I was asked by David Arrowsmith to give a presentation to the meeting of HoDoMS (Heads of Departments of Mathematical Sciences) in April with the daunting title "Mathematics in the UK: an overview". My hope was to bring a perspective both internal, having been Head of Mathematics at UEA for five years, and external, having been primarily located outside a Mathematics Department since 2008.

I want to touch on five themes. In my talk these were accompanied by a large number of images, but copyright problems frees the reader to find her own. Finding these images provided a simple example of a one-way function: using Google to find images from names is far easier than using visual memory to recall the names from the images (though automating that is getting easier – an interesting mathematical problem itself, with huge practical implications).

I'd like to start with a challenging anecdote. An old friend was for some years head of a large Mathematics Department in the USA. In this role he attended a national event for faculty deans and heads of science departments. At the end of one plenary session, a panel of deans of science from about ten large state universities were asked which department caused the most headaches of an inter-personal sort. One after another they answered "Mathematics". I don't think that reflects something awful about mathematicians as people, but I do think it means something about how the pursuit of high individual standards can sometimes end up damaging us collectively.

History and successes

There is no need to tell this audience that the UK has a long history of contributing to mathematical knowledge with greater power than most nations of this size. The international reviews of 2004 and 2010 highlighted strategic issues to address, but did so within a context of success on the international stage. Since 2007 the number of A-level Mathematics entries has risen by 42.7%, and in Further Mathematics the increase is even greater at 68%. The Deloitte study commissioned by EPSRC and the CMS of the economic benefits of mathematical sciences research came up with figures like 16% of UK gross value added, and most strikingly 6% of the world's articles, 11% of citations, and 14% of highly cited articles – all done by 4% of the world's researchers.

It is important to understand that this strength in mathematics extends in many different dimensions. It reaches far back into our history - to Isaac Newton (1642-1727) of course, but arguably to the Venerable Bede (673-735) and beyond. It extends across most of the UK, with research and mathematical education of high quality in many different institutions. It spans many age groups, with much excellent work being done in schools at all stages. It reaches into the highest levels of internationally significant research, as evidenced by our Fields medallists. I want to highlight this because it is easy to let the many problems we face crowd out any celebration of the many positives.

Looking deeper into the citation metrics shows that not only are UK research outputs highly cited, but they have a relatively low rate of self-citation. The evidence suggests a community of highly active researchers performing well, who are perhaps not very eager to blow their own trumpets.

A habit of modesty?

Is there indeed an ingrained culture of modesty? How many mathematicians would have the audacity – or the dazzling feel for phrases that capture the imagination – to come up with "The God particle"? It's a phrase that notoriously irks physicists now, but it has done its job in raising the profile of one little piece of particle physics. Might this modesty feed into, for example, the relatively slight historic representation by mathematicians on bodies like Research Council boards? I sometimes feel that mathematicians are so used to looking with deference or even awe at exceptional talents, or being daunted by the immense difficulty of hard mathematical problems, that we can slip into a collective self-effacement.

We have in this generation some brilliant communicators of mathematics – in subjects close to my interests I would single out Ian Stewart and Marcus du Sautoy, but there are many others. For all the recent improvement in the visibility of mathematics, it can feel like uphill work – more BBC3 than BBC2. We don't seem to find it easy to create a Brian Cox effect. Mathematics broadly interpreted – a theme I will come back to – has its share of interesting figures who could combine star appeal with intellectual contribution. How did we make so little of Hedy Lamarr's insights into frequency hopping? Why is Ada King, arguably the creator of the concept of the algorithm, not a household name?

There is a clichéd view that says we simply can't communicate well, and there is no denying the fact that mathematical research can be arcane and difficult to communicate. I would suggest that some of this frustrating lack of public engagement comes from the best of motives – a determination to avoid inaccuracy, and to speak honestly and precisely. Mathematics is little without its rigour and precision, but surely there is room for a bit more boldness in claiming the public space? Mathematicians are used to being scrupulous, and hesitate to lay claim to the moon landing, the large hadron collider, the relentless improvement in car fuel efficiency, the way we have mapped the astonishing landscape of infinite cardinals, the smartphone, the classification of finite simple groups, as being triumphs of mathematics – but they all are.

Intellectual inertia

We have all seen the cartoons – "you must not be a very good mathematician because you seem rather normal to me", the Venn diagram locating mathematician in the intersection of the intelligent, the socially inept, and the obsessive (or, even less courteously, in the intersection of nerd, dweeb, and geek). I see them on doors of mathematicians and of our PhD students. Some are clever, many are funny – but I don't see them very often anywhere else. They don't reflect any reality I recognise. The mathematicians I know are diverse, and the words that spring to mind as I think through them include literate, fun, cultured, fond of wine, political, musical, athletic, pleasant, gossipy – in short, normal. Do we sometimes take the self-deprecation too far and actually internalise prejudice? As a community we are sometimes competing for the public space – or the political ear, or the funding stream – with other disciplines that seem better able to educate their young practitioners into confidence not diffidence.

Most striking of all are the instances of what I would call deliberate internal destructiveness. There is an image doing the rounds on the internet showing a slide from a large lecture with the following text:

> *Which of the following is the most different from the others?*
> *(A) A PhD in Mathematical Biology*
> *(B) A PhD in Theoretical Mathematics*
> *(C) A PhD in Statistics*
> *(D) A large pepperoni pizza*
> *Answer: (B). The other three can all feed a family of four.*

This irritated me in about a dozen ways: The implicit suggestion is not true: The distinctions being made – apart from that between (D) and the other three - mean little outside a small world: What on earth is "Theoretical" mathematics anyway? I guess this phrase is meant as a contrast between the concrete physicality of, say, the Navier-Stokes equation for an incompressible fluid in a cylinder being studied numerically on the one hand, and the ethereal unreality of, err, the relationship between growth conditions and the existence and smoothness of solutions to the Navier-Stokes equation on the other.

We may never have a deeper answer beyond "it just works" to the question of why mathematics seems to describe with such extraordinary power the workings of the physical universe – or perhaps captures the limits of what we can ever hope to understand about the physical universe, but "theoretical mathematics"? Large cardinals, set theory, numerical analysis, models of cell division, variational calculus, ocean modelling – if it is more complicated than counting, it is all in part "theoretical" mathematics.

This sort of shallow pursuit of a narrowly defined "impact" risks undermining deep knowledge creation. We certainly do have an obligation to keep explaining why mathematics of all sorts matters so much, but it pains me to see any mathematician grabbing onto the short-term and narrow presumption that the only questions that matter are those than originate in an application. What good does it do to the discipline in all its wonderful diversity to undermine those parts of mathematics with a different motivation or toolbox to mine?

Looking beyond mathematics, it sometimes feels that the UK still labours under the shadow of C. P. Snow's famous lecture of 1959, and the ensuing dispute with F. R. Leavis. This was perhaps best précised as the "two cultures" debate, about which Roger Kimball said "The phrase has lived on as a vague popular shorthand for the rift – a matter of incomprehension tinged with hostility – that has grown up between scientists and literary intellectuals in the modern world." The problems we face, arguably now more than ever, are of such complexity and scale that they require the active engagement of all disciplines. Simple examples might include the hazards of multiply drug-resistant infectious diseases, the challenge of internet security and the danger of cyber-warfare, developing economic models that might work in a post-growth world, and responding to anthropogenic climate change. Who would confidently write down a prescriptive list of the disciplines needed in order to help society respond to these problems? "Applied" mathematical techniques certainly put

in an appearance – but who would preclude any of the natural sciences, economics, politics, the humanities, "theoretical" mathematics? If we knew what disciplines were really needed we would be half way to a solution, and the one certainty is we are far from solutions. We need to find productive ways to contribute to a modern version of the "Age of Wonder" – it may not be feasible to generate polymaths in the eighteenth Century sense, but we all have a part to play in poly-mathematical collaborations.

Disconnection from reality: Bertolt Brecht

I have no wish to make light of the uprising in East Berlin in 1953, which led to the death of many hundreds of people, but Bertolt Brecht's satiric poem "The solution" has something to say about mathematics.

> *The Secretary of the Authors' Union*
> *Had leaflets distributed in the Stalin Alley*
> *Which said that the people*
> *Had forfeited the government's confidence*
> *And could only win it back*
> *By redoubled labour. Wouldn't it*
> *Be simpler in that case if the government*
> *Dissolved the people and*
> *Elected another?*

Nor do I wish to make light of the many complex challenges in mathematics education at school level – but I do want to share how a clichéd conversation some of us can fall into looks from outside.

We (largely) train the teachers of high school mathematics in our departments. We set the entry criteria for mathematics degree programmes. We define the hoops through which our students must jump. We often influence the school curriculum. Then we complain about their skill set on arrival. Worse, we sometimes complain about their skill set after we have been teaching them for a year or two. Is there a time to reverse the argument and start from a willingness to meet the students that we select, according to criteria we define, where they are – not where we wish they were? And not where we fantasize they once were? The students choose to study mathematics or choose not to. How can it make sense to ever wish they were someone other than who they are?

There are of course real problems in school level mathematics (and all disciplines would say the same) – but some of them grow from decades in which schools, teachers, and the curriculum have been political footballs. While I can see much good work being done between university mathematics departments, schools, and teachers, and the various exam bodies, I think we should be very cautious about how we join in this year's round of political kicking of schools and teachers. We always have a wish list for incoming students – mine might include A-level Latin, so that they are used to thinking of the structure of a sentence – but we need to be very careful to not be enjoined by political forces with their own agenda driven by this session's politics. We had to engage with the agenda laid out by Michael Gove's letter to OFQUAL on March 30th 2012, but we need to keep pointing out that some of the problems we are

grappling with come from league table pressures, endless changes, and political interference – none of which is being addressed by the current thinking.

Politics or the lack thereof: Elsevier, Gold, and Machiavelli.

Mathematical publishing is more or less invisible in the world of commercial academic publishing. We – and, for example, the London Mathematical Society or IMA publishing arms – are a relative detail. The Elsevier back catalogue, for example, has approximately 85000 mathematics articles, 400000 articles in *The Lancet* alone, 850000 in other Medicine and Dentistry, and so on. We may have been a voice in the debate about excessive library charges for journals, but I can't help feeling we were not very astute in that debate. The medium-term outcomes have included: approximately £35 million diverted from national research spend – largely going to the publishers – with unknown amounts in the future, which will come out of already inadequate research council budgets; a strong political preference for the gold open access route with potential double financial hits on universities; confusion all round. It would have been both useful and possible for mathematics in the UK to lead the pack in making green open access work. If we had been able to say, hand on heart, that all mathematical research outputs in the UK could be easily found on the arXiv (for example), we would have all been in a better position politically to avoid the current mess.

More generally, how does "mathematics in the UK" look as a body? The American Mathematical Society has over 30000 members, SIAM over 10000; the LMS has about 2000, and the IMA about 4300 paying members. At the HODOMS meeting we were joined by a member of parliament with a long involvement in education, a specific interest in mathematics education, and a history of being supportive of mathematics in the UK. She did not know about the CMS. I'm not blaming anyone – gaining visibility is hard, long, expensive, work. I also don't want to reopen some of the painful discussions around the proposed merger between the LMS and the IMA. I just want to remind us all that each of the three learned societies (IMA, LMS, RSS) does great work but is undeniably small, and mathematics as a discipline has less visibility and lobbying clout than the other sciences. It certainly has less visible clout than it merits both economically and intellectually.

We need to punch our weight – mathematics in the UK is a shining jewel. We are demonstrably delivering international research excellence. My suggestion is that we are at times losing out by being self-deprecating and over-scrupulous. We owe it to the UK to express the power and diversity of our subject more fluently and in a more united way. We could also do with being at times a little more Machiavellian!