Consumer Search and Product Returns in E-Commerce

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Abstract

E-commerce has led to a surge in products being returned after purchase. We analyze product returns as resulting from a trade-off between the social waste of returns and the search efficiency gains of being able to inspect a product's value after purchase. We find that whenever returns are efficient, the market generates too few returns as the parties involved in the transaction do not internalize the welfare benefit of consumers continuing their search, generating profits for other firms. We also show that, despite their consumer friendly appearance and the private cost of returns, firms may benefit and capture the gains from less costly search.

Keywords: product returns, consumer search, search efficiencies, product matches

JEL codes: D40, D83, L10

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E-commerce has come with a sharp increase in products being returned after purchase. In the USA the National Retail Federation estimates that across different retail channels $428 billion of merchandise value is returned in 2020, which is around 10% of total retail sales. Focusing on online retail only, these numbers are approximately $102 billion and 18% of online sales. They also mention that "online returns more than doubled and are a major driver of the overall growth of returns".[1] Given this development, it is important to treat product returns strategically by considering optimal return policies. Moreover, product returns are a social concern in that they potentially point to a large social cost of e-commerce, including environmental costs that are paid by agents not involved in the transaction.[2]

Product returns arise in both online and offline markets, but an important source for higher return rates in online markets, we argue, is that online consumer search differs from that in traditional markets along two dimensions. First, search for some information such as prices and objective product features is much easier online. However, and second, it is not that easy to learn subjective features of product match online: How will a certain pair of glasses fit, or, how easy is it to operate a digital camera? To overcome this latter aspect of online search, firms may choose lenient return policies providing incentives to buy the product without spending effort *before* purchase to determine whether the product fits their needs. Instead, consumers may check product fit in a more comfortable home environment and return the product if they learn the fit is not good enough.

Product returns are also important for regulatory agencies whose policies towards returns differ across the world. In the USA, there is no general regulation in place, but some individual states mandate that retailers give refunds on products that are returned within 20 or 30 days of purchase if they themselves do not provide a clearly stated policy.[3] In the European Union, refunds on online purchases are mandatory.[4]

[1] See, e.g., https://nrf.com/research/customer-returns-retail-industry.

[2] These environmental costs include greenhouse gas emissions, non-recycled packaging and products filling up landfills (see, e.g. Tian and Sarkis (2022)), where some websites estimate that only 54 percent of all packaging gets recycled and 5 billion pounds of returned goods end up in landfills each year.

[3] See, e.g., https://www.findlaw.com/consumer/consumer-transactions/customer-returns-and-refund-laws-by-state.html

[4] See, https://europa.eu/youreurope/citizens/consumers/shopping/guarantees-returns.

Even though full refunds seem to be the norm, the small print may make all the difference as refund policies differ according to (i) who should pay for the return cost and (ii) whether firms charge a so-called restocking fee (which can be up to 20% of the purchase price), which are typically not regulated.

This leads us to the following questions. In the absence of regulations, is it optimal for firms to stimulate product returns? Are product returns socially wasteful or do they provide an efficient solution to the trade-off between search cost reduction and cost increases due to returns? Are regulations redundant if firms voluntarily offer product returns?

To answer these questions, we build on the seminal consumer search paper by Wolinsky (1986). Our methodological contribution consists of extending it in a number of fundamental ways. First, firms choose two prices: the price at which consumers purchase the product and the refund consumers get when they return the product, i.e., firms think strategically about their return policy. Second, we divide the search cost into two parts: a very small cost of learning a firm's price and refund policy and a much larger inspection cost to learn the personal match value. Third, we allow consumers to purchase a product without inspecting its match value before purchase. If they do so, they will typically inspect the product after purchase and can do so at a lower than before-purchase inspection cost or not inspect at all. Fourth, if consumers return the product, it has a salvage value for the firm that is typically smaller than the production cost. Thus, there are two important aspects to offering product returns:[5] (i) the difference between production cost and salvage value represents the (social) cost of product returns, while (ii) the difference in search costs before and after purchase represents a potential (social) benefit.

We have two main sets of results. We start with an equilibrium characterization and show that if the inspection cost is not too large, there always exists a unique equilibrium with trade. Depending on the parameter values, it is such that either (*i*) all firms offer a refund resulting in consumers inspecting after purchase, or (*ii*) all firms effectively incentivize consumers to inspect before purchase (which results in

[5] We abstract away from fraud. Especially in online transactions, fraud may come from both sides of the market: firms may ship broken or otherwise non-functional products, while consumers may buy products for a certain occasion and then return them. Our model deals with firms that care about their reputation and who keep track of -and ban- consumers engaged in fraudulent behavior.

the Wolinsky equilibrium), or (*iii*) some firms offer a return policy, while others do not. We use this characterization results to indicate in what type of markets we may expect more product returns and why product returns are more frequent in online markets.[6] One interesting property of the equilibrium with refunds is that when the search efficiency gains of inspecting at home get larger, firms capture the gains from less costly search by increasing the difference between price and refund.

Our second set of results concerns welfare. Standard results show that, without product returns, the market outcome is efficient (Wolinsky, 1986). In contrast, we show that whenever the market provides returns, the outcome is generically inefficient, with either too few or too many returns. In a market context, efficiency requires that for every unit that is bought and then returned, the consumer pays the cost related to the return and firms make no profit, i.e., the difference between price and refund should equal the difference between production cost and salvage value. In the market equilibrium the difference between price and refund generally does not satisfy this condition. In particular, the market provides too few returns if, and only if, the difference between price and refund is larger than the difference between production cost and salvage value.

This evaluation of product returns in a market context is very different from that under a single-product monopoly. First, the notion of efficiency itself is significantly different as under monopoly efficiency requires that the refund that is offered equals the salvage value. The difference lies in the fact that if a consumer does not buy at the monopolist, they do not buy at all, whereas in a market they continue to search, potentially generating both more profits for firms and more value for consumers. Second, in a search market, but not under monopoly, firms may offer a refund that is inefficiently low. By marginally increasing the refund, a firm increases sales at other firms, an externality they do not take into account. These firms make profit over these consumers even if they do not buy at the firm if the difference between

[6]The National Retail Federation reports that retail categories such as apparel and footwear have a relatively high product return rate, whereas other categories such as beauty and health care have fewer returns (see, https://9d4f6e00179f3c3b57f1-4eec5353d4ae74185076baef01cb1fa1.ssl.cf5.rackcdn.com/Customer%20Returns%20in%20the%20Retail. For the first two categories, product match is important and large efficiencies can be gained by inspecting at home, while the salvage value is reasonably close to the production cost. For the second set of categories, the salvage value is almost zero and return policies are therefore unattractive.

price and refund is larger than the difference between production cost and salvage value.

Our welfare result naturally leads to the important question of whether regulators can fix the inefficiency issue by mandating firms to offer a minimal threshold percentage of the sales price as a refund. We show that such a policy, if it is chosen just marginally above the market equilibrium outcome, may actually backfire and harm consumers, while leaving social welfare unaffected. The reason is that firms react to the regulation by increasing prices.

The above results indicate that inspection after purchase and product returns are important in online and offline markets. However, as the difference in inspection cost between before and after purchase is probably larger in online markets, product returns are key to understand the functioning of e-commerce. The results explain that product returns are an integral part of online markets and that, despite the appearance of inefficiency, they can be efficient in reducing search/inspection cost and improving match values. However, the market outcome is generally inefficient as there are either too many or too few returns.

The existing literature on product returns typically focuses on a single product monopoly where consumers do not have an outside option. As discussed above, our results show that studying refunds in a market context yields very different insights. The paper in the monopoly literature that is closest to ours is Matthews and Persico (2007). They show that if a monopolist offers refunds, it always promotes too many returns from a social welfare perspective. Even though in a market context firms also offer a refund that is larger than the salvage value, this is only one aspect of the efficiency considerations and it does not imply the market offers too many returns. Other papers in this literature include Che (1996), Inderst and Ottaviani (2013), Inderst and Tirosh (2015) and Jerath and Ren (2022).[7] Che (1996) shows that a monopolist may offer a generous refund to induce risk-averse buyers to buy, whereas Inderst and Ottaviani (2013), Inderst and Tirosh (2015) study an alternative reason why a firm may offer refunds, namely as a way to signal product quality. Jerath

[7] Hinnosaar and Kawai (2020) explore robust pricing with refunds, where consumers' value either match or do not match with the monopolist's product and where the firm does not know the consumer's prior over his match value.

and Ren (2022) consider whether a monopolist has an incentive to facilitate search before purchase in relation to the complexity of the return policies they choose and show that their model may account for different refund practices.

Our paper is also related to several recent branches of the consumer search literature. First, Armstrong (2017), Choi et al. (2018), Haan et al. (2018), among others, build on Wolinsky (1986), but allow consumers to direct their search based on the prices firms charge. This literature is inspired by the observation that in online markets price information is easy to acquire. These papers stick, however, to the standard consumer search set-up that consumers cannot buy without inspecting before purchase. Doval (2018) is the first paper in this literature to introduce the option of buying without inspection, but she only considers the optimal consumer search problem and studies how the optimal stopping rule differs from the classic Weitzman (1979) rule, while Chen et al. (2021) introduce this option in the Choi et al. (2018) model.[8] None of these papers allows consumers to inspect the product after purchase, however, or the question whether the market stimulates firms to offer lenient return policies leading to product returns. Petrikaitė (2018) introduces the option of product returns in a model where consumers learn one component of their match value after purchase, but the key issues of this paper, namely whether firms stimulate product returns and whether this enhances market efficiency, are not addressed in her paper as firms cannot choose their return policy and the salvage value is assumed to be equal to the production cost.

The rest of the paper is organized as follows. The next section introduces the model, while Section II. characterizes the different market equilibria and when they exist. Section III. analyzes welfare by discussing the efficient allocation, whether the market provides too little or too many returns and the effects of regulation. Section IV. concludes with a discussion, while proofs withheld from the text are given in the (online) Appendix.

[8]Fishman and Lubensky (2016) also introduce the option of purchasing without inspection in a Wolinsky-type model, but like Chen et al. (2021) they do not study product returns.

I. Model and Preliminary Results

The market is comprised of a unit mass of consumers with a unit demand for a product and a unit mass of firms who supply the product. Following the consumer search literature based on Wolinsky (1986), a consumer's match value v with a firm is drawn from a distribution G with support $[\underline{v}, \bar{v}] \subset \overline{\mathbb{R}}$, density g, with both G and $1 - G$ logconcave. Match values are independent across firms and consumers. At the start of the game, firms simultaneously set not only their price $p \geq 0$, but also their *refund* $\tau \geq 0$, which is the amount of money given back to a consumer who purchases the product and then decides to return it. We often refer to the pair (p, τ) as the firm's "contract". Firms face a constant marginal cost of production $c \geq 0$ and a *salvage value* $\eta \in [0, c]$ for items that have been purchased and subsequently returned. The difference $c - \eta$ captures the cost of product returns.[9] As it cannot be optimal for firms to offer a $\tau > p$ we will restrict attention to contracts with $\tau \leq p$.

Consumers are uncertain of their match values as well as firms' contracts, but they can learn these through costly search. A consumer starts out by incurring a small but positive *search cost* $\epsilon > 0$ to visit a firm to learn the price it charges and the return policy it offers. The consumer then has three options: He can incur an *inspection cost* $s > 0$ to learn his match value with the firm before buying the product,[10] he can buy the product without first inspecting it, or he can decide to leave and visit another firm (or leave the market altogether for a payoff of zero). If the consumer buys the product without first inspecting it, he has the option to incur an inspection cost of βs, with $\beta \in [0, 1]$, to learn his match value after purchase. If the consumer learns the match value after purchase, he can decide whether to keep the product or to return it to the firm, receiving τ and then possibly continue his search at another firm. The term $(1 - \beta)s$ measures the search efficiency of inspecting after purchase and captures the reduction in inspection cost if the consumer inspects after purchase relative to inspecting before purchase. Let $\Omega \subset \mathbb{R}^5_+$ denote the set of

[9] Thus, the salvage value is not the profit a firm receives from reselling it, but the savings from not having to produce another item for the next purchase. It captures all costs that need to be made to be able to resell a returned product.

[10] If the consumer would choose this option, the equilibrium would be exactly as in the original Wolinsky model and the splitting of the search cost into two parts does not affect that equilibrium.

parameter values $\omega = (c, \beta, \eta, s, \epsilon)$.

Throughout the paper we focus on Perfect Bayesian Equilibria where firms choose their strategies to maximize expected profits given their information and consumers choose an optimal sequential search strategy. In addition, if consumers observe a firm's deviation from equilibrium behavior, they continue to believe that firms that are not yet visited play their equilibrium strategies as these do not depend on the information consumers have.

A. Some Comments on the Interpretation of the Model

We now discuss different aspects of our model. First, how our model relates to online and/or offline markets can be interpreted in different ways. For example, one may interpret s as the cost of going to a brick-and-mortar store and βs as the cost of inspecting at home after having purchased the product online. Inspection at home is (much) easier, but possible not for free given that a product typically has to be returned within a certain narrow time period.[11] Another interpretation is that all possible inspection (both before and after purchase) is performed online, taking into account that consumers may learn about products from online reviews. Also in this interpretation, it often is easier to inspect a product when having the product at hand after purchase.

Second, and in line with the above second interpretation, our model can capture consumers acquiring information about their match values upon incurring the initial search cost. For example, adopting an approach used by Anderson and Renault (2021) and Nocke and Rey (2023), suppose that when incurring the search cost ϵ, a consumer learns whether or not the product is a "match" for his needs. No match delivers a value of $v = 0$, while other products offer a match value drawn from the distribution G specified above. We could also allow for a consumer's match value to be decomposed into two components, i.e., $v = v_1 + v_2$, whereby v_1 is the value of objective features of a product that can take on a finite set of magnitudes and be learned together with price. With this modification and keeping the setting

[11] We do not explicitly address issues related to the length of the period in which consumers can return the product, but see Lyu (2022) for some of the relevant considerations.

that firms will choose one value of p and τ and not offer a menu of contracts, the only change to our model is that the search cost must factor in the expected cost of visiting firms that fail to provide the (best) match.[12]

Third, we may also consider that there are some product features that are prohibitively costly or even impossible to learn online. For example, we could interpret the decomposition of a consumer's match value differently and say that v_1 is the value offered by features that can be learned by inspecting the product, while v_2 is the value of features that are only discovered once the product is at hand. In this case, there remains uncertainty about the product's value even if one has inspected the v_1 features before purchase, and the analysis becomes more complicated, but our main insights remain.

Fourth, an implication of the assumption that consumers incur an arbitrarily small but positive search cost to learn a firm's price and return policy is that they cannot direct their search to firms with lower prices and/or a more favorable return policy. We know from Diamond (1971) that small search costs may have very different implications from search costs being equal to zero and we think this assumption is also appropriate in many online markets where consumers do not often buy from the same shop and they do not know a return policy in advance. If consumers would know a firm's return policy in advance, then they could update their beliefs about firms' prices. For example, higher refunds may be associated with higher prices. Depending on these beliefs, different effects are possible. for example, if consumers believe refunds are uninformative of prices, then they will direct their search effort towards firms with more favorable return policies, creating Bertrand competition in refunds resulting in full refunds.

Fifth, the model does not explicitly define nor address who pays the transportation costs, which are clearly important for online purchases. These aspects are easily interpreted in our model as follows. First, the cost of shipping itself is one reason why the salvage value is smaller than the production cost. Second, the difference between the price and refund is a measure for how these costs are divided between firms and consumers. For example, if the difference $c - \eta$ is entirely due to the cost

[12]Letting $\rho > 0$ denote the probability of having a match with a firm, the expected search costs incurred before reaching a firm that provides a match is $\epsilon' = \frac{1}{1-\rho}\epsilon$.

of shipping, then a full refund $\tau = p$ implies that firms pay the cost while a contract satisfying $p - \tau = c - \eta$ implies that consumers pay for it.

Sixth, the model is flexible enough to cover environments where consumers incur an additional “hassle cost" to return a product. If we denote this cost by γ, then our model is isomorphic to this new model, by replacing the refund with $\tilde{\tau} \equiv \tau - \gamma$ and replacing the salvage value with $\tilde{\eta} \equiv \eta - \gamma$.

Seventh, we model search as a sequential decision problem. Alternatively, one may consider that the possibility to return a product after purchase introduces a delay for the next search and that this makes it optimal for consumers to engage in simultaneous search (cf., Morgan and Manning (1985)), In the supplementary material we show that as the delay often is not substantial, sequential search continues to be optimal. In the last section of the paper we discuss a more promising alternative for modeling simultaneous search in online markets.

B. Consumers

We now present some preliminary findings, starting with the consumer side. Consider the consumer’s problem at the moment he has incurred the search cost ϵ to visit a firm and learned its price and refund (p, τ). Denote the consumer’s outside option by a. The outside option is endogeneous to the model and depends on the equilibrium contracts offered by other firms. The consumer has four possible paths of play. (*i*) He may decide to buy the product and neither inspect it before nor after purchasing it, yielding the payoff $v - p$. (*ii*) If he buys the product and inspects afterwards, his payoff is $v - p - \beta s$ if his match value exceeds the payoff from returning the product $v \geq a + \tau$, and otherwise his payoff is $a + \tau - p - \beta s$. (*iii*) If he decides to inspect the product before purchase, his payoff is $v - p - s$ if the net value exceeds the outside option $v - p \geq a$ and otherwise the payoff is $a - s$. (*iv*) Finally, if he decides to leave the current firm, he gets a. As $\tau \leq p$, the option for buying and then returning the product without inspecting is never chosen. Thus, the

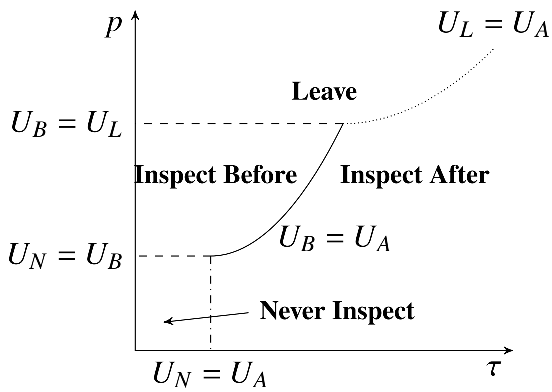

Figure 1: Inspection choices for a given price p and return policy τ.

expected utility from each of these options is given by the following expressions.[13]

$$\begin{cases} \text{never inspect} & U_N = E(v) - p \\ \text{inspect after} & U_A = E(\max\{v, \tau + a\}) - p - \beta s \\ \text{inspect before} & U_B = E(\max\{v - p, a\}) - s \\ \text{leave} & U_L = a \end{cases} \tag{1}$$

When presented with a price and return policy, the consumer selects the inspection option yielding the highest expected payoff. Figure 1 illustrates how the optimal choice depends on the price and return policy of the firm that is visited. When the return policy is unfavorable and the price is moderate, consumers adopt the usual search strategy of inspecting before purchase. When the return policy is unfavorable and the observed price is too high, they leave the firm without inspecting the product, while if the product is sufficiently cheap they buy without ever intending to inspect the good. When the return policy is more favorable, the consumer will opt to first buy the product and then inspect it, returning it if it turns out that his match value is low. As we shall discuss, the sizes of the different regions depend on the underlying search parameters s and β as well as on the outside option a.

[13]The expressions for utility in (1) are functions of the price, refund, outside option, and model primitives: $U_i(p, \tau, a, \omega)$ for $i \in \{N, A, B, L\}$. Throughout the paper, we suppress these arguments when convenient.

Importantly, the figure shows that that there exists a region "Inspect Before" where firms offer a positive refund that is never used by consumers as it is too low. Thus, offering such a contract results in the same market outcome as not offering a refund. In the rest of the paper, we refer to policies where firms offer a refund as those that induce consumers to inspect after purchase and there is a positive probability that the consumer returns the product. For the region "Inspect Before" we say that no refund is given.

To characterize consumer behavior, it is useful to let $S \equiv E(v - \underline{v})$ and introduce the *reservation price* $r : [0, S) \longrightarrow [\underline{v}, \bar{v}]$, implicitly defined by

$$E(\max\{v - r(x), 0\}) = x. \tag{2}$$

Intuitively, $r(x)$ is the price at which consumers are indifferent between incurring the inspection cost x and taking the outside option of zero. Letting $S' \equiv E(\bar{v} - v)$, we follow Doval (2018) and introduce the *backup price* $b : [0, S') \longrightarrow [\underline{v}, \bar{v}]$, implicitly defined by

$$E(\max\{b(x) - v, 0\}) = x. \tag{3}$$

Similarly, $b(x)$ corresponds to the price making consumers indifferent between incurring the cost x to inspect a firm's product and buying it without inspection when the outside option is zero. Throughout the paper, we maintain that the sum of the search and inspection costs is less than the unique value s^* equating the reservation and backup prices $r(s^*) = b(s^*)$, *i.e.*, $0 < s + \epsilon < s^*$.[14]. In the standard Wolinsky model where match values are uniformly distributed over $[0, 1]$ $s^* = 1/8$.

C. Firms

Given the inspection strategy adopted by consumers, we now turn to the firms' problem. Denote the probability a consumer continues to search after visiting

[14]Appendix S.1 verifies the existence of a unique $0 < s^* < \min\{S, S'\}$ equating $r(s^*) = b(s^*)$. Thus, our analysis is restricted to the set $\Omega = \{(c, \beta, \eta, s, \epsilon) \in \mathbb{R}^5_+ : 0 \leq \beta \leq 1, 0 \leq \eta \leq c, 0 < s + \epsilon < s^*\}$

another, randomly drawn, firm by q. Then, if the consumer's outside option is a in each round of search, a firm's expected profit from offering a particular price and refund is determined by the consumers' inspection decisions as follows.

$$\begin{cases} \text{never inspect} & \pi_N = \frac{p-c}{1-q} \\ \text{inspect after} & \pi_A = \frac{p-c+(\eta-\tau)G(a+\tau)}{1-q} \\ \text{inspect before} & \pi_B = \frac{(p-c)(1-G(a+p))}{1-q} \\ \text{leave} & \pi_L = 0 \end{cases} \tag{4}$$

If the consumer never inspects, the firm's profit is simply equal to $p - c$ over all consumers that visit it. If consumers inspect afterwards, the firm has to give the refund τ back to all consumers who return the product in exchange for the salvage value η. From the consumer's problem it is clear that a fraction $G(a + \tau)$ returns the product. Finally, if consumers inspect before purchase, the firm's profit is simply the Wolinsky profit.

The terms $1 - q$ in the different profit expressions may appear to be relatively unimportant scaling factors. That appearance is false, however, as we shall show in the next sections. The probability q endogenously depends on contracts that firms offer and thereby on the parameters in Ω. In particular, some of the comparative statics effects on profits and on welfare arise, because the market generates more product returns. For example, firms may make more profits overall despite the fact that on each direct sale they generate less profits, simply because it is possible to make money over products that are returned.

As an equilibrium requires a firm's best response to be well-defined, we focus on strategy profiles in which consumers break indifference between inspection options in favor of the firm. To find possible best replies, we should examine all points of discontinuity whereby $U_j(p, \tau) = U_k(p, \tau)$ for $j, k \in \{N, A, B, L\}$ and $k \neq j$ and also consider interior optima, where the consumer strictly prefers an inspection option j and $\nabla \pi_j = 0$. In principle, this gives six possible classes of best responses at boundary regions where consumers are indifferent between at least two options and four possible classes of interior solutions.

II. Equilibria

In this section we first characterize properties of equilibria where firms offer a refund such that consumers inspect the product after purchase and claim the refund if their match value is relatively small. We use these results in the next subsection to show when such refund equilibria exist and when other types of equilibria exist. These results are used in Section III. to characterize the efficiency of market outcomes.

A. Refund Equilibria

Allowing purchased items to be returned leads to a simple trade-off. Consumers benefit from waiting to inspect a product until after buying it, when it is easier to do so, and returning it when dissatisfied (i.e. when $\beta < 1$). Firms absorb a loss when returned products lose value (i.e. when the salvage value is less than the production cost $\eta < c$). In this section, we detail how this trade-off determines the return policies that are offered in the market place.

The following proposition provides two of the most basic features of refund equilibria.

Proposition 1. *When a symmetric refund equilibrium exists, it is unique and has the following properties: (i) the price and refund are set so that consumers are indifferent between inspecting products before and after purchasing them,* $U_A(p, \tau, a) = U_B(p, a)$*, and (ii) the refund exceeds the salvage value,* $\tau > \eta$*.*

The first property is easily understood. In symmetric refund equilibria, a more generous refund makes consumers not only return the product more often, it also transfers more money to consumers who make a return, reducing profit. Thus, a firm would not want to offer a more generous refund than is strictly necessary for consumers to be willing to inspect afterwards. As Figure 1 illustrates, this means that equilibrium contracts must lie along one of the boundary regions equating $U_A = U_L$, $U_A = U_B$, or $U_A = U_N$. Reasoning along the lines of Diamond (1971), as consumers do not yet know their match value before buying, there cannot be an active market with consumers being indifferent with leaving the market. Likewise, an argument akin to Diamond's rules out an active market with consumers being indifferent between inspecting after purchase and not inspecting at all (see Lemma B.5): otherwise the firm could optimally lower τ slightly and induce no inspection, which would strictly raise profits (as the consumer is not willing to pay the search cost ϵ to visit another firm). Thus, in symmetric refund equilibria consumers must be indifferent between inspecting before and after making a purchase and strictly prefer these to the other options.

The second property shows that firms incentivize consumers to return products by offering a refund that is higher than their salvage value of returned products. This can be understood by examining the firm's optimal choice of refund along the price curve in Figure 1 for which $U_A = U_B$ binds. Moving up the curve involves both an increase in the refund and the price and thus must also involve a reduction in utility since U_B is decreasing in price. By the envelope theorem, the reduction in utility arises as the expected expenditures for consumers increases, holding the choice of when to return fixed. For the firm, this means that increasing the refund along $U_A = U_B$ has two effects: (i) it yields a larger profit over consumers whose return decision is unchanged, but (ii) it also intentivizes more consumers to place a return with a net effect on profits of $\eta - \tau$ for these consumers. As at an interior optimum, it must be that these two effects sum to zero, returns must be associated with a loss, i.e., $\tau > \eta$.[15]

As in case of a refund equilibrium, the production cost is a sunk cost from the firm's perspective and therefore additively enters the firm's profit function (cf., equation (4)), while the condition $U_A = U_B$ is unaffected by production cost, it immediately follows that in a refund equilibrium firm fully absorbs increases in the production cost. This will be useful to highlight some of the results in the next section.

The constraint $U_A = U_B$ on shaping equilibrium outcomes under refund contracts can be understood as a (credible) threat from the consumer to inspect before purchase if a worse refund or a higher price is offered. When s is relatively large and β is relatively small, this threat is almost non-existent as it is too costly for consumers to inspect before. In such markets, firms could offer almost no refund, while keeping

[15]Formally, let $p(\tau)$ be the price binding the consumer participation constraint $U_A = U_B$ for a given refund τ. The firm's first order condition for an optimal refund is $\tau - \eta = \frac{1-G(a+p(\tau))}{G(a+p(\tau))} \frac{G(a+\tau)}{g(a+\tau)} > 0$.

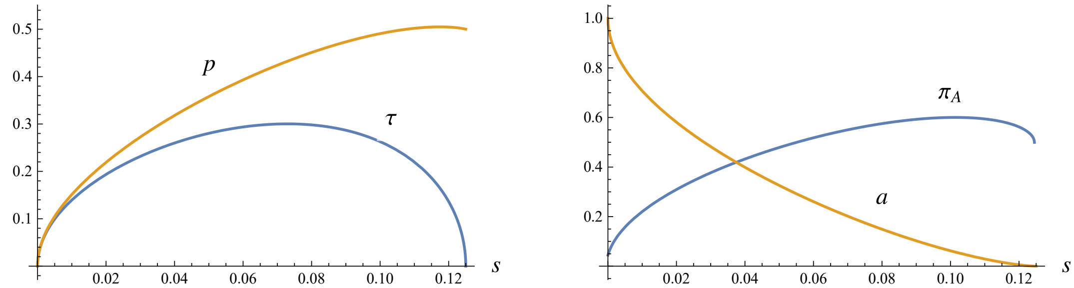

Figure 2: Price p and refund τ, consumer surplus a and profit π_A as functions of s in a refund equilibrium. Values are uniformly distributed and $\beta, c, \eta, \epsilon \longrightarrow 0$.

a high price and consumers will still only inspect after purchase. The next result formalizes this insight by showing that in a candidate refund equilibrium, firms can expropriate almost all ex-ante surplus (that is without knowing consumers' match values) by setting prices close to $E(v)$, while offering almost no refund.

Proposition 2. *Assume that inspection after purchase requires zero cost $\beta = 0$ and that $E(v) - \underline{v} > c - \eta$. Then for ϵ small enough, welfare in a refund equilibrium is decreasing in the pre-purchase inspection cost s in a neighborhood of s^*. Moreover, as $s \longrightarrow s^*$ and $\epsilon \longrightarrow 0$, firms capture the ex ante surplus, i.e., $p \longrightarrow E(v) - \underline{v} + \eta$ and $\tau \longrightarrow \eta$.*

What is interesting about this result is that even though $\beta = 0$ so that the inspection cost is never paid in a refund equilibrium, it plays an important role in shaping the equilibrium. The reason welfare is decreasing in s lies in the fact that as it becomes increasingly unattractive for consumers to inspect before purchase, firms lower their refunds, implying consumers generally keep products with lower match values. Whether firms, consumers or both are worse off depends on how firms change their prices as s increases towards s^*, which in turn depends on the shape of the match value distribution. Figure 2 confirms that for the uniform distribution both firms and consumers are worse off when the inspection cost is larger. Even though the equilibrium generates almost full *ex-ante* surplus if s is close to s^* and firms extract this surplus, it is not the case that the first best optimum is achieved. If β is small, it is socially efficient for consumers to inspect products and return them even if their match value is not very low. When s is smaller than s^*, firms are forced to offer a much more generous refund policy and as they make a profit of $p - \tau - (c - \eta)$ on items that are first bought and then returned, they benefit overall if more consumers return their products. Consumers also benefit even if prices are first increasing when s decreases starting from s^* as the match values of the products they eventually buy are much higher. Thus, this result illustrates the importance of taking a market perspective on product returns: both firms and consumers may benefit from products being returned.

B. The Structure of Equilibrium Contracts

In this subsection, we characterize what type of equilibria exist and for which parameter values they exist. To this end, let us define for each value a, $\mathcal{R}(a) \equiv \{(p, \tau) \in \mathbb{R}^2_+ : U_A(p, \tau, a) = U_B(p, a) \geq U_N(p), U_L(a)\}$ to be the set of contracts making consumers indifferent between inspecting before and after and preferring these to the other inspection options. Similarly, define $\mathcal{W}(a) = \{(p, \tau) \in \mathbb{R}^2_+ : U_B(p, a) \geq U_A(p, \tau, a), U_N(p), U_L(a)\}$ as the set of contracts where consumers inspect before purchase and that they prefer this to other inspection options. For each τ, let $p(\tau)$ denote the price in the pair $(p(\tau), \tau) \in \mathcal{R}(a)$. The lowest $\underline{\tau}$ and highest $\bar{\tau}$ values the refund takes in the set satisfy $p(\underline{\tau}) = (b(s) - a)_+$ and $p(\bar{\tau}) = (r(s) - a)_+$.[16] Throughout this section we maintain the following assumption.

Assumption. $\pi_A(p(\tau), \tau, a)$ is quasiconcave in τ when $\eta < \tau < \bar{\tau}$ and $0 \leq a$.[17]

Lemma B.1 shows that under this assumption *if* for a given a firms find it optimal to offer a refund inducing consumers to inspect after purchasing the product, there is always a unique way to do so. This goes beyond Proposition 1 as the latter only shows that if a *symmetric* refund equilibrium exists it is unique, but it does not guarantee that individual firms would not find it profitable to offer a different refund contract.

[16] The set $\mathcal{R}(a)$ is nonempty since $r(s) > b(s)$ for all $s < s^*$. Consequently, the set $\mathcal{W}(a)$ is also nonempty as it includes all contracts with a refund of zero and a price between $(b(s) - a)_+$ and $(r(s) - a)_+$.

[17] Because $p(\tau)$ is only implicitly defined, it is very difficult to analytically show that $\pi_A(p(\tau), \tau, a)$ is quasiconcave in τ for commonly employed search cost distributions. Numerical analysis for the uniform distribution shows that it is.

We denote the *refund contract* by $(p_R(a), \tau_R(a))$ and the profit it yields by $\Pi_R(a)$. Define a *candidate refund equilibrium* to be the unique triple $(p_R(a_R), \tau_R(a_R), a_R)$ for which consumer surplus satisfies $a_R = U_A(p_R(a_R), \tau_R(a_R), a_R) - \epsilon$.

Within the set of contracts inducing inspection before purchase $\mathcal{W}(a)$, the logconcavity of $1 - G$ guarantees that firms have a unique optimal price. As indicated in Section I., consumers will inspect before purchase if a firm selects a contract within $\mathcal{W}(a)$ and therefore will not use the refund option in this case. As the refund policy is redundant, we assume that within this set the firm chooses the refund to be equal to zero. Refer to the contract charging this optimal price as the *Wolinsky price*, and we denote it by $p_W(a)$. Similarly, denote the profit from charging the Wolinsky price by $\Pi_W(a)$. By standard results, there is a unique consumer surplus solving $a_W = U_B(p_W(a_W), a_W) - \epsilon$. Refer to the pair $(p_W(a_W), a_W)$ as the *candidate Wolinsky equilibrium*.

We focus on the nonempty subset of parameter values $\Omega^* \subset \Omega$ where markets are active and consumers get positive utility from inspecting before purchase when all firms charge the Wolinsky price and from inspecting after purchase when all firms play their part in a candidate symmetric refund equilibrium. Lemmas B.3 and B.4 in the appendix formally show active markets with positive consumer utility exist if s, ϵ, c and η are small enough. However, increasing the inspection, search, and production costs far enough beyond the boundary of Ω^* leads the market to be inactive as tentative equilibria will either deliver negative utility to consumers or negative profit to firms. Lemma B.2 shows that for these parameters, if all firms charge the Wolinsky price and consumers inspect before purchase, no firm has an incentive to deviate to cut the price to such an extent that it induces consumers to buy without inspection.

Proposition 3. *For parameters $\omega \in \Omega^*$, equilibria can be characterized by two continuous functions $\underline{c}(\beta)$ and $\bar{c}(\beta)$ and a constant $c^* > \eta$ whereby $\eta < \underline{c} < \bar{c}$ for all $0 \leq \beta < 1$. For each point with $\beta < 1$ and $c \leq \max\{\bar{c}, c^*\}$ there is a unique equilibrium with trade:*

1. *For $\eta \leq c \leq \underline{c}$, all firms offer a refund contract.*

2. *For $\underline{c} < c < \bar{c}$, a fraction of firms offer a refund contract.*

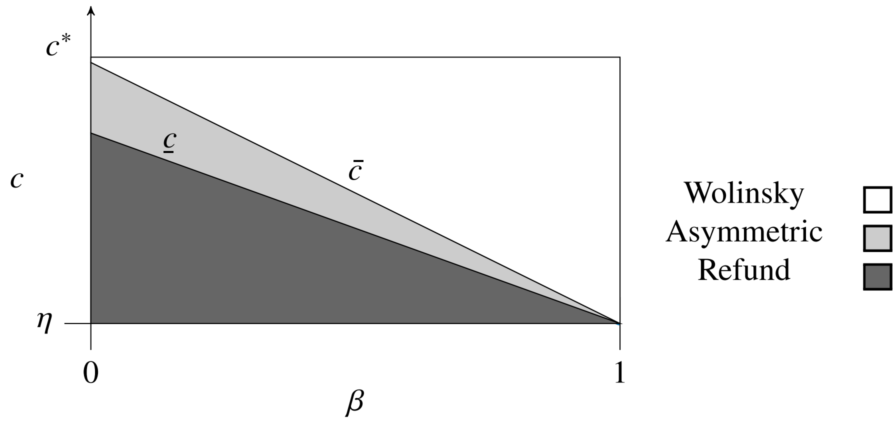

Figure 3: Equilibrium characterization.

3. *For $\bar{c} \leq c \leq c^*$, no firm offers a refund contract.*

The proposition is depicted in Figure 3. At $\beta = 1$ and $c = \eta$ one clearly sees that all three types of equilibria come together as the optimal refund contract has full refunds and mimics market behavior in a Wolinsky equilibrium. Moving horizontally or vertically only a refund equilibrium, respectively a Wolinsky equilibrium exists. This is quite intuitive, as when $\beta = 1$ and $c > \eta$ there is no benefit from consumers inspecting products after purchase and firms do not have an incentive to stimulate this option. Similarly, when $c = \eta$ and $\beta < 1$ there is no downside for firms to offer returns, while consumers are willing to pay a higher price if they are offered even a small refund if they do not like the product. Finally, for any $\beta < 1$ an asymmetric equilibrium with some, but not all, firms offering a refund contract mitigates between the parameter regions where a refund equilibrium or a Wolinsky equilibrium exist. At cost slightly above $\bar{c}$, given that other firms offer a refund contract each firm individually finds it optimal not to offer a refund (and a higher price) as it becomes too costly to do so. However, if all firms would do so, then consumers have a lower outside option to continue to search and each firm individually finds it optimal to offer a refund contract. This creates an asymmetric equilibrium where firms are indifferent between offering and not offering refunds.

The equilibrium characterization shows when the market generates product returns in the absence of regulation and fraud. Despite that regulation and fraud exist in real world brick-and-mortar and online markets, we believe that our equilibrium characterization reveals some important underlying forces that explain why product returns are more prevalent online and also more observed in some industries than in others. In particular, the difference in convenience between inspecting before and after purchase is probably larger in online markets, reflected by a smaller β in our model. The equilibrium characterization predicts that markets where β is smaller are more likely to be in a refund equilibrium. In addition, while product match is important for apparel and footwear where large efficiencies can be gained by inspecting at home, and returned products can be resold, our model predicts that these markets feature product returns. On the other hand, for categories such as beauty and health care, our model predicts that offering return policies is unattractive as the salvage value is too small relative to the production cost.

III. Efficiency

The key to understanding how return policies affect competitive search markets is to examine their impact on social welfare, defined as the sum of industry profit and consumer surplus. In the classic Wolinsky (1986) framework with many firms and without returns, consumers fully internalize the social cost of search, thereby ensuring that the market outcome is efficient. What we show is that this conclusion substantially changes when consumers are allowed to perform product returns.

To this end, we begin by supposing that all firms offer the same contract (p, τ) and ask which contracts yield the most efficient outcome given that consumers search optimally. Our first main result of this section (Proposition 4) shows that there is a simple contract with an intuitive economic interpretation that yields the first best outcome. The online appendix verifies that considering symmetric contracts is without loss in generality. Our second main result (Proposition 5) demonstrates how a market that allows for returns typically is inefficient. In our third result (Proposition 6), we show how practical regulatory policies can achieve the first best outcome in settings where return policies are socially beneficial, but can also inadvertently harm consumers if only marginal changes relative to the market equilibrium are considered.

A. Efficient Prices and Refunds

We begin by identifying which contracts are most efficient given that consumers adopt the inspection strategy of buying a good, inspecting it, and then deciding whether to keep it or return it and search further. In this case, social welfare takes the form of $\mathbf{S}_R = \pi_A(p, \tau, a) + a$, where a satisfies $a = U_A - \epsilon$. A consumer who has purchased an item and inspects afterwards will decide to place a return if their match value v lies below the sum of the refund plus continuation value, $a + \tau - v > 0$, and keep the good otherwise. Returning the good offers a salvage value to the firm who sold the good, but also requires it to pay a refund, providing the firm a net change in profit of $\eta - \tau$. In a monopoly, these two effects are the only two welfare considerations and thus efficiency is achieved if the consumer opts to place a return ($a + \tau - v > 0$) if and only if the effect on welfare is positive ($\eta - \tau + a + \tau - v > 0$). Thus in a monopoly, returns can only be efficient if the refund equals the salvage value $\tau = \eta$. Given that, in a monopoly, just as in a competitive market (Proposition 1), the refund offered is larger than the salvage value, there are too many returns from an efficiency vantage point.

In a competitive market, beyond the parties involved in the transaction, a return also impacts the firms not involved in the transaction as the consumer will continue searching firms not previously visited. Specifically, when placing a return, the expected profit accrued by the remaining firms is the profit to the firm whose good the consumer decides to keep, $p - c$, plus the expected profit to those firms whose goods the consumer buys and returns, $\frac{p-c-(\tau-\eta)}{1-G(a+\tau)} \cdot G(a + \tau)$. Therefore, the total change in welfare resulting from placing a return is

$$\underbrace{\eta - \tau}_{\text{Transacting Firm}} \quad + \overbrace{p - c + \frac{p - c - (\tau - \eta)}{1 - G(a + \tau)} \cdot G(a + \tau)}^{\text{Nontransacting Firms}} + \underbrace{a + \tau - v}_{\text{Consumer}}. \tag{5}$$

Thus, returns yield two competing effects: (i) a loss for the firm involved in the transaction and (ii) a positive expected profit for the remainder of the industry. Simplifying this expression, a consumer places returns efficiently if and only if the equilibrium contracts satisfy $p - c - (\tau - \eta) = 0$. At this contract, the expression for welfare simplifies to $\mathbf{S}_R = r(c - \eta + \beta s + \epsilon) - \eta$.

Given the constraint on search costs $s + \epsilon < s^*$, the only two inspection options performed by consumers when contracts are symmetric across firms is to inspect goods either before or after purchasing them.[18] Turning to the case where consumers inspect goods before purchase, social welfare is constant in the price and refund and takes the standard form found in the Wolinsky framework $\mathbf{S}_W = r(s + \epsilon) - c$.

Finally, we now consider how the firms' contract determines the inspection strategy adopted by consumers. Observe, that if firms offer a contract satisfying $p - c = \tau - \eta$, then profit equals $\tau - \eta$ regardless of whether consumers inspect before or after purchase. Thus, this contract not only leads consumers' return decisions to be socially optimal *given* that they opt to inspect goods after purchase, it also aligns consumers' incentives so that they choose the inspection option that is socially optimal. Denoting $\psi(\omega) = \mathbf{S}_R - \mathbf{S}_W$, we summarize our above discussion in the following proposition, where the requirement that the price satisfies $c \leq \hat{p} \leq \max\{\psi(\omega), 0\} + r(s + \epsilon)$ ensures that profit and consumer surplus are both nonnegative.

Proposition 4. *The social optimum is achieved by having all firms offer a contract* $(\hat{p}, \hat{\tau})$ *with* $\hat{p} - \hat{\tau} = c - \eta$ *and* $c \leq \hat{p} \leq \max\{\psi(\omega), 0\} + r(s + \epsilon)$. *At the social optimum, (i) if* $\psi(\omega) > 0$, *consumers inspect a good after purchasing it, (ii) if* $\psi(\omega) < 0$, *consumers inspect a good before purchasing it, and (iii) if* $\psi(\omega) = 0$, *consumers either inspect a good before or after purchasing it.*

Intuitively, this result states that a social planner who is able to determine the terms of trade in the market can achieve efficiency by setting the difference between the price and refund equal to the difference between the production cost and salvage value. For example, interpreting $c - \eta$ as the transportation cost related to product returns, then the social optimum is achieved by having consumers pay this cost. Note that the contracts described in Proposition 4 are uniquely optimal when $\psi(\omega) > 0$, whereas any contract ensuring an active market and inspection before purchase is

[18] Buying and not inspecting yields an expected payoff of $E(v) - p$. This is exceeded by the payoff from always inspecting before purchase $r(s + \epsilon) - p$ when $s + \epsilon < s^*$.

optimal when $\psi(\omega) < 0$.

The welfare condition stated in Proposition 4 only takes the various costs borne by the individual market participants into account. In reality, repeatedly purchasing and returning products might, however, create additional social costs, for example in the form of pollution due to excessive transportation or the waste from discarding returned items (Tian and Sarkis, 2022). These aspects can easily be incorporated by including a social (environmental) cost from each sale $e_s \geq 0$ and each return $e_r \geq 0$. If the social costs of market activity are too large, then efficiency naturally requires the market to be inactive. For less extreme externalities, when returns are efficient, the welfare maximizing contract is computed to be $\hat{p} - \hat{\tau} = c - \eta_e$, where η_e is the *social salvage value* $\eta^e = \eta - e_r - e_s$.

B. Market Efficiency

Having characterized efficient contracts, we now ask whether the market achieves efficiency. What we find is that the market is nearly always inefficient in its provision of returns.

Proposition 5. *Refund equilibria are generically inefficient, generating either too many or too few returns.*

The disconnect between equilibrium outcomes and efficient outcomes can be best seen by observing how, in a refund equilibrium, firms set the terms of trade to make consumers indifferent between inspecting goods before and after purchase, while the efficient contract requires that consumers internalize the externality of returns so that they most prefer whichever inspection option is efficient. The proof of Proposition 5 follows from observing that (i) By Proposition 4, a refund equilibrium can only be efficient if $p_R - \tau_R = c - \eta$, while (ii) the equilibrium refund contract is constant in the marginal cost of production c.

To identify when the market generates too many or too few returns, it is useful to render the efficient benchmark of Proposition 4 in graphical terms and relate it to the societal costs and benefits of returns as well as to the equilibrium characterization given in Section II.. To do this, let $f(\beta)$ be the strictly decreasing function

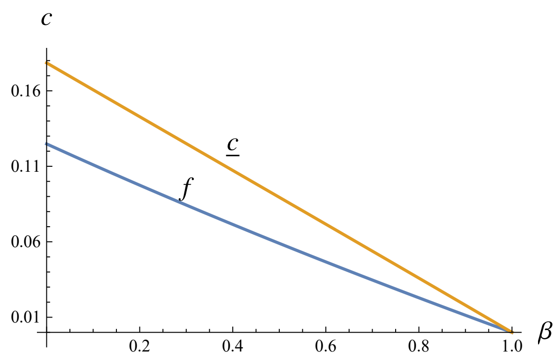

Figure 4: The curves $f(\beta)$ and $\underline{c}(\beta)$ when match values are uniformly distributed, with $\eta = 0$, $s = 0.07$, and $\epsilon = 0.001$.

corresponding to the cost c solving $\psi(f(\beta), \beta, \cdot) = 0$. Proposition 4 implies that it is socially optimal to have all firms stimulate returns if (β, c) lies below the graph of f, while if (β, c) lies above the graph of f, no firm should stimulate returns.

Figure 4 compares the market outcome with what is socially efficient for the uniform distribution. Recall from Proposition 3 that the equilibrium of the market is a refund equilibrium if, and only if, the cost lies below $\underline{c}$. Combining the proposition with the above corollary, one immediately sees from 4 that the market gets it "roughly right": there is a large overlap between the regions where it is optimal (not) to have returns and where the markets offers (no) returns. In particular, when the market is active and production costs exceed $\max\{f(\beta), \bar{c}(\beta)\}$, then the market equilibrium achieves efficiency by not stimulating returns. Also, if production costs are smaller than $\min\{f(\beta), \bar{c}(\beta)\}$, then the market offers returns and it is efficient to have them.

Below this surface, there are however important inefficiencies that arise. First, Figure 4 depicts a set of production costs $f(\beta) < c < \underline{c}(\beta)$ for which a social planner would opt for consumers to inspect before purchase, but the market stimulates returns, with firms collectively incurring a loss on each item that is bought and then returned. Second, asymmetric equilibria are evidently inefficient as it is efficient to have either all or no firms offering refunds for all production costs, aside from the non-generic case where $c = f(\beta)$. Third, the market equilibrium with returns is generically inefficient as even when it is efficient to have returns, the market provides too few returns. To see why, consider that the market is in a refund equilibrium, $c \leq \underline{c}$, $\beta < 1$ and the production cost is precisely equal to $c = f$. Market efficiency requires that $p - \tau = c - \eta$. As the refund contract is not affected by the production cost c, if we lower cost, then it must be that $p - \tau > c - \eta$ and thus the market stimulates too few returns.

C. Regulating Refunds

We finally address the practical regulatory question of whether market outcomes can be improved by requiring firms to offer a more generous return policy, but letting them set their own prices. Regulators may mandate firms to offer consumers the possibility to return their purchased item and get a refund, but they (typically) do not have a mandate to intervene with firms' pricing decisions. Our first result suggests regulators should proceed with caution in mandating product returns. Requiring firms to offer a refund above but close to the market level can be welfare neutral while resulting in a reduction in consumer surplus. Our second and more optimistic result shows that, when the societal benefits from returns strongly outweigh their costs, regulators can achieve the first best with a sufficiently bold policy.

Reconsider the social planner's problem, assuming now that he chooses the minimum (threshold) fraction of the price that firms must offer back as a refund. Specifically, when the planner selects the refund threshold θ, firms are only permitted to offer contracts in the set $X(\theta) = \{(p, \tau) \in \mathbb{R}^2_+ : \tau \geq \theta \cdot p\}$. Consider the interesting case where the unique equilibrium in the market is a refund equilibrium and the social planner can improve welfare by stimulating more returns.[19]

For a given θ, we analyze the *constrained equilibrium* in which firms and consumers both play best replies subject to the constraint that contracts belong to $X(\theta)$. Given a and $\theta < 1$, the region of permitted contracts can be represented in Figure 1 by the region to the right of the function $p = \frac{1}{\theta}\tau$ beginning at the origin and cutting through the curve $\mathcal{R}(a)$ at most once.

There are only two possible types of symmetric constrained equilibria: those in

[19]For a given $\omega \in \Omega^*$, the parameter lies in $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon), \beta < 1, c < \underline{c}(\beta), 0 < \psi(\omega')\}$.

which contracts continue to lie on the boundary $\mathcal{R}(a)$ and others in which contracts lie within the interior of the region in which consumers prefer to inspect after purchase. In general, both types of equilibria may exist for the same refund threshold. If contracts lie on the boundary in a constrained equilibrium, then they must satisfy $(1-\theta)p = p_R - \tau_R$ where (p_R, τ_R) is the unconstrained refund equilibrium contract identified in Proposition 3. This is because there is a unique difference δ between the price and refund that makes consumers indifferent between inspecting before and after purchase. Thus, given the threshold θ, the price in a boundary equilibrium can be expressed as the function $p(\theta) = \frac{\delta}{1-\theta}$. From the indifference condition, consumer surplus can likewise be expressed as $a(\theta) = r(s+\epsilon) - p(\theta)$, which is decreasing in θ. Therefore, if contracts remain on the boundary in a constrained equilibrium, lifting the mandatory refund rate is harmful for consumers. As argued in the first part of the next proposition, whenever θ is above but close enough to the equilibrium ratio τ_R/p_R, such an equilibrium exists. The second part of the proposition states that bolder policies, requiring a larger θ can achieve efficiency.

Proposition 6. *Consider a refund equilibrium.*

1. *For every refund threshold θ sufficiently close to the equilibrium ratio τ_R/p_R, there is a constrained equilibrium generating higher profit and lower consumer surplus, while keeping social welfare unchanged.*
2. *If the difference $c - \eta$ is not too large, then there is a refund threshold such that there is a constrained equilibrium that achieves maximal social welfare.*

The above proposition demonstrates the existence of particular constrained equilibria that can result from regulation, though multiple constrained equilibria may exist. This equilibrium multiplicity itself presents one challenge for introducing regulation. From the first part of the proposition, we see that for a refund threshold to offer any improvement on market outcomes, the threshold must lead firms to offer a contract in which consumers strictly prefer to inspect goods after purchasing them rather than inspect them beforehand. Otherwise, regulation that requires more generous refunds only induces firms to charge higher prices and extract surplus from consumers. From the second part of the proposition, we find that as long as the difference $c - \eta$ is not too large, the planner can set a refund threshold for which there is a constrained equilibrium that achieves the social optimum, namely satisfying $(1 - \hat{\theta})p = c - \eta$. This threshold must be larger than the market equilibrium refund as otherwise the regulation is not effective. As long as there are some costs associated with returns, $c - \eta > 0$, the efficient regulating satisfies $\hat{\theta} < 1$, implying that full refunds are inefficient.

IV. Conclusion

In this paper, we have addressed the trade-off that arises when firms offer consumers the possibility to receive a refund when they return a product after they have purchased it. The return option allows consumers to more easily evaluate whether their purchase satisfies their preferences. However, product returns also come at a cost as the salvage value is typically lower than the production cost. To study this trade-off, we have made a methodological contribution to the consumer search literature by augmenting the seminal search model by Wolinsky (1986) in several dimensions. We have characterized the equilibrium outcomes and have shown that the equilibrium is always unique. We have shown that whenever returns are efficient, the market generates too few returns as firms do not internalize the welfare benefit of consumers returning low match value products and continuing their search. In these cases a regulator can improve upon the market outcome.

We think our paper opens several directions for future research. First, throughout the paper we have maintained the assumption that consumers do not learn any product features unless they incur a more substantial inspection cost. Alternatively, one could make a distinction between objective product features that, certainly in online markets, are (like price) quite readily available to consumers, and other more subjective features that require a consumer to inspect more thoroughly. Consumers may then learn part of their match value without incurring the inspection cost and this will affect their decision whether to continue to inspect the product beforehand or to buy and inspect afterwards. Second, multi-product firms may stimulate consumers to engage in simultaneous search by offering to return all items they do not like. Even if consumers only want to buy one unit, this option allows them to keep the product they like best. Firms may prefer such a strategy as their price can take into account that consumers buy the best product, while the additional cost of returning multiple items is small.

These alternative ways of modeling the consumer search process and the way multi-product firms affect search may change the specific trade-offs that are studied in this paper, but the underlying theme will remain the same: there are costs and benefits of offering refund policies and for a proper understanding of especially online markets, it is important to know whether the market provides proper incentives for an efficient resolution of this trade-off.

A Appendix: Equilibria with Returns

Proof of Proposition 1. As the text argues, in a symmetric refund equilibrium, consumers are indifferent between inspecting goods before and after purchasing them and strictly prefer these to the other inspection options, i.e. $U_A = U_B > U_L, U_N$. Claims A.1 and A.2 below prove that a unique price, refund, and consumer surplus satisfies the first order conditions to be an optimal contract in this region, and thus a symmetric equilibrium with returns is unique. Examining the firm's first order conditions, Claim A.3 verifies $\underline{v} < a + \tau$ and $a + p < \bar{v}$, thus an equilibrium requires the refund to exceed the salvage value $\tau > \eta$.

Claim A.1. *There exists a unique triple $(p^*, \tau^*, a^*) \in \mathbb{R}^3$ satisfying the first order conditions for an interior maximum to*

$$\pi_A(p^*, \tau^*, a^*) = \max_{\{(p,\tau):U_A(p,\tau,a^*)=U_B(p,a^*)\}} \pi_A(p, \tau, a)$$

with consumer's expected utility being $a^* = U_A(p^*, \tau^*, a^*) - \epsilon$. Moreover, the solution (p^*, τ^*, a^*) is continuously differentiable in all parameters.

As a refund equilibrium requires consumers to be indifferent between inspecting before and after purchase, both of the following equations hold.

$$a + \tau = r(p - \tau + \beta s + \epsilon) \tag{6}$$

$$a + p = r(s + \epsilon). \tag{7}$$

Taken together, these conditions implicitly define the equilibrium difference between the price and refund by $\delta^* = p^* - \tau^*$ by

$$r(\delta^* + \beta s + \epsilon) + \delta^* = r(s + \epsilon). \tag{8}$$

We now show that there exists a unique δ^* solving (8). Differentiating, the left side is found to be strictly decreasing in δ, is equal to $r(\beta s + \epsilon) \geq r(s + \epsilon)$ when $\delta = 0$, and converges to $E(v) - \beta s - \epsilon < r(s + \epsilon)$ for any increasing sequence $(\delta_n)_{n\in\mathbb{N}}$ converging to $S - \beta s - \epsilon$.[20] Thus a unique δ^* solves (8).

As before, let $p(\tau)$ denote the price keeping consumers indifferent between inspecting before and after purchase for a given refund, i.e. $U_A(p(\tau), \tau, a) = U_B(p(\tau), a)$. From the firm's first order conditions obtained by differentiating $\pi_A(p(\tau), \tau, a)$ in τ, a unique refund τ^* solves

$$\tau^* - \eta = \frac{1 - G(r(s + \epsilon))}{G(r(s + \epsilon))} \frac{G(r(\delta^* + \beta s + \epsilon))}{g(r(\delta^* + \beta s + \epsilon))}.$$

Given (δ^*, τ^*), there is a unique price satisfying $p^* = \delta^* + \tau^*$ and unique $a^* = r(\delta^* + \beta s + \epsilon) - \tau^*$. Continuous differentiability follows from the Implicit Function Theorem.

Claim A.2. *Given the solution (p^*, τ^*, a^*), we have $U_A > U_L$ and $U_A > U_N$.*

The first inequality holds trivially whenever $\epsilon > 0$. By writing the utility functions explicitly, the second inequality is seen to hold if and only if $b(s) < a^* + p = r(s + \epsilon)$, which must hold since $s + \epsilon < s^*$ implies $b(s) < E(v) < r(s + \epsilon)$.

Claim A.3. *The solution (p^*, τ^*, a^*) admits a positive probability of returns $G(a^* + \tau^*) > 0$.*

[20] For ease of reference, basic properties of the reservation value are detailed in Lemma S.2 of the online appendix.

Having shown in the first claim that $\delta^* + \beta s + \epsilon < S$ in equilibrium, we have $a^* + \tau^* = r(\delta + \beta s + \epsilon) > \underline{v}$, and thus $G(a^* + \tau^*) > 0$. □

Proof of Proposition 2. An increase in the inspection cost leads to a larger difference between the price and refund $\frac{d\delta^*}{ds} > 0$ (see Lemma S.2). The change in the welfare with respect to s is

$$\frac{d\mathbf{S}_R}{ds} = -(\delta^* - c + \eta)\frac{g(a^* + \tau^*)}{(1 - G(a^* + \tau^*))^3}\frac{d\delta}{ds} \tag{9}$$

which is negative if and only if the profit earned from returned items is positive. Given that $\lim_{(s,\epsilon)\to(s^*,0)} r(s + \epsilon) = E(v)$, (8) and Lemma S.2 provide that $\delta^* \to S$, thereby completing the proof. Finally, to verify that firms extract the ex ante surplus, note that (6) and Lemma S.2 give $a^* + \tau^* \to \underline{v}$, the firms' optimal refund condition yields $\tau^* \to \eta$, and thus (7) provides that $p^* \to E(v) - \underline{v} + \eta$. □

B Appendix: Structure of Equilibria

A. Technical Lemmas for Proposition 3

Lemma B.1. *There is a unique best reply in the set of points inducing inspection after purchase and it lies in $\mathcal{R}(a)$.*

Proof. Firstly, the fact that $\mathcal{R}(a)$ can admit at most one best reply follows from the objective π_A being strictly increasing in the refund when $\tau \leq \eta$ and quasiconcave thereafter.

A best reply must lie on either the boundary $\mathcal{R}(a)$ or the set making consumers indifferent with inspecting after purchase and leaving $[U_A = U_L] = \{(p, \tau) \in \mathbb{R}^2_+ : U_A(p, \tau, a) = U_L(a)\}$, otherwise a firm can increase its profit by raising its price by a small amount. In the region $[U_A = U_L]$, there are only two potential critical values for firms: one interior with $\tau = \eta$ and the other at the boundary point equating $U_A = U_B = U_L$. The second critical value belongs to $\mathcal{R}(a)$. As depicted in Figure 1, since the first critical value offers a lower refund than would be optimal in $\mathcal{R}(a)$, it must enter the region where consumers prefer to inspect before purchase. For completeness, we analytically verify what the figure depicts to complete the proof.

The regions $[U_A = U_B \geq U_L] = \{(p,\tau) : U_A(p,\tau,a) = U_B(p,a) \geq U_L(a)\}$ and $[U_A = U_L \geq U_B] = \{(p,\tau) : U_A(p,\tau,a) = U_L(a) \geq U_B(p,a)\}$ contain ordered pairs of contracts with the property that if (p,τ) and (p',τ') either both lie in the first or both lie in the second set and $\tau < \tau'$, then $p < p'$. Moreover, in the region $[U_A = U_B \geq U_L]$, pairs with higher prices deliver less utility as U_B is decreasing. As a result, refunds in $[U_A = U_L \geq U_B]$ must exceed those in $[U_A = U_B \geq U_L]$. It follows that the critical value $\tau = \eta$ lies in the region $[U_A = U_L < U_B]$ and so consumers will opt to inspect the product before purchase if a firm makes this deviation. □

A..1 Active Market

To ensure equilibria can support an active market, we must limit the magnitude of the search and production costs. The following three lemmas give the bounds that are needed. First, we characterize the points for which a firm would prefer to play its part in a Wolinsky equilibrium (or equivalently a refund equilibrium when $c = \eta$ and $\beta = 1$) rather than cut its price to incentivize consumers to buy without inspection. In terms of notation, denote the elements in two generic points by $\omega = (c, \beta, \eta, s, \epsilon)$ and $\omega' = (c', \beta', \eta', s', \epsilon')$.

Lemma B.2. *Let $\Omega_N \subset \Omega$ be the points for which the profit when all firms charge the Wolinsky price and consumers inspect before purchase exceeds the profit from cutting the price to induce no inspection. Then, (a) Ω_N is nonempty, and (b) if $\omega \in \Omega_N$ and $\omega' \in \Omega$ is another point with $s' \leq s$ and $\epsilon' \leq \epsilon$, then $\omega' \in \Omega_N$.*

Proof. Offering the Wolinsky price yields profit $\pi_B = p - c$ while cutting to the optimal price inducing no inspection yields profit $\pi_N = \frac{b(s)-a-c}{1-G(a+p)}$. Making use of the Envelope Theorem, we differentiate $(1 - G(a + p))(\pi_B - \pi_N)$ in s to obtain

$$\begin{aligned}\frac{d}{ds}(1 - G(a + p))\,(\pi_B - \pi_N) &= -(p - c)g(a + p)\frac{da}{ds} - \left(\frac{db}{ds} - \frac{da}{ds}\right) \\ &= G(a + p)\frac{da}{ds} - \frac{db}{ds}\end{aligned}$$

which is necessarily negative because $\frac{da}{ds} < 0 < \frac{db}{ds}$. Similarly, differentiating in ϵ

$$\frac{d}{d\epsilon}(1 - G(a + p))(\pi_B - \pi_N) = G(a + p)\frac{da}{d\epsilon}$$

is negative because $\frac{da}{d\epsilon} < 0$. Substituting $a + p = r(s + \epsilon)$ and $p = \frac{1-G(a+p)}{g(a+p)} + c$ we see that both expressions for profit are constant in all parameters except for s and ϵ. Finally, to prove that Ω_N is nonempty, from

$$\begin{aligned}(1 - G(a + p))(\pi_B - \pi_N) &= \frac{(1 - G(r(s + \epsilon)))^2}{g(r(s + \epsilon))} - b(s) + r(s + \epsilon) - \frac{1 - G(r(s + \epsilon))}{g(r(s + \epsilon))} \\ &= -b(s) + r(s + \epsilon) - \frac{(1 - G(r(s + \epsilon)))\, G(r(s + \epsilon))}{g(r(s + \epsilon))}\end{aligned}$$

the difference goes to $\bar{v} - \underline{v} > 0$ as $\epsilon + s \to 0$ and if the difference is positive at $(\epsilon, s) = (0, s_0)$ then it must also be positive at $(\epsilon, s) = (s_0, 0)$. □

Next, consumers must receive nonnegative utility to be willing to participate in a Wolinsky equilibrium.

Lemma B.3. *Let $\Omega_W \subset \Omega$ be the points for which consumers yield positive utility from inspecting before purchase when all firms charge the Wolinsky price. Then, (a) Ω_W is nonempty, and (b) if $\omega \in \Omega_W$ and $\omega' \in \Omega$ is another point with $c' \leq c$ and $s' + \epsilon' \leq s + \epsilon$, then $\omega' \in \Omega_W$.*

Proof. Consumer surplus is $r(s+\epsilon) - \frac{1-G(r(s+\epsilon))}{g(r(s+\epsilon))} - c$ when all firms charge the Wolinsky price. Surplus is positive when $(c, s, \epsilon) \to (0, 0, 0)$ and by the logconcavity of $1 - G$, it is strictly decreasing in these three arguments. □

While the parameters (β, η) continue to have no effect when consumers inspect before purchase, consumers fully absorb the production cost c. Therefore, for consumer surplus to be positive in a candidate Wolinsky equilibrium, the search costs along with the production cost cannot be too large.

Finally, consumers must receive nonnegative utility to be willing to participate in a refund equilibrium. Recall that a candidate refund equilibrium corresponds to the unique triple (p, τ, a) in which all firms choose the same contract in $\mathcal{R}(a)$ satisfying the first order conditions for a maximum and a is the utility from inspecting products after purchase when these are the prices $a = U_A(p, \tau, a) - \epsilon$.

Lemma B.4. *Let $\Omega_R \subset \Omega$ be the points for which consumers yield positive utility from inspecting after purchase when all firms play their part in a candidate refund equilibrium. Then, (a) Ω_R is nonempty, and (b) there is a nonempty subset $\Omega'_R \subset \Omega_R$ such that, if $\omega \in \Omega'_R$ and $\omega' \in \Omega$ is another point with $\eta' \leq \eta$, $s' \leq s$ and $\epsilon' \leq \epsilon$, then $\omega' \in \Omega'_R$.*

Proof. With $\hat{r} = r(\delta + \beta s + \epsilon)$, a symmetric refund equilibrium offers consumer surplus

$$a = \hat{r} - \tau = \hat{r} - \frac{1 - G(\hat{r} + \delta)}{G(\hat{r} + \delta)} \frac{G(\hat{r})}{g(\hat{r})} - \eta \geq \hat{r} - \frac{1 - G(\hat{r})}{g(\hat{r})} - \eta. \tag{10}$$

Define Ω'_R to be the set of points for which, if we replace the value for β with 0, the right side of (10) is positive. That the set Ω'_R is nonempty follows from noting that $\hat{r} \to \bar{v}$ as $s + \epsilon \to 0$. Observe that the right side of (10) is increasing in β and decreasing in η, ϵ, and s as a result of the logconcavity of $1 - G$ and differentiating (the explicit derivatives are contained in Section S.2 of the supplementary material). From this, if the right side of (10) is positive at any point with $\beta = 0$, then it is also positive at any other point with a (weakly) smaller salvage value, search cost, and inspection cost. □

Our equilibrium characterization will begin at a point in the subset satisfying the conditions of the three preceding lemmas $\Omega^* = \Omega_N \cap \Omega_W \cap \Omega_R$. That Ω^* is nonempty follows simply from taking any $\omega \in \Omega_W$, possibly lowering (η, s, ϵ) to obtain a point in $\Omega'_R \cap \Omega_W$, and then possibly lowering (s, ϵ) again to obtain a point in $\Omega_N \cap \Omega'_R \cap \Omega_W$.

A..2 Equilibria

For the remainder of this section, we fix a point $\omega \in \Omega^*$ and characterize equilibria as we vary the production cost c and the post-purchase inspection parameter β. That is, given ω, we characterize equilibria for the set of points $\{\omega' \in \Omega : (\eta', s', \epsilon') =$

$(\eta, s, \epsilon)\}$. We start by demonstrating the nonexistence of a symmetric equilibrium in which consumers make a purchase without ever inspecting the good.

Lemma B.5. *There does not exist an equilibrium with an active market in which, at each firm, consumers prefer to buy without inspection.*

Proof. Consider an active market where consumers prefer to buy without inspection. It is clear that there cannot be such an equilibrium with $U_N > U_B, U_L$ as each firm would have an incentive to raise its price. Thus, in such an equilibrium, it must be that $U_N = U_j$ for a $j \in \{B, L\}$. However, from inspecting the consumer's pay-off of the different options it follows that upon visiting a firm the consumer prefers to buy without inspection if $a \leq b(s) - p$, to inspect before purchase if $b(s) - p < a \leq r(s) - p$, and to leave if $r(s) - p < a$.[21] Thus, as the consumer cannot be indifferent between not inspecting at all and leaving, an equilibrium without inspection must equate $U_N = U_B$ so that the price is $p = b(s) - a$.

Assuming all other firms charge an average price p' and all induce consumers to not inspect the product, a consumer's outside option is simply to incur the search cost to visit another firm and buy without inspection, hence, $a = E(v) - p' - \epsilon$. But this implies that a firm must charge a smaller price than the competition $p = b(s) - E(v) + \epsilon + p' < p'$[22] to ensure that consumers do not inspect the product. As each firm's best reply is to charge strictly less than the average market price, such an equilibrium cannot exist. □

Lemma B.6. *$U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ and $U_B(p_W(a), a) - \epsilon - a$ have unique roots at a_R and a_W, are positive when $a < a_i$ and negative when $a > a_i$ for $i = R, W$ respectively.*

Proof. Given that there is a unique candidate Wolinsky equilibrium and candidate refund equilibrium, the functions $U_B(p_W(a), a) - \epsilon - a$ and $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ have unique roots at a_W and a_R. The Berge Maximum Theorem[23] guarantees the Wolinsky price and refund contract to be continuous in a and thus $U_B(p_W(a), a) -$

[21] Proposition 0 in Doval (2018) identifies this to be the optimal decision rule.

[22] Lemma S.3 verifies $b(s) - E(v) + \epsilon < 0$ when $s + \epsilon < s^*$.

[23] Theorem 17.31 in Aliprantis and Border (2006).

$\epsilon - a$ and $U_A(p_R(a), \tau_R(a), a) - \epsilon - a$ are also continuous in a. Finally to show that the functions cross zero from above, for $a \geq r(s)$, $p_W(a) = p_R(a) = 0$ and thus $U_A(p_R(a), \tau_R(a), a) - \epsilon - a = U_B(p_R(a), a) - \epsilon - a = U_B(p_W(a), a) - \epsilon - a = -\epsilon < 0$. □

Lemma B.7. *Consider an equilibrium with $\beta < 1$, $\rho_R + \rho_W = 1$ and consumer surplus a^*. If $\Pi_R(a^*) \leq \Pi_W(a^*)$, then (a) $p_R(a^*) < p_W(a^*)$, (b) $a^* \in [a_W, a_R]$ with $a_W < a_R$, and (c) If $\rho_R \in (0, 1)$, then $a_W < a^* < a_R$.*

Proof. An equilibrium must feature $a^* < r(s)$ by virtue of the fact that firms will not charge negative prices. We first argue that $p_R(a^*) < p_W(a^*)$. To the contrary, suppose $p_R \geq p_W$. If the Wolinsky price lies on the boundary $p_W = r(s) - a^*$, then by assumption $p_R = r(s) - a^*$, implying a contradiction as consumer surplus can then be computed to be $a^* - \epsilon$. Now suppose that the Wolinsky price lies in the interior. Then, because τ_R is optimally chosen, we have $\tau_R - \eta$ is weakly less than

$$\frac{1 - G(a^* + p_R)}{g(a^* + p_R)} \cdot \frac{g(a^* + p_R)}{G(a^* + p_R)} \cdot \frac{G(a^* + \tau_R)}{g(a^* + \tau_R)} < \frac{1 - G(a^* + p_R)}{g(a^* + p_R)} \leq \frac{1 - G(a^* + p_W)}{g(a^* + p_W)}$$

which is equal to $p_W - c$. The first inequality follows from the logconcavity of G and $\tau_R < p_R$ and the second inequality follows from the logconcavity of $1 - G$ and the assumption $p_R \geq p_W$. Thus we have $\tau_R - \eta < p_W - c$, further implying $\tau_R < p_W$. Equilibrium profit therefore satisfies

$$\Pi_R(a^*) = \frac{p_R - c - (\tau_R - \eta)G(a^* + \tau_R)}{1 - q} > \frac{(p_W - c)(1 - G(a^* + p_W))}{1 - q} = \Pi_W(a^*)$$

contradicting $\Pi_R(a^*) \leq \Pi_W(a^*)$. Hence, we must have $p_W(a^*) > p_R(a^*)$, proving part (a). For (b), the implication of this is that visiting the firms charging the Wolinsky price yields less utility than visiting firms offering the refund contract.

$$U_B(p_W(a^*), a^*) < U_B(p_R(a^*), a^*) = U_A(p_R(a^*), \tau_R(a^*), a^*).$$

The conclusion for (c) follows from the equilibrium condition $a^* = \rho_R\,(U_B(p_R(a^*), a^*) - \epsilon) + \rho_W\,(U_B(p_W(a^*), a^*) - \epsilon)$ and Lemma B.6. □

Lemma B.8. *Differentiating profit in the consumer surplus yields* $\frac{d}{da}(1-q)\cdot\Pi_R(a) = -1+G(a+\tau_R)$*,* $\frac{d}{da}(1-q)\cdot\Pi_W(a) = -1+G(a+p_W)$*, and* $\frac{d}{da}(1-q)\cdot\Pi_N(a) = -1$*.*

The computations supporting this lemma are standard and can be found in Section S.6 of the supplementary material. Next we simplify the problem so that we need only examine a restricted version of the game. Define the *restricted game* to be the same as the original game, except that firms can only offer either the refund contract or charge the Wolinsky price. In other words, the restricted game rules out equilibria and deviations to no inspection.

Lemma B.9. *A strategy profile is an equilibrium if and only if it is an equilibrium of the restricted game.*

Proof. We begin by showing that any equilibrium must involve $\rho_N = 0$ and thus constitutes an equilibrium of the restricted game. Consider an equilibrium in which a fraction ρ_R offer the refund contract, ρ_W charge the Wolinsky price, and ρ_N induce no inspection. Supposing a nonzero fraction of firms offer each contract, $\rho_i > 0$ for $i \in \{W, R, N\}$, and equilibrium utility is a^* we have $p_N(a^*) \leq p_R(a^*) \leq p_W(a^*)$. The relationship $p_R(a^*) \leq p_W(a^*)$ follows from Lemma B.7 and the inequality $p_N(a^*) \leq p_R(a^*)$ results from consumers preferring to inspect before purchase over not inspecting at all when the price is $p_R(a^*)$ and $U_N(p, a^*) - U_B(p, a^*)$ is strictly decreasing in the price. Moreover, the inequality $p_N(a^*) < p_W(a^*)$ must be strict or else firms charging $p_N(a^*)$ yield strictly higher profit than those charging $p_W(a^*)$.

Due to the order of prices, the utility from visiting a firm charging the Wolinsky price must lie strictly below a^*. Thus, from Lemma B.6, we know $a_W < a^*$. From Lemma S.4 it follows that the difference $(1-q)\,(\Pi_W(a) - \Pi_N(a))$ is increasing in a. Given that the parameter is at a point in Ω_N, we have $\Pi_W(a_W) \geq \Pi_N(a_W)$, implying $\Pi_W(a^*) > \Pi_N(a^*)$, contradicting $\Pi_W(a^*) = \Pi_N(a^*)$. In other words, firms charging the Wolinsky price must yield strictly higher profit than those inducing no inspection. By an analogous argument, there can be no equilibrium with $\rho_N > 0$ where $\rho_R = 0$ and $\rho_W > 0$ or $\rho_R > 0$ and $\rho_W = 0$. Finally, Lemma B.5 proves that there can be no such equilibrium when $\rho_R = \rho_W = 0$, i.e., no equilibrium in which all firms induce no inspection.

Next, we show that any equilibrium of the restricted game constitutes an equilibrium of the original game. To do so, it is sufficient to show that there is no incentive for firms to deviate from such an equilibrium by cutting the price to induce no inspection.

By an analogous argument to that above, if the equilibrium involves $\rho_W > 0$, then profit from the Wolinsky price is demonstrably higher than deviating and cutting the price to induce no inspection. Consider $\rho_W = 0$, so that firms are playing a symmetric refund equilibrium. Proposition 1 provides that the contract lies in the interior of $\mathcal{R}(a_R)$. By Lemma B.2, if $\beta = 1$ and $c = \eta$, there is no incentive to deviate given that the parameter belongs to Ω_N. The difference $(1-q)\cdot(\Pi_R(a_R) - \Pi_N(a_R))$ is demonstrably decreasing in β and constant in c; hence, there is is no incentive to deviate for any other value $\beta < 1$ or $c > \eta$. □

This lemma allows us to draw two conclusions. First, firms only offer either the refund contract or the Wolinsky price in equilibrium. Second, when considering whether a strategy profile in which one or both of these contracts are offered is an equilibrium, the only possible deviations are indeed to one of these two contracts.

B. Equilibrium Structure of Contracts

Proof of Proposition 3. Fixing $\omega \in \Omega^*$, the proof proceeds by characterizing the equilibria that arise as we vary β and c. Throughout, we use Lemma B.9 and characterize equilibria of the restricted game. Finally, we also consider the variable a to satisfy $0 \leq a < r(s)$ as this must hold in any equilibrium. Let $c^* = c^*(s, \epsilon)$ denote the production cost at which $a_W = 0$.

The difference in profit between the two strategies is expressed by

$$\Pi_R(a) - \Pi_W(a) = \frac{p_R - c - (\tau_R - \eta)G(a + \tau_R)}{1 - q} - \frac{(p_W - c)(1 - G(a + p_W))}{1 - q}.$$

As the denominator does not influence which contract is more profitable, we need only compare the numerators to identify the best reply. Differentiating their difference, Lemma B.8 yields $\frac{d}{da}(1 - q) \cdot (\Pi_R(a) - \Pi_W(a)) = G(a + \tau_R) - G(a + p_W)$.

When $\beta < 1$, Lemma B.7 guarantees $p_W > \tau_R$ when $\Pi_R(a) - \Pi_W(a)$ is non-positive and thus strictly decreasing in this region. Hence, there exists at most one root $\Pi_R(a^*) - \Pi_W(a^*) = 0$ with the difference positive for $a < a^*$ and negative for $a^* < a$. We refer to this fact as *single-crossing*. Using single-crossing and drawing repeatedly from Lemma B.7, we now prove that if there exists an equilibrium at a point with $\beta < 1$, then it is unique.

- If there is a Wolinsky equilibrium, then $\Pi_R(a_W) - \Pi_W(a_W) \leq 0$ and $a_W < a_R$. There cannot be a refund equilibrium as $a_W < a_R$ implies $\Pi_R(a_R) - \Pi_W(a_R) < 0$. Similarly, an asymmetric equilibrium requires surplus to satisfy $a^* > a_W$, implying $\Pi_R(a^*) - \Pi_W(a^*) < 0$.

- Suppose there is an asymmetric equilibrium with surplus a^*. Because $\Pi_R(a) - \Pi_W(a)$ has a unique root, there can only be one asymmetric equilibrium. Also, because $a_W < a^* < a_R$, $\Pi_R(a_W) - \Pi_W(a_W) > 0 > \Pi_R(a_R) - \Pi_W(a_R)$ and so neither a Wolinsky nor a refund equilibrium exist.

- If there is a refund equilibrium, then from the last two points it is the only equilibrium.

Claim B.1. *When $\beta = 1$ and $c = \eta$, there exists a refund equilibrium, a Wolinsky equilibrium, and an asymmetric equilibrium. These equilibria are equivalent in the sense that they yield the same profit and deliver the same consumer surplus.*

When $\beta = 1$, for a contract to induce returns (i.e. $(p, \tau) \in \mathcal{R}(a)$), it must offer a full refund $\tau = p$. When also $c = \eta$, then granting a full refund yields profit

$$\begin{aligned}(1-q)\pi_A(p,\tau,a) &= p - c - (\tau - \eta)G(a+\tau) = p - c - (p-c)G(a+p) \\ &= (p-c)(1 - G(a+p)) = (1-q)\pi_B(p,a).\end{aligned}$$

Thus, $p_R(a) = p_W(a)$ and $\Pi_R(a) = \Pi_W(a)$ for all a and also $a_R = a_W$. Thus any strategy profile with $\rho_R \in [0,1]$ and $\rho_W = 1 - \rho_R$ constitutes an equilibrium yielding surplus $a^* = a_R = a_W$.

Claim B.2. *When $\beta = 1$ and $\eta < c \leq c^*$, there only exists a Wolinsky equilibrium.*

As before, any contract in $\mathcal{R}(a)$ must offer a full refund when $\beta = 1$. For any price and a full refund, firms would strictly prefer that consumers inspect before purchase as

$$\begin{aligned}(1-q)\pi_A(p,\tau,a) &= p - c - (\tau-\eta)G(a+\tau) = p - c - (p-\eta)G(a+p)\\ &< (p-c)(1-G(a+p)) = (1-q)\pi_B(p,a)\end{aligned}$$

so that firms always strictly prefer the Wolinsky price to the refund contract $\Pi_W(a) > \Pi_R(a)$. Hence, only a Wolinsky equilibrium exists.

Claim B.3. *When $\beta < 1$ and $c = \eta$, there only exists a refund equilibrium.*

When $\beta < 1$, consumers are willing to take less than a full refund to inspect after purchase: $(p,\tau) \in \mathcal{R}(a)$ implies $\tau < p$. Thus, when $c = \eta$

$$\begin{aligned}\max_{(p',\tau')\in\mathcal{R}(a)} \pi_A(p',\tau',a) &> \max_{p'\in[(b(s)-a)_+,r(s)-a]} \pi_A(p',p',a)\\ &= \max_{p'\in[(b(s)-a)_+,r(s)-a]} \pi_B(p',a)\end{aligned}$$

so that firms yield strictly higher profit from the refund contract than the Wolinsky price. Thus, there only exists a refund equilibrium.

Claim B.4. *When $\beta < 1$, there is a production cost $\underline{c}(\beta) > \eta$ for which only a refund equilibrium exists when $c \in [\eta, \underline{c}(\beta)]$.*

Recall that $a_R > 0$ throughout since varying c has no effect on consumer surplus in a refund equilibrium. At $c = \eta$, we know $\Pi_R(a_R;c) > \Pi_W(a_R;c)$. Differentiating the difference yields $\frac{d}{dc}(\Pi_R(a_R;c) - \Pi_W(a_R;c)) = -\frac{G(a_R+p_W)}{1-q}$. As $\lim_{c\to\infty}\Pi_R(a_R;c) = -\infty$, there is some cost $\underline{c}(\beta) > \eta$ for which firms prefer to stick to the refund equilibrium when $c \in [\eta, \underline{c}(\beta)]$ and deviate to the Wolinsky price when $c > \underline{c}(\beta)$. The single-crossing property provides that the refund equilibrium is the unique equilibrium in this region.

Claim B.5. *When $\beta < 1$, there exists $\bar{c}(\beta) > \underline{c}(\beta)$ for which only a unique asymmetric equilibrium exists when $c \in (\underline{c}(\beta), \bar{c}(\beta))$.*

First, we show that the difference $\Pi_R(a_W;c) - \Pi_W(a_W;c)$ is decreasing in c. Because profit in the candidate of Wolinsky equilibrium does not depend on the production cost, we have $\frac{d}{dc}\Pi_W(a_W;c) = 0$. Also $G(a_W + p_W) = G(r(s+\epsilon))$ is independent of c, thus the probability that a consumer continues to search after visiting a firm charging the Wolinsky price is independent of the production cost. Using $\frac{da_W}{dc} = -1$ and differentiating the deviation profit

$$\frac{d}{dc}\Pi_R(a_W;c) = \frac{\partial \Pi_R}{\partial a_W}\frac{da_W}{dc} + \frac{\partial \Pi_R}{\partial c} = -\frac{1 - G(a_W + \tau_R)}{1-q}(-1) - \frac{1}{1-q} < 0.$$

Let $\hat{c}$ satisfy $\Pi_W(a_W, \hat{c}) = \Pi_R(a_W, \hat{c})$. As the previous claim guaranteed no Wolinsky equilibrium at $\underline{c}$, we have $\Pi_W(a_W, \underline{c}) < \Pi_R(a_W, \underline{c})$ and thus $\underline{c} < \hat{c}$. From this and the previous claim, no Wolinsky equilibrium nor refund equilibrium exists in this region.

Now we show that there is an asymmetric equilibrium. In the region $c \in (\underline{c}, \hat{c})$, the absence of a Wolinsky equilibrium or refund equilibrium implies $\Pi_R(a_R) - \Pi_W(a_R) < 0 < \Pi_R(a_W) - \Pi_W(a_W)$ and so the unique root must belong to $a^* \in (a_W, a_R)$. As Lemma B.7 guarantees $p_W(a^*) > p_R(a^*)$, we have

$$U_B(p_W(a^*), a^*) - \epsilon < a^* < U_A(p_R(a^*), \tau_R(a^*), a^*) - \epsilon$$

and thus there exists a unique $\rho_R \in (0,1)$ for which $a^* = \rho_R(U_B(p_W(a^*), a^*) - \epsilon) + (1-\rho_R)(U_A(p_R(a^*), \tau_R(a^*), a^*) - \epsilon)$.

Finally, we need to ensure that consumers are willing to participate in the market. For a given a, increasing the production cost has the effect

$$\frac{d}{dc}(1-q) \cdot (\Pi_R(a;c) - \Pi_W(a;c)) = -G(a + p_W) < 0.$$

As a consequence of single-crossing, if $a^*(c) \in (a_W(c), a_R)$ is the unique root of $\Pi_R(a;c) - \Pi_W(a;c)$, then $a^*(c)$ is continuous and strictly decreasing in a neighborhood of c. Let $\hat{c}'$ be the production cost at which $a^*(\hat{c}') = 0$. As $a^*(\underline{c}) > 0$, it must be that $\underline{c} < \hat{c}'$. Define $\bar{c}$ to be the cost at which either the asymmetric equilibrium becomes Wolinsky or consumer surplus in the asymmetric equilibrium drops to zero, i.e. $\bar{c} = \min\{\hat{c}, \hat{c}'\}$. As previously argued, $\bar{c} > \underline{c}$.

Claim B.6. *When $\bar{c}(\beta) < c \leq c^*$, only a Wolinsky equilibrium exists.*

As the previous claim demonstrated, $\Pi_R(a_W; c) - \Pi_W(a_W; c)$ is strictly decreasing in c and thus $\Pi_R(a_W) - \Pi_W(a_W) < 0$ in this region; hence, a Wolinsky equilibrium exists as long as it delivers nonnegative consumer surplus. □

C Appendix: Market Efficiency

Proof of Proposition 6. Part 1. First, let us show that for θ close enough to τ_R/p_R, there is a constrained equilibrium in which all firms charge the boundary price. Consider a firm's problem when all other firms charge the boundary price $p(\theta) = \frac{\delta}{1-\theta}$, consumer surplus is set according to these contracts $a(\theta) = r(s+\epsilon) - p(\theta)$, and it must offer a contract in the set $X(\theta)$.

Suppose the planner requires refund rate $\theta_R = \tau_R/p_R$. Given that a refund equilibrium exists at this point, charging p_R is the firm's unique best reply. Moreover, computing the change in profit in the price along the ray $\{(p,\tau) \in \mathbb{R}^2_+ : \tau = \theta_R \cdot p\}$ at this point and recalling that $\tau_R - \eta = \frac{1-G(a_R+p_R)}{G(a_R+p_R)} \frac{G(a_R+\tau_R)}{g(a_R+\tau_R)}$

$$
\begin{aligned}
\frac{\partial}{\partial p}\pi_A(p, \theta_R \cdot p, a_R)|_{p=p_R} &\propto \frac{\partial}{\partial p}\left(p - c - (\theta_R p - \eta)G(a_R + \theta_R p)\right)|_{p=p_R} \\
&= 1 - \theta_R G(a_R + \theta_R p_R) - (\theta_R p_R - \eta)\theta_R g(a_R + \theta_R p_R) \\
&= 1 - \theta_R \frac{G(a_R + \tau_R)}{G(a_R + p_R)} > 0.
\end{aligned}
$$

Let P be a compact neighborhood of p_R for which profit is increasing in the price $\frac{\partial}{\partial p}\pi_A(p, \theta_R \cdot p, a_R)|_{p=\tilde{p}} > 0$ for all $\tilde{p} \in P$. Let Θ be a neighborhood of θ_R for which $\min_{\tilde{p}\in P} \frac{\partial}{\partial p}\pi_A(p, \theta \cdot p, a(\theta))|_{p=\tilde{p}} > 0$ for all $\theta \in \Theta$. The Berge Maximum Theorem provides that there is a neighborhood $\Theta' \subset \Theta$ of θ_R for which $\theta \in \Theta'$ implies that the firm's best replies are contained in P. Thus, when $\theta \in \Theta'$, the firm's unique best reply is on the boundary.

Finally, it is immediate that consumers are made worse off since, on the boundary, the sum $a(\theta) + p(\theta) = r(s+\epsilon)$ is constant and $p(\theta)$ is increasing in θ. In this region, because $(1-\theta)p(\theta) = \delta$ is constant, social welfare

$$\begin{aligned}\mathbf{S}(\theta) &= p(\theta) - c + \frac{\delta - c + \eta}{1 - G(r(\delta + \beta s + \epsilon))} G(r(\delta + \beta s + \epsilon)) + a(\theta) \\ &= r(s + \epsilon) - c + \frac{\delta - c + \eta}{1 - G(r(\delta + \beta s + \epsilon))} G(r(\delta + \beta s + \epsilon))\end{aligned}$$

is likewise constant, implying that profit is increasing in θ.

Part 2. The proof proceeds by constructing a symmetric constrained equilibrium in which contracts lie interior to the region for which consumers prefer to inspect after purchase. For a symmetric constrained equilibrium to achieve the social optimum, it is necessary that the price p, threshold θ, and consumer surplus a satisfy the firm's first order conditions for an interior optimum $\theta p - \eta = \frac{1-\theta G(a+\theta p)}{\theta g(a+\theta p)}$, the equilibrium condition $a = r((1-\theta)p + \beta s + \epsilon) - \theta p$, and the optimality condition $(1-\theta)p = c - \eta$. Letting $r^* = (c - \eta + \beta s + \epsilon)$, substitute consumer surplus $a = r^* - \theta p$ and the price $p = \frac{c-\eta}{1-\theta}$ into the firm's first order condition

$$\frac{\theta}{1-\theta}(c - \eta) - \eta = \frac{1 - \theta G(r^*)}{\theta g(r^*)}. \tag{11}$$

When $c > \eta$, the left side is strictly increasing in θ and explodes to infinity, while the right side is strictly decreasing and arbitrarily large when θ is small, implying that there exists a unique $\hat{\theta}$ solving the equation. Then $\hat{p} = \frac{1}{1-\hat{\theta}}(c - \eta)$ is the price and $a^* = r^* - \frac{1-\hat{\theta}G(r^*)}{\hat{\theta}g(r^*)}$ is the surplus. The price must be below the curve $\mathcal{R}(a^*)$ as consumers strictly prefer to inspect after purchase. To verify that consumers are not tempted to forgo inspection altogether, the price and consumer surplus satisfy $a^* + \hat{p} > r(s + \epsilon) > b(s)$; hence, we have $U_N(\hat{p}) < U_B(\hat{p}, a^*) < U_A(\hat{p}, \hat{\theta}\hat{p}, a^*)$.

We now show that firms have no incentive to deviate from this contract as long as the production cost is not too large. As $c \to \eta$, the optimal threshold $\hat{\theta}$ solving (11) converges continuously to one from below. Rearranging (11) finds that the corresponding price $\hat{p} = \frac{c-\eta}{1-\hat{\theta}}$ converges to the finite limit $\eta + \frac{1-G(r(\beta s+\epsilon))}{g(r(\beta s+\epsilon))}$. At $c = \eta$, and $\theta = \hat{\theta} = 1$, a firm's profit function is precisely equal to the Wolinsky profit, but with the sum of inspection and search costs equal to $\beta s + \epsilon$. For one, this means that profit is logconcave in the price, implying that $\hat{p}$ is the unique price satisfying the firm's first order conditions for an optimum within the region inducing inspection after purchase. Given the parameter belongs to a point in $\{\omega' \in \Omega : (\eta', s', \epsilon') = (\eta, s, \epsilon)\}$ for some $\omega \in \Omega^*$, profit strictly exceeds that from any contract inducing no inspection (see Lemma B.2). Thus for c close to η, continuity in the profit functions and the variables $(a^*, \hat{\theta}, \hat{p})$ provides that there is no temptation to deviate to induce no inspection and that $\hat{p}$ is the unique maximizer of $\pi_A(p, \hat{\theta}p, a^*)$.

Because $\beta < 1$, when $\hat{\theta} = 1$, consumers never prefer to inspect before purchase for any price. This is to say that $X(1)$ never intersects $R(a^*)$. As the variables $(a^*, \hat{\theta}, \hat{p})$ are continuous in the production cost, for c in a neighborhood of η, $X(\hat{\theta})$ remains bounded away from $\mathcal{R}(a^*)$; hence, there are no contracts the firm could offer to induce inspection before purchase. □

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