Distributional effects of public policy choices

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Abstract

This paper examines the effects of a budget-neutral public spending allocation between public investment and private investment subsidy on inequality dynamics and intergenerational mobility in an environment with heterogenous households and incomplete capital market.

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Public investment . investment subsidy . inequality dynamics . elasticity of substitution . incomplete credit market
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1. Introduction

In practically all governments invest in public capital (such as roads, public transport, and irrigation), which are used for the production of goods and services along with private inputs. Most governments also devote considerable resources towards subsidising private investment. For instance, in the UK, until recently, about 35% of universities' total funding came from the government.[1] In India, the government's subsidy to agricultural input has risen by more than ninety times in the last three decades (Fan et al., 2008). In general, the efficiency implications of productive public spending have been well studied in growth literature.[2] However, it is an open question of how

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[1]This has helped to keep student tuition fees significantly down. However, now, amid major budget cuts, the majority of the universities are planning to triple their tuition fees within the coming years (BBC, June 27, 2011).

[2]See, for example, Barro (1990), Futagami et al. (1993) and Glomm and Ravikumar (1994) among many others.

these programs impact the distribution and efficiency of the economy when they are jointly provided. This paper analytically examines the effects of a budget-neutral public spending allocation between public investment and subsidy on inequality dynamics and growth in incomplete capital markets.

The basic intuition behind the theory is as follows: When the credit markets are imperfect, individuals' investment opportunities are limited to the resources they have in hand. The elasticity of substitution between factors then becomes an important determinant of inequality dynamics and aggregate efficiency due to its impact on individual households' productivity and resource constraints. If production at an individual level takes place using public and private inputs that are *substitutable*, for instance, an increase in public investment could relax resource constraints that impede investment opportunities of poor households. Conversely, if the inputs are *complementary*, the rich are the ones who benefit the most from a productivity increase due to the increase in public investment as they own much of the production resources in the economy. In this case, rather a subsidy to private investment benefits the poor, more. The poor have, already, a relatively higher marginal productivity due to diminishing return to private investment but, lack resources to take advantage of it.

The next section provides an overlapping generation growth model that captures this idea. In the model, agents are heterogenous in terms of initial wealth. Credit markets are missing. The government subsidises private investment and also invests in public capital. We show that the elasticity of substitution between public and private factors determines the distributional effect of a budget-neutral public spending allocation between public investment and subsidy. If the elasticity of substitution is greater than unity, an increase in public investment mitigates the persistence of inequality or speeds up the intergenerational mobility. Conversely, if it is less than unity, it is rather an increase in subsidy that mitigates inequality persistence. A higher inequality is also shown to lower growth when the imperfection in the market prevents the efficient amount of investment to be undertaken in the economy as in Loury (1981).

This paper, mainly, contributes to the literature of public investment and growth (see Footnote 2) and inequality and growth with imperfect credit market (e.g., Loury, 1981; Galor and Zeira, 1993; Benabou, 2002) through linking productive public spending to inequality and, hence, growth. It is also related to the limited literature of public investment and distribution (e.g., Garcia-Penalosa and Turnovsky, 2007; Getachew, 2010). There is also

a vast literature that studies the relationship between public education and income distribution, which this paper may also be related to (e.g., Glomm and Ravikumar, 1992). However, these bodies of literature do not focus on the allocation of government expenditure.

2. The Model

2.1. Preference and Technology

There is a continuum two-period-lived heterogenous households, in terms of initial wealth. Agent i who is born at $t-1$ enters to the market at t with k_t^i amount of capital, which is accumulated through her parent's investment (e_{t-1}^i), out of the "joy of giving", and public subsidy $(e_{t-1}^i \psi)$. She employs this capital in a privately-held firm and earns income.[3] She allocates after tax income $((1-\tau) y_t^i)$ between consumption (c_t^i) and saving (e_t^i).

Agent ith utility function at t is given by,

$$U\left(c_t^i,\, k_{t+1}^i\right) \equiv \ln c_t^i + \beta \ln k_{t+1}^i \tag{1}$$

subject to the budget constraint,[4]

$$c_t^i + e_t^i = y_t^i \left(1 - \tau\right) \tag{2}$$

The ith offspring capital is given by,

$$k_{t+1}^i = B e_t^i \left(1 + \psi\right) \tag{3}$$

where τ is a fixed flat rate tax; ψ represents the rate the government subsidises private investment.

The production function of the ith firm is *general* Cobb-Douglas (Revankar, 1971),

$$y_t^i = A \left(k_t^i\right)^\alpha \left(G_t\right)^{1-\alpha} + \epsilon G_t \tag{4}$$

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[3]A similar type of individual entrepreneurship is used in Benabou (2002) and Angeletos and Calvet (2006).

[4]A unitary intertemporal elasticity of substitution utility function and altruistic agents with a "joy of giving" motive are common in growth literature (see, for instance, Glomm and Ravikumar, 1992 and Galor and Zeira, 1993).

where G_t denotes public capital and ϵ is a parameter.[5] (4) has both the technological *flexibility* and *tractability* that combine the important characteristics of the CES (of Arrow et. al., 1961) and Cobb-Douglas, respectively. Similar to the latter, we can easily aggregate it:

$$y_t = A\,(k_t)^\alpha\,(G_t)^{1-\alpha}\exp\left(0.5\sigma_t^2\alpha\,(\alpha-1)\right) + \epsilon G_t \tag{5}$$

where $y_t \equiv \int_i y_t^i$ and $k_t \equiv \int_i k_t^i$.[6] Similar to the former, the elasticity of substitution (δ) is different from unity:

$$\delta_t = 1 + \epsilon\frac{\alpha}{1-\alpha}\frac{G_t}{y_t} \tag{6}$$

The sign of ϵ determines δ_t: When $\epsilon = 0$, the production function is reduced to the specific Cobb-Douglas. But, in general, if $\epsilon \gtreqqless 0$, then $\delta_t \gtreqqless 1$. When $\epsilon \neq 0$, and the output-capital ratio is constant – typical to Ak growth models, it features a constant elasticity of substitution.[7]

The government has a balanced budget:

$$\tau\int_i y_t^i = \psi\int_i e_t^i + I_t^g \tag{7}$$

where

$$G_{t+1} \equiv I_t^g = (1-\theta)\,\tau\int_i y_t^i \tag{8}$$

$$\psi\int_i e_t^i = \theta\tau\int_i y_t^i \tag{9}$$

and, I_t^g and θ denote public investment and the fraction of government revenue that goes to investment subsidy respectively. We, thus, assume complete depreciation of the capital stock.

The ith individual of generation t optimisation problem, from (1), (2) and (3), is given by

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[5]The marginal product of k_t^i is always nonnegative under the standard assumption $0 < \alpha < 1$. But, the production function has nonnegative marginal product of G_t when ϵ has a lower bound, $\epsilon > y_t\,(\alpha-1)/G_t$. See Revankar (1971) for details.

[6]See Appendix A for details on the aggregation.

[7]Note that, the main results do *not* depend on the use of specific production function. In fact, any neoclassical production function, with a factor elasticity of substitution different from unity, may lead to the same result as far as one can deal with aggregation. Results based on the CES (of Arrow et. al., 1961) production function, with a log-linear approximation, are available from the author upon request.

$$\max_{e_t^i} \ln\left((1-\tau)\, y_t^i - e_t^i\right) + \beta\left[\ln B\left(1+\psi\right) e_t^i\right]$$

taking as given τ, θ, k_t^i and G_{t+1}. The FOC gives,

$$e_t^i = a(1-\tau)y_t^i \tag{10}$$

where $a \equiv \beta/\left(1+\beta\right)$. (10) is agent ith optimal saving as a fraction of her after tax income, which is standard. From (9) and (10), following Benabou (2002), we obtain,

$$\psi = \theta\tau/\left(a\left(1-\tau\right)\right) \tag{11}$$

3. Inequality persistence and capital dynamics

The dynamics of private capital at an individual level is given by, from (3), (4), (10) and (11),

$$k_{t+1}^i = B\left(a(1-\tau)+\theta\tau\right) G_t\left(A\left(\varphi_t^i\right)^\alpha + \epsilon\right) \tag{12}$$

from which the aggregate capital is obtained,

$$k_{t+1} \equiv \int_i k_{t+1}^i = B\left(a(1-\tau)+\theta\tau\right) G_t\left(A\left(\varphi_t\right)^\alpha \exp\left(0.5\sigma_t^2\alpha\left(\alpha-1\right)\right) + \epsilon\right) \tag{13}$$

where $\varphi_t^i \equiv k_t^i/G_t$, $\varphi_t \equiv k_t/G_t$ and $\varphi \equiv k/G$.[8]

From (5) and (8), the public capital dynamics is given by,

$$G_{t+1} = (1-\theta)\,\tau G_t\left(A\left(\varphi_t\right)^\alpha \exp\left(0.5\sigma_t^2\alpha\left(\alpha-1\right)\right) + \epsilon\right) \tag{14}$$

Note that, inequality at t has a negative impact on aggregate private and public capital at $t+1$ as $\partial k_{t+1}/\partial\sigma_t^2 < 0$ and $\partial G_{t+1}/\partial\sigma_t^2 < 0$.

Then, the aggregate capital ratio is easily computed from (13) and (14),

$$\varphi_{t+1} = \varphi = B\left(a(1-\tau)+\theta\tau\right)/\left((1-\theta)\,\tau\right) \tag{15}$$

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[8]See Appendix A.

Thus, the private-public capital ratio (φ) and, hence, the output-capital ratios are constant during transition and at equilibrium,[9] as typical to Ak growth models.

It is straightforward that a higher θ increases φ. On the other hand, a higher τ increases public investment. But, τ also distorts private investment although, it increases the government's subsidy rate (ψ). The net effect is that φ is lower when τ is large.

By taking the variance of (12), we get the dynamics of inequality:

$$\sigma_{t+1}^2 = \ln\left(\left(\exp\left(\alpha^2\sigma_t^2\right) - 1\right)\left(\omega\left(\varphi,\sigma_t^2\right)/\left(\omega\left(\varphi,\sigma_t^2\right)+\epsilon\right)\right)^2 + 1\right) \qquad (16)$$

where $\omega\left(\varphi,\ \sigma_t^2\right) \equiv A\varphi^\alpha \exp\left(0.5\sigma_t^2\alpha\left(\alpha-1\right)\right)$.[10] Note that, if $\epsilon = 0$, the Cobb-Douglas case ($\delta = 1$), then (16) reduces to $\sigma_{t+1}^2 = \alpha^2\sigma_t^2$. In this case, the dynamics of inequality is independent of the capital ratio and any of the policy parameters.

We can rewrite (16) after substituting φ from (15),

$$\sigma_{t+1}^2 = \ln\left(\left(\exp\left(\alpha^2\sigma_t^2\right) - 1\right)\left(\omega\left(\tau,\theta,\ \sigma_t^2\right)/\left(\omega\left(\tau,\theta,\ \sigma_t^2\right)+\epsilon\right)\right)^2 + 1\right) \qquad (17)$$

redefining $\omega\left(\tau,\ \theta,\ \sigma_t^2\right) \equiv A\left(B\left(a(1-\tau)+\theta\tau\right)/\left((1-\theta)\tau\right)\right)^\alpha \exp\left(0.5\sigma_t^2\alpha\left(\alpha-1\right)\right)$. Therefore, the dynamics of inequality depends on the policy parameters τ and θ when $\delta \neq 1$.

Proposition 1. *From (16) and (17), if $\delta > 1$ ($\delta < 1$), the persistence of inequality is lower the higher (the lower) is the public spending GDP ratio τ, the lower (the higher) is the private-public capital ratio φ and/or the lower (the higher) is the subsidy rate θ.*

Proof. First, recall that if $\epsilon \gtreqless 0$, $\delta \gtreqless 1$. Then, from (16), one sees that $\partial\sigma_{t+1}^2/\partial\varphi \gtrless 0$ if $\epsilon \gtrless 0$. And, from (17), $\partial\sigma_{t+1}^2/\partial\tau \lessgtr 0$ or $\partial\sigma_{t+1}^2/\partial\theta \gtrless 0$ if $\epsilon \gtrless 0$. ∎

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[9]To see this, first divide (5) by k_t or G_t, and then, use (15).

[10]See Appendix B for details on the derivation.

4. Growth rate

From (14) and (15), one obtains the growth rate of the economy,

$$G_{t+1}/G_t \equiv \gamma_{t+1} + 1 = (1 - \theta)\,\tau\left(A\varphi^\alpha \exp\left(0.5\sigma_t^2\alpha\,(\alpha - 1)\right) + \epsilon\right) \qquad (18)$$

Thus, σ_t^2 has an effect on the growth rate during transition. Considering $\partial\gamma_{t+1}/\partial\sigma_t^2 < 0$, a higher inequality lowers growth. Moreover, a higher elasticity of substitution leads to a higher growth, as in Klump and de la Grandville (2000). Similar to the literature in public capital and growth (e.g., Barro 1990), public investment affects growth directly through its effect on the productivity of private capital. In contrast, it also affects growth *indirectly* through an effect on the distribution dynamics (σ_t^2).

However, as we see soon, the inequality will be vanished $(\sigma^2 = 0)$ in the long run and, hence, $\gamma + 1 = (1 - \theta)\,\tau\,(A\varphi^\alpha + \epsilon)$. Thus, the effect of policy on the steady-state growth is limited to the standard productivity effect.

5. Steady-state inequality

The steady-state distribution is, from (17),

$$\exp\left(\sigma^2\right) - 1 = \left(\exp\left(\alpha^2\sigma^2\right) - 1\right)\left(\omega\left(\varphi,\,\sigma^2\right) / \left(\omega\left(\varphi,\,\sigma^2\right) + \epsilon\right)\right)^2 \qquad (19)$$

Eq. (19) is satisfied only if $\sigma^2 = 0$. This is intuitive: There are no factors (such as an uninsured idiosyncratic shock) in this model that lead to a non-degenerate distribution of wealth. Individuals with relatively lower k_t^i rapidly accumulate wealth due to their relatively high marginal productivity, which, in turn, due to diminishing returns to investment. Therefore, the economy features declining wealth mobility along the transition to a steady-state.

6. Conclusion

The paper has examined how the allocation of government expenditure between public investment and private investment subsidy affects inequality dynamics in an economy with heterogenous agents and incomplete credit market. It has argued that if public and private factors are substitutable (complementary), increased spending on public investment (private investment subsidy) may lead to declining inequality dynamics.

Appendix A. Aggregation

With respect to aggregating (4), note first that,

$$y_t \equiv \int_i y_t^i = \mathrm{E}\left[y_t^i\right] = A\left(G_t\right)^{1-\alpha}\mathrm{E}\left[\left(k_t^i\right)^\alpha\right] + \epsilon G_t \qquad \text{(Appendix A.1)}$$

Considering $\ln k_t^i \sim N\left(\mu, \sigma_t^2\right)$, then $\mathrm{E}\left[\left(k_t^i\right)^\alpha\right]$ is computed as follows:

$$\begin{aligned}
\ln \mathrm{E}\left[\left(k_t^i\right)^\alpha\right] &= \mathrm{E}\left[\ln\left(k_t^i\right)^\alpha\right] + 0.5\,\mathrm{var}\left[\ln\left(k_t^i\right)^\alpha\right] \\
&= \alpha\,\mathrm{E}\left[\ln\left(k_t^i\right)\right] + 0.5\alpha^2\sigma_t^2 \\
&= \alpha\left(\ln \mathrm{E}\left[k_t^i\right] - 0.5\sigma_t^2\right) + 0.5\alpha^2\sigma_t^2 \\
&= \ln\left(k_t\right)^\alpha + 0.5\sigma_t^2\alpha\left(\alpha - 1\right) \qquad \text{(Appendix A.2)}
\end{aligned}$$

Finally, substitute (A.2) into (A.1) to obtain (5). We aggregate (12) similarly.

Appendix B. Distribution dynamics

To derive the distribution dynamics in (16), we take the variance from both side of (12) to obtain,

$$\mathrm{var}\left(k_{t+1}^i\right) = \left(B\left(a(1-\tau) + \theta\tau\right)AG_t\right)^2\mathrm{var}\left[\left(\varphi_t^i\right)^\alpha\right] \qquad \text{(Appendix B.1)}$$

Then, based on the normal-lognormal distribution relationship,

$$\begin{aligned}
\mathrm{var}\left(k_{t+1}^i\right) &= \exp\left(2\,\mathrm{E}\left[\ln k_{t+1}^i\right]\right)\left(\exp\left(\sigma_{t+1}^2\right)\left(\exp\left(\sigma_{t+1}^2\right) - 1\right)\right) \\
&= \left(k_{t+1}\right)^2\left(\exp\left(\sigma_{t+1}^2\right) - 1\right) \qquad \text{(Appendix B.2)}
\end{aligned}$$

We used the fact that $\mathrm{E}\left[\ln k_{t+1}^i\right] = \ln \mathrm{E}\left[k_{t+1}^i\right] - \frac{1}{2}\sigma_{t+1}^2$.

Similarly,

$$\begin{aligned}
\mathrm{var}\left[\left(\varphi_t^i\right)^\alpha\right] &= \exp\left(2\alpha\,\mathrm{E}\left[\ln\varphi_t^i\right]\right)\left(\exp\left(\sigma_t^2\alpha^2\right)\left(\exp\left(\sigma_t^2\alpha^2\right) - 1\right)\right) \\
&= \left(\varphi_t\right)^{2\alpha}\left(\exp\left(\sigma_t^2\alpha\left(\alpha - 1\right)\right)\right)\left(\exp\left(\sigma_t^2\alpha^2\right) - 1\right) \quad \text{(Appendix B.3)}
\end{aligned}$$

Combining (B.1), (B.2) and (B.3), we obtain

$$\begin{aligned}
\left(k_{t+1}\right)^2\left(\exp\left(\sigma_{t+1}^2\right) - 1\right) = {} &\left(B\left(a(1-\tau) + \theta\tau\right)AG_t\right)^2 \\
&\left(\varphi_t\right)^{2\alpha}\left(\exp\left(\sigma_t^2\alpha\left(\alpha - 1\right)\right)\right)\left(\exp\left(\sigma_t^2\alpha^2\right) - 1\right)
\end{aligned}$$
$$\text{(Appendix B.4)}$$

Substituting (13) into (B.4), and after some manipulation, we obtain (16).

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