capital costs and net operating revenues for the shipping oil tankers application (with and w/o Time-to-Build)

from \ *to*	CAPITAL COST											
	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
	Initial Capital Cost											
W	-	70.000	-	115.500	140.000	297.500	595.000	-	-	-	-	0.000
S_1	-	-	-	45.500	70.000	227.500	525.000	-	-	-	-	-17.500
N_{S1}	-	-	-	45.500	70.000	227.500	525.000	-	-	-	-	-17.500
	Switching Capital Cost											
C	-	-	-	-	111.125	268.625	566.125	0.082	-	-	-	-28.875
B	-	-	-	80.500	-	262.500	560.000	-	0.200	-	-	-35.000
E_1	-	-	-	41.125	65.625	-	520.625	-	-	0.329	-	-74.375
E_2	-	-	-	-33.250	-8.750	148.750	-	-	-	-	0.635	-148.750
N_C	-	-	-	0.325	111.125	268.625	566.125	-	-	-	-	-28.875
N_B	-	-	-	80.500	0.790	262.500	560.000	-	-	-	-	-35.000
N_{E1}	-	-	-	41.125	65.625	1.301	520.625	-	-	-	-	-74.375
N_{E2}	-	-	-	-33.250	-8.750	148.750	2.509	-	-	-	-	-148.750

	NET OPERATING REVENUES											
	W	S_1	N_{S1}	C	B	E_1	E_2	N_C	N_B	N_{E1}	N_{E2}	A
Benchmark - B	*Expansion factors for the Operating Revenues (S)*											
	-	-	-	**-88.7%**	**0.0%**	**49.9%**	**115.6%**	-	-	-	-	-
	0.000	-	-	0.412	1.000	1.647	3.176	-	-	-	-	-
	Expansion factors for the Operating Costs (X)											
	-	-	-	**-88.7%**	**0.0%**	**49.9%**	**115.6%**	-	-	-	-	-
	0.000	-	-	0.412	1.000	1.647	3.176	0.234	0.234	0.234	0.234	-

Note: Operating modes $\{C, B, E_1, E_2\}$ are denoted as in Figure 1. State S_1 refers to an initial construction phase under the assumption of needed time-to-build. Mothballing states are denoted by N. The matrices show capital cost to switch from a mode to another and net operating revenues in each mode.

Figure 3a shows the level of net revenues as a function of the underlying asset (state-variable) S and the initial and switching capital costs in the first application, while Figure 3b illustrates the relevant variables for the second application. In both cases, when the firm operates in a given mode (or technology), it receives net cash flows of $R(S) - X$. An initial capital investment cost I is incurred, depending on the choice of operating mode B (base case), C (small/contracted scale), E_1 (expanded scale) and E_2 (very expanded scale). This cost differs if the previous mode is W (wait to invest) or the firm is already operating in any of the modes B, C, E_1, and E_2. If operation comes to an idle mode, a maintenance cost N (with subscript specific to the exiting mode) is incurred. When there is a delay associated with time-to-build, operations (from wait mode W) must first enter the initial stage (construction or infrastructure) S_1. An option to abandon provides an alternative use (or salvage) value A. This is treated as a reversal (or negative cost) since part of the initial capital is recovered. The expansion factors for the operating revenues R and operating costs X are calculated for the contracted case C (relative to the base case B) in Figure 3a from exp(-0.50) = 0.607 and in Figure 3b from exp(-0.887) = 0.412.

Here the simpler case is considered where I and A are constant. In the next section, these are utilization-dependent and stochastic. In the first application, the net revenue function in operating mode j (underlying asset is S_t) at time t is

$$R_t^j(S_t, t) - X_t^j(S_t, t) = f_S^j S_t - f_X^j X \tag{5a}$$

where f are expansion factors that depend on operating mode j (for simplicity here both equal f^j). In the second application, for comparability with Dixit and Pindyck (1994), we use as state variable price P (per ton), with $S_t = S_t(P_t)$ being a linear function of price.

Finally, given the complexity of our model, we refer to Figure 3c that shows determinants of irreversibility (when the threshold to invest is high) drawing on the input from Figures 3a and 3b. In general, high (fixed or operating) costs, low abandonment and low operating costs enhance irreversibility. We also investigate the impact of operating constraints and utilization-dependent economic depreciation. Our results demonstrate that volatility and return shortfall (dividend yield) do not always yield results consistent with traditional options literature.

Figure 3c.

Determinants of irreversibility

Costs (or revenues) that enhance irreversibility:	
	High initial capital investment to enter a mode of operation (C, B, W_1, W_2)

	High switching costs to change from one operating mode to another
	High capital cost (S_1) when there is time-to-build (incurred before initial capital investment costs)
	High mothballing costs to stay inactive in the operating or construction mode (NC, NB, NE_1, NE_2, N_{S1}) or get out of mothballing into an operating mode (NC, NB, NE_1, NE_2)
	Low fixed abandonment value (A)
	Low expansion factor for operating revenues
	High expansion factor for operating costs
Other determinants of irreversibility:	
	Operating constraints
	High economic (utilization dependent) depreciation
Determinants that may or may not enhance irreversibility:	
	High volatility
	High return shortfall (dividend yield)

Figures 4 and 5 show how investment option value and optimal investment policy (the critical thresholds of S whereby decisions change/switch) vary with asset value S, standard deviation σ and the rate of return shortfall ("dividend yield") δ in each of the two applications. Here we use six decision stages (five before maturity and one at maturity) and five lattice steps between decisions for increased accuracy and efficiency. This numerical choice affords a reasonable trade-off between accuracy and efficiency.[2] For the first application involving learning-by-doing, the following base-case parameter values are used: $\sigma = 0.20$ per year, $\delta = 0.10$, $r = 0.05$, total time to maturity $T = 5$ years, investment outlay (or operating cost) $X = 100$. The results in Figures 4a and 4b in terms of the impact of asset uncertainty (σ) and asset payout or opportunity cost (δ) on value (V) are consistent with standard option literature; an increase in uncertainty (σ) and a decrease in the opportunity cost (δ) increase value (see also Bar-Ilan et al., 2002, and Brekke and Schieldrop, 2000). However, note here that flexibility value, generated by switching decision choices, increases the net value of the investment opportunity and may increase the propensity to make early investment. As a result of the involved path-dependencies, in cases that involve significant staging flexibility (thus reduced irreversibility) higher uncertainty tends to expedite - rather than delay - investment, in contrast to the standard real option literature. For standard literature on the sign of the investment-uncertainty relationship, see Caballero (1991), Cortazar and Schwartz (1993), etc. The reversal noted herein may be partly due to the possibility that early investment opens up future switching options whose value also increases with uncertainty. The

[2] Using ten time steps instead of five between decisions improves investment option value accuracy insignificantly only. In general, denser grids are relatively more important for out-of-the-money options.

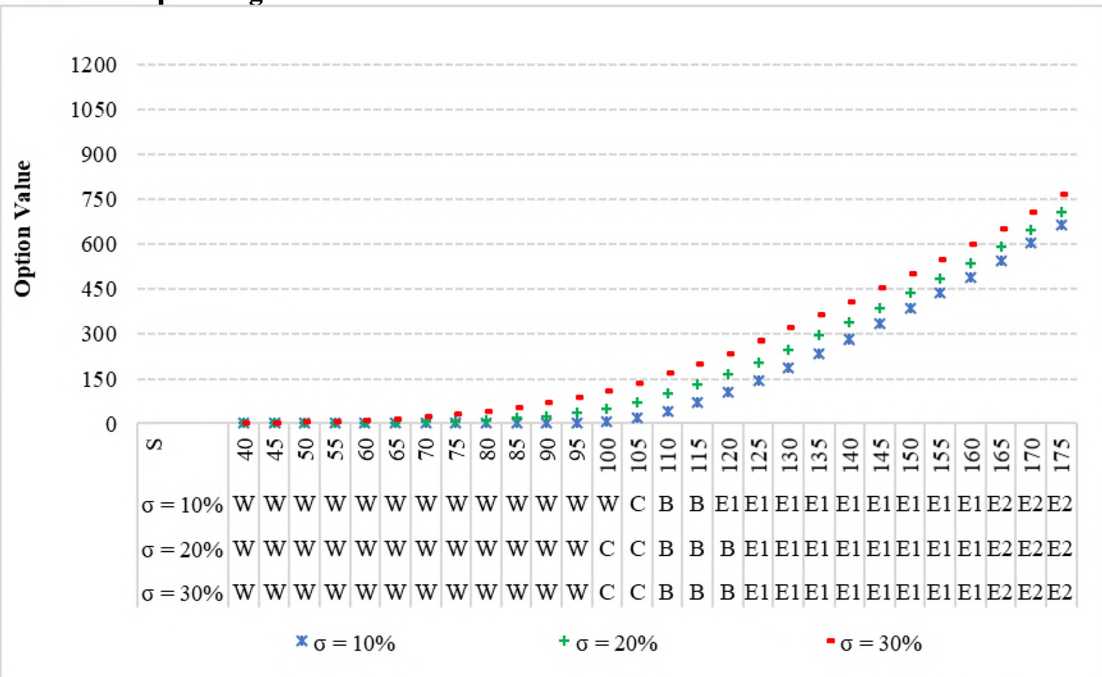
decision to invest early often occurs in a lower mode of operation (e.g., at a reduced capacity level or with less expensive technology). This accords with common real options intuition to start small and scale up (expand) later on. Due to the sequential nature of the investment decision process, an increase in the uncertainty (of asset S) increases the option value of the investment opportunity (V) that embeds the value of future switching opportunities (like a complex compound option). When yearly revenues S increase, the firm may scale up and when they decrease it may contract or abandon, especially when switching costs are not high, as in this first case study. The results differ in the next shipping case study where switching costs are high and the effect of volatility (and dividend yield) are more conventional. In other models like Abel et al. (1996), the non-conventional effects of volatility are mainly attributed to the abandonment option (see also discussions in Kandel and Pearson, 2002).

Analogous non-conventional results justifying investing earlier are also observed with a lower (or zero) return shortfall or "dividend yield" (δ). A zero δ would never justify early exercise of a standard American call option (as the alternative of waiting and investing the exercise cost would yield the higher risk-free rate $r > 0$). The non-conventional δ effect observed here can be partly attributed to the sequential nature of the investment and the growth rate in the value of the cash flows. A lower (or negative) shortfall δ implies a high effective growth rate (capital gains) for the value of cash flows. Thus, deferring the investment penalizes investment value considerably due to the higher lost revenues. The above "anomalous" impact of volatility and return shortfall (or dividend yield) δ is also present when there is time-to-build delay (e.g., see Figure 4c). With time-to-build delay, production and cash flow generation can only start after the initial build-up stage is completed (in our setup, one decision stage later), which effectively places a constraint on operations and reduces the investment opportunity's value and may further expedite investment (see also discussions in Bar-Ilan and Strange, 1998).

Figures 4d and 4e report results incorporating explicit operating or resource constraints. Such constraints may arise from a fixed and expiring economic life or from the exploitation of a fixed amount of exhaustible natural resources. The more constrained the operations, the lower the investment option value and the longer the investment delay (with preference for potentially investing later at possibly higher capacity levels). With operating constraints, the flexibility to switch among alternative modes is reduced. Moreover, higher uncertainty σ and lower return shortfall δ tend to delay investment (now expected to occur at a higher threshold level). Effectively, the presence of operating or resource constraints limits the value of future growth opportunities arising from future switching, enhances the irreversible aspects of the investment, and delays investment decisions. The above non-conventional effects concerning the sensitivity of an American-type investment option to uncertainty and the return shortfall, depend on the trade-off between key factors that increase or decrease the value of flexibility: The higher the

Figure 4a

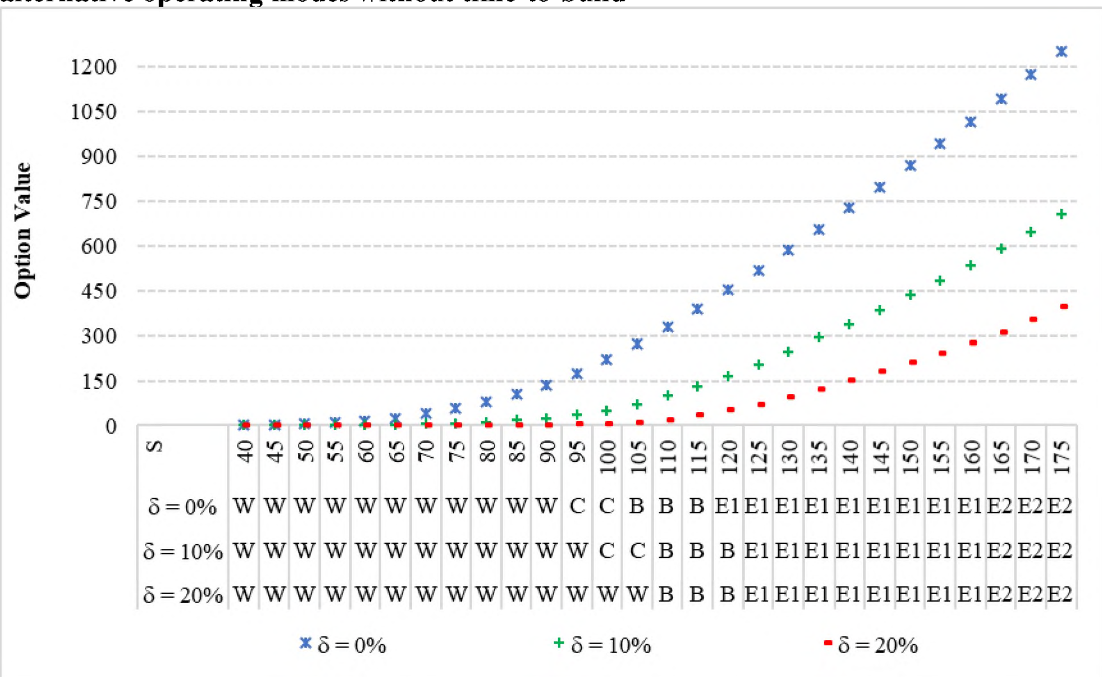
Sensitivity analysis of option value and optimal operating policy vs. volatility σ with four alternative operating modes without time-to-build



Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model without time-to-build. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. Operating cost X = 100, S are base-scale operating revenues, riskless rate r = 5%, Δt = 1 per period.

Figure 4b

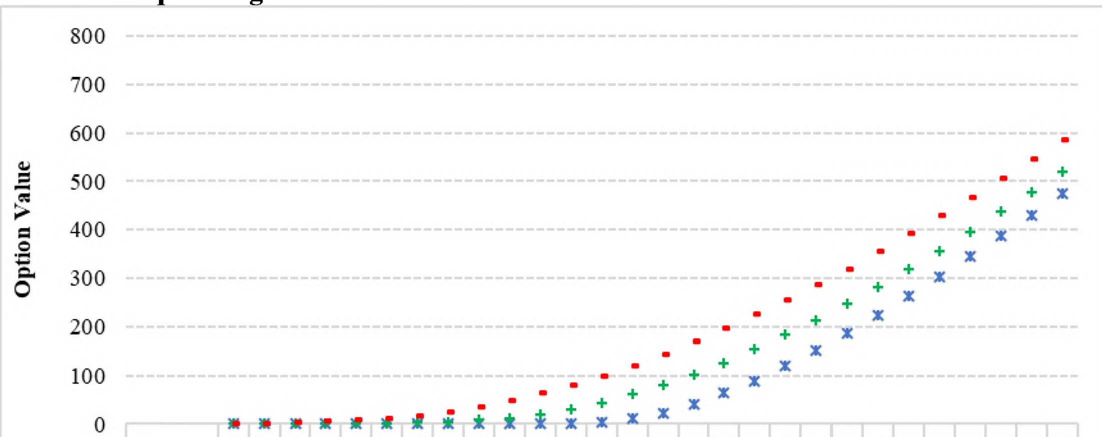
Sensitivity analysis of option value and optimal operating policy vs. payout yield with four alternative operating modes without time-to-build



Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model without time-to-build. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. Operating cost X = 100, S are base-scale operating revenues, riskless rate r = 5%, Δt = 1 per period.

Figure 4c

Sensitivity analysis of option value and optimal operating policy vs. volatility σ with four alternative operating modes with time-to-build

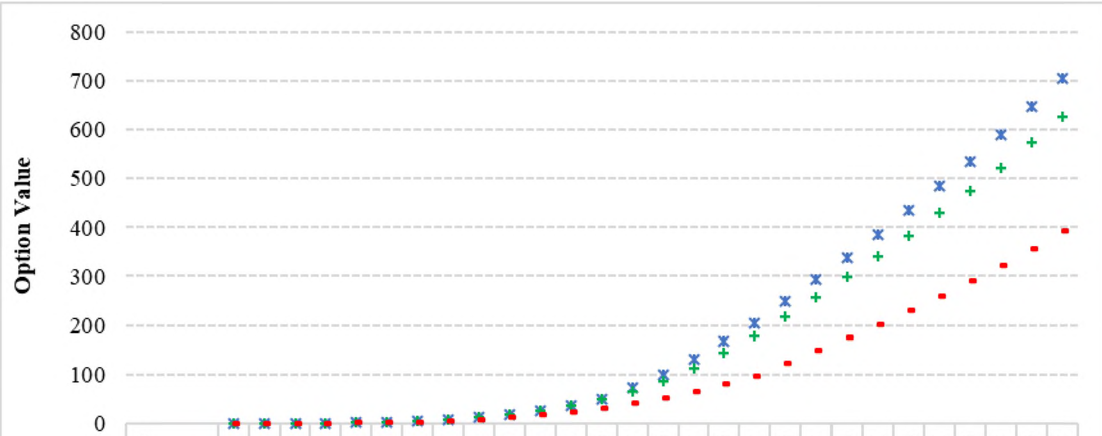


S	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175
σ = 10%	W	W	W	W	W	W	W	W	W	W	W	W	W	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1
σ = 20%	W	W	W	W	W	W	W	W	W	W	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1
σ = 30%	W	W	W	W	W	W	W	W	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1	S1

Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model with time-to-build. S_1 refers to an initial construction phase under the assumption of needed time-to-build. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. Operating cost $X = 100$, S are base-scale operating revenues, riskless rate $r = 5\%$, $\Delta t = 1$ per period. Exceeding the threshold we start construction which takes time-to-build.

Figure 4d

Impact of operating constraints on option value and optimal operating policy for four alternative modes without time-to-build

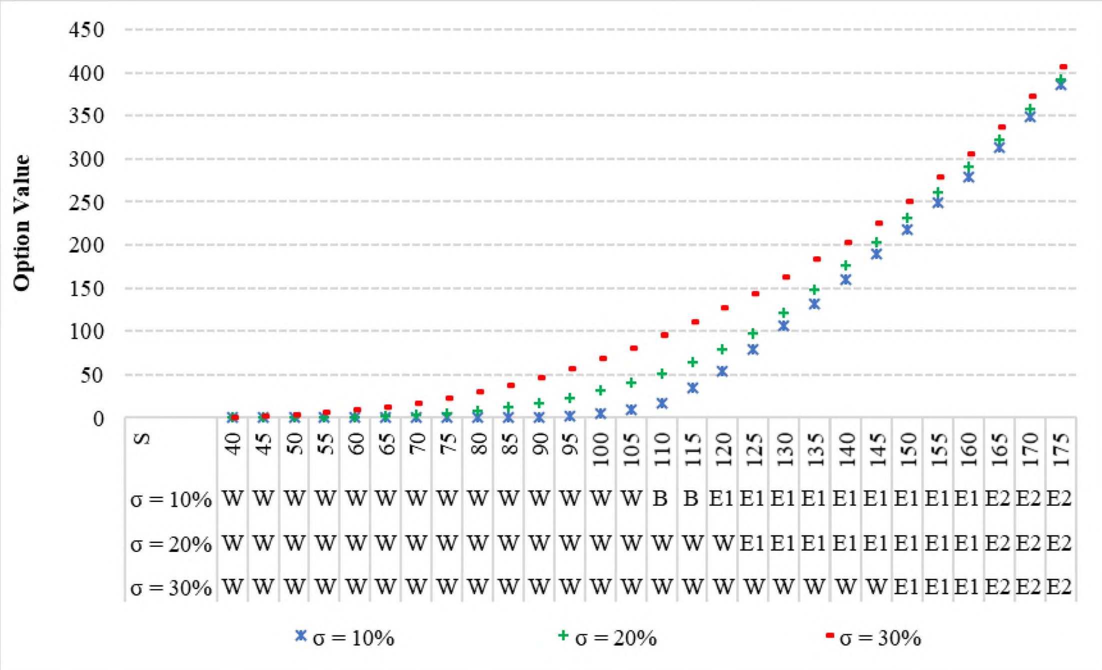


S	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175
no const	W	W	W	W	W	W	W	W	W	W	W	W	C	C	B	B	B	E1	E1	E1	E1	E1	E1	E1	E1	E2	E2	E2
max = 5	W	W	W	W	W	W	W	W	W	W	W	W	W	W	B	B	B	E1	E1	E1	E1	E1	E1	E1	E1	E2	E2	E2
max = 3	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	E1	E1	E1	E1	E1	E1	E1	E1	E2	E2	E2

Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model without time-to-build but with constraints on the maximum number of operations till option maturity. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales. Operating cost $X = 100$, S are base-scale operating revenues, riskless rate $r = 5\%$, $\Delta t = 1$ per period.

Figure 4e

Sensitivity of option value and optimal operating policy vs. volatility σ for four alternative operating modes without time-to-build but with an operating constraint



S	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140	145	150	155	160	165	170	175
σ = 10%	W	W	W	W	W	W	W	W	W	W	W	W	W	W	B	B	E1	E1	E1	E1	E1	E1	E1	E1	E1	E2	E2	E2
σ = 20%	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	E1	E1	E1	E1	E1	E1	E1	E1	E2	E2	E2
σ = 30%	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	W	E1	E1	E1	E2	E2	E2

Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model without time-to-build but with a constraint of maximum times of operation= 3. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales (having B as the benchmark). Operating cost X = 100, S are base-scale operating revenues, riskless rate r = 5%, Δt = 1 per period.

degree of embedded switching optionality (e.g., when switching costs are not high or there are no constraints on operation), the more likely these non-conventional "reverse" effects become. The opposite occurs when flexibility is restricted, e.g., due to high switching costs or added constraints on operations (like exhaustible natural resources or contractual limitations). The use of a switching cost matrix (Figures 3a and 3b) allows effectively for differing impact on irreversibility.

In the second application in the shipping industry, the following base-case parameter values are adopted: $\sigma = 0.15$ per year, $\delta = 0$, $r = 0.05$, total time to maturity $T = 10$ years, and $X = 8.8$. Here, flexibility to switch is a-priori reduced because of relatively high switching costs. These high costs are due to a low resale (or scrap) value of each tanker type. The net revenues here are given by (for each two-year period)

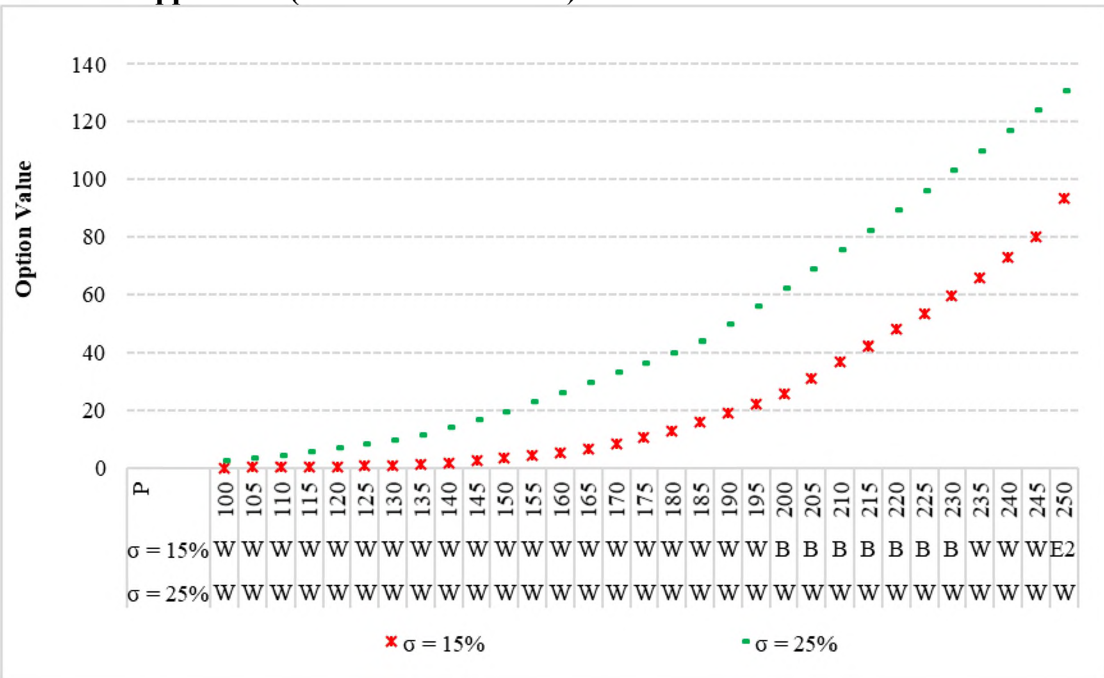
$$f_S^j S_t(P_t) - f_X^j X = \frac{2 f_S^j 85000 P_t}{1000000} - f_X^j 8.8 \text{ million USD} \tag{5b}$$

assuming a capacity of 85000 deadweight tons for the base case (prices are in USD per ton). The data inputs used in Dixit and Pindyck (1994, pp. 237-241) are adopted here, also retaining the same constant of proportionality for technologies other than the base case.

Figures 5a – 5b show analogous numerical results as those reported earlier, except that the relatively high switching costs used here reduce the value of flexibility and bring the sensitivity of optimal policy to volatility σ and return shortfall δ closer to the standard American call option case. Time-to-build delay, similarly with the first application, reduces option value. Results further confirm

Figure 5a

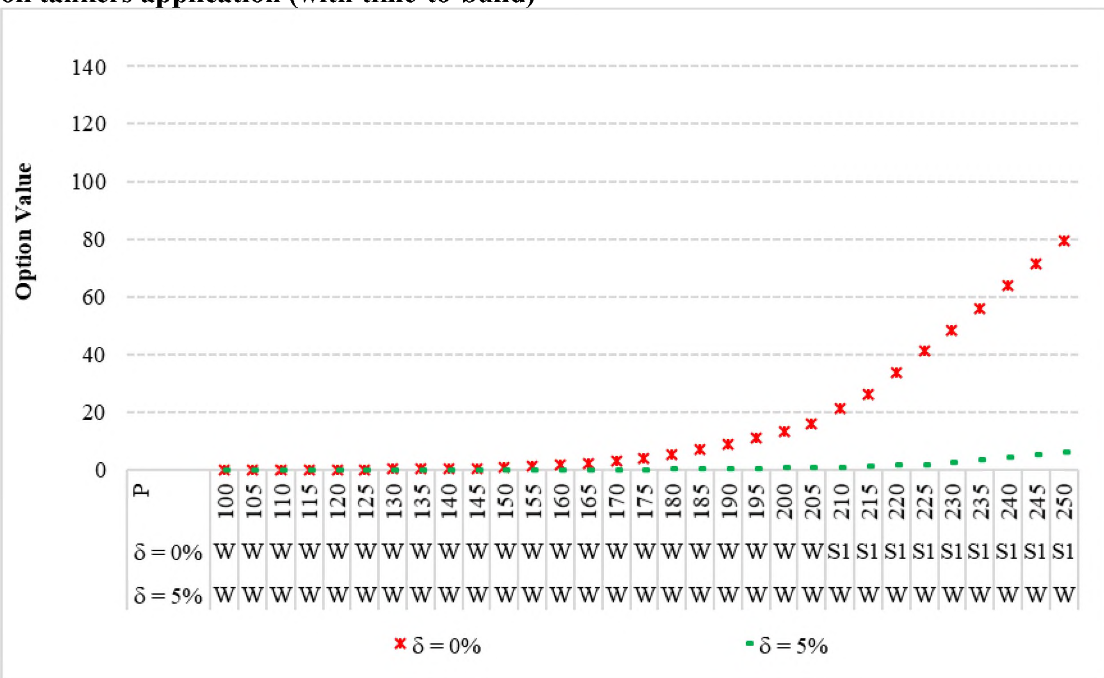
Sensitivity of option value and optimal operating policy vs. volatility σ for the shipping oil tankers application (without time-to-build)



Shipping oil tankers for a (5+1)-stage model without time-to-build. C denotes the small tanker, B the medium tanker, E_1 the large tanker, and E_2 the very large carrier. Operating cost X = 8.8, riskless rate r = 5%, Δt = 2 per period. For the higher volatility case, threshold is at P = 270 and optimal decision is E_2.

Figure 5b

Sensitivity of option value and optimal operating policy vs. payout yield δ for the shipping oil tankers application (with time-to-build)



Shipping oil tankers for a (5+1)-stage model with time-to-build. S_1 refers to an initial construction phase under the assumption of needed time-to-build. C denotes the small tanker, B the medium tanker, E_1 the large tanker, and E_2

the very large carrier. Operating cost X = 8.8, riskless rate r = 5%, Δt = 2 per period. For the higher dividend yield case, threshold is at P = 261. Exceeding the threshold we start construction that takes time-to-build.

that technologies B and E_2 are dominant. For the given investment setup configuration, the other two alternative modes (technologies C and E_1) are not economically attractive. This result holds when the value of the investment (as a function of S) increases reasonably outside the present range. This also illustrates the usefulness of the above analysis in determining whether there is a market niche for a new tanker technology. If these new technologies are analyzed as isolated investment options, their value might appear to be more significant. Such a naive analysis, however, can be very misleading, due to prevailing option interactions (e.g., see Trigeorgis, 1993). All technologies (C, B, E_1 and E_2) should be analysed in combination as part of a network or portfolio of investment options and not in isolation.

4. Utilization-dependent (Stochastic) Salvage and Switching Costs

We have assumed thus far that switching costs and abandonment values are deterministic (constant). In many real-life applications, however, these variables are functions of actual utilization in the current or alternative-use technology. Ramey and Shapiro (2001), in their study of aerospace plant closings, examine the process of shifting physical capital to new uses, noting that age-based accounting depreciation does not accurately reflect the market value of displaced capital. In the following, we enhance our model by considering use-based economic depreciation which can directly affect abandonment values or switching costs. Accounting for actual use-based economic depreciation makes these values path-dependent and stochastic. We show that this affects both investment option values and investment decisions.

Consider the case where abandonment or salvage value depreciates according to $A_i e^{-c_i n_i}$ where A_i is maximum recovery (in immediate abandonment), counter n_i tracks the actual usage of the current technology, and parameter c_i determines the extent of economic depreciation and salvage value recovery. A lower c value implies lower depreciation per period of actual usage and thus higher salvage value, while the opposite is true for a higher c value. For example, if $c = 0.30$ and the technology has been in actual use for 3 periods, recovery is 40.7% of the maximum abandonment value A. A parameter value $c = 0$ implies that abandonment values are constant and do not depend on the utilization of capital (as is the case in our previous results reported in Figures 4a-4e and 5a-5b).

For the first application with learning-by-doing, Tables 1 and 2 provide numerical results with stochastic abandonment value but with constant switching costs. In both tables, the upper panel results are calculated with higher depreciation $c = 0.70$ and the bottom panel results are calculated with lower depreciation $c = 0.30$. Consistentl with traditional intuition, higher volatility and lower dividend yield produce higher investment option values. But we can now see the importance of using accurate economic depreciation. When the value of economic depreciation (c)

is low (implying a higher salvage value), investment option values are significantly higher. Differences in valuation are particularly striking for at- or out-of-the-money options. This finding is of high significance, since the range of at- or near out-of-the-money options is most important for the economic evaluation of new corporate investments, for the adoption of new technologies, and for the addition of operating capacities. It is rather unlikely that managers will forgo extremely profitable opportunities. Investment option values depend on the extent to which the current level of the state variable (S) is in a range where investment is likely to be followed by switching and/or abandonment in case of new market developments (in demand, product prices etc.). Like in Figures 4a-4e, critical (asset) investment thresholds (a counterintuitive result) are lower for higher volatility and lower dividend yield values. Accounting for the true economic depreciation, we again see that the investment thresholds are affected by the increased irreversibility induced by higher depreciation rates (c), in which case investment may be further delayed.

For the second application on shipping, the assumption that switching costs are constant is now relaxed. As with abandonment values, the switching costs from mode i to mode j ($I^{i \to j}$) may be

Table 1
Sensitivity of option value and optimal operating policy vs. volatility σ with four alternative operating modes without time-to-build but with utilization-dependent abandonment values

S	OPTIMAL INITIAL DECISION			OPTION VALUE		
	σ = 10%	σ = 20%	σ = 30%	σ = 10%	σ = 20%	σ = 30%
	c = 0.70					
90	W	W	W	1.502	25.902	70.858
95	W	W	W	3.906	37.075	87.580
96	W	W	**C**	4.579	39.631	**91.666**
99	W	W	C	7.184	47.716	106.713
100	W	**C**	C	8.417	**51.378**	111.874
102	W	C	C	11.234	59.867	122.264
103	**C**	C	C	**13.102**	64.242	127.483
104	C	**B**	C	16.262	**69.115**	132.753
105	C	B	**B**	19.586	74.326	**138.585**
106	**B**	B	B	**23.752**	79.640	144.740
110	B	B	B	43.124	101.782	170.205
115	B	B	B	70.305	133.999	202.656
118	B	B	B	90.114	154.492	224.141
119	**E1**	B	B	**97.333**	161.472	231.415
120	E1	B	B	104.978	168.460	238.726
121	E1	**E1**	B	112.701	**175.580**	246.208
122	E1	E1	B	120.453	183.316	253.693
123	E1	E1	**E1**	128.218	191.080	**261.786**
125	E1	E1	E1	145.001	207.104	278.455
130	E1	E1	E1	188.859	249.415	321.109
…	…	…	…	…	…	…
162	E1	E1	E1	510.375	556.379	620.164
163	**E2**	**E2**	**E2**	**521.481**	**566.978**	**630.886**
	c = 0.30					
90	W	W	W	3.712	30.458	74.945
93	W	W	W	5.819	37.075	85.292
94	W	W	**C**	6.754	39.571	**89.282**
95	W	W	C	7.781	42.086	93.994
97	W	W	C	10.178	47.622	103.623
98	**W**	**C**	C	**11.703**	**51.354**	108.603
99	**C**	C	C	**14.494**	55.338	113.784
100	**B**	C	C	**17.493**	59.349	119.033
102	B	C	C	24.579	67.715	129.604
103	B	**B**	C	28.681	72.605	135.045
104	B	B	C	32.783	78.346	140.572
105	B	B	**B**	36.885	84.100	**146.429**
110	B	B	B	60.140	113.302	177.257
115	B	B	B	86.778	143.805	211.374
118	B	B	B	104.052	163.079	232.342
119	**E1**	B	B	**111.225**	169.655	239.457
120	E1	**E1**	B	118.630	**176.707**	246.678
121	E1	E1	B	126.051	184.258	254.068
122	E1	E1	**E1**	133.536	192.162	**261.820**
125	E1	E1	E1	155.992	216.924	286.346
130	E1	E1	E1	195.941	258.845	328.509
…	…	…	…	…	…	…
162	E1	E1	E1	510.928	560.248	625.199
163	**E2**	E1	E1	**521.896**	570.359	635.120
164	E2	E1	E1	533.643	580.652	645.063
165	E2	**E2**	**E2**	545.500	**591.265**	**655.442**

Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model without time-to-build but with utilization-dependent abandonment values. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales and c is the economic depreciation factor. Operating cost X = 100, S are base-scale operating revenues, riskless rate r = 5%, Δt = 1 per period. Values in bold indicate optimal operating thresholds.

Table 2
Sensitivity of option value and optimal operating policy vs. payout yield δ with four alternative operating modes without time-to-build but with utilization-dependent abandonment values

S	OPTIMAL INITIAL DECISION			OPTION VALUE		
	δ = 0%	δ = 10%	δ = 20%	δ = 0%	δ = 10%	δ = 20%
	c = 0.70					
90	W	W	W	136.690	25.902	2.900
92	W	W	W	150.019	30.034	3.482
93	**C**	W	W	**158.359**	32.266	3.784
95	C	W	W	175.920	37.075	4.497
99	C	W	W	212.464	47.716	6.916
100	C	**C**	W	222.139	**51.378**	7.586
101	C	C	W	232.005	55.575	8.271
102	**B**	C	W	**242.011**	59.867	9.001
103	B	C	W	252.480	64.242	9.795
104	B	**B**	W	263.134	**69.115**	10.638
105	B	B	**C**	273.866	74.326	**12.294**
106	B	B	**B**	284.752	79.640	**14.569**
110	B	B	B	329.829	101.782	24.502
115	B	B	B	389.523	133.999	39.873
119	B	B	B	438.854	161.472	53.082
120	**E1**	B	B	**451.711**	168.460	56.407
121	E1	**E1**	B	464.818	**175.580**	59.887
122	E1	E1	**E1**	477.939	183.316	**83.255**
125	E1	E1	E1	517.925	207.104	75.514
130	E1	E1	E1	586.110	249.415	99.176
…	…	…	…	…	…	…
162	E1	E1	E1	1046.376	556.379	292.688
163	**E2**	**E2**	E1	**1061.632**	**566.978**	299.478
164	E2	E2	**E2**	1077.521	578.505	**306.981**
	c = 0.30					
90	W	W	W	141.656	30.458	4.197
91	W	W	W	148.160	32.537	4.715
92	**C**	W	W	**155.179**	34.721	5.323
95	C	W	W	181.396	42.086	7.324
97	C	W	W	199.513	47.622	8.686
98	C	**C**	W	208.738	**51.354**	9.376
99	C	C	**C**	218.129	55.338	**10.400**
100	C	C	C	227.571	59.349	12.620
101	C	C	C	237.148	63.378	14.839
102	C	C	**B**	246.766	67.715	**17.448**
103	**B**	**B**	B	**257.064**	**72.605**	20.061
105	B	B	B	278.095	84.100	25.313
110	B	B	B	333.361	113.302	39.230
115	B	B	B	392.446	143.805	54.352
118	B	B	B	429.194	163.079	65.419
119	B	B	**E1**	441.507	169.655	**69.673**
120	**E1**	**E1**	E1	**454.165**	**176.707**	74.201
125	E1	E1	E1	519.890	216.924	96.864
130	E1	E1	E1	587.573	258.845	120.161
…	…	…	…	…	…	…
162	E1	E1	E1	1046.741	560.248	306.569
163	**E2**	E1	E1	**1061.831**	570.359	313.181
164	E2	E1	E1	1077.718	580.652	319.793
165	E2	**E2**	E1	1093.629	**591.265**	326.417
…	…	…	…	…	…	…
171	E2	E2	E1	1189.122	660.614	367.609
172	E2	E2	**E2**	1205.044	672.189	**375.271**

Four alternative operating scales (C, B, E_1, E_2) for a (5+1)-stage model without time-to-build but with utilization-dependent abandonment values. C denotes the small operating scale, B the base case, E_1 and E_2 the expanded operating scales and c is the economic depreciation factor. Cperating cost X = 100, S are base-scale operating revenues, riskless rate r = 5%, Δt = 1 per period. Values in bold indicate cptimal operating thresholds.

Table 3

Sensitivity of option value and optimal operating policy vs. volatility σ for the shipping oil tanker application without time-to-build but with utilization-dependent stochastic switching and abandonment values

P	OPTIMAL INITIAL DECISION		OPTION VALUE (in $ mn)	
(in $ mn)	σ = 15%	σ = 25%	σ = 15%	σ = 25%
		c = 0.70		
165	W	W	6.287	28.856
…	…	…	…	…
185	W	W	15.060	43.847
190	W	W	18.239	48.052
195	W	W	21.585	52.642
200	W	W	24.973	59.014
205	W	W	29.175	65.392
207	W	W	30.943	68.094
208	**B**	W	**32.006**	69.462
210	B	W	34.292	72.224
215	B	W	40.566	79.191
220	B	W	46.840	86.172
225	B	W	53.123	93.166
230	B	W	59.418	100.218
…	…	…	…	…
244	B	W	77.200	120.218
245	**W**	W	**78.692**	121.685
250	W	W	86.583	129.134
252	W	W	89.740	132.236
253	**E2**	W	**92.197**	133.786
…	…	…	…	…
280	E2	W	179.671	182.812
281	E2	**E2**	182.911	**185.005**
285	E2	E2	195.871	197.835
290	E2	E2	212.070	213.872
		c = 0.30		
165	W	W	8.783	35.976
…	…	…	…	…
185	W	W	19.139	54.005
190	W	W	22.524	60.200
195	W	W	26.045	66.395
200	W	W	29.648	72.658
203	W	W	32.188	76.535
204	**B**	W	**33.271**	77.830
205	B	W	34.418	79.126
210	B	W	40.274	85.614
215	B	W	46.455	92.201
…	…	…	…	…
250	B	W	99.287	143.925
253	B	W	104.507	149.217
254	**E2**	W	**106.329**	151.160
…	…	…	…	…
265	E2	W	139.321	172.538
270	E2	W	154.815	182.255
271	E2	**E2**	157.928	**184.682**
275	E2	E2	170.544	196.217
…	…	…	…	…
290	E2	E2	217.895	239.647

Shipping oil tanker application for a (5+1)-stage model without time-to-build, using utilization-dependent switching and abandonment values. C denotes the small tanker, B the medium tanker, E_1 the large tanker, and E_2 the very large carrier. Operating cost X = 8.8, riskless rate r = 5%, Δt = 2 per period. Values in bold indicate optimal operating thresholds.

utilization-dependent (path-dependent) and stochastic. Specifically, it is assumed that $I^{i \to j} = I^{W \to j} - A_i e^{-c_i n_i}$ i.e., the switching cost (from i to j) depends on the initial capital outlay to arrive at operating mode j minus the stochastic abandonment value at mode i, A_i, where $A_i = I^{W \to i}$. Numerical results for the shipping application are presented in Table 3. We confirm again that higher volatility (and also lower dividend yield), lead to higher investment option values, similar to Figures 5a-5b. The impact of volatility here is closer to conventional since in this application we have rather high switching costs (higher irreversibility). Still, we can see the impact of considering true economic depreciation. When depreciation rate c is high (top panel), investment option value is significantly lower (especially for at- and out-of-the-money options). For a high depreciation rate, irreversibility is strong and investment decisions may be further delayed.

Our overall results based on both applications involving learning-by-doing and shipping confirm that lower economic depreciation or higher recovery rate result in a higher investment option value, often exceeding 20%. This is most clear when asset value S is in a low range from which it is more likely that switching and abandonment will occur. The extent of capital recovery also affects the optimal investment thresholds and optimal decisions. For alternatives that have higher degrees of economic depreciation, irreversibility is stronger and (as can be confirmed in all three tables) investment will be further delayed. Ignoring the effects of true economic depreciation of installed capital in real option analysis may lead to significant mis-valuation of sequential investment opportunities and lead decision makers astray. Our results also demonstrate that management should consider, from the design stage, to enhance the potential of future economic recovery of the capital (salvage value).

5. Conclusion

We have examined sequential investment and production decisions under uncertainty within a real switching-options framework that incorporates operating flexibility within a configuration of partially reversible decisions, while facing operating constraints arising from limited or exhaustible resources. We have accounted for stochastic utilization-dependent recovery of capital and asymmetric stochastic switching costs, construction lags associated with time-to-build as well as operational and resource constraints. Costly switching among several operating modes along with economic depreciation under operating constraints induces significant path-dependencies. Our real options framework enables studying sequential inter-dependent investment decisions in alternative production technologies or involving alternative capacity (scale) choices. It can be extended to study many related problems, such as those involving choices among mutually exclusive technologies. Our approach is also suitable for various path-dependent problems, as it allows keeping track of the history of past path-dependent decisions.

We find that optionality to switch among several operating modes and/or abandon early considerably impacts on investment option values and investment decisions. In a general setting with high such optionality, we find that an increase in uncertainty often leads to investing earlier rather than waiting. Analogous non-conventional results are found with a decrease in the rate of return shortfall ("dividend yield") in the context of interdependent sequential decisions. Factors that constrain switching flexibility (such as high switching costs and operating constraints) increase irreversibility of investment decisions and may reduce (or eliminate) these non-conventional effects. Our modelling framework also allows the analysis of the market niche potential for newly developed technologies. When new technologies are dominated by existing ones, it may not be economically appealing to bring them to market. An existing technology may also become economically obsolete in light of the emergence of new superior alternative technologies. Such investment (or disinvestment) decisions concerning the adoption of new technologies can be analysed as part of a configuration of interrelated sequential decisions. We have also analysed the impact of utilization-dependent economic depreciation policies giving rise to stochastic switching costs and recovery values. Higher economic depreciation rates reduce option values and increase irreversibility, thus delaying the investment decision. Accounting for true economic depreciation enhances the value and accuracy of real option analysis. Management should thus try to enhance, at the design stage, the potential for the future recovery (salvage) of capital.

Future extensions of this work may incorporate other realistic organizational aspects of corporate decision making, such as agency issues (e.g., Bhattacharya et al., 2015), financing considerations (e.g., Lambrecht and Myers, 2008; Morellec et al., 2015) and game-theoretic aspects arising in a duopolistic industry such as in airplane manufacturing (see Chevalier-Roignant et. al, 2011; Azevedo and Paxson, 2014, for recent reviews). Another direction for future research concerns extensions in strategic management (strategic intent), such as an examination of how likely a strategic expansion path or how attractive a technological leapfrog strategy might be.

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