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The Two-Loop QCD Matrix Element for $e^+e^- \to 3$ Jets

L.W. Garland[a], T. Gehrmann[b], E.W.N. Glover[a], A. Koukoutsakis[a] and E. Remiddi[c]

[a] *Department of Physics, University of Durham, Durham DH1 3LE, England*

[b] *Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

[c] *Dipartimento di Fisica, Università di Bologna and INFN, Sezione di Bologna, I-40126 Bologna, Italy*

Abstract

We compute the $\mathcal{O}(\alpha_s^3)$ virtual QCD corrections to the $\gamma^* \to q\bar{q}g$ matrix element arising from the interference of the two-loop with the tree-level amplitude and from the self-interference of the one-loop amplitude. The calculation is performed by reducing all loop integrals appearing in the two-loop amplitude to a small set of known master integrals. Infrared and ultraviolet divergences are both regularized using conventional dimensional regularization, and the ultraviolet renormalization is performed in the $\overline{\rm MS}$ scheme. The infrared pole structure of the matrix elements agrees with the prediction made by Catani using an infrared factorization formula. The analytic result for the finite terms of both matrix elements is expressed in terms of one- and two-dimensional harmonic polylogarithms.

1 Introduction

Among jet observables, the three-jet production rate in electron–positron annihilation plays an outstanding role. The initial experimental observation of three-jet events at PETRA [1], in agreement with the theoretical prediction [2], provided first evidence for the gluon, and thus strong support for the theory of Quantum Chromodynamics (QCD). Subsequently the three-jet rate and related event shape observables were used for the precise determination of the QCD coupling constant α_s (see [3] for a review). Especially at LEP, three-jet observables were measured to a very high precision and the error on the extraction of α_s from these data is dominated by the uncertainty inherent in the theoretical next-to-leading order (NLO) calculation [4–8] of the jet observables. The planned TESLA [9] linear e^+e^- collider will allow precision QCD studies at even higher energies than at LEP. Given the projected luminosity of TESLA, one again expects the experimental errors to be well below the uncertainty of the NLO calculation.

Related to $e^+e^- \to 3$ jets by crossing symmetry are (2+1)-jet production in deep inelastic ep scattering and vector-boson-plus-jet production at hadron colliders. The experimental data from HERA on $ep \to (2+1)$ jets and related event shape observables have already reached a level of precision demanding predictions beyond the present NLO accuracy; a further improvement on these data is expected soon from the HERA high luminosity programme. Similarly, vector-boson production at large transverse momentum is a classic test of QCD in hadron–hadron collisions and demands the theoretical prediction to be as precise as possible. In this case, it is also an important background in searches for new physics at the Tevatron and the LHC.

Besides its phenomenological importance, the three-jet rate has also served as a theoretical testing ground for the development of new techniques for higher order calculations in QCD: both the subtraction [4] and the phase-space slicing [5] methods for the extraction of infrared singularities from NLO real radiation processes were developed in the context of the first three-jet calculations. The systematic formulation of phase-space slicing [7] as well as the dipole subtraction [8] method were also first demonstrated for three-jet observables, before being applied to other processes. It is very likely that similar techniques at higher orders will first be developed in the context of jet production in e^+e^- annihilation, which in contrast to hadron–hadron collisions or electron–proton scattering does not pose the additional difficulty of the regularization of initial state singularities.

The calculation of next-to-next-to-leading order (NNLO), i.e. $\mathcal{O}(\alpha_s^3)$, corrections to the three-jet rate in e^+e^- annihilation has been considered as a highly important project for a long time [10]. In terms of matrix elements, it requires the computation of three contributions: the tree level $\gamma^* \to 5$ partons amplitudes [11–13], the one-loop corrections to the $\gamma^* \to 4$ partons amplitudes [14–17], and the two-loop (as well as the one-loop times one-loop) corrections to the $\gamma^* \to 3$ partons matrix elements. While the former two contributions have been known for some time already, the two-loop amplitudes have presented an obstacle that prevented further progress on this calculation up to now.

This calculation has now become tractable owing to various technical developments over the last two years. In particular, the systematic application of integration-by-parts [18–20] and Lorentz invariance [21] identities allowed the large number of Feynman integrals appearing in two-loop four-point matrix elements to be reduced to a small number of so-called master integrals. The use of these techniques already allowed the calculation of two-loop QED and QCD corrections to many $2 \to 2$ scattering processes with massless on-shell external particles [22–27], which require master integrals corresponding to massless four-point functions with all legs on-shell [28–33]. The master integrals relevant in the context of the present work are massless four-point functions with three legs on-shell and one leg off-shell. The complete set of these integrals was computed in [34], earlier partial results had been presented in [35, 36].

The master integrals in [34] are expressed in terms of two-dimensional harmonic polylogarithms (2dHPLs). The 2dHPLs are an extension of the harmonic polylogarithms (HPLs) of [37]. All HPLs and 2dHPLs that appear in the divergent parts of the planar master integrals have weight ≤ 3 and can be related to the more commonly known Nielsen generalized polylogarithms [38, 39] of suitable arguments. The functions of weight 4 appearing in the finite parts of the master integrals can all be represented, by the very definition, as one-dimensional integrals over 2dHPLs of weight 3, hence of Nielsen's generalized polylogarithms of suitable arguments according to the above remark. A table with all relations is included in the appendix of [34]. Numerical routines providing an evaluation of the HPLs [40] and 2dHPLs [41] are available.

In this paper, we present the $\mathcal{O}(\alpha_s^3)$ corrections to the $\gamma^* \to q\bar{q}g$ matrix element. At this order, two combinations of amplitudes contribute: the interference of two-loop and tree amplitudes and the self-interference of the one-loop amplitude. We work in conventional dimensional regularization [18,42,43], with $d = 4-2\epsilon$ space-time dimensions, where all external particles are d-dimensional. Ultraviolet renormalization is performed in the $\overline{\mathrm{MS}}$ scheme. The infrared pole structure of the two-loop corrections to the $\gamma^* \to q\bar{q}g$ matrix element was predicted by Catani [44], using an infrared factorization formula. We confirm Catani's prediction with our explicit calculation, and we use the formalism introduced in [44] to present the infrared poles and the finite parts of the $\gamma^* \to q\bar{q}g$ matrix elements in a compact form.

The paper is structured as follows. In Section 2, we define the notation and kinematics used in the paper. Section 3 explains the method we used to reduce the two-loop diagrams to master integrals. The result for the two-loop QCD contribution to the $\gamma^* \to q\bar{q}g$ matrix element, decomposed into infrared-divergent and infrared-finite parts according to the prescription derived in [44], is given in Section 4. Finally, Section 5 contains the conclusions and an outlook on future steps needed for the completion of a full NNLO calculation of three-jet production in e^+e^- annihilation.

2 Notation

We consider the decay of a virtual photon into a quark–antiquark–gluon system:

$$\gamma^*(q) \longrightarrow q(p_1) + \bar{q}(p_2) + g(p_3) \; . \tag{2.1}$$

The kinematics of this process is fully described by the invariants

$$s_{12} = (p_1+p_2)^2 \; , \qquad s_{13} = (p_1+p_3)^2 \; , \qquad s_{23} = (p_2+p_3)^2 \; , \tag{2.2}$$

which fulfil

$$q^2 = s_{12} + s_{13} + s_{23} \equiv s_{123} \; . \tag{2.3}$$

It is convenient to define the dimensionless invariants

$$x = s_{12}/s_{123} \; , \qquad y = s_{13}/s_{123} \; , \qquad z = s_{23}/s_{123} \; , \tag{2.4}$$

with $x+y+z=1$.

Our calculation is performed in conventional dimensional regularization [18,42,43] with $d = 4-2\epsilon$, and all external particle states are taken to be d-dimensional. Renormalization of ultraviolet divergences is performed in the $\overline{\mathrm{MS}}$ scheme. The renormalized amplitude can be written as

$$|\mathcal{M}\rangle = \sqrt{4\pi\alpha}e_q\sqrt{4\pi\alpha_s}\left[|\mathcal{M}^{(0)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)|\mathcal{M}^{(1)}\rangle + \left(\frac{\alpha_s}{2\pi}\right)^2|\mathcal{M}^{(2)}\rangle + \mathcal{O}(\alpha_s^3)\right] \; , \tag{2.5}$$

where α denotes the electromagnetic coupling constant, e_q the quark charge, α_s the QCD coupling constant at the renormalization scale μ, and the $|\mathcal{M}^{(i)}\rangle$ are the i-loop contributions to the renormalized amplitude. They are vectors in colour space.

The squared amplitude, summed over spins, colours and quark flavours, is denoted by

$$\langle\mathcal{M}|\mathcal{M}\rangle = \sum |\mathcal{M}(\gamma^* \to q\bar{q}g)|^2 = \mathcal{T}(x,y,z) \; . \tag{2.6}$$

The perturbative expansion of $\mathcal{T}(x,y,z)$ at renormalization scale $\mu^2 = q^2 = s_{123}$ reads:

$$\begin{aligned}\mathcal{T}(x,y,z) \quad &= \quad 16\pi^2\alpha\sum_q e_q^2\alpha_s(q^2)\left[\mathcal{T}^{(2)}(x,y,z) + \left(\frac{\alpha_s(q^2)}{2\pi}\right)\mathcal{T}^{(4)}(x,y,z)\right.\\ &\qquad \left. + \left(\frac{\alpha_s(q^2)}{2\pi}\right)^2\mathcal{T}^{(6)}(x,y,z) + \mathcal{O}(\alpha_s^3(q^2))\right] \; ,\end{aligned} \tag{2.7}$$

where

$$\mathcal{T}^{(2)}(x,y,z) = \langle \mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle = 4V(1-\epsilon)\left[(1-\epsilon)\left(\frac{y}{z}+\frac{z}{y}\right)+\frac{2(1-y-z)-2\epsilon yz}{yz}\right], \tag{2.8}$$

$$\mathcal{T}^{(4)}(x,y,z) = \langle \mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle + \langle \mathcal{M}^{(1)}|\mathcal{M}^{(0)}\rangle, \tag{2.9}$$

$$\mathcal{T}^{(6)}(x,y,z) = \langle \mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle + \langle \mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle + \langle \mathcal{M}^{(2)}|\mathcal{M}^{(0)}\rangle, \tag{2.10}$$

where $V = N^2 - 1$, with N the number of colours. $\mathcal{T}^{(4)}(x,y,z)$ was first derived in [4,5]; we quote an explicit expression for it in Section 4.1. In the following, we present the contribution to $\mathcal{T}^{(6)}(x,y,z)$ from the interference of two-loop and tree diagrams

$$\mathcal{T}^{(6,[2\times 0])}(x,y,z) = \langle \mathcal{M}^{(0)}|\mathcal{M}^{(2)}\rangle + \langle \mathcal{M}^{(2)}|\mathcal{M}^{(0)}\rangle, \tag{2.11}$$

as well as the one-loop self-interference

$$\mathcal{T}^{(6,[1\times 1])}(x,y,z) = \langle \mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle . \tag{2.12}$$

At the same order in α_s, one finds also a contribution to three-jet final states from the self-interference of the $\gamma^* \to ggg$ amplitude. The matrix element for this process does not contain infrared or ultraviolet divergences; it was computed long ago and can be found in [45,46].

For the remainder of this paper we will set the renormalization scale $\mu^2 = q^2$. The full scale dependence of the perturbative expansion is given by

$$\begin{aligned}\mathcal{T}(x,y,z) = 16\pi^2\alpha \sum_q e_q^2 \alpha_s(\mu^2)\Bigg\{&\mathcal{T}^{(2)}(x,y,z)\\ &+\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)\left[\mathcal{T}^{(4)}(x,y,z)+b_0\mathcal{T}^{(2)}(x,y,z)\ln\left(\frac{\mu^2}{q^2}\right)\right]\\ &+\left(\frac{\alpha_s(\mu^2)}{2\pi}\right)^2\left[\mathcal{T}^{(6)}(x,y,z)+\left(2b_0\mathcal{T}^{(4)}(x,y,z)+b_1\mathcal{T}^{(2)}(x,y,z)\right)\ln\left(\frac{\mu^2}{q^2}\right)\right.\\ &\left.+b_0^2\mathcal{T}^{(2)}(x,y,z)\ln^2\left(\frac{\mu^2}{q^2}\right)\right]+\mathcal{O}(\alpha_s^3)\Bigg\}.\end{aligned} \tag{2.13}$$

3 Method

The Feynman diagrams contributing to the i-loop amplitude $|\mathcal{M}^{(i)}\rangle$ $(i = 0, 1, 2)$ were all generated using QGRAF [47]. There are two diagrams at tree-level, 13 diagrams at one loop and 229 diagrams at two loops. We then project $|\mathcal{M}^{(2)}\rangle$ by $\langle\mathcal{M}^{(0)}|$ and $|\mathcal{M}^{(1)}\rangle$ by $\langle\mathcal{M}^{(1)}|$, and perform the summation over colours and spins using the computer algebra programs MAPLE [48], FORM2 [49] and FORM3 [50]. When summing over the polarizations of the external gluon and off-shell photon, we use the Feynman gauge:

$$\sum_{\text{spins}} \epsilon_i^{\mu}\epsilon_i^{\nu *} = -g^{\mu\nu}. \tag{3.1}$$

This is valid because the gluon always couples to a conserved fermionic current, which selects only the physical degrees of polarization. The use of an axial polarization sum to project out the transverse polarizations (as applied in [24,25]) is therefore not needed.

The one-loop self-interference contribution $\mathcal{T}^{(6,[1\times 1])}$ is computed by reducing all tensorial loop integrals according to the standard Passarino–Veltman procedure [51] to scalar one-loop two-point, three-point and four-point integrals. It has been known for a long time that those three-point integrals can be further reduced to linear combinations of two-point integrals using integration-by-parts identities. After this reduction, $\mathcal{T}^{(6,[1\times 1])}$ is expressed as a bilinear combination of only two integrals: the one-loop box and the one-loop bubble, which are listed in Appendix A.

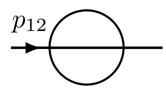
The computation of $\mathcal{T}^{(6,[2\times 0])}$ is by far less straightforward. It is explained in detail in the following two subsections.

3.1 Master Integrals

The integrals appearing in the individual two-loop diagrams contain up to seven propagators in the denominator, and up to four irreducible scalar products in the numerator (i.e. scalar products which can not be expressed as linear combinations of the occurring propagators). Each integral is classified by the number of different denominators t, the total number of denominators r and the number of irreducible scalar products s. The set of t different denominators defines the topology of the diagram.

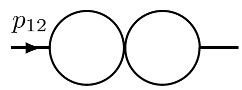
Using the reduction procedure described in Section 3.2 below, all of the two-loop Feynman diagrams can be reduced to a basis set of *master* integrals. Owing to the presence of the extra scale, there are considerably more master topologies than in the on-shell case. Altogether there are 14 planar topologies and 5 non-planar topologies requiring a total of 24 master integrals, as five topologies possess two master integrals (see below). For each topology, we take one of the master integrals (or the master integral, when it is unique) to be equal to the full scalar amplitude with the first power on all propagators. The simpler master integrals are the single scale integrals [52–54], which can be written in terms of Γ functions,

$\text{Sunrise}(s_{12}) =$



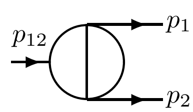
,

$\text{Glass}(s_{12}) =$



,

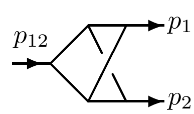
$\text{Dart}_1(s_{12}) =$



,

as well as the more complicated

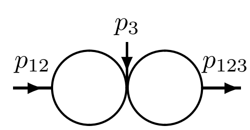
$\text{Xtri}_1(s_{12}) =$



.

The two-scale master integrals can be written in terms of Γ functions,

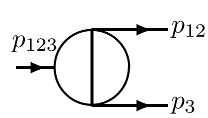
$\text{Tglass}(s_{12}, s_{123}) =$



,

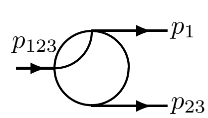
or as generalized polylogarithms or one-dimensional harmonic polylogarithms (see e.g. [34]),

$\text{Dart}_2(s_{12}, s_{123}) =$



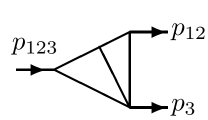
,

$\text{Dart}_2(s_{123}, s_{23}) =$



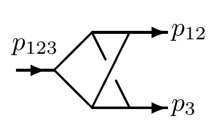
,

$\text{Plane}(s_{12}, s_{123}) =$



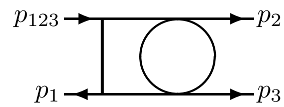
,

$\text{Xtri}_2(s_{123}, s_{12}) =$

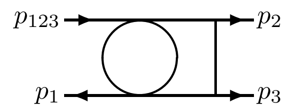


.

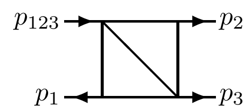
The three-scale master integrals can be written in terms of two-dimensional harmonic polylogarithms [34]. There are the planar graphs,

$$\mathrm{Abox}_1(s_{23}, s_{13}, s_{123}) \quad = \quad$$


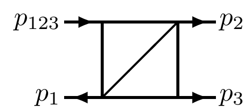
,

$$\mathrm{Abox}_2(s_{23}, s_{13}, s_{123}) \quad = \quad$$


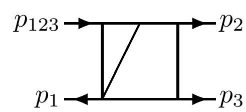
,

$$\mathrm{Cbox}_1(s_{23}, s_{13}, s_{123}) \quad = \quad$$


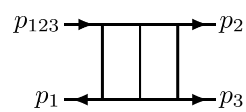
,

$$\mathrm{Cbox}_2(s_{23}, s_{13}, s_{123}) \quad = \quad$$


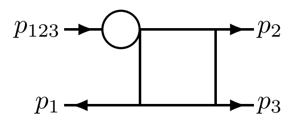
,

$$\mathrm{Tbox}_1(s_{23}, s_{13}, s_{123}) \quad = \quad$$


,

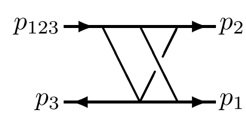
$$\mathrm{Pbox}_1(s_{23}, s_{13}, s_{123}) \quad = \quad$$


,

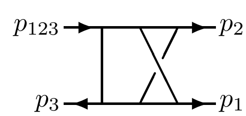
$$\mathrm{Bbox}(s_{23}, s_{13}, s_{123}) \quad = \quad$$


,

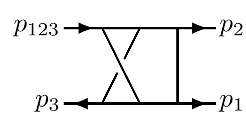
and the non-planar graphs,

$$\mathrm{Ebox}_1(s_{12}, s_{13}, s_{123}) \quad = \quad$$


,

$$\mathrm{Xbmo}_1(s_{12}, s_{13}, s_{123}) \quad = \quad$$


,

$$\mathrm{Xbmi}_1(s_{12}, s_{13}, s_{123}) \quad = \quad$$


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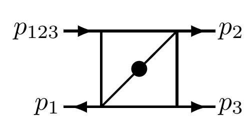
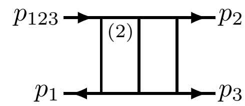
It turns out that for the Cbox_2, Pbox, Ebox, Xbmo and Xbmi topologies, a second master integral is required for expressing all the occurring integrals. One is free to choose the precise form of these additional master integrals in terms of additional powers of a propagator or as a tensor integral, provided the second master integral is not directly related by LI and IBP identities to the first one. As in [34] we choose as second master integral

$$\mathrm{Cbox}_{2A}(s_{23}, s_{13}, s_{123}) \quad = \quad$$


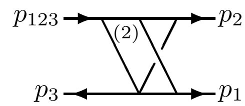
,

$$\mathrm{Pbox}_2(s_{23}, s_{13}, s_{123}) \quad = $$



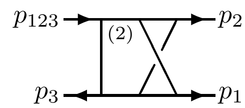
,

$$\mathrm{Ebox}_2(s_{12}, s_{13}, s_{123}) \quad = $$



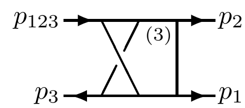
,

$$\mathrm{Xbmo}_2(s_{12}, s_{13}, s_{123}) \quad = $$



,

$$\mathrm{Xbmi}_2(s_{12}, s_{13}, s_{123}) \quad = $$



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3.2 Reduction to master integrals

The reduction of the two-loop diagrams to master integrals using integration-by-parts (IBP) [18–20] and Lorentz invariance (LI) [21] identities was performed with two independent programs, which allowed a comparison diagram by diagram.

One program was based on the reduction algorithm already described in [21]. In this program, reduction equations are derived by applying the differentiation operators of the IBP and LI identities to the following integrands: $t = 7, r = 7, s \leq 4$; $3 \leq t \leq 6, t \leq r \leq 7, s \leq 3$ (note that it is not necessary to include $s = 4$ integrands with $t < 7$). As a result, one obtains a linear system of equations, which contains, besides integrals of the class t, r, s and simpler integrals, also more complicated integrals of the classes $t, r+1, s$ and $t, r+1, s+1$. The number of equations does, however, exceed the number of integrals, thus yielding an apparently overconstrained system of equations. By solving this system of equations using the computer algebra programs MAPLE [48] and FORM2 [49], one expresses integrals of a given class t, r, s as linear combinations of simpler integrals. The irreducible integrals, which are left over after this procedure, are the master integrals. Compared to the algorithm used in [21], one major modification was made. For integrals corresponding to those topologies which do not possess an independent master integral (i.e. can be fully reduced to integrals with one less propagator by the IBP and LI identities), it is straightforward to obtain a symbolic solution of the IBP and LI identities. This solution takes the form of a single equation decreasing the power on one or more of the propagators (see [55] for an example). These symbolic reduction equations for all non-master topologies were derived automatically using a FORM2 [49] program. The use of these symbolic identities did lead to a considerable speed-up of the reduction of the amplitudes of such diagrams to master integrals.

The second approach derived the reduction equations using the IBP and LI identities for three auxiliary diagrams — one planar and two non-planar. These auxiliary diagrams are obtained by mapping the nine possible dot-products involving the three independent external momenta (p_1, p_2 and p_3) and the two loop momenta as propagators. Scalar products in the numerator are replaced by propagators raised to negative powers. For illustration, the planar auxiliary diagram is shown in Fig. 1, with each propagator i raised to the arbitrary power ν_i. An advantage of this method is that the 10 IBP and 3 LI identities are written in terms of arbitrary ν_i and are valid for all subtopologies of the auxiliary diagram ($\nu_i \to 0$) and all tensor integrals ($\nu_i < 0$). Typically we encounter subtopologies with up to 7 propagators ($t = 7$) and tensor integrals up to fourth rank ($s = 4$). For a given value of t, and assignment of propagators, we generate equations using tensor integrals with $s \leq 4$ as seeds for the generic IBP and LI equations. These seed integrals are those that appear in the actual calculation of Feynman diagrams. For example, for $t = 7$, there are 15 seed integrals yielding 195 equations. The equations are stored and solved one by one according to a ranking similar to that proposed by Laporta [56] using MAPLE [48] and FORM3 [50]. At each stage, more complicated integrals are decomposed in terms of simpler integrals of the same topology and pinched integrals. As in the first method, the remaining irreducible integrals are master integrals. Of course, the system of equations contains many

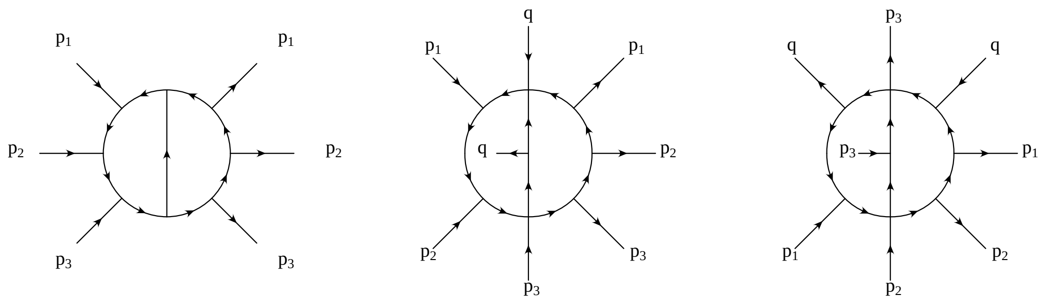

Figure 1: The planar and two non-planar auxiliary diagrams. All nine possible dot-products involving the loop momenta and the external particle momenta are mapped onto the nine propagators.

more complicated integrals (with, for example, $r > 7$ or $s > 4$), which cannot be reduced to master integrals with this approach. However, they are not needed for the evaluation of the Feynman diagrams and can be eliminated from the system of equations.

In both methods, one observes that on the face of it, the planar and non-planar graphs appear to be of equal difficulty — requiring the same depth of tensor reduction with the same numbers of seed integrals and the same number of IBP and LI equations. However the complexity of the non-planar graphs is first revealed in the number of terms in each equation and then in the work done to eliminate the unwanted integrals from the system of equations.

Both methods have their advantages. The major advantage of the first method is that the reduction equations for each topology containing master integrals have to be evaluated only once, while the second method requires their repeated evaluation each time a topology appears as subtopology of the auxiliary diagram. As a consequence, the first method needed less computer time for the derivation of all required reduction equations. An advantage of the second method is that it needs less human intervention, since mapping of subtopologies to master topologies is not necessary, and the generalization to more complicated graphs with additional energy scales requires only the modification of the 13 IBP and LI equations. The solution of the system and identification of master integrals is only limited by computer resources.

3.3 Expansion of master integrals

The two-loop master integrals relevant to the $\gamma^* \to q\bar{q}g$ matrix element are two-loop four-point functions with one leg off-shell. These functions were all computed in [34] in the framework of dimensional regularization with $d = 4-2\epsilon$ space-time dimensions. The results of [34] take the form of a Laurent series in ϵ, starting at ϵ^{-4}, with coefficients containing one- and two-dimensional harmonic polylogarithms [37], which are a generalization of Nielsen's polylogarithms [38]. All master integrals in [34] were given for one particular configuration of the external momenta. They were expressed in a form where the argument of the 2dHPLs was always y, while z appeared in the index vector of the 2dHPLs; y and z appeared also as argument of the HPLs.

Each master integral can occur in six kinematic configurations (corresponding to the permutations of (p_1, p_2, p_3)). To avoid hidden zeros (arising from cancellations occurring in the combinations of HPLs and 2dHPLs with different arguments and different variables in the index vector), we express the master integrals for all kinematic configurations in a unique form, which is the same as in [34]: the argument of the 2dHPLs is always y, the variable in their index vector is z, which appears also as argument of the HPLs. To obtain this unique form, we apply the following relations to the 2dHPLs, which are all derived following the same lines as the formulae for the interchange of arguments discussed in the appendix of [34]:

1. Interchange of arguments: $\mathrm{G}(\vec{m}(y); z) \to \mathrm{G}(\vec{m}(z); y) + \mathrm{H}(\vec{n}; z)$.

2. Replacement of argument variable: $\mathrm{G}(\vec{m}(z); 1-y-z) \to \mathrm{G}(\vec{m}(z); y) + \mathrm{H}(\vec{n}; z)$.

3. Replacement of index variable: $\mathrm{G}(\vec{m}(1-y-z); y) \to \mathrm{G}(\vec{m}(z); y) + \mathrm{H}(\vec{n}; z)$.

The relations required for the two remaining combinations $\mathrm{G}(\vec{m}(y); 1-y-z)$ and $\mathrm{G}(\vec{m}(1-y-z); z)$ are obtained by applying the interchange of argument relations followed by the appropriate replacement relations.

The master integrals in [34] were derived in the kinematical situation of a (space-like) $1 \to 3$ decay, which corresponds to the $\gamma^* \to q\bar{q}g$, such that the only analytic continuation of them required here is the expansion of the overall factor in the time-like region

$$\mathrm{Re}(-1)^{-2\epsilon} = 1 - 2\pi^2\epsilon^2 + 2/3\pi^4\epsilon^4 + \mathcal{O}(\epsilon^6) \; . \tag{3.2}$$

The analytic continuation of the master integrals to other kinematical regions is discussed in the appendix of [34].

3.4 Ultraviolet renormalization

The renormalization of the matrix element is carried out by replacing the bare coupling α_0 with the renormalized coupling $\alpha_s \equiv \alpha_s(\mu^2)$, evaluated at the renormalization scale μ^2

$$\alpha_0 \mu_0^{2\epsilon} S_\epsilon = \alpha_s \mu^{2\epsilon} \left[1 - \frac{\beta_0}{\epsilon}\left(\frac{\alpha_s}{2\pi}\right) + \left(\frac{\beta_0^2}{\epsilon^2} - \frac{\beta_1}{2\epsilon}\right)\left(\frac{\alpha_s}{2\pi}\right)^2 + \mathcal{O}(\alpha_s^3) \right] \; , \tag{3.3}$$

where

$$S_\epsilon = (4\pi)^\epsilon e^{-\epsilon\gamma} \qquad \text{with Euler constant } \gamma = 0.5772\ldots$$

and μ_0^2 is the mass parameter introduced in dimensional regularization [18,42,43] to maintain a dimensionless coupling in the bare QCD Lagrangian density; β_0 and β_1 are the first two coefficients of the QCD β-function:

$$\beta_0 = \frac{11 C_A - 4 T_R N_F}{6}, \qquad \beta_1 = \frac{17 C_A^2 - 10 C_A T_R N_F - 6 C_F T_R N_F}{6} \; , \tag{3.4}$$

with the QCD colour factors

$$C_A = N, \qquad C_F = \frac{N^2-1}{2N}, \qquad T_R = \frac{1}{2} \; . \tag{3.5}$$

We denote the i-loop contribution to the unrenormalized amplitudes by $|\mathcal{M}^{(i),\mathrm{un}}\rangle$, using the same normalization as for the decomposition of the renormalized amplitude (2.5). The renormalized amplitudes are then obtained as

$$\begin{aligned}
|\mathcal{M}^{(0)}\rangle &= |\mathcal{M}^{(0),\mathrm{un}}\rangle \; , \\
|\mathcal{M}^{(1)}\rangle &= S_\epsilon^{-1}|\mathcal{M}^{(1),\mathrm{un}}\rangle - \frac{\beta_0}{2\epsilon}|\mathcal{M}^{(0),\mathrm{un}}\rangle \; , \\
|\mathcal{M}^{(2)}\rangle &= S_\epsilon^{-2}|\mathcal{M}^{(2),\mathrm{un}}\rangle - \frac{3\beta_0}{2\epsilon}S_\epsilon^{-1}|\mathcal{M}^{(1),\mathrm{un}}\rangle - \left(\frac{\beta_1}{4\epsilon} - \frac{3\beta_0^2}{8\epsilon^2}\right)|\mathcal{M}^{(0),\mathrm{un}}\rangle \; .
\end{aligned} \tag{3.6}$$

4 The matrix element

We further decompose the renormalized one- and two-loop contributions to $\mathcal{T}^{(6)}$ as a sum of two terms

$$\mathcal{T}^{(6,[i\times j])}(x,y,z) = \mathcal{P}oles^{(i\times j)}(x,y,z) + \mathcal{F}inite^{(i\times j)}(x,y,z). \tag{4.1}$$

$\mathcal{P}oles$ contains infrared singularities that will be analytically cancelled by those occurring in radiative processes of the same order (ultraviolet divergences are removed by renormalization). $\mathcal{F}inite$ is the renormalized remainder, which is finite as $\epsilon \to 0$. In this section we first give explicit expressions for the infrared pole structure using the procedure advocated by Catani [44] and then give the analytic results for the finite remainders. For simplicity we set the renormalization scale $\mu^2 = s_{123}$ and restore the renormalization scale dependence using Eq. (2.13).

4.1 Infrared factorization

Catani [44] has shown how to organize the infrared pole structure of the two-loop contributions renormalized in the $\overline{\text{MS}}$ scheme in terms of the tree and renormalized one-loop amplitudes, $|\mathcal{M}^{(0)}\rangle$ and $|\mathcal{M}^{(1)}\rangle$ respectively, as

$$\begin{aligned}\mathcal{P}oles^{(2\times 0)} = 2\Re\Bigg[& -\frac{1}{2}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \frac{\beta_0}{\epsilon}\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle \\ & + \langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(1)}\rangle \\ & + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon}+K\right)\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)}(2\epsilon)|\mathcal{M}^{(0)}\rangle \\ & + \langle\mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle\Bigg] \end{aligned} \tag{4.2}$$

and

$$\mathcal{P}oles^{(1\times 1)} = \Re\left[2\langle\mathcal{M}^{(1)}|\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle - \langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)\dagger}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon)|\mathcal{M}^{(0)}\rangle\right] \tag{4.3}$$

where the constant K is

$$K = \left(\frac{67}{18} - \frac{\pi^2}{6}\right)C_A - \frac{10}{9}T_R N_F. \tag{4.4}$$

It should be noted that, in this prescription, part of the finite terms in $\mathcal{T}^{(6,[i\times j])}$ are accounted for by the $\mathcal{O}(\epsilon^0)$ expansion of $\mathcal{P}oles^{(i\times j)}$.

For this particular process, there is only one colour structure present at tree level which, in terms of the gluon colour a and the quark and antiquark colours i and j, is simply $\boldsymbol{T}^a_{ij}$. Adding higher loops does not introduce additional colour structures, and the amplitudes are therefore vectors in a one-dimensional space. Similarly, the infrared singularity operator $\boldsymbol{I}^{(1)}(\epsilon)$ is a 1×1 matrix in the colour space and is given by

$$\boldsymbol{I}^{(1)}(\epsilon) = -\frac{e^{\epsilon\gamma}}{2\Gamma(1-\epsilon)}\left[N\left(\frac{1}{\epsilon^2}+\frac{3}{4\epsilon}+\frac{\beta_0}{2N\epsilon}\right)(\mathtt{S}_{13}+\mathtt{S}_{23}) - \frac{1}{N}\left(\frac{1}{\epsilon^2}+\frac{3}{2\epsilon}\right)\mathtt{S}_{12}\right], \tag{4.5}$$

where (since we have set $\mu^2 = s_{123}$)

$$\mathtt{S}_{ij} = \left(-\frac{s_{123}}{s_{ij}}\right)^{\epsilon}. \tag{4.6}$$

Note that on expanding $\mathtt{S}_{ij}$, imaginary parts are generated, the sign of which is fixed by the small imaginary part $+i0$ of s_{ij}. Other combinations such as $\langle\mathcal{M}^{(0)}|\boldsymbol{I}^{(1)\dagger}(\epsilon)$ are obtained by using the hermitian conjugate operator $\boldsymbol{I}^{(1)\dagger}(\epsilon)$, where the only practical change is that the sign of the imaginary part of $\mathtt{S}$ is reversed. The origin of the various terms in Eq. (4.5) is straightforward. Each parton pair ij in the event forms a radiating antenna of scale s_{ij}. Terms proportional to $\mathtt{S}_{ij}$ are cancelled by real radiation emitted from leg i and absorbed by leg j. The soft singularities $\mathcal{O}(1/\epsilon^2)$ are independent of the identity of the participating partons and are universal. However, the collinear singularities depend on the identities of the participating partons. For each quark we find a contribution of $3/(4\epsilon)$ and for each gluon we find a contribution of $\beta_0/(2\epsilon)$ coming from the integral over the collinear splitting function.

Finally, the last term of Eq. (4.2) that involves $\boldsymbol{H}^{(2)}(\epsilon)$ produces only a single pole in ϵ and is given by

$$\langle\mathcal{M}^{(0)}|\boldsymbol{H}^{(2)}(\epsilon)|\mathcal{M}^{(0)}\rangle = \frac{e^{\epsilon\gamma}}{4\,\epsilon\,\Gamma(1-\epsilon)}H^{(2)}\langle\mathcal{M}^{(0)}|\mathcal{M}^{(0)}\rangle, \tag{4.7}$$

where the constant $H^{(2)}$ is renormalization-scheme-dependent. As with the single pole parts of $\boldsymbol{I}^{(1)}(\epsilon)$, the process-dependent $H^{(2)}$ can be constructed by counting the number of radiating partons present in the event. In our case, there is a quark–antiquark pair and a gluon present in the final state, so that

$$H^{(2)} = 2H^{(2)}_q + H^{(2)}_g \tag{4.8}$$

where in the $\overline{\mathrm{MS}}$ scheme

$$H_g^{(2)} = \left(\frac{1}{2}\zeta_3 + \frac{5}{12} + \frac{11\pi^2}{144}\right) N^2 + \frac{5}{27} N_F^2 + \left(-\frac{\pi^2}{72} - \frac{89}{108}\right) N N_F - \frac{N_F}{4N}, \tag{4.9}$$

$$\begin{aligned} H_q^{(2)} &= \left(\frac{7}{4}\zeta_3 + \frac{409}{864} - \frac{11\pi^2}{96}\right) N^2 + \left(-\frac{1}{4}\zeta_3 - \frac{41}{108} - \frac{\pi^2}{96}\right) + \left(-\frac{3}{2}\zeta_3 - \frac{3}{32} + \frac{\pi^2}{8}\right)\frac{1}{N^2} \\ &\quad + \left(\frac{\pi^2}{48} - \frac{25}{216}\right)\frac{(N^2-1)N_F}{N}, \end{aligned} \tag{4.10}$$

so that

$$\begin{aligned} H^{(2)} &= \left(4\zeta_3 + \frac{589}{432} - \frac{11\pi^2}{72}\right) N^2 + \left(-\frac{1}{2}\zeta_3 - \frac{41}{54} - \frac{\pi^2}{48}\right) + \left(-3\zeta_3 - \frac{3}{16} + \frac{\pi^2}{4}\right)\frac{1}{N^2} \\ &\quad + \left(-\frac{19}{18} + \frac{\pi^2}{36}\right) N N_F + \left(-\frac{1}{54} - \frac{\pi^2}{24}\right)\frac{N_F}{N} + \frac{5}{27} N_F^2. \end{aligned} \tag{4.11}$$

The factors $H_q^{(2)}$ and $H_g^{(2)}$ are directly related to those found in gluon–gluon scattering [25], quark–quark scattering [23] and quark–gluon scattering [24] (which each involve four partons) as well as in the quark form factor [52, 57–59]. We also note that (on purely dimensional grounds) one might expect terms of the type $\mathtt{S}_{ij}^2$ to be present in $H^{(2)}$. Of course such terms are $1 + \mathcal{O}(\epsilon)$ and therefore leave the pole part unchanged and only modify the finite remainder. At present it is not known how to systematically include these effects.

The renormalized interference of tree and one-loop amplitudes also appears in Eq. (4.2). This can be written to all orders in ϵ using the relation

$$\langle \mathcal{M}^{(0)} | \mathcal{M}^{(1)} \rangle = S_\epsilon^{-1} \langle \mathcal{M}^{(0),un} | \mathcal{M}^{(1),un} \rangle - \frac{\beta_0}{2\epsilon} \langle \mathcal{M}^{(0),un} | \mathcal{M}^{(0),un} \rangle, \tag{4.12}$$

where

$$\langle \mathcal{M}^{(0),un} | \mathcal{M}^{(1),un} \rangle = V \left(N f_1(y,z) + \frac{1}{N} f_2(y,z) + (y \leftrightarrow z) \right).$$

The functions $f_1(y,z)$ and $f_2(y,z)$ can be written in terms of the one-loop bubble integral and the one-loop box integral in $d = 6 - 2\epsilon$ dimensions, Box^6:

$$\begin{aligned} f_1(y,z) &= \\ &\frac{1}{yz}\Bigg((-3+\epsilon+2\epsilon^2)\mathrm{Bub}(s_{123}) + \left(-\frac{4}{\epsilon} + 12 - 8\epsilon\right)\mathrm{Bub}(ys_{123})\Bigg) \\ &+\frac{y}{z}\Bigg(\left(-\frac{2}{\epsilon} + 8 - 10\epsilon + 3\epsilon^2 + \epsilon^3\right)\mathrm{Bub}(zs_{123}) + (-3+4\epsilon+\epsilon^2-2\epsilon^3)\mathrm{Bub}(s_{123}) \\ &\qquad\qquad + \left(-\frac{2}{\epsilon} + 8 - 10\epsilon + 4\epsilon^2\right)\mathrm{Bub}(ys_{123})\Bigg) \\ &+\frac{1}{z}\Bigg(\left(\frac{4}{\epsilon} - 12 + 9\epsilon - \epsilon^2\right)\mathrm{Bub}(zs_{123}) + (6-2\epsilon-4\epsilon^2)\mathrm{Bub}(s_{123}) + \left(\frac{4}{\epsilon} - 12 + 8\epsilon\right)\mathrm{Bub}(ys_{123})\Bigg) \\ &+\frac{y}{(1-z)^2}(1-\epsilon)\Big(\mathrm{Bub}(zs_{123}) - \mathrm{Bub}(s_{123})\Big) \\ &+\frac{y}{(1-z)}\Big((3-5\epsilon+2\epsilon^3)\mathrm{Bub}(zs_{123}) + (-3+4\epsilon+\epsilon^2-2\epsilon^3)\mathrm{Bub}(s_{123})\Big) \\ &+\frac{1}{(1-z)}(4-3\epsilon-3\epsilon^2-2\epsilon^3)\Big(\mathrm{Bub}(s_{123}) - \mathrm{Bub}(zs_{123})\Big) \end{aligned}$$

$$+(4-9\epsilon+6\epsilon^2-\epsilon^3)\mathrm{Bub}(zs_{123})$$

$$+s_{123}\mathrm{Box}^6(ys_{123},zs_{123},s_{123})(1-2\epsilon)\Bigg(\frac{1}{z}8(-1+\epsilon)+\frac{y^2}{z}(-2+4\epsilon-2\epsilon^2)+\frac{y}{z}(6-8\epsilon+2\epsilon^2)$$

$$+z(-2+2\epsilon-8\epsilon^2)+(4-3\epsilon+3\epsilon^2)+\frac{1}{yz}2(1-\epsilon)\Bigg)\,, \tag{4.13}$$

$$f_2(y,z)=$$

$$\frac{1}{yz}\Bigg((3-\epsilon-2\epsilon^2)\mathrm{Bub}(s_{123})+\Bigg(\frac{2}{\epsilon}-6+4\epsilon\Bigg)\mathrm{Bub}((1-y-z)s_{123})\Bigg)$$

$$+\frac{y}{z}\Bigg(-\epsilon^2(1-\epsilon)\mathrm{Bub}(zs_{123})+(3-4\epsilon-\epsilon^2+2\epsilon^3)\mathrm{Bub}(s_{123})$$

$$+\Bigg(\frac{2}{\epsilon}-8+10\epsilon-4\epsilon^2\Bigg)\mathrm{Bub}((1-y-z)s_{123})\Bigg)$$

$$+\frac{1}{z}\Bigg(\epsilon(1-\epsilon)\mathrm{Bub}(zs_{123})+(-6+2\epsilon+4\epsilon^2)\mathrm{Bub}(s_{123})+\Bigg(-\frac{4}{\epsilon}+12-8\epsilon\Bigg)\mathrm{Bub}((1-y-z)s_{123})\Bigg)$$

$$+\frac{1}{(y+z)^2}2\Bigg(\mathrm{Bub}((1-y-z)s_{123})-\mathrm{Bub}(s_{123})\Bigg)+\frac{1}{(y+z)}2\epsilon\Bigg(\mathrm{Bub}(s_{123})-2\mathrm{Bub}((1-y-z)s_{123})\Bigg)$$

$$+\frac{y}{(1-z)^2}(1-\epsilon)\Bigg(\mathrm{Bub}(s_{123})-\mathrm{Bub}(zs_{123})\Bigg)$$

$$+\frac{y}{(1-z)}\Bigg((3-4\epsilon-\epsilon^2+2\epsilon^3)\mathrm{Bub}(s_{123})+(-3+5\epsilon-2\epsilon^3)\mathrm{Bub}(zs_{123})\Bigg)$$

$$+\frac{1}{(1-z)}(2+\epsilon-5\epsilon^2-2\epsilon^3)\Bigg(\mathrm{Bub}(zs_{123})-\mathrm{Bub}(s_{123})\Bigg)$$

$$+(2-7\epsilon+2\epsilon^2+3\epsilon^3)\mathrm{Bub}(zs_{123})+(-4+10\epsilon-4\epsilon^2)\mathrm{Bub}((1-y-z)s_{123})$$

$$+s_{123}\mathrm{Box}^6((1-y-z)s_{123},zs_{123},s_{123})(1-2\epsilon)\Bigg((8-4\epsilon)-\frac{(1-y)}{z}4(1-\epsilon)$$

$$+(y+z)(-4+4\epsilon-6\epsilon^2-2\epsilon^3)+\frac{y^2}{z}(-2+4\epsilon-2\epsilon^2)\Bigg). \tag{4.14}$$

Explicit formulae for the bubble and box integrals are given in the appendix. The square of the Born amplitude is given in Eq. (2.8).

4.2 The finite part

The finite remainders of the one- and two-loop contributions to $\mathcal{T}^{(6)}$ can be decomposed according to their colour structure and to their dependence on the number of quark flavours N_F. In the two-loop contribution, one finds moreover a term proportional to the charge-weighted sum of the quark flavours $N_{F,\gamma}$; this equals, in the case of purely electromagnetic interactions:

$$N_{F,\gamma}=\frac{\left(\sum_q e_q\right)^2}{\sum_q e_q^2}\,. \tag{4.15}$$

This term originates from diagrams containing a closed quark loop coupling to the virtual photon and which first appear at the two-loop level.

The tree-level combination of invariants

$$T=\frac{y}{z}+\frac{z}{y}+\frac{2}{yz}-\frac{2}{y}-\frac{2}{z} \tag{4.16}$$

frequently occurs in the finite part. We therefore extracted this combination by expressing $1/(yz)$ by T according to the above equation.

4.2.1 Two-loop contribution to $\mathcal{T}^{(6)}$

The finite remainder of the interference of the two-loop amplitude with the tree-level amplitude is decomposed as

$$\begin{aligned}
\mathcal{F}inite^{(2\times 0)}(x,y,z) \quad = \quad & V\Bigg[N^2\left(A_{20}(y,z)+A_{20}(z,y)\right)+\left(B_{20}(y,z)+B_{20}(z,y)\right)\\
&+\frac{1}{N^2}\left(C_{20}(y,z)+C_{20}(z,y)\right)+NN_F\left(D_{20}(y,z)+D_{20}(z,y)\right)\\
&+\frac{N_F}{N}\left(E_{20}(y,z)+E_{20}(z,y)\right)+N_F^2\left(F_{20}(y,z)+F_{20}(z,y)\right)\\
&+N_{F,\gamma}\left(\frac{4}{N}-N\right)\left(G_{20}(y,z)+G_{20}(z,y)\right)\Bigg], \qquad (4.17)
\end{aligned}$$

with

$$\begin{aligned}
&A_{20}(y,z) =\\
&\quad \frac{z}{12y}\big[2\pi^2+6\pi^2\mathrm{H}(0;z)-12\pi^2\mathrm{G}(1;y)-72\zeta_3+8\mathrm{H}(0;z)-36\mathrm{H}(0;z)\mathrm{G}(1,0;y)-36\mathrm{H}(0,1,0;z)\\
&\quad +39\mathrm{H}(1,0;z)+39\mathrm{G}(1,0;y)+72\mathrm{G}(1,1,0;y)\big]+\frac{1}{2y(y+z)}\big[17\mathrm{H}(1,0;z)+17\mathrm{G}(1,0;y)\big]+\frac{1}{36y}\big[-12\pi^2\\
&\quad -24\pi^2\mathrm{H}(0;z)+48\pi^2\mathrm{G}(1;y)+288\zeta_3+457-84\mathrm{H}(0;z)-36\mathrm{H}(0;z)\mathrm{G}(0;y)+144\mathrm{H}(0;z)\mathrm{G}(1,0;y)\\
&\quad +144\mathrm{H}(0,1,0;z)-306\mathrm{H}(1,0;z)-192\mathrm{G}(0;y)-234\mathrm{G}(1,0;y)-288\mathrm{G}(1,1,0;y)\big]+\frac{z}{36(1-y)^2}\big[-\pi^2\\
&\quad +6\pi^2\mathrm{H}(0;z)+6\pi^2\mathrm{H}(1;z)-6\pi^2\mathrm{G}(1-z;y)+18\pi^2\mathrm{G}(0;y)-12\pi^2\mathrm{G}(1;y)+36\zeta_3-36\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)\\
&\quad +60\mathrm{H}(0;z)\mathrm{G}(0;y)+72\mathrm{H}(0;z)\mathrm{G}(0,0;y)+36\mathrm{H}(0,1,0;z)-36\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+36\mathrm{H}(1,0;z)\mathrm{G}(0;y)\\
&\quad +36\mathrm{H}(1,1,0;z)+36\mathrm{G}(1-z,1,0;y)-355\mathrm{G}(0;y)+270\mathrm{G}(0,0;y)-108\mathrm{G}(0,1,0;y)+6\mathrm{G}(1,0;y)\\
&\quad -72\mathrm{G}(1,0,0;y)+72\mathrm{G}(1,1,0;y)\big]+\frac{z}{36(1-y)}\big[-33\pi^2+18\pi^2\mathrm{H}(0;z)+18\pi^2\mathrm{H}(1;z)-18\pi^2\mathrm{G}(1-z;y)\\
&\quad +54\pi^2\mathrm{G}(0;y)-36\pi^2\mathrm{G}(1;y)+108\zeta_3-277+60\mathrm{H}(0;z)-108\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+216\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&\quad +216\mathrm{H}(0;z)\mathrm{G}(0,0;y)+108\mathrm{H}(0,1,0;z)+36\mathrm{H}(1,0;z)-108\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+108\mathrm{H}(1,0;z)\mathrm{G}(0;y)\\
&\quad +108\mathrm{H}(1,1,0;z)+108\mathrm{G}(1-z,1,0;y)-615\mathrm{G}(0;y)+594\mathrm{G}(0,0;y)-324\mathrm{G}(0,1,0;y)+198\mathrm{G}(1,0;y)\\
&\quad -216\mathrm{G}(1,0,0;y)+216\mathrm{G}(1,1,0;y)\big]+\frac{z}{(y+z)^3}\big[\frac{11\pi^2}{2}\mathrm{H}(1;z)-\frac{11\pi^2}{2}\mathrm{G}(1-z;y)-33\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)\\
&\quad -33\mathrm{H}(0,1,0;z)-33\mathrm{H}(1,0;z)-33\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+33\mathrm{H}(1,0;z)\mathrm{G}(0;y)+33\mathrm{H}(1,1,0;z)\\
&\quad +33\mathrm{G}(1-z,1,0;y)+33\mathrm{G}(0,1,0;y)-33\mathrm{G}(1,0;y)\big]+\frac{z}{2(y+z)^2}\big[-11\pi^2-\frac{22\pi^2}{3}\mathrm{H}(1;z)\\
&\quad +\frac{22\pi^2}{3}\mathrm{G}(1-z;y)+33\mathrm{H}(0;z)+44\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-66\mathrm{H}(0;z)\mathrm{G}(0;y)+44\mathrm{H}(0,1,0;z)\\
&\quad -22\mathrm{H}(1,0;z)+44\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-44\mathrm{H}(1,0;z)\mathrm{G}(0;y)-44\mathrm{H}(1,1,0;z)-44\mathrm{G}(1-z,1,0;y)\\
&\quad -33\mathrm{G}(0;y)-44\mathrm{G}(0,1,0;y)+110\mathrm{G}(1,0;y)\big]+\frac{z}{2(y+z)}\big[\frac{11\pi^2}{6}-11\mathrm{H}(0;z)+11\mathrm{H}(0;z)\mathrm{G}(0;y)
\end{aligned}$$

$$+11\mathrm{H}(1,0;z)+11\mathrm{G}(0;y)-11\mathrm{G}(1,0;y)\Big]+\frac{z^2}{(y+z)^4}\Big[-\frac{11\pi^2}{2}\mathrm{H}(1;z)+\frac{11\pi^2}{2}\mathrm{G}(1-z;y)$$
$$+33\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+33\mathrm{H}(0,1,0;z)+33\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-33\mathrm{H}(1,0;z)\mathrm{G}(0;y)-33\mathrm{H}(1,1,0;z)$$
$$-33\mathrm{G}(1-z,1,0;y)-33\mathrm{G}(0,1,0;y)\Big]+\frac{z^2}{(y+z)^3}\Big[\frac{11\pi^2}{2}+\frac{11\pi^2}{3}\mathrm{H}(1;z)-\frac{11\pi^2}{3}\mathrm{G}(1-z;y)$$
$$-22\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+33\mathrm{H}(0;z)\mathrm{G}(0;y)-22\mathrm{H}(0,1,0;z)+33\mathrm{H}(1,0;z)-22\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$+22\mathrm{H}(1,0;z)\mathrm{G}(0;y)+22\mathrm{H}(1,1,0;z)+22\mathrm{G}(1-z,1,0;y)+22\mathrm{G}(0,1,0;y)-33\mathrm{G}(1,0;y)\Big]$$
$$+\frac{z^2}{2(y+z)^2}\Big[-\frac{11\pi^2}{6}-11\mathrm{H}(0;z)\mathrm{G}(0;y)-11\mathrm{H}(1,0;z)+11\mathrm{G}(1,0;y)\Big]+\frac{1}{18(1-y)}\Big[+23\pi^2-12\pi^2\mathrm{H}(0;z)$$
$$-12\pi^2\mathrm{H}(1;z)+12\pi^2\mathrm{G}(1-z;y)-36\pi^2\mathrm{G}(0;y)+24\pi^2\mathrm{G}(1;y)-72\zeta_3+72\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$-120\mathrm{H}(0;z)\mathrm{G}(0;y)-144\mathrm{H}(0;z)\mathrm{G}(0,0;y)-72\mathrm{H}(0,1,0;z)-18\mathrm{H}(1,0;z)+72\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$-72\mathrm{H}(1,0;z)\mathrm{G}(0;y)-72\mathrm{H}(1,1,0;z)-72\mathrm{G}(1-z,1,0;y)+515\mathrm{G}(0;y)-432\mathrm{G}(0,0;y)+216\mathrm{G}(0,1,0;y)$$
$$-138\mathrm{G}(1,0;y)+144\mathrm{G}(1,0,0;y)-144\mathrm{G}(1,1,0;y)\Big]+\frac{1}{(y+z)^2}\Big[-\frac{7\pi^2}{3}\mathrm{H}(1;z)+\frac{7\pi^2}{3}\mathrm{G}(1-z;y)$$
$$+14\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+14\mathrm{H}(0,1,0;z)+14\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-14\mathrm{H}(1,0;z)\mathrm{G}(0;y)-14\mathrm{H}(1,1,0;z)$$
$$-14\mathrm{G}(1-z,1,0;y)-14\mathrm{G}(0,1,0;y)\Big]+\frac{1}{4(y+z)}\Big[\frac{28\pi^2}{3}+\frac{14\pi^2}{3}\mathrm{H}(1;z)-\frac{14\pi^2}{3}\mathrm{G}(1-z;y)-22$$
$$+11\mathrm{H}(0;z)-28\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+56\mathrm{H}(0;z)\mathrm{G}(0;y)-28\mathrm{H}(0,1,0;z)+56\mathrm{H}(1,0;z)$$
$$-28\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+28\mathrm{H}(1,0;z)\mathrm{G}(0;y)+28\mathrm{H}(1,1,0;z)+28\mathrm{G}(1-z,1,0;y)+11\mathrm{G}(0;y)$$
$$+28\mathrm{G}(0,1,0;y)-56\mathrm{G}(1,0;y)\Big]+\frac{T\pi^2}{216}\Big[-1045+147\mathrm{H}(0;z)+36\mathrm{H}(0;z)\mathrm{G}(1-z;y)+108\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$-36\mathrm{H}(0;z)\mathrm{G}(1;y)+72\mathrm{H}(0,1;z)+54\mathrm{H}(1;z)+72\mathrm{H}(1;z)\mathrm{G}(1-z;y)-72\mathrm{H}(1;z)\mathrm{G}(-z;y)$$
$$-36\mathrm{H}(1;z)\mathrm{G}(1;y)+72\mathrm{H}(1,0;z)+36\mathrm{H}(1,1;z)-186\mathrm{G}(1-z;y)+36\mathrm{G}(1-z,0;y)-72\mathrm{G}(1-z,1;y)$$
$$+72\mathrm{G}(-z,1-z;y)-72\mathrm{G}(0,1-z;y)+147\mathrm{G}(0;y)-72\mathrm{G}(0,1;y)+36\mathrm{G}(1,1-z;y)+132\mathrm{G}(1;y)$$
$$-108\mathrm{G}(1,0;y)+72\mathrm{G}(1,1;y)\Big]+\frac{T}{216}\Big[-\frac{99931}{12}+\frac{132\pi^4}{5}+4776\zeta_3-216\zeta_3\mathrm{H}(0;z)+1080\zeta_3\mathrm{H}(1;z)$$
$$-864\zeta_3\mathrm{G}(1-z;y)-216\zeta_3\mathrm{G}(0;y)-216\zeta_3\mathrm{G}(1;y)+304\mathrm{H}(0;z)-1116\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$-216\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)+432\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)-432\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)$$
$$-144\mathrm{H}(0;z)\mathrm{G}(0;y)+1512\mathrm{H}(0;z)\mathrm{G}(0,0;y)-216\mathrm{H}(0;z)\mathrm{G}(0,1,0;y)+216\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)$$
$$-36\mathrm{H}(0;z)\mathrm{G}(1,0;y)-432\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)+1920\mathrm{H}(0,0;z)+1512\mathrm{H}(0,0;z)\mathrm{G}(0;y)$$
$$+432\mathrm{H}(0,0;z)\mathrm{G}(0,0;y)+864\mathrm{H}(0,0,1,0;z)+1008\mathrm{H}(0,1,0;z)-216\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)$$
$$+432\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)+216\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)-216\mathrm{H}(0,1,0;z)\mathrm{G}(1;y)+432\mathrm{H}(0,1,1,0;z)$$
$$-1095\mathrm{H}(1,0;z)-1116\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+216\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)+432\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)$$
$$-432\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)-432\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)+1152\mathrm{H}(1,0;z)\mathrm{G}(0;y)$$
$$+216\mathrm{H}(1,0;z)\mathrm{G}(1,1-z;y)-216\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)+1512\mathrm{H}(1,0,0;z)+432\mathrm{H}(1,0,0;z)\mathrm{G}(0;y)$$
$$+864\mathrm{H}(1,0,1,0;z)+324\mathrm{H}(1,1,0;z)+432\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)-432\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)$$
$$-216\mathrm{H}(1,1,0;z)\mathrm{G}(1;y)+432\mathrm{H}(1,1,0,0;z)+216\mathrm{H}(1,1,1,0;z)+216\mathrm{G}(1-z,0,1,0;y)$$

$$
\begin{aligned}
&+1116\mathrm{G}(1-z,1,0;y)+432\mathrm{G}(1-z,1,1,0;y)-432\mathrm{G}(-z,1-z,1,0;y)-432\mathrm{G}(-z,0,1,0;y)\\
&+432\mathrm{G}(0,1-z,1,0;y)+304\mathrm{G}(0;y)+1920\mathrm{G}(0,0;y)-432\mathrm{G}(0,0,1,0;y)-1008\mathrm{G}(0,1,0;y)\\
&+432\mathrm{G}(0,1,1,0;y)-216\mathrm{G}(1,1-z,1,0;y)+1095\mathrm{G}(1,0;y)-1512\mathrm{G}(1,0,0;y)+648\mathrm{G}(1,0,1,0;y)\\
&-792\mathrm{G}(1,1,0;y)+432\mathrm{G}(1,1,0,0;y)-432\mathrm{G}(1,1,1,0;y)\Big]+\frac{1}{2}\Big[-\frac{7\pi^2}{3}+1-14\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&-14\mathrm{H}(1,0;z)+14\mathrm{G}(1,0;y)\Big]\,, \qquad (4.18)
\end{aligned}
$$

$$
\begin{aligned}
&B_{20}(y,z)=\\
&\frac{z}{y^2}\Big[-3\mathrm{H}(0;z)\mathrm{G}(1-z;y)-3\mathrm{H}(1;z)\mathrm{G}(-z;y)+3\mathrm{G}(-z,1-z;y)\Big]+\frac{z^2}{y^2}\Big[\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+\mathrm{H}(1;z)\mathrm{G}(-z;y)-\mathrm{G}(-z,1-z;y)\Big]+\frac{1}{y^2}\Big[2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1;z)\mathrm{G}(-z;y)-2\mathrm{G}(-z,1-z;y)\Big]\\
&+\frac{z\pi^2}{18y}\Big[3\mathrm{H}(0;z)-24\mathrm{H}(0;z)\mathrm{G}(1-z;y)+21\mathrm{H}(1;z)-12\mathrm{H}(1;z)\mathrm{G}(1-z;y)+12\mathrm{G}(1-z,1-z;y)\\
&+\mathrm{G}(1-z;y)+12\mathrm{G}(1-z,1;y)+6\mathrm{G}(0,1-z;y)+36\mathrm{G}(1;y)\Big]+\frac{z}{9y}\Big[-27\zeta_3-90\zeta_3\mathrm{G}(1-z;y)\\
&-36\mathrm{H}(0;z)-84\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+18\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)\\
&+18\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)+152\mathrm{H}(0;z)\mathrm{G}(1-z;y)-57\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)\\
&+36\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)+78\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-18\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)\\
&-18\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)-3\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)\\
&+18\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)+54\mathrm{H}(0;z)\mathrm{G}(1,0;y)-18\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)\\
&-36\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-18\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)-18\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)\\
&-9\mathrm{H}(0,0,1;z)+54\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)-72\mathrm{H}(0,0,1;z)\mathrm{G}(-z;y)-9\mathrm{H}(0,1;z)\\
&+54\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)+9\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-18\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)\\
&-108\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+6\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+18\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)+18\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)\\
&-36\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)+9\mathrm{H}(0,1,0;z)-36\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)+18\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)\\
&+72\mathrm{H}(1;z)\mathrm{G}(1-z,-z,-z;y)-75\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-18\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)\\
&-18\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)+36\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)-84\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&+36\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-108\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)+84\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)\\
&+36\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)+143\mathrm{H}(1;z)\mathrm{G}(-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-18\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)\\
&-18\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+36\mathrm{H}(1;z)\mathrm{G}(0,-z,0;y)+9\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&+27\mathrm{H}(1;z)\mathrm{G}(1,0;y)+36\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)-18\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)\\
&+57\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-18\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)-18\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)\\
&-78\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+36\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)-18\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)-18\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)\\
&-36\mathrm{H}(1,0,1;z)-18\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)-36\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+84\mathrm{H}(1,1;z)\mathrm{G}(-z;y)\\
&+18\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)-18\mathrm{H}(1,1,0;z)-54\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+18\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)\\
&-18\mathrm{G}(1-z,1-z,1,0;y)+75\mathrm{G}(1-z,-z,1-z;y)+18\mathrm{G}(1-z,-z,1-z,0;y)\\
&-72\mathrm{G}(1-z,-z,-z,1-z;y)+18\mathrm{G}(1-z,-z,0,1-z;y)+18\mathrm{G}(1-z,0,-z,1-z;y)-9\mathrm{G}(1-z,0;y)
\end{aligned}
$$

$$
\begin{aligned}
&+18\mathrm{G}(1-z,0,1,0;y)+54\mathrm{G}(1-z,1,0;y)-36\mathrm{G}(1-z,1,1,0;y)+84\mathrm{G}(-z,1-z,1-z;y)\\
&-36\mathrm{G}(-z,1-z,-z,1-z;y)-143\mathrm{G}(-z,1-z;y)-24\mathrm{G}(-z,1-z,0;y)+18\mathrm{G}(-z,1-z,1,0;y)\\
&-36\mathrm{G}(-z,-z,1-z,1-z;y)-84\mathrm{G}(-z,-z,1-z;y)-36\mathrm{G}(-z,-z,1-z,0;y)\\
&+108\mathrm{G}(-z,-z,-z,1-z;y)-36\mathrm{G}(-z,-z,0,1-z;y)-24\mathrm{G}(-z,0,1-z;y)+18\mathrm{G}(-z,0,1,0;y)\\
&-9\mathrm{G}(0,1-z;y)+18\mathrm{G}(0,-z,1-z,1-z;y)+3\mathrm{G}(0,-z,1-z;y)-36\mathrm{G}(0,-z,1-z,0;y)\\
&+18\mathrm{G}(0,-z,-z,1-z;y)-36\mathrm{G}(0,-z,0,1-z;y)+27\mathrm{G}(0,1,0;y)-27\mathrm{G}(1,1-z,0;y)\\
&-27\mathrm{G}(1,0,1-z;y)+9\mathrm{G}(1,0;y)-108\mathrm{G}(1,1,0;y)\big]+\frac{z^2}{y}\big[2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&-2\mathrm{G}(-z,1-z;y)\big]+\frac{1}{y(y+z)}\big[2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&-2\mathrm{H}(0,1;z)-6\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-6\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+2\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&+6\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-6\mathrm{H}(1,1,0;z)-2\mathrm{G}(1-z,0;y)+6\mathrm{G}(1-z,1,0;y)\\
&-6\mathrm{G}(-z,1-z,0;y)-6\mathrm{G}(-z,0,1-z;y)-2\mathrm{G}(0,1-z;y)+6\mathrm{G}(0,-z,1-z;y)+6\mathrm{G}(0,1,0;y)\\
&+2\mathrm{G}(1,0;y)\big]+\frac{\pi^2}{9y}\big[-3+3\mathrm{H}(0;z)+24\mathrm{H}(0;z)\mathrm{G}(1-z;y)-15\mathrm{H}(1;z)+12\mathrm{H}(1;z)\mathrm{G}(1-z;y)\\
&-12\mathrm{G}(1-z,1-z;y)-7\mathrm{G}(1-z;y)-12\mathrm{G}(1-z,1;y)-6\mathrm{G}(0,1-z;y)-24\mathrm{G}(1;y)\big]+\frac{1}{9y}\big[\frac{45}{2}\mathrm{H}(1;z)\\
&-\frac{45}{2}\mathrm{G}(1-z;y)-54\zeta_3+180\zeta_3\mathrm{G}(1-z;y)+139+57\mathrm{H}(0;z)+132\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)\\
&-36\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)-36\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)-250\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+96\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-72\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)-210\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)\\
&+36\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)+96\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&-72\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)-36\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)-9\mathrm{H}(0;z)\mathrm{G}(0;y)-72\mathrm{H}(0;z)\mathrm{G}(1,0;y)\\
&+36\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)+144\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)+36\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)\\
&+36\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)-18\mathrm{H}(0,0,1;z)-108\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)+144\mathrm{H}(0,0,1;z)\mathrm{G}(-z;y)\\
&+36\mathrm{H}(0,1;z)-108\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)+36\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)+216\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)\\
&+42\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-36\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)-36\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)+72\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)\\
&+18\mathrm{H}(0,1,0;z)+72\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-36\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)-144\mathrm{H}(1;z)\mathrm{G}(1-z,-z,-z;y)\\
&+132\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+36\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)+36\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)\\
&-72\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+132\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-72\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)\\
&+216\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)-168\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-72\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)\\
&-214\mathrm{H}(1;z)\mathrm{G}(-z;y)-30\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+36\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)+36\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)\\
&+96\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-72\mathrm{H}(1;z)\mathrm{G}(0,-z,0;y)-27\mathrm{H}(1;z)\mathrm{G}(0;y)-18\mathrm{H}(1;z)\mathrm{G}(1,0;y)-18\mathrm{H}(1,0;z)\\
&-72\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)+36\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)-168\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)\\
&+36\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)+36\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)+210\mathrm{H}(1,0;z)\mathrm{G}(-z;y)\\
&-72\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)+36\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)+36\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)+36\mathrm{H}(1,0,1;z)\\
&+36\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)+72\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-132\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-36\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)
\end{aligned}
$$

$$+72\mathrm{H}(1,1,0;z)+108\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)-36\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)+36\mathrm{G}(1-z,1-z,1,0;y)$$
$$-132\mathrm{G}(1-z,-z,1-z;y)-36\mathrm{G}(1-z,-z,1-z,0;y)+144\mathrm{G}(1-z,-z,-z,1-z;y)$$
$$-36\mathrm{G}(1-z,-z,0,1-z;y)-36\mathrm{G}(1-z,0,-z,1-z;y)+27\mathrm{G}(1-z,0;y)-36\mathrm{G}(1-z,0,1,0;y)$$
$$-108\mathrm{G}(1-z,1,0;y)+72\mathrm{G}(1-z,1,1,0;y)-132\mathrm{G}(-z,1-z,1-z;y)+72\mathrm{G}(-z,1-z,-z,1-z;y)$$
$$+214\mathrm{G}(-z,1-z;y)+30\mathrm{G}(-z,1-z,0;y)-36\mathrm{G}(-z,1-z,1,0;y)+72\mathrm{G}(-z,-z,1-z,1-z;y)$$
$$+168\mathrm{G}(-z,-z,1-z;y)+72\mathrm{G}(-z,-z,1-z,0;y)-216\mathrm{G}(-z,-z,-z,1-z;y)$$
$$+72\mathrm{G}(-z,-z,0,1-z;y)+30\mathrm{G}(-z,0,1-z;y)-36\mathrm{G}(-z,0,1,0;y)+27\mathrm{G}(0,1-z;y)$$
$$-36\mathrm{G}(0,-z,1-z,1-z;y)-96\mathrm{G}(0,-z,1-z;y)+72\mathrm{G}(0,-z,1-z,0;y)-36\mathrm{G}(0,-z,-z,1-z;y)$$
$$+72\mathrm{G}(0,-z,0,1-z;y)-48\mathrm{G}(0;y)-54\mathrm{G}(0,1,0;y)+18\mathrm{G}(1,1-z,0;y)+18\mathrm{G}(1,0,1-z;y)$$
$$-18\mathrm{G}(1,0;y)+144\mathrm{G}(1,1,0;y)\Big]+\frac{z}{9(1-y)^2}\Big[-11\pi^2-\frac{27\pi^2}{2}\mathrm{H}(0;z)-\frac{27\pi^2}{2}\mathrm{H}(1;z)+\frac{27\pi^2}{2}\mathrm{G}(1-z;y)$$
$$-6\pi^2\mathrm{G}(0;y)+6\pi^2\mathrm{G}(1;y)+225\zeta_3+\frac{81}{2}\mathrm{H}(1;z)\mathrm{G}(0;y)-\frac{81}{2}\mathrm{G}(1-z,0;y)-\frac{81}{2}\mathrm{G}(0,1-z;y)$$
$$+9\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-72\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(0;y)-9\mathrm{H}(0;z)\mathrm{G}(0,0;y)+81\mathrm{H}(0,0,1;z)$$
$$+36\mathrm{H}(0,1;z)-81\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-9\mathrm{H}(0,1;z)\mathrm{G}(0;y)-81\mathrm{H}(0,1,0;z)-81\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)$$
$$+36\mathrm{H}(1;z)\mathrm{G}(-z;y)-81\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+27\mathrm{H}(1;z)\mathrm{G}(0,0;y)-36\mathrm{H}(1,0;z)+81\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$-9\mathrm{H}(1,0;z)\mathrm{G}(0;y)+81\mathrm{H}(1,0,1;z)-81\mathrm{H}(1,1,0;z)+81\mathrm{G}(1-z,-z,1-z;y)-27\mathrm{G}(1-z,0,0;y)$$
$$-36\mathrm{G}(-z,1-z;y)-27\mathrm{G}(0,1-z,0;y)+81\mathrm{G}(0,-z,1-z;y)+37\mathrm{G}(0;y)-27\mathrm{G}(0,0,1-z;y)$$
$$-45\mathrm{G}(0,0;y)+45\mathrm{G}(0,1,0;y)+30\mathrm{G}(1,0;y)+36\mathrm{G}(1,0,0;y)-36\mathrm{G}(1,1,0;y)\Big]$$
$$+\frac{z}{18(1-y)}\Big[-9\pi^2\mathrm{H}(0;z)-9\pi^2\mathrm{H}(1;z)+9\pi^2\mathrm{G}(1-z;y)-36\pi^2\mathrm{G}(0;y)+36\pi^2\mathrm{G}(1;y)+54\zeta_3+62$$
$$+60\mathrm{H}(0;z)-144\mathrm{H}(0;z)\mathrm{G}(1-z;y)+54\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-108\mathrm{H}(0;z)\mathrm{G}(0;y)-54\mathrm{H}(0;z)\mathrm{G}(0,0;y)$$
$$+54\mathrm{H}(0,0,1;z)-18\mathrm{H}(0,1;z)-54\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-54\mathrm{H}(0,1;z)\mathrm{G}(0;y)-54\mathrm{H}(0,1,0;z)$$
$$+117\mathrm{H}(1;z)-54\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-162\mathrm{H}(1;z)\mathrm{G}(-z;y)-54\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+99\mathrm{H}(1;z)\mathrm{G}(0;y)$$
$$+162\mathrm{H}(1;z)\mathrm{G}(0,0;y)-18\mathrm{H}(1,0;z)+54\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-54\mathrm{H}(1,0;z)\mathrm{G}(0;y)+54\mathrm{H}(1,0,1;z)$$
$$-54\mathrm{H}(1,1,0;z)+54\mathrm{G}(1-z,-z,1-z;y)-117\mathrm{G}(1-z;y)-99\mathrm{G}(1-z,0;y)-162\mathrm{G}(1-z,0,0;y)$$
$$+162\mathrm{G}(-z,1-z;y)-99\mathrm{G}(0,1-z;y)-162\mathrm{G}(0,1-z,0;y)+54\mathrm{G}(0,-z,1-z;y)+132\mathrm{G}(0;y)$$
$$-162\mathrm{G}(0,0,1-z;y)-198\mathrm{G}(0,0;y)+270\mathrm{G}(0,1,0;y)+18\mathrm{G}(1,0;y)+216\mathrm{G}(1,0,0;y)-216\mathrm{G}(1,1,0;y)\Big]$$
$$+\frac{z}{(y+z)^3}\Big[-2\pi^2\mathrm{H}(1;z)+2\pi^2\mathrm{G}(1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(0,1,0;z)+12\mathrm{H}(1,0;z)$$
$$+12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0;y)-12\mathrm{H}(1,1,0;z)-12\mathrm{G}(1-z,1,0;y)-12\mathrm{G}(0,1,0;y)$$
$$+12\mathrm{G}(1,0;y)\Big]+\frac{z}{(y+z)^2}\Big[2\pi^2+\frac{4\pi^2}{3}\mathrm{H}(1;z)-\frac{4\pi^2}{3}\mathrm{G}(1-z;y)-6\mathrm{H}(0;z)-8\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(0;y)-8\mathrm{H}(0,1,0;z)+4\mathrm{H}(1,0;z)-8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+8\mathrm{H}(1,0;z)\mathrm{G}(0;y)$$
$$+8\mathrm{H}(1,1,0;z)+8\mathrm{G}(1-z,1,0;y)+6\mathrm{G}(0;y)+8\mathrm{G}(0,1,0;y)-20\mathrm{G}(1,0;y)\Big]+\frac{z}{y+z}\Big[-\frac{\pi^2}{3}$$

$$
\begin{aligned}
&+2\mathrm{H}(0;z)-2\mathrm{H}(0;z)\mathrm{G}(0;y)-2\mathrm{H}(1,0;z)-2\mathrm{G}(0;y)+2\mathrm{G}(1,0;y)\big]+\frac{z^2}{(1-y)^3}\big[+\frac{2\pi^2}{3}\mathrm{H}(0;z)\\
&+\frac{2\pi^2}{3}\mathrm{H}(1;z)-\frac{2\pi^2}{3}\mathrm{G}(1-z;y)-12\zeta_3+4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-4\mathrm{H}(0,0,1;z)+4\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&+4\mathrm{H}(0,1,0;z)+4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(1,0,1;z)\\
&+4\mathrm{H}(1,1,0;z)-4\mathrm{G}(1-z,-z,1-z;y)-4\mathrm{G}(0,-z,1-z;y)\big]+\frac{z^2}{(1-y)^2}\big[+4\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+4\mathrm{H}(1;z)\mathrm{G}(-z;y)-4\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{1-y}\big[+2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&-2\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{(y+z)^4}\big[2\pi^2\mathrm{H}(1;z)-2\pi^2\mathrm{G}(1-z;y)-12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(0,1,0;z)\\
&-12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+12\mathrm{H}(1,0;z)\mathrm{G}(0;y)+12\mathrm{H}(1,1,0;z)+12\mathrm{G}(1-z,1,0;y)+12\mathrm{G}(0,1,0;y)\big]\\
&+\frac{z^2}{(y+z)^3}\big[-2\pi^2-\frac{4\pi^2}{3}\mathrm{H}(1;z)+\frac{4\pi^2}{3}\mathrm{G}(1-z;y)+8\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&+8\mathrm{H}(0,1,0;z)-12\mathrm{H}(1,0;z)+8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(1,0;z)\mathrm{G}(0;y)-8\mathrm{H}(1,1,0;z)\\
&-8\mathrm{G}(1-z,1,0;y)-8\mathrm{G}(0,1,0;y)+12\mathrm{G}(1,0;y)\big]+\frac{z^2}{(y+z)^2}\big[\frac{\pi^2}{3}+2\mathrm{H}(0;z)\mathrm{G}(0;y)+2\mathrm{H}(1,0;z)\\
&-2\mathrm{G}(1,0;y)\big]+\frac{1}{1-y-z}\big[\frac{\pi^2}{6}+\mathrm{H}(0;z)\mathrm{G}(0;y)+\mathrm{H}(1,0;z)-\mathrm{G}(1,0;y)\big]+\frac{1}{9(1-y)}\big[10\pi^2+12\pi^2\mathrm{H}(0;z)\\
&+9\pi^2\mathrm{H}(1;z)-9\pi^2\mathrm{G}(1-z;y)+12\pi^2\mathrm{G}(0;y)-21\pi^2\mathrm{G}(1;y)-234\zeta_3+54\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&+21\mathrm{H}(0;z)\mathrm{G}(0;y)+18\mathrm{H}(0;z)\mathrm{G}(0,0;y)-18\mathrm{H}(0;z)\mathrm{G}(1,0;y)-90\mathrm{H}(0,0,1;z)-63\mathrm{H}(0,1;z)\\
&+90\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+36\mathrm{H}(0,1;z)\mathrm{G}(0;y)+54\mathrm{H}(0,1,0;z)+90\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)\\
&-63\mathrm{H}(1;z)\mathrm{G}(-z;y)+90\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-18\mathrm{H}(1;z)\mathrm{G}(0;y)-72\mathrm{H}(1;z)\mathrm{G}(0,0;y)+63\mathrm{H}(1,0;z)\\
&-54\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-90\mathrm{H}(1,0,1;z)+54\mathrm{H}(1,1,0;z)-90\mathrm{G}(1-z,-z,1-z;y)+18\mathrm{G}(1-z,0;y)\\
&+72\mathrm{G}(1-z,0,0;y)-36\mathrm{G}(1-z,1,0;y)+63\mathrm{G}(-z,1-z;y)+18\mathrm{G}(0,1-z;y)+72\mathrm{G}(0,1-z,0;y)\\
&-90\mathrm{G}(0,-z,1-z;y)+55\mathrm{G}(0;y)+72\mathrm{G}(0,0,1-z;y)+108\mathrm{G}(0,0;y)-108\mathrm{G}(0,1,0;y)+3\mathrm{G}(1,0;y)\\
&-90\mathrm{G}(1,0,0;y)+126\mathrm{G}(1,1,0;y)\big]+\frac{1}{9(y+z)^2}\big[+18\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+27\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&-18\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-72\mathrm{H}(0,0,1;z)+39\mathrm{H}(0,1;z)+36\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&-90\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+18\mathrm{H}(0,1;z)\mathrm{G}(0;y)+18\mathrm{H}(0,1,0;z)-18\mathrm{H}(0,1,1;z)+230\mathrm{H}(1;z)\\
&+54\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-102\mathrm{H}(1;z)\mathrm{G}(1-z;y)-18\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)+36\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&-108\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+66\mathrm{H}(1;z)\mathrm{G}(-z;y)+18\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-18\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)\\
&+18\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-27\mathrm{H}(1;z)\mathrm{G}(0;y)-27\mathrm{H}(1,0;z)-18\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+18\mathrm{H}(1,0;z)\mathrm{G}(-z;y)\\
&-36\mathrm{H}(1,0,1;z)+102\mathrm{H}(1,1;z)-36\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+18\mathrm{H}(1,1;z)\mathrm{G}(0;y)+18\mathrm{H}(1,1,0;z)\\
&+102\mathrm{G}(1-z,1-z;y)+18\mathrm{G}(1-z,1-z,0;y)-54\mathrm{G}(1-z,-z,1-z;y)-230\mathrm{G}(1-z;y)\\
&+18\mathrm{G}(1-z,0,1-z;y)+27\mathrm{G}(1-z,0;y)-36\mathrm{G}(-z,1-z,1-z;y)-66\mathrm{G}(-z,1-z;y)\\
&-18\mathrm{G}(-z,1-z,0;y)+108\mathrm{G}(-z,-z,1-z;y)-18\mathrm{G}(-z,0,1-z;y)+18\mathrm{G}(0,1-z,1-z;y)
\end{aligned}
$$

$$
\begin{aligned}
&+27\mathrm{G}(0,1-z;y)-18\mathrm{G}(0,-z,1-z;y)\big]+\frac{1}{9(y+z)}\big[-\frac{3\pi^2}{2}\mathrm{H}(1;z)+\frac{3\pi^2}{2}\mathrm{G}(1-z;y)-170\\
&-18\mathrm{H}(0;z)\mathrm{G}(1-z;y)+9\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-72\mathrm{H}(0,1;z)+9\mathrm{H}(0,1,0;z)-123\mathrm{H}(1;z)\\
&-90\mathrm{H}(1;z)\mathrm{G}(-z;y)+18\mathrm{H}(1;z)\mathrm{G}(0;y)+18\mathrm{H}(1,0;z)+9\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-9\mathrm{H}(1,0;z)\mathrm{G}(0;y)\\
&-9\mathrm{H}(1,1,0;z)+123\mathrm{G}(1-z;y)-18\mathrm{G}(1-z,0;y)-9\mathrm{G}(1-z,1,0;y)+90\mathrm{G}(-z,1-z;y)\\
&-18\mathrm{G}(0,1-z;y)-9\mathrm{G}(0,1,0;y)\big]+\frac{T\pi^2}{72}\big[-115-24\mathrm{H}(0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(0;z)\mathrm{G}(1;y)-12\mathrm{H}(0,1;z)\\
&+7\mathrm{H}(1;z)-48\mathrm{H}(1;z)\mathrm{G}(1-z;y)+36\mathrm{H}(1;z)\mathrm{G}(0;y)-12\mathrm{H}(1;z)\mathrm{G}(1;y)-12\mathrm{H}(1,0;z)+12\mathrm{H}(1,1;z)\\
&+48\mathrm{G}(1-z,1-z;y)+19\mathrm{G}(1-z;y)-24\mathrm{G}(1-z,0;y)-36\mathrm{G}(0,1-z;y)+48\mathrm{G}(0,1;y)\\
&+12\mathrm{G}(1,1-z;y)-26\mathrm{G}(1;y)+36\mathrm{G}(1,0;y)-48\mathrm{G}(1,1;y)\big]+\frac{T}{54}\big[-\frac{3\pi^4}{8}+\frac{15251}{12}-\frac{357}{2}\zeta_3\\
&+108\zeta_3\mathrm{H}(1;z)-270\zeta_3\mathrm{G}(1-z;y)+162\zeta_3\mathrm{G}(1;y)-360\mathrm{H}(0;z)-198\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)\\
&+108\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)+108\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)+78\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+54\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)-180\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-108\mathrm{H}(0;z)\mathrm{G}(1-z,0,0;y)\\
&+54\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)+108\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)+297\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)\\
&-108\mathrm{H}(0;z)\mathrm{G}(-z,-z,1-z;y)-108\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)-180\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&+216\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)-216\mathrm{H}(0;z)\mathrm{G}(0;y)+54\mathrm{H}(0;z)\mathrm{G}(0,1,0;y)-54\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)\\
&-108\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+9\mathrm{H}(0;z)\mathrm{G}(1,0;y)+108\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)-378\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)\\
&-108\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)-108\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)+18\mathrm{H}(0,0,1;z)+216\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)\\
&-432\mathrm{H}(0,0,1;z)\mathrm{G}(-z;y)-108\mathrm{H}(0,0,1;z)\mathrm{G}(0;y)+54\mathrm{H}(0,0,1;z)\mathrm{G}(1;y)+216\mathrm{H}(0,0,1,0;z)\\
&+348\mathrm{H}(0,1;z)+324\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)-279\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-162\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)\\
&+108\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)-540\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)-63\mathrm{H}(0,1;z)\mathrm{G}(-z;y)\\
&+108\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)+54\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)-108\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)-117\mathrm{H}(0,1;z)\mathrm{G}(0;y)\\
&+108\mathrm{H}(0,1;z)\mathrm{G}(0,0;y)-54\mathrm{H}(0,1;z)\mathrm{G}(1,1-z;y)+54\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)-252\mathrm{H}(0,1,0;z)\\
&-216\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)+108\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)-54\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)-54\mathrm{H}(0,1,0;z)\mathrm{G}(1;y)\\
&+198\mathrm{H}(0,1,1;z)-108\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)+108\mathrm{H}(0,1,1,0;z)+17\mathrm{H}(1;z)\\
&+432\mathrm{H}(1;z)\mathrm{G}(1-z,-z,-z;y)-477\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-108\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)\\
&+297\mathrm{H}(1;z)\mathrm{G}(1-z;y)-108\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)+198\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)\\
&+162\mathrm{H}(1;z)\mathrm{G}(1-z,1,0;y)+216\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)-396\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&-108\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)+216\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-648\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)\\
&+234\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+108\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)+426\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&-108\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)+108\mathrm{H}(1;z)\mathrm{G}(-z,0,-z;y)-297\mathrm{H}(1;z)\mathrm{G}(-z,0;y)\\
&-54\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)+198\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-108\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)\\
&+108\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)-297\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+216\mathrm{H}(1;z)\mathrm{G}(0,-z,0;y)-78\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&+108\mathrm{H}(1;z)\mathrm{G}(0,0,-z;y)+378\mathrm{H}(1;z)\mathrm{G}(0,0;y)-54\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)-54\mathrm{H}(1;z)\mathrm{G}(1,1-z,-z;y)\\
&-54\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)-81\mathrm{H}(1;z)\mathrm{G}(1,0;y)-108\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)-81\mathrm{H}(1,0;z)
\end{aligned}
$$

$$+216\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)-108\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)+117\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$-162\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)-108\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)+108\mathrm{H}(1,0;z)\mathrm{G}(-z,-z;y)$$
$$-297\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-216\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)+171\mathrm{H}(1,0;z)\mathrm{G}(0;y)$$
$$+108\mathrm{H}(1,0;z)\mathrm{G}(0,0;y)+54\mathrm{H}(1,0;z)\mathrm{G}(1,1-z;y)+54\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)-108\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)$$
$$-54\mathrm{H}(1,0,0,1;z)+360\mathrm{H}(1,0,1;z)-162\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)-108\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)$$
$$+54\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)+54\mathrm{H}(1,0,1;z)\mathrm{G}(1;y)+108\mathrm{H}(1,0,1,0;z)-297\mathrm{H}(1,1;z)$$
$$-216\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+396\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+108\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)+108\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)$$
$$-198\mathrm{H}(1,1;z)\mathrm{G}(0;y)+81\mathrm{H}(1,1,0;z)-378\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+108\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)$$
$$+108\mathrm{H}(1,1,0;z)\mathrm{G}(0;y)-54\mathrm{H}(1,1,0;z)\mathrm{G}(1;y)+108\mathrm{H}(1,1,0,1;z)+162\mathrm{H}(1,1,1,0;z)$$
$$-297\mathrm{G}(1-z,1-z;y)-198\mathrm{G}(1-z,1-z,0;y)-216\mathrm{G}(1-z,1-z,1,0;y)+477\mathrm{G}(1-z,-z,1-z;y)$$
$$+108\mathrm{G}(1-z,-z,1-z,0;y)-432\mathrm{G}(1-z,-z,-z,1-z;y)+108\mathrm{G}(1-z,-z,0,1-z;y)$$
$$-17\mathrm{G}(1-z;y)-198\mathrm{G}(1-z,0,1-z;y)+108\mathrm{G}(1-z,0,-z,1-z;y)+78\mathrm{G}(1-z,0;y)$$
$$-378\mathrm{G}(1-z,0,0;y)+108\mathrm{G}(1-z,0,1,0;y)-162\mathrm{G}(1-z,1,1-z,0;y)-162\mathrm{G}(1-z,1,0,1-z;y)$$
$$+81\mathrm{G}(1-z,1,0;y)+108\mathrm{G}(1-z,1,0,0;y)+396\mathrm{G}(-z,1-z,1-z;y)+108\mathrm{G}(-z,1-z,1-z,0;y)$$
$$-216\mathrm{G}(-z,1-z,-z,1-z;y)-426\mathrm{G}(-z,1-z;y)+108\mathrm{G}(-z,1-z,0,1-z;y)+297\mathrm{G}(-z,1-z,0;y)$$
$$-216\mathrm{G}(-z,-z,1-z,1-z;y)-234\mathrm{G}(-z,-z,1-z;y)-108\mathrm{G}(-z,-z,1-z,0;y)$$
$$+648\mathrm{G}(-z,-z,-z,1-z;y)-108\mathrm{G}(-z,-z,0,1-z;y)+108\mathrm{G}(-z,0,1-z,1-z;y)$$
$$+297\mathrm{G}(-z,0,1-z;y)-108\mathrm{G}(-z,0,-z,1-z;y)-198\mathrm{G}(0,1-z,1-z;y)+54\mathrm{G}(0,1-z,-z,1-z;y)$$
$$+78\mathrm{G}(0,1-z;y)-378\mathrm{G}(0,1-z,0;y)+162\mathrm{G}(0,1-z,1,0;y)+108\mathrm{G}(0,-z,1-z,1-z;y)$$
$$+297\mathrm{G}(0,-z,1-z;y)-216\mathrm{G}(0,-z,1-z,0;y)-108\mathrm{G}(0,-z,-z,1-z;y)-216\mathrm{G}(0,-z,0,1-z;y)$$
$$-360\mathrm{G}(0;y)-378\mathrm{G}(0,0,1-z;y)-108\mathrm{G}(0,0,-z,1-z;y)+108\mathrm{G}(0,0,1,0;y)+54\mathrm{G}(0,1,1-z,0;y)$$
$$+54\mathrm{G}(0,1,0,1-z;y)+333\mathrm{G}(0,1,0;y)-216\mathrm{G}(0,1,1,0;y)+54\mathrm{G}(1,1-z,-z,1-z;y)$$
$$+81\mathrm{G}(1,1-z,0;y)+108\mathrm{G}(1,1-z,0,0;y)+81\mathrm{G}(1,0,1-z;y)+108\mathrm{G}(1,0,1-z,0;y)$$
$$+54\mathrm{G}(1,0,-z,1-z;y)+3\mathrm{G}(1,0;y)+108\mathrm{G}(1,0,0,1-z;y)+378\mathrm{G}(1,0,0;y)-216\mathrm{G}(1,0,1,0;y)$$
$$+117\mathrm{G}(1,1,0;y)-216\mathrm{G}(1,1,0,0;y)+216\mathrm{G}(1,1,1,0;y)\big]+\frac{\pi^2}{18}\big[11+9\mathrm{H}(0;z)-24\mathrm{H}(0;z)\mathrm{G}(1-z;y)$$
$$-6\mathrm{H}(0,1;z)+8\mathrm{H}(1;z)-36\mathrm{H}(1;z)\mathrm{G}(1-z;y)+24\mathrm{H}(1;z)\mathrm{G}(0;y)+6\mathrm{H}(1;z)\mathrm{G}(1;y)+24\mathrm{H}(1,1;z)$$
$$+24\mathrm{G}(1-z,1-z;y)+2\mathrm{G}(1-z;y)-24\mathrm{G}(1-z,0;y)+12\mathrm{G}(1-z,1;y)-6\mathrm{G}(0,1-z;y)+9\mathrm{G}(0;y)$$
$$+12\mathrm{G}(0,1;y)-6\mathrm{G}(1,1-z;y)-10\mathrm{G}(1;y)+24\mathrm{G}(1,0;y)-6\mathrm{G}(1,1;y)\big]+\frac{1}{18}\big[288\zeta_3+180\zeta_3\mathrm{H}(1;z)$$
$$-360\zeta_3\mathrm{G}(1-z;y)+180\zeta_3\mathrm{G}(1;y)-188\mathrm{H}(0;z)-150\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)$$
$$+72\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)+72\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)+295\mathrm{H}(0;z)\mathrm{G}(1-z;y)$$
$$-300\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-36\mathrm{H}(0;z)\mathrm{G}(1-z,0,0;y)+72\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)$$
$$+216\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-72\mathrm{H}(0;z)\mathrm{G}(-z,-z,1-z;y)-36\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)$$
$$-156\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)+144\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)+78\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$-72\mathrm{H}(0;z)\mathrm{G}(0,0,1-z;y)+18\mathrm{H}(0;z)\mathrm{G}(0,0;y)+36\mathrm{H}(0;z)\mathrm{G}(0,1,0;y)-72\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)$$

$$
\begin{aligned}
&-36\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+132\mathrm{H}(0;z)\mathrm{G}(1,0;y)+36\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)+36\mathrm{H}(0;z)\mathrm{G}(1,1,0;y)\\
&+36\mathrm{H}(0,0;z)-36\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)-108\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-36\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)\\
&-36\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)+18\mathrm{H}(0,0;z)\mathrm{G}(0;y)-108\mathrm{H}(0,0,0,1;z)+84\mathrm{H}(0,0,1;z)\\
&+180\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)-288\mathrm{H}(0,0,1;z)\mathrm{G}(-z;y)+36\mathrm{H}(0,0,1;z)\mathrm{G}(1;y)+108\mathrm{H}(0,0,1,0;z)\\
&-36\mathrm{H}(0,0,1,1;z)+289\mathrm{H}(0,1;z)+216\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)-222\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&-72\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)+72\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)-360\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)\\
&+192\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+72\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)-72\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)-60\mathrm{H}(0,1;z)\mathrm{G}(0;y)\\
&+36\mathrm{H}(0,1;z)\mathrm{G}(0,0;y)-36\mathrm{H}(0,1;z)\mathrm{G}(1,1-z;y)-114\mathrm{H}(0,1,0;z)-144\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)\\
&+72\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)-72\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)+36\mathrm{H}(0,1,0;z)\mathrm{G}(1;y)-72\mathrm{H}(0,1,0,1;z)\\
&+150\mathrm{H}(0,1,1;z)-72\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)+36\mathrm{H}(0,1,1;z)\mathrm{G}(0;y)+36\mathrm{H}(0,1,1,0;z)-376\mathrm{H}(1;z)\\
&+288\mathrm{H}(1;z)\mathrm{G}(1-z,-z,-z;y)-372\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-72\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)\\
&+204\mathrm{H}(1;z)\mathrm{G}(1-z;y)-72\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)+150\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)\\
&+36\mathrm{H}(1;z)\mathrm{G}(1-z,0,0;y)+36\mathrm{H}(1;z)\mathrm{G}(1-z,1,0;y)+144\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)\\
&-300\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-72\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)+144\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)\\
&-432\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)+408\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+72\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)\\
&+584\mathrm{H}(1;z)\mathrm{G}(-z;y)-72\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)+72\mathrm{H}(1;z)\mathrm{G}(-z,0,-z;y)-216\mathrm{H}(1;z)\mathrm{G}(-z,0;y)\\
&-36\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)+150\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+36\mathrm{H}(1;z)\mathrm{G}(0,1-z,0;y)\\
&-72\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)+72\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)-216\mathrm{H}(1;z)\mathrm{G}(0,-z;y)\\
&+144\mathrm{H}(1;z)\mathrm{G}(0,-z,0;y)-295\mathrm{H}(1;z)\mathrm{G}(0;y)+36\mathrm{H}(1;z)\mathrm{G}(0,0,1-z;y)-36\mathrm{H}(1;z)\mathrm{G}(0,0,-z;y)\\
&+108\mathrm{H}(1;z)\mathrm{G}(0,0;y)-36\mathrm{H}(1;z)\mathrm{G}(1,1-z,-z;y)-36\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)-72\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)\\
&+15\mathrm{H}(1,0;z)+108\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)-72\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)+6\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)\\
&-72\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)-72\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)+72\mathrm{H}(1,0;z)\mathrm{G}(-z,-z;y)\\
&-216\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+108\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)-144\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)+168\mathrm{H}(1,0;z)\mathrm{G}(0;y)\\
&+36\mathrm{H}(1,0;z)\mathrm{G}(0,0;y)-36\mathrm{H}(1,0;z)\mathrm{G}(1,1-z;y)+72\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)+18\mathrm{H}(1,0,0;z)\\
&-36\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)-72\mathrm{H}(1,0,0,1;z)+222\mathrm{H}(1,0,1;z)-36\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)\\
&-72\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)+36\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)+36\mathrm{H}(1,0,1;z)\mathrm{G}(1;y)+72\mathrm{H}(1,0,1,0;z)\\
&-204\mathrm{H}(1,1;z)-144\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+300\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+72\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)\\
&+72\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)-150\mathrm{H}(1,1;z)\mathrm{G}(0;y)-36\mathrm{H}(1,1;z)\mathrm{G}(0,0;y)+54\mathrm{H}(1,1,0;z)\\
&-216\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+72\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)+36\mathrm{H}(1,1,0;z)\mathrm{G}(0;y)+36\mathrm{H}(1,1,0;z)\mathrm{G}(1;y)\\
&+108\mathrm{H}(1,1,1,0;z)-204\mathrm{G}(1-z,1-z;y)-150\mathrm{G}(1-z,1-z,0;y)-36\mathrm{G}(1-z,1-z,0,0;y)\\
&-108\mathrm{G}(1-z,1-z,1,0;y)+372\mathrm{G}(1-z,-z,1-z;y)+72\mathrm{G}(1-z,-z,1-z,0;y)\\
&-288\mathrm{G}(1-z,-z,-z,1-z;y)+72\mathrm{G}(1-z,-z,0,1-z;y)+376\mathrm{G}(1-z;y)-150\mathrm{G}(1-z,0,1-z;y)\\
&-36\mathrm{G}(1-z,0,1-z,0;y)+72\mathrm{G}(1-z,0,-z,1-z;y)+295\mathrm{G}(1-z,0;y)-36\mathrm{G}(1-z,0,0,1-z;y)\\
&-108\mathrm{G}(1-z,0,0;y)+108\mathrm{G}(1-z,0,1,0;y)-36\mathrm{G}(1-z,1,1-z,0;y)-36\mathrm{G}(1-z,1,0,1-z;y)\\
&+144\mathrm{G}(1-z,1,0;y)+72\mathrm{G}(1-z,1,0,0;y)-72\mathrm{G}(1-z,1,1,0;y)+300\mathrm{G}(-z,1-z,1-z;y)
\end{aligned}
$$

$$
\begin{aligned}
&+72\mathrm{G}(-z,1-z,1-z,0;y)-144\mathrm{G}(-z,1-z,-z,1-z;y)-584\mathrm{G}(-z,1-z;y)\\
&+72\mathrm{G}(-z,1-z,0,1-z;y)+216\mathrm{G}(-z,1-z,0;y)-144\mathrm{G}(-z,-z,1-z,1-z;y)\\
&-408\mathrm{G}(-z,-z,1-z;y)-72\mathrm{G}(-z,-z,1-z,0;y)+432\mathrm{G}(-z,-z,-z,1-z;y)\\
&-72\mathrm{G}(-z,-z,0,1-z;y)+72\mathrm{G}(-z,0,1-z,1-z;y)+216\mathrm{G}(-z,0,1-z;y)\\
&-72\mathrm{G}(-z,0,-z,1-z;y)-150\mathrm{G}(0,1-z,1-z;y)-36\mathrm{G}(0,1-z,1-z,0;y)\\
&+36\mathrm{G}(0,1-z,-z,1-z;y)+295\mathrm{G}(0,1-z;y)-36\mathrm{G}(0,1-z,0,1-z;y)-108\mathrm{G}(0,1-z,0;y)\\
&+72\mathrm{G}(0,1-z,1,0;y)+72\mathrm{G}(0,-z,1-z,1-z;y)+216\mathrm{G}(0,-z,1-z;y)-144\mathrm{G}(0,-z,1-z,0;y)\\
&-72\mathrm{G}(0,-z,-z,1-z;y)-144\mathrm{G}(0,-z,0,1-z;y)-188\mathrm{G}(0;y)-36\mathrm{G}(0,0,1-z,1-z;y)\\
&-108\mathrm{G}(0,0,1-z;y)+36\mathrm{G}(0,0,-z,1-z;y)+36\mathrm{G}(0,0;y)+108\mathrm{G}(0,0,1,0;y)+6\mathrm{G}(0,1,0;y)\\
&-72\mathrm{G}(0,1,1,0;y)+36\mathrm{G}(1,1-z,-z,1-z;y)+72\mathrm{G}(1,1-z,0,0;y)+72\mathrm{G}(1,1-z,1,0;y)\\
&+72\mathrm{G}(1,0,1-z,0;y)+36\mathrm{G}(1,0,-z,1-z;y)-310\mathrm{G}(1,0;y)+72\mathrm{G}(1,0,0,1-z;y)\\
&+90\mathrm{G}(1,0,0;y)-144\mathrm{G}(1,0,1,0;y)+60\mathrm{G}(1,1,0;y)-108\mathrm{G}(1,1,0,0;y)+36\mathrm{G}(1,1,1,0;y)\big]\,, \qquad (4.19)
\end{aligned}
$$

$$
\begin{aligned}
&C_{20}(y,z)=\\
&+\frac{z}{y^2}\big[6\mathrm{H}(0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z;y)-6\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{y^2}\big[-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&-2\mathrm{H}(1;z)\mathrm{G}(-z;y)+2\mathrm{G}(-z,1-z;y)\big]+\frac{1}{y^2}\big[-4\mathrm{H}(0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&+4\mathrm{G}(-z,1-z;y)\big]+\frac{z\pi^2}{6y}\big[-1+4\mathrm{H}(0;z)+10\mathrm{H}(0;z)\mathrm{G}(1-z;y)+\mathrm{H}(1;z)+6\mathrm{H}(1;z)\mathrm{G}(1-z;y)\\
&+10\mathrm{H}(1;z)\mathrm{G}(-z;y)-16\mathrm{G}(1-z,1-z;y)+\mathrm{G}(1-z;y)+2\mathrm{G}(1-z,0;y)+4\mathrm{G}(1-z,1;y)\\
&-10\mathrm{G}(-z,1-z;y)\big]+\frac{z}{4y}\big[-36\zeta_3-16\zeta_3\mathrm{G}(1-z;y)-4\mathrm{H}(0;z)+6\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)\\
&-40\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)+9\mathrm{H}(0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)\\
&-24\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-16\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)+46\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)\\
&-40\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)-10\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1,0;y)\\
&+16\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)+40\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)-4\mathrm{H}(0,0,1;z)+17\mathrm{H}(0,1;z)\\
&+24\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y)-14\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-8\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)\\
&-8\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)-24\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+14\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-8\mathrm{H}(0,1,0;z)\\
&+32\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-40\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)-6\mathrm{H}(0,1,1;z)+8\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)\\
&+24\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-16\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)\\
&-6\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&+8\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)\\
&+60\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)+26\mathrm{H}(1;z)\mathrm{G}(-z;y)+8\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)\\
&+18\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-6\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-10\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-17\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&+24\mathrm{H}(1;z)\mathrm{G}(1,0;y)-13\mathrm{H}(1,0;z)-40\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)+4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)\\
&-24\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-46\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+40\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)+40\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)
\end{aligned}
$$

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\begin{aligned}
&-10\mathrm{H}(1,0,1;z) - 16\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y) + 8\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y) + 24\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)\\
&-12\mathrm{H}(1,1;z)\mathrm{G}(-z;y) - 8\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y) + 6\mathrm{H}(1,1;z)\mathrm{G}(0;y) - 20\mathrm{H}(1,1,0;z)\\
&+8\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y) + 24\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y) - 24\mathrm{G}(1-z,1-z,-z,1-z;y)\\
&+6\mathrm{G}(1-z,1-z,0;y) + 48\mathrm{G}(1-z,1-z,1,0;y) + 8\mathrm{G}(1-z,-z,1-z;y) + 6\mathrm{G}(1-z,0,1-z;y)\\
&+16\mathrm{G}(1-z,0,-z,1-z;y) + 17\mathrm{G}(1-z,0;y) - 48\mathrm{G}(1-z,0,1,0;y) + 18\mathrm{G}(1-z,1,0;y)\\
&-16\mathrm{G}(1-z,1,1,0;y) - 12\mathrm{G}(-z,1-z,1-z;y) - 8\mathrm{G}(-z,1-z,1-z,0;y)\\
&+24\mathrm{G}(-z,1-z,-z,1-z;y) - 26\mathrm{G}(-z,1-z;y) - 8\mathrm{G}(-z,1-z,0,1-z;y) - 18\mathrm{G}(-z,1-z,0;y)\\
&+64\mathrm{G}(-z,1-z,1,0;y) + 24\mathrm{G}(-z,-z,1-z,1-z;y) - 60\mathrm{G}(-z,-z,1-z;y)\\
&-24\mathrm{G}(-z,-z,1-z,0;y) + 24\mathrm{G}(-z,-z,-z,1-z;y) - 24\mathrm{G}(-z,-z,0,1-z;y)\\
&-8\mathrm{G}(-z,0,1-z,1-z;y) - 18\mathrm{G}(-z,0,1-z;y) + 64\mathrm{G}(-z,0,1,0;y) + 6\mathrm{G}(0,1-z,1-z;y)\\
&+17\mathrm{G}(0,1-z;y) + 10\mathrm{G}(0,-z,1-z;y) + 18\mathrm{G}(0,1,0;y) - 24\mathrm{G}(1,1-z,0;y) - 24\mathrm{G}(1,0,1-z;y)\\
&-30\mathrm{G}(1,0;y)\big] + \frac{z^2}{y}\big[-2\mathrm{H}(0;z)\mathrm{G}(1-z;y) - 2\mathrm{H}(1;z)\mathrm{G}(-z;y) + 2\mathrm{G}(-z,1-z;y)\big]\\
&+\frac{1}{2y(y+z)}\big[-6\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y) - 17\mathrm{H}(0;z)\mathrm{G}(1-z;y) + 18\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)\\
&-18\mathrm{H}(0;z)\mathrm{G}(0,1-z;y) + 17\mathrm{H}(0,1;z) + 6\mathrm{H}(0,1;z)\mathrm{G}(1-z;y) - 18\mathrm{H}(0,1;z)\mathrm{G}(-z;y) - 6\mathrm{H}(0,1,1;z)\\
&-6\mathrm{H}(1;z)\mathrm{G}(1-z,0;y) + 18\mathrm{H}(1;z)\mathrm{G}(-z,0;y) - 6\mathrm{H}(1;z)\mathrm{G}(0,1-z;y) - 18\mathrm{H}(1;z)\mathrm{G}(0,-z;y)\\
&-17\mathrm{H}(1;z)\mathrm{G}(0;y) - 17\mathrm{H}(1,0;z) + 24\mathrm{H}(1,0;z)\mathrm{G}(1-z;y) - 18\mathrm{H}(1,0;z)\mathrm{G}(-z;y) - 6\mathrm{H}(1,0,1;z)\\
&+6\mathrm{H}(1,1;z)\mathrm{G}(0;y) - 24\mathrm{H}(1,1,0;z) + 6\mathrm{G}(1-z,1-z,0;y) + 6\mathrm{G}(1-z,0,1-z;y) + 17\mathrm{G}(1-z,0;y)\\
&+18\mathrm{G}(1-z,1,0;y) - 18\mathrm{G}(-z,1-z,0;y) - 18\mathrm{G}(-z,0,1-z;y) + 6\mathrm{G}(0,1-z,1-z;y)\\
&+17\mathrm{G}(0,1-z;y) + 18\mathrm{G}(0,-z,1-z;y) + 18\mathrm{G}(0,1,0;y) - 34\mathrm{G}(1,0;y)\big] + \frac{\pi^2}{3y}\big[-4\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+\mathrm{H}(1;z) - 6\mathrm{H}(1;z)\mathrm{G}(1-z;y) - 4\mathrm{H}(1;z)\mathrm{G}(-z;y) + 10\mathrm{G}(1-z,1-z;y) - 6\mathrm{G}(1-z;y)\\
&-2\mathrm{G}(1-z,0;y) - 4\mathrm{G}(1-z,1;y) + 4\mathrm{G}(-z,1-z;y) + \mathrm{G}(1;y)\big] + \frac{1}{4y}\big[-8\zeta_3 + 80\zeta_3\mathrm{G}(1-z;y) + 19\\
&+8\mathrm{H}(0;z) - 12\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y) + 32\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y) + 26\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+16\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y) + 40\mathrm{H}(0;z)\mathrm{G}(1-z,0;y) + 32\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)\\
&-108\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y) + 32\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y) + 68\mathrm{H}(0;z)\mathrm{G}(0,1-z;y) - 16\mathrm{H}(0;z)\mathrm{G}(1,0;y)\\
&-32\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y) - 32\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y) - 8\mathrm{H}(0,0,1;z) - 26\mathrm{H}(0,1;z)\\
&-48\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y) + 36\mathrm{H}(0,1;z)\mathrm{G}(1-z;y) + 16\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)\\
&+16\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y) + 48\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y) - 28\mathrm{H}(0,1;z)\mathrm{G}(-z;y) + 16\mathrm{H}(0,1,0;z)\\
&-16\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y) + 32\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y) + 12\mathrm{H}(0,1,1;z) - 16\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y) - 2\mathrm{H}(1;z)\\
&-48\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y) + 24\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y) + 32\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)\\
&+12\mathrm{H}(1;z)\mathrm{G}(1-z,0;y) + 48\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y) - 24\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&-16\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y) + 48\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y) + 48\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)\\
&-136\mathrm{H}(1;z)\mathrm{G}(-z,-z;y) - 48\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y) - 16\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y) - 20\mathrm{H}(1;z)\mathrm{G}(-z,0;y)
\end{aligned}
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\begin{aligned}
&+12\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+68\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+30\mathrm{H}(1;z)\mathrm{G}(0;y)-24\mathrm{H}(1;z)\mathrm{G}(1,0;y)+26\mathrm{H}(1,0;z)\\
&+32\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)-56\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+108\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-32\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)\\
&-32\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(1,0,1;z)+32\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)-16\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)\\
&-48\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+24\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+16\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-12\mathrm{H}(1,1;z)\mathrm{G}(0;y)\\
&+56\mathrm{H}(1,1,0;z)-16\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+48\mathrm{G}(1-z,1-z,-z,1-z;y)-12\mathrm{G}(1-z,1-z,0;y)\\
&-48\mathrm{G}(1-z,1-z,1,0;y)-24\mathrm{G}(1-z,-z,1-z;y)+2\mathrm{G}(1-z;y)-12\mathrm{G}(1-z,0,1-z;y)\\
&-32\mathrm{G}(1-z,0,-z,1-z;y)-30\mathrm{G}(1-z,0;y)+48\mathrm{G}(1-z,0,1,0;y)-36\mathrm{G}(1-z,1,0;y)\\
&+32\mathrm{G}(1-z,1,1,0;y)+24\mathrm{G}(-z,1-z,1-z;y)+16\mathrm{G}(-z,1-z,1-z,0;y)\\
&-48\mathrm{G}(-z,1-z,-z,1-z;y)+16\mathrm{G}(-z,1-z,0,1-z;y)+20\mathrm{G}(-z,1-z,0;y)-80\mathrm{G}(-z,1-z,1,0;y)\\
&-48\mathrm{G}(-z,-z,1-z,1-z;y)+136\mathrm{G}(-z,-z,1-z;y)+48\mathrm{G}(-z,-z,1-z,0;y)\\
&-48\mathrm{G}(-z,-z,-z,1-z;y)+48\mathrm{G}(-z,-z,0,1-z;y)+16\mathrm{G}(-z,0,1-z,1-z;y)+20\mathrm{G}(-z,0,1-z;y)\\
&-80\mathrm{G}(-z,0,1,0;y)-12\mathrm{G}(0,1-z,1-z;y)-30\mathrm{G}(0,1-z;y)-68\mathrm{G}(0,-z,1-z;y)-36\mathrm{G}(0,1,0;y)\\
&+24\mathrm{G}(1,1-z,0;y)+24\mathrm{G}(1,0,1-z;y)+60\mathrm{G}(1,0;y)-8\mathrm{G}(1,1,0;y)\big]+\frac{z}{4(1-y)^2}\big[+5\pi^2\\
&+\frac{14\pi^2}{3}\mathrm{H}(0;z)+\frac{14\pi^2}{3}\mathrm{H}(1;z)-\frac{14\pi^2}{3}\mathrm{G}(1-z;y)+\frac{2\pi^2}{3}\mathrm{G}(0;y)-\frac{4\pi^2}{3}\mathrm{G}(1;y)-92\zeta_3\\
&+28\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(0;y)+4\mathrm{H}(0;z)\mathrm{G}(0,0;y)-32\mathrm{H}(0,0,1;z)-16\mathrm{H}(0,1;z)\\
&+32\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,1;z)\mathrm{G}(0;y)+28\mathrm{H}(0,1,0;z)+32\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)\\
&-16\mathrm{H}(1;z)\mathrm{G}(-z;y)+32\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+6\mathrm{H}(1;z)\mathrm{G}(0;y)-4\mathrm{H}(1;z)\mathrm{G}(0,0;y)+16\mathrm{H}(1,0;z)\\
&-28\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-32\mathrm{H}(1,0,1;z)+28\mathrm{H}(1,1,0;z)-32\mathrm{G}(1-z,-z,1-z;y)-6\mathrm{G}(1-z,0;y)\\
&+4\mathrm{G}(1-z,0,0;y)-4\mathrm{G}(1-z,1,0;y)+16\mathrm{G}(-z,1-z;y)-6\mathrm{G}(0,1-z;y)+4\mathrm{G}(0,1-z,0;y)\\
&-32\mathrm{G}(0,-z,1-z;y)+23\mathrm{G}(0;y)+4\mathrm{G}(0,0,1-z;y)-10\mathrm{G}(0,0;y)-8\mathrm{G}(0,1,0;y)-14\mathrm{G}(1,0;y)\\
&-8\mathrm{G}(1,0,0;y)+8\mathrm{G}(1,1,0;y)\big]+\frac{z}{4(1-y)}\big[\frac{11\pi^2}{3}+6\pi^2\mathrm{H}(0;z)+6\pi^2\mathrm{H}(1;z)-6\pi^2\mathrm{G}(1-z;y)\\
&+2\pi^2\mathrm{G}(0;y)-4\pi^2\mathrm{G}(1;y)-132\zeta_3+17+4\mathrm{H}(0;z)+28\mathrm{H}(0;z)\mathrm{G}(1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&+12\mathrm{H}(0;z)\mathrm{G}(0,0;y)-48\mathrm{H}(0,0,1;z)+4\mathrm{H}(0,1;z)+48\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+12\mathrm{H}(0,1;z)\mathrm{G}(0;y)\\
&+36\mathrm{H}(0,1,0;z)-2\mathrm{H}(1;z)+48\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+32\mathrm{H}(1;z)\mathrm{G}(-z;y)+48\mathrm{H}(1;z)\mathrm{G}(0,-z;y)\\
&-22\mathrm{H}(1;z)\mathrm{G}(0;y)-12\mathrm{H}(1;z)\mathrm{G}(0,0;y)-36\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-48\mathrm{H}(1,0,1;z)+36\mathrm{H}(1,1,0;z)\\
&-48\mathrm{G}(1-z,-z,1-z;y)+2\mathrm{G}(1-z;y)+22\mathrm{G}(1-z,0;y)+12\mathrm{G}(1-z,0,0;y)-12\mathrm{G}(1-z,1,0;y)\\
&-32\mathrm{G}(-z,1-z;y)+22\mathrm{G}(0,1-z;y)+12\mathrm{G}(0,1-z,0;y)-48\mathrm{G}(0,-z,1-z;y)+39\mathrm{G}(0;y)\\
&+12\mathrm{G}(0,0,1-z;y)-22\mathrm{G}(0,0;y)-24\mathrm{G}(0,1,0;y)-26\mathrm{G}(1,0;y)-24\mathrm{G}(1,0,0;y)+24\mathrm{G}(1,1,0;y)\big]\\
&+\frac{z}{(y+z)^3}\big[-6\pi^2\mathrm{H}(1;z)+6\pi^2\mathrm{G}(1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+36\mathrm{H}(0,1,0;z)+36\mathrm{H}(1,0;z)\\
&+36\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-36\mathrm{H}(1,0;z)\mathrm{G}(0;y)-36\mathrm{H}(1,1,0;z)-36\mathrm{G}(1-z,1,0;y)-36\mathrm{G}(0,1,0;y)\\
&+36\mathrm{G}(1,0;y)\big]+\frac{z}{(y+z)^2}\big[+6\pi^2+4\pi^2\mathrm{H}(1;z)-4\pi^2\mathrm{G}(1-z;y)-18\mathrm{H}(0;z)-24\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)\\
&+36\mathrm{H}(0;z)\mathrm{G}(0;y)-24\mathrm{H}(0,1,0;z)+12\mathrm{H}(1,0;z)-24\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+24\mathrm{H}(1,0;z)\mathrm{G}(0;y)
\end{aligned}
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\begin{aligned}
&+24\mathrm{H}(1,1,0;z)+24\mathrm{G}(1-z,1,0;y)+18\mathrm{G}(0;y)+24\mathrm{G}(0,1,0;y)-60\mathrm{G}(1,0;y)\big]+\frac{z}{y+z}\big[-\pi^2\\
&+6\mathrm{H}(0;z)-6\mathrm{H}(0;z)\mathrm{G}(0;y)-6\mathrm{H}(1,0;z)-6\mathrm{G}(0;y)+6\mathrm{G}(1,0;y)\big]+\frac{z^2}{(1-y)^3}\big[-\frac{2\pi^2}{3}\mathrm{H}(0;z)\\
&-\frac{2\pi^2}{3}\mathrm{H}(1;z)+\frac{2\pi^2}{3}\mathrm{G}(1-z;y)+12\zeta_3-4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+4\mathrm{H}(0,0,1;z)-4\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&-4\mathrm{H}(0,1,0;z)-4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-4\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(1,0,1;z)\\
&-4\mathrm{H}(1,1,0;z)+4\mathrm{G}(1-z,-z,1-z;y)+4\mathrm{G}(0,-z,1-z;y)\big]+\frac{z^2}{(1-y)^2}\big[-4\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&-4\mathrm{H}(1;z)\mathrm{G}(-z;y)+4\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{1-y}\big[-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&+2\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{(y+z)^4}\big[+6\pi^2\mathrm{H}(1;z)-6\pi^2\mathrm{G}(1-z;y)-36\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-36\mathrm{H}(0,1,0;z)\\
&-36\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+36\mathrm{H}(1,0;z)\mathrm{G}(0;y)+36\mathrm{H}(1,1,0;z)+36\mathrm{G}(1-z,1,0;y)+36\mathrm{G}(0,1,0;y)\big]\\
&+\frac{z^2}{(y+z)^3}\big[-6\pi^2-4\pi^2\mathrm{H}(1;z)+4\pi^2\mathrm{G}(1-z;y)+24\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-36\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&+24\mathrm{H}(0,1,0;z)-36\mathrm{H}(1,0;z)+24\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-24\mathrm{H}(1,0;z)\mathrm{G}(0;y)-24\mathrm{H}(1,1,0;z)\\
&-24\mathrm{G}(1-z,1,0;y)-24\mathrm{G}(0,1,0;y)+36\mathrm{G}(1,0;y)\big]+\frac{z^2}{(y+z)^2}\big[\pi^2+6\mathrm{H}(0;z)\mathrm{G}(0;y)+6\mathrm{H}(1,0;z)\\
&-6\mathrm{G}(1,0;y)\big]+\frac{1}{1-y-z}\big[-\frac{\pi^2}{3}-2\mathrm{H}(0;z)\mathrm{G}(0;y)-2\mathrm{H}(1,0;z)+2\mathrm{G}(1,0;y)\big]+\frac{1}{2(1-y)}\big[-2\pi^2\\
&-\frac{8\pi^2}{3}\mathrm{H}(0;z)-\frac{4\pi^2}{3}\mathrm{H}(1;z)+\frac{4\pi^2}{3}\mathrm{G}(1-z;y)+\frac{2\pi^2}{3}\mathrm{G}(0;y)+\frac{8\pi^2}{3}\mathrm{G}(1;y)+64\zeta_3-8\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)\\
&-16\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(0;y)+4\mathrm{H}(0;z)\mathrm{G}(0,0;y)+8\mathrm{H}(0;z)\mathrm{G}(1,0;y)+20\mathrm{H}(0,0,1;z)\\
&+12\mathrm{H}(0,1;z)-20\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-4\mathrm{H}(0,1;z)\mathrm{G}(0;y)-8\mathrm{H}(0,1,0;z)-20\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)\\
&+12\mathrm{H}(1;z)\mathrm{G}(-z;y)-20\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-2\mathrm{H}(1;z)\mathrm{G}(0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,0;y)-10\mathrm{H}(1,0;z)\\
&+8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+8\mathrm{H}(1,0;z)\mathrm{G}(0;y)+20\mathrm{H}(1,0,1;z)-8\mathrm{H}(1,1,0;z)+20\mathrm{G}(1-z,-z,1-z;y)\\
&+2\mathrm{G}(1-z,0;y)-4\mathrm{G}(1-z,0,0;y)+12\mathrm{G}(1-z,1,0;y)-12\mathrm{G}(-z,1-z;y)+2\mathrm{G}(0,1-z;y)\\
&-4\mathrm{G}(0,1-z,0;y)+20\mathrm{G}(0,-z,1-z;y)-5\mathrm{G}(0;y)-4\mathrm{G}(0,0,1-z;y)-4\mathrm{G}(0,0;y)-16\mathrm{G}(1,1,0;y)\big]\\
&+\frac{1}{2(y+z)^2}\big[\frac{22\pi^2}{3}\mathrm{H}(1;z)-\frac{22\pi^2}{3}\mathrm{G}(1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+3\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&-44\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-2\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+2\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+3\mathrm{H}(0,1;z)\\
&-2\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-44\mathrm{H}(0,1,0;z)+2\mathrm{H}(0,1,1;z)+27\mathrm{H}(1;z)\\
&-4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+6\mathrm{H}(1;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-4\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&-4\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z;y)+2\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+2\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)\\
&+2\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(0;y)-3\mathrm{H}(1,0;z)-44\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1,0;z)\mathrm{G}(-z;y)\\
&+44\mathrm{H}(1,0;z)\mathrm{G}(0;y)+2\mathrm{H}(1,0,1;z)-6\mathrm{H}(1,1;z)+4\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-2\mathrm{H}(1,1;z)\mathrm{G}(0;y)\\
&+44\mathrm{H}(1,1,0;z)-6\mathrm{G}(1-z,1-z;y)-2\mathrm{G}(1-z,1-z,0;y)+4\mathrm{G}(1-z,-z,1-z;y)-27\mathrm{G}(1-z;y)
\end{aligned}
$$

$$-2\mathrm{G}(1-z,0,1-z;y)+3\mathrm{G}(1-z,0;y)+46\mathrm{G}(1-z,1,0;y)+4\mathrm{G}(-z,1-z,1-z;y)-6\mathrm{G}(-z,1-z;y)$$
$$-2\mathrm{G}(-z,1-z,0;y)+4\mathrm{G}(-z,-z,1-z;y)-2\mathrm{G}(-z,0,1-z;y)-2\mathrm{G}(0,1-z,1-z;y)$$
$$+3\mathrm{G}(0,1-z;y)-2\mathrm{G}(0,-z,1-z;y)+46\mathrm{G}(0,1,0;y)\big]+\frac{1}{2(y+z)}\big[-\frac{22\pi^2}{3}-2\pi^2\mathrm{H}(1;z)$$
$$+2\pi^2\mathrm{G}(1-z;y)-17-5\mathrm{H}(0;z)-4\mathrm{H}(0;z)\mathrm{G}(1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-44\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$-4\mathrm{H}(0,1;z)+12\mathrm{H}(0,1,0;z)+4\mathrm{H}(1;z)-4\mathrm{H}(1;z)\mathrm{G}(1-z;y)-8\mathrm{H}(1;z)\mathrm{G}(-z;y)+4\mathrm{H}(1;z)\mathrm{G}(0;y)$$
$$-42\mathrm{H}(1,0;z)+12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0;y)+4\mathrm{H}(1,1;z)-12\mathrm{H}(1,1,0;z)$$
$$+4\mathrm{G}(1-z,1-z;y)-4\mathrm{G}(1-z;y)-4\mathrm{G}(1-z,0;y)-12\mathrm{G}(1-z,1,0;y)+8\mathrm{G}(-z,1-z;y)$$
$$-4\mathrm{G}(0,1-z;y)-5\mathrm{G}(0;y)-12\mathrm{G}(0,1,0;y)+46\mathrm{G}(1,0;y)\big]+\frac{T\pi^2}{24}\big[+29+6\mathrm{H}(1;z)$$
$$+8\mathrm{H}(1;z)\mathrm{G}(1-z;y)-16\mathrm{G}(1-z,1-z;y)+12\mathrm{G}(1-z;y)+8\mathrm{G}(1-z,1;y)+8\mathrm{G}(0,1-z;y)$$
$$-8\mathrm{G}(0,1;y)-18\mathrm{G}(1;y)+8\mathrm{G}(1,1;y)\big]+\frac{T}{8}\big[-\frac{22\pi^4}{45}+\frac{255}{4}-60\zeta_3-16\zeta_3\mathrm{H}(1;z)-16\zeta_3\mathrm{G}(1-z;y)$$
$$+32\zeta_3\mathrm{G}(1;y)+18\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+15\mathrm{H}(0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)$$
$$-16\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)+42\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-16\mathrm{H}(0;z)\mathrm{G}(-z,-z,1-z;y)$$
$$+16\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)-42\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+16\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)$$
$$-16\mathrm{H}(0;z)\mathrm{G}(0,0,1-z;y)+16\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)-16\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)$$
$$+8\mathrm{H}(0,0,1;z)\mathrm{G}(1;y)+16\mathrm{H}(0,0,1,1;z)+15\mathrm{H}(0,1;z)+32\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y)$$
$$+6\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+8\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)-16\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)$$
$$-16\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+42\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+8\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)-8\mathrm{H}(0,1;z)\mathrm{G}(1,1-z;y)$$
$$-8\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)-8\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)+16\mathrm{H}(0,1,0,1;z)-18\mathrm{H}(0,1,1;z)$$
$$+16\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)-16\mathrm{H}(0,1,1;z)\mathrm{G}(0;y)+16\mathrm{H}(0,1,1,0;z)-52\mathrm{H}(1;z)$$
$$+32\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y)+24\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-18\mathrm{H}(1;z)\mathrm{G}(1-z;y)$$
$$-18\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-16\mathrm{H}(1;z)\mathrm{G}(1-z,0,0;y)-8\mathrm{H}(1;z)\mathrm{G}(1-z,1,0;y)$$
$$-32\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+36\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)+16\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)$$
$$-32\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-32\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)+84\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)$$
$$+16\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)+30\mathrm{H}(1;z)\mathrm{G}(-z;y)+16\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)$$
$$+16\mathrm{H}(1;z)\mathrm{G}(-z,0,-z;y)-42\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+8\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)$$
$$-18\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-16\mathrm{H}(1;z)\mathrm{G}(0,1-z,0;y)+16\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)$$
$$+16\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)-42\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-15\mathrm{H}(1;z)\mathrm{G}(0;y)-16\mathrm{H}(1;z)\mathrm{G}(0,0,1-z;y)$$
$$-16\mathrm{H}(1;z)\mathrm{G}(0,0,-z;y)+8\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)-8\mathrm{H}(1;z)\mathrm{G}(1,1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)$$
$$+16\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)+13\mathrm{H}(1,0;z)+24\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+16\mathrm{H}(1,0;z)\mathrm{G}(-z,-z;y)$$
$$-42\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+16\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)-16\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)+8\mathrm{H}(1,0,0,1;z)$$
$$-6\mathrm{H}(1,0,1;z)-24\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)+16\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-8\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)$$
$$+8\mathrm{H}(1,0,1;z)\mathrm{G}(1;y)+18\mathrm{H}(1,1;z)+32\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-36\mathrm{H}(1,1;z)\mathrm{G}(-z;y)$$
$$-16\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-16\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)+18\mathrm{H}(1,1;z)\mathrm{G}(0;y)+16\mathrm{H}(1,1;z)\mathrm{G}(0,0;y)$$

$$
\begin{aligned}
&+12\mathrm{H}(1,1,0;z)-8\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+16\mathrm{H}(1,1,0,1;z)+16\mathrm{H}(1,1,1,0;z)\\
&-32\mathrm{G}(1-z,1-z,-z,1-z;y)+18\mathrm{G}(1-z,1-z;y)+18\mathrm{G}(1-z,1-z,0;y)\\
&+16\mathrm{G}(1-z,1-z,0,0;y)-24\mathrm{G}(1-z,-z,1-z;y)+52\mathrm{G}(1-z;y)+18\mathrm{G}(1-z,0,1-z;y)\\
&+16\mathrm{G}(1-z,0,1-z,0;y)+15\mathrm{G}(1-z,0;y)+16\mathrm{G}(1-z,0,0,1-z;y)-8\mathrm{G}(1-z,0,1,0;y)\\
&+8\mathrm{G}(1-z,1,1-z,0;y)+8\mathrm{G}(1-z,1,0,1-z;y)-42\mathrm{G}(1-z,1,0;y)-16\mathrm{G}(1-z,1,0,0;y)\\
&-16\mathrm{G}(1-z,1,1,0;y)-36\mathrm{G}(-z,1-z,1-z;y)-16\mathrm{G}(-z,1-z,1-z,0;y)\\
&+32\mathrm{G}(-z,1-z,-z,1-z;y)-30\mathrm{G}(-z,1-z;y)-16\mathrm{G}(-z,1-z,0,1-z;y)+42\mathrm{G}(-z,1-z,0;y)\\
&+16\mathrm{G}(-z,1-z,1,0;y)+32\mathrm{G}(-z,-z,1-z,1-z;y)-84\mathrm{G}(-z,-z,1-z;y)\\
&-16\mathrm{G}(-z,-z,1-z,0;y)+32\mathrm{G}(-z,-z,-z,1-z;y)-16\mathrm{G}(-z,-z,0,1-z;y)\\
&-16\mathrm{G}(-z,0,1-z,1-z;y)+42\mathrm{G}(-z,0,1-z;y)-16\mathrm{G}(-z,0,-z,1-z;y)+16\mathrm{G}(-z,0,1,0;y)\\
&+18\mathrm{G}(0,1-z,1-z;y)+16\mathrm{G}(0,1-z,1-z,0;y)-8\mathrm{G}(0,1-z,-z,1-z;y)+15\mathrm{G}(0,1-z;y)\\
&+16\mathrm{G}(0,1-z,0,1-z;y)-8\mathrm{G}(0,1-z,1,0;y)-16\mathrm{G}(0,-z,1-z,1-z;y)+42\mathrm{G}(0,-z,1-z;y)\\
&-16\mathrm{G}(0,-z,-z,1-z;y)+16\mathrm{G}(0,0,1-z,1-z;y)+16\mathrm{G}(0,0,-z,1-z;y)-8\mathrm{G}(0,1,1-z,0;y)\\
&-8\mathrm{G}(0,1,0,1-z;y)-42\mathrm{G}(0,1,0;y)+16\mathrm{G}(0,1,1,0;y)+8\mathrm{G}(1,1-z,-z,1-z;y)\\
&-16\mathrm{G}(1,1-z,0,0;y)+8\mathrm{G}(1,1-z,1,0;y)-16\mathrm{G}(1,0,1-z,0;y)+8\mathrm{G}(1,0,-z,1-z;y)-28\mathrm{G}(1,0;y)\\
&-16\mathrm{G}(1,0,0,1-z;y)+8\mathrm{G}(1,0,1,0;y)+36\mathrm{G}(1,1,0;y)+16\mathrm{G}(1,1,0,0;y)-16\mathrm{G}(1,1,1,0;y)\big]\\
&+\frac{\pi^2}{6}\big[9-7\mathrm{H}(0;z)+6\mathrm{H}(0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(1;y)+4\mathrm{H}(0,1;z)-24\mathrm{H}(1;z)\\
&+12\mathrm{H}(1;z)\mathrm{G}(1-z;y)+8\mathrm{H}(1;z)\mathrm{G}(-z;y)-4\mathrm{H}(1;z)\mathrm{G}(0;y)-6\mathrm{H}(1;z)\mathrm{G}(1;y)-2\mathrm{H}(1,0;z)-6\mathrm{H}(1,1;z)\\
&-20\mathrm{G}(1-z,1-z;y)+34\mathrm{G}(1-z;y)+6\mathrm{G}(1-z,0;y)+8\mathrm{G}(1-z,1;y)-8\mathrm{G}(-z,1-z;y)\\
&+8\mathrm{G}(0,1-z;y)-7\mathrm{G}(0;y)-4\mathrm{G}(0,1;y)+6\mathrm{G}(1,1-z;y)-10\mathrm{G}(1;y)-4\mathrm{G}(1,0;y)\big]\\
&+\frac{1}{4}\big[40\zeta_3+40\zeta_3\mathrm{H}(1;z)-80\zeta_3\mathrm{G}(1-z;y)+40\zeta_3\mathrm{G}(1;y)-2-29\mathrm{H}(0;z)+20\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)\\
&-32\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)-4\mathrm{H}(0;z)\mathrm{G}(1-z;y)+32\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)\\
&+16\mathrm{H}(0;z)\mathrm{G}(1-z,0,0;y)+16\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)-24\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)\\
&+52\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-32\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)-24\mathrm{H}(0;z)\mathrm{G}(-z,-z,1-z;y)\\
&+24\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)+8\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)-44\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&+16\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)+24\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)+40\mathrm{H}(0;z)\mathrm{G}(0;y)-24\mathrm{H}(0;z)\mathrm{G}(0,0,1-z;y)\\
&-20\mathrm{H}(0;z)\mathrm{G}(0,0;y)+16\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)-8\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1,0;y)\\
&-16\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)-16\mathrm{H}(0;z)\mathrm{G}(1,1,0;y)+20\mathrm{H}(0,0;z)+16\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)\\
&-4\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)+16\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)-20\mathrm{H}(0,0;z)\mathrm{G}(0;y)+36\mathrm{H}(0,0,1;z)\\
&-16\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)+16\mathrm{H}(0,0,1;z)\mathrm{G}(1;y)-16\mathrm{H}(0,0,1,0;z)+16\mathrm{H}(0,0,1,1;z)\\
&+48\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y)-52\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-24\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)\\
&-24\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+84\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+8\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)-8\mathrm{H}(0,1;z)\mathrm{G}(0;y)\\
&-16\mathrm{H}(0,1;z)\mathrm{G}(1,1-z;y)-8\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)-28\mathrm{H}(0,1,0;z)+32\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)
\end{aligned}
$$

$-32\mathrm{H}(0,1,0;z)\mathrm{G}(-z;y)+16\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)-24\mathrm{H}(0,1,0;z)\mathrm{G}(1;y)+16\mathrm{H}(0,1,0,1;z)$
$-20\mathrm{H}(0,1,1;z)+24\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)-16\mathrm{H}(0,1,1;z)\mathrm{G}(0;y)+32\mathrm{H}(0,1,1,0;z)-58\mathrm{H}(1;z)$
$+48\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y)-32\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-4\mathrm{H}(1;z)\mathrm{G}(1-z;y)$
$-20\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-16\mathrm{H}(1;z)\mathrm{G}(1-z,0,0;y)-8\mathrm{H}(1;z)\mathrm{G}(1-z,1,0;y)$
$-48\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+40\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)$
$-48\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-48\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)+136\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)$
$+24\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)-4\mathrm{H}(1;z)\mathrm{G}(-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,0,-z;y)$
$-52\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+16\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)-20\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)$
$-16\mathrm{H}(1;z)\mathrm{G}(0,1-z,0;y)+24\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)+24\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)$
$-52\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(0;y)-16\mathrm{H}(1;z)\mathrm{G}(0,0,1-z;y)-24\mathrm{H}(1;z)\mathrm{G}(0,0,-z;y)$
$+4\mathrm{H}(1;z)\mathrm{G}(0,0;y)+8\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)-16\mathrm{H}(1;z)\mathrm{G}(1,1-z,-z;y)-16\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)$
$+24\mathrm{H}(1;z)\mathrm{G}(1,0;y)+16\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)+60\mathrm{H}(1,0;z)-40\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)$
$+80\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+16\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)-32\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)$
$+24\mathrm{H}(1,0;z)\mathrm{G}(-z,-z;y)-52\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+32\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)+32\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)$
$-24\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)-36\mathrm{H}(1,0;z)\mathrm{G}(0;y)-16\mathrm{H}(1,0;z)\mathrm{G}(0,0;y)+24\mathrm{H}(1,0;z)\mathrm{G}(1,1-z;y)$
$-16\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)-20\mathrm{H}(1,0,0;z)+16\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)+16\mathrm{H}(1,0,0,1;z)+28\mathrm{H}(1,0,1;z)$
$-40\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)+24\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-8\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)+16\mathrm{H}(1,0,1;z)\mathrm{G}(1;y)$
$-24\mathrm{H}(1,0,1,0;z)+4\mathrm{H}(1,1;z)+48\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-40\mathrm{H}(1,1;z)\mathrm{G}(-z;y)$
$-24\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-24\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)+20\mathrm{H}(1,1;z)\mathrm{G}(0;y)+16\mathrm{H}(1,1;z)\mathrm{G}(0,0;y)$
$-64\mathrm{H}(1,1,0;z)+16\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+32\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)-16\mathrm{H}(1,1,0;z)\mathrm{G}(0;y)$
$-24\mathrm{H}(1,1,0;z)\mathrm{G}(1;y)+24\mathrm{H}(1,1,0,1;z)-8\mathrm{H}(1,1,1,0;z)-48\mathrm{G}(1-z,1-z,-z,1-z;y)$
$+4\mathrm{G}(1-z,1-z;y)+20\mathrm{G}(1-z,1-z,0;y)+16\mathrm{G}(1-z,1-z,0,0;y)+40\mathrm{G}(1-z,1-z,1,0;y)$
$+32\mathrm{G}(1-z,-z,1-z;y)+58\mathrm{G}(1-z;y)+20\mathrm{G}(1-z,0,1-z;y)+16\mathrm{G}(1-z,0,1-z,0;y)$
$-4\mathrm{G}(1-z,0;y)+16\mathrm{G}(1-z,0,0,1-z;y)-4\mathrm{G}(1-z,0,0;y)-48\mathrm{G}(1-z,0,1,0;y)$
$+8\mathrm{G}(1-z,1,1-z,0;y)+8\mathrm{G}(1-z,1,0,1-z;y)-100\mathrm{G}(1-z,1,0;y)-32\mathrm{G}(1-z,1,0,0;y)$
$-32\mathrm{G}(1-z,1,1,0;y)-40\mathrm{G}(-z,1-z,1-z;y)-24\mathrm{G}(-z,1-z,1-z,0;y)$
$+48\mathrm{G}(-z,1-z,-z,1-z;y)+4\mathrm{G}(-z,1-z;y)-24\mathrm{G}(-z,1-z,0,1-z;y)$
$+52\mathrm{G}(-z,1-z,0;y)+56\mathrm{G}(-z,1-z,1,0;y)+48\mathrm{G}(-z,-z,1-z,1-z;y)$
$-136\mathrm{G}(-z,-z,1-z;y)-24\mathrm{G}(-z,-z,1-z,0;y)+48\mathrm{G}(-z,-z,-z,1-z;y)$
$-24\mathrm{G}(-z,-z,0,1-z;y)-24\mathrm{G}(-z,0,1-z,1-z;y)+52\mathrm{G}(-z,0,1-z;y)-24\mathrm{G}(-z,0,-z,1-z;y)$
$+56\mathrm{G}(-z,0,1,0;y)+20\mathrm{G}(0,1-z,1-z;y)+16\mathrm{G}(0,1-z,1-z,0;y)-16\mathrm{G}(0,1-z,-z,1-z;y)$
$-4\mathrm{G}(0,1-z;y)+16\mathrm{G}(0,1-z,0,1-z;y)-4\mathrm{G}(0,1-z,0;y)-24\mathrm{G}(0,1-z,1,0;y)$
$-24\mathrm{G}(0,-z,1-z,1-z;y)+52\mathrm{G}(0,-z,1-z;y)-24\mathrm{G}(0,-z,-z,1-z;y)-29\mathrm{G}(0;y)$
$+16\mathrm{G}(0,0,1-z,1-z;y)-4\mathrm{G}(0,0,1-z;y)+24\mathrm{G}(0,0,-z,1-z;y)+20\mathrm{G}(0,0;y)-16\mathrm{G}(0,0,1,0;y)$
$-8\mathrm{G}(0,1,1-z,0;y)-8\mathrm{G}(0,1,0,1-z;y)-20\mathrm{G}(0,1,0;y)+16\mathrm{G}(0,1,1,0;y)$

$$+16\mathrm{G}(1,1-z,-z,1-z;y)-24\mathrm{G}(1,1-z,0;y)-16\mathrm{G}(1,1-z,0,0;y)-8\mathrm{G}(1,1-z,1,0;y)$$
$$-24\mathrm{G}(1,0,1-z;y)-16\mathrm{G}(1,0,1-z,0;y)+16\mathrm{G}(1,0,-z,1-z;y)-56\mathrm{G}(1,0;y)-16\mathrm{G}(1,0,0,1-z;y)$$
$$+24\mathrm{G}(1,0,0;y)+24\mathrm{G}(1,0,1,0;y)+40\mathrm{G}(1,1,0;y)+32\mathrm{G}(1,1,0,0;y)\Big], \tag{4.20}$$

$$D_{20}(y,z)=$$
$$\frac{z}{3y}\Big[-2\mathrm{H}(0;z)-3\mathrm{H}(1,0;z)-3\mathrm{G}(1,0;y)\Big]+\frac{1}{y(y+z)}\Big[-2\mathrm{H}(1,0;z)-2\mathrm{G}(1,0;y)\Big]+\frac{1}{18y}\Big[-74$$
$$+15\mathrm{H}(0;z)+36\mathrm{H}(1,0;z)+15\mathrm{G}(0;y)+36\mathrm{G}(1,0;y)\Big]+\frac{z}{18(1-y)^2}\Big[2\pi^2-3\mathrm{H}(0;z)\mathrm{G}(0;y)+50\mathrm{G}(0;y)$$
$$-18\mathrm{G}(0,0;y)-12\mathrm{G}(1,0;y)\Big]+\frac{z}{18(1-y)}\Big[6\pi^2+38-3\mathrm{H}(0;z)-9\mathrm{H}(0;z)\mathrm{G}(0;y)+87\mathrm{G}(0;y)$$
$$-54\mathrm{G}(0,0;y)-36\mathrm{G}(1,0;y)\Big]+\frac{z}{(y+z)^3}\Big[-2\pi^2\mathrm{H}(1;z)+2\pi^2\mathrm{G}(1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$+12\mathrm{H}(0,1,0;z)+12\mathrm{H}(1,0;z)+12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0;y)-12\mathrm{H}(1,1,0;z)$$
$$-12\mathrm{G}(1-z,1,0;y)-12\mathrm{G}(0,1,0;y)+12\mathrm{G}(1,0;y)\Big]+\frac{z}{(y+z)^2}\Big[2\pi^2+\frac{4\pi^2}{3}\mathrm{H}(1;z)-\frac{4\pi^2}{3}\mathrm{G}(1-z;y)$$
$$-6\mathrm{H}(0;z)-8\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(0;y)-8\mathrm{H}(0,1,0;z)+4\mathrm{H}(1,0;z)$$
$$-8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+8\mathrm{H}(1,0;z)\mathrm{G}(0;y)+8\mathrm{H}(1,1,0;z)+8\mathrm{G}(1-z,1,0;y)+6\mathrm{G}(0;y)$$
$$+8\mathrm{G}(0,1,0;y)-20\mathrm{G}(1,0;y)\Big]+\frac{z}{y+z}\Big[-\frac{\pi^2}{3}+2\mathrm{H}(0;z)-2\mathrm{H}(0;z)\mathrm{G}(0;y)-2\mathrm{H}(1,0;z)-2\mathrm{G}(0;y)$$
$$+2\mathrm{G}(1,0;y)\Big]+\frac{z^2}{(y+z)^4}\Big[2\pi^2\mathrm{H}(1;z)-2\pi^2\mathrm{G}(1-z;y)-12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(0,1,0;z)$$
$$-12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+12\mathrm{H}(1,0;z)\mathrm{G}(0;y)+12\mathrm{H}(1,1,0;z)+12\mathrm{G}(1-z,1,0;y)+12\mathrm{G}(0,1,0;y)\Big]$$
$$+\frac{z^2}{(y+z)^3}\Big[-2\pi^2-\frac{4\pi^2}{3}\mathrm{H}(1;z)+\frac{4\pi^2}{3}\mathrm{G}(1-z;y)+8\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$+8\mathrm{H}(0,1,0;z)-12\mathrm{H}(1,0;z)+8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(1,0;z)\mathrm{G}(0;y)-8\mathrm{H}(1,1,0;z)$$
$$-8\mathrm{G}(1-z,1,0;y)-8\mathrm{G}(0,1,0;y)+12\mathrm{G}(1,0;y)\Big]+\frac{z^2}{3(y+z)^2}\Big[\pi^2+6\mathrm{H}(0;z)\mathrm{G}(0;y)+6\mathrm{H}(1,0;z)$$
$$-6\mathrm{G}(1,0;y)\Big]+\frac{1}{9(1-y)}\Big[-4\pi^2+6\mathrm{H}(0;z)\mathrm{G}(0;y)-70\mathrm{G}(0;y)+36\mathrm{G}(0,0;y)+24\mathrm{G}(1,0;y)\Big]$$
$$+\frac{1}{(y+z)^2}\Big[\frac{2\pi^2}{3}\mathrm{H}(1;z)-\frac{2\pi^2}{3}\mathrm{G}(1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-4\mathrm{H}(0,1,0;z)-4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$+4\mathrm{H}(1,0;z)\mathrm{G}(0;y)+4\mathrm{H}(1,1,0;z)+4\mathrm{G}(1-z,1,0;y)+4\mathrm{G}(0,1,0;y)\Big]+\frac{1}{y+z}\Big[-\frac{2\pi^2}{3}-\frac{\pi^2}{3}\mathrm{H}(1;z)$$
$$+\frac{\pi^2}{3}\mathrm{G}(1-z;y)+2-\mathrm{H}(0;z)+2\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-4\mathrm{H}(0;z)\mathrm{G}(0;y)+2\mathrm{H}(0,1,0;z)-4\mathrm{H}(1,0;z)$$
$$+2\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(0;y)-2\mathrm{H}(1,1,0;z)-2\mathrm{G}(1-z,1,0;y)-\mathrm{G}(0;y)-2\mathrm{G}(0,1,0;y)$$
$$+4\mathrm{G}(1,0;y)\Big]+\frac{T\pi^2}{216}\Big[+431-12\mathrm{H}(0;z)+24\mathrm{G}(1-z;y)-12\mathrm{G}(0;y)-24\mathrm{G}(1;y)\Big]+\frac{T}{18}\Big[\frac{4345}{36}-38\zeta_3$$
$$+31\mathrm{H}(0;z)+12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+10\mathrm{H}(0;z)\mathrm{G}(0;y)-18\mathrm{H}(0;z)\mathrm{G}(0,0;y)+3\mathrm{H}(0;z)\mathrm{G}(1,0;y)$$
$$-41\mathrm{H}(0,0;z)-18\mathrm{H}(0,0;z)\mathrm{G}(0;y)-3\mathrm{H}(0,1,0;z)+29\mathrm{H}(1,0;z)+12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$

$$-15\mathrm{H}(1,0;z)\mathrm{G}(0;y)-18\mathrm{H}(1,0,0;z)-12\mathrm{G}(1-z,1,0;y)+31\mathrm{G}(0;y)-41\mathrm{G}(0,0;y)+3\mathrm{G}(0,1,0;y)$$

$$-29\mathrm{G}(1,0;y)+18\mathrm{G}(1,0,0;y)+12\mathrm{G}(1,1,0;y)\big]+\frac{\pi^2}{3}+2\mathrm{H}(0;z)\mathrm{G}(0;y)+2\mathrm{H}(1,0;z)-2\mathrm{G}(1,0;y)\ ,\quad(4.21)$$

$$E_{20}(y,z)=$$

$$\frac{z}{9y}\big[-2\pi^2\mathrm{G}(1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-47\mathrm{H}(0;z)\mathrm{G}(1-z;y)+3\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$

$$-15\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+3\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+18\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)+9\mathrm{H}(0,1;z)$$

$$+3\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+12\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)$$

$$-38\mathrm{H}(1;z)\mathrm{G}(-z;y)+3\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+3\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-9\mathrm{H}(1;z)\mathrm{G}(0;y)+9\mathrm{H}(1,0;z)$$

$$-12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+15\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-12\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-12\mathrm{G}(1-z,-z,1-z;y)$$

$$+9\mathrm{G}(1-z,0;y)-12\mathrm{G}(-z,1-z,1-z;y)+38\mathrm{G}(-z,1-z;y)-3\mathrm{G}(-z,1-z,0;y)$$

$$+12\mathrm{G}(-z,-z,1-z;y)-3\mathrm{G}(-z,0,1-z;y)+9\mathrm{G}(0,1-z;y)-3\mathrm{G}(0,-z,1-z;y)\big]$$

$$+\frac{1}{y(y+z)}\big[-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(0,1;z)-2\mathrm{H}(1;z)\mathrm{G}(0;y)+2\mathrm{H}(1,0;z)+2\mathrm{G}(1-z,0;y)$$

$$+2\mathrm{G}(0,1-z;y)\big]+\frac{1}{18y}\big[8\pi^2\mathrm{G}(1-z;y)-38+3\mathrm{H}(0;z)-48\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)$$

$$+188\mathrm{H}(0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+60\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-12\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$

$$-72\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-36\mathrm{H}(0,1;z)-12\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-48\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)$$

$$-48\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)+48\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+152\mathrm{H}(1;z)\mathrm{G}(-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,0;y)$$

$$-12\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+36\mathrm{H}(1;z)\mathrm{G}(0;y)-36\mathrm{H}(1,0;z)+48\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$

$$-60\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+48\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+48\mathrm{G}(1-z,-z,1-z;y)-36\mathrm{G}(1-z,0;y)$$

$$+48\mathrm{G}(-z,1-z,1-z;y)-152\mathrm{G}(-z,1-z;y)+12\mathrm{G}(-z,1-z,0;y)-48\mathrm{G}(-z,-z,1-z;y)$$

$$+12\mathrm{G}(-z,0,1-z;y)-36\mathrm{G}(0,1-z;y)+12\mathrm{G}(0,-z,1-z;y)+15\mathrm{G}(0;y)\big]+\frac{z}{18(1-y)^2}\big[-2\pi^2$$

$$+3\mathrm{H}(0;z)\mathrm{G}(0;y)-50\mathrm{G}(0;y)+18\mathrm{G}(0,0;y)+12\mathrm{G}(1,0;y)\big]+\frac{z}{18(1-y)}\big[-6\pi^2-38+3\mathrm{H}(0;z)$$

$$+9\mathrm{H}(0;z)\mathrm{G}(0;y)-87\mathrm{G}(0;y)+54\mathrm{G}(0,0;y)+36\mathrm{G}(1,0;y)\big]+\frac{1}{9(1-y)}\big[2\pi^2-3\mathrm{H}(0;z)\mathrm{G}(0;y)$$

$$+38\mathrm{G}(0;y)-18\mathrm{G}(0,0;y)-12\mathrm{G}(1,0;y)\big]+\frac{1}{9(y+z)^2}\big[-9\mathrm{H}(0;z)\mathrm{G}(1-z;y)-3\mathrm{H}(0,1;z)-26\mathrm{H}(1;z)$$

$$+12\mathrm{H}(1;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z;y)+9\mathrm{H}(1;z)\mathrm{G}(0;y)+9\mathrm{H}(1,0;z)-12\mathrm{H}(1,1;z)$$

$$-12\mathrm{G}(1-z,1-z;y)+26\mathrm{G}(1-z;y)-9\mathrm{G}(1-z,0;y)+12\mathrm{G}(-z,1-z;y)-9\mathrm{G}(0,1-z;y)\big]$$

$$+\frac{1}{9(y+z)}\big[+38-9\mathrm{H}(0;z)-12\mathrm{H}(1;z)+12\mathrm{G}(1-z;y)-9\mathrm{G}(0;y)\big]+\frac{T\pi^2}{36}\big[-7+\mathrm{H}(1;z)$$

$$-5\mathrm{G}(1-z;y)+4\mathrm{G}(1;y)\big]+\frac{T}{108}\big[-\frac{4085}{6}+6\zeta_3+72\mathrm{H}(0;z)+72\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)$$

$$-147\mathrm{H}(0;z)\mathrm{G}(1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-108\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+36\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$

$$-18\mathrm{H}(0;z)\mathrm{G}(1,0;y)+108\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-36\mathrm{H}(0,0,1;z)-201\mathrm{H}(0,1;z)$$

$$+72\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-36\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+72\mathrm{H}(0,1;z)\mathrm{G}(0;y)+18\mathrm{H}(0,1,0;z)-72\mathrm{H}(0,1,1;z)$$

$$
\begin{aligned}
&+68\mathrm{H}(1;z)+144\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-108\mathrm{H}(1;z)\mathrm{G}(1-z;y)-72\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)\\
&+144\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-144\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-348\mathrm{H}(1;z)\mathrm{G}(-z;y)+108\mathrm{H}(1;z)\mathrm{G}(-z,0;y)\\
&-72\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+108\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+147\mathrm{H}(1;z)\mathrm{G}(0;y)-108\mathrm{H}(1;z)\mathrm{G}(0,0;y)\\
&-81\mathrm{H}(1,0;z)-72\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+108\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-18\mathrm{H}(1,0;z)\mathrm{G}(0;y)-72\mathrm{H}(1,0,1;z)\\
&+108\mathrm{H}(1,1;z)-144\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+72\mathrm{H}(1,1;z)\mathrm{G}(0;y)+108\mathrm{G}(1-z,1-z;y)\\
&+72\mathrm{G}(1-z,1-z,0;y)-144\mathrm{G}(1-z,-z,1-z;y)-68\mathrm{G}(1-z;y)+72\mathrm{G}(1-z,0,1-z;y)\\
&-147\mathrm{G}(1-z,0;y)+108\mathrm{G}(1-z,0,0;y)-144\mathrm{G}(-z,1-z,1-z;y)+348\mathrm{G}(-z,1-z;y)\\
&-108\mathrm{G}(-z,1-z,0;y)+144\mathrm{G}(-z,-z,1-z;y)-108\mathrm{G}(-z,0,1-z;y)+72\mathrm{G}(0,1-z,1-z;y)\\
&-147\mathrm{G}(0,1-z;y)+108\mathrm{G}(0,1-z,0;y)-108\mathrm{G}(0,-z,1-z;y)+72\mathrm{G}(0;y)+108\mathrm{G}(0,0,1-z;y)\\
&-18\mathrm{G}(0,1,0;y)+228\mathrm{G}(1,0;y)-108\mathrm{G}(1,0,0;y)-72\mathrm{G}(1,1,0;y)\big]+\frac{1}{9}\big[2\pi^2+2\pi^2\mathrm{H}(1;z)\\
&-4\pi^2\mathrm{G}(1-z;y)+2\pi^2\mathrm{G}(1;y)+19\mathrm{H}(0;z)+12\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-29\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+6\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-18\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-3\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&-3\mathrm{H}(0;z)\mathrm{G}(1,0;y)-9\mathrm{H}(0,0;z)+18\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0,0,1;z)-35\mathrm{H}(0,1;z)\\
&+12\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+12\mathrm{H}(0,1;z)\mathrm{G}(0;y)+3\mathrm{H}(0,1,0;z)-12\mathrm{H}(0,1,1;z)\\
&+38\mathrm{H}(1;z)+24\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-12\mathrm{H}(1;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)\\
&+24\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-64\mathrm{H}(1;z)\mathrm{G}(-z;y)+18\mathrm{H}(1;z)\mathrm{G}(-z,0;y)\\
&-12\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+18\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+29\mathrm{H}(1;z)\mathrm{G}(0;y)-18\mathrm{H}(1;z)\mathrm{G}(0,0;y)-3\mathrm{H}(1,0;z)\\
&-12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+18\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-3\mathrm{H}(1,0;z)\mathrm{G}(0;y)-12\mathrm{H}(1,0,1;z)+12\mathrm{H}(1,1;z)\\
&-24\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+12\mathrm{H}(1,1;z)\mathrm{G}(0;y)+12\mathrm{G}(1-z,1-z;y)+12\mathrm{G}(1-z,1-z,0;y)\\
&-24\mathrm{G}(1-z,-z,1-z;y)-38\mathrm{G}(1-z;y)+12\mathrm{G}(1-z,0,1-z;y)-29\mathrm{G}(1-z,0;y)\\
&+18\mathrm{G}(1-z,0,0;y)-24\mathrm{G}(-z,1-z,1-z;y)+64\mathrm{G}(-z,1-z;y)-18\mathrm{G}(-z,1-z,0;y)\\
&+24\mathrm{G}(-z,-z,1-z;y)-18\mathrm{G}(-z,0,1-z;y)+12\mathrm{G}(0,1-z,1-z;y)-29\mathrm{G}(0,1-z;y)\\
&+18\mathrm{G}(0,1-z,0;y)-18\mathrm{G}(0,-z,1-z;y)+19\mathrm{G}(0;y)+18\mathrm{G}(0,0,1-z;y)-9\mathrm{G}(0,0;y)\\
&-3\mathrm{G}(0,1,0;y)+32\mathrm{G}(1,0;y)-18\mathrm{G}(1,0,0;y)-12\mathrm{G}(1,1,0;y)\big]\,, \qquad (4.22)
\end{aligned}
$$

$$
\begin{aligned}
&F_{20}(y,z)=\\
&\quad\frac{T}{108}\big[-17\pi^2-20\mathrm{H}(0;z)+3\mathrm{H}(0;z)\mathrm{G}(0;y)+15\mathrm{H}(0,0;z)-20\mathrm{G}(0;y)+15\mathrm{G}(0,0;y)\big]\,, \qquad (4.23)
\end{aligned}
$$

$$
\begin{aligned}
&G_{20}(y,z)=\\
&\quad\frac{z}{y^2}\big[-9\mathrm{H}(0;z)\mathrm{G}(1-z;y)-9\mathrm{H}(1;z)\mathrm{G}(-z;y)+9\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{y^2}\big[3\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&\quad+3\mathrm{H}(1;z)\mathrm{G}(-z;y)-3\mathrm{G}(-z,1-z;y)\big]+\frac{1}{y^2}\big[6\mathrm{H}(0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z;y)-6\mathrm{G}(-z,1-z;y)\big]\\
&\quad+\frac{z\pi^2}{9y}\big[12\mathrm{H}(1;z)\mathrm{G}(1-z;y)-12\mathrm{G}(1-z,1-z;y)-2\mathrm{G}(1-z;y)+12\mathrm{G}(1-z,0;y)-2\mathrm{G}(1;y)\big]\\
&\quad+\frac{z}{3y}\big[-72\zeta_3\mathrm{G}(1-z;y)-9\mathrm{H}(0;z)+12\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-18\mathrm{H}(0;z)\mathrm{G}(1-z;y)
\end{aligned}
$$

$$-4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-8\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(1,0;y)+24\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)$$
$$+24\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y)-8\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-24\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)$$
$$-24\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)-24\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+4\mathrm{H}(0,1,1;z)+24\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)$$
$$+24\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-24\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)$$
$$+4\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+8\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)$$
$$+24\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)$$
$$-8\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)-18\mathrm{H}(1;z)\mathrm{G}(-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)$$
$$-8\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-4\mathrm{H}(1;z)\mathrm{G}(1,0;y)-12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$+8\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+4\mathrm{H}(1,0,1;z)-24\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)+24\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)$$
$$+24\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-8\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-24\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-4\mathrm{H}(1,1;z)\mathrm{G}(0;y)$$
$$+4\mathrm{H}(1,1,0;z)-24\mathrm{G}(1-z,1-z,-z,1-z;y)-4\mathrm{G}(1-z,1-z,0;y)+24\mathrm{G}(1-z,1-z,1,0;y)$$
$$-4\mathrm{G}(1-z,-z,1-z;y)-4\mathrm{G}(1-z,0,1-z;y)+24\mathrm{G}(1-z,0,-z,1-z;y)-24\mathrm{G}(1-z,0,1,0;y)$$
$$-8\mathrm{G}(-z,1-z,1-z;y)-24\mathrm{G}(-z,1-z,1-z,0;y)+24\mathrm{G}(-z,1-z,-z,1-z;y)$$
$$+18\mathrm{G}(-z,1-z;y)-24\mathrm{G}(-z,1-z,0,1-z;y)+8\mathrm{G}(-z,1-z,0;y)+24\mathrm{G}(-z,1-z,1,0;y)$$
$$+24\mathrm{G}(-z,-z,1-z,1-z;y)+8\mathrm{G}(-z,-z,1-z;y)-24\mathrm{G}(-z,-z,1-z,0;y)$$
$$+24\mathrm{G}(-z,-z,-z,1-z;y)-24\mathrm{G}(-z,-z,0,1-z;y)-24\mathrm{G}(-z,0,1-z,1-z;y)+8\mathrm{G}(-z,0,1-z;y)$$
$$+24\mathrm{G}(-z,0,1,0;y)-4\mathrm{G}(0,1-z,1-z;y)+4\mathrm{G}(1,1-z,0;y)+4\mathrm{G}(1,0,1-z;y)+4\mathrm{G}(1,1,0;y)\big]$$
$$+\frac{z^2}{y}\big[3\mathrm{H}(0;z)\mathrm{G}(1-z;y)+3\mathrm{H}(1;z)\mathrm{G}(-z;y)-3\mathrm{G}(-z,1-z;y)\big]+\frac{1}{3y(1-y-z)}\big[\frac{2\pi^2}{3}\mathrm{G}(1;y)$$
$$+4\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1,0;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,0;y)$$
$$+4\mathrm{H}(1;z)\mathrm{G}(1,0;y)-4\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-4\mathrm{G}(-z,1-z,0;y)-4\mathrm{G}(-z,-z,1-z;y)$$
$$-4\mathrm{G}(-z,0,1-z;y)-4\mathrm{G}(1,1-z,0;y)-4\mathrm{G}(1,0,1-z;y)-4\mathrm{G}(1,1,0;y)\big]+\frac{1}{3y(1-z)}\big[\frac{2\pi^2}{3}\mathrm{G}(1;y)$$
$$+4\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1,0;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,0;y)$$
$$+4\mathrm{H}(1;z)\mathrm{G}(1,0;y)-4\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-4\mathrm{G}(-z,1-z,0;y)-4\mathrm{G}(-z,-z,1-z;y)$$
$$-4\mathrm{G}(-z,0,1-z;y)-4\mathrm{G}(1,1-z,0;y)-4\mathrm{G}(1,0,1-z;y)-4\mathrm{G}(1,1,0;y)\big]$$
$$+\frac{1}{3y(y+z)}\big[8\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-8\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+8\mathrm{H}(0,1,1;z)+8\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)$$
$$+8\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+8\mathrm{H}(1,0,1;z)-8\mathrm{H}(1,1;z)\mathrm{G}(0;y)+8\mathrm{H}(1,1,0;z)$$
$$-8\mathrm{G}(1-z,1-z,0;y)-8\mathrm{G}(1-z,0,1-z;y)-8\mathrm{G}(0,1-z,1-z;y)\big]+\frac{\pi^2}{9y}\big[-6\mathrm{H}(1;z)\mathrm{G}(1-z;y)$$
$$+6\mathrm{G}(1-z,1-z;y)+\mathrm{G}(1-z;y)-6\mathrm{G}(1-z,0;y)+\mathrm{G}(1;y)\big]+\frac{1}{3y}\big[36\zeta_3\mathrm{G}(1-z;y)+18\mathrm{H}(0;z)$$
$$-12\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+11\mathrm{H}(0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-2\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)$$
$$+6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+2\mathrm{H}(0;z)\mathrm{G}(1,0;y)-12\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)-12\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y)$$
$$+4\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+12\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)$$

$$+12\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+6\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-8\mathrm{H}(0,1,1;z)-12\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)+9\mathrm{H}(1;z)$$
$$-12\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+12\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)$$
$$-8\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)-4\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)$$
$$-12\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)$$
$$+4\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)+11\mathrm{H}(1;z)\mathrm{G}(-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)$$
$$-2\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1,0;y)$$
$$+6\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-8\mathrm{H}(1,0,1;z)+12\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)$$
$$-12\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-12\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+4\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+12\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)$$
$$+8\mathrm{H}(1,1;z)\mathrm{G}(0;y)-8\mathrm{H}(1,1,0;z)+12\mathrm{G}(1-z,1-z,-z,1-z;y)+8\mathrm{G}(1-z,1-z,0;y)$$
$$-12\mathrm{G}(1-z,1-z,1,0;y)+8\mathrm{G}(1-z,-z,1-z;y)-9\mathrm{G}(1-z;y)+8\mathrm{G}(1-z,0,1-z;y)$$
$$-12\mathrm{G}(1-z,0,-z,1-z;y)+12\mathrm{G}(1-z,0,1,0;y)+4\mathrm{G}(-z,1-z,1-z;y)$$
$$+12\mathrm{G}(-z,1-z,1-z,0;y)-12\mathrm{G}(-z,1-z,-z,1-z;y)-11\mathrm{G}(-z,1-z;y)$$
$$+12\mathrm{G}(-z,1-z,0,1-z;y)+2\mathrm{G}(-z,1-z,0;y)-12\mathrm{G}(-z,1-z,1,0;y)$$
$$-12\mathrm{G}(-z,-z,1-z,1-z;y)-4\mathrm{G}(-z,-z,1-z;y)+12\mathrm{G}(-z,-z,1-z,0;y)$$
$$-12\mathrm{G}(-z,-z,-z,1-z;y)+12\mathrm{G}(-z,-z,0,1-z;y)+12\mathrm{G}(-z,0,1-z,1-z;y)$$
$$+2\mathrm{G}(-z,0,1-z;y)-12\mathrm{G}(-z,0,1,0;y)+8\mathrm{G}(0,1-z,1-z;y)-6\mathrm{G}(0,-z,1-z;y)$$
$$-2\mathrm{G}(1,1-z,0;y)-2\mathrm{G}(1,0,1-z;y)-2\mathrm{G}(1,1,0;y)\big]+\frac{z}{3(1-y-z)}\big[-\frac{2\pi^2}{3}\mathrm{H}(1;z)-\frac{2\pi^2}{3}\mathrm{G}(1;y)$$
$$-8\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(1,0;y)+8\mathrm{H}(0,0,1;z)$$
$$+8\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-4\mathrm{H}(0,1;z)\mathrm{G}(0;y)+4\mathrm{H}(0,1,0;z)-8\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-4\mathrm{H}(1;z)\mathrm{G}(1,0;y)$$
$$+8\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-4\mathrm{H}(1,0;z)\mathrm{G}(0;y)-4\mathrm{H}(1,0,1;z)-8\mathrm{H}(1,1,0;z)+8\mathrm{G}(-z,1-z,0;y)$$
$$+8\mathrm{G}(-z,0,1-z;y)-4\mathrm{G}(0,1,0;y)+4\mathrm{G}(1,1-z,0;y)+4\mathrm{G}(1,0,1-z;y)+4\mathrm{G}(1,1,0;y)\big]$$
$$+\frac{z\pi^2}{3(1-y)^2}\big[-3-\mathrm{H}(0;z)-\mathrm{H}(1;z)+\mathrm{G}(1-z;y)\big]+\frac{z}{(1-y)^2}\big[6\zeta_3-2\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$
$$+2\mathrm{H}(0,0,1;z)+6\mathrm{H}(0,1;z)-2\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0,1,0;z)-2\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)$$
$$+6\mathrm{H}(1;z)\mathrm{G}(-z;y)-2\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-6\mathrm{H}(1,0;z)+2\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1,0,1;z)$$
$$-2\mathrm{H}(1,1,0;z)+2\mathrm{G}(1-z,-z,1-z;y)-6\mathrm{G}(-z,1-z;y)+2\mathrm{G}(0,-z,1-z;y)\big]$$
$$+\frac{z}{1-y}\big[3\mathrm{H}(0;z)-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+3\mathrm{H}(1;z)-2\mathrm{H}(1;z)\mathrm{G}(-z;y)-3\mathrm{G}(1-z;y)+2\mathrm{G}(-z,1-z;y)\big]$$
$$+\frac{z}{3(y+z)^3}\big[-8\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-24\mathrm{H}(0;z)\mathrm{G}(1-z;y)+24\mathrm{H}(0,1;z)+8\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)$$
$$-8\mathrm{H}(0,1,1;z)-8\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-24\mathrm{H}(1;z)\mathrm{G}(0;y)+24\mathrm{H}(1,0;z)$$
$$+8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(1,0,1;z)+8\mathrm{H}(1,1;z)\mathrm{G}(0;y)-8\mathrm{H}(1,1,0;z)+8\mathrm{G}(1-z,1-z,0;y)$$
$$+8\mathrm{G}(1-z,0,1-z;y)+24\mathrm{G}(1-z,0;y)+8\mathrm{G}(0,1-z,1-z;y)+24\mathrm{G}(0,1-z;y)\big]$$
$$+\frac{z}{3(y+z)^2}\big[-24\mathrm{H}(0;z)+4\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+16\mathrm{H}(0;z)\mathrm{G}(1-z;y)-16\mathrm{H}(0,1;z)$$
$$-4\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,1,1;z)+4\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)$$

$$
\begin{aligned}
&+16\mathrm{H}(1;z)\mathrm{G}(0;y)-16\mathrm{H}(1,0;z)-4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(1,0,1;z)-4\mathrm{H}(1,1;z)\mathrm{G}(0;y)\\
&+4\mathrm{H}(1,1,0;z)-4\mathrm{G}(1-z,1-z,0;y)-4\mathrm{G}(1-z,0,1-z;y)-16\mathrm{G}(1-z,0;y)-4\mathrm{G}(0,1-z,1-z;y)\\
&-16\mathrm{G}(0,1-z;y)+24\mathrm{G}(0;y)\big]+\frac{z}{3(y+z)}\big[8\mathrm{H}(0;z)-8\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+8\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&-8\mathrm{H}(0,1,1;z)-8\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+8\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(1,0,1;z)\\
&+8\mathrm{H}(1,1;z)\mathrm{G}(0;y)-8\mathrm{H}(1,1,0;z)+8\mathrm{G}(1-z,1-z,0;y)+8\mathrm{G}(1-z,0,1-z;y)+8\mathrm{G}(0,1-z,1-z;y)\\
&-8\mathrm{G}(0;y)\big]+\frac{z^2}{(1-y)^3}\big[\pi^2\mathrm{H}(0;z)+\pi^2\mathrm{H}(1;z)-\pi^2\mathrm{G}(1-z;y)-18\zeta_3+6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&-6\mathrm{H}(0,0,1;z)+6\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+6\mathrm{H}(0,1,0;z)+6\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+6\mathrm{H}(1;z)\mathrm{G}(0,-z;y)\\
&-6\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(1,0,1;z)+6\mathrm{H}(1,1,0;z)-6\mathrm{G}(1-z,-z,1-z;y)-6\mathrm{G}(0,-z,1-z;y)\big]\\
&+\frac{z^2}{(1-y)^2}\big[6\mathrm{H}(0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z;y)-6\mathrm{G}(-z,1-z;y)\big]+\frac{z^2}{1-y}\big[3\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+3\mathrm{H}(1;z)\mathrm{G}(-z;y)-3\mathrm{G}(-z,1-z;y)\big]+\frac{1}{3(1-y-z)(1-y)}\big[-\frac{2\pi^2}{3}\mathrm{H}(1;z)-4\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)\\
&+4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+8\mathrm{H}(0,0,1;z)+8\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-4\mathrm{H}(0,1;z)\mathrm{G}(0;y)+4\mathrm{H}(0,1,0;z)\\
&+4\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-4\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+4\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-4\mathrm{H}(1,0;z)\mathrm{G}(0;y)-4\mathrm{H}(1,0,1;z)\\
&-8\mathrm{H}(1,1,0;z)+4\mathrm{G}(-z,1-z,0;y)-4\mathrm{G}(-z,-z,1-z;y)+4\mathrm{G}(-z,0,1-z;y)-4\mathrm{G}(0,1,0;y)\big]\\
&+\frac{1}{3(1-y-z)}\big[\frac{2\pi^2}{3}+\frac{\pi^2}{2}\mathrm{H}(1;z)-\frac{\pi^2}{2}\mathrm{G}(1;y)-3\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&-3\mathrm{H}(0;z)\mathrm{G}(1,0;y)-6\mathrm{H}(0,0,1;z)-6\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+3\mathrm{H}(0,1;z)\mathrm{G}(0;y)-3\mathrm{H}(0,1,0;z)\\
&-6\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(1,0;y)+4\mathrm{H}(1,0;z)+3\mathrm{H}(1,0;z)\mathrm{G}(0;y)+3\mathrm{H}(1,0,1;z)\\
&+6\mathrm{H}(1,1,0;z)+6\mathrm{G}(-z,-z,1-z;y)+3\mathrm{G}(0,1,0;y)+3\mathrm{G}(1,1-z,0;y)+3\mathrm{G}(1,0,1-z;y)\\
&-4\mathrm{G}(1,0;y)+3\mathrm{G}(1,1,0;y)\big]+\frac{1}{1-y}\big[-\frac{\pi^2}{2}+\frac{2\pi^2}{3}\mathrm{H}(0;z)+\frac{2\pi^2}{3}\mathrm{H}(1;z)-\frac{2\pi^2}{3}\mathrm{G}(1-z;y)-12\zeta_3\\
&+4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-\mathrm{H}(0;z)\mathrm{G}(0;y)-4\mathrm{H}(0,0,1;z)+\mathrm{H}(0,1;z)+4\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&+4\mathrm{H}(0,1,0;z)+4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+\mathrm{H}(1;z)\mathrm{G}(-z;y)+4\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&-\mathrm{H}(1,0;z)-4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(1,0,1;z)+4\mathrm{H}(1,1,0;z)-4\mathrm{G}(1-z,-z,1-z;y)\\
&+\mathrm{G}(1-z,0;y)-\mathrm{G}(-z,1-z;y)+\mathrm{G}(0,1-z;y)-4\mathrm{G}(0,-z,1-z;y)+3\mathrm{G}(0;y)+2\mathrm{G}(1,0;y)\big]\\
&+\frac{1}{3(y+z)^2}\big[-\pi^2\mathrm{H}(1;z)+\pi^2\mathrm{G}(1-z;y)-6\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-6\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+6\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-6\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-6\mathrm{H}(0,1;z)\\
&-6\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+6\mathrm{H}(0,1,0;z)+6\mathrm{H}(0,1,1;z)+24\mathrm{H}(1;z)\\
&-12\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&-12\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+6\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)\\
&+6\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+6\mathrm{H}(1;z)\mathrm{G}(0;y)+6\mathrm{H}(1,0;z)+6\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1,0;z)\mathrm{G}(-z;y)\\
&-6\mathrm{H}(1,0;z)\mathrm{G}(0;y)+6\mathrm{H}(1,0,1;z)+8\mathrm{H}(1,1;z)+12\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-6\mathrm{H}(1,1;z)\mathrm{G}(0;y)\\
&-6\mathrm{H}(1,1,0;z)+8\mathrm{G}(1-z,1-z;y)-6\mathrm{G}(1-z,1-z,0;y)+12\mathrm{G}(1-z,-z,1-z;y)-24\mathrm{G}(1-z;y)
\end{aligned}
$$

$$-6\mathrm{G}(1-z,0,1-z;y)-6\mathrm{G}(1-z,0;y)+12\mathrm{G}(-z,1-z,1-z;y)+12\mathrm{G}(-z,1-z;y)$$
$$-6\mathrm{G}(-z,1-z,0;y)+12\mathrm{G}(-z,-z,1-z;y)-6\mathrm{G}(-z,0,1-z;y)-6\mathrm{G}(0,1-z,1-z;y)$$
$$-6\mathrm{G}(0,1-z;y)-6\mathrm{G}(0,-z,1-z;y)\big]+\frac{1}{3(y+z)}\big[\pi^2+\frac{\pi^2}{2}\mathrm{H}(1;z)-\frac{\pi^2}{2}\mathrm{G}(1-z;y)-12\mathrm{H}(0;z)$$
$$-12\mathrm{H}(0;z)\mathrm{G}(1-z;y)-3\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+6\mathrm{H}(0;z)\mathrm{G}(0;y)-18\mathrm{H}(0,1;z)-3\mathrm{H}(0,1,0;z)$$
$$-32\mathrm{H}(1;z)-8\mathrm{H}(1;z)\mathrm{G}(1-z;y)-30\mathrm{H}(1;z)\mathrm{G}(-z;y)+12\mathrm{H}(1;z)\mathrm{G}(0;y)+12\mathrm{H}(1,0;z)$$
$$-3\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+3\mathrm{H}(1,0;z)\mathrm{G}(0;y)+8\mathrm{H}(1,1;z)+3\mathrm{H}(1,1,0;z)+8\mathrm{G}(1-z,1-z;y)$$
$$+32\mathrm{G}(1-z;y)-12\mathrm{G}(1-z,0;y)+3\mathrm{G}(1-z,1,0;y)+30\mathrm{G}(-z,1-z;y)-12\mathrm{G}(0,1-z;y)$$
$$-12\mathrm{G}(0;y)+3\mathrm{G}(0,1,0;y)\big]+\frac{T}{6}\big[-\frac{2\pi^2}{3}\mathrm{G}(1-z;y)+\frac{2\pi^2}{3}\mathrm{G}(1;y)+4\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)$$
$$-4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+4\mathrm{H}(0;z)\mathrm{G}(1,0;y)-4\mathrm{H}(0,1,1;z)-9\mathrm{H}(1;z)+4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)$$
$$-4\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)+8\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-4\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+4\mathrm{H}(1;z)\mathrm{G}(1,0;y)$$
$$-4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(1,0,1;z)-8\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+4\mathrm{H}(1,1;z)\mathrm{G}(0;y)-4\mathrm{H}(1,1,0;z)$$
$$+4\mathrm{G}(1-z,1-z,0;y)-4\mathrm{G}(1-z,-z,1-z;y)+9\mathrm{G}(1-z;y)+4\mathrm{G}(1-z,0,1-z;y)$$
$$-8\mathrm{G}(-z,1-z,1-z;y)+4\mathrm{G}(0,1-z,1-z;y)-4\mathrm{G}(1,1-z,0;y)-4\mathrm{G}(1,0,1-z;y)-4\mathrm{G}(1,1,0;y)\big]$$
$$+\frac{\pi^2}{18}\big[25-12\mathrm{H}(0;z)+6\mathrm{H}(0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0;z)\mathrm{G}(1;y)-23\mathrm{H}(1;z)+12\mathrm{H}(1;z)\mathrm{G}(1-z;y)$$
$$-6\mathrm{H}(1;z)\mathrm{G}(1;y)-6\mathrm{H}(1,0;z)-6\mathrm{H}(1,1;z)-12\mathrm{G}(1-z,1-z;y)+22\mathrm{G}(1-z;y)+6\mathrm{G}(1-z,0;y)$$
$$-12\mathrm{G}(0;y)+6\mathrm{G}(1,1-z;y)+\mathrm{G}(1;y)\big]+\frac{1}{3}\big[72\zeta_3+18\zeta_3\mathrm{H}(1;z)-36\zeta_3\mathrm{G}(1-z;y)+18\zeta_3\mathrm{G}(1;y)$$
$$-5\mathrm{H}(0;z)+11\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+5\mathrm{H}(0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)$$
$$+4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-6\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)+9\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)$$
$$-6\mathrm{H}(0;z)\mathrm{G}(-z,-z,1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)$$
$$-22\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)+\mathrm{H}(0;z)\mathrm{G}(0;y)-6\mathrm{H}(0;z)\mathrm{G}(0,0,1-z;y)$$
$$-6\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(1,0;y)-2\mathrm{H}(0,0,1;z)+6\mathrm{H}(0,0,1;z)\mathrm{G}(1;y)-11\mathrm{H}(0,1;z)$$
$$+12\mathrm{H}(0,1;z)\mathrm{G}(1-z,1-z;y)-9\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)$$
$$-6\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)-6\mathrm{H}(0,1;z)\mathrm{G}(-z,-z;y)+7\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+13\mathrm{H}(0,1;z)\mathrm{G}(0;y)$$
$$-6\mathrm{H}(0,1;z)\mathrm{G}(1,1-z;y)-10\mathrm{H}(0,1,0;z)+6\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0,1,0;z)\mathrm{G}(1;y)$$
$$-11\mathrm{H}(0,1,1;z)+6\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)-10\mathrm{H}(1;z)+12\mathrm{H}(1;z)\mathrm{G}(1-z,1-z,-z;y)$$
$$+2\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+16\mathrm{H}(1;z)\mathrm{G}(1-z;y)-11\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)$$
$$-12\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+22\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)$$
$$-12\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,-z,-z;y)+16\mathrm{H}(1;z)\mathrm{G}(-z,-z;y)$$
$$+6\mathrm{H}(1;z)\mathrm{G}(-z,-z,0;y)-6\mathrm{H}(1;z)\mathrm{G}(-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z,0,-z;y)$$
$$-9\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+6\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)-11\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)+6\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)$$
$$+6\mathrm{H}(1;z)\mathrm{G}(0,-z,-z;y)-9\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-5\mathrm{H}(1;z)\mathrm{G}(0;y)-6\mathrm{H}(1;z)\mathrm{G}(0,0,-z;y)$$
$$-6\mathrm{H}(1;z)\mathrm{G}(1,1-z,-z;y)-6\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)-2\mathrm{H}(1;z)\mathrm{G}(1,0;y)+\mathrm{H}(1,0;z)$$

$$
\begin{aligned}
&-6\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)+13\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1,0;z)\mathrm{G}(-z,-z;y)-9\mathrm{H}(1,0;z)\mathrm{G}(-z;y)\\
&-6\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(0;y)+6\mathrm{H}(1,0;z)\mathrm{G}(1,1-z;y)+6\mathrm{H}(1,0,0,1;z)+11\mathrm{H}(1,0,1;z)\\
&-12\mathrm{H}(1,0,1;z)\mathrm{G}(1-z;y)+6\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)+6\mathrm{H}(1,0,1;z)\mathrm{G}(1;y)-6\mathrm{H}(1,0,1,0;z)\\
&-16\mathrm{H}(1,1;z)+12\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-22\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-6\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)\\
&-6\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)+11\mathrm{H}(1,1;z)\mathrm{G}(0;y)-12\mathrm{H}(1,1,0;z)+6\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)\\
&-6\mathrm{H}(1,1,0;z)\mathrm{G}(1;y)+6\mathrm{H}(1,1,0,1;z)-6\mathrm{H}(1,1,1,0;z)-12\mathrm{G}(1-z,1-z,-z,1-z;y)\\
&-16\mathrm{G}(1-z,1-z;y)+11\mathrm{G}(1-z,1-z,0;y)+6\mathrm{G}(1-z,1-z,1,0;y)-2\mathrm{G}(1-z,-z,1-z;y)\\
&+10\mathrm{G}(1-z;y)+11\mathrm{G}(1-z,0,1-z;y)+5\mathrm{G}(1-z,0;y)-6\mathrm{G}(1-z,0,1,0;y)-24\mathrm{G}(1-z,1,0;y)\\
&-22\mathrm{G}(-z,1-z,1-z;y)-6\mathrm{G}(-z,1-z,1-z,0;y)+12\mathrm{G}(-z,1-z,-z,1-z;y)\\
&+6\mathrm{G}(-z,1-z;y)-6\mathrm{G}(-z,1-z,0,1-z;y)+9\mathrm{G}(-z,1-z,0;y)+6\mathrm{G}(-z,1-z,1,0;y)\\
&+12\mathrm{G}(-z,-z,1-z,1-z;y)-16\mathrm{G}(-z,-z,1-z;y)-6\mathrm{G}(-z,-z,1-z,0;y)\\
&+12\mathrm{G}(-z,-z,-z,1-z;y)-6\mathrm{G}(-z,-z,0,1-z;y)-6\mathrm{G}(-z,0,1-z,1-z;y)\\
&+9\mathrm{G}(-z,0,1-z;y)-6\mathrm{G}(-z,0,-z,1-z;y)+6\mathrm{G}(-z,0,1,0;y)+11\mathrm{G}(0,1-z,1-z;y)\\
&-6\mathrm{G}(0,1-z,-z,1-z;y)+5\mathrm{G}(0,1-z;y)-6\mathrm{G}(0,-z,1-z,1-z;y)+9\mathrm{G}(0,-z,1-z;y)\\
&-6\mathrm{G}(0,-z,-z,1-z;y)-5\mathrm{G}(0;y)+6\mathrm{G}(0,0,-z,1-z;y)+\mathrm{G}(0,1,0;y)+6\mathrm{G}(1,1-z,-z,1-z;y)\\
&+2\mathrm{G}(1,1-z,0;y)+2\mathrm{G}(1,0,1-z;y)+6\mathrm{G}(1,0,-z,1-z;y)-6\mathrm{G}(1,0;y)-\mathrm{G}(1,1,0;y)\big]\,. \qquad (4.24)
\end{aligned}
$$

4.2.2 One-loop contribution to $\mathcal{T}^{(6)}$

The finite remainder of the self-interference of the one-loop amplitude is decomposed as

$$
\begin{aligned}
\mathcal{F}inite^{(1\times1)}(x,y,z) \;=\; & V\Big[N^2\left(A_{11}(y,z)+A_{11}(z,y)\right)+\left(B_{11}(y,z)+B_{11}(z,y)\right)\\
&+\frac{1}{N^2}\left(C_{11}(y,z)+C_{11}(z,y)\right)+NN_F\left(D_{11}(y,z)+D_{11}(z,y)\right)\\
&+\frac{N_F}{N}\left(E_{11}(y,z)+E_{11}(z,y)\right)+N_F^2\left(F_{11}(y,z)+F_{11}(z,y)\right)\Big] \qquad (4.25)
\end{aligned}
$$

with

$$
\begin{aligned}
&A_{11}(y,z)=\\
&\quad\frac{1}{2}+\frac{z}{4y}+\frac{1}{6y}\big[-\pi^2-24-10\mathrm{H}(0;z)-6\mathrm{H}(0;z)\mathrm{G}(0;y)-6\mathrm{H}(1,0;z)-10\mathrm{G}(0;y)+6\mathrm{G}(1,0;y)\big]\\
&\quad+\frac{z}{6(1-y)^2}\big[\pi^2\mathrm{G}(0;y)+10\mathrm{H}(0;z)\mathrm{G}(0;y)+12\mathrm{H}(0;z)\mathrm{G}(0,0;y)+6\mathrm{H}(1,0;z)\mathrm{G}(0;y)+21\mathrm{G}(0;y)\\
&\quad+23\mathrm{G}(0,0;y)-6\mathrm{G}(0,1,0;y)-12\mathrm{G}(1,0,0;y)\big]+\frac{z}{6(1-y)}\big[\pi^2+3\pi^2\mathrm{G}(0;y)+21+10\mathrm{H}(0;z)\\
&\quad+36\mathrm{H}(0;z)\mathrm{G}(0;y)+36\mathrm{H}(0;z)\mathrm{G}(0,0;y)+6\mathrm{H}(1,0;z)+18\mathrm{H}(1,0;z)\mathrm{G}(0;y)+73\mathrm{G}(0;y)+33\mathrm{G}(0,0;y)\\
&\quad-18\mathrm{G}(0,1,0;y)-6\mathrm{G}(1,0;y)-36\mathrm{G}(1,0,0;y)\big]+\frac{1}{3(1-y)}\big[-2\pi^2\mathrm{G}(0;y)-20\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&\quad-24\mathrm{H}(0;z)\mathrm{G}(0,0;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0;y)-42\mathrm{G}(0;y)-28\mathrm{G}(0,0;y)+12\mathrm{G}(0,1,0;y)+24\mathrm{G}(1,0,0;y)\big]\\
&\quad+\frac{T\pi^2}{72}\big[\pi^2+169+20\mathrm{H}(0;z)+12\mathrm{H}(0;z)\mathrm{G}(0;y)+12\mathrm{H}(1,0;z)+20\mathrm{G}(0;y)-12\mathrm{G}(1,0;y)\big]
\end{aligned}
$$

$$+\frac{T}{9}\big[+72+60\mathrm{H}(0;z)+61\mathrm{H}(0;z)\mathrm{G}(0;y)+30\mathrm{H}(0;z)\mathrm{G}(0,0;y)-9\mathrm{H}(0;z)\mathrm{G}(0,1,0;y)$$
$$-15\mathrm{H}(0;z)\mathrm{G}(1,0;y)-18\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)+25\mathrm{H}(0,0;z)+30\mathrm{H}(0,0;z)\mathrm{G}(0;y)+18\mathrm{H}(0,0;z)\mathrm{G}(0,0;y)$$
$$+15\mathrm{H}(0,1,0;z)+9\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)+36\mathrm{H}(1,0;z)+15\mathrm{H}(1,0;z)\mathrm{G}(0;y)-9\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)$$
$$+30\mathrm{H}(1,0,0;z)+18\mathrm{H}(1,0,0;z)\mathrm{G}(0;y)+9\mathrm{H}(1,0,1,0;z)+18\mathrm{H}(1,1,0,0;z)+60\mathrm{G}(0;y)$$
$$+25\mathrm{G}(0,0;y)-15\mathrm{G}(0,1,0;y)-36\mathrm{G}(1,0;y)-30\mathrm{G}(1,0,0;y)+9\mathrm{G}(1,0,1,0;y)+18\mathrm{G}(1,1,0,0;y)\big] \quad (4.26)$$

$$B_{11}(y,z) =$$
$$+\frac{z}{6y}\big[-2\pi^2\mathrm{H}(0;z)\mathrm{G}(1-z;y)-2\pi^2\mathrm{H}(1;z)\mathrm{G}(-z;y)+2\pi^2\mathrm{G}(-z,1-z;y)+3-42\mathrm{H}(0;z)\mathrm{G}(1-z;y)$$
$$-20\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)+2\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)-20\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)$$
$$-4\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-24\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)-24\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)-2\mathrm{H}(0,1;z)\mathrm{G}(-z;y)$$
$$-12\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)-12\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)-12\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-42\mathrm{H}(1;z)\mathrm{G}(-z;y)$$
$$-20\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)-20\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+12\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)$$
$$+12\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)+12\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)$$
$$-12\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)-24\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-24\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)$$
$$+42\mathrm{G}(-z,1-z;y)+20\mathrm{G}(-z,1-z,0;y)-12\mathrm{G}(-z,1-z,1,0;y)+20\mathrm{G}(-z,0,1-z;y)$$
$$-12\mathrm{G}(-z,1,1-z,0;y)-12\mathrm{G}(-z,1,0,1-z;y)+20\mathrm{G}(0,-z,1-z;y)-12\mathrm{G}(1,-z,1-z,0;y)$$
$$-12\mathrm{G}(1,-z,0,1-z;y)-12\mathrm{G}(1,0,-z,1-z;y)\big]+\frac{1}{6y}\big[-\pi^2+4\pi^2\mathrm{H}(0;z)\mathrm{G}(1-z;y)$$
$$+4\pi^2\mathrm{H}(1;z)\mathrm{G}(-z;y)-4\pi^2\mathrm{G}(-z,1-z;y)-10\mathrm{H}(0;z)+90\mathrm{H}(0;z)\mathrm{G}(1-z;y)+40\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$-24\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)-16\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-24\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)$$
$$-24\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)+40\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-24\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)-6\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$-24\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)-24\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+32\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)$$
$$+48\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)+48\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)+6\mathrm{H}(0,1;z)+16\mathrm{H}(0,1;z)\mathrm{G}(-z;y)$$
$$+24\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)+24\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)+24\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-9\mathrm{H}(1;z)$$
$$+96\mathrm{H}(1;z)\mathrm{G}(-z;y)+40\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-24\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)+40\mathrm{H}(1;z)\mathrm{G}(0,-z;y)$$
$$-6\mathrm{H}(1;z)\mathrm{G}(0;y)-24\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)-24\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)-6\mathrm{H}(1,0;z)$$
$$-24\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)+16\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+24\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)+24\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)$$
$$+48\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)+24\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)+48\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)+9\mathrm{G}(1-z;y)$$
$$+6\mathrm{G}(1-z,0;y)-96\mathrm{G}(-z,1-z;y)-40\mathrm{G}(-z,1-z,0;y)+24\mathrm{G}(-z,1-z,1,0;y)$$
$$-40\mathrm{G}(-z,0,1-z;y)+24\mathrm{G}(-z,1,1-z,0;y)+24\mathrm{G}(-z,1,0,1-z;y)+6\mathrm{G}(0,1-z;y)$$
$$-40\mathrm{G}(0,-z,1-z;y)-10\mathrm{G}(0;y)+24\mathrm{G}(1,-z,1-z,0;y)+24\mathrm{G}(1,-z,0,1-z;y)$$
$$+24\mathrm{G}(1,0,-z,1-z;y)\big]+\frac{z}{6(1-y)^2}\big[-\pi^2\mathrm{G}(0;y)-6\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$
$$-10\mathrm{H}(0;z)\mathrm{G}(0;y)-12\mathrm{H}(0;z)\mathrm{G}(0,0;y)-6\mathrm{H}(0,1;z)\mathrm{G}(0;y)-12\mathrm{H}(1;z)\mathrm{G}(-z,0;y)$$

$$-12\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+9\mathrm{H}(1;z)\mathrm{G}(0;y)+12\mathrm{H}(1;z)\mathrm{G}(0,0;y)-6\mathrm{H}(1,0;z)\mathrm{G}(0;y)-9\mathrm{G}(1-z,0;y)$$
$$-12\mathrm{G}(1-z,0,0;y)+12\mathrm{G}(-z,1-z,0;y)+12\mathrm{G}(-z,0,1-z;y)-9\mathrm{G}(0,1-z;y)-12\mathrm{G}(0,1-z,0;y)$$
$$+12\mathrm{G}(0,-z,1-z;y)-42\mathrm{G}(0;y)-12\mathrm{G}(0,0,1-z;y)-26\mathrm{G}(0,0;y)+12\mathrm{G}(0,1,0;y)+24\mathrm{G}(1,0,0;y)\big]$$
$$+\frac{z}{6(1-y)}\big[-\pi^2-3\pi^2\mathrm{G}(0;y)-42-10\mathrm{H}(0;z)-6\mathrm{H}(0;z)\mathrm{G}(1-z;y)-18\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$-18\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-36\mathrm{H}(0;z)\mathrm{G}(0;y)-36\mathrm{H}(0;z)\mathrm{G}(0,0;y)-6\mathrm{H}(0,1;z)-18\mathrm{H}(0,1;z)\mathrm{G}(0;y)$$
$$+9\mathrm{H}(1;z)-12\mathrm{H}(1;z)\mathrm{G}(-z;y)-36\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-36\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+33\mathrm{H}(1;z)\mathrm{G}(0;y)$$
$$+36\mathrm{H}(1;z)\mathrm{G}(0,0;y)-6\mathrm{H}(1,0;z)-18\mathrm{H}(1,0;z)\mathrm{G}(0;y)-9\mathrm{G}(1-z;y)-33\mathrm{G}(1-z,0;y)$$
$$-36\mathrm{G}(1-z,0,0;y)+12\mathrm{G}(-z,1-z;y)+36\mathrm{G}(-z,1-z,0;y)+36\mathrm{G}(-z,0,1-z;y)-33\mathrm{G}(0,1-z;y)$$
$$-36\mathrm{G}(0,1-z,0;y)+36\mathrm{G}(0,-z,1-z;y)-136\mathrm{G}(0;y)-36\mathrm{G}(0,0,1-z;y)-6\mathrm{G}(0,0;y)$$
$$+36\mathrm{G}(0,1,0;y)+12\mathrm{G}(1,0;y)+72\mathrm{G}(1,0,0;y)\big]+\frac{1}{(1-y)^2(y+z)^2}\big[-\mathrm{H}(1;z)\mathrm{G}(0;y)+\mathrm{G}(1-z,0;y)$$
$$+\mathrm{G}(0,1-z;y)\big]+\frac{1}{(1-y)^2(y+z)}\big[2\mathrm{H}(1;z)\mathrm{G}(0;y)-2\mathrm{G}(1-z,0;y)-2\mathrm{G}(0,1-z;y)+\mathrm{G}(0;y)\big]$$
$$+\frac{1}{(1-y)^2}\big[-\mathrm{H}(1;z)\mathrm{G}(0;y)+\mathrm{G}(1-z,0;y)+\mathrm{G}(0,1-z;y)-\mathrm{G}(0;y)\big]+\frac{1}{(1-y)(y+z)^2}\big[-\mathrm{H}(1;z)$$
$$-2\mathrm{H}(1;z)\mathrm{G}(0;y)+\mathrm{G}(1-z;y)+2\mathrm{G}(1-z,0;y)+2\mathrm{G}(0,1-z;y)\big]+\frac{1}{(1-y)(y+z)}\big[1+2\mathrm{H}(1;z)$$
$$+2\mathrm{H}(1;z)\mathrm{G}(0;y)-2\mathrm{G}(1-z;y)-2\mathrm{G}(1-z,0;y)-2\mathrm{G}(0,1-z;y)+2\mathrm{G}(0;y)\big]+\frac{1}{3(1-y)}\big[\pi^2\mathrm{G}(0;y)$$
$$-3+12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+10\mathrm{H}(0;z)\mathrm{G}(0;y)+12\mathrm{H}(0;z)\mathrm{G}(0,0;y)$$
$$+6\mathrm{H}(0,1;z)\mathrm{G}(0;y)-3\mathrm{H}(1;z)+18\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+18\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-18\mathrm{H}(1;z)\mathrm{G}(0;y)$$
$$-12\mathrm{H}(1;z)\mathrm{G}(0,0;y)+6\mathrm{H}(1,0;z)\mathrm{G}(0;y)+3\mathrm{G}(1-z;y)+18\mathrm{G}(1-z,0;y)+12\mathrm{G}(1-z,0,0;y)$$
$$-18\mathrm{G}(-z,1-z,0;y)-18\mathrm{G}(-z,0,1-z;y)+18\mathrm{G}(0,1-z;y)+12\mathrm{G}(0,1-z,0;y)$$
$$-18\mathrm{G}(0,-z,1-z;y)+66\mathrm{G}(0;y)+12\mathrm{G}(0,0,1-z;y)+14\mathrm{G}(0,0;y)-12\mathrm{G}(0,1,0;y)-24\mathrm{G}(1,0,0;y)\big]$$
$$+\frac{1}{6(y+z)^2}\big[-2\pi^2\mathrm{H}(1;z)+2\pi^2\mathrm{G}(1-z;y)+11\mathrm{H}(0;z)\mathrm{G}(1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-11\mathrm{H}(0,1;z)-12\mathrm{H}(0,1;z)\mathrm{G}(0;y)-42\mathrm{H}(1;z)-11\mathrm{H}(1;z)\mathrm{G}(0;y)$$
$$+12\mathrm{H}(1;z)\mathrm{G}(1,0;y)-11\mathrm{H}(1,0;z)+12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0;y)-12\mathrm{H}(1,0,1;z)$$
$$-24\mathrm{H}(1,1,0;z)+42\mathrm{G}(1-z;y)+11\mathrm{G}(1-z,0;y)-12\mathrm{G}(1-z,1,0;y)+11\mathrm{G}(0,1-z;y)$$
$$-12\mathrm{G}(1,1-z,0;y)-12\mathrm{G}(1,0,1-z;y)\big]+\frac{1}{6(y+z)}\big[2\pi^2+42+11\mathrm{H}(0;z)+12\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$+12\mathrm{H}(1,0;z)+11\mathrm{G}(0;y)-12\mathrm{G}(1,0;y)\big]+\frac{T\pi^2}{12}\big[-8-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0,1;z)+3\mathrm{H}(1;z)$$
$$-4\mathrm{H}(1;z)\mathrm{G}(-z;y)+2\mathrm{H}(1;z)\mathrm{G}(0;y)-3\mathrm{G}(1-z;y)-2\mathrm{G}(1-z,0;y)+4\mathrm{G}(-z,1-z;y)$$
$$-2\mathrm{G}(0,1-z;y)+2\mathrm{G}(1,0;y)\big]+\frac{T}{6}\big[-96-40\mathrm{H}(0;z)-39\mathrm{H}(0;z)\mathrm{G}(1-z;y)-29\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$-12\mathrm{H}(0;z)\mathrm{G}(1-z,0,0;y)+6\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)+20\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)-29\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$

$$-12\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)-24\mathrm{H}(0;z)\mathrm{G}(0;y)-12\mathrm{H}(0;z)\mathrm{G}(0,0,1-z;y)$$
$$+6\mathrm{H}(0;z)\mathrm{G}(0,1,0;y)+6\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)+6\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+10\mathrm{H}(0;z)\mathrm{G}(1,0;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)-20\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)$$
$$-20\mathrm{H}(0,0,1;z)-12\mathrm{H}(0,0,1;z)\mathrm{G}(0;y)-9\mathrm{H}(0,1;z)-20\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-12\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)$$
$$-12\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)+9\mathrm{H}(0,1;z)\mathrm{G}(0;y)+12\mathrm{H}(0,1;z)\mathrm{G}(0,0;y)+6\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)$$
$$-10\mathrm{H}(0,1,0;z)-6\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)-6\mathrm{H}(0,1,0,1;z)-12\mathrm{H}(0,1,1,0;z)$$
$$+36\mathrm{H}(1;z)-48\mathrm{H}(1;z)\mathrm{G}(-z;y)-20\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)-20\mathrm{H}(1;z)\mathrm{G}(0,-z;y)$$
$$+39\mathrm{H}(1;z)\mathrm{G}(0;y)+20\mathrm{H}(1;z)\mathrm{G}(0,0;y)-6\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)+12\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)$$
$$+12\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)-9\mathrm{H}(1;z)\mathrm{G}(1,0;y)-12\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)-9\mathrm{H}(1,0;z)$$
$$-9\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-6\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-20\mathrm{H}(1,0;z)\mathrm{G}(-z;y)$$
$$-12\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)-6\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)+19\mathrm{H}(1,0;z)\mathrm{G}(0;y)$$
$$+12\mathrm{H}(1,0;z)\mathrm{G}(0,0;y)+6\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)-12\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0,0,1;z)$$
$$+9\mathrm{H}(1,0,1;z)-12\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)+6\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)-6\mathrm{H}(1,0,1,0;z)+18\mathrm{H}(1,1,0;z)$$
$$-24\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)+12\mathrm{H}(1,1,0;z)\mathrm{G}(0;y)-36\mathrm{G}(1-z;y)-39\mathrm{G}(1-z,0;y)-20\mathrm{G}(1-z,0,0;y)$$
$$+6\mathrm{G}(1-z,0,1,0;y)+9\mathrm{G}(1-z,1,0;y)+12\mathrm{G}(1-z,1,0,0;y)+48\mathrm{G}(-z,1-z;y)$$
$$+20\mathrm{G}(-z,1-z,0;y)-12\mathrm{G}(-z,1-z,1,0;y)+20\mathrm{G}(-z,0,1-z;y)-12\mathrm{G}(-z,1,1-z,0;y)$$
$$-12\mathrm{G}(-z,1,0,1-z;y)-39\mathrm{G}(0,1-z;y)-20\mathrm{G}(0,1-z,0;y)+6\mathrm{G}(0,1-z,1,0;y)$$
$$+20\mathrm{G}(0,-z,1-z;y)-40\mathrm{G}(0;y)-20\mathrm{G}(0,0,1-z;y)+6\mathrm{G}(0,1,1-z,0;y)+6\mathrm{G}(0,1,0,1-z;y)$$
$$+10\mathrm{G}(0,1,0;y)+9\mathrm{G}(1,1-z,0;y)+12\mathrm{G}(1,1-z,0,0;y)-12\mathrm{G}(1,-z,1-z,0;y)$$
$$-12\mathrm{G}(1,-z,0,1-z;y)+9\mathrm{G}(1,0,1-z;y)+12\mathrm{G}(1,0,1-z,0;y)-12\mathrm{G}(1,0,-z,1-z;y)$$
$$+48\mathrm{G}(1,0;y)+12\mathrm{G}(1,0,0,1-z;y)+20\mathrm{G}(1,0,0;y)-12\mathrm{G}(1,0,1,0;y)-24\mathrm{G}(1,1,0,0;y)\big]$$
$$+\frac{\pi^2}{6}\big[\mathrm{H}(0;z)-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0,1;z)+2\mathrm{H}(1;z)-4\mathrm{H}(1;z)\mathrm{G}(-z;y)+2\mathrm{H}(1;z)\mathrm{G}(0;y)$$
$$-2\mathrm{G}(1-z;y)-2\mathrm{G}(1-z,0;y)+4\mathrm{G}(-z,1-z;y)-2\mathrm{G}(0,1-z;y)+\mathrm{G}(0;y)+2\mathrm{G}(1,0;y)\big]$$
$$+\frac{1}{6}\big[21\mathrm{H}(0;z)-53\mathrm{H}(0;z)\mathrm{G}(1-z;y)-52\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-24\mathrm{H}(0;z)\mathrm{G}(1-z,0,0;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)+22\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+24\mathrm{H}(0;z)\mathrm{G}(-z,1-z,0;y)$$
$$+24\mathrm{H}(0;z)\mathrm{G}(-z,0,1-z;y)-52\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-24\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y)$$
$$+24\mathrm{H}(0;z)\mathrm{G}(0,-z,1-z;y)+20\mathrm{H}(0;z)\mathrm{G}(0;y)-24\mathrm{H}(0;z)\mathrm{G}(0,0,1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(0,0;y)$$
$$+12\mathrm{H}(0;z)\mathrm{G}(0,1,0;y)+12\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)+12\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+14\mathrm{H}(0;z)\mathrm{G}(1,0;y)$$
$$+24\mathrm{H}(0;z)\mathrm{G}(1,0,0;y)+2\mathrm{H}(0,0;z)-4\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-24\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y)$$
$$-24\mathrm{H}(0,0;z)\mathrm{G}(0,1-z;y)+12\mathrm{H}(0,0;z)\mathrm{G}(0;y)-40\mathrm{H}(0,0,1;z)-24\mathrm{H}(0,0,1;z)\mathrm{G}(0;y)-31\mathrm{H}(0,1;z)$$
$$-22\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-24\mathrm{H}(0,1;z)\mathrm{G}(-z,0;y)-24\mathrm{H}(0,1;z)\mathrm{G}(0,-z;y)+30\mathrm{H}(0,1;z)\mathrm{G}(0;y)$$
$$+24\mathrm{H}(0,1;z)\mathrm{G}(0,0;y)+12\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)-14\mathrm{H}(0,1,0;z)-12\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)$$
$$-12\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)-12\mathrm{H}(0,1,0,1;z)-24\mathrm{H}(0,1,1,0;z)+42\mathrm{H}(1;z)-84\mathrm{H}(1;z)\mathrm{G}(-z;y)$$

$$
\begin{aligned}
&-22\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)-22\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+53\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&+4\mathrm{H}(1;z)\mathrm{G}(0,0;y)-12\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)+24\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)+24\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)\\
&-12\mathrm{H}(1;z)\mathrm{G}(1,0;y)-24\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)+11\mathrm{H}(1,0;z)-12\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)\\
&-12\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)+24\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-22\mathrm{H}(1,0;z)\mathrm{G}(-z;y)\\
&-24\mathrm{H}(1,0;z)\mathrm{G}(-z,0;y)-12\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)-24\mathrm{H}(1,0;z)\mathrm{G}(0,-z;y)+38\mathrm{H}(1,0;z)\mathrm{G}(0;y)\\
&+24\mathrm{H}(1,0;z)\mathrm{G}(0,0;y)+12\mathrm{H}(1,0;z)\mathrm{G}(1,0;y)+12\mathrm{H}(1,0,0;z)-24\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)\\
&-24\mathrm{H}(1,0,0,1;z)+12\mathrm{H}(1,0,1;z)-24\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)+12\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)-12\mathrm{H}(1,0,1,0;z)\\
&+24\mathrm{H}(1,1,0;z)-48\mathrm{H}(1,1,0;z)\mathrm{G}(-z;y)+24\mathrm{H}(1,1,0;z)\mathrm{G}(0;y)-42\mathrm{G}(1-z;y)-53\mathrm{G}(1-z,0;y)\\
&-4\mathrm{G}(1-z,0,0;y)+12\mathrm{G}(1-z,0,1,0;y)+12\mathrm{G}(1-z,1,0;y)+24\mathrm{G}(1-z,1,0,0;y)\\
&+84\mathrm{G}(-z,1-z;y)+22\mathrm{G}(-z,1-z,0;y)-24\mathrm{G}(-z,1-z,1,0;y)+22\mathrm{G}(-z,0,1-z;y)\\
&-24\mathrm{G}(-z,1,1-z,0;y)-24\mathrm{G}(-z,1,0,1-z;y)-53\mathrm{G}(0,1-z;y)-4\mathrm{G}(0,1-z,0;y)\\
&+12\mathrm{G}(0,1-z,1,0;y)+22\mathrm{G}(0,-z,1-z;y)+21\mathrm{G}(0;y)-4\mathrm{G}(0,0,1-z;y)+2\mathrm{G}(0,0;y)\\
&+12\mathrm{G}(0,1,1-z,0;y)+12\mathrm{G}(0,1,0,1-z;y)-4\mathrm{G}(0,1,0;y)+12\mathrm{G}(1,1-z,0;y)+24\mathrm{G}(1,1-z,0,0;y)\\
&-24\mathrm{G}(1,-z,1-z,0;y)-24\mathrm{G}(1,-z,0,1-z;y)+12\mathrm{G}(1,0,1-z;y)+24\mathrm{G}(1,0,1-z,0;y)\\
&-24\mathrm{G}(1,0,-z,1-z;y)+42\mathrm{G}(1,0;y)+24\mathrm{G}(1,0,0,1-z;y)-8\mathrm{G}(1,0,0;y)-24\mathrm{G}(1,0,1,0;y)\\
&-48\mathrm{G}(1,1,0,0;y)\big]\,, \qquad (4.27)
\end{aligned}
$$

$$
\begin{aligned}
&C_{11}(y,z)=\\
&\frac{z(1-z)^2}{y^3}\big[-2\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)\\
&+4\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)+2\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)+2\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)\\
&-2\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)-4\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)+2\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)\\
&+2\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+2\mathrm{G}(-z,1-z,-z,1-z;y)\\
&+4\mathrm{G}(-z,-z,1-z,1-z;y)\big]+\frac{z}{y^2}\big[-8\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+12\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)\\
&+24\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-24\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)\\
&+4\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)+4\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)\\
&-12\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)+2\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)\\
&+12\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)+24\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)\\
&-12\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)+4\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-12\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)\\
&+2\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-24\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+8\mathrm{H}(1,1;z)\mathrm{G}(-z;y)+4\mathrm{G}(1-z,-z,1-z;y)\\
&+8\mathrm{G}(-z,1-z,1-z;y)-12\mathrm{G}(-z,1-z,-z,1-z;y)-24\mathrm{G}(-z,-z,1-z,1-z;y)\big]\\
&+\frac{z^2}{y^2}\big[4\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-8\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)-16\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)\\
&+2\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+16\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)-4\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)\\
&+8\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)-2\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+8\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)\\
&-2\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)
\end{aligned}
$$

$$-16\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)+8\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)$$
$$+8\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+16\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-4\mathrm{H}(1,1;z)\mathrm{G}(-z;y)$$
$$-2\mathrm{G}(1-z,-z,1-z;y)-4\mathrm{G}(-z,1-z,1-z;y)+8\mathrm{G}(-z,1-z,-z,1-z;y)$$
$$+16\mathrm{G}(-z,-z,1-z,1-z;y)\big]+\frac{1}{y^2}\big[+4\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)$$
$$-8\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)+8\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)+4\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)$$
$$-2\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)$$
$$-4\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)$$
$$+4\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)$$
$$+8\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-4\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-2\mathrm{G}(1-z,-z,1-z;y)-4\mathrm{G}(-z,1-z,1-z;y)$$
$$+4\mathrm{G}(-z,1-z,-z,1-z;y)+8\mathrm{G}(-z,-z,1-z,1-z;y)\big]+\frac{z}{y}\big[\frac{1}{4}+14\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)$$
$$+4\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)-14\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)+5\mathrm{H}(0;z)\mathrm{G}(1-z;y)$$
$$+4\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)-28\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)$$
$$+7\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)$$
$$-2\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+2\mathrm{H}(0,0;z)+24\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)-14\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)$$
$$+4\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)+2\mathrm{H}(0,1;z)+14\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)-7\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)$$
$$-2\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)+12\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)-7\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-2\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)$$
$$+2\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)+7\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)$$
$$-\mathrm{H}(1;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)-16\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)$$
$$+14\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)-32\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)$$
$$+7\mathrm{H}(1;z)\mathrm{G}(-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)-2\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)+2\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)$$
$$+4\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)-2\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)-2\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)+2\mathrm{H}(1,0;z)$$
$$+14\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)-7\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)$$
$$+14\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-7\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)$$
$$+2\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)+\mathrm{H}(1,1;z)+32\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)-14\mathrm{H}(1,1;z)\mathrm{G}(-z;y)$$
$$-4\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-4\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)+\mathrm{G}(1-z,1-z;y)-7\mathrm{G}(1-z,-z,1-z;y)$$
$$-2\mathrm{G}(1-z,-z,1-z,0;y)-2\mathrm{G}(1-z,-z,0,1-z;y)-2\mathrm{G}(1-z,0,-z,1-z;y)$$
$$-14\mathrm{G}(-z,1-z,1-z;y)-4\mathrm{G}(-z,1-z,1-z,0;y)+16\mathrm{G}(-z,1-z,-z,1-z;y)$$
$$-7\mathrm{G}(-z,1-z;y)-4\mathrm{G}(-z,1-z,0,1-z;y)+2\mathrm{G}(-z,1-z,1,0;y)+32\mathrm{G}(-z,-z,1-z,1-z;y)$$
$$-4\mathrm{G}(-z,0,1-z,1-z;y)+2\mathrm{G}(-z,1,1-z,0;y)+2\mathrm{G}(-z,1,0,1-z;y)-2\mathrm{G}(0,1-z,-z,1-z;y)$$
$$-4\mathrm{G}(0,-z,1-z,1-z;y)+2\mathrm{G}(1,-z,1-z,0;y)+2\mathrm{G}(1,-z,0,1-z;y)+2\mathrm{G}(1,0,-z,1-z;y)\big]$$
$$+\frac{1}{y}\big[+4-20\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)-8\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)+16\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)$$
$$-13\mathrm{H}(0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)$$
$$+32\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-8\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)$$

$$
\begin{aligned}
&+4\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)+4\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)-24\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)\\
&+8\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)-\mathrm{H}(0,1;z)-16\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)\\
&+10\mathrm{H}(0,1;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,1;z)\mathrm{G}(1-z,0;y)-12\mathrm{H}(0,1;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(0,1;z)\mathrm{G}(-z;y)\\
&+4\mathrm{H}(0,1;z)\mathrm{G}(0,1-z;y)-4\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)-8\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)-\frac{3}{2}\mathrm{H}(1;z)\\
&-10\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)-4\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z;y)-4\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)\\
&+20\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)-20\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)\\
&+40\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)-14\mathrm{H}(1;z)\mathrm{G}(-z;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)\\
&-4\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)-8\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)-\mathrm{H}(1;z)\mathrm{G}(0;y)+4\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)\\
&+4\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)-2\mathrm{H}(1,0;z)-16\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)+10\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)\\
&+4\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)-16\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+4\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)\\
&-4\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-2\mathrm{H}(1,1;z)-40\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)+20\mathrm{H}(1,1;z)\mathrm{G}(-z;y)\\
&+8\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)+8\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)-2\mathrm{G}(1-z,1-z;y)+10\mathrm{G}(1-z,-z,1-z;y)\\
&+4\mathrm{G}(1-z,-z,1-z,0;y)+4\mathrm{G}(1-z,-z,0,1-z;y)+\frac{3}{2}\mathrm{G}(1-z;y)+4\mathrm{G}(1-z,0,-z,1-z;y)\\
&+\mathrm{G}(1-z,0;y)+20\mathrm{G}(-z,1-z,1-z;y)+8\mathrm{G}(-z,1-z,1-z,0;y)-20\mathrm{G}(-z,1-z,-z,1-z;y)\\
&+14\mathrm{G}(-z,1-z;y)+8\mathrm{G}(-z,1-z,0,1-z;y)-4\mathrm{G}(-z,1-z,1,0;y)-40\mathrm{G}(-z,-z,1-z,1-z;y)\\
&+8\mathrm{G}(-z,0,1-z,1-z;y)-4\mathrm{G}(-z,1,1-z,0;y)-4\mathrm{G}(-z,1,0,1-z;y)+4\mathrm{G}(0,1-z,-z,1-z;y)\\
&+\mathrm{G}(0,1-z;y)+8\mathrm{G}(0,-z,1-z,1-z;y)-4\mathrm{G}(1,-z,1-z,0;y)-4\mathrm{G}(1,-z,0,1-z;y)\\
&-4\mathrm{G}(1,0,-z,1-z;y)-\mathrm{G}(1,0;y)\big]+\frac{z}{2(1-y)^2}\big[2\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+2\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&+2\mathrm{H}(0,1;z)\mathrm{G}(0;y)+4\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(0;y)-4\mathrm{H}(1;z)\mathrm{G}(0,0;y)\\
&+3\mathrm{G}(1-z,0;y)+4\mathrm{G}(1-z,0,0;y)-4\mathrm{G}(-z,1-z,0;y)-4\mathrm{G}(-z,0,1-z;y)+3\mathrm{G}(0,1-z;y)\\
&+4\mathrm{G}(0,1-z,0;y)-4\mathrm{G}(0,-z,1-z;y)+7\mathrm{G}(0;y)+4\mathrm{G}(0,0,1-z;y)+1\mathrm{G}(0,0;y)-2\mathrm{G}(0,1,0;y)\\
&-4\mathrm{G}(1,0,0;y)\big]+\frac{z}{2(1-y)}\big[7+2\mathrm{H}(0;z)\mathrm{G}(1-z;y)+6\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)+6\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&+2\mathrm{H}(0,1;z)+6\mathrm{H}(0,1;z)\mathrm{G}(0;y)-3\mathrm{H}(1;z)+4\mathrm{H}(1;z)\mathrm{G}(-z;y)+12\mathrm{H}(1;z)\mathrm{G}(-z,0;y)\\
&+12\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-11\mathrm{H}(1;z)\mathrm{G}(0;y)-12\mathrm{H}(1;z)\mathrm{G}(0,0;y)+3\mathrm{G}(1-z;y)+11\mathrm{G}(1-z,0;y)\\
&+12\mathrm{G}(1-z,0,0;y)-4\mathrm{G}(-z,1-z;y)-12\mathrm{G}(-z,1-z,0;y)-12\mathrm{G}(-z,0,1-z;y)+11\mathrm{G}(0,1-z;y)\\
&+12\mathrm{G}(0,1-z,0;y)-12\mathrm{G}(0,-z,1-z;y)+21\mathrm{G}(0;y)+12\mathrm{G}(0,0,1-z;y)-9\mathrm{G}(0,0;y)-6\mathrm{G}(0,1,0;y)\\
&-2\mathrm{G}(1,0;y)-12\mathrm{G}(1,0,0;y)\big]+\frac{1}{(1-y)^2(y+z)^2}\big[\mathrm{H}(1;z)\mathrm{G}(0;y)-\mathrm{G}(1-z,0;y)-\mathrm{G}(0,1-z;y)\big]\\
&+\frac{1}{(1-y)^2(y+z)}\big[-2\mathrm{H}(1;z)\mathrm{G}(0;y)+2\mathrm{G}(1-z,0;y)+2\mathrm{G}(0,1-z;y)-\mathrm{G}(0;y)\big]\\
&+\frac{1}{(1-y)^2}\big[+\mathrm{H}(1;z)\mathrm{G}(0;y)-\mathrm{G}(1-z,0;y)-\mathrm{G}(0,1-z;y)+\mathrm{G}(0;y)\big]\\
&+\frac{1}{(1-y)(y+z)^2}\big[+\mathrm{H}(1;z)-\mathrm{G}(1-z;y)\big]+\frac{1}{(1-y)(y+z)}\big[-1-2\mathrm{H}(1;z)+2\mathrm{G}(1-z;y)\big]
\end{aligned}
$$

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\begin{aligned}
&+\frac{1}{1-y}\big[1-2\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-2\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+\mathrm{H}(1;z)-2\mathrm{H}(1;z)\mathrm{G}(-z,0;y)\\
&-2\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+3\mathrm{H}(1;z)\mathrm{G}(0;y)-\mathrm{G}(1-z;y)-3\mathrm{G}(1-z,0;y)+2\mathrm{G}(-z,1-z,0;y)\\
&+2\mathrm{G}(-z,0,1-z;y)-3\mathrm{G}(0,1-z;y)+2\mathrm{G}(0,-z,1-z;y)-8\mathrm{G}(0;y)-2\mathrm{G}(0,0;y)\big]\\
&+\frac{1}{2(y+z)^2}\big[-4\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+\mathrm{H}(0;z)\mathrm{G}(1-z;y)-\mathrm{H}(0,1;z)+4\mathrm{H}(0,1,1;z)+14\mathrm{H}(1;z)\\
&-4\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)+8\mathrm{H}(1;z)\mathrm{G}(1-z;y)+4\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&+4\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-\mathrm{H}(1;z)\mathrm{G}(0;y)-2\mathrm{H}(1;z)\mathrm{G}(1,0;y)-\mathrm{H}(1,0;z)+2\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)\\
&+2\mathrm{H}(1,0,1;z)-8\mathrm{H}(1,1;z)+8\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-4\mathrm{H}(1,1;z)\mathrm{G}(0;y)-8\mathrm{G}(1-z,1-z;y)\\
&-4\mathrm{G}(1-z,1-z,0;y)+4\mathrm{G}(1-z,-z,1-z;y)-14\mathrm{G}(1-z;y)-4\mathrm{G}(1-z,0,1-z;y)+\mathrm{G}(1-z,0;y)\\
&+2\mathrm{G}(1-z,1,0;y)+8\mathrm{G}(-z,1-z,1-z;y)-4\mathrm{G}(0,1-z,1-z;y)+\mathrm{G}(0,1-z;y)+2\mathrm{G}(1,1-z,0;y)\\
&+2\mathrm{G}(1,0,1-z;y)\big]+\frac{1}{2(y+z)}\big[-14+\mathrm{H}(0;z)-2\mathrm{H}(0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0,1;z)+6\mathrm{H}(1;z)\\
&-4\mathrm{H}(1;z)\mathrm{G}(-z;y)+2\mathrm{H}(1;z)\mathrm{G}(0;y)-6\mathrm{G}(1-z;y)-2\mathrm{G}(1-z,0;y)+4\mathrm{G}(-z,1-z;y)\\
&-2\mathrm{G}(0,1-z;y)+\mathrm{G}(0;y)+2\mathrm{G}(1,0;y)\big]+T\big[8+3\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)\\
&+2\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)-2\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1-z;y)\\
&+2\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)-\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)-4\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)\\
&+2\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)-\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)-\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)\\
&+2\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)+2\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)+2\mathrm{H}(0,0,1,1;z)+4\mathrm{H}(0,1;z)\\
&+2\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)-\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)+\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)+\mathrm{H}(0,1,0,1;z)\\
&-3\mathrm{H}(0,1,1;z)+4\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)-2\mathrm{H}(0,1,1;z)\mathrm{G}(0;y)-6\mathrm{H}(1;z)+3\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)\\
&+2\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)+2\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)\\
&-2\mathrm{H}(1;z)\mathrm{G}(1-z,0,0;y)-4\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+6\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&+4\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)+8\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&+4\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)-2\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)+2\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)\\
&-2\mathrm{H}(1;z)\mathrm{G}(0,1-z,0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)-4\mathrm{H}(1;z)\mathrm{G}(0;y)-2\mathrm{H}(1;z)\mathrm{G}(0,0,1-z;y)\\
&+\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)-2\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)-2\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)+2\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)\\
&+2\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)-\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)+2\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)\\
&-\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)+2\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)+8\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)\\
&-6\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-4\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-4\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)+3\mathrm{H}(1,1;z)\mathrm{G}(0;y)\\
&+2\mathrm{H}(1,1;z)\mathrm{G}(0,0;y)+3\mathrm{G}(1-z,1-z,0;y)+2\mathrm{G}(1-z,1-z,0,0;y)-3\mathrm{G}(1-z,-z,1-z;y)\\
&-2\mathrm{G}(1-z,-z,1-z,0;y)-2\mathrm{G}(1-z,-z,0,1-z;y)+6\mathrm{G}(1-z;y)+3\mathrm{G}(1-z,0,1-z;y)\\
&+2\mathrm{G}(1-z,0,1-z,0;y)-2\mathrm{G}(1-z,0,-z,1-z;y)+4\mathrm{G}(1-z,0;y)+2\mathrm{G}(1-z,0,0,1-z;y)\\
&-\mathrm{G}(1-z,0,1,0;y)-2\mathrm{G}(1-z,1,0,0;y)-6\mathrm{G}(-z,1-z,1-z;y)-4\mathrm{G}(-z,1-z,1-z,0;y)\\
&+4\mathrm{G}(-z,1-z,-z,1-z;y)-8\mathrm{G}(-z,1-z;y)-4\mathrm{G}(-z,1-z,0,1-z;y)+2\mathrm{G}(-z,1-z,1,0;y)\\
&+8\mathrm{G}(-z,-z,1-z,1-z;y)-4\mathrm{G}(-z,0,1-z,1-z;y)+2\mathrm{G}(-z,1,1-z,0;y)+2\mathrm{G}(-z,1,0,1-z;y)
\end{aligned}
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\begin{aligned}
&+3\mathrm{G}(0,1-z,1-z;y)+2\mathrm{G}(0,1-z,1-z,0;y)-2\mathrm{G}(0,1-z,-z,1-z;y)+4\mathrm{G}(0,1-z;y)\\
&+2\mathrm{G}(0,1-z,0,1-z;y)-\mathrm{G}(0,1-z,1,0;y)-4\mathrm{G}(0,-z,1-z,1-z;y)+2\mathrm{G}(0,0,1-z,1-z;y)\\
&-\mathrm{G}(0,1,1-z,0;y)-\mathrm{G}(0,1,0,1-z;y)-2\mathrm{G}(1,1-z,0,0;y)+2\mathrm{G}(1,-z,1-z,0;y)\\
&+2\mathrm{G}(1,-z,0,1-z;y)-2\mathrm{G}(1,0,1-z,0;y)+2\mathrm{G}(1,0,-z,1-z;y)-4\mathrm{G}(1,0;y)-2\mathrm{G}(1,0,0,1-z;y)\\
&+\mathrm{G}(1,0,1,0;y)+2\mathrm{G}(1,1,0,0;y)+\frac{11}{4}\mathrm{H}(1,1;z)-\frac{11}{4}\mathrm{H}(1;z)\mathrm{G}(1-z;y)+\frac{11}{4}\mathrm{G}(1-z,1-z;y)\\
&+\frac{3}{2}\mathrm{H}(1;z)\mathrm{G}(1,0;y)-\frac{3}{2}\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-\frac{3}{2}\mathrm{H}(1,0,1;z)-\frac{3}{2}\mathrm{G}(1-z,1,0;y)-\frac{3}{2}\mathrm{G}(1,1-z,0;y)\\
&-\frac{3}{2}\mathrm{G}(1,0,1-z;y)\Big]-\frac{1}{2}-\frac{7}{2}\mathrm{H}(0;z)+10\mathrm{H}(0;z)\mathrm{G}(1-z,1-z;y)+8\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y)\\
&-8\mathrm{H}(0;z)\mathrm{G}(1-z,-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1-z;y)+8\mathrm{H}(0;z)\mathrm{G}(1-z,0,1-z;y)\\
&-2\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-4\mathrm{H}(0;z)\mathrm{G}(1-z,1,0;y)-16\mathrm{H}(0;z)\mathrm{G}(-z,1-z,1-z;y)\\
&+5\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+8\mathrm{H}(0;z)\mathrm{G}(0,1-z,1-z;y)-2\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)\\
&-4\mathrm{H}(0;z)\mathrm{G}(1,1-z,0;y)-4\mathrm{H}(0;z)\mathrm{G}(1,0,1-z;y)+\mathrm{H}(0;z)\mathrm{G}(1,0;y)+3\mathrm{H}(0,0;z)\\
&+8\mathrm{H}(0,0;z)\mathrm{G}(1-z,1-z;y)-8\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-2\mathrm{H}(0,0,1;z)+8\mathrm{H}(0,0,1;z)\mathrm{G}(1-z;y)\\
&+8\mathrm{H}(0,0,1,1;z)+10\mathrm{H}(0,1;z)+8\mathrm{H}(0,1;z)\mathrm{G}(1-z,-z;y)-5\mathrm{H}(0,1;z)\mathrm{G}(-z;y)-3\mathrm{H}(0,1;z)\mathrm{G}(0;y)\\
&-4\mathrm{H}(0,1;z)\mathrm{G}(1,0;y)-\mathrm{H}(0,1,0;z)+4\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,1,0,1;z)-10\mathrm{H}(0,1,1;z)\\
&+16\mathrm{H}(0,1,1;z)\mathrm{G}(-z;y)-8\mathrm{H}(0,1,1;z)\mathrm{G}(0;y)-7\mathrm{H}(1;z)+10\mathrm{H}(1;z)\mathrm{G}(1-z,-z;y)\\
&+8\mathrm{H}(1;z)\mathrm{G}(1-z,-z,0;y)-6\mathrm{H}(1;z)\mathrm{G}(1-z;y)+8\mathrm{H}(1;z)\mathrm{G}(1-z,0,-z;y)-10\mathrm{H}(1;z)\mathrm{G}(1-z,0;y)\\
&-8\mathrm{H}(1;z)\mathrm{G}(1-z,0,0;y)-16\mathrm{H}(1;z)\mathrm{G}(-z,1-z,-z;y)+20\mathrm{H}(1;z)\mathrm{G}(-z,1-z;y)\\
&+16\mathrm{H}(1;z)\mathrm{G}(-z,1-z,0;y)-32\mathrm{H}(1;z)\mathrm{G}(-z,-z,1-z;y)+14\mathrm{H}(1;z)\mathrm{G}(-z;y)\\
&+16\mathrm{H}(1;z)\mathrm{G}(-z,0,1-z;y)-5\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(-z,1,0;y)+8\mathrm{H}(1;z)\mathrm{G}(0,1-z,-z;y)\\
&-10\mathrm{H}(1;z)\mathrm{G}(0,1-z;y)-8\mathrm{H}(1;z)\mathrm{G}(0,1-z,0;y)+16\mathrm{H}(1;z)\mathrm{G}(0,-z,1-z;y)-5\mathrm{H}(1;z)\mathrm{G}(0,-z;y)\\
&-4\mathrm{H}(1;z)\mathrm{G}(0;y)-8\mathrm{H}(1;z)\mathrm{G}(0,0,1-z;y)+8\mathrm{H}(1;z)\mathrm{G}(0,0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,1,0;y)\\
&-8\mathrm{H}(1;z)\mathrm{G}(1,-z,0;y)-8\mathrm{H}(1;z)\mathrm{G}(1,0,-z;y)+5\mathrm{H}(1;z)\mathrm{G}(1,0;y)+8\mathrm{H}(1;z)\mathrm{G}(1,0,0;y)+3\mathrm{H}(1,0;z)\\
&+8\mathrm{H}(1,0;z)\mathrm{G}(1-z,-z;y)-5\mathrm{H}(1,0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)\\
&+8\mathrm{H}(1,0;z)\mathrm{G}(-z,1-z;y)-5\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-4\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)+\mathrm{H}(1,0;z)\mathrm{G}(0;y)\\
&-5\mathrm{H}(1,0,1;z)+8\mathrm{H}(1,0,1;z)\mathrm{G}(-z;y)-4\mathrm{H}(1,0,1;z)\mathrm{G}(0;y)+6\mathrm{H}(1,1;z)+32\mathrm{H}(1,1;z)\mathrm{G}(-z,-z;y)\\
&-20\mathrm{H}(1,1;z)\mathrm{G}(-z;y)-16\mathrm{H}(1,1;z)\mathrm{G}(-z,0;y)-16\mathrm{H}(1,1;z)\mathrm{G}(0,-z;y)+10\mathrm{H}(1,1;z)\mathrm{G}(0;y)\\
&+8\mathrm{H}(1,1;z)\mathrm{G}(0,0;y)+6\mathrm{G}(1-z,1-z;y)+10\mathrm{G}(1-z,1-z,0;y)+8\mathrm{G}(1-z,1-z,0,0;y)\\
&-10\mathrm{G}(1-z,-z,1-z;y)-8\mathrm{G}(1-z,-z,1-z,0;y)-8\mathrm{G}(1-z,-z,0,1-z;y)+7\mathrm{G}(1-z;y)\\
&+10\mathrm{G}(1-z,0,1-z;y)+8\mathrm{G}(1-z,0,1-z,0;y)-8\mathrm{G}(1-z,0,-z,1-z;y)+4\mathrm{G}(1-z,0;y)\\
&+8\mathrm{G}(1-z,0,0,1-z;y)-8\mathrm{G}(1-z,0,0;y)-4\mathrm{G}(1-z,0,1,0;y)-5\mathrm{G}(1-z,1,0;y)\\
&-8\mathrm{G}(1-z,1,0,0;y)-20\mathrm{G}(-z,1-z,1-z;y)-16\mathrm{G}(-z,1-z,1-z,0;y)\\
&+16\mathrm{G}(-z,1-z,-z,1-z;y)-14\mathrm{G}(-z,1-z;y)-16\mathrm{G}(-z,1-z,0,1-z;y)\\
&+5\mathrm{G}(-z,1-z,0;y)+8\mathrm{G}(-z,1-z,1,0;y)+32\mathrm{G}(-z,-z,1-z,1-z;y)-16\mathrm{G}(-z,0,1-z,1-z;y)
\end{aligned}
$$

$$+5\mathrm{G}(-z,0,1-z;y)+8\mathrm{G}(-z,1,1-z,0;y)+8\mathrm{G}(-z,1,0,1-z;y)+10\mathrm{G}(0,1-z,1-z;y)$$
$$+8\mathrm{G}(0,1-z,1-z,0;y)-8\mathrm{G}(0,1-z,-z,1-z;y)+4\mathrm{G}(0,1-z;y)+8\mathrm{G}(0,1-z,0,1-z;y)$$
$$-8\mathrm{G}(0,1-z,0;y)-4\mathrm{G}(0,1-z,1,0;y)-16\mathrm{G}(0,-z,1-z,1-z;y)+5\mathrm{G}(0,-z,1-z;y)$$
$$-\frac{7}{2}\mathrm{G}(0;y)+8\mathrm{G}(0,0,1-z,1-z;y)-8\mathrm{G}(0,0,1-z;y)+3\mathrm{G}(0,0;y)-4\mathrm{G}(0,1,1-z,0;y)$$
$$-4\mathrm{G}(0,1,0,1-z;y)+4\mathrm{G}(0,1,0;y)-5\mathrm{G}(1,1-z,0;y)-8\mathrm{G}(1,1-z,0,0;y)+8\mathrm{G}(1,-z,1-z,0;y)$$
$$+8\mathrm{G}(1,-z,0,1-z;y)-5\mathrm{G}(1,0,1-z;y)-8\mathrm{G}(1,0,1-z,0;y)+8\mathrm{G}(1,0,-z,1-z;y)-7\mathrm{G}(1,0;y)$$
$$-8\mathrm{G}(1,0,0,1-z;y)+8\mathrm{G}(1,0,0;y)+4\mathrm{G}(1,0,1,0;y)+8\mathrm{G}(1,1,0,0;y)\;, \tag{4.28}$$

$$D_{11}(y,z) =$$
$$\frac{1}{6y}\big[\mathrm{H}(0;z)+\mathrm{G}(0;y)\big]+\frac{z}{6(1-y)^2}\big[-\mathrm{H}(0;z)\mathrm{G}(0;y)-2\mathrm{G}(0,0;y)\big]+\frac{z}{6(1-y)}\big[-\mathrm{H}(0;z)$$
$$-3\mathrm{H}(0;z)\mathrm{G}(0;y)-\mathrm{G}(0;y)-6\mathrm{G}(0,0;y)\big]+\frac{1}{3(1-y)}\big[2\mathrm{H}(0;z)\mathrm{G}(0;y)+4\mathrm{G}(0,0;y)\big]$$
$$+\frac{T\pi^2}{36}\big[-22-\mathrm{H}(0;z)-\mathrm{G}(0;y)\big]+\frac{T}{18}\big[-12\mathrm{H}(0;z)-10\mathrm{H}(0;z)\mathrm{G}(0;y)-6\mathrm{H}(0;z)\mathrm{G}(0,0;y)$$
$$+3\mathrm{H}(0;z)\mathrm{G}(1,0;y)-10\mathrm{H}(0,0;z)-6\mathrm{H}(0,0;z)\mathrm{G}(0;y)-3\mathrm{H}(0,1,0;z)-3\mathrm{H}(1,0;z)\mathrm{G}(0;y)$$
$$-6\mathrm{H}(1,0,0;z)-12\mathrm{G}(0;y)-10\mathrm{G}(0,0;y)+3\mathrm{G}(0,1,0;y)+6\mathrm{G}(1,0,0;y)\big]\;, \tag{4.29}$$

$$E_{11}(y,z) =$$
$$\frac{z}{3y}\big[\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)+2\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)$$
$$+\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+\mathrm{H}(1;z)\mathrm{G}(0,-z;y)+\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-\mathrm{G}(-z,1-z,0;y)$$
$$-\mathrm{G}(-z,0,1-z;y)-\mathrm{G}(0,-z,1-z;y)\big]+\frac{1}{6y}\big[\mathrm{H}(0;z)-4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)$$
$$+4\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)-4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-8\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)-4\mathrm{H}(0,1;z)\mathrm{G}(-z;y)$$
$$-4\mathrm{H}(1;z)\mathrm{G}(-z,0;y)-4\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-4\mathrm{H}(1,0;z)\mathrm{G}(-z;y)+4\mathrm{G}(-z,1-z,0;y)$$
$$+4\mathrm{G}(-z,0,1-z;y)+4\mathrm{G}(0,-z,1-z;y)+\mathrm{G}(0;y)\big]+\frac{z}{6(1-y)^2}\big[\mathrm{H}(0;z)\mathrm{G}(0;y)+2\mathrm{G}(0,0;y)\big]$$
$$+\frac{z}{6(1-y)}\big[\mathrm{H}(0;z)+3\mathrm{H}(0;z)\mathrm{G}(0;y)+\mathrm{G}(0;y)+6\mathrm{G}(0,0;y)\big]+\frac{1}{3(1-y)}\big[-\mathrm{H}(0;z)\mathrm{G}(0;y)$$
$$-2\mathrm{G}(0,0;y)\big]+\frac{1}{3(y+z)^2}\big[-\mathrm{H}(0;z)\mathrm{G}(1-z;y)+\mathrm{H}(0,1;z)+\mathrm{H}(1;z)\mathrm{G}(0;y)+\mathrm{H}(1,0;z)$$
$$-\mathrm{G}(1-z,0;y)-\mathrm{G}(0,1-z;y)\big]+\frac{1}{3(y+z)}\big[-\mathrm{H}(0;z)-\mathrm{G}(0;y)\big]+\frac{T}{12}\big[8\mathrm{H}(0;z)$$
$$+3\mathrm{H}(0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-4\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+4\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)$$
$$-2\mathrm{H}(0;z)\mathrm{G}(1,0;y)+4\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)+4\mathrm{H}(0,0,1;z)-3\mathrm{H}(0,1;z)+4\mathrm{H}(0,1;z)\mathrm{G}(-z;y)$$
$$+2\mathrm{H}(0,1,0;z)+4\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+4\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-3\mathrm{H}(1;z)\mathrm{G}(0;y)-4\mathrm{H}(1;z)\mathrm{G}(0,0;y)$$
$$-3\mathrm{H}(1,0;z)+4\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-2\mathrm{H}(1,0;z)\mathrm{G}(0;y)+3\mathrm{G}(1-z,0;y)+4\mathrm{G}(1-z,0,0;y)$$
$$-4\mathrm{G}(-z,1-z,0;y)-4\mathrm{G}(-z,0,1-z;y)+3\mathrm{G}(0,1-z;y)+4\mathrm{G}(0,1-z,0;y)-4\mathrm{G}(0,-z,1-z;y)$$
$$+8\mathrm{G}(0;y)+4\mathrm{G}(0,0,1-z;y)-2\mathrm{G}(0,1,0;y)-4\mathrm{G}(1,0,0;y)\big]+\frac{1}{3}\big[\mathrm{H}(0;z)\mathrm{G}(1-z;y)$$

$$
\begin{aligned}
&+2\mathrm{H}(0;z)\mathrm{G}(1-z,0;y)-2\mathrm{H}(0;z)\mathrm{G}(-z,1-z;y)+2\mathrm{H}(0;z)\mathrm{G}(0,1-z;y)-\mathrm{H}(0;z)\mathrm{G}(0;y)\\
&-\mathrm{H}(0;z)\mathrm{G}(1,0;y)-\mathrm{H}(0,0;z)+2\mathrm{H}(0,0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(0,0,1;z)-\mathrm{H}(0,1;z)\\
&+2\mathrm{H}(0,1;z)\mathrm{G}(-z;y)+\mathrm{H}(0,1,0;z)+2\mathrm{H}(1;z)\mathrm{G}(-z,0;y)+2\mathrm{H}(1;z)\mathrm{G}(0,-z;y)-\mathrm{H}(1;z)\mathrm{G}(0;y)\\
&-2\mathrm{H}(1;z)\mathrm{G}(0,0;y)-\mathrm{H}(1,0;z)+2\mathrm{H}(1,0;z)\mathrm{G}(-z;y)-\mathrm{H}(1,0;z)\mathrm{G}(0;y)+\mathrm{G}(1-z,0;y)\\
&+2\mathrm{G}(1-z,0,0;y)-2\mathrm{G}(-z,1-z,0;y)-2\mathrm{G}(-z,0,1-z;y)+\mathrm{G}(0,1-z;y)+2\mathrm{G}(0,1-z,0;y)\\
&-2\mathrm{G}(0,-z,1-z;y)+2\mathrm{G}(0,0,1-z;y)-\mathrm{G}(0,0;y)-\mathrm{G}(0,1,0;y)-2\mathrm{G}(1,0,0;y)\Big]\,, \qquad (4.30)
\end{aligned}
$$

$$
\begin{aligned}
&F_{11}(y,z)=\\
&\qquad\frac{T}{36}\left[2\pi^2+\mathrm{H}(0;z)\mathrm{G}(0;y)+\mathrm{H}(0,0;z)+\mathrm{G}(0,0;y)\right]\,. \qquad (4.31)
\end{aligned}
$$

5 Conclusions and Outlook

In this paper, we have presented analytic formulae for the two-loop virtual corrections to the process $\gamma^* \to q\bar{q}g$, which arise from the interference of the two-loop with the tree amplitude and from the self-interference of the one-loop amplitude. Together with the contribution from the self-interference of the one-loop amplitudes for $\gamma^* \to ggg$ [45, 46], these form the full $\mathcal{O}(\alpha_s^3)$ corrections to the three-parton subprocess contribution to $e^+e^- \to 3$ jets at NNLO.

Similar results can in principle be obtained for $(2+1)$-jet production in deep inelastic ep scattering or $(V+1)$-jet production in hadron–hadron collisions. However, the complexity of the cut structure of the non-planar graphs together with the rather different domains of convergence of the one- and two-dimensional harmonic polylogarithms makes this a non-trivial task, and we defer this to a later paper.

It must also be kept in mind that these virtual corrections form only part of a full NNLO calculation, which also has to include the one-loop corrections to $\gamma^* \to 4$ partons [14–17] where one of the partons becomes collinear or soft, as well as tree-level processes $\gamma^* \to 5$ partons [11–13] with two soft or collinear partons. Only after summing all these contributions (and including terms from the renormalization of parton distributions for processes with partons in the initial state), do the divergent terms cancel among one another. The factorization properties of both the one-loop, one-unresolved-parton contribution [60–65] and the tree-level, two-unresolved-parton contributions [66–69] have been studied, but a systematic procedure for isolating the infrared singularities has not been established. Although this is still an open and highly non-trivial issue, significant progress is anticipated in the near future.

The remaining finite terms must then be combined into a numerical program implementing the experimental definition of jet observables and event-shape variables. A first calculation involving the above features was presented for the case of photon-plus-one-jet final states in electron–positron annihilation in [70, 71], thus demonstrating the feasibility of this type of calculations. A prerequisite for such a numerical program is a stable and efficient next-to-leading order four-jet program, where the infrared singularities for the one-loop $\gamma^* \to 4$ partons are combined with the tree-level $\gamma^* \to 5$ parton with one parton unresolved. Four such programs currently exist [72–75], each of which could be used as a starting point for a full $\mathcal{O}(\alpha_s^3)$ NNLO three-jet program.

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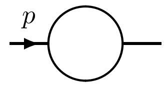
A One-loop master integrals

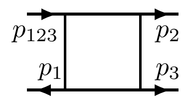
In this appendix, we list the expansions for the one-loop master integrals appearing in $\langle\mathcal{M}^{(0)}|\mathcal{M}^{(1)}\rangle$ and $\langle\mathcal{M}^{(1)}|\mathcal{M}^{(1)}\rangle$. These squared amplitudes can be expressed in terms of only two master integrals evaluated at $d = 4-2\epsilon$,

$$\mathrm{Bub}(p^2) \quad = $$



$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2(k-p)^2}, \tag{A.1}$$

$$\mathrm{Box}(s_{23}, s_{13}, s_{123}) \quad = $$



$$= \int \frac{\mathrm{d}^d k}{(2\pi)^d} \frac{1}{k^2(k-p_2)^2(k-p_2-p_3)^2(k-p_1-p_2-p_3)^2} . \tag{A.2}$$

Note that in Eqs. (4.13) and (4.14) we have written the one-loop functions f_1 and f_2 in terms of the one-loop box integral in $d = 6-2\epsilon$. This is straightforwardly related to the box integral in $d = 4-2\epsilon$ dimensions by

$$\begin{aligned}\mathrm{Box}^6(s_{23}, s_{13}, s_{123}) \quad = \quad & -\frac{s_{13}s_{23}}{2(d-3)s_{12}}\mathrm{Box}(s_{23}, s_{13}, s_{123}) \\ & -\frac{2}{s_{12}(d-4)}\left(\mathrm{Bub}(s_{13}) + \mathrm{Bub}(s_{23}) - \mathrm{Bub}(s_{123})\right) .\end{aligned} \tag{A.3}$$

Closed expressions for these integrals for symbolic d in terms of Γ-functions and the ${}_2F_1$ hypergeometric function have been known for a long time (see e.g. [7, 21]). The bubble integral reads

$$\mathrm{Bub}(s_{12}) = i\frac{(4\pi)^\epsilon}{16\pi^2}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-s_{12}\right)^{-\epsilon}\frac{1}{\epsilon(1-2\epsilon)} . \tag{A.4}$$

In the present context, an expansion of the box integral to the second order in $\epsilon = (4-d)/2$ is required.

$$\mathrm{Box}(s_{23}, s_{13}, s_{123}) = i\frac{(4\pi)^\epsilon}{16\pi^2}\frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(-s_{123}\right)^{-2-\epsilon}\frac{1}{yz}\sum_{i=-2}^{2}\frac{l_{4.1,i}\left(\frac{s_{13}}{s_{123}}, \frac{s_{23}}{s_{123}}\right)}{\epsilon^i} + \mathcal{O}(\epsilon^3), \tag{A.5}$$

with

$$l_{4.1,2}(y,z) \quad = \quad 2 , \tag{A.6}$$

$$l_{4.1,1}(y,z) \quad = \quad -2\mathrm{H}(0;z) - 2\mathrm{G}(0;y) , \tag{A.7}$$

$$l_{4.1,0}(y,z) \quad = \quad 2\mathrm{H}(0;z)\mathrm{G}(0;y) + 2\mathrm{H}(0,0;z) + 2\mathrm{H}(1,0;z) + 2\mathrm{G}(0,0;y) - 2\mathrm{G}(1,0;y) + \frac{\pi^2}{3} , \tag{A.8}$$

$$\begin{aligned}l_{4.1,-1}(y,z) \quad = \quad & 2\mathrm{H}(0;z)\mathrm{G}(1-z,0;y) - 2\mathrm{H}(0;z)\mathrm{G}(0,0;y) - 2\mathrm{H}(0,0;z)\mathrm{G}(0;y) - 2\mathrm{H}(0,0,0;z) \\ & -2\mathrm{H}(0,1,0;z) + 2\mathrm{H}(1,0;z)\mathrm{G}(1-z;y) - 2\mathrm{H}(1,0;z)\mathrm{G}(0;y) - 2\mathrm{H}(1,0,0;z) \\ & -2\mathrm{H}(1,1,0;z) - 2\mathrm{G}(1-z,1,0;y) - 2\mathrm{G}(0,0,0;y) + 2\mathrm{G}(0,1,0;y) + 2\mathrm{G}(1,0,0;y)) \\ & +\frac{\pi^2}{3}\left[-\mathrm{H}(0;z) - \mathrm{H}(1;z) + \mathrm{G}(1-z;y) - \mathrm{G}(0;y)\right] ,\end{aligned} \tag{A.9}$$

$$\begin{aligned}l_{4.1,-2}(y,z) \quad = \quad & 2\mathrm{H}(0;z)\mathrm{G}(1-z,1-z,0;y) - 2\mathrm{H}(0;z)\mathrm{G}(1-z,0,0;y) - 2\mathrm{H}(0;z)\mathrm{G}(0,1-z,0;y) \\ & +2\mathrm{H}(0;z)\mathrm{G}(0,0,0;y) - 2\mathrm{H}(0,0;z)\mathrm{G}(1-z,0;y) + 2\mathrm{H}(0,0;z)\mathrm{G}(0,0;y) \\ & +2\mathrm{H}(0,0,0;z)\mathrm{G}(0;y) + 2\mathrm{H}(0,0,0,0;z) + 2\mathrm{H}(0,0,1,0;z) - 2\mathrm{H}(0,1,0;z)\mathrm{G}(1-z;y)\end{aligned}$$

$$
\begin{aligned}
&+2\mathrm{H}(0,1,0;z)\mathrm{G}(0;y)+2\mathrm{H}(0,1,0,0;z)+2\mathrm{H}(0,1,1,0;z)+2\mathrm{H}(1,0;z)\mathrm{G}(1-z,1-z;y)\\
&-2\mathrm{H}(1,0;z)\mathrm{G}(1-z,0;y)-2\mathrm{H}(1,0;z)\mathrm{G}(0,1-z;y)+2\mathrm{H}(1,0;z)\mathrm{G}(0,0;y)\\
&-2\mathrm{H}(1,0,0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1,0,0;z)\mathrm{G}(0;y)+2\mathrm{H}(1,0,0,0;z)+2\mathrm{H}(1,0,1,0;z)\\
&-2\mathrm{H}(1,1,0;z)\mathrm{G}(1-z;y)+2\mathrm{H}(1,1,0;z)\mathrm{G}(0;y)+2\mathrm{H}(1,1,0,0;z)+2\mathrm{H}(1,1,1,0;z)\\
&-2\mathrm{G}(1-z,1-z,1,0;y)+2\mathrm{G}(1-z,0,1,0;y)+2\mathrm{G}(1-z,1,0,0;y)+2\mathrm{G}(0,1-z,1,0;y)\\
&+2\mathrm{G}(0,0,0,0;y)-2\mathrm{G}(0,0,1,0;y)-2\mathrm{G}(0,1,0,0;y)-2\mathrm{G}(1,0,0,0;y)+\frac{7\pi^4}{180}\\
&+\frac{\pi^2}{3}\Big[-\mathrm{H}(0;z)\mathrm{G}(1-z;y)+\mathrm{H}(0;z)\mathrm{G}(0;y)+\mathrm{H}(0,0;z)+\mathrm{H}(0,1;z)-\mathrm{H}(1;z)\mathrm{G}(1-z;y)\\
&+\mathrm{H}(1;z)\mathrm{G}(0;y)+\mathrm{H}(1,0;z)+\mathrm{H}(1,1;z)+\mathrm{G}(1-z,1-z;y)-\mathrm{G}(1-z,0;y)\\
&-\mathrm{G}(0,1-z;y)+\mathrm{G}(0,0;y)\Big]\ . \qquad \text{(A.10)}
\end{aligned}
$$

B Harmonic polylogarithms

The generalized polylogarithms $\mathrm{S}_{n,p}(x)$ of Nielsen [38] turn out to be insufficient for the computation of multi-scale integrals beyond one loop. To overcome this limitation, one has to extend generalized polylogarithms to harmonic polylogarithms [34, 37].

Harmonic polylogarithms are obtained by the repeated integration of rational factors. If these rational factors contain, besides the integration variable, only constants, the resulting functions are one-dimensional harmonic polylogarithms (or simply harmonic polylogarithms, HPLs) [37,40]. If the rational factors depend on a further variable, one obtains two-dimensional harmonic polylogarithms (2dHPLs) [34, 41]. In the following, we define both classes of functions, and summarize their properties.

B.1 One-dimensional harmonic polylogarithms

The HPLs, introduced in [37], are one-variable functions $\mathrm{H}(\vec{a};x)$ depending, besides the argument x, on a set of indices, grouped for convenience into the vector $\vec{a}$, whose components can take one of the three values $(1,0,-1)$ and whose number is the weight w of the HPL. More explicitly, the three HPLs with $w=1$ are defined as

$$
\begin{aligned}
\mathrm{H}(1;x) &= \int_0^x \frac{\mathrm{d}x'}{1-x'} = -\ln(1-x)\ ,\\
\mathrm{H}(0;x) &= \ln x\ ,\\
\mathrm{H}(-1;x) &= \int_0^x \frac{\mathrm{d}x'}{1+x'} = \ln(1+x)\ ; \qquad \text{(B.1)}
\end{aligned}
$$

their derivatives can be written as

$$
\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{H}(a;x) = \mathrm{f}(a;x)\ , \qquad a=1,0,-1\ , \qquad \text{(B.2)}
$$

where the 3 rational fractions $\mathrm{f}(a;x)$ are given by

$$
\begin{aligned}
\mathrm{f}(1;x) &= \frac{1}{1-x}\ ,\\
\mathrm{f}(0;x) &= \frac{1}{x}\ ,\\
\mathrm{f}(-1;x) &= \frac{1}{1+x}\ . \qquad \text{(B.3)}
\end{aligned}
$$

For weight w larger than 1, write $\vec{a} = (a, \vec{b})$, where a is the leftmost component of $\vec{a}$ and $\vec{b}$ stands for the vector of the remaining $(w-1)$ components. The harmonic polylogarithms of weight w are then defined as follows: if all the w components of $\vec{a}$ take the value 0, $\vec{a}$ is said to take the value $\vec{0}_w$ and

$$\mathrm{H}(\vec{0}_w; x) = \frac{1}{w!} \ln^w x \ , \tag{B.4}$$

while, if $\vec{a} \neq \vec{0}_w$,

$$\mathrm{H}(\vec{a}; x) = \int_0^x \mathrm{d}x' \ \mathrm{f}(a; x') \ \mathrm{H}(\vec{b}; x') \ . \tag{B.5}$$

In any case the derivatives can be written in the compact form

$$\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{H}(\vec{a}; x) = \mathrm{f}(a; x)\mathrm{H}(\vec{b}; x) \ , \tag{B.6}$$

where, again, a is the leftmost component of $\vec{a}$ and $\vec{b}$ stands for the remaining $(w-1)$ components.

It is immediate to see, from the very definition Eq. (B.5), that there are 3^w HPLs of weight w, and that they are linearly independent. The HPLs are generalizations of Nielsen's polylogarithms [38]. The function $\mathrm{S}_{n,p}(x)$, in Nielsen's notation, is equal to the HPL whose first n indices are all equal to 0 and the remaining p indices all equal to 1:

$$\mathrm{S}_{n,p}(x) = \mathrm{H}(\vec{0}_n, \vec{1}_p; x) \ ; \tag{B.7}$$

in particular the Euler polylogarithms $\mathrm{Li}_n(x) = \mathrm{S}_{n-1,1}(x)$ correspond to

$$\mathrm{Li}_n(x) = \mathrm{H}(\vec{0}_{n-1}, 1; x) \ . \tag{B.8}$$

As shown in [37], the product of two HPLs of a same argument x and weights p, q can be expressed as a combination of HPLs of that argument and weight $r = p + q$, according to the product identity

$$\mathrm{H}(\vec{p}; x)\mathrm{H}(\vec{q}; x) \quad = \sum_{\vec{r} = \vec{p} \uplus \vec{q}} \mathrm{H}(\vec{r}; x) \ , \tag{B.9}$$

where $\vec{p}, \vec{q}$ stand for the p and q components of the indices of the two HPLs, while $\vec{p} \uplus \vec{q}$ represents all mergers of $\vec{p}$ and $\vec{q}$ into the vector $\vec{r}$ with r components, in which the relative orders of the elements of $\vec{p}$ and $\vec{q}$ are preserved.

The explicit formulae relevant up to weight 4 are

$$\mathrm{H}(a; x) \ \mathrm{H}(b; x) = \mathrm{H}(a, b; x) + \mathrm{H}(b, a; x) \ , \tag{B.10}$$

$$\begin{aligned} \mathrm{H}(a; x) \ \mathrm{H}(b, c; x) &= \mathrm{H}(a, b, c; x) + \mathrm{H}(b, a, c; x) + \mathrm{H}(b, c, a; x) \ , \\ \mathrm{H}(a; x) \ \mathrm{H}(b, c, d; x) &= \mathrm{H}(a, b, c, d; x) + \mathrm{H}(b, a, c, d; x) + \mathrm{H}(b, c, a, d; x) + \mathrm{H}(b, c, d, a; x) \ , \end{aligned} \tag{B.11}$$

and

$$\begin{aligned} \mathrm{H}(a, b; x) \ \mathrm{H}(c, d; x) &= \mathrm{H}(a, b, c, d; x) + \mathrm{H}(a, c, b, d; x) + \mathrm{H}(a, c, d, b; x) \\ &+ \mathrm{H}(c, a, b, d; x) + \mathrm{H}(c, a, d, b; x) + \mathrm{H}(c, d, a, b; x) \ , \end{aligned} \tag{B.12}$$

where a, b, c, d are indices taking any of the values $(1, 0, -1)$. The formulae can be easily verified, one at a time, by observing that they are true at some specific point (such as $x = 0$, where all the HPLs vanish except in the otherwise trivial case in which all the indices are equal to 0), then taking the x-derivatives of the two sides according to Eq. (B.6) and checking that they are equal (using when needed the previously established lower-weight formulae).

Another class of identities is obtained by integrating (B.4) by parts. These integration-by-parts (IBP) identities read:

$$\begin{aligned} \mathrm{H}(m_1, \ldots, m_q; x) &= \mathrm{H}(m_1; x)\mathrm{H}(m_2, \ldots, m_q; x) - \mathrm{H}(m_2, m_1; x)\mathrm{H}(m_3, \ldots, m_q; x) \\ &+ \ldots + (-1)^{q+1}\mathrm{H}(m_q, \ldots, m_1; x) \ . \end{aligned} \tag{B.13}$$

These identities are not fully linearly independent from the product identities.

A numerical implementation of the HPLs up to weight $w = 4$ is available [40].

B.2 Two-dimensional harmonic polylogarithms

The 2dHPLs family is obtained by the repeated integration, in the variable y, of rational factors chosen, in any order, from the set $1/y$, $1/(y-1)$, $1/(y+z-1)$, $1/(y+z)$, where z is another independent variable (hence the 'two-dimensional' in the name). In full generality, let us define the rational factor as

$$\mathrm{g}(a;y) = \frac{1}{y-a} \ , \tag{B.14}$$

where a is the *index*, which can depend on z, $a = a(z)$; the rational factors which we consider for the 2dHPLs are then

$$\begin{aligned} \mathrm{g}(0;y) &= \frac{1}{y} \ , \\ \mathrm{g}(1;y) &= \frac{1}{y-1} \ , \\ \mathrm{g}(1-z;y) &= \frac{1}{y+z-1} \ , \\ \mathrm{g}(-z;y) &= \frac{1}{y+z} \ . \end{aligned} \tag{B.15}$$

With the above definitions the index takes one of the values $0, 1, (1-z)$ and $(-z)$.

Correspondingly, the 2dHPLs at weight $w = 1$ (i.e. depending, besides the variable y, on a single further argument, or *index*) are defined to be

$$\begin{aligned} \mathrm{G}(0;y) &= \ln y \ , \\ \mathrm{G}(1;y) &= \ln(1-y) \ , \\ \mathrm{G}(1-z;y) &= \ln\left(1 - \frac{y}{1-z}\right) \ , \\ \mathrm{G}(-z;y) &= \ln\left(1 + \frac{y}{z}\right) \ . \end{aligned} \tag{B.16}$$

The 2dHPLs of weight w larger than 1 depend on a set of w indices, which can be grouped into a w-dimensional *vector* of indices $\vec{a}$. By writing the vector as $\vec{a} = (a, \vec{b})$, where a is the leftmost component of $\vec{a}$ and $\vec{b}$ stands for the vector of the remaining $(w-1)$ components, the 2dHPLs are then defined as follows: if all the w components of $\vec{a}$ take the value 0, $\vec{a}$ is written as $\vec{0}_w$ and

$$\mathrm{G}(\vec{0}_w;y) = \frac{1}{w!} \ln^w y \ , \tag{B.17}$$

while, if $\vec{a} \neq \vec{0}_w$,

$$\mathrm{G}(\vec{a};y) = \int_0^y \mathrm{d}y' \ \mathrm{g}(a;y') \ \mathrm{G}(\vec{b};y') \ . \tag{B.18}$$

In any case the derivatives can be written in the compact form

$$\frac{\mathrm{d}}{\mathrm{d}y}\mathrm{G}(\vec{a};y) = \mathrm{g}(a;y)\mathrm{G}(\vec{b};y) \ , \tag{B.19}$$

where, again, a is the leftmost component of $\vec{a}$ and $\vec{b}$ stands for the remaining $(w-1)$ components.

It should be noted that the notation for the 2dHPLs employed here is the notation of [41], which is different from the original definition proposed in [34]. Detailed conversion rules between different notations, as well as relations to similar functions in the mathematical literature (hyperlogarithms and multiple polylogarithms) can be found in the appendix of [41].

Algebra and reduction equations of the 2dHPLs are the same as for the ordinary HPLs. The product of two 2dHPLs of a same argument y and weights p, q can be expressed as a combination of 2dHPLs of that argument and weight $r = p + q$, according to the product identity

$$\mathrm{G}(\vec{p};x)\mathrm{G}(\vec{q};x) = \sum_{\vec{r}=\vec{p}\uplus\vec{q}} \mathrm{G}(\vec{r};x) \; , \tag{B.20}$$

where $\vec{p}, \vec{q}$ stand for the p and q components of the indices of the two 2dHPLs, while $\vec{p} \uplus \vec{q}$ represents all possible mergers of $\vec{p}$ and $\vec{q}$ into the vector $\vec{r}$ with r components, in which the relative orders of the elements of $\vec{p}$ and $\vec{q}$ are preserved. The explicit product identities up to weight $w = 4$ are identical to those for the HPLs (B.10)–(B.12), with all H replaced by G.

The integration-by-parts identities read:

$$\begin{aligned} \mathrm{G}(m_1,\ldots,m_q;x) &= \mathrm{G}(m_1;x)\mathrm{G}(m_2,\ldots,m_q;x) - \mathrm{G}(m_2,m_1;x)\mathrm{G}(m_3,\ldots,m_q;x) \\ &\quad + \ldots + (-1)^{q+1}\mathrm{G}(m_q,\ldots,m_1;x) \; . \end{aligned} \tag{B.21}$$

A numerical implementation of the 2dHPLs up to weight $w = 4$ is available [41].

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