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DARK CLUSTERS IN GALACTIC HALOS?

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ABSTRACT

We propose that the invisible mass in galactic halos may consist of $10^6\ M_\odot$ dark clusters. Such clusters would be able to heat the stellar disks in galaxies, just as in the Lacey and Ostriker supermassive black hole scenario, but dynamical friction would not necessarily drag too many of them into the galactic nucleus and one would avoid accretion of interstellar gas making them excessively luminous as they traverse the disk. The dynamical friction problem can be circumvented because the clusters may be disrupted by encounters before the drag can be effective. This works, providing the halo core radius is less than 2–4 kpc, the clusters have a size of ~ 1 pc, and the components of the clusters have masses less than $\sim 10\ M_\odot$. We suggest a variety of ways in which the clusters required in this model could arise.

Subject headings: black holes — galaxies: stellar content — galaxies: structure

I. INTRODUCTION

Lacey and Ostriker (1985) have proposed that the dark matter in galactic halos could consist of black holes with a mass of $\sim 10^6\ M_\odot$. This possibility was also considered by Ipser and Semenzato (1985). The prime attraction of this proposal is that the holes could naturally explain the amount of heating of the stellar disk which is observed in our Galaxy. In particular, the model explains (1) why the velocity dispersion of the disk stars increases with age as $t^{1/2}$ for large t; (2) the relative dispersions in the radial, azimuthal, and vertical directions; and (3) the existence of a tail of high-energy disk stars. It is also interesting that $10^6\ M_\odot$ is the mass of the first objects expected to form in many cosmological scenarios.

However, Lacey and Ostriker also highlight the potential problems with the supermassive black hole scenario. First, all holes within a galactocentric radius of 2 kpc would have drifted into the galactic nucleus through dynamical friction. This would lead to an excessive amount of mass in the nucleus (cf. Carr 1978) unless the holes could be ejected through the gravitational slingshot mechanism. It would also tend to evaporate stars within a galactocentric radius of ~ 10 pc, whereas the observed stellar population rises in density all the way into 0.1 pc. Second, holes of $10^6\ M_\odot$ would accrete during their passage through the disk, and the resulting luminosity could violate either background light constraints (Carr 1979) or discrete source count limits (Ipser and Price 1977; McDowell 1985).

A possible way around these problems is to suppose that the halo objects are not single $10^6\ M_\odot$ black holes but $10^6\ M_\odot$ clusters of smaller objects. The disk-heating argument would still apply in this situation, whereas the dynamical friction and accretion arguments might not. One can circumvent the dynamical friction problem, providing the clusters are disrupted by gravitational encounters or by the galactic tidal field before they get too close to the galactic nucleus. One can circumvent the accretion problem, even if the cluster components are themselves black holes, since for Bondi accretion the total accretion rate of each cluster is reduced in proportion to the black hole mass. Thus putting the dark matter in clusters could retain the advantages of the Lacey-Ostriker scenario, while avoiding its disadvantages.

It is, in any case, perhaps more plausible that the dark matter should be in composite objects. For although it is quite natural to envisage a large fraction of the universe going into gravitationally bound clouds of $10^6\ M_\odot$, one might expect these clouds to fragment rather than collapse to a single supermassive object. It is difficult to predict the fragment mass *a priori*. Some people have argued that the fragments could be very large ($\gg 1\ M_\odot$) due to the absence of heavy elements (Silk 1977), the effects of the microwave background radiation (Kashlinsky and Rees 1983), and the absence of initial substructure (Tohline 1980). Other people have argued that the fragments could be very small ($\ll 1\ M_\odot$) as a result of enhanced formation of molecular hydrogen at early times (Palla *et al.* 1984). Nevertheless, it is hard to construct a scenario in which *no* fragmentation occurs.

Although one cannot predict the fragment mass, it is considerably constrained by observations if one wants the fragments to comprise the dark matter in galactic halos. On the basis of background light and nucleosynthetic considerations, Carr, Bond, and Arnett (1984) argue that the dark objects must be either "Jupiters" smaller than $0.1\ M_\odot$ or the black hole remnants of massive stars. Because of its more dramatic cosmological consequences, Carr *et al.* emphasize the massive star scenario. However, from the point of view of the dark matter problem alone, the Jupiter scenario is just as plausible. Indeed, in a sense, it is more attractive since it complicates the standard cosmological scenario as little as possible.

In this paper we will examine in some detail the possibility that the halo objects could be dark clusters. In § II we discuss the various ways in which the clusters might form and estimate their likely radii; this parameter is important since, if it is too small, the clusters will evaporate through internal relaxation before they can have any interesting dynamical effects, and, if it is too large, the clusters will be easily disrupted. In § III we review the disk heating arguments and show how they can be applied to clusters. In § IV we examine how the dynamical friction of the spheroid would affect the dark matter distribution; these considerations apply independently of whether the dark matter comprises composite or single objects. In § V we examine the mechanisms which may disrupt clusters: collisions, tides, and internal relaxation. We find that there is a narrow range of cluster radii for which the clusters are collisionally disrupted before they become subject to dynamical friction, while still heating the disk down to fairly small radii. In § VI we show how the cluster picture can circumvent the accretion problem mentioned by Lacey and Ostriker, and we examine how the proposed scenario can be tested by VLBI searches for gravitational lensing effects.

II. THE FORMATION OF THE CLUSTERS

There are at least three scenarios in which a large fraction of the universe could go into $10^6\ M_\odot$ gravitationally bound clouds. If the initial density fluctuations were *isothermal* and the universe's density is baryon dominated, one would expect the first objects to form to have a mass similar to the baryon Jeans mass just after decoupling:

$$m_{Jb} = \frac{4\pi}{3}\rho_b\left(\frac{5\pi kT}{3G\mu_p\rho_b}\right)^{3/2} \approx 1\times 10^6\Omega^{-1/2}h^{-1}\ M_\odot\,, \tag{2.1}$$

where Ω_b is the baryon density (ρ_b) in units of the critical density, μ_p is the proton mass, and h is the Hubble constant H_0 in units of 100 km s^{-1} Mpc^{-1}. (For $\Omega_b = 1$ and $H_0 = 50$ this is exactly the mass required in the Lacey-Ostriker model.) One can show that fluctuations on scales below this would be damped out because the thermal and dynamical decoupling only occurs gradually (Peebles 1969; Carr and Rees 1984). The redshift z_B at which the clouds bind depends upon the amplitude of the fluctuations at decoupling, but if the fluctuation spectrum is assumed to be a power law, the form of the spectrum required to explain the galaxy correlation function (Peebles 1974) suggests that z_B would be in the range 10^2–10^3. When the clouds first virialize, they should have a density which is just $18\pi^2$ times the background cosmological density if $\Omega \approx 1$ at $z = z_B$. If they dissipate and collapse by a further factor of Δ in radius before fragmenting, the final radius of each cluster will be

$$r_c = (36\ \text{pc})\left(\frac{z_B}{100}\right)^{-1}\left(\frac{m_c}{10^6\ M_\odot}\right)^{1/3}\left(\frac{\Omega_b h^2}{0.1}\right)^{-1/3}\Delta^{-1}\,. \tag{2.2}$$

Here m_c is the mass of the cloud, and Ω_b is normalized to the value required to explain the dark matter in galactic halos. Equation (2.2) implies that r_c could be at most of order 100 pc, while—for large Δ—it could be arbitarily small.

If the initial density fluctuations were *adiabatic* and the universe's density is dominated by cold particles like axions, the axion fluctuations would have begun to grow as soon as the axions dominated the density at $z_{\text{eq}} \approx 3\times 10^4\Omega_a h^2$ (where Ω_a is the axion density in units of the critical density). In this case, the axionic fluctuations could survive down to very small scales, and the resulting axion clouds would have a spectrum of sizes $r_c(m_c)$ given by equation (2.2) but with z_B itself a function of m_c and with $\Omega_b \to \Omega_a$. For example, if the fluctuations at z_{eq} (denoted by δ_{eq}) scale as $M^{-\alpha}$, we get

$$r_c \approx (130\ \text{pc})\left(\frac{m_c}{10^6\ M_\odot}\right)^{\alpha+1/3}\left[\frac{(1+z_{\text{eq}})\delta_{\text{eq}}(10^6\ M_\odot)}{20}\right]^{-1}(\Omega_a h^2)^{-1/3}\,. \tag{2.3}$$

We have here normalized $(1+z_{\text{eq}})\delta_{\text{eq}}$ to the value 20 used by Peebles (1984) to fit the observations of large-scale structure in the $\Omega_a = 1$ case; in general, this value will itself depend upon Ω_a. For $m_c \approx 10^6\ M_\odot$, we have $\alpha \approx -0.1$. Note that since axions are dissipationless, one expects $\Delta = 1$ in this situation. However, one could still have $\Delta \gg 1$ for any gas which sinks into the gravitational potential wells of the axion clouds. One can show (Carr and Rees 1984) that this happens only for axion clouds larger than

$$m_c \approx 10^6\Omega_a^{-1/2}h^{-1}\ M_\odot\,. \tag{2.4}$$

The corresponding baryon mass is smaller than this by a factor Ω_b/Ω_a, so the characteristic mass of the first baryon clouds is now closer to $10^5\ M_\odot$ for $\Omega_a = 1$ and $\Omega_b = 0.1$ (although this mass is only a rough guide). This scenario therefore involves a hybrid scheme in which the dark matter is partly baryonic and partly axionic. However, we will find in § V that the axionic component of the clusters would almost certainly be disrupted within the visible radius of the galaxy, so only the baryonic component could be in clusters there.

A third scenario for cluster formation has been proposed by Fall and Rees (1985) and applies independently of the form of the initial density fluctuations. They argue that globular cluster type objects will form naturally at the epoch of galaxy formation ($z \approx 10$) since the Jeans mass of pressure-confined gas at 10^4 K is then $\sim 10^6\ M_\odot$. Although they invoke this process to put a *small* fraction of the galactic mass into visible clusters, it is also possible that it could result in a large fraction of the galactic mass going into dark clusters. (Fall and Rees do not attempt to calculate either the fragment mass itself or the efficiency of cluster formation.) In any case, their argument leads to a cluster radius and mass

$$r_c = (19\ \text{pc})\alpha^{-1}\beta^{-1/2}\left(\frac{R}{\text{kpc}}\right)^{1/2}\Delta^{-1}, \qquad m_c \approx (6\times 10^5\ M_\odot)\alpha^{-1}\beta^{-1/2}\left(\frac{R}{\text{kpc}}\right)^{1/2}, \tag{2.5}$$

where R is the galactocentric radius of the cluster and α and β are constants of order unity.

In all of these situations, one expects clusters of mass $m_c \approx 10^6\ M_\odot$ to form with a radius r_c in the range 0.1–10^2 pc, if we limit the effect of dissipation to $\Delta \le 10$. Another crucial question concerns the mass m_* of the fragments within the cluster. We have seen that it is very hard to predict this mass *a priori*, although observational constraints require $m_* < 0.1\ M_\odot$ or $m_* > m_{\text{BH}}$, where m_{BH} is the mass above which a star can be expected to collapse to a black hole without appreciable metal ejection. Carr, Bond, and Arnett

(1984) argue that only VMOs could avoid metal ejection and infer that $m_{\rm BH} \approx 200\ M_\odot$. However, this conclusion is not definite, and $m_{\rm BH}$ could, in principle, be much smaller. Indeed, the dynamical constraints discussed in § V will be found to exclude the $m_* > 200\ M_\odot$ possibility. For consistency with the dynamical constraints, we would have to assume that $m_{\rm BH}$ could be as low as 10 $M_\odot$.

Finally we must enquire whether it is possible to make the $10^6\ M_\odot$ clusters evolve to single supermassive holes, as required in the Lacey-Ostriker picture. One might envisage the core of the cluster forming a black hole of rather modest mass ($m_{\rm H}$), which then swallows the rest of the cluster by accretion. If the rest of the cluster remains gaseous and is accreted at the Bondi rate, it would be accreted on the time scale.

$$t_{\rm acc} \approx (10^6\ {\rm yr})\left(\frac{m_{\rm H}}{10^2\ M_\odot}\right)^{-1}\left(\frac{m_c}{10^6\ M_\odot}\right)^{-1}\left(\frac{r_c}{\rm pc}\right)^3\left(\frac{T}{10^4\ {\rm K}}\right)^{3/2} . \tag{2.6}$$

(The dependence on r_c comes from the fact that the gas density in the cluster is taken to be $3m_c/4\pi r_c^3$.) Assuming that T does not fall below 10^4 K, $t_{\rm acc}$ may be much less than the age of the galaxy, so it would seem that the initial hole would have plenty of time to swallow the entire cluster. However, this argument would require modification in the presence of rotation or turbulence, and it would be invalidated if the gas fragmented into "stars" first. Another possibility would be to assume that the cluster *does* at first fragment into stars but that it undergoes core collapse. If the stars are then shattered by collisions to form a single amorphous gas cloud, a supermassive black hole might eventually form. The problem with this suggestion is that a large fraction of the initial cluster mass will be evaporated in the process, so the final black hole will be much smaller than $10^6\ M_\odot$ unless accretion is again invoked.

An entirely different way of making supermassive black holes might be to propose that they are primordial in origin (i.e. forming in the very early universe). However, primordial holes cannot exceed the horizon size at formation, and therefore those of $10^6\ M_\odot$ would have to form at least 10 s after the big bang (Carr 1975). It is rather unlikely that the large inhomogeneities required for black hole formation could exist at this time in view of the cosmological nucleosynthesis constraints. On the other hand, it might be possible to make a lot of smaller primordial holes: for example, Jupiter mass holes might form at the quark-hadron phase transition (Crawford and Schramm 1983). In this situation it would be quite natural that the first baryonic objects to form would be $10^6\ M_\odot$ gas clouds with halos of black holes (Freese, Price, and Schramm 1983). However, as for the axion case, the black hole component of the clusters would probably be disrupted within the visible radius of the galaxy.

III. DISK HEATING BY HALO OBJECTS

The effect on the velocity distribution of Galactic disk stars of gravitational two-body scattering by massive halo black holes is discussed by Lacey and Ostriker (1985) and Ipser and Semenzato (1985). Their analysis assumes that the halo objects can be regarded as point masses. If the halo objects are clusters, the stellar velocity change in close encounters is reduced because of the finite size of the clusters. However, this does not significantly change the evolution of the disk velocity dispersion, because most of the disk heating comes from distant encounters. (On the other hand, it does affect the production of a high-velocity tail of disk stars because this depends on the close encounters, as we discuss below.) We can therefore take over directly the Lacey-Ostriker results for the disk velocity dispersion and scale height. The velocity dispersion is predicted to increase with age as $t^{1/2}$ for large t, in good agreement with observations. If *all* of the disk heating observed in the solar neighborhood is attributed to this mechanism, as we will assume in the bulk of this paper, one derives a mass for the halo clusters of $m_c \approx 2 \times 10^6\ M_\odot$; this assumes a Galactic age $t_g \approx 1.5 \times 10^{10}$ yr. If other mechanisms contribute to disk heating, then this is an upper limit on m_c.

We assume that the radial dependence of the halo density is roughly that of an isothermal sphere, which we approximate as

$$\rho_H(R) = \begin{cases} \rho_c & (R < R_c)\,, \\ \rho_c\left(\dfrac{R_c}{R}\right)^2 & (R > R_c)\,, \end{cases} \tag{3.1}$$

where R_c is the halo core radius. We can write ρ_c in terms of the asymptotic circular velocity V_∞:

$$\rho_c = \frac{V_\infty^2}{4\pi G R_c^2} . \tag{3.2}$$

For making numerical estimates in the context of the Galaxy, we adopt $V_\infty = 220$ km s^{-1}. The value of R_c as determined by fitting Galactic mass models (e.g., Ostriker and Caldwell 1983; Bahcall, Schmidt, and Soneira 1983) probably lies in the range $2 < R_c < 8$ kpc. The Galactic rotation curve (due to dark and luminous matter together) is roughly flat for $R > (0.1$–$1)$ kpc, so the halo one-dimensional velocity dispersion in this range is roughly

$$\sigma_H \approx \frac{V_\infty}{\sqrt{2}} . \tag{3.3}$$

The radial dependence of the disk velocity dispersion would be given by $\sigma_D^2 \propto \rho_H/\sigma_H$ if the cluster population did not evolve, so one would expect σ_D to increase inward for $R > R_c$ and then flatten off for $R < R_c$.

An observational investigation by Lewis (1986) indicates that the disk velocity dispersion increases with decreasing galactocentric radius down to radii perhaps as small as $R \approx 0.1$ kpc. This would appear to require a very small core radius for the halo. However, there are uncertainties; in particular, the observations may be contaminated by spheroid stars in the inner few kpc. Also, disk heating at small R may be modified by the effects of cluster disruption or dynamical friction, as discussed later. Thus, a generous upper limit is $R_c \leq 4$ kpc.

We now consider the generation of a high-velocity tail of disk stars. For small-angle scattering of disk stars by halo clusters, the velocity change in an encounter with impact parameter l at relative velocity $V_{\rm rel}$ is

$$\Delta v \approx \frac{2Gm_c}{V_{\rm rel}\, l}, \qquad l \geq \max\left(r_c, \frac{Gm_c}{V_{\rm rel}^2}\right). \tag{3.4}$$

For smaller impact parameters, Δv flattens off or decreases. Thus the maximum possible velocity change in a single encounter is

$$\Delta V_{\rm max} \approx \begin{cases} 2V_{\rm rel}, & r_c < \dfrac{Gm_c}{V_{\rm rel}^2}, \\ \dfrac{2Gm_c}{V_{\rm rel} r_c} \approx 10\,\dfrac{\sigma_c^2}{V_{\rm rel}}, & r_c > \dfrac{Gm_c}{V_{\rm rel}^2}. \end{cases} \tag{3.5}$$

Here σ_c is the one-dimensional velocity dispersion of the cluster stars; if we model each cluster as a uniform sphere, this is

$$\sigma_c = \left(\frac{Gm_c}{5r_c}\right)^{1/2} \approx (41\ {\rm km\ s^{-1}})\left(\frac{m_c}{2\times 10^6\ M_\odot}\right)^{1/2}\left(\frac{r_c}{\rm pc}\right)^{-1/2}. \tag{3.6}$$

Taking $V_{\rm rel} \approx V_c \approx V_\infty$, we find

$$\Delta v_{\rm max} \approx 80\ {\rm km\ s^{-1}}\left(\frac{m_c}{2\times 10^6\ M_\odot}\right)\left(\frac{r_c}{\rm pc}\right)^{-1}, \qquad r_c > 0.2\ {\rm pc}\left(\frac{m_c}{2\times 10^6\ M_\odot}\right). \tag{3.7}$$

The high-velocity disk population one seeks to explain has peculiar velocities $V \gtrsim 100$ km s^{-1}, as discussed by Lacey and Ostriker (1985), and so could only be produced by scattering off halo clusters for

$$r_c \lesssim 1\ {\rm pc}\left(\frac{m_c}{2\times 10^6\ M_\odot}\right). \tag{3.8}$$

The lower cutoff in the Coulomb logarithm, $\ln \Lambda = \ln (l_{\rm max}/l_{\rm min})$ which enters the expression for the disk heating rate (Lacey and Ostriker; eq. [27]) is increased to $l_{\rm min} = r_c$ for $r_c > Gm_c/V_{\rm rel}^2$, but this is only a small correction. If condition (3.8) is violated, one would have to account for high-velocity disk stars by a different mechanism.

IV. THE EFFECT OF DYNAMICAL FRICTION ON HALO OBJECTS

Encounters between the halo clusters and lower mass objects will steal energy from the clusters, with the result that they will tend to drift towards the centre of the galaxy. Lacey and Ostriker assume that the dominant source of drag will be the stars in the spheroid; these are taken to have a density profile

$$\rho_S(R) = \begin{cases} \rho_1\left(\dfrac{R_1}{R}\right)^3 & (R > R_1), \\ \rho_1\left(\dfrac{R_1}{R}\right)^{1.8} & (R < R_1), \end{cases} \tag{4.1}$$

where $R_1 \approx 800$ pc and $\rho_1 \approx 1.8\ M_\odot$ pc^{-3}. The subscript "1" will always refer to values at the spheroid characteristic radius. If the halo objects are taken to be on circular orbits, their orbital radius will then shrink according to

$$\frac{dR}{dt} = -\frac{4\pi G^2 m_c (\ln \Lambda) B(V_c/\sqrt{2}\,\sigma) R \rho_S(R)}{(1 + d\ln V_c/d\ln R) V_c(R)^3}, \tag{4.2}$$

where $V_c(R)$ is the circular velocity at galactocentric radius R,

$$B(x) = {\rm erf}\,(x) - x\,{\rm erf}'\,(x), \qquad \Lambda = RV_c^2/Gm_c\,.$$

As shown in Appendix A, this implies

$$\frac{dR}{dt} \approx \begin{cases} -\left[\dfrac{12\sqrt{6}\,B(2/\sqrt{5})}{11\sqrt{5}}\right](\ln \Lambda)\left(\dfrac{m_c}{4\pi R_1^3 \rho_1}\right)\left(\dfrac{R_1}{R}\right)^{11/10} R_1(4\pi G\rho_1)^{1/2} & (R < R_1), \\ -\left[\dfrac{12\sqrt{6}\,B(\sqrt{2})}{5\sqrt{5}}\right](\ln \Lambda)\left(\dfrac{m_c}{4\pi R_1^3 \rho_1}\right)\dfrac{(R_1/R)^{1/2}(4\pi G\rho_1)^{1/2} R_1}{[11/5 + 6/5\ln (R/R_1)][1 + 6/5 \ln (R/R_1)]^{1/2}} & (R > R_1), \end{cases} \tag{4.3}$$

where

$$\begin{aligned} \ln \Lambda &\approx \frac{6}{5}\ln\left(\frac{R}{R_1}\right) - \ln\left(\frac{6}{5}\frac{m_c}{4\pi R_1^3 \rho_1}\right) && (R < R_1), \\ \ln \Lambda &= \ln\left(1 + \frac{6}{5}\ln\frac{R}{R_1}\right) - \ln\left(\frac{6}{5}\frac{m_c}{4\pi R_1^3\rho_1}\right) && (R > R_1). \end{aligned} \tag{4.4}$$

For the values of ρ_1 and R_1 chosen, we have

$$4\pi\rho_1 R_1^3 \approx 1.2 \times 10^{10}\, M_\odot\,, \qquad (4\pi G\rho_1)^{-1/2} \approx 3.2 \times 10^6 \text{ yr}\,, \qquad \ln\Lambda \approx -\ln\left(\frac{3m_c}{10\pi\rho_1 R_1^3}\right) \approx 8 \text{ for } m_c \approx 2 \times 10^6\, M_\odot\,. \quad (4.5)$$

We also have $B(2/\sqrt{5}) = 0.34$ and $B(\sqrt{2}) = 0.74$. The time to drift into the galactic centre from an initial galactocentric radius R is

$$t_{\rm df} = \begin{cases} \dfrac{10}{21}\dfrac{R}{|dR/dt|} & (R < R_1)\,, \\[2ex] \dfrac{2}{3}\dfrac{R}{|dR/dt|} & (R > R_1)\,. \end{cases} \quad (4.6)$$

Thus, if the spheroid is the dominant source of friction, the clusters will drift into the nucleus within the age of the Galaxy ($t_g \approx 1.5 \times 10^{10}$ yr) providing R is initially less than

$$R_{\rm df} = \begin{cases} 1.8 \text{ kpc}\left(\dfrac{m_c}{2 \times 10^6\, M_\odot}\right)^{10/21}\left(\dfrac{t_g}{1.5 \times 10^{10} \text{ yr}}\right)^{10/21} & (R_{\rm df} < R_1)\,, \\[2ex] 5.6 \text{ kpc}\left(\dfrac{m_c}{2 \times 10^6\, M_\odot}\right)^{2/3}\left(\dfrac{t_g}{1.5 \times 10^{10} \text{ yr}}\right)^{2/3} \Big/ \left[\dfrac{11}{5} + \dfrac{6}{5}\ln\left(\dfrac{R_{\rm df}}{R_1}\right)\right]^{2/3}\left[1 + \dfrac{6}{5}\ln\left(\dfrac{R_{\rm df}}{R_1}\right)\right]^{1/3} & (R_{\rm df} > R_1)\,. \end{cases} \quad (4.7)$$

For $m_c \approx 2 \times 10^6\, M_\odot$ and $t_g \approx 1.5 \times 10^{10}$ yr, this gives $R_{\rm df} \approx 2.0$ kpc.

We stress that several approximations have gone into equation (4.7). First, we have assumed that the halo clusters move on circular orbits; in fact, the result for a realistic initial distribution of orbital eccentricities probably does not differ very much. Second, we have neglected the dynamical effects of the disk and the halo. A more precise derivation of the dynamical friction rate would have to include these. Since the combined density of the spheroid, disk, and halo is such as to produce a roughly flat rotation curve for $R \geq 1$ kpc, one can derive approximate results by using the dynamical friction formula appropriate for a single-component isothermal sphere with constant velocity (Tremaine, Ostriker, and Spitzer 1975). Such a calculation (Carr and Sakellariadou 1986) shows that the estimate for $R_{\rm df}$ given by equation (4.7) is not appreciably affected for clusters of $2 \times 10^6\, M_\odot$, so we will neglect this complication in the present discussion.

The effect of the spheroid's drag on the density profile of the halo objects is discussed in Appendix A and summarized in Figure 1.

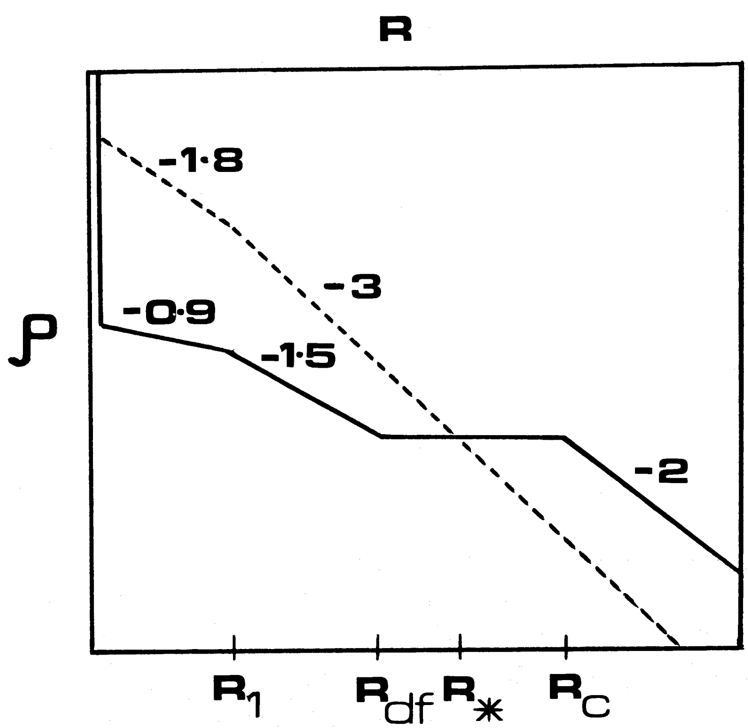

FIG. 1.—Effect of the spheroid's dynamical drag on the dark halo distribution. The initial halo density profile is assumed to fall off as R^{-2} outside some core radius R_c. The current halo profile is modified inside some radius $R_{\rm df}$, as indicated by the solid line. The quantity $R_{\rm df}$ is close to 2 kpc for halo objects of $2 \times 10^6\, M_\odot$. In addition, there will be a cusp at $R = 0$ formed from all the halo material which was originally within $R_{\rm df}$. The spheroid density (*broken line*) will exceed the modified halo density everywhere in the regime $R_* > R > 0$, although not at the origin itself. Figure assumes $R_c > R_{\rm df}$. If $R_c < R_{\rm df}$, there will be no constant density regime in the final halo profile.

The profile is nearly unaffected outside R_{df}, but it falls off as $R^{-3/2}$ for $R_1 < R < R_{df}$ and as $R^{-9/10}$ for $R < R_1$. In particular, this means that the magnitude of the Lacey-Ostriker disk-heating effect is modified within R_{df} (see § III). The velocity dispersion induced in the disk stars scales as

$$\sigma_D^2 \propto \int \rho_H(R, t)dt/\sigma_H(R) \propto \rho_H(R, t_g)/\sigma_H(R) . \tag{4.8}$$

The last expression follows, providing $\rho_H(R, t)$ does not decrease faster than t^{-1} at fixed R. The quantity σ_H should vary roughly as the circular speed V_c associated with the spheroid. From equation (A4), this scales as $R^{-1/2}$ for $R > R_1$ and $R^{1/10}$ for $R < R_1$. The behavior of $\rho_H(R)$ and equation (4.8) therefore imply

$$\sigma_D^2 \propto \begin{cases} R^{-1} & (R < R_{df}) , \\ R^0 & (R_{df} < R < R_c) , \\ R^{-2} & (R > R_c) , \end{cases} \tag{4.9}$$

where the R^0 part pertains only if $R_c > R_{df}$. Thus dynamical friction alone should cause the disk velocity dispersion to increase as one goes to smaller radii. However, this result may be modified by cluster disruption, as discussed in § V.

The drag of the spheroid will not only change the halo profile for $R_{df} > R > 0$; it will also produce a large density excess at $R = 0$ associated with the halo objects which have already reached there. If $R_c > R_{df}$, this mass is

$$M_N \approx \frac{4\pi}{3} \rho_c R_{df}^3 \approx \frac{V_\infty^2 R_{df}^3}{3GR_c^2} \approx 2 \times 10^9 \left(\frac{m_c}{2 \times 10^6 M_\odot}\right)^2 \left(\frac{t_g}{1.5 \times 10^{10} \text{ yr}}\right)^2 \left(\frac{R_c}{4 \text{ kpc}}\right)^{-2} M_\odot , \tag{4.10}$$

where we have substituted for R_{df} from equation (4.7), assuming $R_{df} > R_1$ and setting the logarithmic factors to be constant. While the value of R_c is uncertain, it seems clear, as stressed by Lacey and Ostriker (1985), that M_N exceeds the upper limit of $4 \times 10^6 M_\odot$ on the nuclear mass derived by Serabyn and Lacy (1985), so there is a potential problem if the halo comprises $2 \times 10^6 M_\odot$ black holes. There are possible ways around this: for example, Lacey and Ostriker (1985) appeal to the gravitational slingshot mechanism to eject supermassive holes from the galactic nucleus. In this way one may never have more than two there at any one time. However, it is not clear that this really works. An alternative is to suppose that the halo objects are composite and are disrupted before dynamical friction can take them into the nucleus. We now consider how such disruption could occur.

V. DISRUPTION OF HALO CLUSTERS

In this section we consider three mechanisms which can disrupt clusters, and then we combine the results with those of the previous sections to set constraints on the cluster properties.

a) *Collisional Disruption*

We first examine collisions between clusters. As discussed in Appendix B, such collisions ultimately lead to disruption if the one-dimensional velocity dispersion of the halo objects σ_H exceeds the internal dispersion σ_c given by equation (3.6). For $R > R_c$, this just requires

$$r_c > \frac{2}{5} \frac{Gm_c}{V_\infty^2} \approx 0.1 \text{ pc} \left(\frac{m_c}{2 \times 10^6 M_\odot}\right) . \tag{5.1}$$

In Appendix B, we show that the time scale for collisional disruption is

$$t_{dis} \approx \frac{3}{5} \sqrt{\frac{\pi}{2}} \frac{\max(R^2, R_c^2)}{V_\infty r_c} . \tag{5.2}$$

Since t_{dis} is constant within the core and increases further outward, clusters will only be disrupted over the lifetime of the galaxy if

$$r_c > \frac{3}{5} \sqrt{\frac{\pi}{2}} \frac{R_c^2}{V_\infty t_g} \approx 0.9 \text{ pc} \left(\frac{R_c}{2 \text{ kpc}}\right)^2 \left(\frac{t_g}{1.5 \times 10^{10} \text{ yr}}\right)^{-1} . \tag{5.3}$$

If this condition is satisfied, clusters will be destroyed within a galactocentric radius

$$R_{dis} \approx \left(\frac{5}{3} \sqrt{\frac{2}{\pi}} V_\infty r_c t_g\right)^{1/2} \approx 2.1 \text{ kpc} \left(\frac{r_c}{\text{pc}}\right)^{1/2} \left(\frac{t_g}{1.5 \times 10^{10} \text{ yr}}\right)^{1/2} . \tag{5.4}$$

If $\sigma_c > \sigma_H$ (i.e. if eq. [5.1] is violated), then cluster collisions lead to merging rather than disruption. However, this case is probably not interesting in practice, because it would require the clusters to be so small that the merging time scale would necessarily exceed t_g. Note that the disk heating will be modified within R_{dis}: since $\sigma_D^2 \propto \rho_H t_{dis}$ and $t_{dis} \propto \rho_H^{-1}$ there, we have $\sigma_D(R < R_{dis}) \approx \sigma_D(R_{dis})$. Thus the disk velocity dispersion should be constant within R_{dis}.

b) *Tidal Disruption*

The galactocentric radius within which a cluster on a circular orbit is tidally disrupted is shown in Appendix B to be given by

$$R_T = \left[\frac{3M(R_T)}{4\pi\rho_{cc}}\right]^{1/3}\left(3 - \frac{d\ln M}{d\ln R}\right)^{1/3}, \tag{5.5}$$

where ρ_{cc} is the density within each cluster. For interesting values of the cluster density, tidal disruption is only important at radii where the spheroid dominates the background density. For $R_T < R_1$, we find

$$R_T = R_1\left(\frac{9\rho_1}{2\rho_{cc}}\right)^{5/9} \qquad \left(\rho_{cc} > \frac{9}{2}\rho_1\right). \tag{5.6}$$

Inserting numerical values gives

$$R_T = 1.8 \text{ pc}\left(\frac{m_c}{2\times10^6\,M_\odot}\right)^{-5/9}\left(\frac{r_c}{\text{pc}}\right)^{5/3}, \qquad r_c < 39 \text{ pc}\left(\frac{m_c}{2\times10^6\,M_\odot}\right)^{1/3}. \tag{5.7}$$

The corresponding expressions for $R_T > R_1$ are not given here, but follow straightforwardly from equation (B13).

c) *Evaporation*

If each cluster is composed of objects of mass m_*, its two-body relaxation time is

$$t_{\text{rel}} = 0.34\,\frac{\sigma_c^3}{G^2 m_* \rho_{cc}\ln\Lambda} \approx 0.13\,\frac{m_c r_c^{3/2}}{G^{1/2} m_* \ln\Lambda}, \tag{5.8}$$

where $\Lambda = r_c/(Gm_*/3\sigma_c^2) \approx 3m_c/5m_*$. Taking $\ln\Lambda = 14$, we find

$$t_{\text{rel}} \approx 2.0\times10^8 \text{ yr}\left(\frac{m_c}{2\times10^6\,M_\odot}\right)^{1/2}\left(\frac{r_c}{\text{pc}}\right)^{3/2}\left(\frac{m_*}{M_\odot}\right)^{-1}. \tag{5.9}$$

A simple calculation for the evaporation rate due to two-body encounters gives $t_{\text{evap}} \approx 100 t_{\text{rel}}$ (Spitzer 1975). Thus the condition $t_{\text{evap}} > t_g$ requires

$$m_* \lesssim 1\,M_\odot\left(\frac{m_c}{2\times10^6\,M_\odot}\right)\left(\frac{r_c}{\text{pc}}\right)^{3/2}\left(\frac{t_g}{1.5\times10^{10}\text{ yr}}\right)^{-1}. \tag{5.10}$$

This neglects core collapse, which occurs after a time $t_{cc} \approx 15 t_{\text{rel}}$. Cluster reexpansion following core collapse may drive most of the mass over the tidal limit faster than two-body evaporation if the tidal radius r_t is not too large compared to the cluster radius (Lee and Ostriker 1986). For $1 > r_c/r_t > 0.1$, the limit (5.10) would have to be reduced by a factor of order $30(r_c/r_t)^{3/2}$.

d) *Discussion*

We now combine the results of this and the previous sections to set constraints on the permissible values of r_c and m_c. The disk velocity dispersion must continue to increase inwards down to some radius R_{heat}, where R_{heat} is at most $\sim$4 kpc and may be considerably smaller. Therefore we require both $R_c < R_{\text{heat}}$ and $R_{\text{dis}} < R_{\text{heat}}$. From equation (5.4) the latter condition implies

$$r_c \leq 4 \text{ pc}\left(\frac{R_{\text{heat}}}{4\text{ kpc}}\right)^2\left(\frac{t_g}{1.5\times10^{10}\text{ yr}}\right)^{-1}. \tag{5.11}$$

Thus the clusters need to be fairly small, and their formation may have to involve considerable dissipation (see the estimates of r_c in § II). Combining equations (5.11) and (5.10) gives

$$m_* < 10\,M_\odot\left(\frac{m_c}{2\times10^6\,M_\odot}\right)\left(\frac{R_{\text{heat}}}{4\text{ kpc}}\right)^3\left(\frac{t_g}{1.5\times10^{10}\text{ yr}}\right)^{-5/2}. \tag{5.12}$$

Thus VMO remnants are ruled out since these must be larger than 200 $M_\odot$. The black hole remnants of ordinary massive stars are marginally permitted, but only if one can circumvent the nucleosynthesis constraints of Carr *et al.* (1984). The most natural conclusion is that the clusters consist of Jupiters with $m_* < 0.1\,M_\odot$.

We next address the problem, raised in § IV, of dynamical friction briging an excessive amount of mass into the galactic center. One way around this would be to suppose that clusters are disrupted by collisions before dynamical friction becomes operative. This requires $R_{\text{dis}} > R_{\text{df}}$, which, from equation (4.7) with $R_{\text{df}} > R_1$ and equation (5.4), gives

$$r_c \geq 0.9 \text{ pc}\left(\frac{m_c}{2\times10^6\,M_\odot}\right)^{4/3}\left(\frac{t_g}{1.5\times10^{10}\text{ yr}}\right)^{1/3}; \tag{5.13}$$

we have here set the logarithmic factors in equation (4.7) to constants. Equations (5.11) and (5.13) together confine r_c to a rather narrow range. In fact, the range vanishes entirely if

$$R_{\text{heat}} < 2 \text{ kpc}\left(\frac{m_c}{2\times10^6\,M_\odot}\right)^{2/3}\left(\frac{t_g}{1.5\times10^{10}\text{ yr}}\right)^{2/3}. \tag{5.14}$$

If $R_c > R_{df}$, then condition (5.13) is replaced by condition (5.3), which restricts the allowed range of r_c even further. In any case, the model requires fine-tuning.

We now consider the alternative possibility that the radii of the clusters are small enough that $t_{dis} > t_g$ at $R = R_{df}$. From equation (B7), the disruption time varies as $t_{dis} \propto \sigma_H/\rho_H$, where ρ_H is the mass density *as modified by dynamical friction.* Thus $t_{dis} \propto R$ for $R < R_{df}$, and for a cluster spiraling in due to dynamical friction, $t_{dis}(R)$ remains larger than $t_{df}(R)$ (see § IV). It will therefore be dragged all the way into the center without being collisionally disrupted. However, one must still consider the possibility of tidal disruption. The condition $t_{dis}(R_{df}) > t_g$ requires the inverse of equation (5.3) if $R_c > R_{df}$, and the inverse of equation (5.13) if $R_c < R_{df}$. Since we also require $R_c < R_{heat} < 4$ kpc, we see that r_c must be fairly small. Equation (5.7) then implies that R_T must also be small.

If $R_T > r_c$, then initially clusters dragged in by dynamical friction will be tidally disrupted at $R = R_T$ and produce a cloud of debris within this radius. This process will continue until the accreted mass in clusters becomes equal to the spheroid mass within R_T. From equations (A3) and (5.7), the latter is

$$M_S(R_T) = 6 \times 10^6 \, M_\odot \left(\frac{m_c}{2 \times 10^6 \, M_\odot}\right)^{-2/3} \left(\frac{r_c}{\text{pc}}\right)^2 . \tag{5.15}$$

Once the accreted mass exceeds this value, the debris cloud will dominate the density close to the center and thus control the radius at which further incoming clusters are tidally disrupted. If we assume that incoming clusters spiral in on roughly circular orbits, then they will be disrupted when the background density becomes comparable to their internal density. One therefore expects to produce a roughly uniform debris cloud, with mean density

$$\rho_N \approx \rho_{cc} \approx \frac{3m_c}{4\pi r_c^3} \approx 5 \times 10^5 \, M_\odot \, \text{pc}^{-3} \left(\frac{m_c}{2 \times 10^6 \, M_\odot}\right)\left(\frac{r_c}{\text{pc}}\right)^{-3} \tag{5.16}$$

and with the mass M_N given by equation (4.10). If $R_T < r_c$ initially, clusters will never be disrupted by the tidal field of the spheroid but will begin the process of merging into a debris cloud immediately. Note that two-body relaxation within the cloud should be unimportant since the two-body relaxation time exceeds that in the original clusters by a factor $\sim (M_N/m_c)$.

These results do not appear to be consistent with observational constraints on the mass distribution close to the Galactic center (Oort 1977; Crawford *et al.* 1985). In particular, Crawford *et al.* find that the total mass within $R = 10$ pc is $M \approx (5\text{–}10) \times 10^6 \, M_\odot$, corresponding to a mean density within that radius of $\bar{\rho} \approx (1\text{–}2) \times 10^3 \, M_\odot \, \text{pc}^{-3}$. This conflicts with equation (5.16) unless $r_c > 6$ pc. But if the clusters were this large, they would be disrupted by collisions before dynamical friction could be effective, as discussed earlier.

We note that the gravitational slingshot mechanism could not eject clusters from the center unless they were compact enough to satisfy

$$r_c \leq \frac{Gm_c}{V_\infty^2} \approx 0.2 \text{ pc} \left(\frac{m_c}{2 \times 10^6 \, M_\odot}\right) \tag{5.17}$$

(see discussion in § III). This seems rather extreme. We conclude that dynamical friction will lead to too much mass accumulating at the Galactic center, unless collisional disruption acts first.

VI. ACCRETION AND LENSING LIMITS

Another reason for believing that the halo objects may not be single supermassive black holes is associated with their accretion effects when they traverse the interstellar medium in the disk. A black hole of mass m_H moving supersonically at speed V_H through gas of density n_g will capture gas from within a radius

$$r_{acc} \approx \frac{2Gm_H}{V_H^2} \approx (0.2 \text{ pc})\left(\frac{m_H}{2 \times 10^6 \, M_\odot}\right) \tag{6.1}$$

and accrete at the Bondi rate

$$\dot{M} \approx 4\pi G^2 m_H^2 n_g \mu_p V_H^{-3} \approx (5 \times 10^{19} \text{ g s}^{-1})\left(\frac{m_H}{2 \times 10^6 \, M_\odot}\right)^2 \left(\frac{n_g}{\text{cm}^{-3}}\right), \tag{6.2}$$

where μ_p is the proton mass. In calculating numerical values, we have taken the typical value $V_H = 300 \text{ km s}^{-1}$ for the speed of halo objects relative to the disk. If the accreted material generates radiation with efficiency ϵ, the associated luminosity is just

$$L = \epsilon \dot{M} c^2 = (5 \times 10^{39} \text{ ergs s}^{-1})\left(\frac{m_H}{2 \times 10^6 \, M_\odot}\right)^2 \left(\frac{n_g}{\text{cm}^{-3}}\right)\left(\frac{\epsilon}{0.1}\right). \tag{6.3}$$

Halo black holes of mass $m_H \approx 10^6 \, M_\odot$ may already be excluded observationally as a result of their accretion luminosity making them detectable as individual sources when they cross the disk (Ipser and Price 1977, 1982; McDowell 1985; Lacey and Ostriker 1985). This conclusion depends somewhat on the model adopted for the accretion process, which determines both the efficiency ϵ and the wavelength of the emitted radiation.

Now suppose that the halo objects are not single black holes but clusters of smaller objects. If the cluster components are Jupiters, the accretion will be negligible. If the components are small black holes with mass $m_H \ll 10^6 \, M_\odot$, there will be some accretion radiation but much less than before. The accretion radius for the cluster as a whole, given by equation (6.1), is smaller than the cluster radius for parameters of interest, so the black holes will accrete individually. The total accretion rate for all the black holes in the cluster is therefore (for $\sigma_c \lesssim V_H$)

$$\dot{M} \approx \frac{4\pi G^2 m_c m_H \mu_p n_g}{V_H^3} , \tag{6.4}$$

which is smaller by a factor m_H/m_c than that for a single black hole of the same mass. The accretion luminosity for the whole cluster is reduced by at least this factor; the efficiency ϵ may also be smaller for smaller m_H. For $m_H \leq 10\ M_\odot$, the limit specified by equation (5.12), one probably does not violate any observational constraints.

If the dark matter in galactic halos really is in the form of $10^6\ M_\odot$ clusters, at least outside the galactocentric radius max (R_T, $R_{\rm dis}$), then these clusters must have a density of about $\Omega_c \approx 0.1$ in units of the critical density. They should therefore have interesting gravitational lensing effects. The probability that an object at $z_S = 1$ will be significantly lensed (with an intensity ratio for the two images exceeding 0.15 and a total amplification exceeding 1.3) is

$$P \approx \begin{cases} 0.2\Omega_c & (\Omega_0 \ll 1) , \\ 0.1\Omega_c & (\Omega_0 = 1) , \end{cases} \tag{6.5}$$

where Ω_0 is the present total density parameter; the typical image separation is

$$\theta \approx 8\left(\frac{m_c}{2 \times 10^6\ M_\odot}\right)^{1/2} h^{1/2}\ \text{mas} \tag{6.6}$$

(Press and Gunn 1973; Turner, Ostriker, and Gott 1984). Thus lensing by dark clusters should in principle be detectable by VLBI techniques.

It is interesting to compare the above results with existing VLBI data. Of the 45 sources which are "core dominated" (i.e., with 95% of the flux coming from a region smaller than 1″), five show double structure in which the components have similar spectra (Pearson and Readhead 1984). These are 0108+388 ($\theta = 6$ mas), 0153+744 ($\theta = 10$ mas), 0710+439 ($\theta = 6$ mas and 18 mas—three images), 2021+614 ($\theta = 8$ mas), and 2352+495 ($\theta = 18$ mas). None of these objects exhibits large-scale double structure. They are all slightly variable (< 10% in 4 yr), and their radio spectra are flat with a single smooth hump peaking at 1–5 GHz. The fraction of sources observed with double structure on the scale of equation (6.6) is thus $\sim 10\%$, compared to the fraction $P \approx 1\%$–2% for $\Omega_c = 0.1$ predicted by equation (6.5). Of course, the double structure does not have to be due to lensing; it could be intrinsic (Phillips, Hodges, and Mutel 1982).

In making use of equations (6.5) and (6.6), we have assumed that the clusters lense like point masses. This is a valid approximation provided $l_{\rm cr} \gtrsim r_c$, where $l_{\rm cr}$, the characteristic impact parameter for a strong lensing event, has the value

$$l_{\rm cr} = \left(\frac{4Gm_c}{c^2}\frac{d_{LS} d_L}{d_S}\right)^{1/2} \approx 11\ \text{pc}\left(\frac{m_c}{2 \times 10^6\ M_\odot}\right)^{1/2} h^{-1/2} , \tag{6.7}$$

where d_L, d_S, and d_{LS} are the angular diameter distances between observer-lens, observer-source, and lens-source, respectively (Turner, Ostriker, and Gott 1984). The specific numerical value is evaluated for $z_S = 1$, $z_L = 0.5$, and $\Omega_0 \leq 1$. (Note that $z_L = 0.5$ corresponds roughly to the most probable lens redshift for $z_S = 1$.) We see that the condition $l_{\rm cr} \gtrsim r_c$ is satisfied for the cluster parameters of interest. Correspondingly, minilensing by the cluster components is unlikely to occur, even if lensing by the cluster as a whole is significant.

Note that Canizares (1982) has argued that a critical density ($\Omega_c = 1$) in compact objects with mass in the range $10^{-2} < M < 10^5\ M_\odot$ can be ruled out because gravitational lensing would cause too large a dispersion in the line-to-continuum ratios of QSOs. However, this constraint is much weaker for $M \approx 10^6\ M_\odot$ objects because the line and continuum regions of a QSO are typically lensed together in that case. In fact, Canizares's argument may not exclude 1/10 of the critical density (which is all that we require) even in the lower mass range.

Lensing by dark clusters within the halo of our Galaxy might also be detectable, although the lensing probability is only $P \approx 10^{-6}$ (Paczyński 1985). In this case, for $d_{LS} \approx d_S \gg d_L$, the characteristic impact parameter for strong lensing is

$$l_{\rm cr} \approx 0.06\ \text{pc}\left(\frac{m_c}{2 \times 10^6\ M_\odot}\right)^{1/2}\left(\frac{d_L}{10\ \text{kpc}}\right)^{1/2} , \tag{6.8}$$

so that lensing by the cluster as a whole is probably not important and the main effects come from minilensing by the cluster components. The effects of minilensing could be seen as intensity fluctuations in the images of stars in nearby galaxies. The probability of minilensing for a line of sight through the cluster (i.e. the "lensing optical depth") is

$$P_c \approx \frac{4Gm_c d_L}{c^2 r_c^2} \approx 3.8 \times 10^{-3}\left(\frac{m_c}{2 \times 10^6\ M_\odot}\right)^{1/2}\left(\frac{d_L}{10\ \text{kpc}}\right)^{1/2}\left(\frac{r_c}{\text{pc}}\right)^{-2} , \tag{6.9}$$

so the cluster components probably only lense singly. In that case, the considerations of observability described by Paczyński apply, and one might reasonably detect masses in the range $10^{-6} \lesssim m_* \lesssim 10^2\ M_\odot$. Thus, in principle, lensing by clusters in distant galaxy halos would provide information about the properties of the clusters as a whole, while lensing by clusters in the halo of our own Galaxy would provide information about the properties of the cluster components.

VII. SUMMARY

We have investigated a modified version of the scenario, originally proposed by Lacey and Ostriker (1985), in which galactic disk heating is due to massive ($m \approx 2 \times 10^6\ M_\odot$) halo objects. In this modified version, the halo objects are dark clusters rather than single supermassive black holes. Dark clusters produce the same disk heating effects as black holes, except that the production of a high-velocity tail of disk stars may not occur if the cluster radius exceeds $r_c \approx 1$ pc. However, the dark cluster model can circumvent some of the problems of the black hole model. In particular, dynamical friction is prevented from building up too much mass at the Galactic center if the clusters are disrupted by mutual collisions before dynamical friction can become operative. This model is only viable if the halo core radius is less than the galactocentric radius down to which the disk velocity dispersion rises. This requirement may already be incompatible with observations; even if it is not, it seems that in our Galaxy the two radii have to be fairly close and that the cluster radius needs to be in a fairly narrow range around $r_c \approx 1$ pc. The model therefore requires some fine-tuning. If the clusters do have radii $r_c \approx 1$ pc, they would have to be composed of objects with $m_* \lesssim 10\ M_\odot$ to avoid evaporating too soon. Therefore, the cluster components would have to be either "Jupiters" with $m_* \lesssim 0.1\ M_\odot$ or perhaps black hole remnants with $m_* \approx 10\ M_\odot$.

With the dark cluster model, one also avoids the problem that the halo objects may generate too much luminosity through accretion as they traverse the disk. The accretion luminosity vanishes entirely if the cluster components are Jupiters, and it is greatly reduced if they are low-mass black holes. On the other hand, the distinctive emission pattern expected from a cluster of accreting holes traversing the disk could provide an important test of the scenario. Another test of the scenario involves its consequences for gravitational lensing. VLBI observations of lensing of compact radio sources by dark clusters in distant galaxies would give information on the masses of the clusters, while observations of lensing due to clusters in the halo of our own Galaxy would give information on the masses of the cluster components. An indirect test of the dark cluster hypothesis would be any observation which probes either the density profile of the halo objects within 2 kpc of the Galactic center or the amount of disk heating at such distances.

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APPENDIX A

DYNAMICAL FRICTION FROM THE SPHEROID

We first evaluate equation (4.2) for the spheroid. We introduce dimensionless variables

$$x = \frac{R}{R_1}, \quad \tilde{\rho} = \frac{\rho_S}{\rho_1}, \quad \tilde{M} = \frac{M}{4\pi\rho_1 R_1^3}, \quad \tilde{V} = \frac{V}{(4\pi G\rho_1 R_1^2)^{1/2}}, \quad \tau = t(4\pi G\rho_1)^{1/2}. \tag{A1}$$

Then equation (4.1) implies for $\{x < 1, x > 1\}$

$$\tilde{\rho} = \{x^{-9/5}, x^{-3}\}, \tag{A2}$$

$$\tilde{M} = \{\tfrac{5}{6}x^{6/5}, \tfrac{5}{6} + \ln x\}, \tag{A3}$$

$$\tilde{V}_c^2 = \frac{\tilde{M}(x)}{x} = \left\{\frac{5}{6}\, x^{1/5}, \frac{1}{6}\left(\frac{5 + 6\ln x}{x}\right)\right\}, \tag{A4}$$

$$1 + \frac{d\ln \tilde{V}_c}{d\ln x} = \left\{\frac{11}{10}, \frac{1}{2}\left(\frac{11 + 6\ln x}{5 + 6\ln x}\right)\right\}, \tag{A5}$$

$$\ln\Lambda = \ln\left(\frac{\tilde{M}}{\tilde{m}_c}\right) = \left\{\frac{6}{5}\ln x - \ln\left(\frac{6\tilde{m}_c}{5}\right), \ln\left(1 + \frac{6}{5}\ln x\right) - \ln\left(\frac{6}{5}\,\tilde{m}_c\right)\right\}. \tag{A6}$$

The velocity dispersion $\sigma(x)$ is determined from

$$\tilde{\sigma}(x)^2 = \frac{1}{\tilde{\rho}(x)}\int_x^\infty \tilde{\rho}(y)\,\frac{\tilde{M}(y)}{y^2}\,dy, \tag{A7}$$

and this gives

$$\tilde{\sigma}^2 = \left\{\frac{25}{48}\,x^{1/5} - \frac{1}{4}\,x^{9/5}, \frac{1}{4}\frac{\ln x}{x} + \frac{13}{48x}\right\}, \tag{A8}$$

$$\frac{\tilde{V}_c}{\sqrt{2}\,\tilde{\sigma}} = \left\{\left(\frac{20}{25 - 12x^{8/5}}\right)^{1/2}, 2\left(\frac{5 + 6\ln x}{13 + 12\ln x}\right)^{1/2}\right\} \approx \left\{\frac{2}{\sqrt{5}}, \sqrt{2}\right\}, \tag{A9}$$

where the last expression applies for $\{x \ll 1, x \gg 1\}$. Using these expressions in equation (4.2) then gives equations (4.3) and (4.4).

We next calculate the effect of the spheroid dynamical friction on the density profile of the dark matter. If objects with initial radius R_0 have drifted to a radius R at time t, conservation of mass implies that the density profile will have evolved from an initial form $\rho_0(R_0)$ to

$$\rho(r, t) = \rho_0(R_0)\left(\frac{R_0}{R}\right)^2\left(\frac{dR_0}{dR}\right) . \tag{A10}$$

If the spheroid alone provides the drag, then equation (4.4) implies

$$R_0 \approx (R^{3/2} + \tfrac{3}{2}\beta t)^{2/3} \qquad (R > R_1) , \tag{A11}$$

where

$$\beta = \left[\frac{12\sqrt{6}\,B(\sqrt{2})}{5\sqrt{5}}\right](\ln \Lambda)\sqrt{G}\,m_c(4\pi\rho_1 R_1^3)^{-1/2}\left(\frac{11}{5} + \frac{6}{5}\ln\frac{R}{R_1}\right)^{-1}\left(1 + \frac{6}{5}\ln\frac{R}{R_1}\right)^{-1/2} , \tag{A12}$$

and $B(\sqrt{2})$ is defined following equation (4.2). Therefore, if $R_c > R_1$, equation (A10) implies

$$\rho(R, t) \approx \begin{cases} \left(1 + \dfrac{3\beta t}{2R^{3/2}}\right)^{-1/3} \rho_c\left(\dfrac{R_c}{R}\right)^2 & (R_0 > R_c) , \\ \left(1 + \dfrac{3\beta t}{2R^{3/2}}\right)\rho_c & (R_1 < R_0 < R_c) . \end{cases} \tag{A13}$$

The profile is essentially unchanged for

$$R > \left(\frac{3\beta t}{2}\right)^{2/3} \approx R_{\rm df}\left(\frac{t}{t_g}\right)^{2/3} , \tag{A14}$$

while it falls off as $R^{-3/2}$ for $R < R_{\rm df}$. For parameters of interest, one typically finds $R_1 < R_{\rm df} < R_c$, so one still has a constant density regime with $\rho = \rho_c$. A similar analysis in the $R < R_1$ regime shows that $\rho(R)$ falls off as $R^{-9/10}$ for $R < R_{\rm df}$. The overall form of $\rho(R)$ is therefore as shown in Figure 1. Since the spheroid density itself increases faster than $\rho(R)$ with decreasing R (viz., as $R^{-9/5}$ for $R < R_1$ and R^{-3} for $R > R_1$), the modified halo density is always less than the spheroid density for the values of R of interest. However, the swept-in halo mass given by equation (4.10) will dominate at the very center since this corresponds to a δ-function in $\rho(R)$ at $R = 0$.

APPENDIX B

CLUSTER COLLISIONS, MERGERS, AND TIDAL DISRUPTION

We consider collisions between identical spherical clusters, which are treated as having uniform density and one-dimensional internal velocity dispersion σ_c given by equation (3.6). We first focus on collisions in which $V_{\rm rel} \gg \sigma_c$, where $V_{\rm rel}$ is the relative velocity of the clusters. For a head-on collision (impact parameter $l = 0$), the change in velocity for a particle at distance D from the collision axis is, in the impulse approximation,

$$\Delta V = \frac{2Gm_c}{V}\frac{\{1 - [1 - (D/r_c)^2]^{3/2}\}}{D} . \tag{B1}$$

(Only the projected perturber mass within distance D contributes.) Integrating $\frac{1}{2}(\Delta V)^2$ over the volume of the cluster gives the total change in either cluster's energy:

$$\Delta E = \int_0^{r_c}(\Delta V)^2 2\pi\rho_c D\sqrt{r_c^2 - D^2}\,dD \approx \left(4\ln 2 - \frac{247}{105}\right)\frac{3G^2m_c^3}{V_{\rm rel}^2 r_c^2} \qquad (l = 0) . \tag{B2}$$

For the opposite case of a distant impulsive collision with impact parameter $l \gg r_c$, the analysis of Spitzer (1958) gives

$$\Delta E \approx \frac{4G^2m_c^3 r_c^2}{5V_{\rm rel}^2 l^2} \qquad (l \gg r_c) . \tag{B3}$$

To calculate the total rate of energy injection into a cluster due to all encounters at relative velocity $V_{\rm rel}$, we assume that equation (B2) applies for $l < r_c$ and equation (B3) for $l > r_c$ (see Gerhard and Fall 1983 for a more precise treatment):

$$\frac{dE}{dt} = \int_0^{\infty} 2\pi n_c V_{\rm rel}(\Delta E) l\,dl \approx \frac{2\pi n_c G^2 m_c^3 \eta}{V_{\rm rel}} , \tag{B4}$$

where n_c is the number density of clusters and

$$\eta = [\tfrac{3}{2}(4\ln 2 - \tfrac{247}{105}) + \tfrac{2}{5}] \approx 1.03 .$$

Since equation (B4) is only approximate anyway, we set $\eta = 1$. We note that most of the contribution to dE/dt comes from collisions with $l < r_c$. Assuming that the cluster system has a Maxwellian velocity distribution with one-dimensional dispersion σ_H, we find

$$\left\langle \frac{1}{V_{\rm rel}} \right\rangle = \sqrt{\frac{2}{\pi}} \frac{1}{\sigma_{\rm rel}} = \frac{1}{\sqrt{\pi}\,\sigma_H} . \tag{B5}$$

The total energy for a uniform cluster is

$$E = -\frac{3Gm_c^2}{10r_c} , \tag{B6}$$

so the time scale for disruption is

$$t_{\rm dis} \equiv \left| \frac{E}{\dot{E}} \right| = \frac{3\sigma_H}{20\sqrt{\pi}\, G n_c m_c r_c} \qquad (\sigma_H \gg \sigma_c) . \tag{B7}$$

Substituting for $n_c m_c$ using equations (3.1) and (3.2) and setting $\sigma_H = V_\infty/\sqrt{2}$, we get equation (5.2). Note that the number of close encounters (with $l < r_c$) required to disrupt the cluster is of order $(\sigma_H/\sigma_c)^2$.

The above analysis neglects gravitational focusing and the possibility that the colliding clusters merge; it also assumes the validity of the impulse approximation. We now examine these assumptions. The impact parameter l and relative velocity $V_{\rm rel}$ at infinity, appearing in expressions (B1)–(B4), should properly be replaced by the distance p and velocity V_p at the pericenter of the relative orbit. These are related by

$$l = p\left(1 + \frac{4Gm_c}{pV_\infty^2}\right)^{1/2} , \qquad V_p = V_{\rm rel}\left(1 + \frac{4Gm_c}{pV_{\rm rel}^2}\right)^{1/2} , \qquad \text{for} \quad p \gtrsim r_c . \tag{B8}$$

(These equations are strictly valid for $p \geq 2r_c$.) Thus gravitational focusing is effective only for $V_{\rm rel} \leq (4Gm_c/p)^{1/2}$, i.e. only for $V_{\rm rel} \leq 2\sqrt{5}\,\sigma_c$ for encounters with $p \leq r_c$. The impulse approximation is valid for $p\omega_c \leq V_p$, where $\omega_c = (Gm_c/r_c^3)^{1/2}$ is the internal orbital frequency of the cluster. For the encounters with $p < r_c$ responsible for most of the energy transfer, this condition is well satisfied for $V_{\rm rel} \gg \sigma_c$, and it is marginally satisfied even for $V_{\rm rel} \ll \sigma_c$. Encounters with $V_{\rm rel} \leq 2\sqrt{5}\,\sigma_c$ and $p \leq r_c$ result in energy transfers ΔE comparable to or larger than the relative orbital energy and are expected to lead to merging (cf. Aarseth and Fall 1970).

In summary, equation (B7) for the disruption time should be valid for $\sigma_H \geq \sigma_c$. For $\sigma_H \lesssim \sigma_c$, encounters with $p \lesssim r_c$ lead to merging. Including the effects of gravitational focusing in this case, we find that the time scale for merging is also given by equation (B7) up to a numerical factor close to 1.

Finally we consider the disruption of the clusters by the galactic tidal field. The tidal radius is determined by

$$r_t^3 = \frac{Gm_c(r_t)}{\Omega_c^2 - d^2\Phi/dR^2} , \tag{B9}$$

where Ω_c is the angular velocity for a circular orbit and Φ is the gravitational potential (King 1962). For a background mass profile $M(R)$ this just gives

$$r_t = \left[m_c(r_t) \Big/ M(R)\left(3 - \frac{d \ln M}{d \ln R}\right) \right]^{1/3} R . \tag{B10}$$

For the spheroidal distribution given by equation (4.1), we have

$$M(R) = 4\pi\rho_1 R_1^3 \left\{ \frac{5}{6}\left(\frac{R}{R_1}\right)^{6/5} , \frac{5}{6} + \ln\left(\frac{R}{R_1}\right) \right\} \tag{B11}$$

for $\{R < R_1, R > R_1\}$, implying

$$3 - \frac{d \ln M}{d \ln R} = \left\{ \frac{9}{5} , \frac{9 + 18 \ln (R/R_1)}{5 + 6 \ln (R/R_1)} \right\} . \tag{B12}$$

Thus equation (B10) becomes

$$r_t = \left[\frac{m_c(r_t)}{6\pi\rho_1}\right]^{1/3} \left\{ \left(\frac{R}{R_1}\right)^{3/5} , \frac{R/R_1}{[1 + 2 \ln (R/R_1)]^{1/3}} \right\} . \tag{B13}$$

For a constant density cluster, this yields equation (5.6) for the value of R at which $r_t = r_c$.

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