NEW HIGHER-ORDER ACTIVE CONTOUR ENERGIES FOR NETWORK EXTRACTION

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ABSTRACT

Using the framework of higher-order active contours, we present a new quadratic *continuation energy* for the extraction of line networks (*e.g.* road, hydrographic, vascular) in the presence of occlusions. Occlusions create gaps in the data that frequently translate to gaps in the extracted network. The new energy penalizes nearby opposing extremities of the network, and thus favours the closure of the gaps created by occlusions. Nearby opposing extremities are identified using a sophisticated interaction between pairs of points on the contour. This new model allows the extraction of fully connected networks, even though occlusions violate common assumptions about the homogeneity of the interior, and high contrast with the exterior, of the network. We present experimental results on real aerial images that demonstrate the effectiveness of the new model for network extraction tasks.

1. INTRODUCTION

The automatic extraction of line networks, *e.g.* road, hydrographic, and vascular networks, from images is an important problem in many applications. It is also very challenging, because such networks often cannot be reliably distinguished from other entities by local measurements such as intensity or texture. On the other hand, by their very nature we have a great deal of prior information about the shape of networks, prior information that is clearly critical for successful extraction. This information is, however, hard to model: networks are not mere variations around a 'mean' shape, and can possess complicated topologies. In order to describe this type of sophisticated prior geometric knowledge, and to provide a general framework for shape modelling, [1] introduced 'higher-order active contours'. The energies defining these models incorporate long-range interactions between tuples of contour points, and can thus describe complex families of shapes. This is in contrast to classical energies, where the interactions are local, being mediated by contour derivatives. Using a specific quadratic instance of this new class of model, [1] showed results extracting road networks. The results were good, and used a generic, hence automatic initialization, but a problem remained. One of the strongest pieces of prior geometric knowledge about networks is 'continuation'. That is, arms of the network do not simply stop, and then continue a little further on in roughly the same direction. If this is how they appear in an image, then there must be an occlusion that explains the interruption. The models of [1] did not directly incorporate this prior knowledge, and in consequence the results showed gaps where the real network was occluded in the image, *e.g.* by cast shadows or directly by trees and buildings. The data in the presence of such occlusions directly supports the idea of a gap because the assumptions of homogeneity in the interior, and high contrast with the exterior, of the network are violated, as can be seen in figure 1. The situation was worsened by the model in [1], because nearby points of the network repelled one another, thus making gap closure even less likely.

Previous work has dealt with this issue in different ways, often without addressing it explicitly. Some semi-automatic methods require user-defined endpoints, which must be connected. These include methods minimizing the optimal path between endpoints [2, 3], and active contour models such as 'ribbon snakes' [4, 5] and 'ziplock snakes' [6]. In this case, the topology is fixed and occlusions are not an issue, but these models cannot succeed when the network has a more complicated topology. One road tracking method [7] uses an 'inertia' term that allows the estimated road to extend a short distance into a gap. Methods using marked point processes [8] penalize isolated extremities, and thus do penalize gaps implicitly, if not directly.

In a preliminary attempt to address this problem, [9] added a nonlocal 'gap closure' force to the gradient descent equation. This term could not be derived from an energy, however, and thus lay outside the framework of higher-order active contours, complicating analysis, and meaning that convergence to an energy minimum could not be guaranteed. In this paper, we present a solution to the gap closure problem within the framework of higher-order active contours by introducing a novel quadratic *continuation energy*. This energy is added to the model used in [1], which is briefly summarized at the beginning of section 2. In order

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to close gaps, we first identify them; we equate gaps with 'nearby opposing extremities' of the network, and then construct a quadratic energy that penalizes such configurations while simultaneously annulling the repulsive effects of the model in [1]. The identification of gaps and the continuation energy are described in sections 2.1 and 2.2 respectively. The resulting continuation energy presents a tough challenge numerically due to the presence of first and second derivatives of the contour curvature in its gradient descent equation. We describe how we deal with this challenge in section 3. In section 4, we show the benefits of the new energy via geometric experiments and the extraction of road networks.

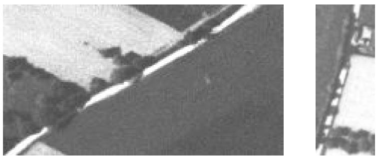

Fig. 1. Aerial images with occlusions.

2. A NEW MODEL

The new continuation energy will be added to the energy used in [1], which we now briefly summarize. Let $I : \Omega \to \mathbb{R}$ be an image, where $\Omega \subset \mathbb{R}^2$. Define a region by its boundary, denoted C, and called a contour. We define a functional on the space of boundaries in Ω of the following form:

$$E(C) = E_g(C) + \lambda E_i(C; I) \ , \tag{1}$$

where $\lambda \in \mathbb{R}$. The geometric part E_g is

$$E_g(C) = \mathcal{L}(C) + \alpha \mathcal{A}(C) - \beta \iint dp\, dp'\, (\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}')\, \Psi(R(p,p')) \ , \tag{2}$$

where $\mathcal{L}$ is the contour length, and $\mathcal{A}$ the interior area; $\vec{\mathbf{t}}$ is the tangent vector to the contour; Ψ is a function with the form of a smoothed hard-core potential ($\Psi(x)$ is 1 if $x < d - \epsilon$, 0 if $x > d + \epsilon$, and $\frac{1}{2}(1 - \frac{x-d}{\epsilon} - \frac{1}{\pi}\sin(\pi \frac{x-d}{\epsilon}))$ otherwise; d and ϵ are two positive parameters); and $R(p,p')$ is the Euclidean distance from $C(p)$ to $C(p')$. Primed variables are evaluated at p' or $C(p')$.

The image part E_i is composed of three terms:

$$E_i(C; I) = \int dp\, \hat{\mathbf{n}} \cdot \nabla I - \lambda_1 \int dp\, G[I](p) - \lambda_2 \iint dp\, dp'\, (\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}')\, (\nabla I \cdot \nabla I')\, \Psi(R(p,p')) \ , \tag{3}$$

where $\hat{\mathbf{n}}$ is the unit outward normal to the contour. The first, linear term favours situations in which the outward normal is opposed to a large image gradient. The second, linear term incorporates a simple line detector filter measurement. The third, *quadratic* term favours situations in which pairs of points that are not too distant, and whose tangent vectors are antiparallel, lie on large image gradients that point in opposite directions. Note that the model describes not just the 1D network topology, but the region occupied by the network in the image.

2.1. Identification of gaps

In order to develop a continuation energy, we identify situations in which continuation is violated, and penalize them. We equate a failure of continuation with configurations in which there are 'nearby opposing extremities' in the network, a situation illustrated in the leftmost part of figure 4. We define three 'switch' functions, which correspond to 'nearby', 'opposing', and 'extremities', and use their product to ensure that only these configurations contribute to the new continuation energy:

$$\begin{aligned} S_{\mathrm{n}}(p,p') &= \Phi(R(p,p')) \ , \\ S_{\mathrm{o}}(p,p') &= H(\hat{R} \cdot \hat{\mathbf{n}}(p)) \ , \\ S_{\mathrm{e}}(p) &= H(\kappa(p) - \epsilon) \ , \\ S(p,p') &= S_{\mathrm{n}}(p,p') S_{\mathrm{o}}(p,p') S_{\mathrm{o}}(p',p) S_{\mathrm{e}}(p) S_{\mathrm{e}}(p') \ . \end{aligned}$$

Here H is a smoothed Heaviside function, κ is the signed curvature, ϵ is a fixed positive threshold, and $\hat{R}$ is the unit vector pointing from $C(p)$ to $C(p')$. The function Φ, shown in figure 2.1, is minimum for $R(p,p') = 0$ and increasing. Thus S_{n} will ensure that the new energy increases with gap size up to a certain point, after which it is constant. The definition of 'nearby' is given by the range of Φ, which is ~ 30 pixels. This should be contrasted with the range of the function Ψ in equation (2) (~ 3 pixels).

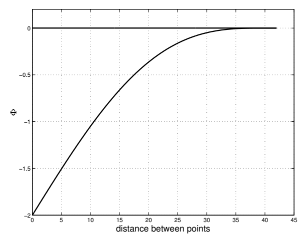

Fig. 2. The function Φ. $\Phi(x) = \frac{x}{r} + \frac{1}{\pi}\sin(\frac{\pi x}{r}) - 1$ if $x < r$, 0 otherwise (r is a positive parameter).

Switch S_{o} is non-zero when p' lies roughly along the outward pointing normal direction from p, which corresponds to the fact that points on 'opposing' extremities lie outside the contour with respect to one another and are roughly aligned. This is illustrated in figure 2.1. Switch S_{e} is non-zero when the curvature at p is greater than ϵ. This corresponds to the fact that 'extremities' have large, positive curvature (the rest of the network has curvatures close to zero or negative).

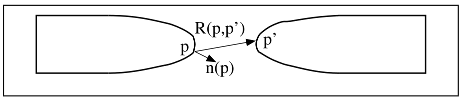

Fig. 3. A pair of 'opposing extremities'.

The product switch, $S(p, p')$, will thus be non-zero only when both p and p' have curvatures greater than ϵ, and when they are mutually exterior and roughly aligned. Due to S_{n}, the value of S decreases as the separation between $C(p)$ and $C(p')$ decreases.

2.2. Continuation energy

The new continuation energy E_{con} is built using the product switch function $S(p, p')$ described above. It takes the form $E_{\mathrm{con}}(C) = E_{\mathrm{can}}(C) + E_{\mathrm{pen}}(C)$. The first term, E_{can}, cancels the third term of equation (2) for nearby opposing extremities, while the second, E_{pen}, penalizes the gap between such extremities:

$$E_{\mathrm{can}}(C) = \beta \iint dp\, dp'\ (\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}')\Psi(R(p, p'))S(p, p') \quad \text{(5a)}$$

$$E_{\mathrm{pen}}(C) = -\beta_2 \iint dp\, dp'\ (\vec{\mathbf{t}} \cdot \vec{\mathbf{t}}')\Phi(R)S(p, p') \quad \text{(5b)}$$

Note that equation (5a) is of the same form as equation (2), but of the opposite sign, and with the integrand multiplied by the product switch that identifies nearby opposing extremities. Its effect is thus simply to annul the third term of equation (2) for nearby opposing extremities. Equation (5b) again takes a similar form, but differs in using the function Φ rather than Ψ. The result is that the interaction expressed by this term is of much longer range than that of the third term of equation (2), and is of the opposite sign, thus favouring the absence of gaps between nearby opposing extremities. As with the third term of equation 2, and all higher-order terms, the force due to E_{con} is nonlocal, being given at each point by an integral over the contour.

3. CONTOUR EVOLUTION

We solve the minimization problem for the new energy using the extended level set techniques necessary for dealing with the nonlocal forces generated by higher-order active contour energies. These techniques are described in detail in [10]. The new energy introduces its own complications, however. The functional derivative of E_{con} contains many terms (which space restrictions prevent us from including here), all of them nonlocal. Some of these contain both first and second derivatives of the contour curvature, which translate into third and fourth derivatives of the level set function, and first, second, and third derivatives of the smoothed Heaviside function H. We adopt specific measures to ameliorate the numerical difficulties that these terms could cause.

First, we adjust the function H so that its derivatives have a similar magnitude to the function itself. Failure to do this results in the derivatives of H having very large magnitudes that slow down the evolution (we use an adaptive time step inversely proportional to the maximum force on the contour), and if sufficiently exaggerated, destabilize it. Note that this is not an approximation, but a choice of interaction. Second, before computing geometric quantities such as the curvature and its derivatives, we apply Gaussian smoothing with $\sigma_x = \sigma_y = 1$ to the level set function. In particular, this smooths the contour curvature, and leads to better recognition of extremities. Third, to improve accuracy, all derivatives are computed from the resulting level set function using fourth order finite differences. Finally, as in [1, 9], in order to compute the nonlocal forces, we have to approximate integrals over the contour by sums over extracted contour points. Because we are dealing with periodic functions, we can improve the precision of this approximation to fourth order by using points separated by equal contour lengths. Thus, in order to compute the force at a point p, we first redistribute the extracted contour points around p so that they are equidistant; we then compute the necessary geometric quantities at these points using the techniques of [1]; and finally perform the numerical integration.

4. EXPERIMENTAL RESULTS

The first experiments we show are purely geometric, *i.e.* the terms in equation (3) are absent, and demonstrate the behaviour of the new quadratic energy term $E_{\mathrm{con}}(C)$. We present the results in figure 4. In the absence of E_{con}, two nearby opposing extremities repel, and develop into two separated networks. This effect is clearly undesirable, since it is more probable that the arms should be connected. Adding E_{con}, the arms extend towards one another and join to produce a network with continuation.

We now show experiments on real images. We start with the simple image in figure 4. The result of extraction using only the model $E(C)$ fails to continue the network across the gap. Adding $E_{\mathrm{con}}(C)$, we find that the contour now continues the network across the gap, thus producing the correct result.

Another result is shown in figure 4. Despite the trees obscuring the network, the road is reconstructed correctly.

Note that in both cases, we took as an initial contour a

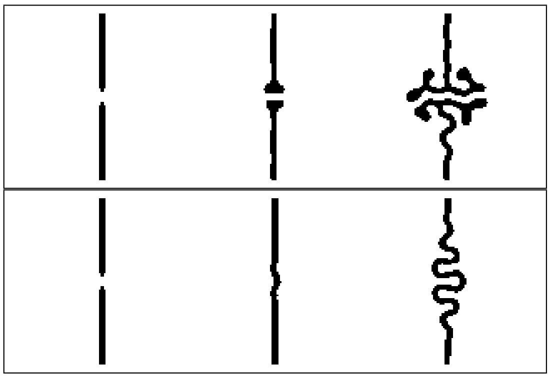

Fig. 4. Two purely geometric evolutions, one without (top) and one with (bottom) the new continuation energy E_{con}.

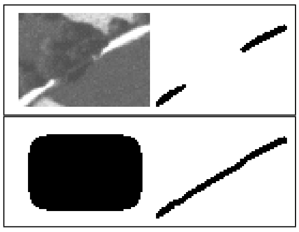

Fig. 5. Top: aerial image and extraction with $E(C)$; bottom: generic initialization and extraction with E_{con} added.

rounded rectangle filling the image domain, as shown in the bottom left of figure 4. This is very far from the solution, and the fact that the model still manages to find the correct result demonstrates the robustness resulting from the inclusion of the sophisticated prior geometric information described by the higher-order terms.

5. CONCLUSION

In this paper, we have tackled the problem generated by occlusions in the extraction of line networks, and in particular, road networks, from images. It often happens that entities such trees, buildings or their shadows obscure the network. Using the framework of higher-order active contours, we can identify situations where pairs of network extremities should be connected, and we have described a new quadratic energy term that favours such continuation. Gradient descent using this new energy is a delicate matter due to the presence of numerous force terms containing higher derivatives, which require special attention. Experiments on remote sensing images demonstrate the efficacy of the new continuation energy, but the model is not limited to road network extraction. It could be applied, for example to medical or biological network extraction.

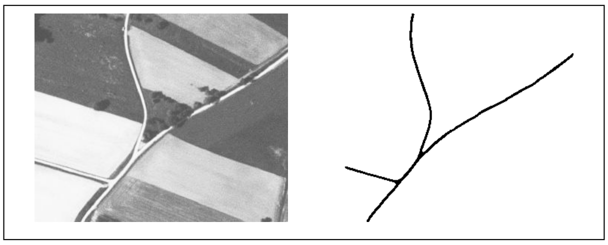

Fig. 6. Aerial image and result of extraction.

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