Optimal public investment, growth, and consumption: Evidence from African countries

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How much does public capital matter for economic growth? How large should it be? This paper attempts to answer these questions, taking the case of SSA countries. It develops and estimates a model that posits a nonlinear relationship between public investment and growth, to determine the growth-maximizing public investment GDP share. It empirically also accounts for the crowding-in and crowding-out effects between public and private investment, with equations estimated separately and simultaneously, using System GMM. The paper further runs simulation and examines the public investment GDP share that maximizes consumption. This is estimated to be between 8.4 percent and 11.0 percent. The results from estimating the growth model are in the middle of this range, which is larger than the observed value of 7.2 percent at the end of the sample period. These outcomes suggest that, on average, there has been public under-investment in Africa, contrary to previous findings.

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1. Introduction

The gap separating the world's rich and poor countries remains startling. In 2007, per-capita income in the United States was at least thirty times higher than in eighteen Sub-Saharan African (SSA) countries.[1] Compared to Ethiopia and Tanzania, for instance, the United States has a per-capita income that is more than thirty-eight and forty-six times larger, respectively, when measured in terms of purchasing power parity. Put differently, a typical individual in Tanzania has to work more than a month and a half to earn what his counterpart in the United States earns in a day. Differences in economic growth rates compounded over long periods of time account for these differences. Fortunately, endogenous growth theory suggests that there is something we can do about it.

One of the most important contributions of the "new" growth theory is the insight into the role of fiscal policy in long-run growth. In his seminal contribution, Barro (1990) argues that when the private rate of return of capital is lower than its social rate, optimal allocation calls for further capital accumulation. In this case, public investment becomes important for long-run growth. A vast theoretical literature on endogenous growth underscores the importance of fiscal policy, in the form of public capital flow and stock, for economic growth (e.g., Ziesemer, 1990, Futagami et al., 1993, Glomm and Ravikumar, 1994, 1997, Turnovsky, 2000, Agenor, 2008).

Existing empirical evidence is mixed, however, due to mainly methodological and model specification issues as well as due to differences in samples. Recent estimates of the elasticity of output with respect to public capital range from zero to a value that is higher than the output elasticity of private capital, for instance.[2] Fedderke and Bogetic (2009) presented five reasons for the contradictory empirical

[1]Based on Penn World Table 6.3 (Heston et al., 2009).

[2]For instance, using cross country data, Canning (1999), Aschauer (2000b), and Demetriades and Mamuneas (2000) estimate elasticity of output to be as large as that for private capital; Miller and Tsoukis (2001) and Kamps (2005) estimate values below private capital's. On the other hand, Milbourne et al. (2003) report insignificant effects of public investment on growth and output. Using country specific data, Everaert and Heylen (2004) and Fedderke et al. (2006) estimate elasticity values of public capital, for Belgium and South Africa, respectively, from 0.3 to 0.5. Luoto (2011) estimates about 0.1 for Finland.

findings: the presence of nonlinearity; crowding out effect; endogeneity; an indirect or complementarity effect (rather than a direct productivity effect); or problems of aggregation. We address in this paper the first four of these reasons while providing a more comprehensive analysis of optimal public investment, with a focus on SSA countries.

The issue of the optimal level of public investment is under-researched for SSA, as much of the discussion in the literature has been on attracting private investment to this region. However, Foster and Briceno-Garmendia (2010) argue that countries in SSA lag behind their developing countries' peers in any measure of infrastructure. According to these authors, there are in particular significant differences among SSA and other low- and middle-income countries in terms of paved roads, telephone mainlines, and power generation, with SSA possessing less than four, seven and eight times the respective infrastructure units than their counterparts. The cost of infrastructure service in SSA is, furthermore, twice more expensive than elsewhere. In contrast, Devarajan et al. (2001, 2003) argue that most African countries have already public over-investment, probably the result of creating rent-seeking opportunities. This ambiguity is likely explained by the implied low 'quality' of public investment due to inefficient public allocation. However, as African governments seem to have improved governance in the more recent period, it is expected that higher quality would now accompany a given quantity of public investment.

Moreover, although the literature on the impact of public capital on economic growth has grown voluminous in the past few decades, only very few studies have addressed Africa (Ayogu, 2007). In particular, the issue of the growth-maximizing levels of public capital for African economies is yet to be addressed, as existing studies tend to employ linear models.[3] Nonlinear models have been applied to other parts

[3]For instance, Fedderke et al. (2006) and Fedderke and Bogetic (2009) study the impact of infrastructure on growth for South Africa, and Ayogu (1999) for Nigeria. Calderon and Serven (2008) and Calderon (2009), using large panel data sets covering over 100 countries, examine the relationship between infrastructure assets and growth in SSA; Estache et al. (2005), applying an augmented Solow model with infrastructure variables, and using pooled OLS study the relationship for 41 SSA countries. Boopen (2006) studies the impact of transport infrastructure on economic growth for SSA countries, using a dynamic panel model of Difference GMM method. Devarajan et al. (2001, 2003) examine the productivity of public investment in Africa employing 2SLS for a

of the world, however.[4]

The relevant question for policy is not only whether public capital is productive, that is, whether or not a unit increment on public capital stock increases output or growth, but also whether public capital is overall growth-enhancing given that it diverts resources from other activities (Romp and de Haan, 2007, Canning and Pedroni, 2008). The reason is that public capital can have a negative as well as a positive effect on the economy. Even though an adequate and efficient supply of public capital promotes output and growth, the burden resulting from financing it may have an adverse effect as well, such as crowding-out of private capital. A highly enhanced transportation system, for instance, could improve the efficiency of trucks, but overly burdensome taxes to finance it could deter the accumulation of these trucks (Aschauer, 1998). Should the private sector not receive a net advantage from the infrastructure development, there would be no increase in output. It is this phenomenon that mainly gives rise to the nonlinearity between public capital and growth.

This paper first develops a simple endogenous growth model in an overlapping-generation framework. It then estimates the implied nonlinear relationship between public investment and economic growth, resulting from a positive public investment and a negative taxation effect. The growth-maximizing level of public investment is determined by employing various nonlinear estimation techniques to dynamic panel data from SSA countries: System GMM, weighted least square (WLS), and seemingly unrelated regression (SUR). All three methods are applied with fixed effects. Estimation of dynamic panel models with fixed effects give consistent estimates implying only a weak bias when there is a sufficiently long time period. Given the relatively small sample in time dimension, however, we also estimate the growth model using non-linear System GMM.[5] In contrast, earlier studies that estimate the elasticity of output of public capital in nonlinear models usually apply simple calibration (e.g.,

cross section of countries.

[4]For example, Aschauer (2000a) and Kamps (2005) examined the optimality of public capital in the United States and European countries, respectively, while Miller and Tsoukis (2001) was on a set of low and middle-income countries.

[5]Section 3 provides a detailed discussion of this phenomenon.

Aschauer, 2000a, Miller and Tsoukis, 2001) or nonlinear least squares methods (e.g., Kamps, 2005), or that simply use cross-country analysis, which runs the risk of taking into account only the short-term effects (see Glomm and Ravikumar 1997).

Limiting the growth impact of public investment to its direct effects may provide a poor indicator of its importance in the economy. This is because public investment is likely to affect other important variables such as private investment. Moreover, policy-induced changes of growth may in turn influence population growth, for instance, with further implications for growth. The current paper attempts to capture these indirect effects through formulating and estimating a system of difference equations that account for the mutual interaction among growth, public and private investments and population growth.

In addition to estimating the growth equation, we regress public investment on private investment and conversely. We examine the crowding-in (complementarity) and the crowding-out effects, using interacting variables. We also treat population growth endogenously and estimate an equation for it. All four equations are estimated separately and also together as a simultaneous equations system in order to account for possible correlation across equations, using System GMM. Finally, we run simulations to further examine the optimality issue with policy experiments using coefficient estimates from both the separate- and simultaneous-equations estimations.

Among our findings is that public investment has a positive effect on growth. Perhaps more interestingly, the growth maximizing public investment GDP percentages is between 9.0 percent and 10.0 percent when applying separate and simultaneous equations estimations, which is larger than the mean of the observed values 7.2 percent at the end of the sample period. Furthermore, from the policy simulation experiment, the sum of the discounted future consumption is maximized when there is an increase in the public investment GDP share in 2015, from 7.2 percent to values between 8.4 percent and 11.0 percent, depending on the discount rate, the accelerator, complementarity and crowding-out effects. When estimates are used from the simultaneous equations regression, the complementarity is much stronger and leads to a value of 11.0 percent (at 4 percent discount rate), for instance.

We organize the rest of the paper as follows. In Section 2, we provide the theoretical model. Sections 3 and 4 present the empirical estimation and the simulations, respectively. Section 5 contains the conclusion.

2. Theoretical model

In neoclassical growth models, exogenous technical progress is the source of long-run growth, leaving no room for policy decisions to have long-term effects on economic growth. Therefore, a shock to the public policy variable will have a transitory effect on the economy, affecting only the level of (long-run) output. By contrast, in endogenous growth models, policies may have a lasting impact on growth rates. Hence, in these models, a shock to public capital may influence both the long-run growth rate and the output level.

In this section, we develop a simple endogenous growth model in an overlapping-generations framework where agents live two periods. The model will form the basis for the empirical analysis in a later section of the paper. Our model allows for the capital stock to be long-lasting, even with a zero depreciation cost. In contrast to standard models (see, for e.g., Barro, 1990, Futagami et. al., 1993, Glomm and Ravikumar, 1994, 1997, Turnovsky, 2000, 2004), aggregate capital may depreciate nonlinearly. Capital is assumed to be heterogeneous, so that current investment may not add to the existing stock on a one-to-one basis.[6] Therefore, the model also allows adjustment cost associated with new investment in the spirit of Lucas and Prescott (1971) and Basu (1987).[7] The model explicitly captures the nonlinear relationship between both the flow and the stock of public capital and economic growth, and their respective growth maximizing levels are derived.

[6]For instance, in the case of public capital, the existing aggregate capital stock consists of past investment in electricity, telecommunication, roads, etc.

[7]However, these bodies of literature do not focus on public capital and growth.

2.1. The model

Consumers

We use an overlapping-generations model with logarithmic preferences and technologies of a representative agent, as in Glomm and Ravikumar (1997). When young, that is, during the first period of life, the individual is endowed with a unit of labor, which she supplies to the representative firm inelastically. Her income is equal to the wage income (w_t) . The government taxes this income at a fixed flat rate tax (ψ), in order to finance public investment. The individual allocates after-tax income between current consumption (c_t) and saving (s_t^k). When she is old, she consumes (c_{t+1}) what she has saved in the previous period plus the after-tax return from saving.[8]

$$u(c_t, c_{t+1}) = \ln c_t + \beta \ln c_{t+1} \tag{1}$$

$$c_t + s_t^k = (1-\psi)w_t \tag{2}$$

$$c_{t+1} = (1 + r_t(1-\psi))\, s_t^k \tag{3}$$

where r_t is the interest rate, net of the depreciation and the adjustment costs of capital. Private capital is accumulated according to the following equation,

$$k_{t+1} = B(k_t)^{1-\delta} \left(k_t(1-\kappa) + s_t^k\right)^{\delta} \tag{4}$$

where κ, δ and k_t represent the depreciation cost, the adjustment cost and the private capital stock, respectively. Therefore, the model explicitly allows installation cost for new investment and depreciation cost. When $\delta = 0$, adjustment cost is too high to change both private and public capital. But when $\delta = 1$, adjustment cost is zero, and capital stocks are accumulated according to the perpetual inventory method

[8]The model is kept simple for sake of tractability and technicality. For instance, population growth is set to zero, as it could result in scaling effects in growth. The applications of log-linear preference and production function, and fixed flat-rate taxes on income and capital (in contrast to alternative financing methods) serve to obtain a tractable solution.

(e.g., $k_{t+1} = k_t\,(1-\kappa) + s_t^k$ if $B = 1$). When $\delta \in (0,1)$, adjustment cost is different from zero. Current investment adds to the stock of capital after adjustment made for installation costs.

Government

The government budget is always balanced and given by,

$$s_t^g = y_t \psi \tag{5}$$

where s_t^g and y_t, are public investment and aggregate income, respectively.

The public capital accumulation equation is given by,

$$G_{t+1} = B\,(G_t)^{1-\delta}\,(G_t\,(1-\kappa) + s_t^g)^{\,\delta} \tag{6}$$

Similar to (4), κ and δ are the depreciation rate and the adjustment cost associated to the public capital stock (G_t).[9]

Firms

The production function of the representative firm has the Cobb-Douglas form:

$$y_t = A\,(G_t)^{\alpha}\,(k_t)^{1-\alpha} \tag{7}$$

where y_t denotes output. There is no population growth, and labor is standardized to be unity ($l_t = 1$).

The firm maximizes profit within a competitive economy setting, taking prices and

[9]We set similar technological parameters for public and private capital in order to avoid unnecessarily complicating the model.

public capital as given,

$$\max_{k_t} \left\{ A \left(G_t\right)^{\alpha} \left(k_t\right)^{1-\alpha} - w_t - R_t k_t \right\} \tag{8}$$

where R_t denotes the cost of capital, including a rental price or interest for a unit of capital paid to households (r_t) and adjustment and depreciation costs. The first-order condition for profit maximization thus gives,

$$R_t = (1-\alpha)\, A \left(G_t\right)^{\alpha} \left(k_t\right)^{-\alpha} \tag{9}$$

And, the zero-profit condition in the competitive economy leads to the wage rate,

$$w_t = \alpha A \left(G_t\right)^{\alpha} \left(k_t\right)^{1-\alpha} \tag{10}$$

Competitive equilibrium

The representative household of period t solves the following problem, obtained by substituting (2) and (3) into (1),

$$\max_{s_t^k} \left\{ \ln\left((1-\psi)\, w_t - s_t^k \right) + \beta \ln \left(1 + (1-\psi) r_t\right) s_t^k \right\} \tag{11}$$

taking prices as given. The optimization yields,

$$s_t^k = (1-\psi) w_t \beta / \left(1+\beta\right) \tag{12}$$

Eq. (12) shows the agent's optimal saving as a function of her wage income. Dividing both sides by (y_t), and using (5) and (10), one obtains

$$s_t^k/y_t = (1 - s_t^g/y_t)\alpha\beta/\,(1+\beta) \tag{13}$$

Thus, eqs. (12) and (13) capture the crowding-out effect of the public variable through taxation.

Capital dynamics and growth

We get the dynamics of the private capital stock, first by substituting eq. (12) into eq. (4), and using (10),

$$k_{t+1} = Bk_t\left(1 - \kappa + A(1-\psi)\chi\left(G_t/k_t\right)^{\alpha}\right)^{\delta} \tag{14}$$

where $\chi \equiv \beta\alpha/\,(1+\beta)$.

The difference equation for the public capital stock is computed, by substituting (5) into (6), and using (7), as:

$$G_{t+1} = BG_t\left(1 - \kappa + A\psi\left(G_t/k_t\right)^{\alpha-1}\right)^{\delta} \tag{15}$$

Equations (14) and (15) characterize the dynamics of the economy during the transition. They explicitly demonstrate complementarities among public and private capital. On the other hand, (14) captures the crowding-out effect of public investment, through a negative relationship between taxation (ψ) and private capital accumulation (k_{t+1}).

From (14) and (15), we obtain the following difference equation for the public-private capital ratio,

$$G_{t+1}/k_{t+1} = \left(G_t/k_t\right)\left(\left(1 - \kappa + A\psi\left(G_t/k_t\right)^{\alpha-1}\right)/\left(1 - \kappa + A\chi(1-\psi)\left(G_t/k_t\right)^{\alpha}\right)\right)^{\delta} \tag{16}$$

The log-linearized version of eq. (16) is shown to be stable in Appendix A.

On the balanced growth path, considering (16), the public-private capital stock ratio is constant:

$$G/k = \psi / \left((1-\psi)\chi \right) \tag{17}$$

Also, from (7), y/k is constant. Therefore, the capital stocks and output grow at the same rate γ_y:

$$\gamma_y = \ln\left(G_{t+1}/G_t\right) = \ln\left(k_{t+1}/k_t\right) = \ln\left(y_{t+1}/y_t\right) \tag{18}$$

Growth maximizing public capital stock and flow

Using (14), γ_y is easily computed,

$$\gamma_y = \ln B + \delta \ln\left(1 - \kappa + A\chi(1-\psi)\left(G_t/k_t\right)^{\alpha}\right) \tag{19}$$

Solving for ψ from (17) and substituting the result into (19), we obtain

$$\gamma_y = \ln B + \delta \ln\left(1 - \kappa + A\chi\left(G/k\right)^{\alpha} / \left(\chi\left(G/k\right) + 1\right)\right) \tag{20}$$

Eq. (20) represents the growth rate of the economy as a function of the steady-state public-private capital *stock* ratio G/k. The last term captures the nonlinear relationship between economic growth and the public-private capital ratio.

The public-private capital stock ratio ($(G/k)^*$) that maximizes the growth rate (20) is,

$$(G/k)^* = (1+\beta) / \left(\beta(1-\alpha)\right) \tag{21}$$

With regard to the *flow* of public capital (public investment), we substitute (17) into (19), and use (5) to replace the tax rate, and obtain

$$\gamma_y = \ln B + \delta \ln \left(1 - \kappa + A\chi^{1-\alpha}(1 - s_t^g/y)^{1-\alpha} \left(s_t^g/y\right)^{\alpha}\right) \tag{22}$$

Eq. (22) shows the growth rate of the economy as a function of the public investment-output ratio (s^g/y). Maximizing it with respect to s^g/y, we get the following familiar result,

$$\left(s^g/y\right)^* = \alpha \tag{23}$$

Therefore, (23) is the growth-maximizing productive government expenditure, which balances the negative taxation and the positive productive effects of public investment on the economy, as does the stock of public capital in eq. (20). This is also the optimal public investment when $\delta = 1$ and $\kappa = 1$ (see, for e.g., Barro, 1990 and Futagami et al., 1993).

Both (22) and (23) will be referred to in the next section for empirical estimation.

3. Estimation

This section empirically examines the nonlinear relationship between the *flow* of public capital (public investment) and growth using panel data from SSA countries, as data on public capital stock are often limited and unreliable.[10] It also analyzes complementarities and cowding-out effects between public and private investment. We estimate not only the growth model of Section 2 but also a system of difference equations involving population growth and economic growth, as well as public investment and private investment. Estimations of equations are conducted both separately and simultaneously using various econometric techniques, including

[10]Construction of public capital stock data depends on rather arbitrary assumptions about depreciation and initial capital stock.

nonlinear System GMM, seemingly unrelated regression (SUR) and weighted least squares (WLS).

The first estimation equation is a growth equation, based on (22), that regresses per capita GDP growth on public investment and other control variables (lagged dependent variable, private investment, and population growth). The second and third estimation equations characterize the dynamics of the capital flows. The fourth is a population growth equation that regresses population growth on lagged population growth and GDP per capita variables. The growth estimation yields the growth-maximizing level of public investment. We compare this estimate with a consumption-maximizing level of public investment from simulating a system of macroeconomic dynamics with the four difference equations that captures the mutual interaction among public investment, private investment, population growth, and economic growth.

The panel data used in the study cover 33 SSA countries, mainly, for the period 1967 to 2008.[11] Table 1 provides summary statistics, definitions and data sources of the variables used in the estimation. The average public investment of these countries over the specified period is 7.1 percent of real GDP while the average growth rate of real GDP per capita is 0.8 percent.

TABLE 1 ABOUT HERE

3.1. Econometric Methods

We estimate the dynamic panel equations, first, separately using System GMM and, second, together, as a simultaneous equations system using WLS, SUR and System GMM. All methods include fixed effects. Although fixed effects estimations of dynamic panel data are biased, the bias approaches zero for a large time-dimension sample size (Bond, 2002). As a general rule, the fixed effects bias is of order $1/T$, where T represents time-dimension. Thus, it is sufficiently small for $T = 30$ or more

[11] Countries are included in the study based on the availability of relatively reliable data. These are: Angola, Benin, Botswana, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Congo, Dem. Rep., Congo, Rep., Cote d'Ivoire, Equatorial Guinea, Ethiopia, The Gambia, Ghana, Guinea, Kenya, Malawi, Mali, Mauritania, Mozambique, Namibia, Niger, Rwanda, Senegal, Sierra Leone, South Africa, Sudan, Tanzania, Togo, Uganda, Zambia, and Zimbabwe.

(see, for e.g., Judson and Owen, 1999, Baltagi, 2008, Ch.8). In our data, the time dimension T could be as little as 21 years, based on the average of 700 observations for 33 countries.[12] Hence the fixed effects estimates could suffer from a downward bias in the coefficient of the lagged dependent variable. The coefficient of the public investment variable may also be affected then.

We, therefore, also present nonlinear estimates based on the System GMM method. The System GMM version uses one equation in first differences with lagged levels as instruments and one within-groups estimator equation in levels using lagged first differences as instruments. The coefficients of the two equations then are restricted to be the same for the level variables and their counterparts in the first difference equation. Alternatively, the first difference equation could be replaced by the Arellano and Bover (1995) method of orthogonal deviations. Implementation of non-linear items is more easily tractable in the first difference version of System GMM given the complexity of the orthogonal deviation model. On the other hand, the orthogonal deviation method has the advantage of losing fewer observations in case of missing values (Roodman, 2006).

3.2. Separate equations estimations with System GMM

3.2.1. Growth equation

We now estimate the possible nonlinear relationship between the flow of public capital and growth using panel data from SSA countries based on equations from the model developed in Section 2. First, we employ eq. (22),[13] with standard control variables - lagged dependent variable, lagged private investment as a share of GDP, population growth rate and time trend - to determine whether there exists a nonlinear relationship between public investment and growth. Then, we obtain an estimate for the output elasticity of public capital (α). Finally, we use the estimated value for α and eq. (23), in order to obtain the growth-maximizing rate of public investment, which can then be compared to the existing value of the panel average at the end of

[12]From the panel period 1967-2008 in the data, the maximum time period is 41 years; however, due to missing data, the effective time period averages 21 years.

[13]We only consider the case when there is complete depreciation of capital and zero adjustment cost, $\kappa = 1$ and $\delta = 1$.

the period and/or a result from a simulation analysis.

We thus rewrite (22) (with no adjustment cost and complete depreciation), including control variables and error terms, in a panel form:

$$\left(\gamma_y\right)_{it} = m_{it} + \mathbf{a}\mathbf{x}_{it-j} + e_i + u_{it} \tag{24}$$

where $\left(\gamma_y\right)_{it} = \ln y_{it} - \ln y_{it-1}$; e_i and u_{it} denote the fixed effects and error terms, respectively; $\left(\gamma_y\right)_{it}$ and $\mathbf{x}_{it-j}$ represent growth rates of GDP per capita and a vector of control variables, respectively; and, m_{it} stands for the function of public investment as a share of GDP, which has a nonlinear relationship with growth rate of GDP per capita (22):

$$m_{it} = \ln\left(\left(1 - \left(s^g/y\right)_{it}\right)^{1-\alpha}\left(\left(s^g/y\right)_{it}\right)^{\alpha}\right) \tag{25}$$

We use eq. (24) to estimate α and **a** applying nonlinear regression methods. The standard formulation for our growth regression, – an elaborated formulation of (24), – then is as follows:

$$\begin{aligned}\ln\left(y_{it}\right) &= a_1 \ln\left(y_{it-1}\right) + (1-\alpha)\ln\left(1 - \left(s^g/y\right)_{it}\right) + \alpha \ln\left(\left(s^g/y\right)_{it}\right) \\ &\quad + a_2 \ln\left(s^k/y\right)_{it-1} + a_3\left(\gamma_p\right)_{it}^2 + a_4\tau_t + e_i + u_{it} \end{aligned} \tag{26}$$

where $\left(\gamma_p\right)_{it}^2$ and τ_t denote the square of population growth and time, respectively.

Eq. (26) shows a dynamic panel data model, where we have rewritten (24) with growth expressed difference in log income levels and have defined the control variables explicitly.[14]

We first estimate (26) separately using the first difference approach to System

[14]Absence of the control variables (and $a_1 = 1$), (26) reduces to the special case (22) with $\kappa = 1$ and $\delta = 1$.

GMM.[15] The result is as follows (t-values in parentheses):[16]

$$\begin{aligned}\ln(y_{it}) \;=\;& \underset{(47.1)}{0.942}\ln(y_{it-1}) + \underset{(7.06)}{0.902}\ln\left(1-(s^g/y)_{it}\right) + \underset{(7.06)}{.098}\ln\left((s^g/y)_{it}\right) \\ & + \underset{(2.04)}{0.029}\ln\left(s^k/y\right)_{it-1} - \underset{(-8.5)}{22.5}\left(\gamma_p\right)^2_{it} + \underset{(1.65)}{0.001}\tau_t + e_i + u_{it} \qquad (26')\end{aligned}$$

The coefficient for the lagged dependent variable is significant, and at 0.94 it indicates the persistency of output. The coefficient of the time trend variable is also positive and significant. The nonlinear coefficient estimate of public investment, – the growth maximizing level of public investment as denoted by α in the theoretical model, – is thus estimated at 0.098. This result suggests, then, the need to increase public investment, as percent of GDP, from its 7.2 percent level at the end of the sample period to 9.8 percent. The coefficient for private investment share is 2.9 percent and is also significant. The population growth rate is significant in its squared form and has a negative coefficient, in line with growth theory. The Sargan p-value of 0.27 is close to the interval of 5 percent to 25 percent as suggested by Roodman (2009). Note that our 9.8 percent estimate of the optimal level of public investment is, in general, smaller than most of those in the recent literature (see Section 1).

3.2.2. Private investment equation

Public investment is believed to have both complementary and crowding-out effects on private investment. In the growth model, eqs. (12) and (13) show that public investment crowds out private investment, as the tax used to finance it distorts private saving. Eqs. (14) and (15), on the other hand, capture complementarities between the stock variables.[17]

Our second estimation equation is a regression of private investment on public investment, both as shares of GDP. We set up the model intended to empirically determine the net effects of crowding-in and crowding-out of public investment. We

[15] We use GMM-HAC (GMM cum heteroscedasticity and autocorrelation consistent standard errors). The econometric formulation of the systems GMM approach and the list of instruments are found in the technical appendix.

[16] This result also appears in the first column of Table 2, equation I.

[17] By construction, eq. (7) implies that increasing public capital (for a given private capital) enhances the productivity of private capital and conversely.

thus include an interaction variable that is useful to empirically examine complementarities among the investment variables.[18]

For estimation, we use System GMM in its orthogonal deviation variant by Arellano and Bover (1995):

$$\begin{aligned}\ln\left(s^k/y\right)_{it} &= b_1 \ln\left(s^k/y\right)_{it-1} + b_2 \ln\left(s^g/y\right)_{it-2} + b_3\left(\gamma_y\right)_{it-1} \\ &\quad + b_4\left(\ln\left(s^k/y\right)_{it-2} * \ln\left(s^g/y\right)_{it-1}\right) + e_i + u_{it} \end{aligned} \tag{27}$$

The first term of equation (27) denotes lagged private investment while the second represents two-periods lagged public investment, which captures the crowding-out effect in the spirit of eq. (13).[19] The third, one-period lagged GDP per capita growth rate, captures the accelerator mechanism; higher lagged growth is expected to lead to a higher level of current investment. The fourth term is the interaction variable, which consists of the first and the second lags of public and private investment, respectively. The effect of public investment is higher (in the case of a positive coefficient) if private investment in the previous period was higher implying complementarity. It thus captures complementarity beyond the log linear structure of the theoretical model, a step also known as leading to a translog function.

The estimation results are shown in column 1 of Table 2 under equation II. The coefficient for the two-period lagged public investment is -0.125 whereas the coefficient of the interaction variable is 0.057. Both are significant at the 1 percent level. Therefore, public investment has both crowding-out and complementarity effects. As the log of the private investment share is in the order of magnitude of about two, the positive complementarity effect and the crowding-out effect of public on private investment are similar in order of magnitude, with slight dominance of the complementarity effect.[20]

[18] The growth model (in Section 2) does not feature such phenomena due to the particular production functional form adopted. However, note that the application of standard production functions is justified technically, as they are well-behaved and, often, provide tractable solutions.

[19] Lagged values of variables are often used as explanatory variables in the literature of dynamic panel data and growth (see, e.g., Arellano and Bond, 1991, Bond et al., 2010). This is intuitive as it may take some time before certain macroeconomic variables have effects on the economy.

[20] Cavallo and Daude (2011) find a negative effect of public investment, but they do not use inter-

TABLE 2 ABOUT HERE

3.2.3. Public investment equation

Our third estimation equation treats public investment as the dependent variable where the lags of public, private and growth rates are the independent variables. This formulation is in consideration of policy responses. Policy makers often react to changes in macroeconomic variables. For instance, an increase in private investment or stronger growth may lead to a change in public investment policy.

Similar to the previous equations we formulate and estimate the public investment equation using flow variables and with a more general specification that includes an interaction term.

$$\begin{aligned}\ln\left(s^g/y\right)_{it} &= c_1 \ln\left(s^g/y\right)_{it-1} + c_2 \ln\left(s^k/y\right)_{it-1} + c_3 \ln\left(s^k/y\right)_{it-2} + c_4\left(\gamma_y\right)_{it-2} \\ &\quad + c_5\left(\ln\left(s^k/y\right)_{it-2} * \ln\left(s^g/y\right)_{it-1}\right) + e_i + u_{it} \qquad (28)\end{aligned}$$

We estimate equation (28) using the orthogonal deviation method of Arellano and Bover (1995) for System GMM. The results are presented in column 1, Table 2, under equation III. Government action is weakly self-perpetuating, as indicated by the low coefficient for the lagged dependent variable of 0.38. Lagged private investment has a net negative effect on public investment as the negative coefficient of the second lag dominates the positive coefficient of the first lag. Productive government spending partly complements private investment as shown in the interaction-term coefficient of 0.16. Finally, GDP per capita growth, lagged two years, has a positive impact.

For the equations (26), (27) and (28), the bottom part of Table 2 shows that the second-order serial correlation is very low, in particular the coefficient is below 0.2, making the Sargan p-value the relevant statistic for judging the validity of the instruments (see Roodman, 2006). The Sargan p-values are indeed in, or close to, the interval of 5 to 25 percent as recommended by Roodman (2009). Whenever there is more than one instrument per regressor, we have applied a Sargan difference test (not reported) to verify that again the values are in the relevant interval of 5 and 25 percent.

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action terms.

3.2.4. Population growth equation

Our fourth estimation equation is a population growth equation. Recall that we want to run simulations of a system that characterizes the macroeconomic dynamics of the economy in order to further examine the optimal public investment, and also analyze its effects on the economy. So far we have three equations (eqs. (26), (27) and (28)) but four variables (income, public and private investment and population growth).

The fourth equation is:

$$\begin{aligned}\left(\gamma_p\right)_{it} &= d_1\left(\gamma_p\right)_{it-1} + d_2\left(\gamma_p\right)_{it-2} + d_3\left(\gamma_p\right)_{it-3} + d_4\left(\gamma_p\right)_{it-7} \\ &\quad + d_5\left(\gamma_p\right)_{it-8} + d_6 \ln y_{it-2} + d_7 \ln y_{it-3} + d_8 \ln y_{it-5} + e_i + u_{it}\end{aligned} \tag{29}$$

The data used for estimating (29) have more than thirty observations per country. Thus, the fixed effects bias is sufficiently small. Therefore, we estimate it with fixed effects, using lagged levels as instruments, while taking into account the period-SUR version of panel corrected standard errors (PCSE) similar to (27) and **(28).** The results appear also in column 1 of Table 2 under equation IV. The coefficients of lagged values of population growth sum up to about 0.96.[21] Although the third-lagged income coefficient is positive, the coefficients of the second and fifth lagged are negative; in addition, the sum of all lagged income coefficients is negative suggesting the standard demographic transition.

[21]Some lagged variables are dropped due to collinearity and insignificance. Panel unit root tests for population growth and income do not deliver clear-cut results, as is typical of cases of near-unit roots. We use the residuals from regression of equation (29) to run panel unit root tests. The unit root hypothesis is always rejected, indicating co-integration of the variables in the equation. Using the Breusch-Godfrey test for serial correlation in the presence of endogeneity, we have also re-run the regression with lagged residuals added to the regression. The lagged residuals turn out to be insignificant, indicating an absence of serial correlation and of the corresponding potential bias in the coefficients.

3.3. A simultaneous equation system with System GMM

We estimate eqs. (26), (27), (28) and (29) as a simultaneous equations system as well. We first estimate it using the WLS method where the reciprocals of the variances of the residuals from least squares are employed as weights. Then, to account for possible correlations across equations, we also estimate the system using the SUR method. Finally, in order to deal with both endogeneity and contemporaneous correlation, we use System GMM.

In the latter case, we set up the system in which we write each of the first three equations as a System GMM estimator model, in first-differences, and the fourth equation as a within-groups estimator (fixed-effects) model. This approach combines the strength of the SUR estimator, taking into account relations between the residuals of the equations, and that of the System GMM estimator, taking into account fixed effects and endogeneity without imposing a normality assumption on the residuals.

The results for the WLS and the SUR methods are reported in columns 2 and 3 of Table 2, respectively. Column 4 reports results from the GMM-HAC.[22] Coefficient estimates for the public investment variable, in the growth equation, using WLS and SUR, are lower than those from the GMM methods. Coefficients estimate for private investment and population growth variables are also lower.

Across the four approaches, in Table 2, all coefficients have the same sign and are significant mostly at the 1 percent level. They differ slightly in magnitude, though. For instance, in the growth regression, the coefficient of public investment is lowest, about 6.3 percent, in the non-instrumented models (WLS and SUR of columns 2 and 3). It is highest, about 9.8 percent, in the separate equations System GMM estimation (column 1), and it is about 9.3 percent when the System GMM simultaneous equations estimation is used (column 4). Private investment and population growth effects are also found to be stronger in the GMM estimations. In all cases we have a significantly positive time trend, suggesting positive long-run growth.

Comparing the GMM estimates, in the private investment estimation equation,

[22]The instruments used for the GMM-HAC procedure are presented in the technical appendix. Again whenever there is more than one instrument per regressor we have applied the Sargan difference test to ensure that p-values are in or close to the interval (5%, 25%).

(equation II of Table 2), most coefficients are larger in absolute terms in the simultaneous GMM estimations; in particular, the accelerator effect is strong here. The complementarity effect relative to the crowding out effect is larger in the simultaneous equation system than under separate GMM estimation. As we will see soon, this will have an implication to the optimal level of public investment derived from the policy experiments conducted in the next section. In the public investment equation, coefficients do not seem that different across GMM models.[23]

4. Simulations

In this section, we simulate the system of four equations ((26), (27), (28) and (29)) and conduct policy experiment in order to determine the public investment GDP share that maximizes discounted consumption and assess the effects on investment, net income, and consumption. Parameter coefficients of the variables are estimates from the GMM-HAC regressions in column 4 of Table 2.[24] Initial values are constructed from regressing the variables on linear-quadratic time trends, in the first five to ten years period. First, we simulate a benchmark economy with values that (roughly) match with the panel average of real economies of SSA, particularly during the end of the sample period.[25] Then, we examine two types of experiments: a one-time percentage shock to public investment, and an increase to a certain constant level of public investment.

4.1. The benchmark economy

The result of the benchmark simulation is shown in Figure 1. Population growth first increases and then decreases. The GDP per capita growth rate increases until 1966 and then starts falling, in particular during the 1970s through the oil crises and in the 1980s through the Latin American debt crisis, both of which hit SSA severely

[23]For equations II and III of Table 2, the panel corrected standard errors in column 1 are very close to the conventional standard errors in column 4.

[24]Later on, we conduct sensitivity analysis using coefficient estimates from the separate equation estimation (column 1 of Table).

[25]The simulation starts in 1960 when the earliest data are available for the estimation of the quadratic time trends. It ends in 3431 just before population growth becomes negative.

and led to a 'lost decade' (Greene, 1989, Humphreys and Underwood, 1989). During the 1982 crisis public investment grows more quickly than GDP and therefore the public investment GDP ratio has a small peak. Part of it goes only into the residuals of our equations because the actual growth rates were slightly lower during the 1982 crisis. After the crisis, growth resumes, and more strongly so after 1990. It is well known that much of this is due to higher natural resource prices, which may also lead to high growth rates in the long run.

FIGURE 1 OVER HERE

The shares of public and private investment are about 7.5 and 14.3 percent, respectively, at the end of the simulation period. The largest values of the public and private shares of GDP, about 7.6 percent and 14.5 percent, are reached in 2080 and 2082, respectively. Population growth approaches zero (but very slowly) at the end of the simulation period.[26] For all variables the simulation values at the end of the sample period are quite close to those of the actual panel average for 2005-2007 presented in the last column of Table 1.

4.2. Counterfactual analysis: Is public investment optimal in SSA countries?

From Table 1, the actual panel-average of public investment is 7.1 percent of real GDP. At the end of the sample period, 2005-2007, the value is 7.2 percent (see Table 1, last column). According to the benchmark simulation it goes up to 7.56 percent where it is from 2060-2110. However, the public investment that maximizes the growth rate from the nonlinear growth regression, in Table 2 column 4 (or column1), is 9.3 percent of GDP (or 9.8 percent). These results imply that on average the public investment share of output in SSA countries is sub-optimal.

To further examine this with policy experiments, consider, first, a one-time increase in public investment in the year 2015 that boosts per capita consumption and net income. Figures 2 and 3 show that the maximum feasible policy increase that can be made is much below 2 percent. Adding a one-time 1.8 percent or stronger shock

[26]There is no steady state as population growth rate keeps changing.

in the year 2015 to the public investment equation makes the model unstable. The interactions between private and public investment are too strong with this shock that make public and private investment to explode and, through taxation and non-linearity in the growth equation, growth to implode after the year 2500 (see Figure 2). However, if we limit the shock to a one-time 1.79 percent, the effect is to increase the GDP per capita and the after-tax per capita income by about 30 percent and per capita consumption by up to 23 percent (see Figure 3a). Public and private investment shares with this shock go slightly beyond 20 percent above the benchmark values for a long period (see Figure 3b) with a peak reached around 2290, indicating that stability is ensured.

FIGURE 2 OVER HERE
FIGURE 3 OVER HERE

Our second policy experiment is to find rather the constant level of the public investment that maximizes the sum of per capita consumption (discounted at 4 percent), from 2015 to 3430. On the basis of the simultaneous equations regression in column 4 of Table 2, this value, as share of GDP, is 11 percent.[27] The results are plotted in Figure 4. The value for the growth rate returns to its benchmark value in the year 2166. The fall in the population growth rate is speeded up slightly but is about 97 percent of the benchmark value after a long period of time. The increases relative to the benchmark are about 39 percent for consumption, 46 percent for public investment shares, 64 percent for after-tax income, 70 percent for GDP per capita, and 75 percent for the share of private investment. These values may seem large; however, they are reached only after a long period of time (more than hundred years).

FIGURE 4 OVER HERE

4.2.1. Sensitivity analysis

We also run the simulation using the estimates from the separate equations estimation (column 1 of Table 2). The simulation results are shown in Figure 5. A

[27] Sensitivity analysis for discount rates of 8 percent and 12 percent yields about 9.3 percent and 8.4 percent optimal public investment, respectively (see Table 3).

relatively lower value of 9.12 percent of public investment (as a share of GDP) maximizes the sum of per capita consumption (discounted at 4 percent). The outcomes are much smaller than the ones we get from the 11 percent increase in the previous simulation. For instance, public investment is raised by only 25 percent from its benchmark value of 7.5 percent, in contrast to the 46 percent rise earlier; GDP per capita by about 8.4 percent; both after- tax income and consumption per capita by about 5.6 percent; and, private investment by only about 1.7 percent, in contrast to the 75 percent rise earlier.

FIGURE 5 OVER HERE

The difference in the simulations' outcomes is apparently due to differences in the estimates of the variables, which in turn depend on the estimation methods employed. Which of the latter are more plausible? Both methods have their own merits. The advantages of the simultaneous equations estimation vis-à-vis the separate is similar to that of the SUR estimation. It takes into account the contemporaneous correlation. However, the orthogonal deviation method used in the separate equation estimation has the advantage of losing fewer observations than first-differences.

In the simultaneous equations estimation, the accelerator, and net complementarity effects are much stronger compared to the separate-equations estimation. This leads to differences in the simulation outcome. However, note that, although the values for optimal public investment differ from each other to some extent, they are all larger than the value of 7.2 percent, which the current data have for the end of the sample period. In addition, they are much smaller than the values, which were reported, by earlier works, for other areas. For instance, Aschauer's (2000a) estimate of the growth maximizing level of public capital for the US is about 30 percent; Miller and Tsoukis's (2001) for a wide range of low and middle income countries is 18 percent; Kamps's (2005) for European and OECD countries is 20 percent.

We also perform sensitivity analysis for using different values of discount rates. Table 3 presents simulation results related to 8 percent and 12 percent discount rates, in addition to the results of using 4 percent discount rate that we discussed earlier.

TABLE 3 ABOUT HERE

In general, the optimal public investment shares decrease at discount rates. The optimal public investment shares that correspond to 4 percent, 8 percent and 12 percent discount rates under the simultaneous equations estimation, for instance, are 11 percent, 9.3 percent and 8.4 percent, respectively. The gap is much smaller under the separate equation estimation. However, note that even at a discount rate of 12 percent the optimal public investment rate in both simulations is about 8.4 percent, which is still higher than the actual end-of-panel average value, 7.2 percent. As the strong effects of higher public investment in the simultaneous estimation model stem from the mutual reinforcement of public and private investment and the accelerator effect of GDP per capita growth and therefore arrive in the later years, higher discount rates also reduce the discrepancies of the effects from the two estimation methods.

5. Conclusion

Economists have long acknowledged the importance of public investment. Many believe public investment enhances productivity and complements private investment, with a positive impact on long-run growth and welfare. Others argue that the higher taxation, for instance, resulting from the larger public investment, lowers growth and welfare as it distorts private saving and efforts. Thus, the relationship between long-run growth and public investment could be non-monotonic, with the likelihood of an optimal level of public investment.

The present paper first developed an endogenous growth model that posited non-linearity in the public capital and growth relationship in SSA countries. Using the panel data from SSA countries, from 1967 to 2008, and applying various econometric techniques, it estimated the model and identified the growth-maximizing level of public investment in the region. It has found that not only does public investment highly matter for economic growth but also that the current level prevailing in SSA is, on average, sub-optimal. Applying separate and simultaneous equations estimation methodologies, we found growth maximizing public investment GDP percentages of between 9.0 percent and 10.0 percent.

An important aspect of public investment is its indirect impact on growth through an effect on private investment, and conversely. To shed light on this phenomenon, we formulated a system of difference equations that captured the relationships among growth, public and private investment and population growth, and conducted estimation both separately and simultaneously using various econometric techniques. Both complementarities and crowding-out effects were detected between public and private investments while accelerator and net complementarity effects were found to be stronger under the simultaneous equations estimation. Applying the estimates from these regressions we then ran simulations to determine the level of public investment that maximizes the sum of discounted consumption. The optimal value was computed to be between 8.4 percent and 11.0 percent, when using discount rates ranging from 4 percent to 12 percent, respectively. The results from estimating the growth model are thus in the middle of this range. These values are larger than the observed value of 7.2 percent at the end of the sample period. The present findings are, therefore, not in concert with the previous finding of public over-investment in the region. Our estimates are, nevertheless, generally much lower than those for other regions and country groups.

A. Appendix

A.1. Stability of the capital ratio dynamics

To examine the stability of (16), first rewrite it, using (17), as:

$$\begin{aligned} & \left(G_{t+1}/k_{t+1}\right)^{\frac{1}{\delta}} \left(1-\kappa\right) / \left(A\psi\right) + \left(G_{t+1}/k_{t+1}\right)^{\frac{1}{\delta}} \left(G_t/k_t\right)^{\alpha} \left(G/k\right)^{-1} \\ = \; & \left(1-\kappa\right) / \left(A\psi\right) \left(G_t/k_t\right)^{\frac{1}{\delta}} + \left(G_t/k_t\right)^{\alpha-1+\frac{1}{\delta}} \end{aligned} \tag{A.1}$$

Then, log-linearize (A.1) near the steady-state capital ratio (G/k), (see Novales, et al. 2010), to obtain

$$z_{t+1} \approx \Theta z_t \tag{A.2}$$

where $z_t \equiv \ln(G_t/k_t) - \ln(G/k)$ and

$$\Theta \equiv 1 - \frac{(\chi(1-\psi)/\psi)^{1-\alpha}}{\frac{1}{\delta}\left((1-\kappa)/(A\psi) + (\chi(1-\psi)/\psi)^{1-\alpha}\right)} \tag{A.3}$$

Thus, the root of the log-linearized eq. (A.2) is stable as long as $0 < \Theta < 1$, which is the case since the denominator of the second term of (A.3) is greater than the nominator while both are positive.

B. Technical appendix

This section demonstrates the formulation of the separate and simultaneous equations estimations conducted in Section 3. It also provides the instruments used for *each* equations under both methods. Four estimation equations are used: the growth equation, the public investment equation, the private investment equation and the population growth equations.

B.1. The simultaneous equation system

For the simultaneous equations estimation, we set up a system in which we write each of the first three equations as a System GMM estimator model (due to limited sample sizes in the time dimension as mentioned earlier). Each equation is written twice, once in first differences and then in levels with sample means subtracted (within-groups estimator). For the population growth equation, however, we have enough observations in the time dimension. Therefore we enter it only as a within-groups estimator, using lagged levels as instruments.

Thus, the first difference and the level equations related to the growth equation (26) are, respectively,

$$\begin{aligned}
\Delta \ln y_{it} \quad = \quad & a_1 (\Delta \ln y_{it-1}) + (1-\alpha)(\Delta \ln (1-(s^g/y)_{it})) \\
& + \alpha (\Delta \ln (s^g/y)_{it}) + a_2 \left(\Delta \ln \left(s^k/y\right)_{it-1}\right) \\
& + a_3 \left(\Delta \left(\gamma_p\right)^2_{it}\right) + a_4 (\Delta \tau_t) + \Delta u_{it}
\end{aligned} \tag{B1}$$

and

$$
\begin{aligned}
\widehat{\ln y_{it}} &= a_1 \widehat{\ln y_{it-1}} + (1-\alpha)\ln(1-\widehat{(s^g/y)_{it}}) \\
&\quad + \alpha \ln \widehat{(s^g/y)_{it}} + a_2 \ln \widehat{(s^k/y)_{it-1}} \\
&\quad + a_3 \widehat{\left(\gamma_p\right)^2_{it}} + a_4 \widehat{\tau}_t + \widehat{u_{it}} \qquad \text{(B2)}
\end{aligned}
$$

where $\Delta \ln x_{it} \equiv \ln x_{it} - \ln x_{it-1}$; $\widehat{\ln x_{it}} \equiv \ln x_{it} - \overline{\ln x_{it}}$; and, $\overline{\ln x_{it}}$ is the average of the variables over time.

In relation to the private investment equation (27), the first difference and the level equations are, respectively,

$$
\begin{aligned}
\Delta \ln \left(s^k/y\right)_{it} &= b_1 \left(\Delta \ln \left(s^k/y\right)_{it-1}\right) + b_2 \left(\Delta \ln (s^g/y)_{it-2}\right) + b_3 \left(\Delta \gamma_y\right)_{it-1} \\
&\quad + b_4 \left(\Delta \left(\ln \left(s^k/y\right)_{it-2} * \ln (s^g/y)_{it-1}\right)\right) + \Delta u_{it} \qquad \text{(B3)}
\end{aligned}
$$

and

$$
\begin{aligned}
\ln \widehat{(s^k/y)_{it}} &= b_1 \ln \widehat{(s^k/y)_{it-1}} + b_2 \ln \widehat{(s^g/y)_{it-2}} + b_3 \widehat{\left(\gamma_y\right)_{it-1}} \\
&\quad + b_4 \ln \widehat{(s^k/y)_{it-2} * \ln} (s^g/y)_{it-1} + \widehat{u_{it}} \qquad \text{(B4)}
\end{aligned}
$$

with respect to to the public investment equation (28),

$$
\begin{aligned}
\Delta \ln (s^g/y)_{it} &= c_1 \left(\Delta \ln (s^g/y)_{it-1}\right) + c_2 \left(\Delta \ln \left(s^k/y\right)_{it-1}\right) \\
&\quad + c_3 \left(\Delta \ln \left(s^k/y\right)_{it-2}\right) + c_4 \left(\Delta \left(\gamma_y\right)_{it-2}\right) \\
&\quad + c_5 \left(\Delta \left(\ln \left(s^k/y\right)_{it-2} * \ln (s^g/y)_{it-1}\right)\right) + \Delta u_{it} \qquad \text{(B5)}
\end{aligned}
$$

and

$$\begin{aligned}
\widehat{\ln\left(s^g/y\right)_{it}} &= c_1\widehat{\ln\left(s^g/y\right)_{it-1}} + c_2\widehat{\ln\left(s^k/y\right)_{it-1}} \\
&\quad + c_3\widehat{\ln\left(s^k/y\right)_{it-2}} + c_4\widehat{\left(\gamma_y\right)_{it-2}} \\
&\quad + c_5\widehat{\ln\left(s^k/y\right)_{it-2} * \ln\left(s^g/y\right)_{it-1}} + \widehat{u_{it}} \qquad \text{(B6)}
\end{aligned}$$

The within groups estimator related to the population growth equation (29) is

$$\begin{aligned}
\widehat{\left(\gamma_p\right)_{it}} &= d_1\widehat{\left(\gamma_p\right)_{it-1}} + d_2\widehat{\left(\gamma_p\right)_{it-2}} + d_3\widehat{\left(\gamma_p\right)_{it-3}} \\
&\quad + d_4\widehat{\left(\gamma_p\right)_{it-7}} + d_5\widehat{\left(\gamma_p\right)_{it-8}} + d_6 \ln y_{it-2} \\
&\quad + d_7\widehat{\ln y_{it-3}} + d_8\widehat{\ln y_{it-5}} + \widehat{u_{it}} \qquad \text{(B7)}
\end{aligned}$$

The instruments used for each of the above equations are a constant and the following (A double index indicates the first and the last lag used as an instrument).:

(B1): $\ln y_{it-2}$, $\ln\left(1-\left(s^g/y\right)_{it-1}\right)$, $\ln\left(s^g/y\right)_{it-1}$, $\ln\left(s^k/y\right)_{it-1}$, $\left(\gamma_p\right)^2_{it-1}$, τ_t

(B2): $\Delta\ln y_{it-2,it-4}$, $\Delta\ln\left(1-\left(s^g/y\right)_{it-1}\right)$, $\Delta\ln\left(s^g/y\right)_{it-1}$, $\Delta\ln\left(s^k/y\right)_{it-1}$, $\Delta\left(\gamma_p\right)^2_{it}$

(B3): $\ln\left(s^k/y\right)_{it-2}$, $\ln\left(s^g/y\right)_{it-2}$, $\left(\gamma_y\right)_{it-1}$, $\ln\left(s^k/y\right)_{it-2} * \ln\left(s^g/y\right)_{it-1}$, c

(B4): $\Delta\ln\left(s^k/y\right)_{it-2,it-4}$, $\Delta\ln\left(1-\left(s^g/y\right)_{it-2}\right)$, $\Delta\left(\gamma_y\right)_{it-3}$, $\Delta\left(\ln\left(s^k/y\right)_{it-3} * \ln\left(s^g/y\right)_{it-2}\right)$

(B5): $\ln\left(s^g/y\right)_{it-2}$, $\ln\left(s^k/y\right)_{it-2,it-3}$, $\left(\gamma_y\right)_{it-2}$, $\ln\left(s^k/y\right)_{it-3} * \ln\left(s^g/y\right)_{it-2}$

(B6): $\Delta\ln\left(1-\left(s^g/y\right)_{it-2}\right)$, $\Delta\ln\left(s^k/y\right)_{it-3}$, $\Delta\left(\gamma_y\right)_{it-2}$, $\Delta\left(\ln\left(s^k/y\right)_{it-3} * \ln\left(s^g/y\right)_{it-2}\right)$

(B7): $\left(\gamma_p\right)_{it-2}$, $\left(\gamma_p\right)_{it-3}$, $\left(\gamma_p\right)_{it-4}$, $\left(\gamma_p\right)_{it-7}$, $\left(\gamma_p\right)_{it-8}$, $\ln y_{it-2}$, $\ln y_{it-3}$, $\ln y_{it-5}$

B.2. Separate estimation

For the separate equation estimations, both first difference and the Arellano-Bover (1995) orthogonal deviation methods are applied. The latter is applied to the investment equations. But, similar to the simultaneous estimation, first difference is used for the growth equation while fixed effects is applied to the population growth equation. The growth equations in the separate equations estimation are therefore similar to (B1) and (B2) whereas the population growth equation is similar to (B7).

The instruments used for each of the equations in the separate estimation method are a constant and the following:

(B1): $\ln y_{it-2}$, $\ln\left(1-(s^g/y)_{it-1}\right)$, $\ln(s^g/y)_{it-1}$, $\ln\left(s^k/y\right)_{it-2}$, $\left(\gamma_p\right)^2_{it-2}$, τ_t

(B2): $\left(\gamma_y\right)_{it-2}$, $\Delta\ln\left(1-(s^g/y)_{it-1}\right)$, $\Delta\ln(s^g/y)_{it-1}$, $\Delta\ln\left(s^k/y\right)_{it-2}$, $\Delta\left(\gamma_p\right)^2_{it-3}$

Private investment equation: $\ln\left(s^k/y\right)_{it-2,t-3}$, $\ln(s^g/y)_{it-2,t-3}$, $\left(\gamma_y\right)_{it-1}$, $\ln\left(s^k/y\right)_{it-2}*\ln(s^g/y)_{it-1}$

Public investment equation: $\ln(s^g/y)_{it-2,t-3}$, $\ln\left(s^k/y\right)_{it-2,t-3}$, $\left(\gamma_y\right)_{it-2}$, $\ln\left(s^k/y\right)_{it-2}*\ln(s^g/y)_{it-1}$

A double index indicates again the first and the last lag used as an instrument.

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Table 1
Summary statistics for 33 SSA countries over the period 1967-2008

Variable	**Mean**	**Standard Deviation**	**Min**	**Max**	**Mean 2005-2007**
GDPPC	2108	2022	312	23444	2997
GDPGR	0.0083	0.08	-0.56	0.78	0.0235
PUB/GDP	7.08	3.77	0.1	20.36	7.17
PRI/GDP[a]	11.48	8.67	0.05	112.35	13.45
POPGR[b]	0.0269	0.01	-0.083	0.1	0.024

Note: GDPPC - GDP per capita (PPP); GDPGR - GDP per capita Growth rate; PUB/GDP - Public investment/GDP; PRI/GDP - Private investment/GDP; POPGR - Population growth rate.
[a]A very high value of private investment corresponds to a high value of growth; but for public investment this is not the case.
[b]The minimum and maximum values are for Rwanda in 1993 and 1998, respectively.
Source: The data for GDP per-capita are obtained from the PWT 6.3 (Heston et al., 2009) while the data for the public and private investment variables are extracted from the African Development Indicators (World Bank, 2010). Population data are from World Bank (2010). Only fixed capital investment by governments and non-financial public enterprises are included here for public investment.

Table 2
Estimation results

Regressors	Separate[a] (1)	WLS (2)	SUR (3)	Simultaneous[b] (4)
Equation I: Dependent variable: Log(GDPPC)				
Log(GDPPC) (L1)	0.942	0.984	0.991	0.962
	(47.1)	(220.4)	(269.5)	(39.71)
Log(PUB/GDP)	0.098	0.063	0.062	0.093
	(7.06)	(15.6)	(17.7)	(6.03)
Log(PRI/GDP)	0.029	0.023	0.019	0.035
	(2.04)*	(5.95)	(6.08)	(2.82)
$POPGR^2$	-22.5	-11.7	-7.8	-25.04
	-(8.5)	-(3.0)	(-2.36)*	-(3.01)
Time	0.00082	0.00078	0.00084	0.00089
	(1.65)**	(3.21)	(4.14)	(1.71)**
Equation II: Dependent variable: Log(PRI/GDP)				
Log(PRI/GDP) (L1)	0.643	0.586	0.686	0.679
	(16.1)	(29.1)	(36.0)	(15.36)
Log(PUB/GDP) (L2)	-0.125	-0.150	-0.126	-0.182
	-(3.45)	-(6.69)	-(5.79)	-(3.67)
Log(GDPGR) (L1)	0.286	0.264	0.273	0.429
	(1.70)**	(2.28)*	(2.35)*	(1.81)**
Log(PRI/GDP) (L2)× Log(PUB/GDP) (L1)	0.057	0.091	0.073	0.106
	(4.2)	(10.1)	(8.1)	(3.49)
Equation III: Dependent variable: Log(PUB/GDP)				
Log(PUB/GDP) (L1)	0.382	0.384	0.559	0.266
	(4.5)	(9.9)	(15.6)	(2.83)
Log(PRI/GDP) (L2)× Log(PUB/GDP) (L1)	0.163	0.154	0.113	0.172
	(4.2)	(8.7)	(6.9)	(4.81)
Log(PRI/GDP) (L1)	0.119	0.132	0.132	0.158
	(3.3)	(5.6)	(5.4)	(3.21)
Log(PRI/GDP) (L2)	-0.383	-0.399	-0.327	-0.382
	-(4.9)	-(11.2)	-(9.4)	-(4.88)
GDPGR (L2)	0.337	0.372	0.385	0.422
	(2.19)*	(3.3)	(3.3)	(2.37)*
Equation IV: Dependent variable: POPGR[c]				
POPGR (L1)	2.658	2.607	2.607	2.620
	(97.5)	(132.1)	(132.1)	(43.19)
POPGR (L2)	-2.707	-2.561	-2.561	-2.559
	-(46.4)	-(66.5)	-(66.5)	-(21.06)
POPGR (L3)	1.059	0.950	0.950	0.936
	(28.7)	(42.4)	(42.4)	(14.10)

Table 2 (continued)

POPGR (L7)	-0.118	-0.039	-0.039	-0.031
	-(7.7)	-(10.1)	-(10.1)	-(4.77)
POPGR (L8)	0.066	0.003	0.003	0.002
	(5.5)	(3.0)	(3.0)	(2.11)*
LOG(GDPPC) (L2)	-0.0009	-0.001	-0.001	-0.001
	-(2.6)	-(3.0)	-(3.0)	-(1.48)***
LOG(GDPPC) (L3)	0.0017	0.002	0.002	0.001
	(3.8)	(4.1)	(4.1)	(1.6)**
LOG(GDPPC) (L5)	-0.0009	-0.001	-0.001	-0.001
	(-3.257)	-(3.6)	-(3.5)	-(1.67)**
Period	[d]	1966-2008	1966-2008	1967-2008
Observation	[e]	1314	1314	1312
J-statistics	[f]	-	-	0.0203
Sargan p-value	[g]	-	-	0.27
2nd order serial correlation	[h]	-	-	[i]

Note: See Table 1 for variables definition. L(1) and L(2) show that the 1st and the 2nd lag of the indicated variable are used, respectively. t-values are in parentheses. All coefficients are significant at the 1% level except asterisked entries: *, ** and *** indicate significance at the 5%, the 10% and the 14% (in one case), respectively. In the first-difference version of System GMM, the J-statistics and the number of instruments are divided by the number of observations. The J-statistic is the quadratic form minimized by GMM. Columns (3) and (4) look identical for equation IV due to rounding.

[a]Equations are estimated separately; System GMM estimator (in its first difference variant) is used for equation I; System GMM estimator (in its orthogonal deviation variant) for equations II and III; fixed effects for equation IV.

[b]A system of simultaneous equations is estimated; for the first three equations, System GMM estimator (in its first difference variant) is applied; GMM-HAC: Kernel: Quadratic, Bandwidth: Variable Newey-West (10), No prewhitening. For the fourth equation, fixed effects is applied.

[c]The population growth regression (column 1) has an intercept of 0.002.

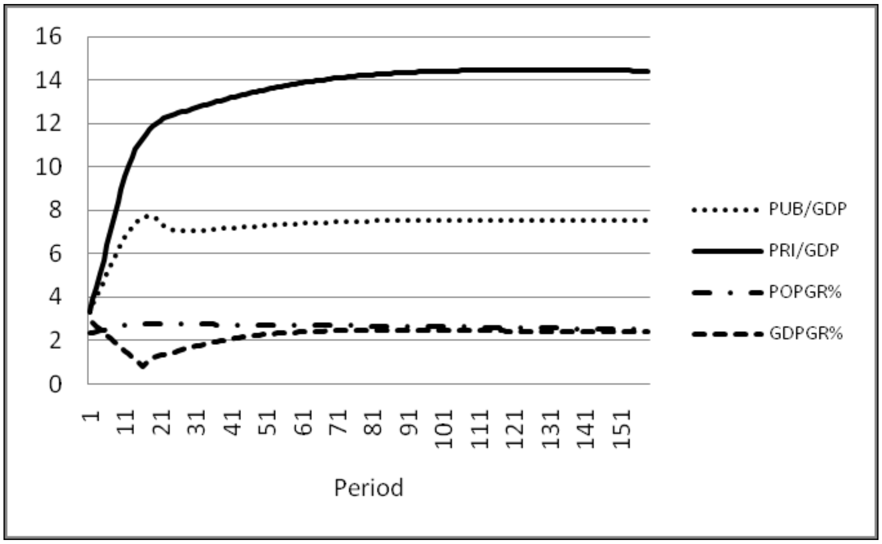
Separate estimation	Equation I	Equation II	Equation III	Equation IV
Period[d]	1967-2007	1968-2008	1968-2008	1969-2008
Observation[e]	733	710	722	1308
J-statistics[f]	0.014	173.18	173.19	-
Instruments	17	115	152	-
Sargan p-value[g]	0.27	0.034	0.069	-
2nd order serial correlation[h]	-0.127	0.085	-0.054	-
(t-value)	(-3.23)	(-1.89)	-(1.14) -	
2nd order serial correlation[i]	-0.127	-0.123	0.002	-0.095
(t-value)	(-3.23)	(-3.20)	(0.045)	(-3.09)

Table 3
Sensitivity analysis: Optimal public investment under different discount rates

Discount rates	**(4%)**		**(8%)**		**(12%)**	
Estimates	**Separate (1)**	**System (2)**	**Separate (3)**	**System (4)**	**Separate (5)**	**System (6)**
Optimal PUB/GDP(%)	9.12	11	8.75	9.27	8.4	8.41
Effects of raising public investment from the benchmark to its optimal level:						
Increase in PUB/GDP	15.4%	45%	20%	22.9%	15.4%	11.6%
Increase in PRI/GDP	1%	75%	1.3%	27.2%	1%	12%
Increase in GDPPC	6%	70%	7.4%	32.1%	6.2%	15.9%
Increase in NETINC	4.9%	64%	5.7%	29.7%	4.9%	14.8%
Increase in CONUM	4.5%	39%	5.3%	22.6%	4.6%	12%

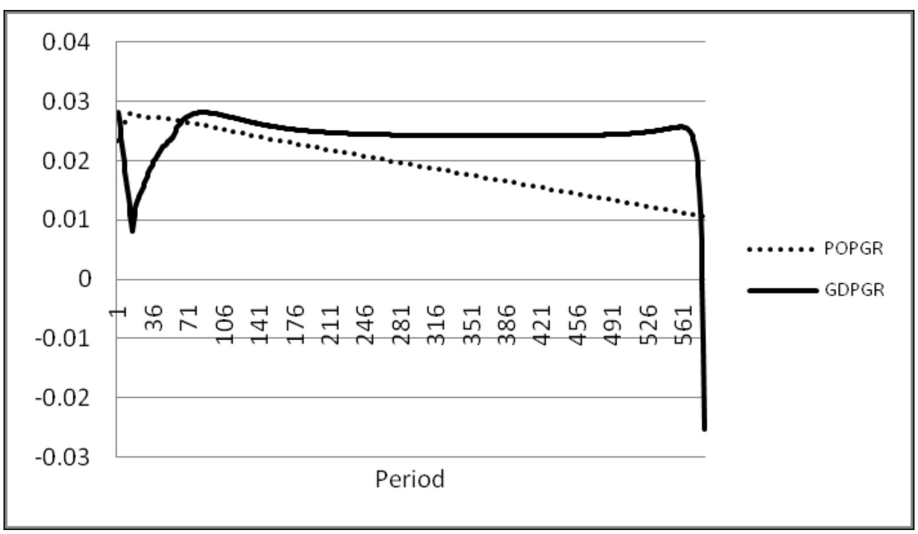
Note: NETINC - Net income per capita (after tax GDPPC); CONSUM - Consumption per capita (NETINC minus PRI/GDP). See Table 1 for the rest of the variables definition.

Figure 1. Benchmark simulation, 1960 to 2900



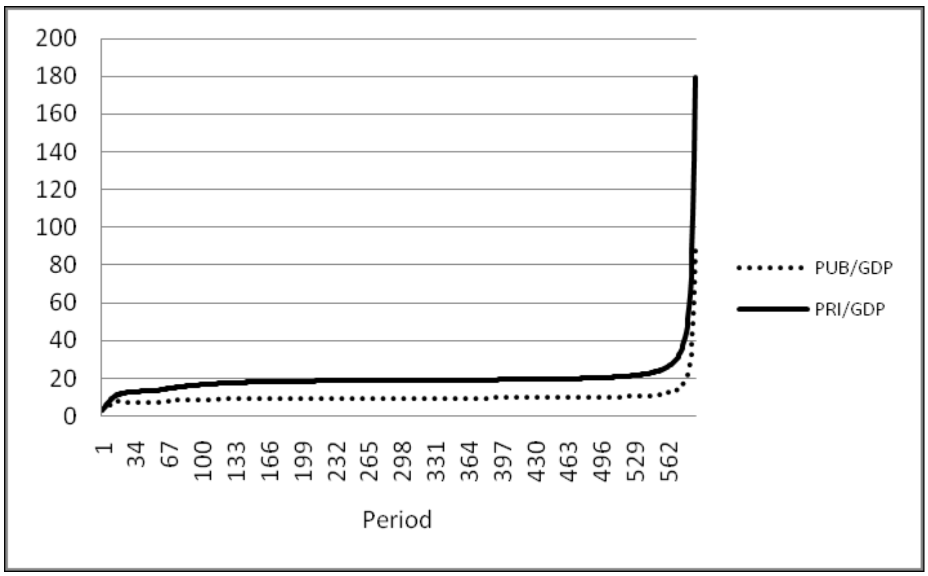
Note: See Table 1 for variables definition. Coefficients estimates from the simultaneous equation estimations of column 4, Table 2, are used.

Figure 2a. The effect of a de-stabilizing public investment shock on growth rate relative to the benchmark



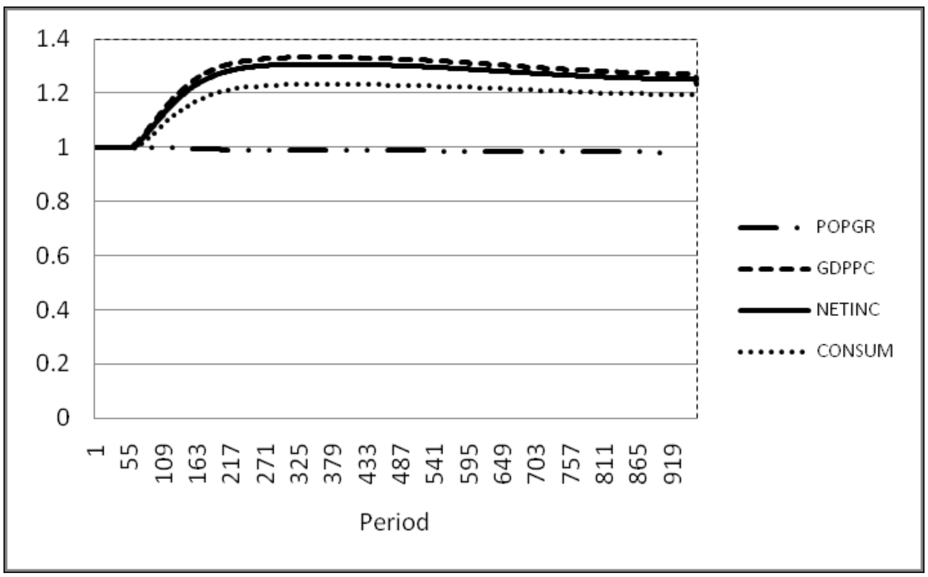
Note: Under the smallest de-stabilizing shock of public investment GDP share of 1.8%, output implodes after 2500.

Figure 2b. The effect of a de-stablizing public investment shock on investments



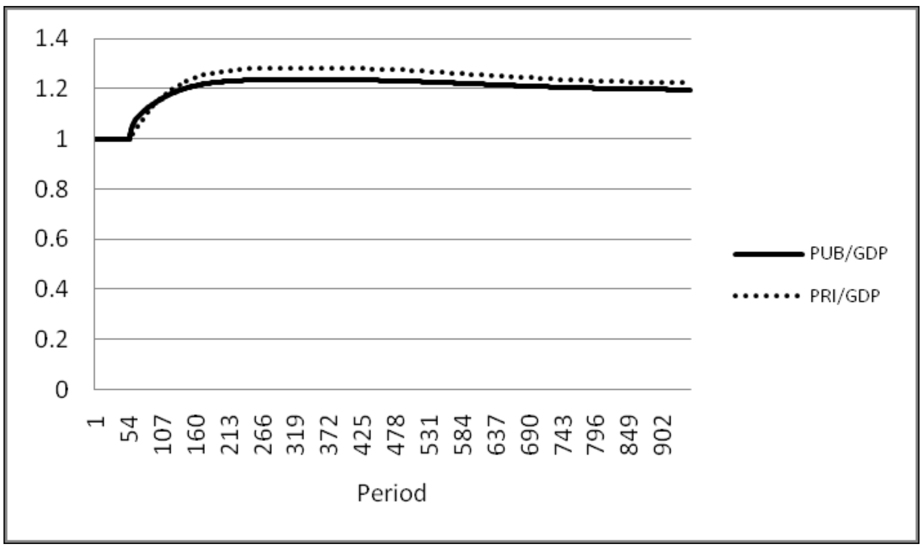
Note: Under the smallest de-stabilizing shock of public investment GDP share of 1.8% , investments explode after 2500.

Figure 3a. The effect of a non de-stabilizing shock relative to the benchmark



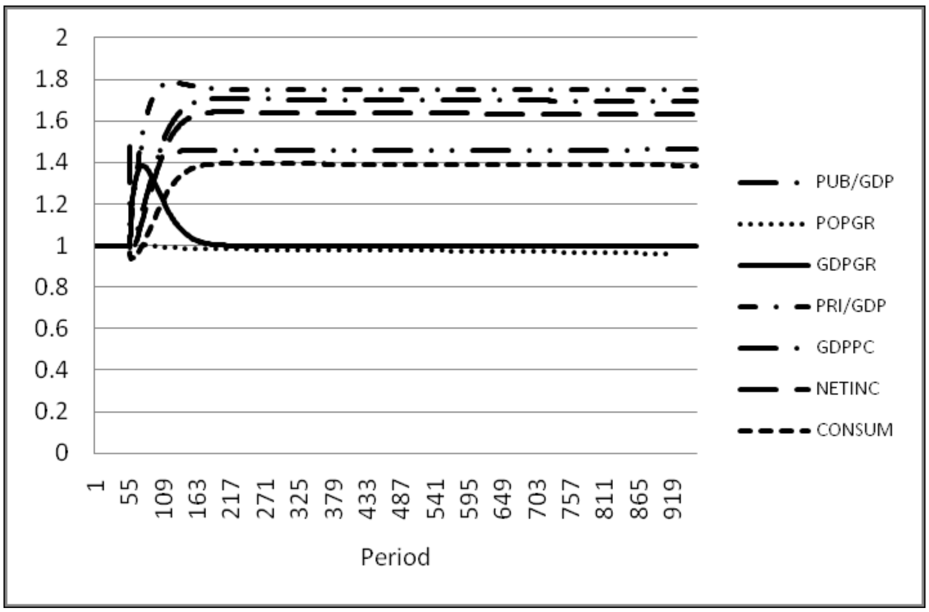
Note: See Table 1 and 3 for variables definition. Under the largest non de-stabilizing shock of public investment GDP share (1.79%), output, net income and consumpiton per capita increase by more than 30 percent after a certain transition period; population growth rates decrease slightly.

Figure 3b. The effect of a non de-stabilizing shock on investments relative to the benchmark



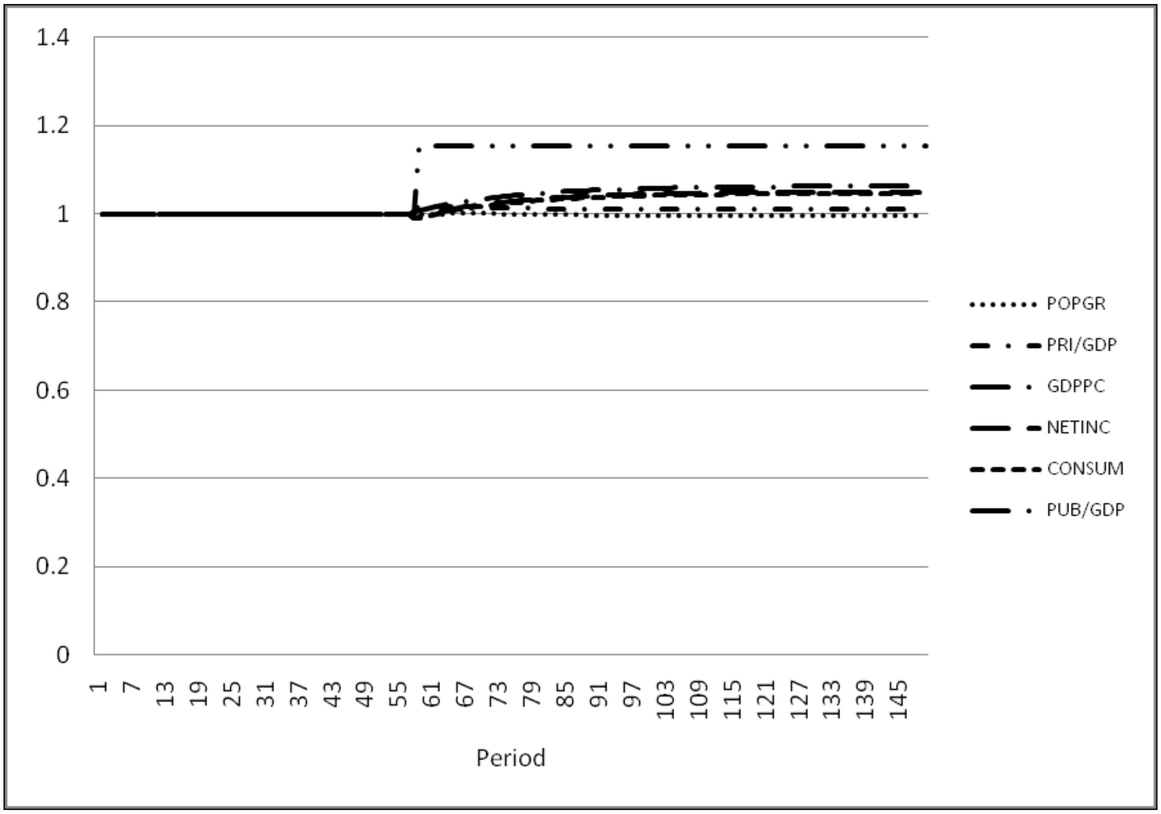
Note: Under the largest non de-stabilizing shock of public investment GDP share (1.79%) in 2015, investments increase by more than 20 percent after a certain transition period.

Figure 4. Effects of raising public investment GDP share to 11% – the optimal value under the simultaneous equations estimation



Note: When raising the public investment GDP share to 11%, – the value which maximizes the net present value of consumption (at 4% discount rate) when using coefficient estimates from the simultaneous equation estimation of columns 4 of Table 2, – from its benchmark values, population growth rates decline, growth rates return to the baseline value after more than hundred years, and all other variables increase by 40% to 80%.

Figure 5. Effects of raising public investment to 9.12% – the optimal value under the separate equations estimation



Note: When raising the public investment GDP share to 9.12%, – the value which maximizes the net present value of consumption at (4% discount rate) when using coefficient estimates from the separate equation estimation of columns 1 of Table 2, – from its benchmark values, population growth rates are lower, public investment shares increase by more than 20%, growth rates return to the baseline value after more than hundred years, and all other variables increase by only less than 10%.