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Tests for Symmetric and Asymmetric Nonlinear Mean Reversion in Real Exchange Rates

New tests, based on smooth transition autoregressive models, for mean reversion in time series of real exchange rates are proposed. One test forces mean reversion to be symmetric about the integrated process central case, while the other permits asymmetry. The tests are applied to monthly series of seventeen real exchange rates against the U.S. dollar and fourteen against the deutsche mark. They reveal stronger evidence against the unit root null hypothesis than does the usual Dickey-Fuller test.

THE THEORETICAL IDEA of purchasing power parity, grounded in the “law of one price,” is discussed by Rogoff (1996). This law states that, for any good i , $P_i = EP_i^*$, where P_i is the domestic currency price of the good, P_i^* is the foreign currency price, and E is the nominal exchange rate as the home currency price of foreign currency. Tariffs, nontariff barriers, and transportation costs can all mitigate against the law of one price, preventing purchasing power parity from holding. However, Rogoff (1996) notes that most economists have an “instinctive belief” in the purchasing power parity hypothesis *in the long run*, implying mean reversion in time series of real exchange rates.

Early analyses of this mean reversion hypothesis, including Meese and Rogoff (1988), Enders (1988), Taylor (1988), and Mark (1990), were based on augmented Dickey-Fuller tests, and generally failed to find strong evidence against the unit root null hypothesis—that is, of mean reversion. Similarly, Edison, Gagnon, and Melick (1994) found only weak evidence of cointegration between nominal exchange rates and relative prices. Frankel (1986) and Lothian and Taylor (1996), arguing that failure to reject the unit root hypothesis could reflect the low power of the test, extended the data sets to cover periods of over one hundred years, then finding apparently

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strong evidence of mean reversion. However, Engel (1996) and Papell (1997) have questioned the validity of employing pre-1973 data, thereby mixing fixed and floating exchange rate periods. The former author demonstrates how such an approach can generate spurious rejections of the unit root null hypothesis. An alternative approach that yields more powerful tests, applied, for example, by Abuaf and Jorion (1990), Levin and Lin (1992), Lothian (1994), Frankel and Rose (1996), Jorion and Sweeney (1996), Oh (1996), Wu (1996), Papell (1997), and Parsley and Popper (2001), is through the use of panel data, the last of these papers also allowing non-linearity in the adjustment mechanism. In many of these studies the unit root null hypothesis was rejected at conventional significance levels. However, O'Connell (1998) has cautioned about the possible size distortions in these tests if cross-sectional correlation is ignored, noting that proper allowance for this factor can lead to tests of reduced power.

In this research we analyze the logarithms of 296 monthly observations, from April 1973 to November 1997, on seventeen real exchange rates of industrialized countries against the U.S. dollar. These time series, constructed from consumer price indices and exchange rates recorded in the *International Financial Statistics* database of the International Monetary Fund, were previously analyzed by Papell (1997) and Bleaney and Leybourne (1998). We seek through new tests evidence of mean reversion *from the analysis of individual series*. Theoretical models of Sercu, Uppal, and Van Hulle (1995), Coleman (1995), and Ohanian and Stockman (1997) suggest that, taking into account the effects of transactions costs on models of goods arbitrage, deviations from the law of one price that are nonlinear in nature may arise. Both Michael, Nobay, and Peel (1997) and Taylor and Sarno (1998) have argued that, in the presence of transactions costs, the possibility of reversion toward purchasing power parity might be assessed through nonlinear time series models, such as the threshold autoregressive models (TAR) by Tong and Lim (1980) and Tong (1983). They note, however, that because exchange rate series are highly aggregated, one is more likely to observe smooth rather than abrupt transitions between regimes, suggesting the appropriateness of the smooth transition autoregressive (STAR) models discussed by Granger and Terasvirta (1993) and Terasvirta (1994).

Our own analysis is also based on adaptations of the STAR model. As an alternative to the integrated generating process, we consider a model in which the extent of any mean reversion is an increasing function of recent squared deviations from the mean. In the central case, where the system is at the mean, it will behave as an integrated process, and will be virtually indistinguishable from such a process when the real exchange rate is close to the mean. However, stronger mean reversion may occur with increasing discrepancies from the mean. The approach followed here differs from previous analyses in two ways. First, we develop tests of mean reversion that are natural extensions to our model of the standard Dickey-Fuller test. Second, there seems no strong a priori reason to believe that transactions costs will necessarily generate symmetric deviations from purchasing power parity, so that we allow for *asymmetric effects* in our STAR models. To do so, we find it more convenient to adapt the logistic transition function rather than the exponential function used by previous

authors. Although considerations of possible asymmetry have arisen in other applied econometric work, as, for example, in the analysis of Neftci (1984) of U.S. unemployment, there are relatively few applications to financial time series. An exception is Enders and Granger (1998), who develop tests of unit roots against asymmetric TAR alternatives, with an application to the short- and long-term interest rate differential. Our tests can be viewed as extensions of theirs to STAR-type models.

In section 1 of the paper we set out the adapted STAR models of mean reversion, against which the unit root null hypothesis is tested, and develop the associated test statistics. Symmetric and asymmetric models are considered separately. Although applied specifically here to the analysis of real exchange rates, our tests are more generally applicable, and we report critical values, obtained through simulation, for different sample sizes. The tests are applied to real exchange rate data in section 2. The main application, to exchange rates against the U.S. dollar, demonstrates the value of allowing for asymmetry. In several cases, when this was done, rejections of the unit root null hypothesis were found when such rejections at conventional significance levels were not obtained either through the standard Dickey-Fuller test, or a test based on a symmetric STAR alternative.

1. POSSIBLE STAR MODELS OF NONLINEAR MEAN REVERSION AND ASSOCIATED TESTS

Let y_t be a given time series of T observations, taken in the next section to be the logarithm of the real exchange rate, possibly reverting to mean μ , estimated in part of our empirical work to be the sample mean. Then, if $z_t = y_t - \mu$ is the series of deviations from the mean, consider the specification

$$\Delta z_t = \alpha S_t(\gamma, z_{t-1}) z_{t-1} + \sum_{i=1}^k \beta_i \Delta z_{t-i} + \varepsilon_t \quad (1)$$

where k is chosen sufficiently large that the error term ε_t is white noise. For $\alpha = 0$, model (1) represents an integrated process, while $S_t(\gamma, z_{t-1}) = 1$ for all t corresponds to the usual Dickey-Fuller regression. More generally, $S_t(\gamma, z_{t-1})$ can be defined to allow increasing degrees of mean reversion the further is z_{t-1} from 0; that is, the further is y_{t-1} from μ . In the symmetric case, we use the modified logistic function

$$S_t(\gamma, z_{t-1}) = \{1 + \exp(-\gamma^2 z_{t-1}^2)\}^{-1} - 0.5 \quad (2)$$

with $\gamma^2 > 0$. Here $S_t(\gamma, 0) = 0$, implying integrated behavior at $y_{t-1} = \mu$, and symmetry results through $S_t(\gamma, -z_{t-1}) = S_t(\gamma, z_{t-1})$. The parameter γ^2 in (2) governs the speed at which the function moves from 0 to 0.5 as z_{t-1}^2 increases: all else equal, the larger is γ^2 , the more rapid the transition. Finally, note that the function (2) can be elaborated to $S_t(\gamma, z_{t-d})$, where the integer d is a delay parameter. In our empirical work, we

tried $d = 1, 2, 3$, finding for every time series the closest fit for $d = 1$, which for convenience we shall retain in the subsequent exposition.

The model (1), (2) can be expanded to allow for asymmetry in the mean reversion process by adding one further parameter. In place of (1), write

$$\Delta z_t = \alpha S_t(\gamma_1, \gamma_2, z_{t-1}) z_{t-1} + \sum_{i=1}^k \beta_i \Delta z_{t-i} + \varepsilon_t. \quad (3)$$

Then using the Heaviside indicator, I_t , defined as

$$I_t = \begin{cases} 1 & \text{if } z_{t-1} > 0 \\ 0 & \text{if } z_{t-1} \leq 0 \end{cases}$$

the function

$$S_t(\gamma_1, \gamma_2, z_{t-1}) = [1 + \exp\{-\gamma_1^2 z_{t-1}^2 I_t - \gamma_2^2 z_{t-1}^2 (1 - I_t)\}]^{-1} - 0.5 \quad (4)$$

with γ_1^2 and $\gamma_2^2 > 0$ provides a natural generalization of (2), but now permitting asymmetry. Of course, the symmetric model is nested in (3), (4) at $\gamma_1^2 = \gamma_2^2$. The advantage of the more general formulation is that it permits, for example, stronger mean reversion when the dollar exchange rate of a currency is above the mean than when it is below the mean by the same proportionate amount. Enders and Granger (1998) exploited the Heaviside indicator in a similar formulation when developing a Dickey-Fuller-type test based on TAR models.

Tests of the unit root null hypothesis $\alpha = 0$ against the symmetric and asymmetric mean reversion alternatives can be based respectively on nonlinear least squares estimation of (1) and (3). The numbers of lagged differences, k , to include in these regressions could be determined through general-to-specific testing. However, in the following section, we wanted to compare results for the two models, both with each other and with those from a standard Dickey-Fuller test, based on (1) but with $S_t(\gamma, z_{t-1}) = 1$ for all t . It seems most reasonable to do so by employing the same k in each of these three cases. This was chosen through general-to-specific testing at the 10 percent level in the Dickey-Fuller regressions. The models (1) and (3) could be estimated directly, and associated standard errors of estimators calculated, through nonlinear least squares algorithms. However, difficulties in achieving convergence in the iterative algorithms can result, particularly when the unit root null hypothesis is true (or even approximately true), since then the γ parameters are (virtually) undefined. Of course, this is precisely the issue faced when attempting to simulate critical values of the test statistics. In constructing the tests, we first replaced μ in $z_t = y_t - \mu$ by the sample mean $\bar{y}$. Then, to speed up computation of the nonlinear least squares estimators of the remaining parameters, we performed a grid search over the single parameter γ in the case of (1) and the two parameters (γ_1, γ_2) in the case of (3),

fitting the remainder of the coefficients in these regressions by ordinary least squares.¹ In that way, the full set of parameter values minimizing the sums of squared residuals can be determined.

Let $\hat{\gamma}$ or $(\hat{\gamma}_1, \hat{\gamma}_2)$ denote the nonlinear least squares estimators of the corresponding parameters γ and (γ_1, γ_2) . We replace these parameters by their nonlinear least squares estimates in (1) or (3) and then fit the equations using ordinary least squares (recalling that μ is replaced by $\bar{y}$), generating unit root test statistics for the null hypothesis $\alpha = 0$ as the ratio of $\hat{\alpha}$ to its standard error as estimated from this final ordinary least squares regression. We denote the unit root test statistic based on (1) by t_S and that based on (3) by t_A .

Critical values for the unit root test statistics t_S and t_A were obtained through simulation, for each of several sample sizes T . These, and subsequent simulations were based on 10,000 replications. In either case, the null generating model was a driftless random walk with standard normal white noise error terms and in estimating the regressions k was set to its correct value of 0.^{2,3} Table 1 gives empirically determined critical values for 10 percent-, 5 percent-, and 1 percent-level tests. Note that, as is common with Dickey-Fuller type tests, these critical values seem to converge quite rapidly as the sample size increases. The critical values for the t_S and t_A tests are seen to be considerably more negative than those for the corresponding standard Dickey-Fuller test, which we denote t_{ADF} , that is, the test based on subtracting the sample mean (or equivalently including a constant, but no trend, in the usual Dickey-Fuller

TABLE 1
CRITICAL VALUES FOR THE t_S AND t_A TEST STATISTICS

T	t_S			t_A		
	10 percent	5 percent	1 percent	10 percent	5 percent	1 percent
50	-2.90	-3.21	-3.87	-3.08	-3.38	-4.04
100	-2.89	-3.19	-3.73	-3.01	-3.32	-3.88
200	-2.86	-3.15	-3.71	-2.94	-3.26	-3.83
300	-2.86	-3.12	-3.70	-2.88	-3.19	-3.79
500	-2.83	-3.11	-3.66	-2.85	-3.15	-3.75

1. In the subsequent simulation and empirical sections of the paper, the grid search for the parameters γ and (γ_1, γ_2) was conducted over the interval 0.01 to 0.5 in steps of 0.01. Experiments using finer grid searches with the same number of increments made negligible difference to the null critical values. However, in some of our empirical analyses, where there was evidence of asymmetric mean reversion, we did find finer grid searches useful.

2. It should be noted that under the null hypothesis the parameter α of (1) or (3) is 0. Thus, neither model involves the transition function $S(\cdot)$. Hence, since critical values are determined from simulation of null generating models (but estimation of the full models), it is not necessary to specify "true" values of the parameters γ and (γ_1, γ_2) as they do not appear in those models.

3. Setting $\beta_i = 0$ in (1) or (3) in the generation of critical values is standard practice for Dickey-Fuller-type tests. Of course, in relatively small samples the critical values will depend to some extent on the parameters β_i . In unreported simulation experiments, we computed t_S and t_A for different values of k and the parameters β_i . As is commonly found for Dickey-Fuller-type tests [for example, one involving smooth transitions in linear trend discussed by Leybourne, Newbold, and Vougas (1998)], only very modest changes to the critical values resulted.

regression). For example, for $T = 300$, the critical value for a 5 percent-level test based on t_{ADF} is -2.88 , compared to -3.12 and -3.19 for t_S and t_A , respectively. We would therefore expect our tests to have less power than the standard Dickey-Fuller test when the true generating process is a stationary linear autoregression. We thus view the new tests as a complement to, not a substitute for, the Dickey-Fuller test. On the other hand, the critical values for the t_S and t_A tests are very close to each other, particularly for the larger sample sizes. This suggests that rather little in terms of power might be lost in allowing for the possibility of asymmetry when the true generating process is, in fact, one that exhibits symmetric STAR mean reversion.

Of course, one would expect that when the true data-generating process was one they were specifically designed to detect, the new tests would be more powerful than the standard Dickey-Fuller test. We illustrate this point through a small simulation study. In conformity with our interest in real exchange rate behavior, we generated data from the model that we found to fit the rate for the German mark against the U.S. dollar. Further details of that analysis are given in the following section, the fitted model being illustrated in Figure 1(a). Specifically, data were generated from the asymmetric STAR model

$$\Delta z_t = -0.43S_A(0.001, 0.031, z_{t-1})z_{t-1} + \varepsilon_t$$

with ε_t standard normal white noise and, as was found for the mark-dollar rate, $k = 0$ in (3). Our test statistics are invariant to μ , which was set to 0 in the data generation process, though this parameter was estimated in calculation of the test statistics using $\bar{y}$. The statistics t_A and t_{ADF} were computed, in each case with k set to its correct value of 0. The upper part of Table 2 shows the percentage of rejections of 5 percent-level and 10 percent-level tests, for samples of size $T = 200, 300$, and 400 . It is clear that the rejection rate is much higher for t_A than for t_{ADF} . This occurs because t_{ADF} is based on an under-specified model that permits no STAR behavior. To complete the picture, we also simulated series from the stationary first-order autoregression

$$z_t = 0.97z_{t-1} + \varepsilon_t$$

TABLE 2
EMPIRICAL POWER OF t_A AND t_{ADF} TEST STATISTICS

	$T = 200$		$T = 300$		$T = 400$	
	10 percent	5 percent	10 percent	5 percent	10 percent	5 percent
Asymmetric STAR DGP						
t_A	35.6	20.2	49.3	30.4	77.8	57.3
t_{ADF}	21.3	12.4	29.6	16.9	49.5	29.7
AR DGP						
t_A	26.8	14.7	41.4	25.3	73.6	53.2
t_{ADF}	27.7	16.6	44.0	26.7	78.8	57.5

and applied the same two tests. The results in the lower part of Table 2 show that t_{ADF} is more powerful than t_A . This occurs because the model (3), on which t_A is based, is overspecified. Notice, however, that the difference in power between the two test is now only minor; a consequence of the fact that overspecification is a much less serious problem than is underspecification.

2. ANALYSIS OF TIME SERIES OF REAL EXCHANGE RATES

The standard Dickey-Fuller test t_{ADF} and the two new tests t_S and t_A were applied to series of 296 monthly observations on the logarithms of real exchange rates against the U.S. dollar for seventeen industrialized countries. Table 3 shows the resulting test statistics. In common with previous authors, we found little evidence of mean reversion from the Dickey-Fuller test. Moreover, allowing for symmetric nonlinear autoregressive mean reversion through the statistic t_S also failed to generate rejections of the unit root null hypothesis at the 10 percent significance level. However, the statistic t_A , which allows for asymmetry, produced rejections at that level, or lower, for six of the seventeen countries. It therefore appears that allowance for asymmetry in the STAR formulation has allowed us to uncover some evidence of mean reversion.

The implications of our data analysis, and in particular the nature of the asymmetry uncovered, can best be illustrated graphically. Writing (3) as

$$z_t = \{1 + \alpha S_t(\gamma_1, \gamma_2, z_{t-1})\} z_{t-1} + \sum_{i=1}^k \beta_i \Delta z_{t-i} + \varepsilon_t \quad (5)$$

TABLE 3
TESTS FOR REAL EXCHANGE RATES AGAINST THE DOLLAR

	k	t_{ADF}	t_S	t_A
Austria	0	-1.80	-1.94	-2.41
Belgium	0	-1.57	-1.58	-2.25
Canada	0	-.67	-.90	-1.38
Denmark	2	-1.88	-1.89	-3.41**
Finland	5	-2.17	-2.51	-2.66
France	0	-1.81	-1.91	-2.92*
Greece	4	-1.47	-1.60	-1.89
Germany	0	-1.74	-1.80	-2.92*
Italy	0	-1.83	-2.07	-2.47
Japan	1	-1.64	-2.05	-2.14
Netherlands	0	-1.88	-1.89	-2.58
Norway	2	-2.33	-2.43	-4.07***
Portugal	0	-1.60	-1.60	-1.76
Spain	0	-1.50	-1.65	-1.96
Sweden	0	-1.66	-1.82	-3.01*
Switzerland	0	-2.11	-2.26	-2.69
U.K.	1	-2.26	-2.64	-2.88*

NOTES: * ** *** denote significance at the 10 percent, 5 percent and 1 percent significance levels respectively, and k is the order of the autoregressive terms included in the nonlinear regression.

where $z_t = y_t - \mu$, the term $\{1 + \alpha S_t(\gamma_1, \gamma_2, z_{t-1})\}$ that multiplies z_{t-1} in (5) provides a description of departures from an integrated generating model. For the six countries for which the statistic t_A showed rejection of the unit root null hypothesis at the 10 percent level or lower, using nonlinear least squares we estimated the model (5) treating the mean μ as an additional parameter to be estimated jointly with the other parameters through minimization of the sum of squared residuals, rather than simply replacing it with the sample mean $\bar{y}$. The extra computational burden this entails is justified as Tong (1983) has shown, in the context of TAR models, that the sample mean is an inconsistent estimator of μ when adjustment is asymmetric.⁴ However, Chan (1993) also proved that a consistent estimator of μ in (5) results from treating it as an additional parameter in the residual sum of squares function. Such a procedure was followed in estimating asymmetric TAR models by Enders and Granger (1998).

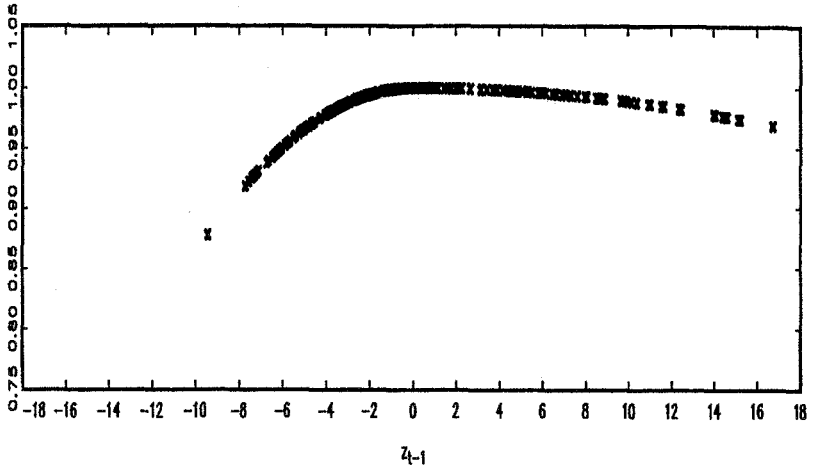
In Figure 1 the estimated term $\{1 + \hat{\alpha} S_t(\hat{\gamma}_1, \hat{\gamma}_2, z_{t-1})\}$, where $z_t = y_t - \hat{\mu}$, is graphed against z_{t-1} for Germany and Norway, the former of which generated only a mild rejection of the unit root null hypothesis, while for the latter the rejection was very strong. The crosses on the graphs correspond to those values of z_{t-1} that actually arise from the data. The extent of the estimated asymmetry in mean reversion is very pronounced indeed. Qualitatively similar results were found for three of the other four countries. The exception, where mean reversion seemed to be only slightly asymmetric, is for the United Kingdom. This might possibly be anticipated from Table 3, however, where we see that this is the most marginal of the six rejections of the null hypothesis and, moreover, that the values of the test statistics t_S and t_A do not differ dramatically in this particular case.

An interesting conclusion from our analysis is that the direction of the estimated asymmetry is the same for all six countries. The estimates are of stronger mean reversion when z_{t-1} is negative—that is, when the real exchange rate is below the mean—than when it is positive. Since in our calculation of the real exchange rates, the nominal exchange rates are denominated in units of foreign currency per U.S. dollar, the implication is that we estimate stronger mean reversion when the foreign currency is overvalued against the dollar relative to historical averages and accounting for inflation differentials than when it is undervalued by the same proportionate amount.

Due to the dramatic rise in U.S. interest rates between 1980 and 1985, this period was one in which, relative to long-term averages, the currencies of the six countries for which the statistic t_A led to rejection of the unit root null hypothesis were weakened against the dollar. In our notation, between 1980 and 1985, for these six countries $z_{t-1} > 0$. Our asymmetric models reveal that in the years immediately following 1985, mean reversion of the real exchange rates of these countries was relatively slower than mean reversion from below the mean throughout the entire sample period. This is understandable given the significant but gradual reduction in U.S. interest rates from 1985. Note that while nonlinearity in the autoregressive representation of real exchange rates may indeed be a consequence of the effects of transactions

4. It should be noted that the tests of the previous section, where $\bar{y}$ is substituted for μ , are valid as there is no asymmetry under the null hypothesis.

(a) Germany



(b) Norway

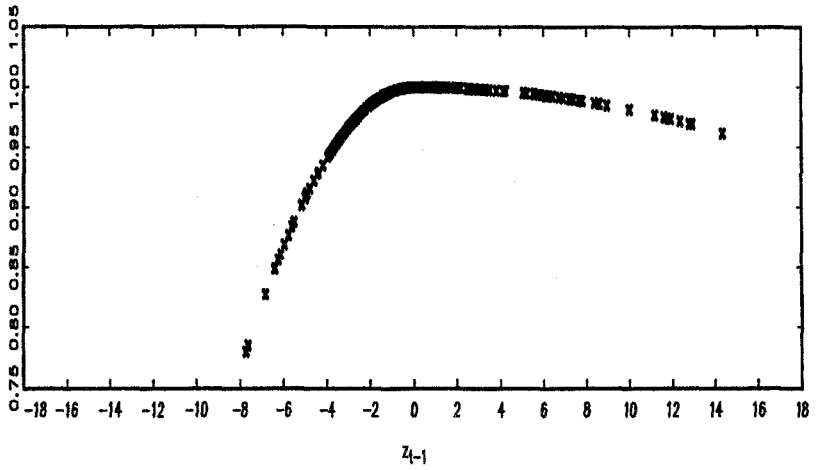


FIG. 1. Estimates of $(1 + \alpha S_A(\gamma_1, \gamma_2, z_{t-1}))$ in asymmetric STAR Models of Mean Reversion of Log of Real Exchange Rates against U.S. \$. Top panel: Germany; bottom: Norway

costs on arbitrage, the nonlinear autoregressive models proposed by the authors mentioned in the introduction cannot then capture the full impact that economic policy may have on the autoregressive representation of these series, such as the asymmetry described above.

Prompted by the insightful comment of a referee, we attempted, following Granger and Terasvirta (1993) and Terasvirta (1998), to estimate a nonlinear error correction model for the German exchange rate against the U.S. dollar to better understand the source of asymmetric nonlinearity revealed in Figure 1(a). Let x_{1t} denote the logarithm of the nominal exchange rate and x_{2t} the logarithm of the ratio of U.S. prices to German prices, so that, as in our previous notation, $(x_{1t} - x_{2t}) = y_t$, the logarithm of the real exchange rate. We tried nonlinear vector error-correction models, selecting the number of lagged changes by general-to-specific testing, which yielded just one lag, so that the fitted model was

$$\begin{aligned} \Delta x_{it} = & c_i + \rho_i y_{t-1} + \alpha_{i1} \Delta x_{1,t-1} + \alpha_{i2} \Delta x_{2,t-1} \\ & + (c_i^* + \rho_i^* y_{t-1} + \alpha_{i1}^* \Delta x_{1,t-1} + \alpha_{i2}^* \Delta x_{2,t-1}) S_i(\gamma_{i1}, \gamma_{i2}, \tilde{y}_{t-1}) + \varepsilon_{it} \end{aligned} \quad (6)$$

for $i = 1, 2$. Here $\tilde{y}_t = y_t - \hat{\mu}$, with $\hat{\mu}$ the estimate of μ from (5), $S_i(\cdot)$ is the asymmetric transition function (4), and the ε_{it} are white noise errors. The two equations of (6) were estimated, one at a time, by nonlinear least squares. Insignificant coefficients were dropped, yielding the results shown in Table 4.

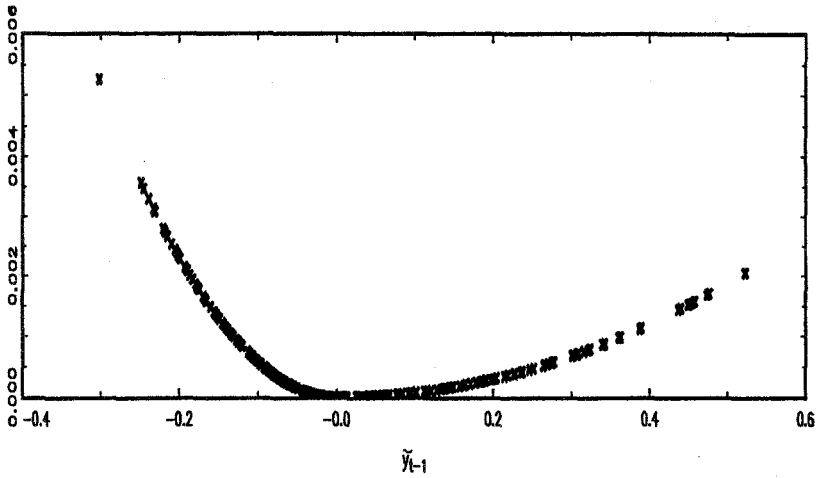
The F -ratios, appropriate for testing the null hypothesis that the coefficients associated with the nonlinear component of the model are zero indicate that the strongest nonlinearity is in the second equation—that is, that equation that determines changes in the logarithms of relative prices. To explore further, we have plotted the transition functions that were estimated for the two equations in Figure 2. Specifically,

TABLE 4
ESTIMATION RESULTS FOR THE NONLINEAR VECTOR ECM

	$i = 1$	$i = 2$
c_i	-.004(-1.73)	
ρ_i		.003(2.66)
α_{i1}	.207(2.94)	
α_{i2}		.429(5.20)
c_i^*	6.60(2.09)	.006(3.97)
ρ_i^*		-.001(-3.16)
α_{i1}^*	-1.89(-3.49)	
α_{i2}^*	-4.56(-2.26)	-.549(-2.10)
γ_{i1}	.03	.62
γ_{i2}	.23	213.10
F -ratio	6.59	11.39

NOTES: Numbers in brackets are t -ratios. " F -ratio" is the F -statistic for testing the null hypothesis that the coefficients associated with the nonlinear terms are all zero.

(a) Function plot from nonlinear ECM for log of nominal exchange rate: DM price of \$



(b) Function plot from nonlinear ECM for log of \$/DM price ratio

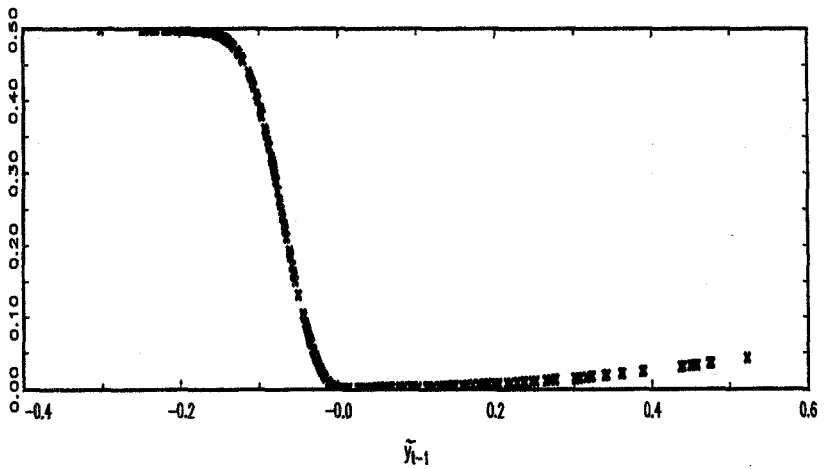


FIG. 2. Top panel: Function Plot from Nonlinear ECM for Log of Nominal Exchange Rate: DM Price of \$. Bottom panel: Function Plot from Nonlinear ECM for Log of \$/DM Price Ratio

$S_1(\hat{y}_{i1}, \hat{y}_{i2}, \tilde{y}_{i-1})$ is plotted against $\tilde{y}_{i-1}$. The graphs reveal very clearly that by far the strongest source of asymmetry arises from the determination of relative prices. Qualitatively similar results were found from the analysis of other series for which our tests indicated asymmetric mean reversion.

The results of Table 3 show no rejections at the 10 percent level for the t_S test. However, it certainly can occur that significant evidence of symmetric STAR-type mean reversion is found in cases where the standard Dickey-Fuller test reveals relatively scant evidence against the unit root null hypothesis. Such cases arose in our analysis of fourteen series of logarithms of real exchange rates for European countries against the deutsche mark, covering the same time period as the U.S. dollar exchange rates. These series too have been previously analysed by Papell (1997) and Bleaney and Leybourne (1998). Table 5 shows results for the three tests t_{ADF} , t_S and t_A . The null hypothesis is rejected by the Dickey-Fuller test for five countries at the 10 percent level or lower. It is similarly rejected for these five plus two other countries by the t_S test, and for those seven plus one other country by the t_A test. It is also apparent that for several series stronger rejections are obtained from the t_S and t_A tests than from the Dickey-Fuller test. Taken together, our analyses of the two sets of exchange rates strongly suggest the value of the new tests in uncovering evidence of mean reversion, and that evidence of departures from the unit root-generating model based on post-1972 data on single series of real exchange rates is more readily available than suggested by the results of many previous studies.

We have shown that fairly straightforward adaptations of Dickey-Fuller tests can uncover such evidence from data of this period. Those adaptations exploit nonlinearity in the adjustment process, as suggested by the results of Michael, Nobay, and Peel (1997) and more recently Parsley and Popper (2001). In particular, in common with these authors, we contemplate processes in which adjustment to equilibrium is fastest when the real exchange rate differs most from that equilibrium. However, by contrast

TABLE 5
TESTS FOR REAL EXCHANGE RATES AGAINST THE DEUTSCHEMARK

	k	t_{ADF}	t_S	t_A
Austria	0	-2.83*	-3.48**	-3.58**
Belgium	4	-2.64*	-3.16**	-3.44**
Denmark	3	-3.15**	-3.19**	-3.96**
Finland	1	-1.72	-2.24	-2.41
France	4	-2.86*	-4.33***	-4.35***
Greece	0	-1.68	-1.76	-2.45
Italy	4	-1.73	-2.87*	-3.34**
Netherlands	3	-2.69*	-3.64**	-3.65**
Norway	2	-2.12	-2.76	-2.77
Portugal	0	-2.07	-2.18	-2.33
Spain	0	-2.28	-2.56	-2.95*
Sweden	1	-2.22	-2.22	-2.72
Switzerland	1	-2.50	-2.70	-2.70
U.K.	1	-1.96	-3.17*	-3.17*

NOTES: *, **, *** denote significance at the 10 percent, 5 percent and 1 percent significance levels respectively, and k is the order of the autoregressive terms included in the nonlinear regression.

with their work, we have found it particularly useful to permit the adjustment process to be asymmetric. This possibility has also been recently considered by Enders and Dibooglu (2001), though these authors work with TAR models, rather than what we consider to be the more intuitively appealing STAR specifications. Although the results of Parsley and Popper (2001) are based on panel data analyses, we regard our own research as adding to the growing body of literature which suggests that evidence of mean reversion in floating exchange rates can be found through the analysis of individual series if nonlinear models are considered. We believe that it can be particularly valuable to permit such models to exhibit asymmetry in the adjustment mechanism. Such a specification does indeed find evidence of mean reversion in a much more recent period than that analyzed by Michael, Nobay, and Peel (1997).

3. SUMMARY

In this paper we have been able, through analysis of individual series of post-1972 data, to uncover quite strong evidence of a type of mean reversion in real exchange rates. This has been possible through two extensions of the standard Dickey-Fuller test to allow for a form of nonlinearity under the alternative hypothesis. In that structure, while the system behaves as an integrated process in equilibrium, the further it moves from equilibrium, the more strongly it is pulled toward mean reversion. One of our tests forces symmetry on this process, so that the impacts of positive and negative discrepancies of the same amount from the mean are identical. However, we have also found it useful to nest this model within one that permits asymmetric effects.

Seventeen series of real exchange rates against the U.S. dollar and fourteen series of real exchange rates against the deutsche mark were analyzed. In the former case the unit root null hypothesis was never rejected at the 10 percent level by the Dickey-Fuller test, though there were five such rejections in the latter case, presumably reflecting European Union agreements on monetary integration. However, further rejections were obtained through our new tests, based on smooth transition autoregressive models. For the U.S. dollar rates, these rejections occurred only when asymmetry in the mean reversion process was allowed, while both the symmetric and asymmetric tests generated further, and stronger, rejections for the deutsche mark series. In no case where there was a Dickey-Fuller test rejection was a rejection at least as strong not found by our new tests.

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