# Probing the structure of $f_{0}(980)$ through radiative $\phi$ decays 

F. De Fazio ${ }^{a}$ and M.R. Pennington ${ }^{b}$<br>${ }^{a}$ Istituto Nazionale di Fisica Nucleare, Sezione di Bari, Italy<br>${ }^{b}$ Institute for Particle Physics Phenomenology, University of Durham, DH1 3LE, Durham, UK


#### Abstract

We consider the radiative transition $\phi \rightarrow f_{0} \gamma$, which is a sensitive probe of the nature of the $f_{0}(980)$ particle. Using the QCD sum-rule technique, we estimate the branching ratio of such decay mode to be: $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=$ $(2.7 \pm 1.1) 10^{-4}$, in fair agreement with present experimental data. As for the structure of the $f_{0}$, the result suggests a sizeable $s \bar{s}$ component; however, this result does not exclude the possibility of further components and allows a more complex structure than indicated by the naive quark model.


The quark model provides a rather good description of hadrons, which fit into suitable multiplets reasonably well. In its simplest version the model then interprets mesons as pure $q \bar{q}$ states. Scalar mesons present a remarkable exception to this successful scheme. Indeed, the nature of these mesons is not established yet [1]. There are more scalars than can fit into one quark model multiplet. Consequently, some of these states could be either glueballs or admixtures of quark and gluonic states, or belong to multiquark multiplets. A particular feature of some of these particles is that they appear to be rather
 such as $K \bar{K}$ or $\pi \pi$. This might suggest that they can be identified as composite systems of hadrons, or that they spend an appreciable part of their lifetimes as such states. This could be the result of hadronic dressing, whereby the strong interaction enriches a $q \bar{q}$ state with other components such as $|K \bar{K}\rangle,|\pi \eta\rangle$, etc. Such a viewpoint could also explain why the scalar mesons seem to contradict the OZI rule. Since the two mesons composing the state in which they spend much of their lifetime may readily annihilate to $q \bar{q}$, leading to a subsequent OZI allowed decay.

In this letter, we focus on the structure of the $f_{0}(980)$ and the possibility of gleaning information about this from radiative $\phi$ decays. According to the quark model, the $f_{0}(980)$ should be an $s \bar{s}$ state, an interpretation supported in Refs. [6, 7, 8]. However, this does not explain its mass degeneracy with the $a_{0}(980)$, that should be a $(u \bar{u}-d \bar{d}) / \sqrt{2}$ state. There are also suggestions that the $f_{0}(980)$ could be a four quark $q q \overline{q q}$ state [9]. In this case, it could either be nucleon-like [10], i.e. a bound state of quarks with symbolic quark structure $s \bar{s}(u \bar{u}+d \bar{d}) / \sqrt{2} \rrbracket$, or deuteron-like, i.e. a bound state of hadrons, which is usually referred to as a $K \bar{K}$ molecule [2, 11, 12, 13]. In the former of these two possibilities, the mesons are treated as point-like objects, while in the latter they should be considered as extended objects. Some objections have been raised against the $K \bar{K}$ molecular model [2, 14]. In particular, such an interpretation requires a width smaller than the binding energy of the molecule itself, which has been estimated to be $\epsilon \simeq 10-20$ MeV [2], in contrast to the measured width lying in the range $40-100 \mathrm{MeV}$ [15]].

Various ways have been suggested of clarifying the situation, such as the analysis of the $f_{0} \rightarrow \gamma \gamma$ decay [16, [17] or of the ratio $\frac{\Gamma\left(\phi \rightarrow a_{0} \gamma\right)}{\Gamma\left(\phi \rightarrow f_{0} \gamma\right)}$ [11]. In the naive quark model, for example, it is expected that $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)$ and $\mathcal{B}\left(\phi \rightarrow a_{0} \gamma\right)$ would differ by a factor of 10. Moreover the rate for $\phi \rightarrow f_{0} \gamma$ may distinguish among the different possibilities [11],

[^0]since, according to the existing theoretical estimates, the expected branching ratio would be as high as $10^{-4}$ in the $q q \overline{q q}$ case, $\mathcal{O}\left(10^{-5}\right)$ in the $s \bar{s}$ case. For a $K \bar{K}$ molecule, the branching ratio clearly depends on its size. For a compact state this is $\sim 7 \cdot 10^{-5}$, while for a diffuse, deuteron-like system, it is down below $10^{-5}$ [11].

From the experimental point of view, the PDG value (15:

$$
\begin{equation*}
\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=(3.4 \pm 0.4) 10^{-4} \tag{1}
\end{equation*}
$$

stems from averaging the results of the CMD2 [18] and SND [19] collaborations, analysing $\pi^{+} \pi^{-} \gamma, \pi^{0} \pi^{0} \gamma$ and $5 \gamma$ final states. What is more, a significant improvement is expected at the $\phi$ factory DA $\Phi$ NE [20], where the first results give:

$$
\begin{equation*}
\mathcal{B}\left(\phi \rightarrow f_{0} \gamma \rightarrow \pi^{0} \pi^{0} \gamma\right)=(0.81 \pm 0.09(\text { stat }) \pm 0.06(\text { syst })) \times 10^{-4} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathcal{B}\left(\phi \rightarrow f_{0} \gamma \rightarrow \pi^{-} \pi^{+} \gamma\right)<1.64 \times 10^{-4} \tag{3}
\end{equation*}
$$

at $90 \%$ C.L. 21].
The present letter is devoted to analysis of the radiative decay $\phi \rightarrow f_{0} \gamma$ using QCD sum-rules [22], which we previously applied to the radiative $\phi$ transitions to $\eta, \eta^{\prime}$ [23]. That the $f_{0}(980)$ couples significantly through $s \bar{s}$ components has long been known $\emptyset$ from its appearance as a peak in $J / \psi \rightarrow \phi f_{0}$ [25] and $D_{s} \rightarrow \pi f_{0}$ [26], as discussed in Refs. [3], and in more detail in [27]. Our calculation relies on the assumed coupling of the $f_{0}$ to the scalar $s \bar{s}$ density. As a preliminary, we evaluate the strength of this coupling using two point QCD sum-rules. The result will then be exploited in the three point QCD sum-rule evaluation of the relevant quantity needed to compute $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)$.

The coupling of the $f_{0}(980)$ to the scalar current $J^{s}=\bar{s} s$ can be parametrized in terms of a constant $\tilde{f}$ :

$$
\begin{equation*}
\langle 0| J^{s}\left|f_{0}(p)\right\rangle=m_{f_{0}} \tilde{f} \tag{4}
\end{equation*}
$$

In order to compute this parameter by QCD sum-rules, we consider the two-point correlator:

$$
\begin{equation*}
T\left(q^{2}\right)=i \int d^{4} x e^{i q \cdot x}\langle 0| T\left[J^{s}(x) J^{s \dagger}(0)\right]|0\rangle, \tag{5}
\end{equation*}
$$

[^1]which is given by the dispersive representation:
\[

$$
\begin{equation*}
T\left(q^{2}\right)=\frac{1}{\pi} \int_{4 m_{s}^{2}}^{\infty} d s \frac{\rho(s)}{s-q^{2}}+\text { subtractions } \tag{6}
\end{equation*}
$$

\]

In the region of low values of $s$, the physical spectral density contains a $\delta$-function term corresponding, in the small width approximation, to the coupling of the $f_{0}$ to the scalar current. Picking up this contribution and dropping possible subtractions which we discuss later, we can write:

$$
\begin{equation*}
T\left(q^{2}\right)=\frac{m_{f_{0}}^{2} \tilde{f}^{2}}{m_{f_{0}}^{2}-q^{2}}+\frac{1}{\pi} \int_{s_{0}}^{\infty} d s \frac{\rho^{h a d}(s)}{s-q^{2}} \tag{7}
\end{equation*}
$$

assuming that the contribution of higher resonances and continuum of states start from an effective threshold $s_{0}$. On the other hand, the correlator $T\left(q^{2}\right)$ can be computed in QCD for large Euclidean values of $q^{2}$, by using the Operator Product Expansion (OPE) to expand the $T$-product in Eq. (5) as the sum of a perturbative contribution plus nonperturbative terms which are proportional to vacuum expectation values of quark and gluon gauge-invariant operators of increasing dimension, the so called vacuum condensates. In practice, only a few condensates are included, the most important contributions coming from the dimension $3<\bar{q} q>$ and dimension $5<\bar{q} g \sigma G q>$.

In the QCD expression for the two-point correlator considered, the perturbative term can also be written dispersively, so that:

$$
\begin{equation*}
T^{Q C D}\left(q^{2}\right)=\frac{1}{\pi} \int_{4 m_{s}^{2}}^{\infty} d s \frac{\rho^{p e r t}(s)}{s-q^{2}}+d_{3}<\bar{s} s>+d_{5}<\bar{s} g \sigma G s>+\ldots, \tag{8}
\end{equation*}
$$

where the spectral function $\rho^{\text {pert }}$ and the coefficients $d_{3}, d_{5}$ can be computed in QCD. The next step consists in assuming quark-hadron duality, which amounts to the claim that the physical and the perturbative spectral densities give the same result when integrated appropriately above some $s_{0}$. This leads to the sum-rule:

$$
\begin{equation*}
\frac{m_{f_{0}}^{2} \tilde{f}^{2}}{m_{f_{0}}^{2}-q^{2}}=\frac{1}{\pi} \int_{4 m_{s}^{2}}^{s_{0}} d s \frac{\rho^{\text {pert }}(s)}{s-q^{2}}+d_{3}<\bar{s} s>+d_{5}<\bar{s} g \sigma G s>+\ldots \tag{9}
\end{equation*}
$$

This expression can be improved by applying to both sides of Eq. (9) a Borel transform, defined as follows:

$$
\begin{equation*}
\mathcal{B}\left[\mathcal{F}\left(Q^{2}\right)\right]=\lim _{Q^{2} \rightarrow \infty, n \rightarrow \infty, \frac{Q^{2}}{n}=M^{2}} \frac{1}{(n-1)!}\left(-Q^{2}\right)^{n}\left(\frac{d}{d Q^{2}}\right)^{n} \mathcal{F}\left(Q^{2}\right), \tag{10}
\end{equation*}
$$

where $\mathcal{F}$ is a generic function of $Q^{2}=-q^{2}$. The application of such a procedure to the sum-rules amounts to exploiting the following result:

$$
\begin{equation*}
\mathcal{B}\left[\frac{1}{\left(s+Q^{2}\right)^{n}}\right]=\frac{\exp \left(-s / M^{2}\right)}{\left(M^{2}\right)^{n}(n-1)!} \tag{11}
\end{equation*}
$$

where $M^{2}$ is known as the Borel parameter. This operation improves the convergence of the series in the OPE by factorials in $n$ and, for suitably chosen values of $M^{2}$, enhances the contribution of low lying states. Moreover, since the Borel transform of a polynomial vanishes, it is correct to neglect subtraction terms in Eq. (6), which are polynomials in $q^{2}$. The final sum-rule reads:

$$
\begin{align*}
& m_{f_{0}}^{2} \tilde{f}^{2} \exp \left(-\frac{m_{f_{0}}^{2}}{M^{2}}\right)=\frac{3}{8 \pi^{2}} \int_{4 m_{s}^{2}}^{s_{0}} d s s\left(1-\frac{4 m_{s}^{2}}{s}\right)^{3 / 2} \exp \left(-\frac{s}{M^{2}}\right) \\
& +m_{s} \exp \left(-\frac{m_{s}^{2}}{M^{2}}\right)\left[<\bar{s} s>\left(3+\frac{m_{s}^{2}}{M^{2}}+\frac{m_{s}^{4}}{M^{4}}\right)+<\bar{s} g \sigma G s>\frac{1}{M^{2}}\left(1-\frac{m_{s}^{2}}{2 M^{2}}\right)\right] \tag{12}
\end{align*}
$$

In the numerical evaluation of Eq. (12) we use $\left\langle\bar{s} s>=0.8<\bar{q} q>,\langle\bar{q} q\rangle=(-0.24)^{3}\right.$ $\mathrm{GeV}^{3},<\bar{s} g \sigma G s>=0.8 \mathrm{GeV}^{2}<\bar{s} s>, m_{f_{0}}=0.980 \mathrm{GeV}$. The strange quark mass is chosen in the range $m_{s}=0.125-0.160 \mathrm{GeV}$, obtained in the same QCD sum-rule framework [28]. The threshold is chosen below a possible $f_{0}(1370)$ pole and varied between $s_{0}=1.6-1.7 \mathrm{GeV}^{2}$. Since the Borel parameter has no physical meaning, we look for a range of its values ("stability window") where the sum-rule is almost independent on $M^{2}$. Such a window is usually sought in a restricted interval of values of the Borel parameter chosen by requiring that the perturbative contribution is at least $20 \%$ of the continuum and additionally requiring that the perturbative term is greater than the non-perturbative contribution. The stability window for $M^{2}$ is selected in $[1.2,2] \mathrm{GeV}^{2}$, as seen in Fig. [1, where, taking into account the uncertainty on $m_{s}$, we obtain the coupling:

$$
\begin{equation*}
\tilde{f}=(0.180 \pm 0.015) \mathrm{GeV} \tag{13}
\end{equation*}
$$

This result will be used in the analysis of the decay $\phi \rightarrow f_{0} \gamma$ as we shall see in the following.

The relevant matrix element describing the transition $\phi \rightarrow f_{0}$ induced by a strange vector current $J_{\mu}=\bar{s} \gamma_{\mu} s$, can be parameterized as follows:

$$
\begin{align*}
\left\langle f_{0}\left(q_{2}\right)\right| J_{\mu}\left|\phi\left(q_{1}, \epsilon_{1}\right)\right\rangle & =F_{1}\left(q^{2}\right)\left(q_{1} \cdot q_{2}\right) \epsilon_{1 \mu}+F_{2}\left(q^{2}\right)\left(\epsilon_{1} \cdot q_{2}\right)\left(q_{1}+q_{2}\right)_{\mu} \\
& +F_{3}\left(q^{2}\right)\left(\epsilon_{1} \cdot q_{2}\right) q_{\mu} \tag{14}
\end{align*}
$$



Figure 1: Coupling of the $f_{0}$ to the scalar current as a function of the Borel parameter $M$, for $m_{s}=0.140 \mathrm{GeV}$. The solid curve corresponds to the higher threshold $s_{0}=1.7 \mathrm{GeV}^{2}$, the dashed curve corresponds to $s_{0}=1.6 \mathrm{GeV}^{2}$.
where $q=q_{1}-q_{2}$. In order to consider the radiative decay $\phi \rightarrow f_{0} \gamma$, one needs the amplitude

$$
\begin{equation*}
\mathcal{A}\left(\phi\left(q_{1}, \epsilon_{1}\right) \rightarrow f_{0}\left(q_{2}\right) \gamma(q, \epsilon)\right)=-\frac{1}{3} e \epsilon^{* \mu}\left[F_{1}(0)\left(q_{1} \cdot q_{2}\right) \epsilon_{1 \mu}+F_{2}(0)\left(\epsilon_{1} \cdot q_{2}\right)\left(q_{1}+q_{2}\right)_{\mu}\right] \tag{15}
\end{equation*}
$$

where the charge of the strange quark has been explicitly written. Eq. (15) shows that only two of the three form factors appearing in Eq. (14) are actually needed. Furthermore, gauge invariance requires that $q^{\mu} \cdot\left[F_{1}(0)\left(q_{1} \cdot q_{2}\right) \epsilon_{1 \mu}+F_{2}(0)\left(\epsilon_{1} \cdot q_{2}\right)\left(q_{1}+q_{2}\right)_{\mu}\right]=0$, which relates the values of $F_{1}$ and $F_{2}$ at $q^{2}=0$ :

$$
\begin{equation*}
F_{2}(0)=F_{1}(0) \frac{m_{\phi}^{2}+m_{f_{0}}^{2}}{2\left(m_{\phi}^{2}-m_{f_{0}}^{2}\right)} \tag{16}
\end{equation*}
$$

In terms of $F_{1}(0)$, the rate for the process we consider becomes:

$$
\begin{equation*}
\Gamma\left(\phi \rightarrow f_{0} \gamma\right)=\alpha\left[F_{1}(0)\right]^{2} \frac{\left(m_{\phi}^{2}-m_{f_{0}}^{2}\right)\left(m_{\phi}^{2}+m_{f_{0}}^{2}\right)^{2}}{216 m_{\phi}^{3}} \tag{17}
\end{equation*}
$$

Three-point QCD sum-rules can be applied to evaluate the form factor $F_{1}\left(q^{2}\right)$. We consider the three-point function:

$$
\begin{equation*}
\Pi_{\mu \nu}\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)=i^{2} \int d^{4} x d^{4} y e^{-i q_{1} \cdot x} e^{i q_{2} \cdot y}\langle 0| T\left[J^{s}(y) J_{\nu}(0) J_{\mu}(x)\right]|0\rangle \tag{18}
\end{equation*}
$$

where $J^{s}$ has been defined above and $J_{\nu}=\bar{s} \gamma_{\nu} s$ is the vector current. The correlator Eq. (18) can be written in terms of invariant structures as follows:

$$
\begin{equation*}
\Pi_{\mu \nu}\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)=\Pi\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right) g_{\mu \nu}+\Pi_{1}\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right) q_{1 \mu} q_{1 \nu}+\cdots \tag{19}
\end{equation*}
$$

and a QCD sum-rule can be built up for the structure $\Pi\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)$. The method closely follows the one described for the two-point sum-rule. We assume $\Pi\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)$ obeys a dispersion relation in both the variables $q_{1}^{2}, q_{2}^{2}$ :

$$
\begin{equation*}
\Pi\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)=\frac{1}{\pi^{2}} \int d s_{1} \int d s_{2} \frac{\rho\left(s_{1}, s_{2}, q^{2}\right)}{\left(s_{1}-q_{1}^{2}\right)\left(s_{2}-q_{2}^{2}\right)} \tag{20}
\end{equation*}
$$

with possible subtractions. Such a representation is true at each order in perturbation theory and, as is standard in QCD sum rule analyses, it is assumed to hold in general. In this case the spectral function contains, for low values of $s_{1}, s_{2}$, a double $\delta$-function corresponding to the transition $\phi \rightarrow f_{0}$. Extracting this contribution, we can write:

$$
\begin{equation*}
\Pi\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)=-\frac{m_{f_{0}} \tilde{f} m_{\phi} f_{\phi} F_{1}\left(q^{2}\right)\left(q_{1} \cdot q_{2}\right)}{\left(m_{\phi}^{2}-q_{1}^{2}\right)\left(m_{f_{0}}^{2}-q_{2}^{2}\right)}+\frac{1}{\pi^{2}} \int_{s_{0}^{\prime}}^{\infty} d s_{1} \int_{s_{0}}^{\infty} d s_{2} \frac{\rho^{h a d}\left(s_{1}, s_{2}, q^{2}\right)}{\left(s_{1}-q_{1}^{2}\right)\left(s_{2}-q_{2}^{2}\right)}, \tag{21}
\end{equation*}
$$

where subtractions are neglected as later they will vanish on taking a Borel transform. The parameter $\tilde{f}$ appearing in the previous equation is just the coupling of the $f_{0}$ to the scalar current, computed previously. Deriving an OPE-based QCD expansion for $\Pi$ for large and negative $q_{1}^{2}, q_{2}^{2}$ and $q^{2}$, one can write:

$$
\begin{equation*}
\Pi\left(q_{1}^{2}, q_{2}^{2}, q^{2}\right)=\frac{1}{\pi^{2}} \int_{4 m_{s}^{2}}^{\infty} d s_{1} \int_{4 m_{s}^{2}}^{\infty} d s_{2} \frac{\rho^{\text {pert }}\left(s_{1}, s_{2}, q^{2}\right)}{\left(s_{1}-q_{1}^{2}\right)\left(s_{2}-q_{2}^{2}\right)}+c_{3}<\bar{s} s>+c_{5}<\bar{s} g \sigma G s>+\ldots \tag{22}
\end{equation*}
$$

Invoking quark-hadron global duality as before, we arrive at the sum-rule:

$$
\begin{align*}
\frac{m_{f_{0}} \tilde{f} m_{\phi} f_{\phi} F_{1}\left(q^{2}\right)\left(q^{2}-m_{\phi}^{2}-m_{f_{0}}^{2}\right)}{2\left(m_{\phi}^{2}-q_{1}^{2}\right)\left(m_{\eta}^{2}-q_{2}^{2}\right)} & =\frac{1}{\pi^{2}} \int_{D} d s_{1} d s_{2} \frac{\rho^{\text {pert }}\left(s_{1}, s_{2}, q^{2}\right)}{\left(s_{1}-q_{1}^{2}\right)\left(s_{2}-q_{2}^{2}\right)} \\
& +c_{3}<\bar{s} s>+c_{5}<\bar{s} g \sigma G s>+\ldots \tag{23}
\end{align*}
$$

where the domain $D$ should now also satisfy the kinematical constraints specified below. After a double Borel transform in the variables $-q_{1}^{2}$ and $-q_{2}^{2}$, we obtain:

$$
\begin{gather*}
\frac{1}{2} m_{f_{0}} \tilde{f} m_{\phi} f_{\phi}\left(q^{2}-m_{\phi}^{2}-m_{f_{0}}^{2}\right) F_{1}\left(q^{2}\right) \exp \left(-\frac{m_{\phi}^{2}}{M_{1}^{2}}-\frac{m_{f_{0}}^{2}}{M_{2}^{2}}\right)= \\
\frac{1}{\pi^{2}} \int_{D} d s_{1} d s_{2} \exp \left(-\frac{s_{1}}{M_{1}^{2}}-\frac{s_{2}}{M_{2}^{2}}\right) \rho^{\text {pert }}\left(s_{1}, s_{2}\right) \tag{24}
\end{gather*}
$$

$$
\begin{aligned}
& +\exp \left(-\frac{m_{s}^{2}}{M_{1}^{2}}-\frac{m_{s}^{2}}{M_{2}^{2}}\right)\left\{<\bar{s} s>\left[q^{2}+2 m_{s}^{2}-\frac{m_{s}^{2} q^{2}}{M_{1}^{2}}\right.\right. \\
& \left.+\frac{m_{s}^{2} q^{2}\left(2 m_{s}^{2}-q^{2}\right)}{2 M_{1}^{2} M_{2}^{2}}+\frac{m_{s}^{4} q^{2}}{2}\left(\frac{1}{M_{1}^{4}}+\frac{1}{M_{2}^{4}}\right)\right] \\
& +<\bar{s} g \sigma G s>\left[-\frac{1}{3}+\frac{q^{2}-m_{s}^{2}}{3 M_{1}^{2}}+\frac{2 q^{2}+m_{s}^{2}}{3 M_{2}^{2}}\right. \\
& \left.\left.-\frac{q^{2}\left(5 m_{s}^{2}-2 q^{2}\right)}{6 M_{1}^{2} M_{2}^{2}}-\frac{m_{s}^{2} q^{2}}{4 M_{2}^{4}}-\frac{m_{s}^{2}\left(3 q^{2}+m_{s}^{2}\right)}{12 M_{1}^{4}}\right]\right\},
\end{aligned}
$$

where:

$$
\begin{array}{r}
\rho^{\text {pert }}\left(s_{1}, s_{2}\right)=\frac{3 m_{s}}{4}\left\{\left(4 m_{s}^{2}+s_{1}-s_{2}+q^{2}\right)\left[\left(s_{1}+s_{2}-q^{2}\right)^{2}-4 s_{1} s_{2}\right]+4 q^{2} s_{1} s_{2}\right\} / \\
{\left[\left(s_{1}+s_{2}-q^{2}\right)^{2}-4 s_{1} s_{2}\right]^{3 / 2}} \tag{25}
\end{array}
$$

The integration domain $D$ over the variables $s_{1}, s_{2}$ depends on the value of $q^{2}$. For $\left(-q^{2}\right)>$ $s_{0}-4 m_{s}^{2}, D$ is specified by: $\left(s_{2}\right)_{-} \leq s_{2} \leq s_{0} 4 m_{s}^{2} \leq s_{1} \leq s_{0}^{\prime}$; while, for $\left(-q^{2}\right)<s_{0}-4 m_{s}^{2}$ $D$ is bounded by: $\left(s_{2}\right)_{-} \leq s_{2} \leq\left(s_{2}\right)_{+}$if $4 m_{s}^{2} \leq s_{1} \leq\left(s_{1}\right)_{-}$and $\left(s_{2}\right)_{-} \leq s_{2} \leq s_{0}$ if $\left(s_{1}\right)_{-} \leq$ $s_{1} \leq s_{0}^{\prime}$, with: $\left(s_{2}\right)_{ \pm}=\left[2 m_{s}^{2} q^{2}+\left(2 m_{s}^{2}-q^{2}\right) s_{1} \pm \sqrt{s_{1} q^{2}\left(q^{2}-4 m_{s}^{2}\right)\left(s_{1}-4 m_{s}^{2}\right)}\right] / 2 m_{s}^{2}$ and $\left(s_{1}\right)_{ \pm}=\left[2 m_{s}^{2} q^{2}+\left(2 m_{s}^{2}-q^{2}\right) s_{0} \pm \sqrt{s_{0} q^{2}\left(q^{2}-4 m_{s}^{2}\right)\left(s_{0}-4 m_{s}^{2}\right)}\right] / 2 m_{s}^{2}$.

Since we consider the form-factor $F_{1}\left(q^{2}\right)$ for arbitrary negative values of $q^{2}$, we could perform a double Borel transform in the two variables $Q_{1}^{2}=-q_{1}^{2}$ and $Q_{2}^{2}=-q_{2}^{2}$, which allows us to remove single poles in the $s_{1}$ and $s_{2}$ channels from the sum-rule. Our procedure is therefore to compute the form-factor $F_{1}\left(q^{2}\right)$ and then to extrapolate the result to $q^{2}=0$. In the numerical analysis we use: $m_{\phi}=1.02 \mathrm{GeV}, f_{\phi}=0.234 \mathrm{GeV}$ (obtained from the experimental datum on the decay to $e^{+} e^{-}$[15]). We compute the result for two values of the $\phi$ threshold: $s_{0}^{\prime}=1.8,1.9 \mathrm{GeV}^{2} . s_{0}$ coincides with the $f_{0}$ threshold chosen as for the two point function. The extrapolation to $q^{2}=0$ shown in Fig. 2 gives:

$$
\begin{equation*}
F_{1}(0)=0.34 \pm 0.07 \tag{26}
\end{equation*}
$$

which, using $\Gamma(\phi)=4.458 \mathrm{MeV}$ [15] and Eq. (17), gives:

$$
\begin{equation*}
\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=(2.7 \pm 1.1) 10^{-4} \tag{27}
\end{equation*}
$$

Both the results Eq. (13) and Eq. (26) have been derived without the inclusion of radiative $\alpha_{s}$ corrections, an approximation which is usually believed more accurate for the three point sum rule, where the $\mathcal{O}\left(\alpha_{s}\right)$ corrections are expected to cancel in the ratio of a three-point and a two-point function.


Figure 2: Form factor $F_{1}\left(q^{2}\right)$. The dashed and dotted lines are the highest and the lowest curves obtained varying the set of parameters entering in the sum rule (24). The isolated point on the right is the result of an extrapolation. The result (26) corresponds to the central point on the right obtained by extrapolating the solid curve.

Our result of Eq. (27) is in reasonable agreement with the outcome of refs. [29, 11], where the decay is supposed to proceed through the chain $\phi \rightarrow K \bar{K} \gamma \rightarrow f_{0} \gamma$, and so depends on the coupling $g_{f_{0} K \bar{K}}$ [ 5 . Their results are: $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=1.9 \times 10^{-4}$ 29] and $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=1.35 \times 10^{-4}$ [11]. On the other hand, QCD spectral sum rules are exploited in ref. [31] to predict $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=1.3 \times 10^{-4}$.

A different strategy is proposed in [16], where the experimental datum is assumed together with the structure $f_{0}(980)=n \bar{n} \cos \theta+s \bar{s} \sin \theta$, where $n \bar{n}=(u \bar{u}+d \bar{d}) / \sqrt{2}$ and $\theta$ is a mixing angle. A theoretical prediction is derived describing the particles $\left(\phi, f_{0}\right)$ through wave functions depending on the radii of the mesons. Such a prediction is then compared to the experimental datum in order to constrain the mixing angle.

Although our result of Eq. (27) is affected by a rather large uncertainty, it is in agreement with the available data [18, 19]. Since our sum rule analysis is based on the hypothesis that the $f_{0}(980)$ couples to the scalar $\bar{s} s$ current, this agreement leads to the conclusion that an $\bar{s} s$ component is present in such a state. However, our branching ratio is an order of magnitude larger than the naive quark model gives for a pure $\bar{s} s$

[^2]state. Our result is consequently consistent with the view that the $f_{0}(980)$ is a meson with a basic $\bar{q} q$ composition, which spends a sizeable part of its lifetime in a two meson state, such as $K \bar{K}$. This is in keeping with the analyses of [6, 8, [17, 32] that attribute such multi-hadron components to dressing. While the effect of $K \bar{K}$ couplings have been studied phenomenologically in Ref. [16] in a range of hadronic reactions, they have been dynamically calculated by Marco et al. [33] explicitly for the radiative decay we study here and found to give $\mathcal{B}\left(\phi \rightarrow f_{0} \gamma\right)=2.4 \times 10^{-4}$, in reassuringly good agreement with our sum-rule result, Eq. (27).

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[^0]:    ${ }^{1}$ Within the same framework the isovector partner $a_{0}(980)$ is written as $s \bar{s}(u \bar{u}+d \bar{d}) / \sqrt{2}$.

[^1]:    ${ }^{2}$ And noticed more recently by Delbourgo et al. 24 for $\phi \rightarrow f_{0} \gamma$.

[^2]:    ${ }^{3}$ Such a coupling is taken from ref. 30.

