A Multi–Level Hypergraph Partitioning Algorithm using Rough Set Clustering

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Abstract. The hypergraph partitioning problem has many applications in scientific computing and provides a more accurate inter-processor communication model for distributed systems than the equivalent graph problem. In this paper, we propose a sequential multi-level hypergraph partitioning algorithm. The algorithm makes novel use of the technique of rough set clustering in categorising the vertices of the hypergraph. The algorithm treats hyperedges as features of the hypergraph and tries to discard unimportant hyperedges to make better clustering decisions. It also focuses on the trade-off to be made between local vertex matching decisions (which have low cost in terms of the space required and time taken) and global decisions (which can be of better quality but have greater costs). The algorithm is evaluated and compared to state-of-the-art algorithms on a range of benchmarks. The results show that it generates better partition quality.

1 Introduction

A hypergraph is a pair: a set of vertices and a set of hyperedges. Each hyperedge is a subset of the vertex set (there is no restriction on its size). The *hypergraph partitioning problem* asks, roughly speaking, for a partition of the vertex set such that the vertices are evenly distributed amongst the parts and the number of hyperedges that intersect multiple parts is minimised. A tool to solve this probem is called a *partitioner*. Hypergraph partitioning has applications in many areas of computer science such as data mining and image processing.

The hypergraph partitioning problem is a generalisation of the graph partitioning problem (in which the edges of a graph are subsets of the vertex set of size two contrasting with hyperedges whose size is unbounded), and provides a more natural way of representing the relationships between objects inherent in many problems [14]. The removal of the constraint on edge size, however, increases the practical difficulty of partitioning [13]. As both the graph and hypergraph variants of the partitioning problem are NP-hard [12], a number of heuristic algorithms have been proposed [10, 17]. In this paper, we propose and evaluate a new algorithm.

Our serial *Feature Extraction Hypergraph Partitioning (FEHG)* algorithm is of a type known as multi-level. It has three distinct phases: coarsening, initial partitioning and uncoarsening. During coarsening vertices are merged to obtain hypergraphs with progressively smaller vertex sets. After the coarsening stage, the partitioning problem is solved on the smaller hypergraph obtained in the initial partitioning. During uncoarsening, the coarsening stage is reversed and the solution obtained on the small hypergraph is used to provide a solution on the input hypergraph. We describe some of the problems of multi-level partitioning that motivate our study:

1. Heuristics for multi-level hypergraph partitioning focus on finding highly-connected clusters of vertices that can be merged to form a coarser hypergraph. This requires a metric of similarity, the evaluation of which requires the recognition of "similar" vertices. As the mean and standard deviation of vertex degrees are usually high (and so the similarity of pairs of vertices is typically low), it is often a problem to define and measure the similarity [9].
2. There can be redundancy in modelling scientific problems with hypergraphs and it is desirable to remove it. In [13], an attempt to reduce the storage overhead of saving and processing hypergraphs is presented, but the strategy can increase either the storage requirement or the running time in some cases.
3. Decision making for matching vertices (that will be merged) is usually done locally. Global decisions are avoided due to their high cost and complexity though they give better results [21]. All proposed heuristics reduce the search domain and try to find the vertices to be matched using some degree of randomness. This degrades the quality of the partitioning by increasing the possibility of getting stuck in a local minimum. A better trade-off is needed between the low cost of local decisions and the high quality of global ones.

Highlights of our contribution:

– We propose a new serial multi-level hypergraph partitioning algorithm which gives significant quality improvements over state-of-the-art algorithms.
– We use rough set based clustering techniques for removing redundant attributes while partitioning and so make better clustering decisions.
– We provide a trade-off between global and local clustering methods by calculating sets of core vertices (a global decision) and then traversing these cores one at a time to find best matchings between vertices (a local decision).
– We show that solely relying on a vertex similarity metric can result in major degradation of the partitioning quality for some hypergraphs and different coarsening methods should be considered.

In the next section, we briefly review partitioning algorithms and software tools. In Sect. 3, we give a technical introduction to the Hypergraph Partitioning Problem. In Sect. 4 we introduce FEHG. In Sect. 5 we evaluate the algorithm and report results of a simulation comparing FEHG to state-of-the-art algorithms. Finally in Sect. 6, we conclude with comments on ongoing and future work.

2 Related Work

We provide a brief review of algorithms, tools, and applications of hypergraph partitioning; the reader is referred to [21] for an extensive survey. We note that, in general, there is no partitioner recognized to perform well for all types of hypergraphs as there are always trade-offs such as those between quality and speed [21]. Partitioning algorithms can be serial [16, 4] or parallel [8], iterative move-based [10] or multi-level [5], static [22] or dynamic [5], recursive [8] or direct [1], and finally they can work directly on hypergraphs [16] or model them as graphs and use graph partitioning algorithms [17].

Few software tools are available for hypergraph partitioning and there is no unified framework for hypergraph processing. One popular tool designed for VLSI circuit partitioning is *hMetis*[1] [16]. The algorithms are based on multi-level partitioning schemes and support recursive bisectioning (*shmetis, hmetis*), and direct *k–way* partitioning (*kmetis*). Examples of tools that are designed for specific applications are *MLPart*[2] and *Mondriaan*[3], designed for VLSI circuit partitioning and rectangular sparse matrix-vector multiplications, respectively. The emphasis of *MLPart* is on simplicity of design and *Mondriaan* uses the idea of 2D matrix partitioning to enhance performance [22]. *PaToH*[4] [4] is a multi-level recursive bipartitioning tool designed for serial hypergraph partitioning. It supports agglomerative (vertex clusters are formed one at a time) and hierarchical (several clusters of vertices can be formed simultaneously) clustering algorithms. *Zoltan*[5] [8] is developed for parallel applications. Its library includes a range of tools for problems such as dynamic load balancing and graph and hypergraph colouring and partitioning. Both static and dynamic hypergraph partitioning are supported as are multi-criteria load balancing and processor heterogeneity.

There are a wide range of applications for hypergraph partitioning (see, for example, [20]) including classifying gene expression data, replication management in distributed databases [6] and high dimensional data clustering [15].

3 Definitions

3.1 Hypergraph Partitioning

A hypergraph $H = (V, E)$ is a pair consisting of a finite set of vertices V, with size $|V| = n$ and a multi–set $E \subseteq 2^n$ of hyperedges with size $|E| = m$. For a hyperedge $e \in E$ and vertex v, we say e contains v, or is incident to v, if $v \in e$; this is represented by $e \triangleright v$. The degree of a vertex is the number of distinct incident hyperedges and the size of a hyperedge $|e|$ is the number of vertices it contains. The hypergraph is simply a graph if every hyperedge has size two.

Definition 1. *Let k be a non–negative integer and let $H = (V, E)$ be a hypergraph. A* **k–way partitioning** *of H is a collection of sets $\Pi = \{P_1, P_2, \cdots, P_k\}$ such that $\cup_{i=1}^{k} P_i = V$, and $\forall P_i, P_j \subset V$, $1 \leqslant i \neq j \leqslant k$, we have $P_i \neq \emptyset$, $P_i \cap P_j = \emptyset$.*

[1] http://glaros.dtc.umn.edu/gkhome/metis/hmetis/overview
[2] http://vlsicad.ucsd.edu/GSRC/bookshelf/Slots/Partitioning/MLPart/
[3] http://www.staff.science.uu.nl/ bisse101/Mondriaan/mondriaan.html
[4] http://bmi.osu.edu/umit/software.html
[5] http://www.cs.sandia.gov/zoltan/

We say that $v \in V$ is *assigned* to a part $P \in \Pi$ if $v \in P$. Let $\omega\colon V \mapsto \mathbb{N}$ and $\gamma\colon E \mapsto \mathbb{N}$ be weight functions for the vertices and hyperedges. The weight of P is defined as $\omega(P) = \sum_{v \in P} \omega(v)$. A hyperedge $e \in E$ is said to be connected to P if $e \cap P \neq \emptyset$. The *connectivity degree* of e is the number of parts connected to e and is denoted by $\lambda_e(H, \Pi)$. A hyperedge is cut if it connects to more than one part. We define the *cost* of a partition Π of H as

$$\sum\nolimits_{e \in E} (\gamma(e) \cdot (\lambda_e(H, \Pi) - 1)).$$

The *connectivity objective* is to find a partition Π of low cost. Let W_{ave} be the average weight of the parts: that is $W_{\text{ave}} = \sum_{v \in V} \omega(v)/k$. The *balancing requirement* asks that all parts of the partition have similar weight: that is, given *imbalance tolerance* $\epsilon \in (0, 1)$, it is required that

$$W_{\text{ave}} \cdot (1 - \epsilon) \leqslant \omega(P) \leqslant W_{\text{ave}} \cdot (1 + \epsilon),\ \forall P \in \Pi. \tag{1}$$

The *hypergraph partitioning problem* is finding a minimum cost partition Π of H that satisfies the balancing requirement.

3.2 Rough Set Clustering

Rough set theory was introduced by Pawlak in 1991 as an approach to understanding fuzzy and uncertain knowledge [19]. It provides a mathematical tool to discover hidden patterns in data; it can be used, for example, for feature selection, data reduction, pattern extraction. It can deal efficiently with large data sets [2] by extracting global information that resides in the data.

Definition 2. *Let $\mathbb{U}$ be a non-empty finite set of objects (called the universe). Let $\mathbf{A}$ be a non-empty finite set of attributes. Let $\mathbf{V}$ be a multi-set of attribute values such that $\mathbf{V}_a \in \mathbf{V}$ is a set of values for each $a \in \mathbf{A}$. Let $\mathcal{F}$ be a mapping function such that $\mathcal{F}(u, a) \mapsto \mathbf{V}_a, \forall (a, u) \in \mathbf{A} \times \mathbb{U}$. Then $\mathfrak{I} = (\mathbb{U}, \mathbf{A}, \mathbf{V}, \mathcal{F})$ is called an* **information system**.

For any $\mathbf{B} \subseteq \mathbf{A}$ there is an associated equivalence relation denoted $\text{IND}(\mathbf{B})$ and called a *B-Indiscernibility* relation:

$$\text{IND}(\mathbf{B}) = \left\{(u, v) \in \mathbb{U}^2 \mid \forall b \in \mathbf{B},\, \mathcal{F}(u, b) = \mathcal{F}(v, b)\right\}. \tag{2}$$

When $(u, v) \in \text{IND}(\mathbf{B})$, it is said that u and v are indiscernible under B and this is represented as $u\mathcal{R}v$. Furthermore, the equivalence class of u with respect to B is $[u]_{\mathbf{B}} = \{v \in \mathbb{U} \mid u\mathcal{R}v\}$. The equivalence relation provides a partitioning of the universe and it is represented as $\mathbb{U}/\text{IND}(\mathbf{B})$ or simply $\mathbb{U}/\text{IND}$. Thus, for every $X \in \mathbb{U}$, and with respect to $\mathbf{B} \subseteq \mathbf{A}$, a $\mathbf{B}$*–lower* and $\mathbf{B}$*–upper* approximation can be defined for X, by, respectively, $\underline{\mathbf{B}X} = \{x \mid [x]_{\mathbf{B}} \subseteq X\}$ and $\overline{\mathbf{B}X} = \{x \mid [x]_{\mathbf{B}} \cap X \neq \emptyset\}$. $\underline{\mathbf{B}X}$ contains objects that belong to X with certainty and $\overline{\mathbf{B}X}$ contains objects that possibly belong to X. We describe a hypergraph $H = (V, E)$ with an information system $\mathcal{I}_H = (V, E, \mathbf{V}, \mathcal{F})$ such that $\mathbf{V}_e \in [0, 1], \forall e \in E$ and the mapping function is defined as:

$$\mathcal{F}(v, e) = \frac{f(e)}{\sum_{\forall e' \rhd v} \gamma(e')},\ \text{where } f(e) = \gamma(e) \text{ if } e \rhd v \text{ and is otherwise } 0.$$

3.3 Hyperedge Connectivity Graph

We use rough set clustering in our algorithm to make better clustering decisions in hypergraphs. We will need a measure of *similarity* of a pair of hyperedges, a function $sim(\cdot)$. Different similarity measures, such as *Jaccard Index* or *Cosine Measure*, can be used. Similarity is scaled according to the weight of hyperedges: for two $e_i, e_j \in E$ the scaling factor is $\frac{\gamma(e_i)+\gamma(e_j)}{2\times\max_{e\in E}(\gamma(e))}$.

Definition 3. *For a given similarity threshold $s \in (0,1)$, the* **Hyperedge Connectivity Graph** *(HCG) of a hypergraph $H = (V, E)$ is a graph $\mathcal{G}^s(\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = E$ and two vertices $v_i, v_j \in \mathcal{V}$ are adjacent if, for the corresponding hyperedges $e_i, e_j \in E$ we have $sim(e_i, e_j) \geqslant s$.*

We discuss the importance of the choosing the similarity threshold in Sect. 5.

4 The Algorithm

The proposed algorithm is a recursive multi–level algorithm composed of coarsening, initial partitioning and uncoarsening phases.

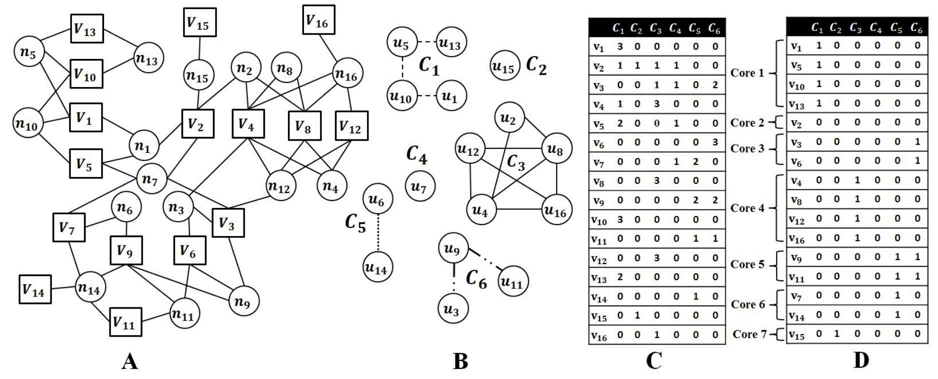
4.1 The Coarsening

The process of coarsening involves finding a sequence of hypergraphs $H = (V, E), H^1 = \left(V^1, E^1\right), \ldots, H^c = (V^c, E^c)$ such that each hypergraph has fewer vertices than its predecessor and the coarsest hypergraph H^c has fewer vertices than a predefined threshold. We say H^i is the hypergraph found at the ith level of coarsening. The compression ratio of successive levels i, j is defined as $\frac{|V^i|}{|V^j|}$. We use vertex matching to match a pair of vertices and merge them to form a coarser vertex. The best pair is chosen using the *Weighted Jaccard Index* defined by:

$$J(u,v) = \frac{\sum_{\{e \triangleright v \,\wedge\, e \triangleright u\}} \gamma(e)}{\sum_{\{e \triangleright v \,\vee\, e \triangleright u\}} \gamma(e)}, \quad v, u \in V \text{ , and } \forall e \in E. \tag{3}$$

This is similar to *non-weighted jaccard index* in *PaToH* which is called *Scaled Heavy Connectivity Matching.* The algorithm first constructs *HCG* graph defined above. by traversing H using Breadth-First Search (the graph itself does not need to be saved). A partition E^{R} of the hyperedges of H is then obtained where each part contains hyperedges that belong to the same connected component of HCG. The size and weight of each $e_{\mathrm{R}} \in E^{\mathrm{R}}$ is the number of hyperedges it contains and the sum of their weights, respectively. If we represent a hypergraph with an information system, a reduced information system $\mathcal{I}_H^{\mathrm{R}}\left(V, E^{\mathrm{R}}, \mathbf{V}^{\mathrm{R}}, \mathcal{F}^{\mathrm{R}}\right)$ is constructed based on E^{R}. A vertex is incident to $e_{\mathrm{R}} \in E^{\mathrm{R}}$ if at least one of its incident edges $e \in H$ is in e_{R}. In addition $\mathbf{V}^{\mathrm{R}}_{e_{\mathrm{R}}} \subseteq \mathbb{N}, \forall e_{\mathrm{R}} \in E^{\mathrm{R}}$ and the mapping function is defined as:

$$\mathcal{F}^{\mathrm{R}}(v, e_{\mathrm{R}}) = \left|\{e \triangleright v \,\wedge\, e \in e_{\mathrm{R}}, \forall e \in E\ \}\right|. \tag{4}$$



	C_1	C_2	C_3	C_4	C_5	C_6
v_1	3	0	0	0	0	0
v_2	1	1	1	1	0	0
v_3	0	0	1	1	0	2
v_4	1	0	3	0	0	0
v_5	2	0	0	1	0	0
v_6	0	0	0	0	0	3
v_7	0	0	0	1	2	0
v_8	0	0	3	0	0	0
v_9	0	0	0	0	2	2
v_{10}	3	0	0	0	0	0
v_{11}	0	0	0	0	1	1
v_{12}	0	0	3	0	0	0
v_{13}	2	0	0	0	0	0
v_{14}	0	0	0	0	1	0
v_{15}	0	1	0	0	0	0
v_{16}	0	0	1	0	0	0

		C_1	C_2	C_3	C_4	C_5	C_6
Core 1	v_1	1	0	0	0	0	0
	v_5	1	0	0	0	0	0
	v_{10}	1	0	0	0	0	0
	v_{13}	1	0	0	0	0	0
Core 2	v_2	0	0	0	0	0	0
Core 3	v_3	0	0	0	0	0	1
	v_6	0	0	0	0	0	1
Core 4	v_4	0	0	1	0	0	0
	v_8	0	0	1	0	0	0
	v_{12}	0	0	1	0	0	0
	v_{16}	0	0	1	0	0	0
Core 5	v_9	0	0	0	0	1	1
	v_{11}	0	0	0	0	1	1
Core 6	v_7	0	0	0	0	1	0
	v_{14}	0	0	0	0	1	0
Core 7	v_{15}	0	1	0	0	0	0

Fig. 1: An example of the coarsening procedure. (a) The sample hypergraph. (b) *HCG* using *weighted jaccard index* in (3) and similarity threshold $s = 0.5$. (c) The reduced information system, and (d) Remaining attributes after removing superfluous attributes for clustering threshold $c = 0.5$.

The next step is to remove superfluous attributes from E^{R}. A clustering threshold $c \in [0, 1]$ is defined and the mapping function of (4) is transformed to:

$$\mathcal{F}^f(v, e_{\mathrm{R}}) = \begin{cases} 1, & \text{if } \frac{\mathcal{F}^{\mathrm{R}}(v, e_{\mathrm{R}})}{|\{e \triangleright v, \forall e \in E\}|} \geqslant c \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

At this point we have a reduced information system $\mathcal{I}^{\mathrm{f}}$ and we use this to find clusters of vertices using rough set clustering techniques. Using the indiscernibility relation defined in (2), the equivalence relation between vertices (Sect. 3.2), and the mapping function $\mathcal{F}^{\mathrm{f}}$ of (5), $\mathbb{U}/\mathrm{IND}(E^{\mathrm{R}})$ provides a partitioning of the vertex set V. The parts are called the cores of the hypergraph. Cores of unit size as well as vertices whose $\mathcal{F}^{\mathrm{f}}(v, e_{\mathrm{R}}) = 0$, $\forall e_{\mathrm{R}} \in E^{\mathrm{R}}$ are categorised as non–core vertices and they will be processed after core vertices. The cores are visited one at a time and they are searched locally to find the best matching pairs according to (3). The larger the mean vertex degree in the hypergraph is, the larger denominator we get in (5) and this makes it difficult to choose a clustering threshold. As a result, large mean vertex degrees produce more cores of unit size and this causes the number of vertices that belong to cores to be small compared to $|V|$.

To maintain a certain compression ratio between two successive levels of the coarsening, we perform a random matching of the non–core vertices. An example of the coarsening procedure is given in Fig. 1.

4.2 Initial Partitioning and Uncoarsening

In the initial partitioning phase, a bipartitioning on the coarsest hypergraph H^c is found using a number of algorithms. An output is selected to be projected back to the original hypergraph: if many outputs fulfill the balancing requirement then the one with lowest cost is chosen else it is the output that comes closest to

Table 1: Tested hypergraphs and their specifications

Hypergraph	Description	Rows	Columns	Non-Zeros	Structure[1]	NSC[2]
CNR–2000	Small web crawl of Italian CNR domain	325,557	325,557	3,216,152	USYM	100,977
AS–22JULY06	Internet routers	22,963	22,963	96,872	SYM	1
CELEGANSNEURAL	Neural Network of Nematode C. Elegans	297	297	2,345	USYM	57
NETSCIENCE	Co-authorship of scientists in Network Theory	1,589	1,589	5,484	SYM	396
PGPGIANTCOMPO	Largest connected component in graph of PGP users	10,680	10,680	48,632	SYM	1
GUPTA1	Linear Programming matrix ($A \times A^T$)	31,802	31,802	2,164,210	SYM	1
MARK3JAC120	Jacobian from MULTIMOD Mark3	54,929	54,929	322,483	USYM	1,921
NOTREDAME	Barabasi's web page network of nd.edu	325,729	325,729	929,849	USYM	231,666
PATENTS–MAIN	Pajek network: mainNBER US Patent Citations	240,547	240,547	560,943	USYM	240,547
STD1–JAC3	Chemical process simulation	21,982	21,982	1,455,374	USYM	1
COND–MAT–2005	Collaboration network, www.arxiv.org	40,421	40,421	351,382	SYM	1,798

[1] NSC stands for the number of strongly connected components.
[2] SYM stands for symmetric and USYM stands for unsymmetric.

meeting the balancing requirement. The algorithms used are random partitioning (randomly assign vertices to parts), linear partitioning (linearly assign vertices to parts), and a modification of the *FM* algorithm [10]. During uncoarsening, we try to refine the quality of the partitioning by moving the vertices across the partition boundary. A vertex is on the boundary if at least one of its incident edges is cut by the bipartitioning. The FM algorithm and its variants have been shown to be successful for the refinement process [8, 16] and we use a modified version of FM or *Boundary FM* algorithm.

5 Evaluation

We have compared our algorithm (*FEHG*) with *PHG* (the Zoltan hypergraph partitioner) [8], *hMetis* [16], and *PaToH* [4]. These algorithms achieve k-way partitioning by recursive bipartitioning. The evaluated hypergraphs listed in Table 1 are from the University of Florida Sparse Matrix Collection [7]. They are from a variety of applications with different specifications and include both symmetric and non–symmetric instances, and hypergraphs with different numbers of strongly connected components, etc. Each matrix in the table is treated as a hypergraph. We use the column-net model where each row of the matrix corresponds to a vertex and each column corresponds to a hyperedge [8]. The weights of vertices and hyperedges are set to unity. The evaluated tools have different input parameters that can be selected by the user. For our case, we use default settings for the comparison: *shmetis* is the default partitioner selected for *hMetis*, *PaToH* is initialised by setting `SBProbType` parameter to `PATOH_SUGPARAM_DEFAULT`, and the coarsening algorithm for *PHG* is set to *agglomerative*. All of them use a variation of *FM* for the refinement and uncoarsening phase.

FEHG has two input parameters: the *similarity threshold* to construct *HCG*, and the clustering threshold from (5). The values chosen for these parameters can have a large impact on the quality of the partitioning. We describe how calculate the similarity threshold when the *Jaccard Index* is used for measuring the similarity between hyperedges.

The Clustering Coefficient (CC) is a graph theory measure determined by the degree to which a node clusters with other nodes of the graph or hypergraph.

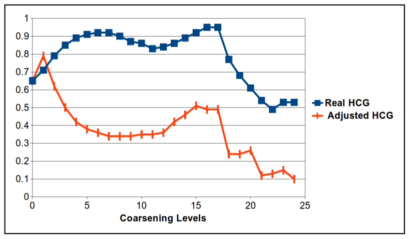
Different methods for finding CC in hypergraphs have been proposed [18]. Given a hypergraph $H = (V, E)$, we define CC for a hyperedge $e \in E$ as:

$$\mathrm{CC}(e) = \begin{cases} \dfrac{\sum_{\{e' \cap e \neq \emptyset\}} \left(\left(1 - \frac{(|e|-1) - |e \cap e'|}{|e|-1} \right) \cdot \gamma(e') \right)}{\sum_{\{v \in e\}} \sum_{\{e'' \triangleright v\}} \gamma(e'')}, \forall e', e'' \in E \backslash e, & \text{if } |e| > 1 \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

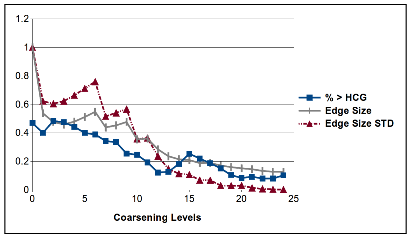
The CC of the hypergraph is calculated as the average CC over all hyperedges. We calculate CC at the start of the algorithm. As the structure of the hypergraph changes at each level of coarsening, we readjust its value instead of recalculation. As proposed in [11] to analyse Facebook social networks and theoretically investigated in [3] on sparse random intersection graphs, the clustering of nodes in hypergraphs is inversely correlated with average vertex degree. Based on this, we readjust CC's value according to the variation of average vertex degree from one level of the coarsening to the next. Finally, CC value of the hypergraph is set as the similarity threshold at each coarsening level.

Figure 2a depicts the variation of *similarity threshold* for each coarsening level of a tested graph CNR–2000. Both the readjusted value and the actual value are shown. The readjusted value provides a lower bound for the actual value and it is about 50% of its value from the third iteration onward which is sufficient for feature reduction. In Fig. 2b, the percentage of the edges whose clustering coefficients are at least equal to the *similarity threshold* along with normalised variation of edge size and its standard deviation (STD) is represented. As the partitioner gets close to the coarsest hypergraph we have small average size of hyperedges (2.09) and small average vertex degrees (2.47) but larger vertex degree standard deviation (14.33); most of the vertices share very few hyperedges so clustering decisions are difficult. As we see, the automatic readjustment still catches the possible similarities. In general we achieve a cut size 50 for CNR–2000.

In our evaluation we found that variation of the similarity threshold has higher impact on the quality of the partitioning than the clustering threshold. The reason is that hyperedges with higher CC value are more likely to cluster with others and they get higher coefficient in (4) and tend to be included in the final reduced information system in (5). This reduces the effects of clustering threshold variations. Therefore, we remove each $e_R \in E^R$ of unit size (refer to Sect. 4.1) and we set the clustering threshold to 0 in (5) for the others. For example, edge partitions C_2 and C_4 are removed from the table in Fig. 1c. For all tested hypergraphs, the algorithms are each run 20 times and the average and best cut sizes are reported. Simulations are done with 2% imbalance tolerance in (1) and the number of parts are $\{2, 4, 8, 16, 32\}$. The final imbalance achieved by the algorithms are not reported because the balancing requirement was always met by all algorithms. The simulation results as well as standard deviation from the average cut are reported in Table 2. The latter could be used as a measure of the robustness of the algorithms specifically when they give close partitioning quality. The values are normalised with the best cut generated among all algorithms except the standard deviation. According to the results, *FEHG* performs very well compared to *Zoltan* and *hMetis* and it is competitive with

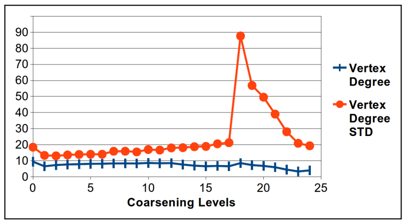


(a) *Similarity Threshold*



(b) Change *vs.* edge size and its STD

Fig. 2: (a) Readjusted *similarity threshold* s for the test hypergraph CNR–2000 according to (6) compared to its recalculation at each coarsening level. (b) The percentage of the hyperedges whose CC is more than s and comparison to normalised edge size edge and its standard deviation (STD).



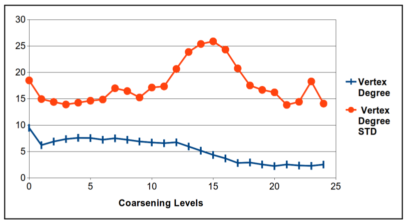


Fig. 3: Variation of vertex degree and its standard deviation when *FEHG* makes extra effort for achieving maximum vertex similarity matching vs normal matching

PaToH. For example in Noterdame and Patents-Main, FEHG achieves a superior quality improvement compared to *Zoltan* and *hMetis*. In another simulation, we investigate whether relying only on a vertex similarity metric is enough to achieve better partition quality. When two vertices are matched, we refer to their similarity degree as the *roughness* of the match and it is calculated using (3). Matching pairs of vertices with higher similarity degree at each level of coarsening means higher average roughness of the matched vertices in that level. According to the algorithms that investigate vertex similarity metrics, an algorithm would be better if it yields higher average roughness for levels of coarsening compared to the others [21]. Furthermore, the decision about the vertex similarity is made locally in those algorithms without collecting global information. In *FEHG*, we refer to the average roughness of core vertices as core roughness. We consider two scenarios for our test while we find a pair match for non–core vertices: in the first one, a match is allowed for a non–core vertex if the roughness of the match is at least equal to the core roughness. In the second scenario we allow non–core vertex to be matched to any vertex as long as the roughness of the match is greater than zero. In the first scenario, the emphasis is on finding vertices with higher similarity as is the case for similarity metric based methods and it guarantees higher average roughness compared to the second scenario during levels of coarsening. The test is done on the hypergraphs and the result

Table 2: Quality comparison of the algorithms for different part sizes and imbalance factor 2% with normalised values.

Graph	Algorithm	Number of Parts														
		2			4			8			16			32		
		AVE	**STD**	**BEST**	**AVE**	**STD**	**BEST**	**AVE**	**STD**	**BEST**	**AVE**	**STD**	**BEST**	**AVE**	**STD**	**BEST**
AS-22JULY06	FEHG	1.11	34	1.00	1.02	32	1.00	1.04	25	1.01	1.01	30	1.00	1.01	28	1.03
	PHG	2.90	86	2.46	1.77	92	1.56	1.64	78	1.36	1.43	87	1.34	1.37	90	1.32
	hMetis	1.34	0	1.95	1.19	7	1.30	1.16	12	1.18	1.04	23	1.06	1.09	27	1.04
	PaToH	1.00	4	1.43	1.00	16	1.03	1.00	20	1.00	1.00	37	1.00	1.00	43	1.00
	Best Value	136	–	93	355	–	319	629	–	599	1051	–	995	1591	–	1529
CELEGANSNEURAL	FEHG	1.00	2	1.00	1.09	9	1.00	1.10	15	1.06	1.11	16	1.08	1.07	17	1.03
	PHG	1.07	6	1.00	1.04	8	1.03	1.02	9	1.00	1.06	12	1.00	1.00	18	1.00
	hMetis	1.17	0	1.21	1.00	5	1.05	1.00	0	1.04	1.00	2	1.02	1.00	6	1.00
	PaToH	1.01	0	1.04	1.00	0	1.06	1.03	0	1.07	1.03	0	1.06	1.05	0	1.05
	Best Value	79	–	77	195	–	184	354	–	342	548	–	536	773	–	769
CNR–2000	FEHG	1.37	63	1.00	1.71	131	1.07	1.59	226	1.41	1.53	218	1.45	1.63	217	1.51
	PHG	35.88	552	45.62	12.48	760	9.17	5.73	569	4.84	3.54	477	2.98	2.42	530	2.02
	hMetis	12.19	74	18.82	8.24	163	8.43	5.08	240	4.71	3.46	238	3.29	2.66	231	2.50
	PaToH	1.00	3	1.71	1.00	37	1.00	1.00	48	1.00	1.00	62	1.00	1.00	85	1.00
	Best Value	81	–	45	244	–	202	569	–	509	1014	–	911	1927	–	1830
COND–MAT–2005	FEHG	1.00	28	1.00	1.00	58	1.00	1.00	87	1.00	1.01	88	1.02	1.01	82	1.00
	PHG	1.17	37	1.17	1.11	84	1.10	1.05	94	1.05	1.03	112	1.03	1.02	105	1.01
	hMetis	1.05	14	1.07	1.11	75	1.12	1.11	81	1.12	1.11	129	1.10	1.01	122	1.01
	PaToH	1.02	39	1.02	1.03	193	1.03	1.00	98	1.00	1.00	153	1.10	1.00	178	1.00
	Best Value	2134	–	2087	5057	–	4951	8609	–	8485	12370	–	12150	16270	–	16150
NETSCIENCE*	FEHG	0.0	0	0.0	0.0	0	0.0	2.00	1	1.50	1.50	2	1.00	2.08	2	1.81
	PHG	0.0	0	0.0	0.0	0	0.0	1.50	1	1.00	1.40	2	1.00	1.87	2	1.5
	hMetis	2.0	0	2.0	5.0	0	5.0	4.22	1	3.50	1.75	0	1.75	1.99	2	1.87
	PaToH	0.0	0	0.0	0.0	0	0.0	1.00	0	1.00	1.00	0	1.00	1.00	0	1.00
	Best Value	0	–	0	0	–	0	2	–	2	8	–	8	16	–	16
PGPGIANTCOMPO	FEHG	2.12	8	1.27	1.00	23	1.00	1.04	18	1.00	1.00	16	1.08	1.00	18	1.00
	PHG	13.23	48	1.83	1.44	65	1.04	1.25	45	1.04	1.02	53	1.00	1.08	46	1.00
	hMetis	9.7	3	9.61	1.46	11	1.71	1.04	13	1.40	1.31	24	1.40	1.26	25	1.27
	PaToH	1.00	0	1.00	1.04	0	1.27	1.00	7	1.04	1.02	2	1.15	1.08	5	1.06
	Best Value	18	–	18	242	–	200	419	–	400	695	–	617	956	–	930
GUPTA1	FEHG	1.00	60	1.00	1.00	55	1.00	1.00	80	1.00	1.00	115	1.00	1.00	15	1.00
	PHG	1.58	67	1.45	1.31	146	1.24	1.15	204	1.04	1.07	253	1.04	1.09	58	1.05
	hMetis	1.73	2	1.82	1.61	10	1.69	1.58	58	1.64	1.60	137	1.57	1.51	643	1.48
	PaToH	1.22	32	1.17	1.08	43	1.09	1.04	84	1.05	1.05	95	1.07	1.08	120	1.09
	Best Value	486	–	462	1466	–	1384	3077	–	2893	5342	–	5134	8965	–	8519
MARK3JAC120	FEHG	1.01	6	1.01	1.02	18	1.01	1.01	23	1.00	1.00	83	1.00	1.06	132	1.07
	PHG	1.00	4	1.01	1.02	15	1.02	1.02	27	1.00	1.00	53	1.00	1.72	106	1.78
	hMetis	1.00	13	1.00	1.00	15	1.02	1.00	29	1.00	1.30	217	1.00	4.20	214	1.78
	PaToH	1.00	0	1.02	1.00	11	1.00	1.00	17	1.00	1.26	248	1.20	1.00	267	1.00
	Best Value	408	–	400	1229	–	1202	2856	–	2835	6317	–	6245	3142	–	2944
NOTREDAME*	FEHG	0	0	0	1.00	9	1.00	1.12	40	1.12	1.09	116	1.03	1.06	119	1.07
	PHG	4326	0	4326	158.56	124	288.69	13.82	67	16.78	2.09	75	3.06	1.72	78	1.78
	hMetis	880	84	707	67.92	65	129.92	10.98	108	12.65	3.36	143	3.37	2.23	129	2.30
	Patoh	24	1	22	1.90	8	3.31	1.00	27	1.00	1.00	52	1.00	1.00	62	1.00
	Best Value	0	–	0	27	–	13	316	–	259	1577	–	1484	3142	–	2944
PATENTS–MAIN	FEHG	1.20	180	1.00	1.03	275	1.01	1.05	270	1.03	1.00	327	1.00	1.00	342	1.00
	PHG	12.49	1286	13.19	2.52	1736	2.30	1.79	1749	1.65	1.42	1575	1.38	1.23	1602	1.18
	hMetis	2.38	36	2.77	1.16	70	1.24	1.26	115	1.43	1.26	161	1.31	1.21	231	1.22
	PaToH	1.00	70	1.02	1.00	145	1.00	1.00	217	1.00	1.00	220	1.00	1.01	306	1.00
	Best Value	643	–	528	3490	–	3198	6451	–	6096	11322	–	10640	16927	–	16460
STD1–JAC3	FEHG	1.01	260	1.00	1.00	246	1.03	1.00	424	1.00	1.00	549	1.00	1.00	557	1.00
	PHG	1.15	227	1.08	1.16	377	1.10	1.18	748	1.13	1.28	768	1.35	1.33	801	1.29
	hMetis	1.05	105	1.00	1.52	1649	1.03	1.54	2057	1.23	1.70	2330	1.53	1.71	2995	1.51
	Patoh	1.00	125	1.00	1.08	506	1.00	1.16	700	1.14	1.00	827	1.26	1.30	945	1.29
	Best Value	1490	–	1371	3735	–	3333	7616	–	6167	13254	–	11710	22242	–	21200

* When the minimum cut for the average or best cases are zero, the values shown are actual cut values rather than normalised values.

for CNR-2000 is reported in Fig. 3. According to the results, the first scenario causes high fluctuations of vertex degree standard deviations while the second scenario produces a smooth change. We achieve average cuts of 490 and 110 for CNR–2000 for the first and second scenarios, respectively.

The agglomerative clustering of *Zoltan* and *hMetis* give 25.54 and 8.89 times worse quality. *PaToH* also produces good average quality of 81 using absorption clustering using pins and hyperedge clustering. The variations in vertex degree or its standard deviation causes problems for clustering algorithms, making it hard to make good clustering decisions because of the increased conflicts between local and global decisions. Consequently, finding vertices with higher similarity for matching can not be relied on for every hypergraph and it does not always gives a better partitioning cut. In addition, gathering some global information before making clustering decisions can give a major quality improvement and decreases the unexpectedness of the partitioning cut as depicted in Table 2.

6 Conclusions and Future Work

We have proposed a multi–level hypergraph partitioning algorithm based on feature extraction and attribute reduction using rough set clustering techniques. The algorithm clusters hyperedges using different similarity metrics and a similarity threshold and tries to removes less important hyperedges. An automated calculation of this similarity threshold is proposed. The hypergraph is then transformed into a reduced information system. Employing the idea of Rough Set clustering, the algorithm calculates the partitioning of the objects in the reduced information system based on indispensability relations and core sets of vertices with globally high similarities. Then cores are searched locally for vertex matchings. Evaluating the algorithm in comparison to the state-of-the-art algorithms has shown improvements in quality of the partitioning for tested hypergraphs. Future work is to implement parallel versions of the algorithm. Using a special distribution of vertices and hyperedges among processors and the ideas of rough set theory, we are focusing on proposing a scalable partitioner.

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