

Spacetime decay of cones at strong coupling

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ABSTRACT: We study time dependent solutions in dilaton gravity which correspond to the decay of conical spacetimes. In string theory this can be interpreted as a strong coupling limit of the decay of a non-supersymmetric orbifold spacetime with localized tachyons.

KEYWORDS: Tachyon condensation, supergravity solutions.

In the last few years progress has been achieved in studying string theory in the presence of instabilities represented by tachyon fields. The problem is particularly well understood when the tachyons appear in the open string spectrum [1]. In this case gravity can be ignored at weak string coupling and the problem can be studied by a variety of means including string field theory [2, 3, 4], world-sheet renormalization group (RG) flow [5] and boundary string field theory [6, 7, 8, 9].

The study of closed string tachyon condensation is both more difficult and more interesting. Open string tachyon condensation can be studied in a fixed, often flat, spacetime background. In contrast, closed string tachyons couple to other closed string modes, including the graviton, and hence the spacetime geometry itself will generically change in the decay process.

In some cases, closed string tachyons may be localized on defects such as NS5-branes or orbifolds [10]. The localization makes the problem more tractable and tachyon condensation in this system has been studied using RG flow [10, 11, 12, 13, 14], D-brane techniques [10] and duality arguments [15]. What has not been done so far is to study this process directly in the supergravity régime by finding time-dependent solutions which interpolate between one conical spacetime at early times and another one at late times. That is what we shall do in this note.

The problem originally considered in [10] involves string propagation on the orbifold spacetime

$$R^{7,1} \times R^2/Z_n \tag{1}$$

That is, on eight-dimensional Minkowski space times a two-dimensional cone. To be concrete we will consider IIA string theory on such a space. String propagation on such an orbifold is exactly soluble and one finds tachyonic modes in the twisted sector of the orbifold. These correspond to instabilities which are localized at the fixed point of the orbifold, that is at the tip of the cone. Any perturbation of the system which excites the tachyon field should lead to a time-dependent solution of string theory with the solution at late times approaching a new solution of string theory. The full time dependent problem is currently too difficult to solve, so what has been done instead is to replace time dependence with renormalization group flow and to then study the corresponding RG equations. One result of these studies is the claim that a Z_n orbifold spacetime can decay to a $Z_{n'}$ spacetime with $n' < n$, that is to a cone with a smaller deficit angle (with $n' = 1$ being the stable case of flat space).

Here we will take a different approach, studying the time-dependent equations but in a low-energy limit. The mass of the tachyons in these theories is typically string scale, so there is no consistent approximation in which we keep the tachyon fields and ignore other string fields. However we can consider a low-energy supergravity action with the tachyon fields simply acting as source terms for the massless fields. The tachyon field will thus provide initial conditions for the massless fields but we

can then study the time-evolution of the massless fields with these initial conditions. The relevant massless fields are the graviton and dilaton. Thus we are led to study the time-dependent equations for gravity coupled to a dilaton field.

To get a feel for the problem, we start by deriving a spacetime which consists of an expanding shell dividing two regions of differing conical deficits. Any two spacetimes can be glued together across a wall, provided the induced metric on the wall is identical in each domain. The energy-momentum of the wall is then given by the jump in extrinsic curvature via the Israel conditions. In general a wall corresponds to a source, which will require some sort of matter; whether or not this can be achieved with the fields at hand will be addressed presently.

Thus consider the conical spacetime

$$ds^2 = dt^2 - dr^2 - dy_i^2 - r^2(1 - \Delta)^2 d\theta^2 \quad (2)$$

Here the y_i are coordinates on R^7 while (r, θ) are polar coordinates for the two-dimensional cone. The angular variable, θ , runs from 0 to 2π on each side of the wall for agreement of the internal wall metric, so if the deficit angle differs, then clearly the value of r on each side of the wall has to be different in order that $g_{\theta\theta}$ be the same. Note that this means $r_+ \propto r_-$.

We suppose that the wall has trajectory

$$x_\pm^\mu(\tau) = (t_\pm(\tau), r_\pm(\tau)) \quad (3)$$

on each side, where $\dot{t}_\pm^2 - \dot{r}_\pm^2 = 1$. This has unit normal

$$n_{\mu\pm} = (\dot{r}_\pm, -\dot{t}_\pm) \quad (4)$$

which gives for the extrinsic curvature

$$K_{\tau\tau} = \ddot{r}/\dot{t} \quad , \quad K_{\theta\theta} = -\dot{t}r(1 - \Delta)^2 \quad (5)$$

The energy-momentum of the wall is given by Israel's equations, $T_{ab} = \delta K_{ab} - \delta K h_{ab}$, and reads:

$$T_y^y = -\frac{\ddot{r}_+}{\dot{t}_+} + \frac{\ddot{r}_-}{\dot{t}_-} - \frac{\dot{t}_+}{r_+} + \frac{\dot{t}_-}{r_-} \quad (6)$$

$$T_\tau^\tau = -\frac{\dot{t}_+}{r_+} + \frac{\dot{t}_-}{r_-} = T_y^y - T_\theta^\theta \quad (7)$$

So, for example, if $r = vt$, $\dot{t} = \gamma = 1/\sqrt{1-v^2}$, and $\dot{r} = v\gamma$. The energy-momentum of this boundary is then

$$T_\tau^\tau = T_y^y = -\frac{\gamma_+}{r_+} + \frac{\gamma_-}{r_-} = (\gamma_-(1 - \Delta_-) - \gamma_+(1 - \Delta_+)) [(1 - \Delta)r]^{-1} \quad (8)$$

$$T_\theta^\theta = 0 \quad (9)$$

If we assume that the velocities on each side are the same (which is probably not necessary) then we have

$$T_\tau^\tau = T_y^y = \frac{\gamma}{(1 - \Delta r)} (\Delta_+ - \Delta_-) \quad (10)$$

Assuming that the exterior conical deficit exceeds the interior one, then this corresponds to a positive tension brane, but smeared over the θ -direction, as it has no $\theta - \theta$ energy momentum.

This corresponds to an expanding region of lower conical deficit spacetime separated from the original spacetime by an expanding shell - a source. Whether or not this can be translated into a genuine solution to low-energy string theory necessitates solving the gravity equations of motion with dilaton source terms. We now show that this is possible in a certain limit of string theory.

Instead of solving the gravity problem in the low-energy limit of ten-dimensional string theory, that is gravity with a dilaton, we consider an M-theory approach using eleven-dimensional gravity which renders the problem purely geometrical. This approach is valid in the strong coupling limit of ten-dimensional IIA string theory.

We look for a solution involving only the metric in eleven dimensions, *i.e.*, the three-form potential $C_{ABC} \equiv 0$. We will also set the Kaluza-Klein (KK) gauge field to zero, consistent with the RG equations, which can be consistently truncated to those for only the metric and dilaton [10].

We take spacetime to be the product of a seven-dimensional Ricci flat manifold with coordinates y^i , the M-theory circle with coordinate ψ and radius determined by the dilaton D in the usual way, and a $2 + 1$ dimensional spacetime with coordinates (t, r, θ) . For the latter we consider the most general time-dependent metric with a single rotational degree of freedom, leading to the ansatz:

$$ds^2 = B^2(t, r) [dt^2 - dr^2] - A^2(t, r) dy_i^2 - C^2(t, r) d\theta^2 - D^2(t, r) d\psi^2 \quad (11)$$

The Ricci curvature of this metric is:

$$R_y^y = -\frac{1}{B^2} \left[\frac{\partial_+ \partial_- A}{A} + 6 \frac{\partial_+ A \partial_- A}{A^2} + \frac{1}{2} \frac{\partial_+ A}{A} \frac{\partial_- (CD)}{CD} + \frac{1}{2} \frac{\partial_- A}{A} \frac{\partial_+ (CD)}{CD} \right] \quad (12)$$

$$R_\theta^\theta = -\frac{1}{B^2} \left[\frac{\partial_+ \partial_- C}{C} + \frac{1}{2} \frac{\partial_+ C}{C} \frac{\partial_- (A^7 D)}{A^7 D} + \frac{1}{2} \frac{\partial_- C}{C} \frac{\partial_+ (A^7 D)}{A^7 D} \right] \quad (13)$$

$$R_\psi^\psi = -\frac{1}{B^2} \left[\frac{\partial_+ \partial_- D}{D} + \frac{1}{2} \frac{\partial_+ D}{D} \frac{\partial_- (A^7 C)}{A^7 C} + \frac{1}{2} \frac{\partial_- D}{D} \frac{\partial_+ (A^7 C)}{A^7 C} \right] \quad (14)$$

$$R_t^t = \frac{1}{B^2} \left[-7 \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\ddot{D}}{D} + \frac{B'}{B} \frac{(A^7 CD)'}{A^7 CD} + \frac{\dot{B}}{B} \frac{(A^7 CD)'}{A^7 CD} - \partial_+ \partial_- (\ln B) \right] \quad (15)$$

$$R_r^r = \frac{1}{B^2} \left[7 \frac{A''}{A} + \frac{C''}{C} + \frac{D''}{D} - \frac{B'}{B} \frac{(A^7 CD)'}{A^7 CD} - \frac{\dot{B}}{B} \frac{(A^7 CD)'}{A^7 CD} - \partial_+ \partial_- (\ln B) \right] \quad (16)$$

$$R_{rt} = -\frac{\dot{C}'}{C} - \frac{\dot{D}'}{D} - 7 \frac{\dot{A}'}{A} + \frac{\dot{B}}{B} \frac{(A^7 CD)'}{A^7 CD} + \frac{B'}{B} \frac{(A^7 CD)'}{A^7 CD} \quad (17)$$

where $\partial_{\pm} = \partial/\partial x_{\pm}$, and $x_{\pm} = (t \pm r)/2$.

Although this form of the Ricci tensor does not look particularly illuminating, it is possible to rewrite the metric functions to produce instead a set of free and interacting fields [16] (one of which can be identified as the dilaton) by defining:

$$\ln A = \frac{\sigma}{7} \quad (18)$$

$$\ln B = \chi - \sigma - 2\phi/3 \quad (19)$$

$$\ln D = 2\phi/3 \quad (20)$$

$$C = \alpha e^{-\sigma} e^{-2\phi/3} \quad (21)$$

The vacuum equations now reduce to the much simpler form

$$\partial_+ \partial_- \alpha = 0 \quad (22)$$

$$\partial_+ \partial_- \phi + \frac{1}{2} \left(\partial_+ \phi \frac{\partial_- \alpha}{\alpha} + \partial_- \phi \frac{\partial_+ \alpha}{\alpha} \right) = 0 \quad (23)$$

$$\partial_+ \partial_- \sigma + \frac{1}{2} \left(\partial_+ \sigma \frac{\partial_- \alpha}{\alpha} + \partial_- \sigma \frac{\partial_+ \alpha}{\alpha} \right) = 0 \quad (24)$$

$$\partial_+ \partial_- \chi + \frac{4}{9} \partial_+ \phi \partial_- \phi + \frac{4}{7} \partial_+ \sigma \partial_- \sigma + \frac{1}{3} \partial_+ \phi \partial_- \sigma + \frac{1}{3} \partial_+ \sigma \partial_- \phi = 0 \quad (25)$$

$$\frac{\partial_{\pm}^2 \alpha}{\alpha} + \frac{8}{9} (\partial_{\pm} \phi)^2 + \frac{8}{7} (\partial_{\pm} \sigma)^2 + \frac{4}{3} \partial_{\pm} \phi \partial_{\pm} \sigma - 2 \partial_{\pm} \chi \frac{\partial_{\pm} \alpha}{\alpha} = 0 \quad (26)$$

One way of viewing this redefinition is via a KK reduction over $\{y_i, \theta, \psi\}$ to two dimensions. The free field, α , represents the overall volume of the internal space, with ϕ and σ representing distortions, or relative volume breathing modes, which couple to α via a friction term. ϕ is of course the dilaton.

This now turns out to be totally prescriptive. Since there are no sources, we can simply make a coordinate choice to fix out the conformal coordinate freedom setting

$$\alpha \equiv r \quad (27)$$

and then both ϕ and σ satisfy cylindrically symmetric wave equations in 2+1 dimensions:

$$\ddot{\phi} - \phi'' - \frac{\phi'}{r} = 0 = \ddot{\sigma} - \sigma'' - \frac{\sigma'}{r} \quad (28)$$

giving

$$\chi' = r \left[\frac{4}{9} (\dot{\phi}^2 + \phi'^2) + \frac{4}{7} (\dot{\sigma}^2 + \sigma'^2) + \frac{2}{3} (\sigma' \phi' + \dot{\phi} \dot{\sigma}) \right] \quad (29)$$

$$\dot{\chi} = 2r \left[\frac{4}{9} \dot{\phi} \phi' + \frac{4}{7} \dot{\sigma} \sigma' + \frac{1}{3} (\sigma' \dot{\phi} + \phi' \dot{\sigma}) \right] \quad (30)$$

These equations have some simple solutions. If we set the dilaton and modulus of the extra seven dimensions to zero, then χ is obviously also a constant. Choosing

$\chi = 0$ gives flat spacetime. Choosing $\chi \neq 0$ results in a conical spacetime, with deficit

$$\Delta = 2\pi(1 - e^{-\chi_0}) \quad (31)$$

We now want to find a solution which is of this form for $r > t$, and has a form for $r < t$ which is asymptotic to a different conical spacetime for $r \ll t$. For simplicity we suppose that the extra y -dimensions in the string frame are inert *i.e.*, $A = D^{-1/2}$ or $\sigma = -7\phi/3$. This gives

$$\chi' = 2r \left(\dot{\phi}^2 + \phi'^2 \right) \quad (32)$$

$$\dot{\chi} = 4r\dot{\phi}\phi' \quad (33)$$

The general cylindrically symmetric solution to the wave equation (28) with initial data $\phi(0, r) = U(r)$, $\dot{\phi}(0, r) = V(r)$ is

$$\phi(\vec{r}, t) = \frac{1}{2\pi} \int_{|\vec{r}-\vec{r}'| \leq t} \frac{V(r')d^2r'}{\sqrt{t^2 - |\vec{r}-\vec{r}'|^2}} + \frac{1}{2\pi} \frac{\partial}{\partial t} \int_{|\vec{r}-\vec{r}'| \leq t} \frac{U(r')d^2r'}{\sqrt{t^2 - |\vec{r}-\vec{r}'|^2}} \quad (34)$$

This shows that for a general dilaton wave, while the wavefront can be largely clustered around the future lightcone, there will nonetheless be a tail inside the future light cone, slowly settling down to a vacuum solution. This is shown in figure 1 where, for illustrative purposes, we take $U(r) = 0$ and $V(r) = e^{-r^2}$. Furthermore, if $U(r)$ and $V(r)$ are localized near the origin, then (34) shows that at late times the fluctuations of the dilaton field disperse to infinity with the the dilaton decaying back to its original value as $1/t$. This is shown in figure 2 for the same U and V as figure 1. Thus the decay process of the conical spacetime does not change the value of the string coupling constant and hence can be studied consistently at strong coupling.

Finally, a general result relating the two deficit angles is obtained by integrating the χ' equation (29). Integrating out from $r = 0$ we find that the right hand side is positive definite, independently of the precise form of ϕ and σ . Thus χ will always be bigger at larger r than at $r = 0$. In other words, geometry mandates the decay of larger deficits into smaller deficits as has been found in other analyses of this problem [10, 11].

It would be interesting to interpret our results in terms of the change in Bondi energy. For a static conical spacetime the Bondi energy is proportional to the deficit angle. Our result on the decrease of the deficit angle under time evolution is suggestive of the decrease of Bondi energy with time in four-dimensional Einstein gravity [17]. In our case the outgoing energy would be carried away by the dilaton pulse rather than by gravitational radiation. However to our knowledge such an analysis has not been done in $(2+1)$ -dimensional dilaton gravity, and indeed the usual analysis of asymptotic flatness is quite different in odd spacetime dimensions [18]. It may

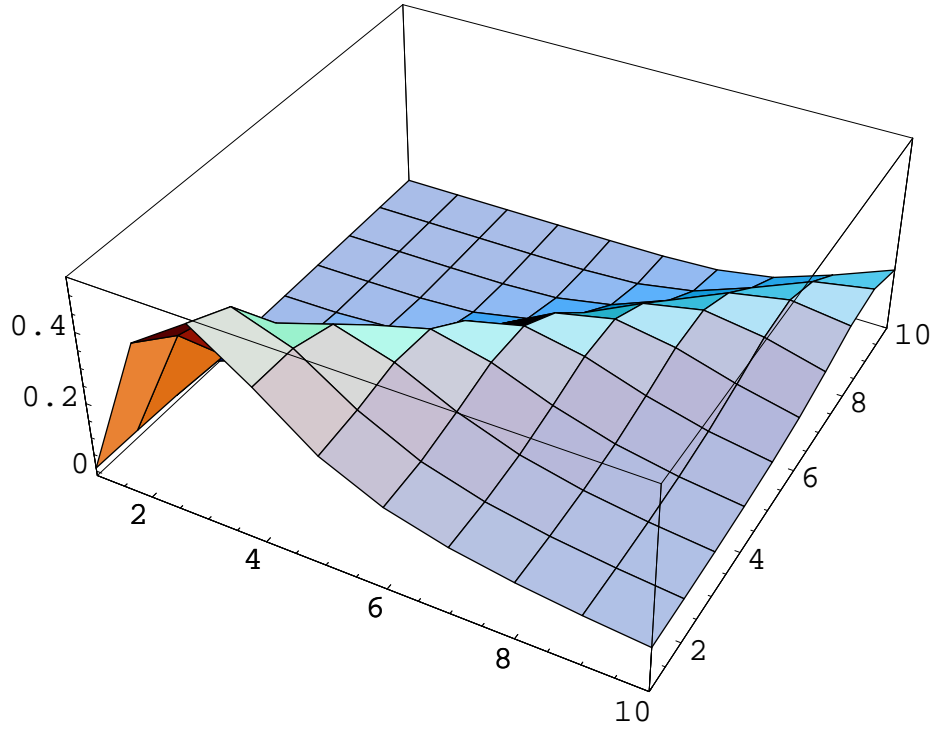


Figure 1: Dilaton pulse as a function of (r, t) for Gaussian initial conditions

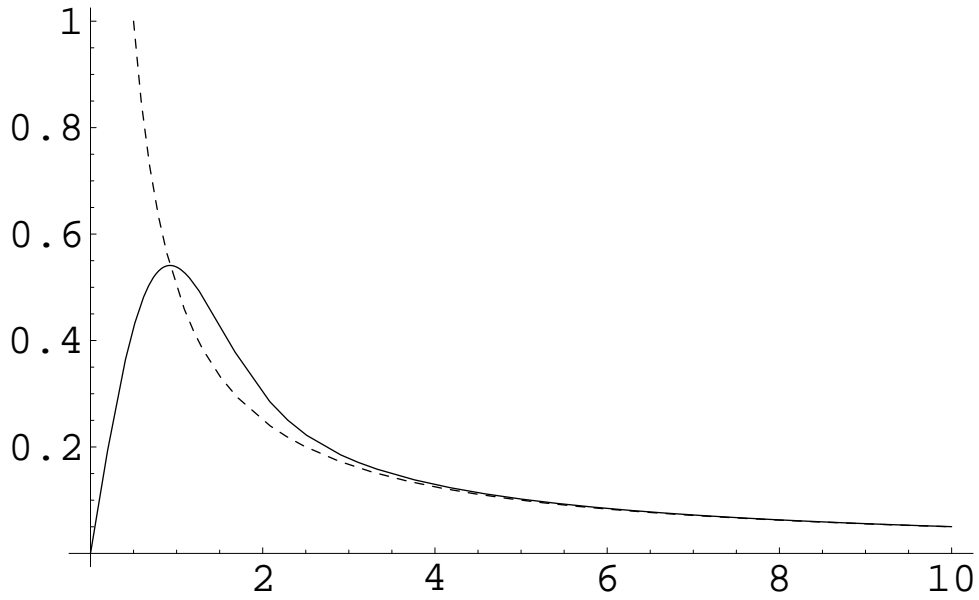


Figure 2: The dilaton at the origin as a function of time. The dotted line represents the $1/t$ asymptotic falloff for late times.

be that the concept of C-energy [19] is the most appropriate tool for analysis of this system.

To summarize: we have studied the decay of conical spacetimes directly as time-dependent solutions to gravity. Our results, which apply at strong string coupling, are consistent with previous results based on RG flow and D-brane probe analysis and show that these results can in part be understood purely in gravitational terms. It would be interesting to extend this analysis to higher dimensions, in particular to the orbifolds $C^2/Z_{n(p)}$ which were studied in [10, 11]. As might be expected from the subtleties in the world-sheet analysis, this case is also much more complicated from the space-time point of view.

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